جامعة البلقاء التطبيقية

كلية الهندسة التكنولوجية

هندسة الطاقة الكهربائية

تلخيص

تحليل أنظمة كهربائية (1)

إعداد: محمود يوسف صالح

بسم الله الرحمن الرحيم

* هذا الملخص يحتوي على تلخيص لمادة تحليل أنظمة كهربائية (1)، وقد وضعت فيه شرح الدروس واثبات كل نظرية حصلنا عليها وجميع المعلومات المتعلقة بها وفي النهاية وضعت أمثلة على شرح المواضيع وأسئلة عامة تجمع بين عدة أفكار لا بد من حلها وفهم فكرتها

* استعنت بكتابة هذا الملخص بدفتر تحليل (1) الذي كنت أتابع به جميع المحاضرات عند المهندس ياسين الشبول وأيضاً ببعض الكتب وتم أسئلة السنوات

* هذا الملخص من إعداد الطالب : محمود يوسف صالح

* المراجع :
1) Modern Power System Analysis
2) Power System Analysis

* وضعت فيه هذا الملخص جميع المعلومات التي تمكنت من مراجعتها والوصول إليها لكن لا بد من العودة إلى الكتاب لقراءته ودراسته وفهمه جيداً ومن ثم حل الأسئلة المتعلقة بالمادة المطلوبة.

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Power System Analysic

* Introduction to power system analysis:-

- Electrical energy is generally produced by:-

1) Burning of fuels (gas, oil, coal, ----).
2) Hydro sources.
3) Nectuar energy.
4) Other sources (solar cells, wind energy, ---)

* By burning of fuels or necluer energy is transformed to mechanical energy through thermodynamic cycle process, then we used that mechanical energy to move aturbins (generator) to get electricity.

Chemical energy (fuel) → [Thermo-cycle process] → mechanical energy → [gen.] → electrical energy

- The thermocycleprocess is unefficient process.
  - 45% mechanical energy
  - 55% heat (losses)

- With increasing the rated power of the generated unit, the efficiency increase

- Generally, transmitting electrical energy is cheaper than transmitting the electrical quantity of fuel. Generated station is build next/near to sources

- What is said before is not valid for necular energy because the amount of fuel for necular station is little so it's build not so far from the load center.

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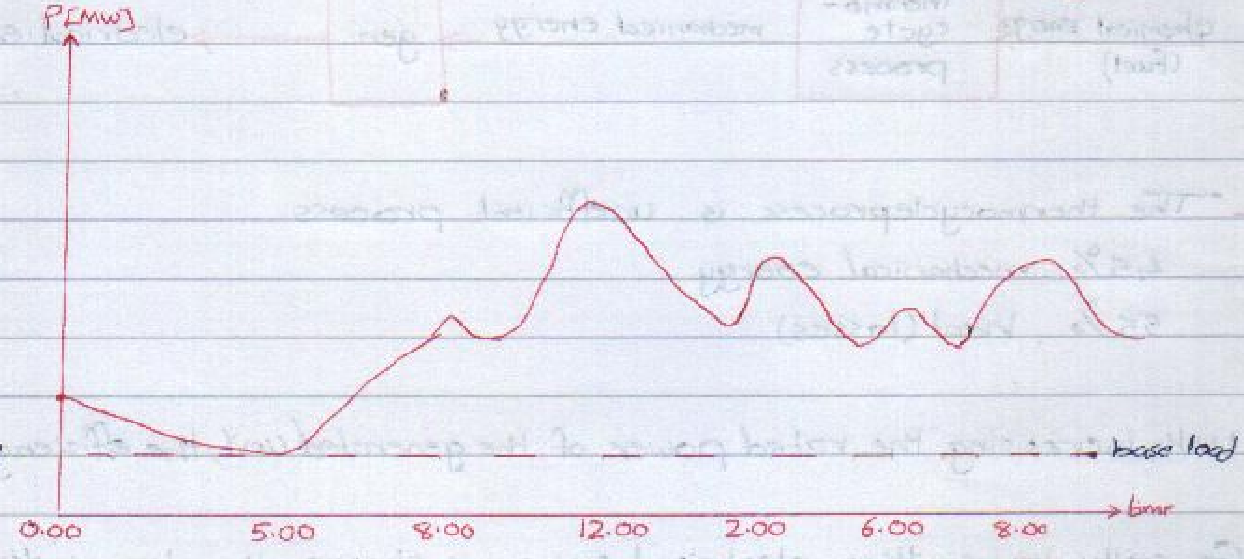
* ملاحظة : يتم تخزين الطاقة الكهربائية عن battery or pumping storage scheme

* The annual consumption of electrical energy is measure of development of the country.

* Characteristic of Electrical Energy :

1) electrical energy cannot be stored ~~eq~~ except in small quantity.
2) electrical companies don't have a control on the demand, except in small quantity but the electrical generator generate active and reactive power as required from the customer.
3) electrical demand not constant, it's variable with time, this change depend on time of day, montly, weekly, -----.

مثلاً لو درست نظام حمل كهربائي في منطقة معينة في أوقات مختلفة



يختلف الحمل الكهربائي باختلاف التوقيت والفصل
نظام الحمل الكهربائي مرتبط بخاصية load curve

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- From this curve we can conclude that the load consist of:

a) Steady part (base load)
b) predicted which can be known from previous history
c) purly random component

- The load curve is described what is called load factor

$$\boxed{\text{load factor} = \frac{\text{av. load}}{\text{peak load}} < 1}$$

load factor: the average power (load) divided by peak load over a period of time less than 1

كلما كان الـ load fac قريب من الـ ١ كان أفضل لأسباب اقتصادية

Tie lines: line that connects the high voltages line between the countries and works to reduce the difference between the averag load and peak load to become more unity (التعريف مهم)

* Construction of the power system

- The components of power system are:-

1) generating stations.
2) transmission lines.
3) distribution systems
4) Other Component (switches, protection appliances, ...)

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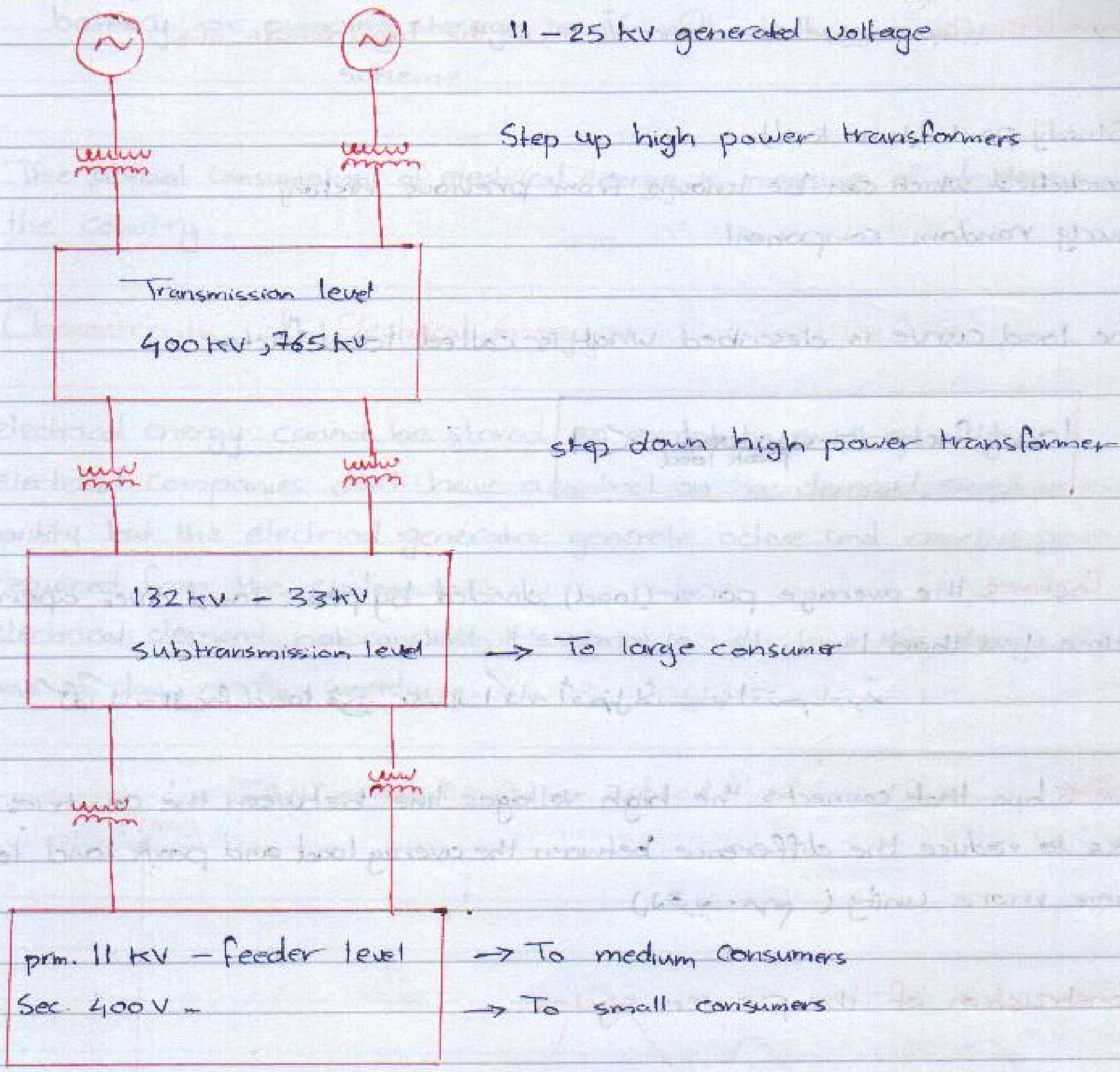

يتم رفع الفولتية لتقليل التيار المار في الأسلاك وبالتالي خفض التكلفة لأن القدرة تصبح أعلى $P = \frac{V^2}{R}$

* مشاكل رفع الفولتية :

١) تفريغ جزئي حول الخط (partial discharge around the conductor) مما يؤدي إلى ضياعات في الطاقة ونشوء ظاهرة Corona (ستشرح لاحقاً)

٢) تداخل المجالات الكهربائية القوية مع موجات البث الراديوي (Radio interface)

٣) صعوبة تصميم عوازل لهذه الفولتية العالية

٤) صعوبة تصميم قواطع لهذه الفولتية العالية Circuit Breaker

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* The most common generating stations:

1) Thermal Stations — تعمل عن طريق حرق الوقود الأحفوري وتسخين المياه والاستفادة من ضغط البخار الذي يصل الى درجات حرارة عالية ويسلط هذا البخار على توربينات تعمل على تحريك المولد مشكلتها: تتطلب وقت كبير لتجهيزها وتشغيلها وبالتالي لا يلبي الاستهلاك في حالة peak load

2) Hydrolic Stations — تعمل بسرعة ولكن تحتاج الى تصميم هندسي صعب وتحتاج وقت لبنائه

3) Nuclear station: — تعتمد على مبدأ الاندماج النووي لذرة اليورانيوم U_{238} (انشطار المواد النووية) تستعمل الطاقة لتسخين المية وتوليد البخار المضغوط

$$U^{235} + \text{neutrons} \longrightarrow U^{238} + \text{heat}$$

$$U^{238} + \text{neutrons} \longrightarrow U^{239}_{(\text{platinum})} + \text{heat}$$

مشاكلها: من الصعب اطفاء هذه المحطة لذا تعمل على الـ base load

حسناتها: توليد الكهرباء غير مكلف وتولد بكميات كبيرة

4) Gas Stations — يستعمل الغاز: ١) استعماله كوقود في محركات الاحتراق الداخلي التي تحرك المولد
٢) تسليطه تحت ضغط عالي على توربينات المولد

5) Diesel engine stations — يتم استعمالها كمحطات دعم/احتياط للشبكة في حالة System down
كفاءتها قليلة ولكن تشغيلها سريع

6) Solar energy sources — يتم التوليد بواسطة تسخين المياه وبالتالي توليد ضغط البخار أو توليد الكهرباء (DC) وتحويلها الى (AC) عن طريق inverter

7) Winds generators — استغلال طاقة الرياح

8) Geothermal Sources — طريقة استغلال ضغط البخار الصاعد من الآبار الجوفية في باطن الأرض أو استغلال الحرارة الصاعدة من البراكين

9) Tidal Sources — استغلال طاقة المد والجزر للبحار

* Basic Concepts

- Electrical quantity can be represented in two forms:-

1) instantaneous values (as a function of time).
2) phasors.

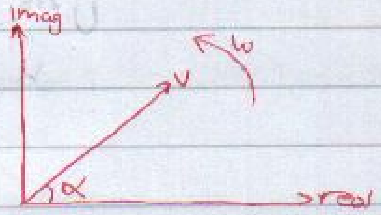
1) as a function:

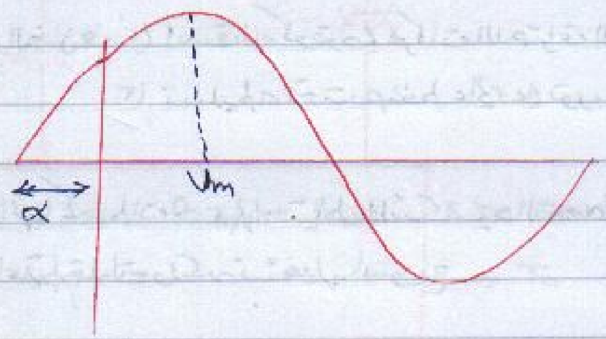
$$V(t) = V_m \sin(\omega t + \alpha)$$ الصورة العامة

حيث : α : phase shift ω : السرعة الزاوية rad/sec t : الزمن (S)

$V = |V| \angle \alpha = |V| e^{j\alpha}$ polar form

$V = |V| [\cos\alpha + j \sin\alpha]$ cartesian form



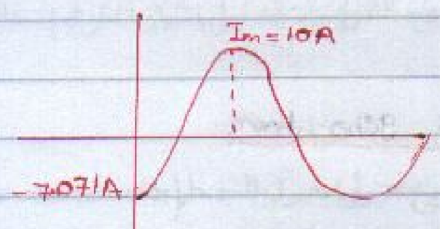


$V(t) = V_m \sin(\omega t + \alpha)$

$= \sqrt{2}\, |V| \sin(\omega t + \alpha)$

$|V| = \dfrac{V_m}{\sqrt{2}}$

e.g: for the following sinusoidal current waveform shown. express the current as a function of time



$i(t) = I_m \sin(\omega t + \alpha)$

$i(t) = 10 \sin(\omega t + \alpha)$

At $\omega t = 0$, $i(t) = -7.071$

$-7.071 = 10 \sin\alpha$

$\sin\alpha = \dfrac{-7.071}{10}$

$\alpha = -\dfrac{\pi}{4}$ $i(t) = 10 \sin(\omega t - \dfrac{\pi}{4})$

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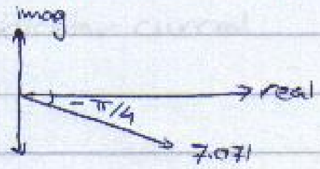
draw the phasor diag of current



express in polar and cartesian form

$$I = 7.071 \angle -\pi/4 = 7.071\, e^{-j\pi/4}$$

$$I = 7.071\left[\cos\left(-\frac{\pi}{4}\right) + j\sin\left(-\frac{\pi}{4}\right)\right] = 5 - j5$$

if the $f = 50\,HZ$ find the paired per unit

$$f = \frac{1}{\tau} \Rightarrow \tau = \frac{1}{50}\sec = 20\,ms$$

find the time t_1 (x-axis لما قطع)

$$2\pi \longrightarrow 20\,ms$$

$$\frac{\pi}{4} \longrightarrow ?? \quad \Rightarrow t_1 = \frac{20\,ms \times \pi/4}{2\pi} = 2.5\,ms$$

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* Power in Single phase AC Circuits

$$v(\omega t) = V_m \sin \omega t$$

$$i(\omega t) = I_m \sin(\omega t - \phi)$$

$$P(\omega t) = v(\omega t) \cdot i(\omega t)$$

$$P(\omega t) = V_m \cdot I_m \sin \omega t \sin(\omega t - \phi)$$ متطابقة

$$= V_m \cdot I_m \sin \omega t \left[\sin \omega t \cos\phi - \cos \omega t \sin\phi \right]$$

$$= V_m \cdot I_m \sin^2 \omega t \cos\phi - V_m I_m \sin\phi \sin \omega t \cos \omega t$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi \left[1 - \cos 2\omega t\right] - \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \sin\phi \sin 2\omega t$$

$$= V \cdot I \cos\phi \left[1 - \cos 2\omega t\right] - V \cdot I \sin\phi \sin 2\omega t \quad \text{------} \; (1)$$

$$P(\omega t) = V \cdot I \cos\phi - V \cdot I \cos\phi \cos 2\omega t - V \cdot I \sin\phi \cdot \sin 2\omega t$$

$$= V \cdot I \cos\phi - V \cdot I \left[\cos\phi \cos 2\omega t + \sin\phi \cdot \sin 2\omega t\right]$$

$$= V \cdot I \cos\phi - V \cdot I \cos(2\omega t - \phi) \quad \text{------} \; (2)$$

المتطابقات المستخدمة :

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

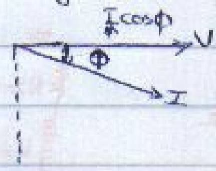
$$\sin^2 \omega t = 1 - \cos 2\omega t$$

$$\sin\phi \cos\phi = \sin 2\phi$$

- from equ. 2 it's clear that the inst power is a pulsating with frequency double of that of the frequency voltage waveform and the averag value equal $V \cdot I \cdot \cos\phi$ which is defined as active power

- from equ. 1 we find that the inst power contain from two terms, first one never goes negative it has average value equal $V \cdot I \cdot \cos\phi$ which called the active powr the second one has average value equal zero and having a max value which defind as a reactive power.

- Reactive power can defined as aproduct of voltage and the component of prependicular current

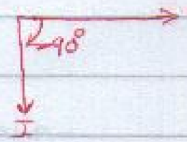


⇒ For resistive load $\phi = 0$

$P = V \cdot I$, $Q = 0$

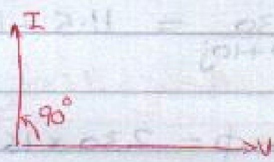
⇒ For inductive load

$P = 0$, $Q < 0$



⇒ For capacitive load

$P = 0$, $Q > 0$



⇒ For res, ind load

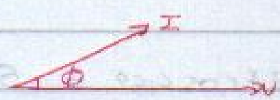
$P > 0$, $Q < 0$

$-90^\circ < \phi < 0$

⇒ For res, cap load

$P > 0$, $Q > 0$

consume — generate



$0 < \phi < 90^\circ$

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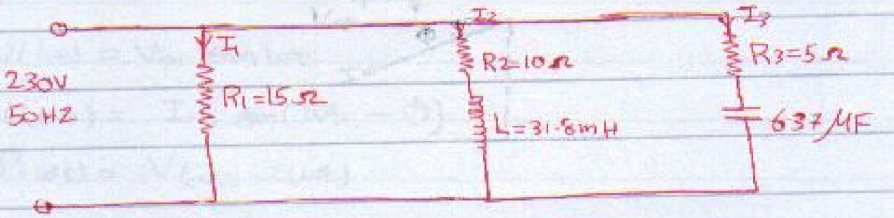
Ex: for the following Circuit Calculate the variable power absorbed (P, Q)



$$I_1 = \frac{230\angle 0^\circ}{15} = 15.33\ A$$

unity $\phi = 0$

$$\left.\begin{aligned} P &= V \cdot I = 230 \times 15.33 = 3526.6\ W \\ Q &= 0 \end{aligned}\right\}\ \text{Res}$$

$$\left.\begin{aligned} Z &= R_2 + j\omega L = 10 + j\,2\pi * 50 * 31.8 \times 10^{-3} = 10 + 10j \\ I &= \frac{V}{Z} = \frac{230}{10+10j} = 11.5 - 11.5j = 16.26\angle -45^\circ \\ P &= V \cdot I \cos\phi = 230 * 16.26 * \cos -45^\circ = 2644.44\ W \\ Q &= V \cdot I \sin\phi = 230 * 16.26 * \sin -45^\circ = -2644.44\ VA \end{aligned}\right\}\ \text{Res, ind}$$

$$Z = R_3 - j/\omega c = R - \frac{j}{2\pi * 50 * 637 * 10^{-6}} = 5 - 5j$$

$$I = \frac{V}{Z} = \frac{230}{5-5j}$$

$$\left.\begin{aligned} P &= 230 * 32.52 \times \cos 45^\circ = 5288.87\ W \\ Q &= 5288.87\ w \end{aligned}\right\}\ \text{Cap, Res}$$

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* The Concept of Complex power

$V = |V| \angle \Phi_V$

$I = |I| \angle \Phi_I$

$S = V \cdot I = |V| \cdot |I| \angle (\Theta_v + \Theta_i)$

but $I^* = |I| \angle \Phi_I$

$$S = V \cdot I^* = |V||I| \angle (\Phi_v - \Phi_i) = \underbrace{|V||I| \cos\Phi}_{P} + j \underbrace{|V||I| \sin\Phi}_{Q}$$

$S = V \cdot I^* = P + jQ$ ...... Complex Power

$|S| = \sqrt{P^2 + Q^2}$ VAR

بعض الصيغ الأخرى

$S = V \cdot I^*$

but $V = I \cdot Z$

$S = I \cdot I^* \cdot Z \Rightarrow S = Z \cdot |I|^2$

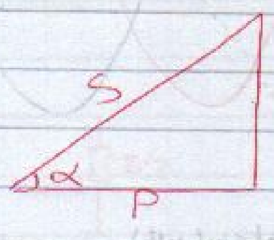
$I = V \cdot Y$

$I^* = Y^* \cdot V^*$

$S = V \cdot V^* \cdot Y^* = Y^* \cdot |V|^2$

$S = \sum S_i = (P_1 + P_2 + \dots) + j(Q_1 + Q_2 + \dots)$

$S = \sqrt{(P_1 + P_2 + \dots)^2 + (Q_1 + Q_2 + \dots)^2} \neq \sqrt{P_1^2 + Q_1^2} + \sqrt{P_n^2 + Q_n^2}$



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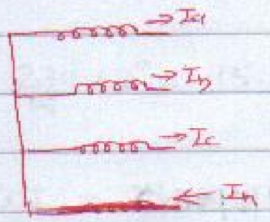
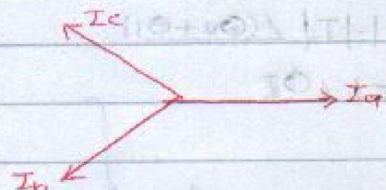
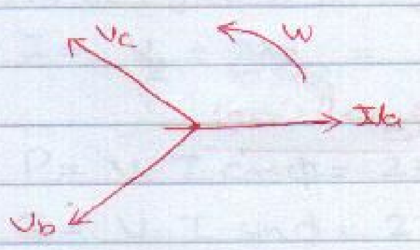
* Voltage and Current in 3-phase

- Usually 3-ph sys are symetrical, it's such mean that the 3-phase sys are equally loaded also in I in 3phase equal in magnitude and phase shift 180°

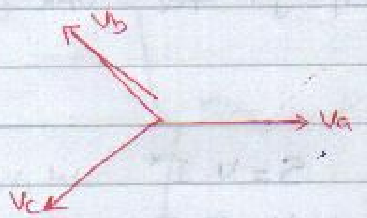




Positive Sequence



Negative Sequence



$V_a = |V_a| \angle 0°$

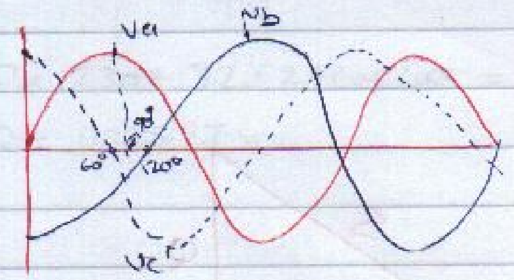
$V_b = |V_a| \angle -120°$

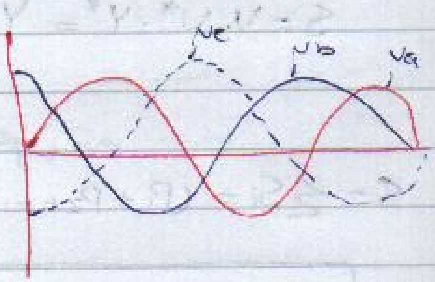
$V_c = |V_a| \angle 120°$

$V_a = |V_a| \angle 0°$

$V_b = |V_a| \angle 120°$

$V_c = |V_a| \angle -120°$





$V_a = \sqrt{2} |V_a| \sin(wt)$

$V_b = \sqrt{2} |V_a| \sin(wt - 120°)$

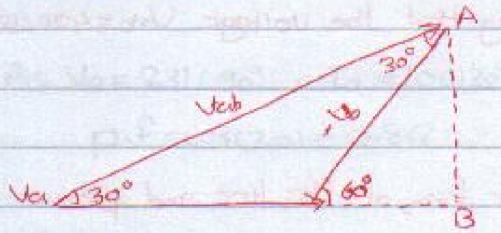
$V_c = \sqrt{2} |V_a| \sin(wt + 120°)$

$V_a = \sqrt{2} |V_a| \sin(wt)$

$V_b = \sqrt{2} |V_a| \sin(wt + 120°)$

$V_c = \sqrt{2} |V_a| \sin(wt - 120°)$

+ve Sequence



نفس المبدأ بالنسبة لـ -ve Seq

$V_{ab} = V_a - V_b$

$|V_b| \sin 60^\circ = |V_{ab}| \sin 30^\circ = \overline{AB}$

$|V_b| * \frac{\sqrt{3}}{2} = |V_{ab}| * \frac{1}{2}$

$|V_{ab}| = \sqrt{3}\, V_b$

$V_{ab} = \sqrt{3}\, |V_a| \angle 30^\circ$

$V_{bc} = V_b - V_c$

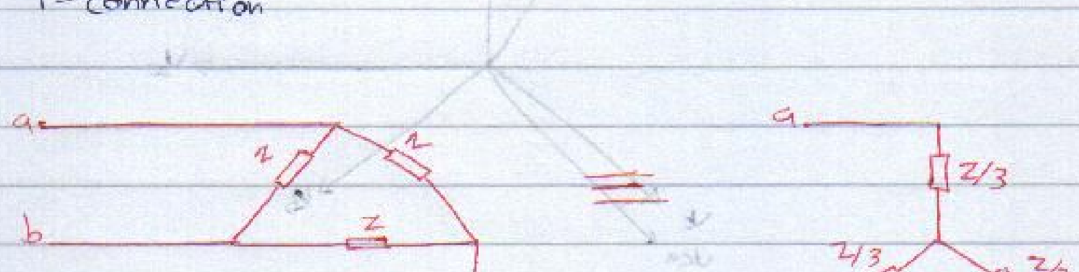
$V_{bc} = \sqrt{3}\, |V_a| \angle -90^\circ = (30^\circ - 120) = -90^\circ$

$V_{ca} = \sqrt{3}\, |V_a| \angle 150^\circ$

$V_{ab} + V_{bc} + V_{ca} = 0$

Usually, all the calculations done in case of normal operation of 3-phase sys and done per phase

All ~~of~~ the transformers and loads which are Δ-connected transfered to equivalent Y-connection



[First 2011/2012]

Ex: A three phase Y-connected balanced load is supplied by a three phase positive Sequence, 50 HZ, symmetrical power system. knowing that the voltage $V_{ab} = 400\angle 120^\circ$
$V_{ab} = 400\angle 120^\circ$, $I_a = 10\angle 120^\circ$

a) Find the phasor expressions in polar and rectangular form, of the line and phase voltages and currents (3 marks)

$V_{ab} = 400\angle 120^\circ = 400(\cos 120^\circ + j\sin 120^\circ) = -200 + j346.4$

$V_{bc} = 400\angle 0^\circ = 400(\cos 0 + j\sin 0) = 400$

$V_{ca} = 400\angle -120^\circ = 400(\cos -120^\circ + j\sin -120^\circ) = -200 - j346.4$

$V_a = \frac{400}{\sqrt{3}}\angle 90^\circ = 231\angle 90^\circ = 231\cos(90^\circ + j\sin 90^\circ) = j231$

$V_b = 231\angle -30^\circ = 231(\cos -30^\circ + j\sin -30^\circ) = 200 - j115.5$

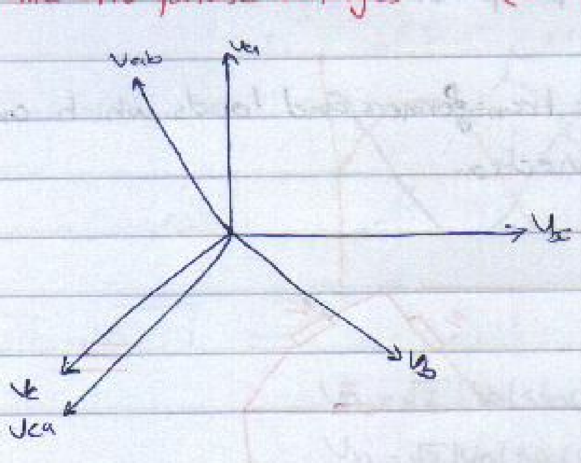
$V_c = 231\angle -150^\circ = 231(\cos(-150^\circ) + j\sin(-150^\circ)) = -200 + j115.5$

$I_a = 10\angle 120^\circ = 10(\cos 120^\circ + j\sin 120^\circ) = -5 + j8.66$

$I_b = 10\angle 0^\circ = 10(\cos 0 + j\sin 0) = 10$

$I_c = 10\angle -120^\circ = 10(\cos -120 + j\sin 120) = -5 - j8.66$

b) Draw the phasor diagram of the line and phase Voltages (2marks)



c) Is the load ind, cap or res and what is the power factor of load (1 mark)

As $V_a = 231\angle 90^\circ$, $I_a = 10\angle 120^\circ$ the load is res. capacitive

$p.f = \cos(30) = 0.866$

d) find the complex, the apparent, the active and reactive powers consumed by the load (2 marks)

$S = 3 V_a . I_a^* = 3(j231)(-5-j8.66) = 6000 - j3465$

$|S| = \sqrt{(6000)^2 + (3465)^2} = 6930\ VA$

$P = 6000\ W$

$Q = -3465\ VAR$

e) Prove that $V_{ab} + V_{ac} = 3V_{an}$ (2 mark)

$V_{ab} = 400\angle 120^\circ = 400(\cos 120^\circ + j\sin 120^\circ) = -200 + j346.4$

$V_{ca} = -200 - j346.4$

$V_{ab} + V_{ac} = 3V_{an}$

*Power in a balanced 3-phase System

$P = 3 V_{ph} \cdot I_{ph} \cos\phi$

for Y - connection

$I_L = I_{ph}$

$V_{ph} = \frac{V_L}{\sqrt{3}}$

$P = 3 \frac{V_{line}}{\sqrt{3}} \cdot I_{line} \cos\phi = \sqrt{3} \cdot V_{line} \cdot I_{line} \cos\phi$

for Δ - connection

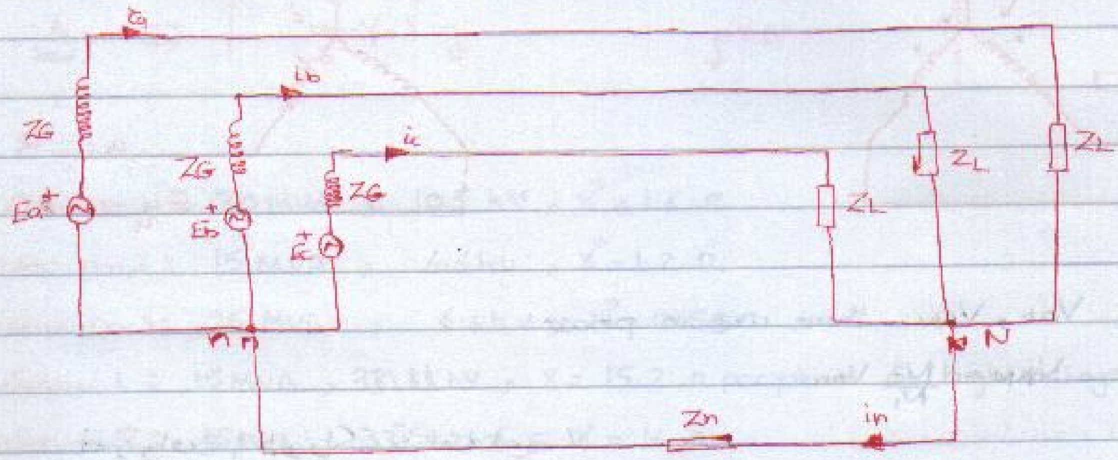
$P = 3 V_{ph} I_{ph} \cos\phi$

$= 3 V_{line} \cdot \frac{I_{line}}{\sqrt{3}} \cos\phi = \sqrt{3} \cdot V_{line} \cdot I_{line} \cos\phi$

Regardless of connection type, the power is same

* Representation of Power System Component

في أي نظام كهربائي يتم إعادة تمثيل عناصره حيث في أي نظام متزن يتم تحويل النظام إلى دائرة equ single pha مما يعمل على تسهيل الحسابات للعناصر والكميات الكهربائية فرضاً يوجد شبكة كهربائية كالتالي:

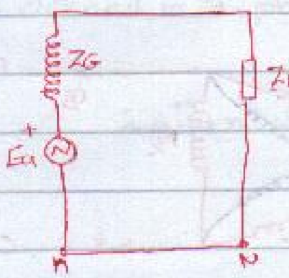


$$i_a + i_b + i_c = i_n = 0$$

$$V_n = V_N$$

يتم تبسيط النظام السابق بأخذ فاز واحد فقط كمرجع وللتوصيل يكون Y لكل النظام

⇒ equivalent circuit per phase

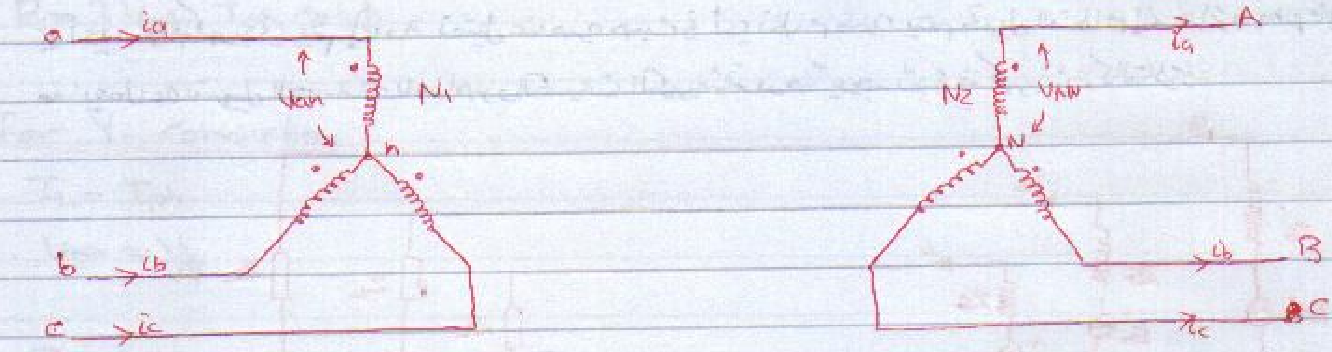


$$I_a = \frac{E_a}{Z_G + Z_L}$$

طريقة الربط سوف تسبب إزاحة في القيم الطورية الحالية

- أما إذا وجد محولات في الشبكة يتم تحويله إلى دارة مكافئة شرطها أن يكون كلا الطرفين للمحول توصيله Y بحيث إذا وجدت توصيله Δ نحولها إلى توصيلة Y وفيما يلي شرح للطريقة:

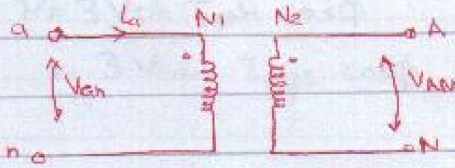
⟹ Transformers (Y–Y)



V_{an}, V_{AN} there are in phase

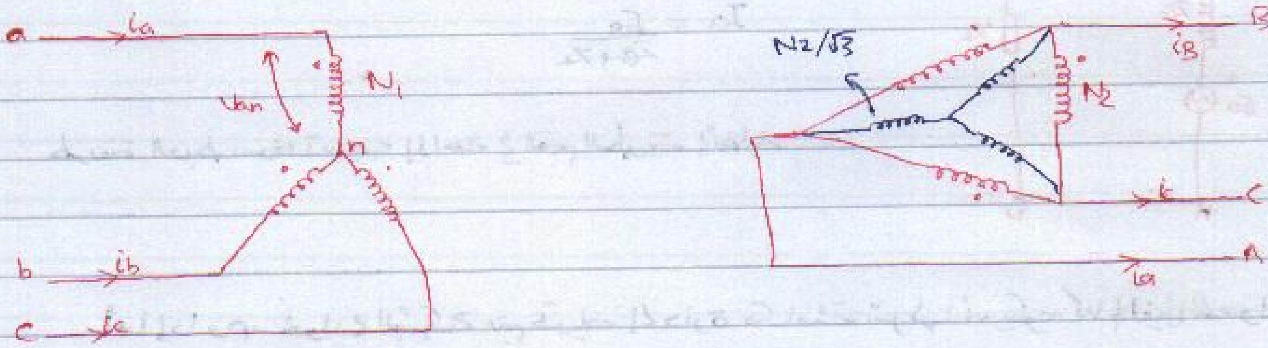
$$V_{AN} = \frac{N_2}{N_1} V_{an}$$

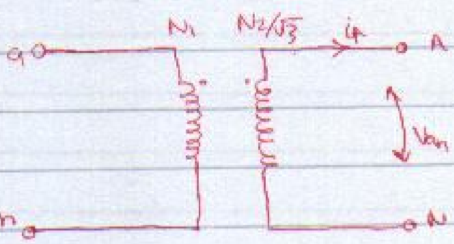
نفس زاوية الطور ولكن تختلف قيمة V



equivalent circuit of (Y–Y) per phase

⟹ Transformers (Y–Δ)

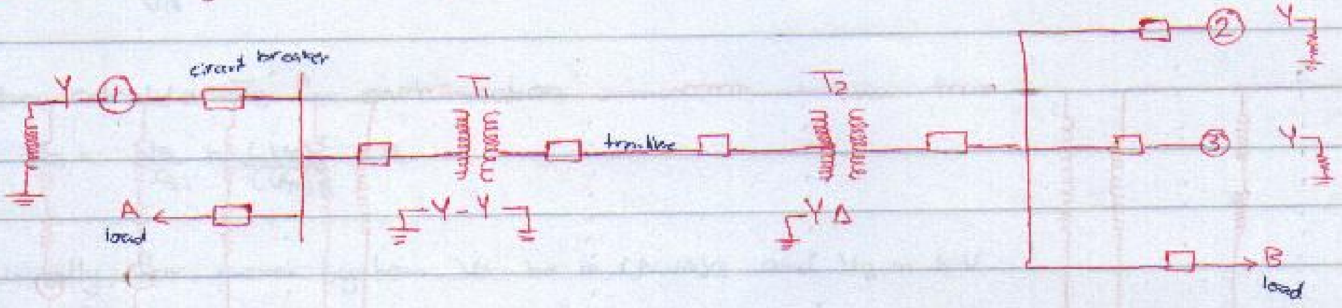




$$V_{AN} = \frac{N_2}{\sqrt{3} N_1} V_{an} \angle 90^\circ$$

where $V_{an} = |V_{an}| \angle 0^\circ$

* One line diagram and impedance or reactance diagram



Generator no. 1 : 30 MVA , 10.5 kV , $X'' = 1.6\ \Omega$

Generator no. 2 : 15 MVA , 6.6 kV , $X'' = 1.2\ \Omega$

Generator no. 3 : 25 MVA , 6.6 kV , $X'' = 0.56\ \Omega$

Transformer 1 : 15 MVA , 33/11 kV , $X = 15.2\ \Omega$ per phase on high voltage side

Transformer 2 : 15 MVA , 33/6.2 kV , $X = 16\ \Omega$

Transmission lines : $20.5\ \Omega$ / per phase

load A : 15 MW , 11 kV , 0.9 lagging P.F

load B : 40 MW , 6.6 kV , 0.85 lagging P.F

- Generator are satisfied in 3 ph power, L-L voltage per phase reactance
- The method of connection of N-point has no effect in case the system in normal operation
- Load is defined in 3-phase power L-L voltage at power factor

يرسم المولد دائماً مع مقاومته الداخلية وشكل التوصيلة Y بحيث تكون n مؤرضة

- ملاحظات على رسم الدائرة المكافئة :-

١) يتم رسم مع المولد مقاومة وممانعة على التوالي (إشارة له P,Q للمولد)

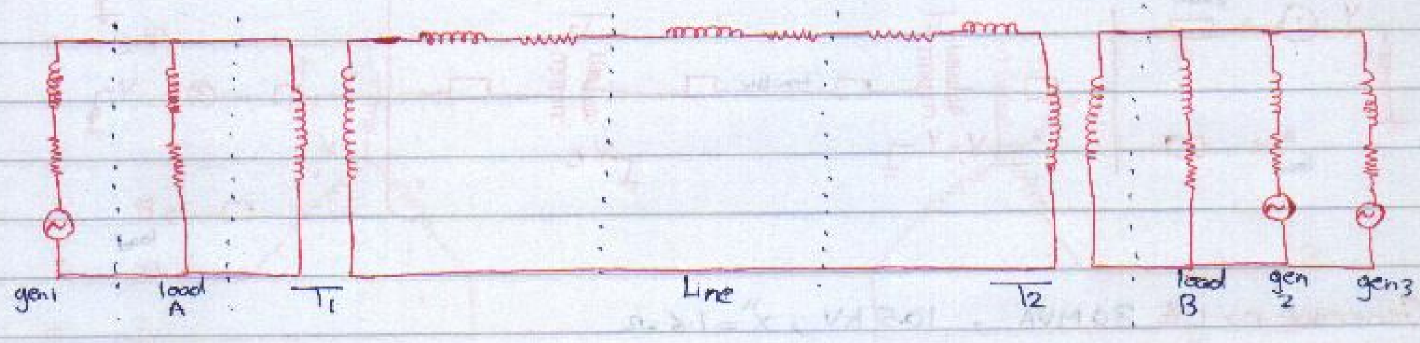
٢) يتم تمثيل الحمل على شكل مقاومة وممانعة على التوالي (إشارة له P,Q للحمل)

٣) رسمة المحول تكون Ideal trans.

٤) يتم رسم مقاومة وممانعة للمحول على الملف الذي له فولتية أعلى

٥) يرسم القاطع على شكل مربع والحمل على شكل مستطيل

⇒ Impedance and reactance diagram per phase



* Per Unit Analysic

This method can be used to simplify the analysic of power system have transformers

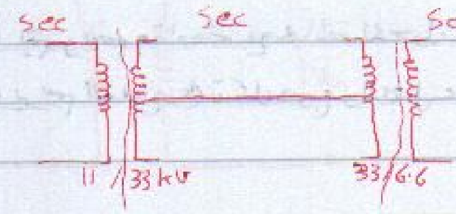
⇒ Advantegese:-

1) All values are given related to their rated values
2) Simply the analysis of the circuits have transformers.
3) V, I values of power system under load operation are around 1 (± error)

$$\boxed{\text{P.U value} = \frac{\text{actual value}}{\text{base value}}}$$

⇒ P.U analysis is Single phase Circuits

$(VA)_B$: Base voltampers is selected for asystem and it's common for all system.
V_B : Base voltage is selected for one section and calculated to other system



كل مقطع يغير الفولتية يعتبر section

$$I_B = \frac{(VA)_B}{V_B} \ A$$

then calculate Z_B for each section

$$Z_B = \frac{V_B}{I_B} = \frac{(V_B)^2}{(VA)_B} \ \Omega$$

usually, for power system V_A be in $(MVA)_B$ and V_B in KV

$$I_B = \frac{1000 * (MVA)_B}{(KV)_B} \ A$$

(ضربنا ب1000) حولنا وحدة ال MVA لوحدة KV ليظهر التيار بـ A

$$Z_B = \frac{V_B}{I_B} = \frac{1000 * (KV)_B}{\frac{1000 * (MVA)_B}{(KV)_B}} = \frac{(KV)_B^2}{(MVA)_B} \ \Omega$$

⇒ For 3-phase System

1) Select totall 3-phase base VA in MVA
2) Select L-L voltage in $(KV)_B$ for one section and calculated for other section according to transformer ratio

$$I_B = \frac{1000 (MVA)_B}{\sqrt{3} * (KV)_B}$$

$$Z_B = \frac{(KV)_B}{\sqrt{3} * I_B} = \frac{(KV)_B * 1000}{\sqrt{3} * \frac{1000 (MVA)_B}{\sqrt{3} (KV)_B}} = \frac{(KV)_B^2}{(MVA)_B}$$

$Z_{actual} = Z_{p.u} * Z_B$ (not depend on the base value)

$$Z_{p.u\,old} \cdot Z_{B\,old} = Z_{p.u\,new} \cdot Z_{B\,new}$$

$$Z_{p.u\,new} = Z_{p.u\,old} \frac{Z_{B\,old}}{Z_{B\,new}}$$

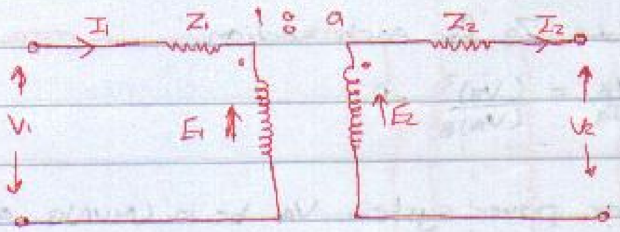
هنا في حالة تغير نسب التحويل في transformer

$$Z_{p.u\,new} = Z_{p.u\,old} \cdot \frac{(KV)^2_{B\,old}}{(KV)^2_{B\,new}} \cdot \frac{(MVA)_{B\,new}}{(MVA)_{B\,old}}$$

* P.U representation for the transformers

$\frac{V_{1B}}{V_{2B}} = \frac{1}{a}$, $\frac{E_{1B}}{E_{2B}} = \frac{1}{a}$

$\frac{I_{1B}}{I_{2B}} = a$



$V_2 = E_2 - Z_2 I_2$

$V_2 = aE_1 - Z_2 I_2$

$E_1 = V_1 - Z_1 I_1$

$V_{2\,actual} = a(V_1 - Z_1 I_1) - Z_2 I_2$ valid for actual value

$\| $

$\left(V_{2p.u} * V_{2B} = a \cdot V_{1pu} V_{1B} - a Z_{1pu} Z_{1B} I_{1pu} I_{1B} - Z_{2pu} Z_{2B} \cdot I_{2pu} \cdot I_{2B}\right)$ V_{2B} قسمة

$V_{2p.u} = a V_{1pu} \cdot \frac{V_{1B}}{V_{2B}} - a Z_{1pu} Z_{1B} \frac{I_{1pu} I_{1B}}{V_{2B}} - Z_{2pu} Z_{2B} I_{2pu} \cdot \frac{I_{2B}}{V_{2B}}$: $\left(a \frac{Z_{1B} I_{1B}}{V_{2B}} = \frac{V_{1B}}{V_{2B}} \cdot a\right)$ $\frac{1}{a} \cdot a$

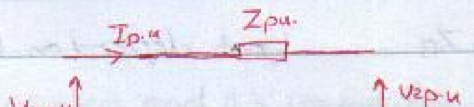
$V_{2p.u} = V_{1p.u} - Z_{1pu} I_{1pu} - Z_{2p.u} I_{2p.u}$

but $I_{2pu} = \frac{I_{2\,actual}}{I_{2B}} = \frac{\frac{I_{1\,actual}}{a}}{\frac{I_{1B}}{a}} = \frac{I_{1\,actual}}{I_{1B}} = I_{1p.u}$

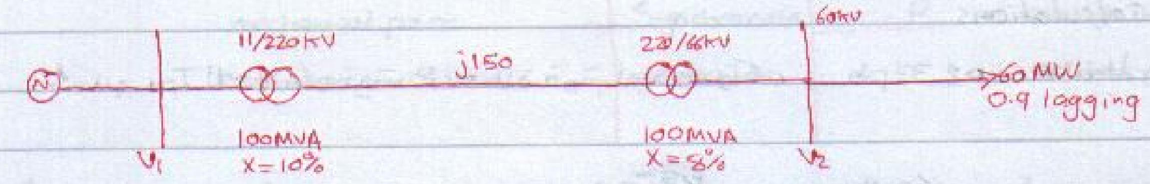
$V_{2p.u} = V_{1p.u} - I_{p.u}(Z_{1pu} + Z_{2pu})$

$V_{2p.u} = V_{1pu} - I_{pu} \cdot Z_{p.u}$

the equivalent Circuit:



e.g: For the 3-phase System. A load 60MW is connected to the 66 kV onto bus, if the voltage at the load bus be mentein as 60 kV. Calaute the terminals voltage at generated bus and the value of V_1 must be to obtain a 60 kV at V_2



يعني كم فولتية V_1 الي تجعل فولتية الـ 60kVload

We choose $(MVA)_B = 100$ MVA as a common for all system.

select $(kV)_B$ for the transmission line = 220 kV

Gen. Sec $(MVA)_B = 100$ MVA , $(kV)_B = 11$ kV

$$I_B = \frac{1000 * (MVA)_B}{\sqrt{3} * (kV)_B} = \frac{1000 * 100}{\sqrt{3} * 11} = 5248.6 \text{ A}$$

$$Z_B = \frac{(kV)_B^2}{(MVA)_B} = \frac{(11)^2}{100} = 1.21 \ \Omega$$

Trn. line $(MVA)_B = 100$ MVA , $(kV)_B = 220$ kV

$$I_B = \frac{1000 * (MVA)_B}{\sqrt{3} * (kV)_B} = \frac{1000 * 100}{\sqrt{3} * 220} = 262.4 \text{ A}$$

$$Z_B = \frac{(kV)_B^2}{(MVA)_B} = \frac{(220)^2}{100} = 484 \ \Omega$$

load Sec $(MVA)_B = 100$ MVA , $(kV)_B = 66$ kV

$$I_B = \frac{1000 * 100}{\sqrt{3} * 66} = 874.7 \text{ A}$$

$$Z_B = \frac{(66)^2}{100} = 43.56 \ \Omega$$

$Z_{T_1}\,p.u = 10\% = 0.1j\ \Omega$

$Z_{T_2}\,p.u = 8\% = 0.08j\ \Omega$

Now, P.U calculations

لحساب Ipu للحمل تكون قيمة P المعطاة قيمة الصرف وتكون بـ 3-ph في الـ rated Volt

$$I_{actual} = \frac{P}{\sqrt{3} * V_L * \cos\phi} = \frac{60\ MW}{\sqrt{3} \times \underset{kV}{66} * 0.9} = 583\ A$$

ملاحظة لو قدرة الحمل بـ MVA تكون القدرة Q ولا ندخل P.F في الحساب

$$I_{pu} = \frac{I_{actual}}{I_B} = \frac{583}{874.7} = 0.667\ p.u \Rightarrow I_{p.u} = 0.667\angle{-25.8^\circ}$$

الآن نريد حساب Ztotal في نظام الـ p.u

$$Z_{p.u\,line} = \frac{150}{484} = 0.31\ p.u$$

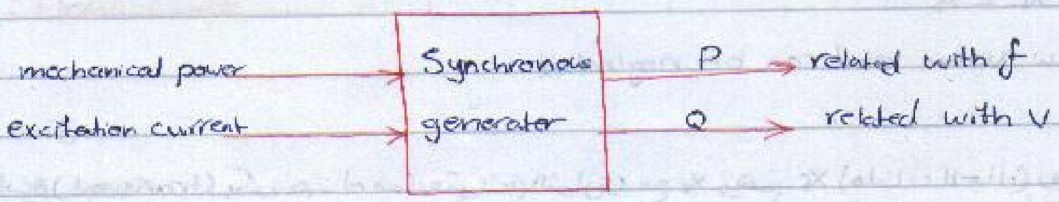
$$Z_{T\,pu} = Z_{T_1} + Z_{T_2} + Z_{line} = 0.49\ pu$$

$$V_{1pu} = V_{2pu} + I_{pu} Z_{p.u}$$

$$= \left(\frac{60}{66}\right) + \underbrace{(0.667\angle{-25.8^\circ})}_{(0.6-0.3j)}(0.49) = 1.20 - 0.142 = 1.09\angle{5.6^\circ}$$

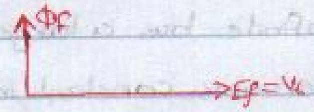
$$V_{1\,actual} = V_{1pu} \cdot V_{1B} = 1.09 * \underset{\text{primary (ع طرف المولد}}{11000} = 11.99\ kV$$

* Synchronous Generator as Element of Power System



As a element of power system, the synch gen. can be assumed as a system have two input and two output

- At no load state, there is no current but we have (EMF) (Φ_f mag field from exciter)



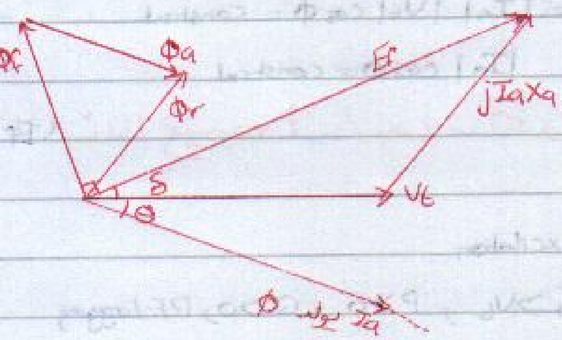
- At load state

Φ_f : فيض ناتج من تيار المجال

Φ_a : الفيض الناتج عن مرور I_a في الأسلاك للحمل

Φ_r : الفيض المحصل بين Φ_f , Φ_a

δ : power / Torque angle

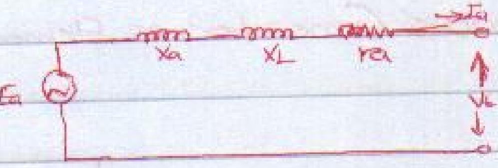


نتيجة دوران الروتر يتولد مجال يدور باتجاه دوران الروتر ويصل إلى الستيتر عبر الفجوة الهوائية ويعمل على توليد armature current وينشأ عن هذا التيار مجال بحيث يكون التيار الناتج له نفس زاوية الطور الذي نتج عنه

في حالة الـ no load تكون الفولتية متأخرة عن المجال بزاوية 90° أما في حالة الـ 3-ph load فيكون مرتبط مع المولد وسيكون ثلاثة مجالات r و a و f حيث r المجال المحصل للمجالين f و a

الزاوية δ هي الزاوية المسؤولة عن القدرة الفعلية المولدة في الشبكة ومقدار الفولتية مسؤولة عن القدرة الردفعلية المتولدة في الشبكة .

⇒ Equivalent Circuit :-



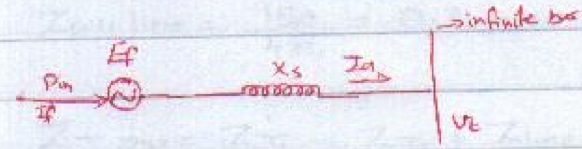
$X_a + X_L = X_s$

r_a has low value and can be neglected

في الحالات العابرة (transient) يكون هناك ممانعتين على التوازي مع X_a وهم X_f (ملفات المجال) وملفات الإخماد X_d
وتحدث عند التشغيل ويهمل تأثيرها بعد فترة قليلة أما في حالة steady st ليس لها تأثير

* Operation generator with infinite bus

Infinite bus a large power system such that the voltage and frequency of the system remains constant irrespective of P delievered from generator.



الدائرة المكافئة مع إهمال وجود الـ r_a والحالات العابرة:
نأخذ هذا الجزء بمعزل عن النظام الكهربائي

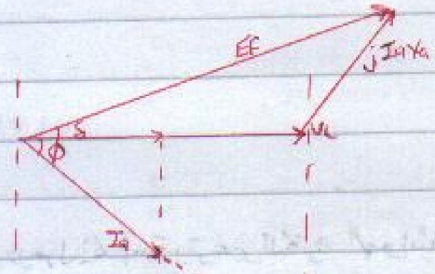
$P_o = |I_a| \cdot |V_t| \cos\phi = \text{constant}$

$\therefore \quad |I_a| \cos\phi = \text{constant}$

من خلال تحكمنا بتيار المجال يمكن التحكم بالفولتية (E_f)

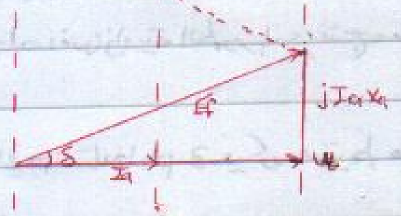
over excitation

$E_f \cos\delta > V_t$, $P > 0$, $Q > 0$, PF lagging



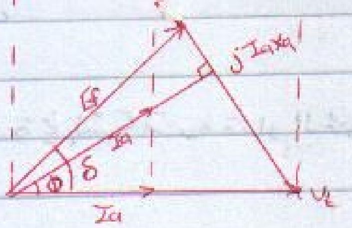
Normal excitation

$\phi = 0$, $E_f \cos\delta = V_t$, PF = 0 , $P > 0$, $Q = 0$



Under excitation تقليل تيار المجال

$E_f \cos\delta < V_t$, $P > 0$, $Q < 0$, PF-leading



عن طريق التحكم بتيار المجال نستطيع التحكم بمقدار E و Q فتنشأ الحالات السابقة.

⇒ Power flow Curve

$\overline{AB} = |E_f| \sin\delta$

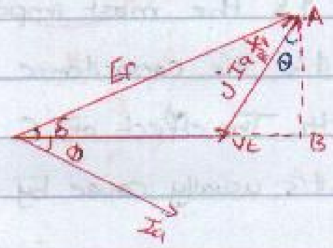
$\overline{AB} = |I_a|\, X_s \cdot \cos\theta$

$|E_f| \sin\delta = |I_a|\, X_s \cdot \cos\theta$ by multiply $|V_t|$

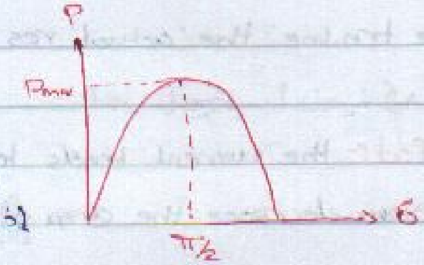
$|E_f||V_t| \sin\delta = |I_a||V_t|\, X_s \cdot \cos\theta$

$|E_f||V_t| \sin\delta = P \cdot X_s$

$$P = \frac{|E_f|\,|V_t| \sin\delta}{X_s}$$



$$P_{max} = \frac{|E_f| \cdot |V_t|}{X_s}$$



إذا كانت δ > 90° تفقد الآلة توافقيتها

الاثبات مهم: أثبت أن Pmax تتناسب عكسياً مع Xs (سؤال سنوات سابقة)

من الآن فصاعداً جميع الاثباتات مهم اثباتها وكيفية الوصول للصيغة النهائية

* Parameter of Power transmission lines:-

The power trn.line has the following parameters-

1) R: it's the reason of losses in trn.line and can be negled.
2) L: it's the most important parameter of the trn.line
3) C: it's the capacitence between different phases of trn.line, also exist between line and earth. The effect of C cable is greater than that overhead trn.line . تتوزع على طول خط النقل
4) G: it's usually cause by leakage current through isolater, very small and always negled

- For the trn.line the actual res is greater than calculated cuz:-

1) Skin effect: the current tends to be constructed in the outer shell of the conductor which mean decrease the area of the conductor and increasing the resistance

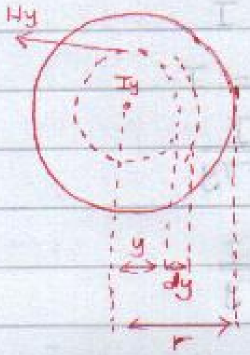
2) Proximity effect: produce by two conductor exists from eachother, then generate between them force so we put the wires with a high distences to avoid this effect emerge

* Inductence of Power transmission line

The flux leakage of an isolated conductor divided into:-

1) internal flux
2) external flux

* Inductence due to internal magnetic flux



موصل بتوزيع التيار (K) سطحه بالتساوي وباستخدام

قانون أمبير

current density

$$\oint Hy \cdot dL = \vec{J} \cdot \pi y^2$$

$$Hy \cdot 2\pi y = \frac{I}{\pi r^2} \cdot \pi y^2$$

$$Hy = \frac{I y}{2\pi r^2}$$ شدة المجال المغناطيسي

$$By = \mu_0 Hy = \frac{\mu_0 I y}{2\pi r^2}$$

$$d\phi = By\, ds = By \cdot dy \quad \text{where } ds = 1 \cdot dy = dy$$ على فرض أن طول الموصل 1m

$$d\phi = \frac{\mu_0 I y}{2\pi r^2} \cdot dy \qquad \text{but } d\lambda = d\phi \cdot \frac{y^2}{r^2}$$

$$d\lambda = \frac{\mu_0 I y}{2\pi r^2} \cdot \frac{y^2}{r^2} = \frac{\mu_0 I y^3}{2\pi r^4}$$

$$\lambda = \int_0^r d\lambda = \int_0^r \frac{\mu_0 I y^3}{2\pi r^4} = \frac{\mu_0 I}{2\pi r^4} \cdot \frac{y^4}{4}\Big]_0^r$$

$$\lambda = \frac{\mu_0 I r^4}{2\pi r^4 \cdot 4} = \frac{4\pi * 10^{-7} I}{2\pi * 4}$$

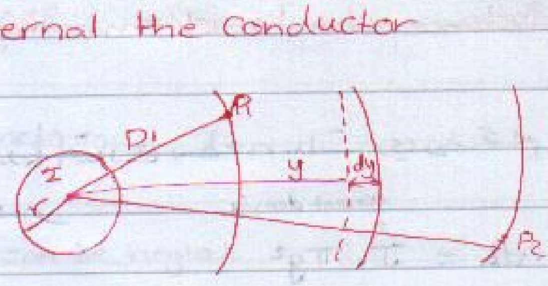
$$\lambda_{int} = 0.5 * 10^{-7} I \quad H/m \quad , \quad L = \frac{\lambda}{I} = 0.5 \times 10^{-7} H/m$$

$$\boxed{\begin{aligned} \lambda_{int} &= 0.5 * 10^{-7} I \ Wb/m \\ L &= 0.5 \times 10^{-7} \ H/m \end{aligned}}$$

flux linkage

inductance

* Flux linkage between two point external the conductor



$$\int H_y \cdot dy = I$$

$$2\pi y \cdot H_y = I$$

$$H_y = \frac{I}{2\pi y}$$

$$B_y = \frac{\mu_0 I}{2\pi y}$$

$$d\phi = B_y \cdot ds = B_y \cdot 1 \cdot dy = \frac{\mu_0 I}{2\pi y} dy$$

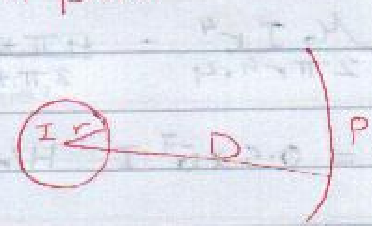
$$d\lambda_{ext} = \frac{\mu_0 I}{2\pi y} dy$$

$$\lambda_{12} = \int_{D_1}^{D_2} \frac{\mu_0 I}{2\pi y} dy = \frac{\mu_0 I}{2\pi} \ln y \Big]_{D_1}^{D_2} = \frac{\mu_0 I}{2\pi} \ln \frac{D_2}{D_1}$$

$$\lambda_{12} = \frac{4\pi * 10^{-7}}{2\pi} I \ln \frac{D_2}{D_1} = 2*10^{-7} \ln \frac{D_2}{D_1} \text{ Wb/m}$$

$$L_{12} = 2*10^{-7} \ln \frac{D_2}{D_1} \text{ H/m}$$

* Flux linkage due to flux up to an external point



total flux linkage has two component as

$$\lambda = \lambda_{int} + \lambda_{ext}$$

$$\lambda_{int} = 0.5 \times 10^{-7} I$$

$$\lambda_{ext} = 2*10^{-7} I \ln \frac{D}{r}$$

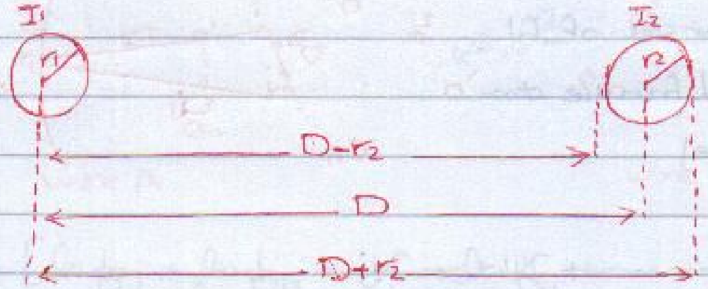
$$\lambda = 0.5*10^{-7} I + 2*10^{-7} I \ln \frac{D}{r} = 2*10^{-7} I \left[\frac{1}{4} + \ln \frac{D}{r} \right]$$

$$\lambda = 2*10^{-7} I \left[\ln e^{1/4} + \ln \frac{D}{r} \right] = 2*10^{-7} I \ln \frac{D}{e^{-1/4} \cdot r}$$

$0.7788 \cdot r = r'$

$$\lambda = 2*10^{-7} I \ln \frac{D}{r'} \quad , \quad L = 2*10^{-7} \ln \frac{D}{r'}$$

* Inductance of a single phase two wire line



- From $0 \rightarrow r_1$, the flux linkage constant and equal $0.5 * 10^{-7} I_1$
- from $r_1 \rightarrow D-r_2$, the flux linkage caused by current I_1.
- From $D-r_2 \rightarrow D+r_2$, the flux linkage a current whose magnitude progressively reduced from $I_1 \rightarrow 0$ along the distance because the effect of negative current of conductor 2.
- Flux beyond $D+r_2$ links a net current of zero.

$I_1 \Rightarrow \lambda_1 = 2*10^{-7} I_1 \ln \frac{D}{r_1'}$, $L_1 = 2*10^{-7} \ln \frac{D}{r_1'}$

$I_2 \Rightarrow L_2 = 2*10^{-7} \ln \frac{D}{r_2'}$

$L = L_1 + L_2 = 2*10^{-7} \left[\ln \frac{D}{r_1'} + \ln \frac{D}{r_2'} \right] = 2*10^{-7} \ln \frac{D^2}{r_1' \cdot r_2'}$

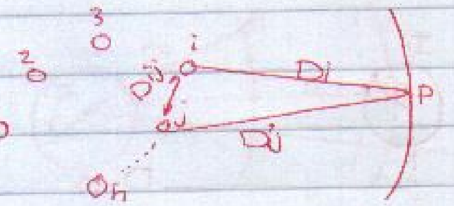
$L = 2*10^{-7} \ln \frac{D^2}{r_1' \cdot r_2'} = 4*10^{-7} \ln \frac{D}{\sqrt{r_1' \cdot r_2'}}$ حاصلجمع المعادلة بعضهم

usually $r_1' = r_2' = r'$

$\therefore L = 4*10^{-7} \ln \frac{D}{r'}$

* Flux linkage of a single Conductor in a group of Conductor

the f.L of conductor i will consist of f.L caused by i conductor (self) and from the other conductors (mutual flux linkage)



$$\lambda_i = \sum_{j=1}^{n} \lambda_{ij} = \lambda_{i1} + \lambda_{i2} + \cdots\cdots + \lambda_{ii} + \cdots \lambda_{ij}$$

$$\lambda_{ii} = 2\times10^{-7}\, I_i \ln \frac{D_i}{r_i'} \quad , \quad \lambda_{j} = 2\times10^{-7}\, I_j \ln \frac{D_j}{D_{ij}}$$

$$\lambda_i = 2\times10^{-7}\left[I_1 \ln \frac{D_1}{D_{i1}} + I_2 \ln \frac{D_2}{D_{i2}} + \cdots\cdots + I_i \ln \frac{D_i}{D_{ii}} + \cdots + I_n \ln \frac{D_n}{D_{in}}\right]$$

$$\lambda_i = 2\times10^{-7}\Big[I_1 \ln D_1 + I_1 \ln \frac{1}{D_{i1}} + I_2 \ln D_2 + I_2 \ln \frac{1}{D_{i2}} + \cdots + I_i \ln D_i + I_i \ln \frac{1}{D_{ii}} + \cdots$$

$$+ \cdots I_n \ln D_n + I_n \ln \frac{1}{D_{in}}\Big]$$

فكينا الأقواس

$$I_n = -(I_1 + I_2 + \cdots\cdots + I_{n-1})$$

$$\lambda_i = 2\times10^{-7}\Big[I_1 \ln D_1 + I_2 \ln D_2 + \cdots I_i \ln D_i - I_1 \ln D_n - I_2 \ln D_n \cdots\cdots I_{n-1} \ln D_n$$

$$+ I_1 \ln \frac{1}{D_{i1}} + I_2 \ln \frac{1}{D_{i2}} + \cdots + I_i \ln \frac{1}{D_{ii}} + \cdots + I_n \ln \frac{1}{D_{in}}\Big]$$

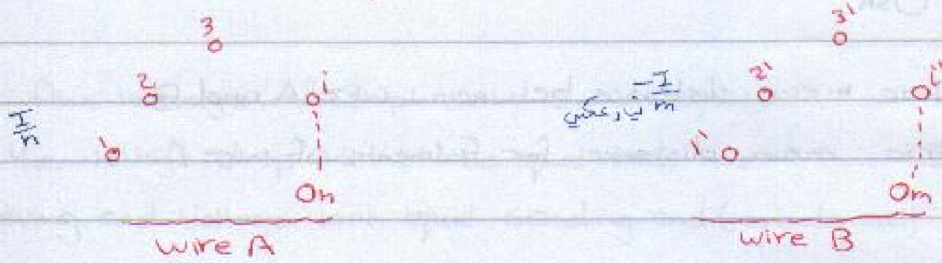
عوضنا بدل I_n القيم حسب المعادلة السابقة

$$\lambda_i = 2\times10^{-7}\left[I_1 \ln \frac{D_1}{D_n} + I_2 \ln \frac{D_2}{D_n} + \cdots\cdots + I_{n-1} \ln \frac{D_{n-1}}{D_n} + I_1 \ln \frac{1}{D_{i1}} + I_2 \ln \frac{1}{D_{i2}} + \cdots I_n \ln \frac{1}{D_{in}}\right]$$

totall f.L will be when $P \longrightarrow \infty$, so $D_1 = D_2 = D_n$ — عند تعويضها يعطي $\ln 1 = 0$

$$\lambda_i = 2\times10^{-7}\left[I_1 \ln \frac{1}{D_{i1}} + I_2 \ln \frac{1}{D_{i2}} + \cdots + I_i \ln \frac{1}{D_{i}} + \cdots I_n \ln \frac{1}{D_{in}}\right]$$

* Inductane of Composite Conductor lines



$$\lambda_1 = 2*10^{-7}\frac{I}{n}\left[\ln\frac{1}{D_{11}} + \ln\frac{1}{D_{12}} + \dots + \ln\frac{1}{D_{1i}} + \dots + \ln\frac{1}{D_{1n}}\right] - 2*10^{-7}\frac{I}{m'}\left[\ln\frac{1}{D_{11'}} + \right.$$

$$\left.\ln\frac{1}{D_{12'}} + \dots \ln\frac{1}{D_{1i'}} + \dots + \ln\frac{1}{D_{1m'}}\right]$$

$$\lambda_1 = -2*10^{-7}\frac{I}{n}\left[\ln D_{11} + \ln D_{12} + \dots \ln D_{1n}\right] + 2*10^{-7}\frac{I}{m'}\left[\ln D_{11'} + \ln D_{12'} + \dots + \ln D_{1m'}\right]$$

$$\lambda_1 = -2*10^{-7}\frac{I}{n}\left[\ln(D_{11} + D_{12} + \dots + D_{1n})\right] + 2*10^{-7}\frac{I}{m'}\left[\ln(D_{11'} + D_{12'} + \dots D_{1m'})\right]$$

$$\lambda_1 = 2*10^{-7} I \ln(D_{11} * D_{12} \dots D_{1n})^{-1/n} + 2*10^{-7}\frac{I}{m'}\ln(D_{11'} \dots D_{1m'})^{1/m'}$$

$$\lambda_1 = 2*10^{-7} I \ln\frac{(D_{11'} \dots D_{1m'})^{1/m}}{(D_{11} \dots D_{1n})^{1/n}}$$

the inductance of filament :-

$$L_1 = \frac{\lambda_1}{\frac{I}{n}} = n.\frac{\lambda_1}{I} = 2*10^{-7} n . \ln\frac{(D_{11'} \dots D_{1m'})^{1/m}}{(D_{11} \dots D_{1n})^{1/n}}$$

the average of filament of conductor A :-

$$L_{avg} = \frac{L_1 + L_2 + L_3 + \dots L_n}{n}$$

$$L_{avg} = 2*10^{-7}\left[\ln\frac{(D_{11'} \dots D_{1m'})^{1/m}}{(D_{11} \dots D_{1n})^{1/n}} + \ln\frac{(D_{21'} \dots D_{2m'})^{1/m'}}{(D_{21} \dots D_{2n})^{1/n}} + \dots + \ln\frac{(D_{n1'} \dots D_{nm'})^{1/m'}}{(D_{n1} \dots D_{nm})^{1/n}}\right]$$

$$L_{avg} = 2*10^{-7}\ln\frac{\left[(D_{11'} \dots D_{1m'})(D_{21'} \dots D_{2m'}) \dots (D_{n1'} \dots D_{nm'})\right]^{1/m'}}{\left[(D_{11} \dots D_{1n}) \cdot (D_{21} \dots D_{2n}) \dots (D_{n1} \dots D_{nn})\right]^{1/n}}$$

$$L_a = \frac{2*10^{-7}}{n}\ln\frac{\left[(D_{11'} \dots D_{1m'})(D_{21'} \dots D_{2n'}) \dots (D_{n1'} \dots D_{nm})\right]^{1/m'}}{\left[(D_{11} \dots D_{1n}) \cdot (D_{21} \dots D_{2n}) \dots (D_{n1} \dots D_{nn})\right]^{1/n}}$$

$$L_a = 2*10^{-7} \ln \frac{D_m}{D_{SA}}$$

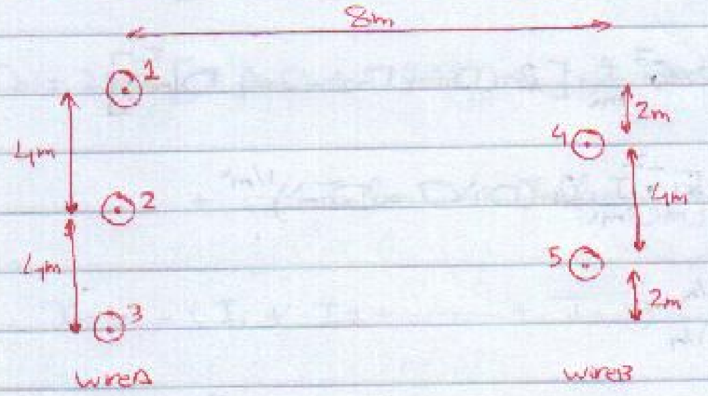
D_m: mutual geometric mean distance between wire A and B

D_s: self geometric mean distance for filaments of wire A

$$L_b = 2*10^{-7} \ln \frac{D_m}{D_{SB}}$$

$$L = L_a + L_b$$

Ex: For a following configuration of a single phase. Find the total inductance of the line. $r_a = 2.5$ mm, $r_b = 5$ mm



$D_{14} = 8^2 + 2^2 = \sqrt{68}$ فيثاغورس

$$L_a = 2*10^{-7} \ln \frac{D_m}{D_{SA}}$$

$$D_m = (D_{14} \cdot D_{15} \cdot D_{24} \cdot D_{25} \cdot D_{34} \cdot D_{35})^{1/6} = (\sqrt{68} * 10 * \sqrt{68} * \sqrt{68} * 10 * \sqrt{68})^{1/6} = 8.8\text{m}$$

$$D_{SA} = (D_{11} \cdot D_{12} \cdot D_{13} \cdot D_{21} \cdot D_{22} \cdot D_{23} \cdot D_{31} \cdot D_{32} \cdot D_{33})^{1/9}$$

$$= ((0.7788 * 0.0025)^3 * (4)^4 * 8^2)^{1/9} = 0.367\text{m}$$

$L_a = 2*10^{-7} \ln \frac{8.8}{0.367} = 0.635$ mH/km الجواب بـ H/m نضربه بـ 1000 ليصبح km

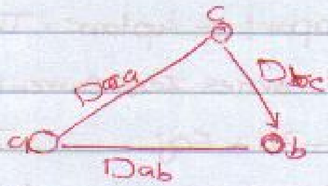
$D_{mB} = (D_{41} \cdot D_{42} \cdot D_{43} \cdot D_{51} \cdot D_{52} \cdot D_{53})^{1/6} = ($ نفس تطبيق ال A $) = 8.8$m

$$D_{SB} = (D_{44} \cdot D_{45} \cdot D_{54} \cdot D_{55})^{1/4} = ((0.7788 * 0.005)^2 * (4)^2)^{1/4} = 0.125\text{m}$$

$$L_B = 2*10^{-7} \ln \frac{8.8}{0.125} = 0.85 \text{ mH/km}$$

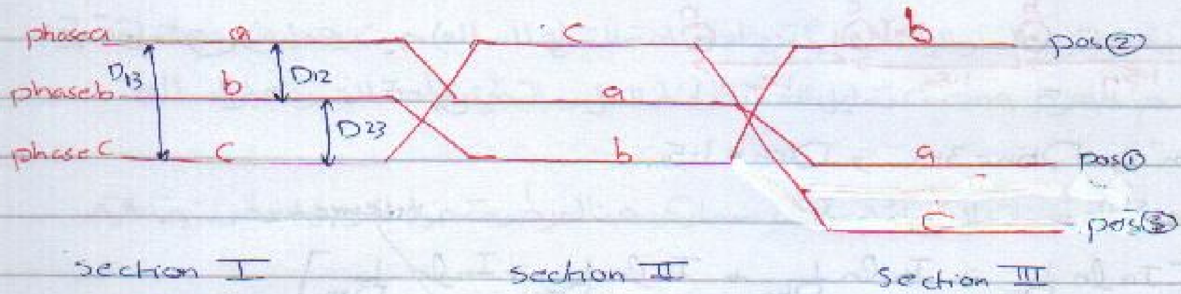
$$L_T = L_a + L_b = 1.485 \text{ mH/km}$$

* Inductane of 3-phase Lines



The flux linkage of 3-phase are not-equal, thus the voltage drop will not be equal, thus the recieving end voltage not equal sending end (unbalanced)

To overcome proplem, we use a transposition تغير أماكن الخطوط



$$\lambda_{a1} = 2*10^{-7}\left[I_a \ln \frac{1}{r_a'} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{13}}\right]$$

$$\lambda_{a2} = 2*10^{-7}\left[I_a \ln \frac{1}{r_a'} + I_b \ln \frac{1}{D_{23}} + I_c \ln \frac{1}{D_{12}}\right]$$

$$\lambda_{a3} = 2*10^{-7}\left[I_a \ln \frac{1}{r_a'} + I_b \ln \frac{1}{D_{13}} + I_c \ln \frac{1}{D_{23}}\right]$$

$$\text{average } \lambda_a = \frac{2}{3}*10^{-7}\left[3 I_a \ln \frac{1}{r_a'} + I_b \ln \frac{1}{D_{12} \cdot D_{23} \cdot D_{13}} + I_c \ln \frac{1}{D_{12} \cdot D_{23} \cdot D_{13}}\right]$$

$$\lambda_a = \frac{2}{3}*10^{-7}\left[3 I_a \ln \frac{1}{r_a'} + (I_b + I_c) \ln \frac{1}{D_{12} \cdot D_{23} \cdot D_{13}}\right]$$

$$I_a + I_b + I_c = 0$$

$$\lambda_a = \frac{2}{3}*10^{-7}\left[3 I_a \ln \frac{1}{r_a'} - I_a \ln \frac{1}{(D_{12} \cdot D_{23} \cdot D_{13})}\right]$$

$$\lambda_a = 2*10^{-7}\left[I_a \ln \frac{1}{r_a'} - I_a \ln \frac{1}{(D_{12} \cdot D_{23} \cdot D_{13})^{1/3}}\right]$$

$$\lambda_a = 2*10^{-7} I_a \ln \frac{(D_{12} \cdot D_{23} \cdot D_{13})^{1/3}}{r_a'} = 2*10^{-7} I_a \ln \frac{D_{eq}}{D_A}$$

$$L = 2*10^{-7} \ln \frac{D_{eq}}{D_A}$$

Ex: A threephase, 50HZ, 15Km long line has four wires (1 cm diameter) spaced horizantly 1.5m apart in aplane. The wires carrying current I_a, I_b, I_c and fourth wire is neutral carries zero current.

$I_a = -30 + 50j$ $\quad I_b = -25 + 55j$ $\quad I_c = 55 - j105$

The line is untransposed

1) Find flux lintage of the neutral wire and voltage
2) Find the drop voltage in each wire

1) $D_{an} = 4.5\,m$, $D_{bn} = 3m$, $D_{cn} = 1.5m$

$$\lambda_n = 2\times10^{-7}\left[I_a \ln\frac{1}{D_{an}} + I_b \ln\frac{1}{D_{bn}} + I_c \ln\frac{1}{D_{cn}} + \cancel{I_n \ln\frac{1}{D_m}}^{\text{no current}=0}\right]$$

$$\lambda_n = 2\times10^{-7}\left[1.51\, I_a + 1.1\, I_b + 0.405\, I_c\right]$$ بالتعويض بالقيم

$$\lambda_n = -0.01 + 0.0187 = 0.021\ \text{Wb/Km}$$

$$V_n = j\omega\lambda_n * L^{\rightarrow \text{طول الخط}}$$
$$= +j2\pi*50 * 0.021*10^{-3} * 15*1000 = 99\ V$$

2) $$\lambda_a = 2\times10^{-7}\left[I_a \ln\frac{1}{r_a'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{2D}\right]$$ نأخذ وجهة النظر a
كأنها مراد حسابه

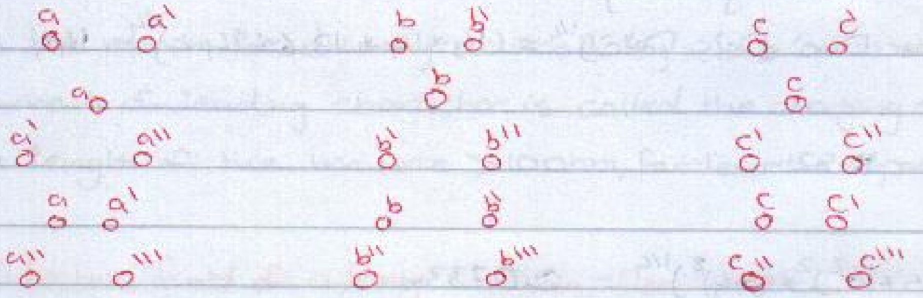
$$\lambda_a = 2\times10^{-7}\left[I_a \ln\frac{1}{0.005\times0.7788} + I_b \ln\frac{1}{1.5} + I_c \ln\frac{1}{3}\right]$$

$$\lambda_a = 2\times10^{-7}\left[5.55\, I_a + (-0.405\, I_b) - 1.098\, I_c\right] = -0.043 + 0.074$$
$$= 0.0855\ \text{Wb/Km}$$

$$\Delta V_a = j\omega\lambda_a * L = j314*(-0.043+0.074)*15 = -348.5 - 202.5j$$
$$= 403\,V$$

ونفس الشيء بالنسبة لـ V_b و V_c

* Bundled Conductors



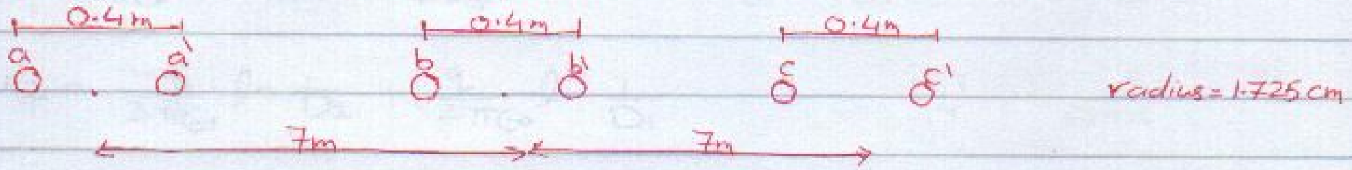
هي موصلات تستخدم لتفريع الفاز الواحد (علماً تزداد مساحة المقطع للسلك) حيث يعمل على رفع الفولتية التي تحدث عندها / عندها ظاهرة corona وتقليل المحاثة لخط النقل وبالتالي زيادة القدرة عبر السلك

corona : تفريغ جزئي للشحنات الكهربائية وتحدث عندما يكون هناك رطوبة حول الموصل عند فولتية عالية

→ bundled adjust by two advantages:

1) increasing the maximum voltage at which corona emerge
2) decrease the ind of the line, thus the voltage drop decrease and the max power that can be transfered through the line is increase.

Ex: Calculate the inductance and reactance for the bundled conductor as shown



$$L = 2\times10^{-7} \ln \frac{D_{eq}}{D_s}$$

$$D_{eq} = (D_{ab} \cdot D_{bc} \cdot D_{ac})^{1/3}$$

$$D_s = (r\grave{a} \cdot d_{aa'} \cdot r\grave{a} \cdot d_{aa'})^{1/4}$$

Mutual GMD $D_{ab} = (D_{ab} \cdot D_{ab'} \cdot D_{a'b} \cdot D_{a'b'})^{1/4} = (7 * 7.4 * 6.6 * 7)^{1/4} = 7m$

$D_{bc} = 7m$ from symmetry

$D_{ac} = (D_{ac} \cdot D_{ac'} \cdot D_{a'c} \cdot D_{a'c'})^{1/4} = (14 * 14 * 13.6 * 14.4)^{1/4} = 14m$

$D_{eq} = (7 * 7 * 14)^{1/3} = 8.82m$

$D_s = ((0.7788 * 1.725 * 10^{-2})^2 * (0.4)^2)^{1/4} = 0.0733m$

$L = 2 * 10^{-7} \ln \frac{8.82}{0.0733} = 0.96 \text{ mH/km}$

$X = 2\pi f L = 0.96 * 2 * \pi * 50 = 0.301\ \Omega/km$

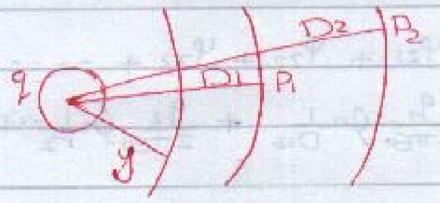
Capacitence

* Capacitence of Power transmission line

is the reason for drawing a current by a line over a line is not loaded. This current of leading character is called the charging current. It has an effect if the length of the trn.line >100km, for less the cap can be neglected.

* Electric field of a long straigh Conductor

q: charge density
the E.F around this conductor will be consider cylindrecal.



According to gauss law $\int D \cdot ds = Q$

$$D \cdot 2\pi y L = q \cdot L \Rightarrow D = \frac{q}{2\pi y} \quad \text{Flux density}$$

$$E = \frac{D}{\epsilon_0} = \frac{q}{2\pi \epsilon_0 y}$$

$$V_{12} = \varphi_1 - \varphi_2 = \int_{D_1}^{D_2} E \cdot dL = \int_{D_1}^{D_2} \frac{q}{2\pi\epsilon_0 y} dL = \frac{q}{2\pi\epsilon_0} \int_{D_1}^{D_2} \frac{1}{y} dy$$

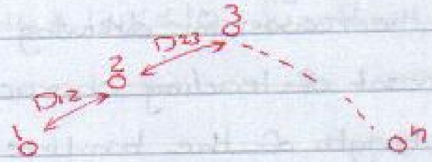
$$V_{12} = \frac{q}{2\pi\epsilon_0} \ln y \Big]_{D_1}^{D_2} = \frac{q}{2\pi\epsilon_0} [\ln D_2 - \ln D_1]$$

$$V_{12} = \frac{-q}{2\pi\epsilon_0} \ln \frac{1}{D_2} + \frac{q}{2\pi\epsilon_0} \ln \frac{1}{D_1}$$

$$= \underbrace{\frac{q}{2\pi\epsilon_0} \ln \frac{1}{D_1}}_{\varphi_1} - \underbrace{\frac{q}{2\pi\epsilon_0} \ln \frac{1}{D_2}}_{\varphi_2}$$

We can conclude that the potential at any point $= \frac{q}{2\pi\epsilon_0} \ln \frac{1}{D}$ where D is the distance between the point.

* Potential Difference between two conductor in a group of parallel Conductors



$$V_{12} = \varphi_1 - \varphi_2$$

$$\varphi_1 = \varphi_{11} + \varphi_{12} + \varphi_{13} + \cdots\cdots + \varphi_{1n}$$

$$= \frac{q_1}{2\pi\epsilon_0} \ln \frac{1}{r_1} + \frac{q_2}{2\pi\epsilon_0} \ln \frac{1}{D_{12}} + \frac{q_3}{2\pi\epsilon_0} \ln \frac{1}{D_{13}} + \cdots\cdots + \frac{q_n}{2\pi\epsilon_0} \ln \frac{1}{D_{2n}}$$

$$\varphi_2 = \varphi_{21} + \varphi_{22} + \varphi_{23} + \cdots\cdots + \varphi_{2n}$$

$$= \frac{q_1}{2\pi\epsilon_0} \ln \frac{1}{D_{12}} + \frac{q_2}{2\pi\epsilon_0} \ln \frac{1}{r_2} + \frac{q_3}{2\pi\epsilon_0} \ln \frac{1}{D_{23}} + \cdots\cdots + \frac{q_n}{2\pi\epsilon_0} \ln \frac{1}{D_{2n}}$$

$$V_{12} = \varphi_1 - \varphi_2$$

$$= \frac{q}{2\pi\epsilon_0} \ln \frac{D_{12}}{r_1} + \frac{q_2}{2\pi\epsilon_0} \ln \frac{r_2}{D_{12}} + \frac{q_3}{2\pi\epsilon_0} \ln \frac{D_{23}}{D_{13}} + \cdots\cdots + \frac{q_n}{2\pi\epsilon_0} \ln \frac{D_{2n}}{D_{1n}}$$

* Capacitance of two wire line

$$\varphi_1 = \varphi_{11} + \varphi_{12}$$

$$= \frac{q}{2\pi\epsilon_0} \ln \frac{1}{r_1} - \frac{q}{2\pi\epsilon_0} \ln \frac{1}{D}$$

$$\varphi_1 = \frac{q}{2\pi\epsilon_0} \ln \frac{D}{r_1}$$

$$\varphi_{12} = \varphi_{22} + \varphi_{21}$$

$$= \frac{-q}{2\pi\epsilon_0} \ln \frac{1}{r_2} + \frac{q}{2\pi\epsilon_0} \ln \frac{1}{D} = \frac{q}{2\pi\epsilon_0} \ln \frac{r_2}{D}$$

$$V_{12} = \varphi_1 - \varphi_2$$

$$= \frac{q}{2\pi\epsilon_0} \ln \frac{D}{r_1} - \frac{q}{2\pi\epsilon_0} \ln \frac{r_2}{D}$$

$$= \frac{q}{2\pi\epsilon_0} \ln \frac{D}{r_1} + \frac{q}{2\pi\epsilon_0} \ln \frac{D}{r_2} = \frac{q}{2\pi\epsilon_0} \ln \frac{D^2}{r_1 \cdot r_2}$$

usually $r_1 = r_2 = r$

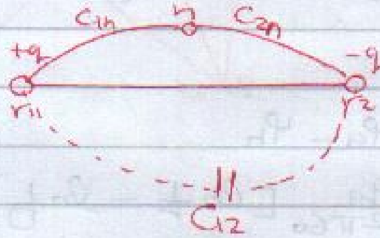
$$V_{12} = \frac{q}{2\pi\epsilon_0} \ln \frac{D^2}{r^2} = \frac{q}{\pi\epsilon_0} \ln \frac{D}{r}$$

but $C = \frac{q}{V} \Rightarrow$

$$C_{12} = \frac{q}{V_{12}} = \frac{\pi\epsilon_0}{\ln \frac{D}{r}} = \frac{3.14 * 8.85 * 10^{-12}}{\ln \frac{D}{r}} = \frac{2.78 * 10^{-11}}{\ln \frac{D}{r}} \, F/m$$

$$C_{12} = \frac{2.78 * 10^{-5}}{\ln \frac{D}{r}} \, \mu F/m = \frac{2.78 * 10^{-2}}{\ln \frac{D}{r}} \, \mu F/km$$

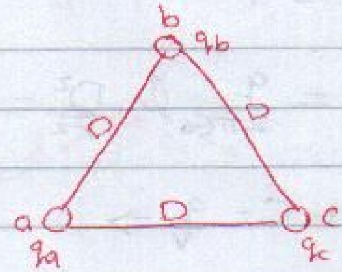
إذا افترضنا وجود الأرض



$C_{1n} = C_{2n} = 2 * C_{12}$

كل سلك والأرض $= \frac{5.56 * 10^{-2}}{\ln \frac{D}{r}} \, \mu F/km$

* Capacitance of 3-phase System



$$\varphi_a = \varphi_{aa} + \varphi_{ab} + \varphi_{ac}$$

$$= \frac{q_a}{2\pi\epsilon_0} \ln\frac{1}{r} + \frac{q_b}{2\pi\epsilon_0} \ln\frac{1}{D} + \frac{q_c}{2\pi\epsilon_0} \ln\frac{1}{D}$$

$$\varphi_b = \varphi_{ba} + \varphi_{bb} + \varphi_{bc}$$

$$= \frac{q_a}{2\pi\epsilon_0} \ln\frac{1}{D} + \frac{q_b}{2\pi\epsilon_0} \ln\frac{1}{r} + \frac{q_c}{2\pi\epsilon_0} \ln\frac{1}{D}$$

$$\varphi_c = \varphi_{ca} + \varphi_{cb} + \varphi_{cc}$$

$$= \frac{q_a}{2\pi\epsilon_0} \ln\frac{1}{D} + \frac{q_b}{2\pi\epsilon_0} \ln\frac{1}{D} + \frac{q_c}{2\pi\epsilon_0} \ln\frac{1}{r}$$

$$V_{ab} = \varphi_a - \varphi_b$$

$$= \frac{q_a}{2\pi\epsilon_0}\left[\ln\frac{1}{r} - \ln\frac{1}{D}\right] + \frac{q_b}{2\pi\epsilon_0}\left[\ln\frac{1}{D} - \ln\frac{1}{r}\right]$$

$$V_{ab} = \frac{q_a}{2\pi\epsilon_0} \ln\frac{D}{r} + \frac{q_b}{2\pi\epsilon_0} \ln\frac{r}{D} \quad \cdots\cdots\cdots (1)$$

$$V_{ac} = \varphi_a - \varphi_c$$

$$= \frac{q_a}{2\pi\epsilon_0}\left[\ln\frac{1}{r} - \ln\frac{1}{D}\right] + \frac{q_c}{2\pi\epsilon_0}\left[\ln\frac{1}{D} - \ln\frac{1}{r}\right]$$

$$V_{ac} = \frac{q_a}{2\pi\epsilon_0} \ln\frac{D}{r} + \frac{q_c}{2\pi\epsilon_0} \ln\frac{r}{D} \quad \cdots\cdots\cdots (2)$$

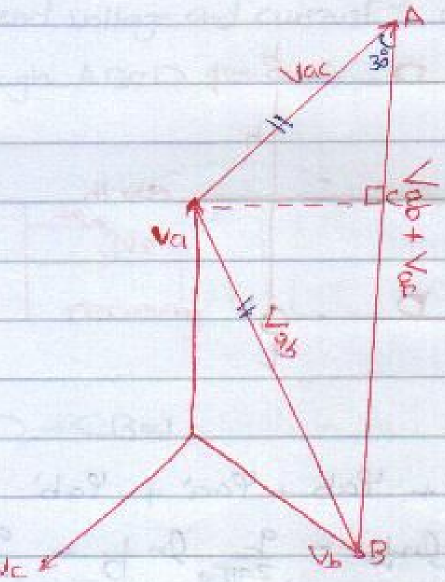
$$V_{ab} + V_{ac} = \frac{q_a}{\pi\epsilon_0} \ln\frac{D}{r} + \frac{q_b + q_c}{2\pi\epsilon_0} \ln\frac{r}{D} \quad \cdots\cdots\cdots (3)$$

$$q_a + q_b + q_c = 0$$

$$q_b + q_c = -q_a \quad \cdots\cdots\cdots (4)$$

$$V_{ab} + V_{ac} = \frac{q_a}{\pi \varepsilon_0} \ln \frac{D}{r} - \frac{q_a}{2\pi \varepsilon_0} \ln \frac{r}{D}$$

$$= \frac{q_a}{\pi \varepsilon_0} \ln \frac{D}{r} + \frac{q_a}{2\pi \varepsilon_0} \ln \frac{D}{r} = \boxed{\frac{3 q_a}{2\pi \varepsilon_0} \ln \frac{D}{r}} \quad \cdots\cdots (5)$$



ملاحظة : مثلث متساوي الساقين ولكن ليس
واضحاً بدقة الرسم

$$|AC| = \frac{|V_{ab} + V_{ac}|}{2} = |V_{ac}| \cdot \frac{\sqrt{3}}{2}$$

$$|V_{ab} + |V_{ac}| = \sqrt{3} |V_{ac}|$$

but $V_{ac} = \sqrt{3}\, V_{an}$

$$V_{ab} + V_{ac} = \sqrt{3} \cdot (\sqrt{3}\, V_{an})$$

$$V_{ab} + V_{ac} = 3\, V_{an}$$

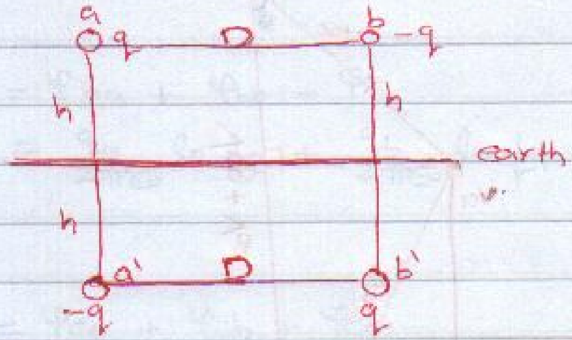
$$3 V_{an} = \frac{3 q_a}{2\pi \varepsilon_0} \ln \frac{D}{r}$$

$$C_{an} = \frac{q_{an}}{V_{an}} = \frac{2\pi \varepsilon_0}{\ln \frac{D}{r}} = \frac{5.56 \times 10^{-2}}{\ln \frac{D}{r}} \ \mu F/km$$

From the previous we can conclude that the capacitance of single phase same as capacitance of three phase.

* The effect of earth

All previous formula of capacitence where derived without taking the presence of earth. For taken the considration of earth, the imagainry method used



نظام تالي مع وجود الأرض وعكس القيم

$$\varphi_a = \varphi_{aa} + \varphi_{ab} + \varphi_{aa'} + \varphi_{ab'}$$

$$= \frac{q}{2\pi\epsilon_0}\ln\frac{1}{r} - \frac{q}{2\pi\epsilon_0}\ln\frac{1}{D} - \frac{q}{2\pi\epsilon_0}\ln\frac{1}{2h} + \frac{q}{2\pi\epsilon_0}\ln\frac{1}{\sqrt{D^2+4h^2}}$$

$$\varphi_b = \varphi_{ba} + \varphi_{bb} + \varphi_{ba'} + \varphi_{bb'}$$

$$= \frac{q}{2\pi\epsilon_0}\ln\frac{1}{D} - \frac{q}{2\pi\epsilon_0}\ln\frac{1}{r} - \frac{q}{2\pi\epsilon_0}\ln\frac{1}{\sqrt{D^2+4h^2}} + \frac{q}{2\pi\epsilon_0}\ln\frac{1}{2h}$$

$$V_{ab} = \varphi_a - \varphi_b = \frac{q}{2\pi\epsilon_0}\left[\ln\frac{D}{r} + \ln\frac{r}{D} + \ln\frac{(4h^2+D^2)^{1/2}}{2h} + \ln\frac{2h}{(4h^2+D^2)^{1/2}}\right]$$

$$V_{ab} = \frac{q}{2\pi\epsilon_0}\ln\frac{2hD}{r(4h^2+D^2)^{1/2}}$$

حصلت على الصيغة بقلب الحدود وتجميعهم مع بعضهم

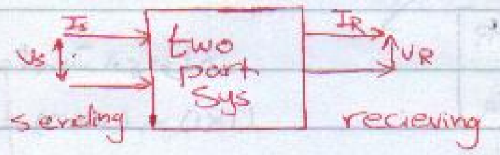
$$C_{ab} = \frac{q_{ab}}{V_{ab}} = \frac{\pi\epsilon_0}{\ln\frac{2hD}{r\sqrt{4h^2(1+\frac{D^2}{4h^2})}}} = \frac{\pi\epsilon_0}{\ln\frac{2hD}{2hr\sqrt{1+\frac{D^2}{4h^2}}}} = \frac{\pi\epsilon_0}{\ln\frac{D}{r\sqrt{1+\frac{D^2}{4h^2}}}}$$

$$C_{ab} = \frac{\pi\epsilon_0}{\ln\frac{D}{r\left(1+\frac{D^2}{4h^2}\right)^{1/2}}}$$

Transmission lines

* Transmission Lines in Normal operation work under balanced

Transmission lines are normally operated with a balanced 3-phase load, the analysis can therefore proceed on a per phase basis. A trn-line on a per phase can be regarded as a two part networks wherein the sending end voltage and current I_s are related to the recieving end voltage V_R and I_R through ABCD constants



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \qquad AD - CB = 1$$

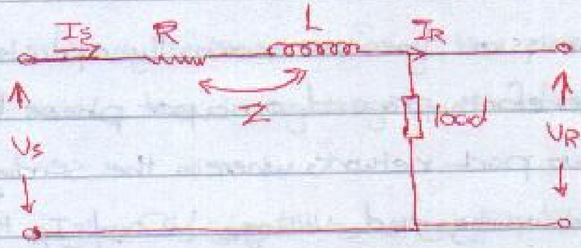
- There are some kinds of trn.line according to the lengths.

1) Short Line $L \leq 100\,km$
2) Medium Line $100\,km < L < 250\,km$
3) Long Line $L > 250\,km$

- Short and medium lines are assume to be lumped parameter
- For long line the parameter are assumed distributed at the line.
- In short line, line represented by res, ind cuz the effect of short line is low
- For medium line, the capacitor is big value so we take the value of it.

1] Short transmission line

the equ cir of short trn. lines



$$V_s = V_R + Z I_R$$

$$I_s = I_R$$

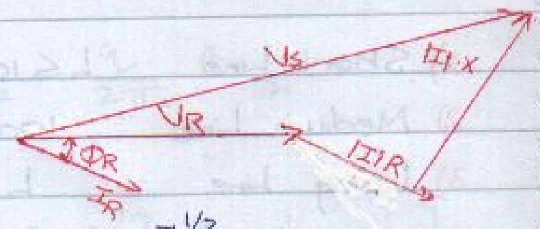
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

دائماً يجب أن يساوي واحد

$$(1\times1) - (Z\times0) = 1$$

to find Z, we multiply it with length of line (Z given by per km)

$$|V_s| = \sqrt{(|V_R|\cos\Phi_R + |I|\cdot R)^2 + (|V_R|\sin\Phi_R + |I|\cdot X)^2}$$



$$|V_s| = \Big[|V_R|^2\cos^2\Phi_R + |I|^2R^2 + 2|V_R||I|R\cos\Phi_R + |V_R|^2\sin^2\Theta_R + |I|^2X^2 + 2|V_R||I|X\sin\Phi_R\Big]^{1/2}$$

$$|V_s| = \Big[|V_R|^2 + |I|^2(X^2+R^2) + 2|V_R||I|(R\cos\Theta_R + X\sin\Theta_R)\Big]^{1/2}$$

$$|V_R|^2\cos^2\Theta_R + |V_R|^2\sin^2\Theta_R = |V_R|^2(\cos^2\Theta + \sin^2\Theta) = |V_R|^2$$ حيث

$$|V_s| = |V_R|\Big[1 + \cancel{\frac{|I|^2(R^2+X^2)}{|V_R|^2}} + \frac{2|I|}{|V_R|}(R\cos\Theta_R + X\sin\Theta_R)\Big]^{1/2}$$

neglect

تيار قليل مقارنة بمربع الفولتية

$$|V_s| = |V_R|\Big[1 + \frac{|I|}{|V_R|}(R\cos\Theta_R + X\sin\Theta_R)\Big]$$

تخلصنا من الجذر و رقم 2 مع الثالث باستخدام متسلسلة تايلور حيث أخذنا الحدود الأولى منها ليصبح الشكل كما سبق

$$|V_S| = |V_R| + |I|\,(R\cos\theta_R + X\sin\theta_R)$$

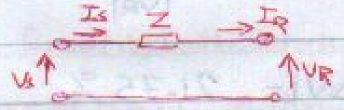
$$\Delta V = |V_S| - |V_R| \approx |I|\cdot(R\cos\phi_R + X\sin\theta_R)$$ ضرب البسط والمقام بـ $|V_R|$

$$\Delta V = |V_S| - |V_R| = \frac{|I|\,|V_R|\,(R\cos\phi_R + X\sin\phi_R)}{|V_R|}$$

$$\Delta V \approx \frac{|I|\,|V_R|\,(R\cos\theta_R + X\sin\theta_R)}{|V_R|}$$ البسط عبارة عن P و Q

$$\boxed{\Delta V = \frac{P\cdot R + Q\cdot X}{|V_R|}}$$

* Voltage Regulation



هنا عندي ال الفولتية نقيس الفولتية بدون وجود حمل

$$\frac{|V_{R0}| - |V_{RL}|}{|V_{RL}|} * 100\%$$

$$V.reg = \frac{|V_S| - |V_R|}{|V_R|} * 100\% = \frac{|\Delta V|}{|V_R|} * 100\% = \frac{P\cdot R + Q\cdot X}{|V_R|^2}$$

$$V.reg = \frac{|V_R||I|\cos\phi_R\cdot R + |V_R||I_R|\sin\phi_R\cdot X}{|V_R|^2}$$

$$V.reg = |I_R|\left[\frac{(R\cos\phi_R + X\sin\phi_R)}{|V_R|}\right]$$

for inductive load, $\phi_R = +ve$. For Capacitive, $\phi_R = -ve$

⇒ For capacitive load: $V_{reg} = |I_R|\left[\frac{R\cos\phi_R - X\sin\phi_R}{|V_R|}\right]$

⇒ Voltage regulation can be zero if $R\cos\phi_R = X\sin\phi_R \Rightarrow \tan\phi_R = \frac{R}{X}$

⇒ Voltage regulation have a relation only with reactive power

Ex: A single phase, 50HZ gen supplies inductive load of 5MW at P.F=0.707 lagging by mean overhead of trn.line $l = 20\,km$, $r = 0.0195\ \Omega/km$, $L = 0.63\,mH/km$ and the voltage at recieving end 10kV.

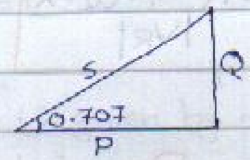
a) find the sending end voltage and voltage regulation of the line

b) find the value of C parallel to load such that V.R reduced to 50% as obtain in a.

$$R = r \cdot l = 0.0195 * 20 = 0.39\ \Omega$$

$$X = 2\pi f \cdot L \cdot l = 314 * 0.63 * 10^{-3} * 20 = 3.96\ \Omega$$

a) $$Q = \frac{P}{\tan(\cos^{-1} 0.707)} \simeq 5\ MVA$$



$$|\Delta V| = \frac{P \cdot R + Q \cdot X}{|V_R|} = \frac{5MW * 0.39 + 5MVA * 3.96}{10kV}$$

$$|\Delta V| = 21.75\ \%$$

$$|V_S| = |V_R| + |\Delta V| = 10000 + 2175 = 12.175\ kV$$

b) $$|\Delta V| = \frac{1}{2} |\Delta V| = 10.9\%$$

$$|\Delta V| = \frac{P \cdot R + Q \cdot X}{|V_R|}$$ هنا تتغير ال Q فقط لأنها علاقة مع V.r

$$10.9\% = \frac{5MW * 0.39 + Q * 3.96}{10kV} \Rightarrow Q = 2.27\ MVA$$

$$Q_t = Q_C + Q_L$$

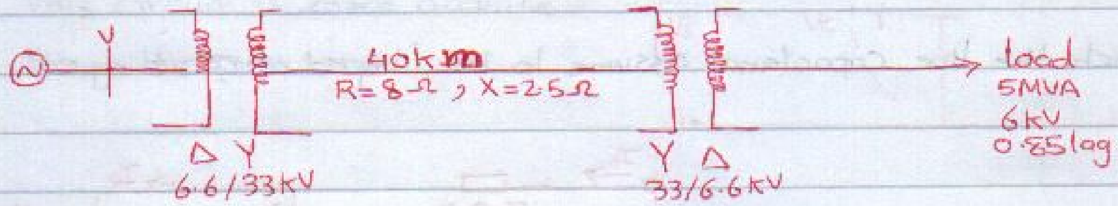
$$Q_C = Q - Q_L = 2.74\ MVA$$

$$Q_C = \frac{|V_R|^2}{X_C} \Rightarrow X_C = \frac{|V_R|^2}{Q_C} = 36.5\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = 87\ \mu F$$

* Exercises

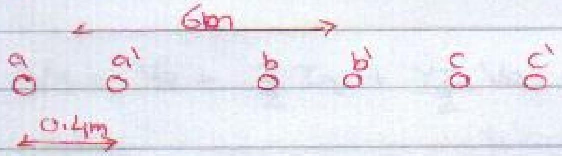
Q1) A load of 5MVA is supply at station by 6kV supply voltage through short line. find the voltages



for Δ winding $R=0.6\Omega$, $X=0.36\Omega$

for Y winding $R=0.5\Omega$, $X=3.75\Omega$

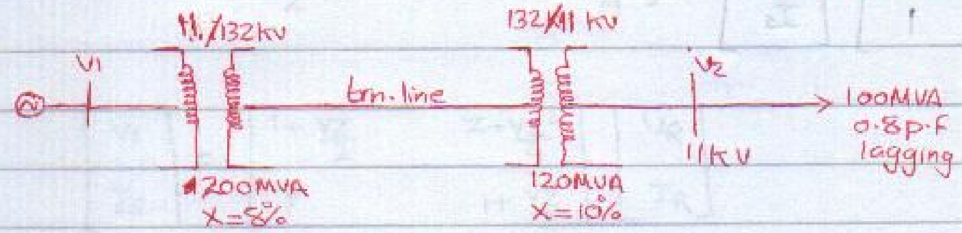
Q2) A 3-phase bundled radial transmission line shown in figure(1), with length of 60km and radius of each conductor of 25mm and resistance of $R=0.02\ \Omega/km$, is supplying a load of 100MVA, 0.8 p.f lagging as shown in figure(2)



$L = 60\ km$

$r = 25\ mm$

$R = 0.02\ \Omega/km$



a) Find the inductive reactance in Ω/km at 50 HZ of trn. line

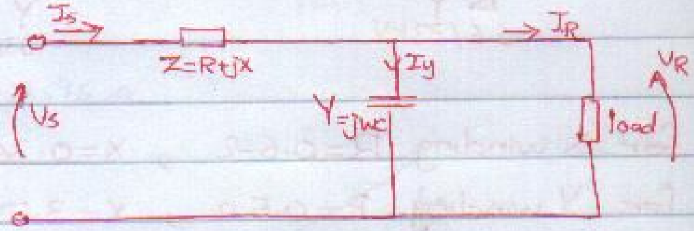
b) If the voltage at load bus required to be kept at 11kV, find the generator terminal voltage and the voltage regulation of the line. choose $(100MVA)_B$ and 132kV of trn. line

c) Find the value of C to be placed in parallel with load such that the line regulat reduced to 50% of obtained in b. What will the generator terminal voltage in this case

* Medium Transmission Line

For line with length $100km < L < 250km$, the capacitance can be consider in the calculations

1) Circuit at which the line capacitance assume to be lumped at recieving end



$$I_S = I_R + Y \cdot V_R$$

$$V_S = V_R + Z \cdot I_S$$

$$V_S = V_R + Z(I_R + Y \cdot V_R)$$

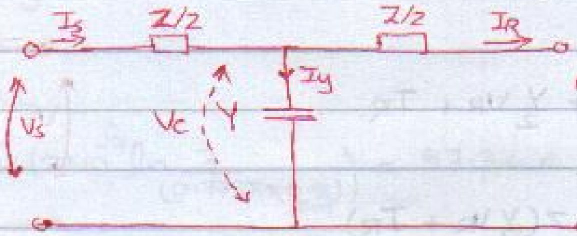
$$V_S = (1 + ZY)V_R + Z I_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

2) T-equivalent Circuits in this circuit the capacitance of line assume to be lumped on the middle of the line

this circuit is more accurate than the previous



$$V_C = V_R + Z/2\, I_R$$
$$I_S = I_R + Y \cdot V_C$$

$$I_S = I_R + Y(V_R + Z/2\, I_R)$$
$$= Y \cdot V_R + I_R\left(1 + \frac{YZ}{2}\right)$$

$$V_S = V_C + \frac{Z}{2} I_S$$
$$V_S = V_R + \frac{Z}{2} I_R + \frac{Z}{2}\left(Y \cdot V_R + \left(1 + \frac{YZ}{2}\right) I_R\right)$$

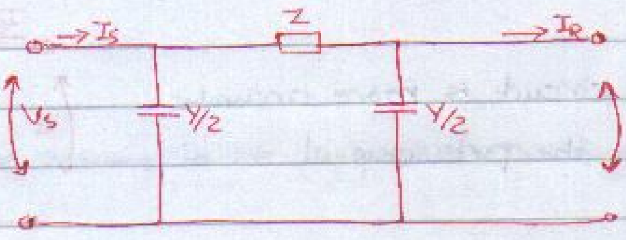
$$V_S = V_R + \frac{Z}{2} I_R + \frac{YZ}{2} V_R + \frac{Z}{2} I_R + \frac{YZ^2}{4} I_R$$

$$V_S = \left(1 + \frac{YZ}{2}\right) V_R + \left(Z + \frac{YZ^2}{4}\right) I_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{YZ}{2} & Z + \frac{YZ^2}{4} \\ Y & 1 + \frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\Delta = \left(1 + \frac{YZ}{2}\right)^2 - Y\left(Z + \frac{YZ^2}{4}\right) = 1$$

3) Π-equivalent Circuit: in this circuit, the cap of line assumed to be divided between recieving end and sending end. This circuit is most popular.



$$I_S = \frac{Y}{2}V_S + \frac{Y}{2}V_R + I_R$$

$$V_S = V_R + Z\left(\frac{Y}{2}V_R + I_R\right)$$

$$V_S = \left(1 + \frac{YZ}{2}\right)V_R + ZI_R$$

$$I_S = \frac{Y}{2}\left[\left(1+\frac{YZ}{2}\right)V_R + ZI_R\right] + \frac{Y}{2}V_R + I_R$$

$$I_S = \frac{Y}{2}V_R + \frac{Y^2Z}{4}V_R + \frac{YZ}{2}I_R + \frac{Y}{2}V_R + I_R$$

$$I_S = \left(Y + \frac{Y^2Z}{4}\right)V_R + \left(1 + \frac{YZ}{2}\right)I_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ Y + \frac{Y^2Z}{4} & 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\Delta = 1$$

All calculations used usually Π-circuit

Ex8 Using π-circuit, find the sending-end voltage and voltage regulation of 250 km 3phase, 50HZ, trn.lin deliver 25 MVA at 0.8 pf to abalanced load at 132 kV (V_R). The line conductors are spaced equilaterally 3m apart, $r = 0.11\ \Omega/km$, diameter 1.6cm

$$R = r \cdot L = 0.11 * 250 = 27.5\ \Omega$$

$$X = 2\pi f L = 2 * 3.14 * 50 * 250km * \left(2\times10^{-7} \ln \frac{3}{(0.7788 * 0.8_{cm})}\right) = 97.34\ \Omega$$

$$Z = R + jX = 27.5 + 97.34j$$

$$Y = jWCL = j314 * 250 * \left(\frac{5.56 \times 10^{-2}}{\ln \frac{D}{r}} \mu F/km\right) = j7.4 * 10^{-4}\ \mho$$

$$I_R = \frac{Q}{\sqrt{3} * V} = \frac{25 MVA}{\sqrt{3} * 132 kV} = 109.35 \angle -36.9^\circ$$ هون عطانا P.F و Q

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{YZ}{2} & Z \\ Y + \frac{Y^2 Z}{4} & 1 + \frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$V_S = \left(1 + \frac{YZ}{2}\right) V_R + Z I_R$$ ملاحظة V_R هي فيز $76.2 = 132/\sqrt{3}$

$$V_S = (73.45 + 0.775j) + (8.8 + 6.7j) = 82.25 + 7.475j = 82.6 \angle 5.2^\circ$$

$$V_S(line) = 82.6 * \sqrt{3} = 143\ kV$$

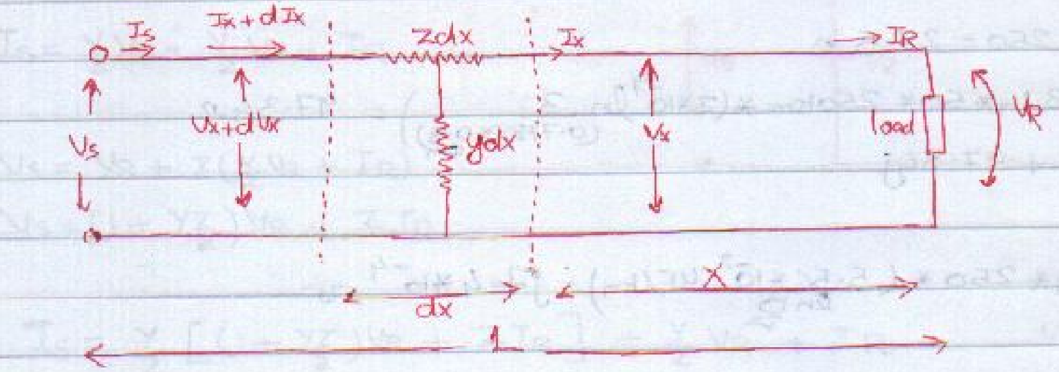
Calculate V.r تذكرو تحسب ع حالة noload

$$V_R = \frac{V_S}{1 + \frac{YZ}{2}} = \frac{143 kV}{0.964} = 148.3\ kV$$

$$\text{Voltage regulation} = \frac{148.3 - 132}{132} * 100\% = 12.3\%$$

* Long transmission line

line > 250km, assume to distributed uniformly along the line



$dV_x = I_x \cdot Z dx$, $dI_x = V_x \cdot y\, dx$

$\frac{dV_x}{dx} = Z \cdot I_x$ $\qquad \frac{dI_x}{dx} = V_x \cdot y$

$\frac{d^2 V_x}{dx^2} = Z \frac{dI_x}{dx} \Rightarrow \frac{d^2 V_x}{dx^2} = Z \cdot y \cdot V_x \Rightarrow \frac{d^2 V_x}{dx^2} - yZ V_x = 0$

this is 2^{nd} order differential equation

the characteristic equ $r^2 - yZ = 0$

$r_{12} = \pm \sqrt{yZ} = \pm \gamma$ (γ معامل الانتشار)

$V_x = C_1 e^{\gamma x} + C_2 e^{-\gamma x}$ by differniate this equ

$\frac{dV_x}{dx} = C_1 \gamma e^{\gamma x} - C_2 \gamma e^{-\gamma x}$

$I_x = \frac{1}{Z}\frac{dV_x}{dx} = C_1 \frac{\gamma}{Z} e^{\gamma x} - C_2 \frac{\gamma}{Z} e^{-\gamma x}$

$I_x = C_1 \frac{\sqrt{yZ}}{Z} e^{\gamma x} - C_2 \frac{\sqrt{yZ}}{Z} e^{-\gamma x}$

$I_x = C_1 \sqrt{y/Z}\, e^{\gamma x} - C_2 \sqrt{y/Z}\, e^{-\gamma x}$

$I_x = \frac{C_1}{\sqrt{y/Z}} e^{\gamma x} - \frac{C_2}{\sqrt{y/Z}} e^{-\gamma x}$

$\sqrt{y/z}$ $\Rightarrow$ Characteristic impedance

$$I_x = \frac{C_1}{Z_c} e^{\gamma x} - \frac{C_2}{Z_c} e^{-\gamma x}$$

the equs which describ the performane of the lines

$$\boxed{\begin{aligned} V_x &= C_1 e^{\gamma x} + C_2 e^{-\gamma x} \\ I_x &= \frac{C_1}{Z_c} e^{\gamma x} - \frac{C_2}{Z_c} e^{-\gamma x} \end{aligned}}$$

Constants C_1, C_2 are found from boundary condition

$\Rightarrow$ When $x = 0 \Rightarrow V_x = V_R$, $I_x = I_R$

$$V_R = C_1 + C_2 \quad \Rightarrow \quad C_1 + C_2 = V_R$$

$$I_x = \frac{C_1}{Z_c} - \frac{C_2}{Z_c} \quad \Rightarrow \quad \underline{C_1 - C_2 = Z_c I_x}$$

$$2C_1 = V_R + Z_c I_x$$

$$C_1 = \frac{V_R + Z_c I_x}{2}, \quad C_2 = \frac{V_R - Z_c I_x}{2}$$

$$V_x = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I_x = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$$

$$V_x = V_R \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2} \right) + Z_c I_R \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right)$$

$$I_x = \frac{V_R}{Z_c} \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right) + I_R \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2} \right)$$

by simplifying these equation

$$V_x = \cosh\gamma x \cdot V_R + \sinh\gamma x \cdot Z_c I_R$$

$$I_x = \frac{V_R}{Z_c}\sinh\gamma x + I_R\cosh\gamma x$$

$$\begin{bmatrix} V_x \\ I_x \end{bmatrix} = \begin{bmatrix} \cosh\gamma x & Z_c\sinh\gamma x \\ \frac{1}{Z_c}\sinh\gamma x & \cosh\gamma x \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

at recieving end $x = l$, thus $x = l$ طولها

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c\sinh(\gamma l) \\ \frac{\sinh(\gamma l)}{Z_c} & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

* بعض التطابقات وتسهيل طرق الحل :

1) $\gamma = \sqrt{yz} = \alpha + j\beta$

2) $\cosh(\gamma l) = \cosh(\alpha l + j\beta l)$

$$= \cosh(\alpha l)\cosh(j\beta l) + \sinh(\gamma l)\sinh(j\beta l)$$

$$= \cosh(\alpha l)\cos(\beta l) + j\sinh(\alpha l)\cdot\sin(\beta l)$$

3) $\sinh(\alpha l + j\beta l) = \sinh(\alpha l)\cosh(j\beta l) + \cosh(\alpha l)\cdot\sinh(j\beta l)$

$$= \sinh(\alpha l)\cdot\cos(\beta l) + j\cosh(\alpha l)\cdot\sin(\beta l)$$

4) $\sinh(\gamma l) = \gamma l + \frac{(\gamma l)^3}{3!} + \frac{(\gamma l)^5}{5!} + \cdots$

5) $\cosh(\gamma l) = 1 + \frac{(\gamma l)^2}{2!} + \frac{(\gamma l)^4}{4!} + \cdots$

ملاحظة مهمة : نظراً لصعوبة التعامل مع hybrolic func مثل cosh , sinh قمت باشتقاق طريقة باستخدام كود توصلت لل صيغ مكافئة نستطيع الحل عليها وهي صحيحة 100%

$$\begin{bmatrix} \cosh(\gamma l) & Z_c\sinh(\gamma l) \\ \frac{\sinh(\gamma l)}{Z_c} & \cosh(\gamma l) \end{bmatrix} \cong \begin{bmatrix} 1 + \frac{YZ}{2} & Z\left(1 + \frac{YZ}{6}\right) \\ Y\left(1 + \frac{YZ}{6}\right) & 1 + \frac{YZ}{2} \end{bmatrix}$$

Exa: 3-phase, 50Hz, $L = 400\,km$, $r = 0.125\,\Omega/km$, $x = 0.4\,\Omega/km$, $y = 2.8 \times 10^{-6}\,\mho/km$. If $V_s = 220\,kV$

Find 1) Sending end current, recieving end voltage when line is not loaded

2) the max permissable length that the voltage on recieving end don't exceed 235 kV

$$R = 0.125 * 400 = 50\,\Omega$$

$$X = 0.4 * 400 = 160\,\Omega$$

$$Z = R + jX = 50 + 160j$$

$$Y = 2.8 * 10^{-6} * 400 = j1.12 * 10^{-3}\,\mho$$

a) not loaded i.e $I_R = 0$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 + \frac{YZ}{2} & Z(1 + \frac{YZ}{6}) \\ Y(1 + \frac{YZ}{6}) & 1 + \frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$V_R = \frac{V_s}{1 + \frac{YZ}{2}} = 241.4 - 7.425j = 241.5\,kV \angle -1.76^\circ$$

$$I_s = Y(1 + \frac{YZ}{6})(V_R) = Y(1 + \frac{YZ}{6})(\frac{V_R}{\sqrt{3}}) = -1.46 + 151.5j = 151.5 \angle -89.44^\circ$$

ممكن الحسابات تطلع قريبة بحلول أي طالب. دائمًا هناك نسبة error في power system analysis

error = ± 5%

b) $V_s = (1 + \frac{YZ}{2}) V_R$

$$\frac{V_s}{V_R} = (1 + \frac{YZ}{2}) \Rightarrow \frac{V_s}{V_R} = 1 + \frac{1}{2} * L^2 * j2.8 * 10^{-6} * (0.125 + 0.4j)$$

$$\frac{220}{235} = 1 + \frac{1}{2} * L^2 * j2.8 * 10^{-6} * (0.125 + 0.4j)$$

by solving this equ $\Rightarrow L = 338\,km$

* Interpretation of Long Line Equation

$$V_x = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I_x = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$$

but $\gamma = \sqrt{yz} = \alpha + j\beta$

$$V_x = \frac{V_R + Z_c I_R}{2} e^{(\alpha + j\beta)x} + \frac{V_R - Z_c I_R}{2} e^{-(\alpha + j\beta)x}$$

$$\frac{V_R + Z_c I_R}{2} = \left|\frac{V_R + Z_c I_R}{2}\right| \angle \phi_1 = \left|\frac{V_R + Z_c I_R}{2}\right| e^{j\phi_1}$$

$$\frac{V_R - Z_c I_R}{2} = \left|\frac{V_R - Z_c I_R}{2}\right| \angle \phi_2 = \left|\frac{V_R - Z_c I_R}{2}\right| e^{j\phi_2}$$

$$V_x = \underbrace{\left|\frac{V_R + Z_c I_R}{2}\right| e^{\alpha x} \cdot e^{j(\beta x + \phi_1)}}_{V_{x_1}} + \underbrace{\left|\frac{V_R - Z_c I_R}{2}\right| e^{-\alpha x} \cdot e^{j(\phi_2 - \beta x)}}_{V_{x_2}}$$

from this, we can see that V_x has two components V_{x_1}, V_{x_2}

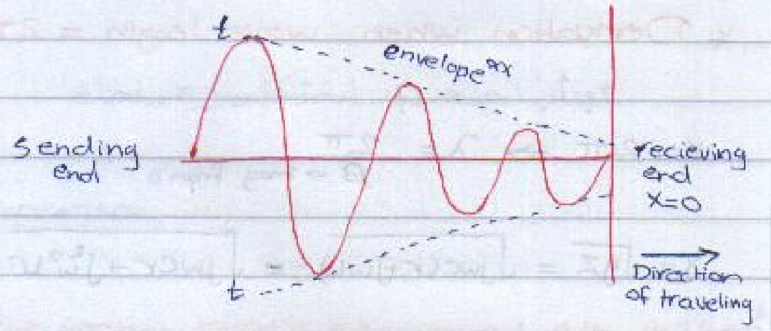
$$V_{x_1} = \underbrace{\left|\frac{V_R + Z_c I_R}{2}\right|}_{\text{amplitude}} e^{\alpha x} \cdot e^{j(\beta x + \phi_1)} = \sqrt{2} \underbrace{\left|\frac{V_R + Z_c I_R}{2}\right| e^{\alpha x}}_{\text{magnitude}} \cdot \cos(\omega t + \beta x + \phi_1)$$

$$V_{x_2} = \left|\frac{V_R - Z_c I_R}{2}\right| e^{-\alpha x} \cdot e^{j(\phi_2 + \beta x)} = \sqrt{2} \left|\frac{V_R - Z_c I_R}{2}\right| e^{-\alpha x} \cdot \cos(\omega t - \beta x + \phi_2)$$

$$\omega \Delta t + \beta \Delta x = 0$$

$$\omega \Delta t = -\beta \Delta x$$

$$\Delta x = \frac{-\omega}{\beta} \Delta t$$

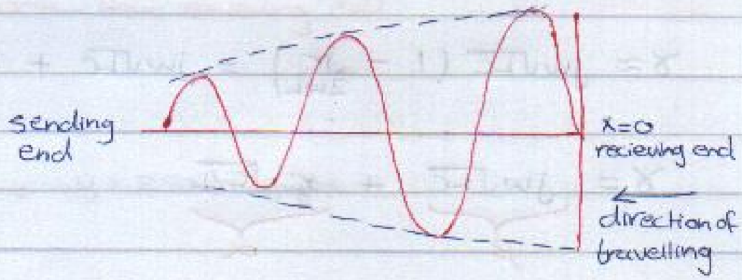


(incident waveform)

$$\omega \Delta t - \beta \Delta x = 0$$

$$\omega \Delta t = \beta \Delta x$$

$$\Delta x = \frac{\omega}{\beta} \Delta t$$



(reflected waveform)

- From this waveform, we can conclude that the voltage has two components:

1) incident waveform as appearing from sending end to recieving end
2) reflected waveform as appearing from recieving end to sending end.

- If the line is loaded by it's characteristic impedance, that mean no reflected exist in the waveform, in this case the line ~~is~~ called infinite bus(line).

- Sometimes, the characteristic impedance of the line is $Z_c = \sqrt{Z/y} = Z_s$ (surage impedance) for overhead line $Z_s = 400\ \Omega$, for cables $= 40\ \Omega$

- Surage impedance:- the line when were it loaded by characteristic impedance

$$S.I = 3 V_{ph} . I_{ph}$$

$$= 3 . \frac{|V_R|}{\sqrt{3}} . \frac{|V_R|}{\sqrt{3} . Z_s} = \frac{|V_R|^2}{Z} \text{ MW} = \frac{1000 * |V_R|^2}{400} = 2.5 |V_R|^2 \text{ kW}$$

* Derivation when wave length = 2π

$x = 2\pi \Rightarrow \lambda = \dfrac{2\pi}{\beta}$ → may from γ

$$\gamma = \sqrt{yZ} = \sqrt{j\omega c(r + j\omega L)} = \sqrt{j\omega c r + j^2\omega^2 LC} = \sqrt{j^2\omega^2 LC\left(1 + \frac{j\omega c r}{j^2\omega^2 LC}\right)}$$

$$\gamma = j\omega\sqrt{LC}\sqrt{1 + \frac{r}{j\omega L}} = j\omega\sqrt{LC}\sqrt{1 - \frac{jr}{\omega L}}$$

$$\gamma \approx j\omega\sqrt{LC}\left(1 - \frac{jr}{2\omega L}\right) = j\omega\sqrt{LC} + \frac{\omega r\sqrt{LC}}{2\omega L}$$

$$\gamma = \underbrace{j\omega\sqrt{LC}}_{\beta} + \underbrace{\frac{r}{2}\sqrt{C/L}}_{\alpha}$$

⇒ if / for cases of losseless line $r = 0$

سرعة الانتشار $v = f \cdot \lambda$ speed of prograpation

$\alpha = 0$, $\beta = \omega\sqrt{LC}$

but $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{2\pi f\sqrt{LC}} = \dfrac{1}{f\sqrt{LC}}$

$v = f \cdot \lambda = \dfrac{1}{\sqrt{LC}}$

$\gamma = j\omega\sqrt{LC}$

$L = \dfrac{\mu_0}{2\pi}\ln\dfrac{D}{r'}$, $C = \dfrac{2\pi\epsilon_0}{\ln\frac{D}{r}}$ نستطيع كتابتها بهذه الصورة

$$v = \frac{1}{\sqrt{\frac{\mu_0}{2\pi}\ln\frac{D}{r'} \cdot \frac{2\pi\epsilon_0}{\ln\frac{D}{r}}}}$$

$$v \approx \frac{1}{\sqrt{\mu_0\epsilon_0}} \cong 3*10^8 \text{ m/s}$$

i.e. waveform prograpate over total line by light speed.

- Practically is less than this value

$\lambda = \frac{v}{f} = \frac{3*10^8}{50} = 6000 km$ اي انها بعد هذه المسافة تكمل دورة كاملة

but this legth is very large as a comparison.

Ex: 3-phase, 50HZ, open circuit overhead line 400km, $r = 0.125 \Omega/km$, $x = 0.4 \Omega/km$, $y = 2.8*10^{-6}$ if $V_R = 220 KV$. find a) insident and reflected voltage to neutral at recieving end.

b) the insident and reflected voltage at 200km from recieving end.

c) find the resultant

$r = 0.125 \Omega/km$, $x = 0.4 \Omega/km$, $y = 2.8*10^{-6} \mho/km$

$Z = (0.125 + j0.4) = 0.42\angle 76.6^\circ \ \Omega/km$ Zcapital عندما تكون R, X و Zsmall عندما تكون r, x

هنا مع طول الخط دون الضرب بطول الخط

$\gamma = \sqrt{yZ} = (j2.8*10^{-6} * 0.42\angle 76.6^\circ)^{1/2} = (0.163 + j1.068)*10^{-3}$

$\therefore \alpha = 0.163*10^{-3}$, $\beta = j1.068*10^{-3}$

(a) at recieving end, opencircuit so $I_R = 0$, x at recieveng end $= 0$

Insident Voltage $V_{x_1} = \frac{V_R + Z_C I_R}{2} = \frac{V_R}{2} = \frac{220/\sqrt{3}}{2} = 63.51$ KV to neutral

Reflected voltage $V_{x_2} = \frac{V_R - Z_C I_R}{2} = \frac{V_R}{2} = 63.51$ KV

(b) At 200 km from recieving end

$V_{x_1} = \frac{V_R}{2} e^{\alpha x} \cdot e^{j\beta x} = 63.51 \exp(0.163*10^{-3}*200) * \exp(1.068*10^{-3}*200j) = 65.5 KV\angle 12.2^\circ$ to neu.

$V_{x_2} = \frac{V_R}{2} e^{-\alpha x} \cdot e^{-j\beta x} = 63.51 * e^{-0.0326} * e^{-j0.2136} = 61.47\angle -12.2^\circ$ KV to neu

ⓒ Resultant $V_x = V_{x1} + V_{x2}$

$$= 65.5\angle 12.2^\circ + 61.47\angle -12.2^\circ = 124.01\angle 0.4^\circ$$

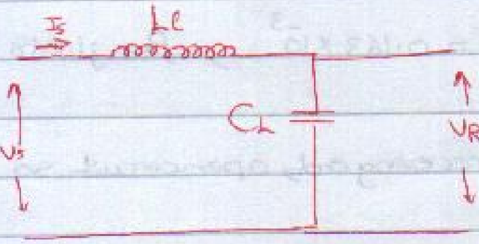
resultant Line to Line at 200km

$124.1 * \sqrt{3} = 215$ km يجب على المحصلة أن تكون قريبة من الـ V_R

* Ferranti Effect

- درسنا سابقاً أن الـ V_R يجب أن تكون أقل من V_s وذلك لوجود الـ drop volt على طول الخط ، لكن في بعض الحالات تكون الـ $V_s < V_R$ وهذه تسمى فيرانتي

- When the line open circuit or lightly loaded, the recieving end voltage can reach values greater than sending end. This can be explain by lumping the capacitance and inductan as shown in the figure.



$L\ell$, $C\ell$ totall inductance and capacitance

i.e $L\ell = L \cdot \ell$ induc * length

$$I_s = \frac{V_s}{jwL\ell + \frac{1}{jwC\ell}}$$

but for practical line $\frac{1}{wC\ell} >> wL\ell$

$$I_s = \frac{V_s}{\frac{1}{jwC\ell}} = jwC\ell \cdot V_s$$

$$V_R = V_s - I_s(jwL\ell)$$

$$V_R = V_s - jwC\ell \cdot V_s(jwL\ell)$$

$$V_R = V_s + V_s w^2 \cdot C\ell \cdot L\ell$$

$$V_R = V_s(1 + w^2 C\ell \cdot L\ell)$$

From this result, we can see that $V_R > V_S$ cuz $w^2 L_\ell C_\ell > 0$

$$|V_R - V_S| = w^2 C_\ell \cdot L_\ell \cdot V_S$$

$$\Delta V = w^2 \cdot C \cdot \ell \cdot L \cdot \ell \cdot V_S$$

$$\Delta V = \frac{w^2 \cdot \ell^2 \cdot V_S}{\frac{1}{LC}}$$

نزلنا LC على مقام المقام حيث أنها نفس الشيء بسط

(مقام المقام = بسط)

$$\Delta V = \frac{w^2 \cdot \ell^2 \cdot V_S}{\left(\frac{1}{\sqrt{LC}}\right)^2} = \frac{w^2 \cdot \ell^2 \cdot V_S}{v^2}$$

ΔV حيث أنها تعتمد على مربع طول الخط

- In order to decrease ferranti effect, sometimes an inductance is connected at the recieving end of the line.

* Tuned Power line

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma \ell) & Z_c \cdot \sinh(\gamma \ell) \\ \frac{\sinh(\gamma \ell)}{Z_c} & \cosh(\gamma \ell) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\gamma = \sqrt{yZ} = \sqrt{jwC \cdot jwL} = jw\sqrt{LC}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(jw\ell\sqrt{LC}) & Z_c \sinh(jw\ell\sqrt{LC}) \\ \frac{\sinh(jw\ell\sqrt{LC})}{Z_c} & \cosh(jw\ell\sqrt{LC}) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\left.\begin{aligned} \cosh(jx) &= \cos(x) \\ \sinh(jx) &= j\sin(x) \end{aligned}\right\} \text{قواعد}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cos(w\ell\sqrt{LC}) & jZ_c \sin(w\ell\sqrt{LC}) \\ \frac{j\sin(w\ell\sqrt{LC})}{Z_c} & \cos(w\ell\sqrt{LC}) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$w\ell\sqrt{LC} = n \cdot \pi$, $\cos(w\ell\sqrt{LC}) = \pm 1$, $\sin(w\ell\sqrt{LC}) = 0$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \qquad \begin{aligned} |V_S| &= |V_R| \\ |I_S| &= |I_R| \end{aligned}$$

This case, the line set to be tuned, no voltage drop exists.

⇒ Condition of tuned line:-

$$w\ell\sqrt{LC} = n \cdot \pi$$

$$\ell = \frac{n \cdot \pi}{w\sqrt{LC}} = \frac{n \cdot \pi}{w} \cdot \frac{1}{\sqrt{LC}} = \frac{n \cdot \pi \cdot v}{2\pi f} = \frac{n \cdot 3 \times 10^8}{2 \times 50} = n \times 3000 \text{ km}$$

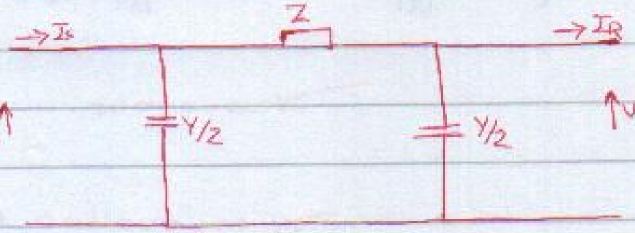
- practically, this is very long line. Tuned can be done industrial by adding series inductance and parallel capacitance to the line, but usually this is not done for power line cuz it's not economical cuz the low frequency of power system, it's done for communication line cuz the high frequency of it

- Nowadays, in order to decrease the effect of line inductance (voltage drop), a series capacitance is added to the line. In order to decrease the effect of line capacitance an inductance (coil or reactor) is added at recieving end of the line to cancel the parallel capacitance of the line

* Equivalent Circuit of Long Lines

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{\sinh(\gamma l)}{Z_c} & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ Y+\frac{Y^2 Z}{4} & 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$



$Z = Z_c \sinh(\gamma l)$ ⇐ بنعمل عملية مساواة بين عناصر حيث Z مصفوفة تقابل $Z_c \sinh(\gamma l)$

(إسقاط كل عنصر من الدائرة على ما يقابله)

$$1 + \frac{YZ}{2} = \cosh(\gamma l)$$

$$\frac{YZ}{2} = \cosh(\gamma l) - 1 \Rightarrow \frac{Y}{2} = \frac{\cosh(\gamma l) - 1}{Z} = \frac{\cosh(\gamma l) - 1}{Z_c \cdot \sinh(\gamma l)}$$

متطابقة

$$\frac{Y}{2} = \frac{\cosh(\gamma l) - 1}{Z_c \sinh(\gamma l)} = \frac{2\sinh^2(\frac{\gamma l}{2}) + 1 - 1}{2 Z_c \sinh\frac{\gamma l}{2} \cdot \cosh\frac{\gamma l}{2}} = \frac{1}{Z_c} \tanh(\frac{\gamma l}{2})$$

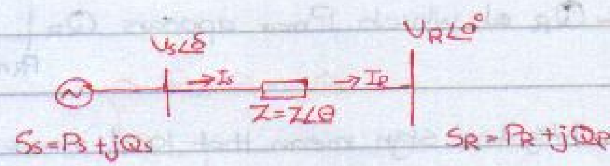
وهكذا حصلنا على عناصر الدائرة

Ex: A 50HZ, 300 km long trn line has a total impedance $Z = 10 + j125$ and total shunt admittance $Y = 10^{-3}$ ℧. the recieving end voltage 220 kV and $P = 50MW$, 0.8 p.F lagging find V_s, I_s, P_s, P.F ?

حل السؤال مشابه بنفس الطرق السابقة للحل ونفس المعادلات ونفس مجموعة الحل

* Power flow (Load flow) through trn. Line

Consider the following two bus system



عادة ما يؤخذ الـ V_R كمرجع لذا تكون زاويته صفر

$$I_S = I_R = \frac{V_S - V_R}{Z} = \frac{|V_S|\angle\delta - |V_R|\angle 0}{|Z|\angle\theta} = \frac{|V_S|}{|Z|}\angle\delta-\theta - \frac{|V_R|}{|Z|}\angle-\theta$$

$$I_S^* = I_R^* = \frac{|V_S|}{|Z|}\angle\theta-\delta - \frac{|V_R|}{|Z|}\angle\theta$$

$$S_R = V_R \cdot I_R^* = P_R + jQ_R = |V_R|\left[\frac{|V_S|}{|Z|}\angle\theta-\delta - \frac{|V_R|}{|Z|}\angle\theta\right]$$

$$S_R = \frac{|V_R||V_S|}{|Z|}\angle\theta-\delta - \frac{|V_R|^2}{|Z|}\angle\theta$$

$$= \frac{|V_R||V_S|}{|Z|}\left[\cos(\theta-\delta) + j\sin(\theta-\delta)\right] + \frac{|V_R|^2}{|Z|}\left[\cos\theta + j\sin\theta\right]$$

$$= \frac{|V_R||V_S|}{|Z|}\cos(\theta-\delta) - \frac{|V_R|^2}{|Z|}\cos\theta + j\left[\frac{|V_R||V_S|}{|Z|}\sin(\theta-\delta) - \frac{|V_R|^2}{|Z|}\sin\theta\right]$$

$$P_R = \frac{|V_R||V_S|}{|Z|}\cos(\theta-\delta) - \frac{|V_R|^2}{|Z|}\cos\theta$$

$$Q_R = \frac{|V_R||V_S|}{|Z|}\sin(\theta-\delta) - \frac{|V_R|^2}{|Z|}\sin\theta$$

$$S_S = V_S \cdot I_S^* = |V_S|\angle\delta\left[\frac{|V_S|}{|Z|}\angle\theta-\delta - \frac{|V_R|}{|Z|}\angle\theta\right]$$

$$S_S = P_S + jQ_S = \frac{|V_S|^2}{|Z|}\angle\theta - \frac{|V_S||V_R|}{|Z|}\angle\theta+\delta$$

$$= \frac{|V_S|^2}{|Z|}\left[\cos\theta + j\sin\theta\right] - \frac{|V_S||V_R|}{|Z|}\left[\cos(\theta+\delta) + j\sin(\theta+\delta)\right]$$

$$P_S = \frac{|V_S|^2}{|Z|}\cos\theta - \frac{|V_S||V_R|}{|Z|}\cos(\theta+\delta)$$

$$Q_S = \frac{|V_S|^2}{|Z|}\sin\theta - \frac{|V_S||V_R|}{|Z|}\sin(\theta+\delta)$$

- P_R max appears when $\theta = \delta$. In this case $P_{R\,max} = \frac{|V_R||V_S|}{|Z|} - \frac{|V_R|^2}{|Z|}\cos\theta$

- Q_R at which P_{max} appears $Q_R\Big|_{P_{R\,max}} = -\frac{|V_R|^2}{|Z|}\sin\theta$ أي أن الـload هو يولد الـQ

- Negative sign mean that load must generate Q_R in order to recieve P_R max.

- Usually, R for the line is neglect as it small with compore to inductance :-

$\theta = 90^\circ$, $|Z| = X$

$$P_R = \frac{|V_S||V_R|}{Z=X} \cdot \sin\delta \quad , \quad Q_R = \frac{|V_S||V_R|}{X}\cos\delta - \frac{|V_R|^2}{X}$$

also in normal cases δ is a small angle $\Rightarrow \cos\delta \approxeq 1$ كلما كانت δ قليلة كان الـcos قريب للواحد

$$Q_R \approx \frac{|V_S||V_R|}{X} - \frac{|V_R|^2}{X} = \frac{|V_R|}{|X|}\left[|V_S| - |V_R|\right]$$

$$Q_R = \frac{|V_R||\Delta V_R|}{X} \Rightarrow \Delta V = \frac{Q_R \cdot X}{|V_R|}$$

- Notes :-

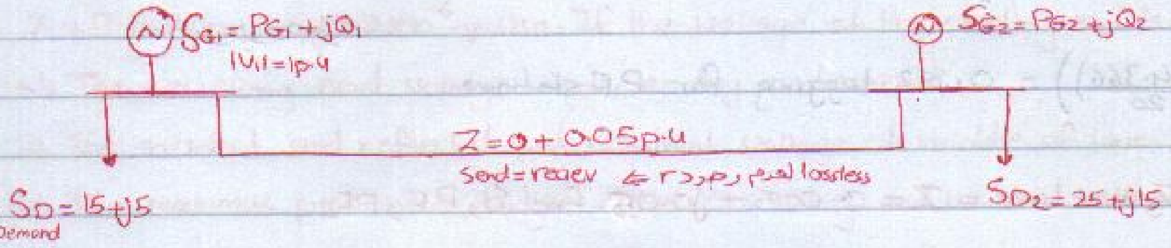
1] the recieved power is a function of angle δ.

2] the max. recieved power is directly proprotional to the square of line voltage

3] P_R is inversly proprotional with X, when increase $P_R$max, X decreas, this is why series capacit-ance is adding to line to decrease the effect

4] $|\Delta V| = \frac{QX}{|V_R|}$ ذكرنا سابقاً أن الـ V.r على علاقة طردية بـ Q

Ex Two generation station are interconnected by a cable link, the desired voltage is flat i.e $(V_1 = V_2) = 1 p.u$



if the active power generated by two station is equal, find δ, $P.F_1$, PF_2

$V_2 = 1\angle 0^\circ$ reference

$V_1 = 1\angle \delta$

$\sum PG = \sum P_D$ power balanced

$P_{G1} + P_{G2} = P_{D1} + P_{D2} + P_{losses}$

$P_{G1} + P_{G2} = 15 + 25 = 40 \Rightarrow P_{G1} = P_{G2} = 20\ p.u$

$$P_S = P_R \Rightarrow P_R = \frac{|V_S|\cdot|V_R|}{|Z|}\cos(\theta - \delta) - \frac{|V_R|^2}{|Z|}\cos\theta$$

$$P_R = P_G - P_D = 5 = \frac{1 * 1}{0.005}\cos(90 - \delta) - 0$$

$$5 = \frac{1}{0.005}\cos(90 - \delta) \Rightarrow \delta = 14.47^\circ$$

$\delta = (10^\circ - 25^\circ)$ غالباً

$$Q_S = \frac{|V_S|^2}{|Z|}\sin\theta - \frac{|V_S||V_R|}{|Z|}\sin(\theta + \delta)$$

$$= \frac{1}{0.05}\sin 90^\circ - \frac{1 * 1}{0.05}\sin(90^\circ + 14.47^\circ) = 0.634\ p.u$$

$$S_{G1} = (P_{S1} + P_{D1}) + j(Q_S + Q_{D1})$$

$$= (15 + 5) + j(5 + 0.634) = 20 + j5.634$$

$\cos\left(\tan^{-1}\left(\frac{Q_G}{P_G}\right)\right) = 0.963$ lagging P.F for first station

$$S_{G2} = (P_{D2} - P_R) + (Q_{D2} - Q_{R=S})$$
$$= (25 - 5) + j(15 - 0.634)$$
$$= 20 + j14.366$$

الاشارة ناقص بين (Q) لأن الـ load هو يطلب reactiv power فيولد demand ويعطي الي فيها send' دي recieve لعدم وجود مكثفات

$$\cos\left(\tan^{-1}\left(\frac{14.366}{20}\right)\right) = 0.812 \text{ lagging for P.F station 2}$$

extra if the Z over line = $Z = 0.005 + j0.05$ find δ, $P.F_1$, $P.F_2$

نفس الحل السابق ولكن الاختلاف يصبح الزاوية θ بدلاً من أنها 90° في الفرع الأول لعدم وجود R يصبح لها قيمة أقل من 90° وبالحسابات نعوض قيمتها $Z = |Z|\angle\theta$

$$Z = |0.05|\angle 84.29^\circ$$

نعوض 84.29° في الحسابات بدلاً من 90°

* Exercises: Years Questions

Q1) 3-phase, 50HZ, open circuit trn-Line 500km along. The line parameter are $r = 0.1\,\Omega/km$ $X = 0.5\,\Omega/km$, $y = 2\times10^{-6}$ ℧/km. If the voltage at the sending end is 400kV.

a) The recieving end voltage and sending end current (4marks)

b) The incident and reflected and resultant voltage at middle of Line. (4marks)

c) The maximus permissible length if the recieving end no-load voltage is not exceed 420kV (2marks)

Q2) Two generation station are interconnected by a cable link with impedance $Z = 0.01 + j0.1\,p.u$, the load demands at two buses $S_{D1} = 10 + j5\,p.u$, and $S_{D2} = 20 + j10\,p.u$. If the active power generated by G_1 is $P_{G1} = 15\,p.u$, and the desired voltage profile is flat $|V_1| = |V_2| = 1\,p.u$

Finds:- a) torque angle of bus 1, assuming bus 2 reference. (4 marks)

b) active and reactive powers generated by each generator (4 marks)

c) the active and reactive losses in the line. (4marks)