ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

تلخيص لمادة:

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جزيل الشكر للطالب:

حمزة أسماعيل

* Numerical Analysis :-

* Numerical methods :-

Numerical methods → Approximate Value.

True Value → Calculations.

* Absolute and Relative True Error :-

True Error = True Value − Approx. value

Absolute True Error = | True Value − Approx. Value |

Relative True Error $\varepsilon_t\% = \left| \dfrac{\text{True Error}}{\text{True Value}} \right| * 100\%$

* Absolute and Relative Approx. Error :-

Approximate Error = New Approx. Value − old Approx. Value.

Relative Approx Error $\varepsilon_a\% = \left| \dfrac{\text{Approx. Error}}{\text{New Approx. Value}} \right| * 100\%$

$\varepsilon_a \downarrow \Rightarrow$ Convergent. , $\varepsilon_a < \varepsilon_s$ acceptance Error.

$\varepsilon_a \uparrow \Rightarrow$ divergent.

* Taylor Series Expansion :-

$$f(x_1) = f(x_0) + f'(x_0)(x_1 - x_0) + \frac{f''(x_0)(x_1 - x_0)^2}{2!} + \frac{f'''(x_0)(x_1 - x_0)^3}{3!} \ldots\ldots$$

$$f(x_1) = \sum_{i=0}^{\infty} \frac{(x_1 - x_0)^i f^{(i)}(x_0)}{i!}$$

* Taylor Series Remainder (Rn) :-

$$R_n = \sum_{i=n+1}^{\infty} \frac{(x_1 - x_0)^i f^{(i)}(x_0)}{i!}$$

↳ Remainde represent the truncation Error &

$\boxed{R_n = \text{Exat Val} - \text{Talyor}}$

* Taylor Series Remainder (Rn) :-

$\underset{\text{exact}}{f(x)} = f_n(x) + R_n$ → نسبة الخطأ

$f_n(x)$: القيم التي بجيبها من ال Taylor

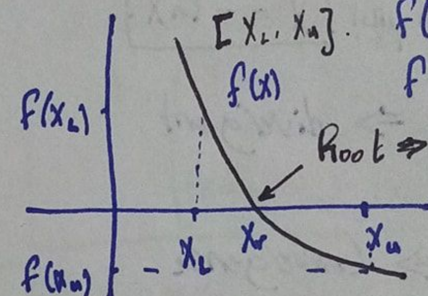
R_n = Truncation Error.

$R_n = f(x)_{\text{exact}} - f_n(x)$.

R_n Relative $= \left| \dfrac{f(x)_{\text{exact}} - f_n(x)}{f(x)_{\text{exact}}} \right| * 100\%$

* Bracketing Methods :- We have Interval.

1. Bisection Methods :-



$[X_L, X_u]$. $f(X_L) = +Ve$

$f(X_u) = -Ve$.

Check :-

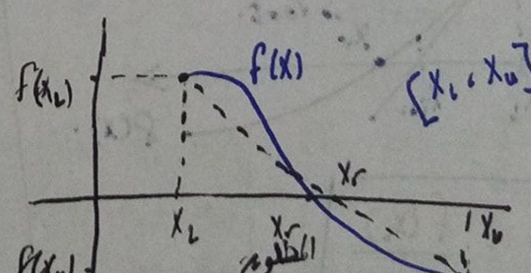
if $f(X_L) * f(X_u) > 0 \Rightarrow$ Method Fails

if $f(X_L) * f(X_u) < 0 \Rightarrow$ Method True

$$\boxed{X_r = \frac{X_L + X_u}{2}}$$

* Bracketing Methods :-

2- False Position Method :-



$[X_L, X_u]$

$$\boxed{x_r = x_u - \frac{f(x_u)(x_u - x_L)}{f(x_u) - f(x_L)}}$$

* Numerical Analysis :-

* Open Methods :- $\boxed{\text{no interval}}$

1- Simple fixed Point Iteration :-

$f(x) = h(x) = 0$.

$\Rightarrow$ rearrange $\boxed{x = g(x)}$

$\boxed{x_{i+1} = g(x_i)}$ $\qquad x_0 \rightarrow$ initial guess : "given"

$x_1 = g(x_0)$

$x_2 = g(x_1)$.

$x_3 = g(x_2)$.

$f(x) = e^{-x} - x$. $\qquad x = g(x)$. $\boxed{x_0 = 0.5}$

$\boxed{x = e^{-x}}$ أيهم أوفق $\boxed{x = -\ln x}$.

$\left|\frac{dg}{dx}\right|_{x=x_0} > 1 \Rightarrow$ divergent

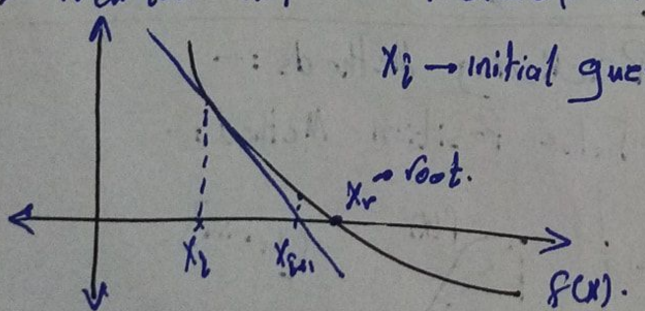
$\left|\frac{dg}{dx}\right|_{x=x_0} < 1 \Rightarrow$ Convergence

$\left|\frac{dg}{dx}\right|_{x=x_0} = 1 \Rightarrow$ Convergence slowly

* Open Methods :-

2- Newton Raphson Method (NR) :-

$x_i \rightarrow$ initial guess.



$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

* Open Methods :-

3- Modified Newton Raphson Method :-

assume that

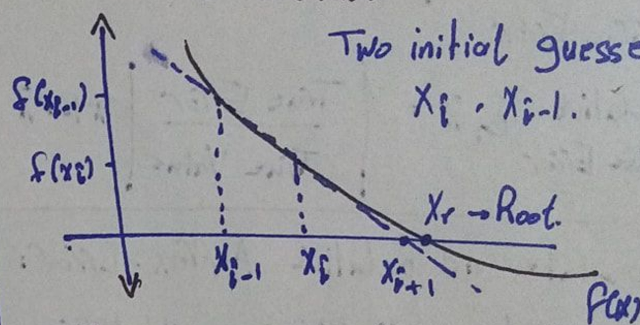
$$U(x) = \frac{f(x)}{f'(x)}$$

$$x_{i+1} = x_i - \frac{U(x)}{U'(x)}$$

$$\boxed{x_{i+1} = x_i - \frac{f.f'}{f'^2 - ff''}}$$

4- Seconed Method :-

Two initial guesses x_i, x_{i-1}.



$f(x)$ is complicated that $f'(x)$ evaluation is hard $\Rightarrow$ Seconed Method is the solution.

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)\,(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}}$$

* Bracketing Methods :-

1. Bisection

$$\boxed{x_r = \frac{x_L + x_u}{2}}$$

2. False Position.

$$\boxed{x_r = x_u - \frac{f(x_u)\,(x_u - x_L)}{f(x_u) - f(x_L)}}$$

* Open Methods :-

1. Simple fixed

$$\boxed{\begin{gathered} f(x) = h(x) = 0 \\ x = g(x) \end{gathered}}$$

2. Newton Raphson

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

3- Modified (NR) :-

$$\boxed{x_{i+1} = x_i - \frac{f.f'}{f'^2 - ff''}}$$

4- Second Method

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)\,(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}}$$

※. Numerical Analysis:-

* Systems of non-linear equations:-

2. Newton Raphson Method:-

given $F_1(X,y)=0$ $\quad F_2(X,y)=0$.

$$J = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}$$

J:- Jocobian matrix.

$$X = \begin{bmatrix} F_1 & \frac{\partial f_1}{\partial y} \\ F_2 & \frac{\partial f_2}{\partial y} \end{bmatrix} \quad Y = \begin{bmatrix} \frac{\partial f_1}{\partial x} & f_1 \\ \frac{\partial f_2}{\partial x} & F_2 \end{bmatrix}$$

$$X_{i+1} = X_i - \frac{|X|_i}{|J|_i}$$

$$Y_{i+1} = Y_i - \frac{|Y|_i}{|J|_i}.$$

* Matrix Operation Review:-

1. Size of matrix:-

$A_{m \times n}$ $\quad m \rightarrow$ number of rows.
$\quad n \rightarrow$ number of columns.

2. Matrix addition and subtraction:-

matrices must have the same size.

$$A_{m\times n} + B_{m\times n} = C_{m\times n}.$$

$$A_{m\times n} - B_{m\times n} = C_{m\times n}.$$

3. Matrix Multiplication:-

$$A_{m\times L} * B_{L\times n} = C_{m\times n}.$$

4. Matrix Inverse (A^{-1}):-

$$A_{m\times n} * A^{-1}_{m\times n} = I_{m\times n}$$

5. Matrix Transpose (A^{T}):-

$$Tra(A_{m\times n}) = A^{T}_{n\times m}$$

* Row Operations:-

Given $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.

1. Row swapping. $R_2 \leftrightarrow R_3$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}.$$

2. Multiplication by factor (K).

$$A = \begin{bmatrix} Ka_{11} & Ka_{12} & Ka_{13} \\ a_{21} & a_{22} & a_{23} \\ Ca_{31} & Ca_{32} & Ca_{33} \end{bmatrix}$$

3. Row addition $(KR_2 + R_3)$:-

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ (Ka_{21}+a_{31}) & (Ka_{22}+a_{32}) & (Ka_{23}+a_{33}) \end{bmatrix}$$

* Kramers Rule:-

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad b = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$$

$$|A| = a_{11}a_{22} - a_{12}a_{21}.$$

$$X_1 = \frac{\begin{vmatrix} b_{11} & a_{12} \\ b_{21} & a_{22} \end{vmatrix}}{|A|} \quad X_2 = \frac{\begin{vmatrix} a_{11} & b_{11} \\ a_{21} & b_{21} \end{vmatrix}}{|A|}$$

* LU Decomposition:-

$$Ax = b.$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}}$$

$$A = LU$$

Solve $Ld = b$ to find (d).

Solve $Ux = d$ to find (X).

*. Numerical Analysis :-

* Matrix inverse Using LU decomp. :-

$A A^{-1} = I \rightsquigarrow$ Identity matrix.

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
\begin{array}{c} \begin{array}{ccc} \underline{x_1} & \underline{x_2} & \underline{x_3} \end{array} \\ \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \end{array}
=
\begin{array}{c} \begin{array}{ccc} \underline{b_1} & \underline{b_2} & \underline{b_3} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}
$$

المصفوفة الأصلية (A) — inverse (A^{-1}) — (I)

Solve :-

$$\left.\begin{aligned} A x_1 &= b_1 \\ A x_2 &= b_2 \\ A x_3 &= b_3 \end{aligned}\right\} \text{by Using LU decomposition.}$$

* Jacobi and Gauss seidel Iteration :-

These method used to Solve linear Sys :-

$$a_{11} X_1 + a_{12} X_2 + a_{13} X_3 = b_1$$
$$a_{21} X_1 + a_{22} X_2 + a_{23} X_3 = b_2 .$$
$$a_{31} X_1 + a_{32} X_2 + a_{33} X_3 = b_3 .$$

$$X_1 = \frac{b_1 - a_{12} X_2 - a_{13} X_3}{a_{11}}$$

$$X_2 = \frac{b_2 - a_{21} X_1 - a_{23} X_3}{a_{22}}$$

$$X_3 = \frac{b_3 - a_{31} X_1 - a_{32} X_2}{a_{33}} .$$

for Jacobi Iteration :-

$$X_{1_{(i+1)}} = \frac{b_1 - a_{12} X_{2_{(i)}} - a_{13} X_{3_{(i)}}}{a_{11}}$$

$$X_{2_{(i+1)}} = \frac{b_2 - a_{21} X_{1_{(i)}} - a_{23} X_{3_{(i)}}}{a_{22}}$$

$$X_{3_{(i+1)}} = \frac{b_3 - a_{31} X_{1_{(i)}} - a_{32} X_{2_{(i)}}}{a_{33}}$$

* Jacobi and Gauss seidel Iteration :-

- for Gauss seidel Iteration :-

نعوض بكل مرة اخر قيمة لـ (X_1 , X_2 , X_3) طلعو معنا .

$$X_{1_{(i+1)}} = \frac{b_1 - a_{12} X_{2_{(i)}} - a_{13} X_{3_{(i)}}}{a_{11}}$$

$$X_{2_{(i+1)}} = \frac{b_2 - a_{21} X_{1_{(i+1)}} - a_{23} X_{3_{(i)}}}{a_{22}}$$

$$X_{3_{(i+1)}} = \frac{b_3 - a_{31} X_{1_{(i+1)}} - a_{32} X_{2_{(i+1)}}}{a_{33}}$$

الفرق بين الـ Jacobi و Gauss بالمعادلتين الثانية والثالثة .

*Ex :-

$$X_1 + 5 X_2 + 4 X_3 = 12 \quad \text{--- (1)}$$
$$6 X_1 + 3 X_2 + 2 X_3 = 17. \quad \text{--- (2)}$$
$$3 X_1 + X_2 + 7 X_3 = 1. \quad \text{--- (3)}$$

- من المعادلة الأولى نحط (X_2) موضوع قانون . لأنه معاملها أكبر من معامل $(X_3 + X_1)$.
- من المعادلة الثانية بحط (X_1) موضوع قانون . لأنه معاملها أكبر من معامل $(X_3 + X_2)$.
- من المعادلة الثالثة . بحط (X_3) موضوع قانون . لأنه معاملها أكبر من معامل $(X_1 + X_2)$.

* Curve fitting :-

* Linear Regression :-

Residual Error (e_i) :-

$$e_i = y_i - y = y_i - (a_0 + a_1 X_i)$$

Total Residual (Sr) :-

$$Sr = \sum (e_i)^2 = \sum (y_i - a_0 - a_1 x_i)^2$$

$$\begin{bmatrix} n & \sum X_i \\ \sum X_i & \sum X_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum X_i y_i \end{bmatrix}$$

n : number of data .

* Numerical Analysis :-

* Curve fitting :-

* Correlation Coefficient (R) :-

$$\boxed{R^2 = \frac{S_t - S_r}{S_t}}, \quad R = \sqrt{R^2}$$

S_t :- Covariance , S_r :- residual error.

1. If $\boxed{R=1} \Rightarrow S_r = 0$
   the Proposed fit is said to be <u>Exact</u>
   (the line will Pass through all the Point).
2. If R approaches 1 $\Rightarrow \boxed{R \simeq 1}$
   the fit is Refered to as Excellent fit.
3. If R approaches 0 $\Rightarrow \boxed{R \simeq 0}$
   the fit is Refered to as Poor.
4. If $\boxed{R=0} \Rightarrow S_t = S_r$
   x and y are Independent.

$$\boxed{R = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}}.$$

* Data linearization :- $\boxed{y = a_0 + a_1 x}$

1. Power :- $y = \alpha x^{B}$
   $\ln y = \ln \alpha + B \ln x$.
   $y^* = a_0 + a_1 x^*$ $\boxed{y^* = \ln y}$, $\boxed{x^* = \ln x}$
2. Exponential :- $y = \alpha e^{Bx}$.
   $\ln y = \ln \alpha + Bx$.
   $y^* = a_0 + a_1 x^*$. $\boxed{y^* = \ln y}$, $\boxed{x^* = x}$
3. Groth Rate :- $y = \frac{\alpha x}{B + x}$
   $\frac{1}{y} = \frac{B}{\alpha x} + \frac{x}{\alpha x}$
   $\frac{1}{y} = \frac{1}{\alpha} + \frac{B}{\alpha}\left[\frac{1}{x}\right] \Rightarrow y^* = a_0 + a_1 x^*$
   $\boxed{y^* = \frac{1}{y}}$, $\boxed{x^* = \frac{1}{x}}$

* Polynomial Regration :-

* Second order Polynomial :-

$$y = a_0 + a_1 x + a_2 x^2.$$

$$m = 2 \Rightarrow n \geq (m+1)$$

عدد المعادلات أكبر من او يساوي عدد المجاهيل.

$$\begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^2 & \sum x^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \\ \sum x^2 y \end{bmatrix}$$

* Multiple linear Regration :-

$$y = a_0 + a_1 x_1 + a_2 x_2$$

$$\begin{bmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 \\ \sum x_2 & \sum x_1 x_2 & \sum x_2^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum x_1 y \\ \sum x_2 y \end{bmatrix}$$

* Interpolating Polynomials :-

1. Newton Divided Differences Interpolating Polynomial :-

x_i	y_i	1st	2nd	3rd
x_0	$y_0 \downarrow b_0$	$\frac{y_1 - y_0}{x_1 - x_0} = a_1 \downarrow b_1$	$\frac{a_2 - a_1}{x_2 - x_0} = c_1 \downarrow b_2$	$\frac{c_2 - c_1}{x_3 - x_0} = d \downarrow b_3$
x_1	y_1	$\frac{y_2 - y_1}{x_2 - x_1} = a_2$	$\frac{a_3 - a_2}{x_3 - x_1} = c_2$	
x_2	y_2	$\frac{y_3 - y_2}{x_2 - x_1} = a_3$		
x_3	y_3			

$$f_3(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2).$$

$$f_4(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2) + b_4(x - x_0)(x - x_1)(x - x_2)(x - x_3).$$

* Numerical Analysis :-

* Interpolating Polynomials :-

2- Lagrange Interpolation Polynomial :-

$$f_n(x) = \sum_{i=0}^{n} L_i(x) * f(x_i).$$

↳ Lagrange Interpolation Polynomial

$$L_i(x) = \prod_{j=0}^{n} \frac{(x - x_i)}{(x_i - x_j)}.$$

↳ Lagrange Polynomial.

Ex :- Find $f_3(2.75)$.

X	x_0 0	x_1 1	x_2 2.5	x_3 3
Y	2	5	9	11

$$L_0 = \left(\frac{x-1}{0-1}\right)\left(\frac{x-2.5}{0-2.5}\right)\left(\frac{x-3}{0-3}\right)$$

$$L_1 = \left(\frac{x-0}{1-0}\right)\left(\frac{x-2.5}{1-2.5}\right)\left(\frac{x-3}{1-3}\right)$$

$$L_2 = \left(\frac{x-0}{2.5-0}\right)\left(\frac{x-1}{2.5-1}\right)\left(\frac{x-3}{2.5-3}\right)$$

$$L_3 = \left(\frac{x-0}{3-0}\right)\left(\frac{x-1}{3-1}\right)\left(\frac{x-2.5}{3-2.5}\right)$$

$L_0(2.75) = 0.0145$.

$L_1(2.75) = -0.05729$.

$L_2(2.75) = 0.6416$.

$L_3(2.75) = 0.4010$.

$f_3(2.75) = 0.0145(2) + -0.05729(5)$
$+ 0.6416(9) + 0.4010(11) = \underline{\underline{9.929}}$

---

*Ex :- e.v

X	Y	1st	2nd
1	2	$\frac{7-2}{2.25-1} = 4$	$\frac{2.6667-4}{3-1} = 0.66665$
2.25	7	$\frac{9-7}{3-2.25} = 2.667$	
3	9		

$f_2(x) = 2 + 4(x-1) + 0.66665(x-1)(x-2.25)$

$f_2'(x) = 4 + 0.6665(2x - 3.25)$.

$f_2''(x) = (2)\,0.66665$.

* Numerical Differentiation :-

1- Equally Spaced Data :-

في هذا النوع المسافة بين كل نقطة والثانية
متساوية وتساوي (h)

$x_{i+1} - x_i = h$

h :- step size.

Other Derivatives :-

1- forward difference

عندك نقطة ونقطة بعدها وصورة النقطة نفسها.

2- Backward difference.

عندك نقطة ونقطة قبلها وصورة النقطة نفسها

3- Centered difference

عندك نقطة قبل ونقطة بعد وما عندك النقطة نفسها.

* for 1st derivative with $O(h)$:-

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} \quad \text{forward.}$$

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} \quad \text{Backward.}$$

* for 1st derivative with $O(h^2)$:-

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}$$
forward.

$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$$
Backward.

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$
Centered.

2- Unequally Spaced Data :-

في هذا النوع المسافة بين كل نقطة والثانية
مش متساوية ☹ .

Use give data to create an Interpolating Polynomial. then use It's derivatives to estimate the required derivatives.

Ex : Estimate $f'(2.25)$

X	1	2.25	3
Y	2	7	9

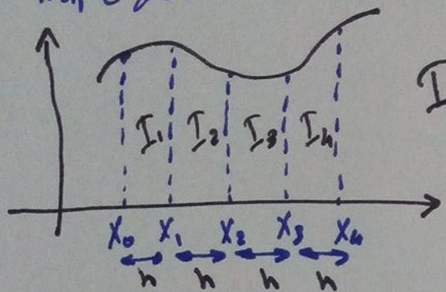
* Numerical Analysis :-

* Numerical Integration :-

* Numerical Integration methods :-

Equally spaced Data :-

1. Trapezoidal Rule.



$I = \sum_{i=1}^{4} I_i.$

n :- number of segment.

$$h = \frac{\Delta x}{n} = \frac{x_f - x_0}{n}.$$

$$I = \frac{h}{2}\left(f(x_0) + 2\sum_{i=1}^{n-1}(f(x_i)) + f(x_n)\right)$$

*Ex:- find $\int_2^5 e^{\cos x}\,dx$ $2 \cdot 3 \cdot 4 \cdot 5$, $n=3$

$$h = \frac{\Delta x}{n} = \frac{5-2}{3} = 1$$

$$I = \frac{h}{2}\left[f(2) + 2f(3) + 2f(4) + f(5)\right].$$

$$= \frac{1}{2}\left[2.71 + 5.42 + 5.42 + 2.7\right].$$

$I = 8.12$ #

2. Simpson's Rule :-

a. $\frac{1}{3}$ Simpson's Rule :-

$$h = \frac{\Delta x}{n} = \frac{x_f - x_i}{n}.$$

n :- number of sigment.

$$I = \frac{h}{3}\left(f(x_0) + 4\sum_{i=1,3,5\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,6\ldots}^{n-2} f(x_i) + f(x_n)\right)$$

for $n = 2$

$$I = \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right)$$

* Numerical Integration methods :-

2. Simpson's Rule :-

b. $\frac{3}{8}$ Simpson's Rule :-

For this rule the minimum number of segment is $\boxed{n=3}$

$$h = \frac{\Delta x}{n} = \frac{x_f - x_i}{n}$$

$$I = \frac{3h}{8}\left(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right)$$

for n intervals :-

$$I = \frac{3h}{8}\left(f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4) + 3f(x_5) + 2f(x_6) - \ldots + f(x_n)\right).$$

* Ordinary differential Equation :-

$$\frac{dy}{dx} = f(x,y) \qquad y(x_0) = y_0.$$

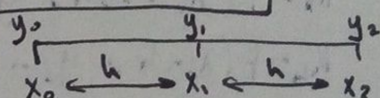
* Numerical methods :-

1. Euler's Methods :-

$$\frac{dy}{dx} = f(x,y) \qquad y(x_0) = y_0$$

$$\boxed{y_{i+1} = y_i + f(x_i, y_i)\,h}$$

h :- step size.



2. Runge-Kutta methods :-

a. Second order Runge Kutta method

1. Heun's Methods :-

$$\boxed{y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)\,h}$$

$k_1 = f(x_i, y_i)$.

$k_2 = f(x_i + h, y_i + k_1 h)$.

2. Mid Point Method

$$\boxed{y_{i+1} = y_i + k_2 h}$$

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + 0.5h, y_i + 0.5k_1 h)$

* Numerical Analysis :-

* Ordinary differential Equations :-

* Numerical Methods :-

2- Runge-Kutta methods :-

a- Second order Runge Kutta method

3- Ralston's method :-

$$\boxed{y_{i+1} = y_i + \frac{1}{3}(k_1 + 2k_2)h}$$

$k_1 = f(x_i , y_i)$.

$k_2 = f(x_i + \frac{3}{4}h , y_i + \frac{3}{4}k_1 h)$.

b- Third order Runge Kutta method

$$\boxed{y_{i+1} = y_i + \frac{1}{6}(k_1 + 4k_2 + k_3)}$$

$k_1 = f(x_i , y_i)$.

$k_2 = f(x_i + 0.5h , y_i + 0.5k_1 h)$.

$k_3 = f(x_i + h , y_i - k_1 h + 2k_2 h)$.

c- fourth order Runge Kutta method

$$\boxed{y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)}$$

$k_1 = f(x_i , y_i)$.

$k_2 = f(x_i + 0.5h , y_i + 0.5k_1 h)$.

$k_3 = f(x_i + 0.5h , y_i + 0.5k_2 h)$.

$k_4 = f(x_i + h , y_i + k_3 h)$.