# تحليل عددي

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للطالبة المبدعة

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إرادة - ثقة - تغيير

# Errors

"بسم الله الرحمن الرحيم"

① True error ($E_T$) ⇝ Absolute $E_T$ = True value − Approx value

② Approx error ($E_a$) ⇝ $E_a$ = New value − old value

③ realtive True error ($\epsilon_T$) ⇝ $\epsilon_T = \left|\dfrac{E_T}{\text{True value}}\right| * 100\%$

%$\epsilon_a\downarrow$ = converge( تحل ) , %$\epsilon_a\uparrow$ = يوقف الحل ومالازم نسبة الخطأ تزيد.

④ realitive approx error ($\epsilon_a$) ⇝ $\epsilon_a = \left|\dfrac{E_a}{\text{new value}}\right| * 100\%$

* accepect error ⇝ ($E_s$, $\epsilon_s$)

*Ex:- we used a method to determine the +ve root For $f(x) = x^2 - 4x - 5$ and resalted in the Following iteriotion

| i | xr |
|---|---|
| 1 | 3.9 |
| 2 | 4.6 |
| 3 | 4.8 |
| 4 | 4.94 |

calculate $E_T$, $\epsilon_T$, $E_a$, $\epsilon_a$ after the 4th iteration.

*$\epsilon_s$ ⇝ accepected error.

* Sol :- (True = root Fun)

① $E_T = x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$, ~~$x = -1$~~ X

⇝ $E_T = 5 - 4.94 = 0.06$

② $\epsilon_T = \left|\dfrac{0.06}{5}\right| * 100\% = 1.2\%$

③ $E_a = 4.94 - 4.83 = 0.11$

④ $\epsilon_a = \left|\dfrac{0.11}{4.94}\right| * 100\% = 2.2\%$

*(Taylor series):-

$5*4*3*2*1 \leftarrow 5! \leftarrow n!$

مضروب $(0!) = 1$

$F^{(n)}$ تدل على المشتقة

*f(x) = .....

*f(1) = 4...

f(3) = ؟؟؟ كل ما زاد عدد ال Terms كل ما كان الجواب أدق

ما ينقص منا ال Terms يعني error truncation.

$$f(x) = \sum_{i=0}^{n} \frac{(x-x_0)^i f^{i}(x)}{z!}$$

* مضروب الصفر

كل شغلنا ال Rad. $(0!) = 1$

$i \approx 0$

$$f(x) = \frac{(x-x_0)^0 f^0(x_0)}{0!} = f(x) = f(x_0)$$ zero order equation عند $\boxed{i=0}$ *ملاحظة:-

$i \approx 1$ $f(x) = f(x_0) + (x-x_0) f'(x_0) \rightsquigarrow 1^{st}$ order equation

$i \approx 2$ $f(x) = f(x_0) + (x-x_0) f'(x_0) + \frac{(x-x_0)^2 f''(x_0)}{2}$

* يطلب مني أشتق 5 مرات ← بشتق بس 4.

Secound order equation

*Ex:- Expend f(x) = sin (x) in atglar series of

if we it to approximate the value of sin (2)

using the value of f(x) of its derivatives at

0 using 4 significant figures. the value

of f(x) of its $1^{st}$ three derivetives at o?

*Sol:- $\sin(x_0) = \sin(0) = 0$

$f'(x) = \cos(0) = 1$

$f''(x) = \sin(0) = 0$

$f'''(x) = \cos(0) = 1$

$$f(x) = 0 + (2-0)1 + \frac{(2-0)^2 \, 0}{2} + \frac{(2-0)^3 * -1}{6} = 0.6667$$

$0.0348 \rightsquigarrow \boxed{0.909}$ ✓

* لازم نحولها rad لحتى نشوف أي اقتران مثلثي (sin cos tan...)

"بسم الله الرحمن الرحيم"

*Ex:- $f(x) = \sin(x) + x'$

$f(\frac{\pi}{3}) \quad x_0 = \frac{\pi}{4}$

* $f(x_0) = f(\frac{\pi}{4}) = 1.324$

$f(x_0)' = \cos(x) + 2x.$

$f'(x_0) = f'(\frac{\pi}{4}) = 2.278$

$f''(x_0) = -\sin(x) + 2$

$f''(\frac{\pi}{4}) = 1.293 \quad f'''(x) = -\cos(x) = -\cos(\frac{\pi}{4}) = -0.707$

$$f(x) = \varepsilon f(x_0) + (x - x_0) f'(x_0) + \frac{(x-x_0)^2}{2} f''(x_0)$$

$$+ \frac{(x-x_0)^3}{6} f'''(x)$$

$$f(\frac{\pi}{3}) = 1.324 + (\frac{\pi}{3} - \frac{\pi}{4}) 2.278 + \frac{(\frac{\pi}{3} - \frac{\pi}{4})^2}{2} 1.293$$

$$+ \frac{(\frac{\pi}{3} - \frac{\pi}{4})^3}{6} -0.707 = 1.849$$

$\boxed{1.962}$

0.909 True

1.953

0.6667 Approx

$E_T = 0.2423 \rightarrow$ remainder $(R_n)$.

*(سؤال مهم):

* $f(x) = x^4 - 3x^2 + 1$

$x_0 = 1 \quad , \quad x = 2$

$f(x) = f(x_0) = -1$

$f'(x) = 4x^3 - 6x = R'(1) = -2$

$f''(x) = 12x^2 - 6 = 6$

$f'''(x) = 24x = 24$

$f(2) = -1 + -2 + \frac{6}{2} + \frac{24}{6} = 4$

$\boxed{R_n = 5 - 4 = 1}$ *

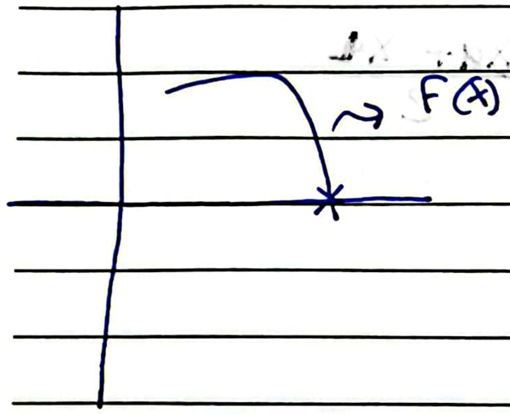
دلالة على انه ما وصلت لل exect.

التاريخ ٥ /٣/ ١٠١ الموضوع : الثلاثاء

"بسم الله الرحمن الرحيم"

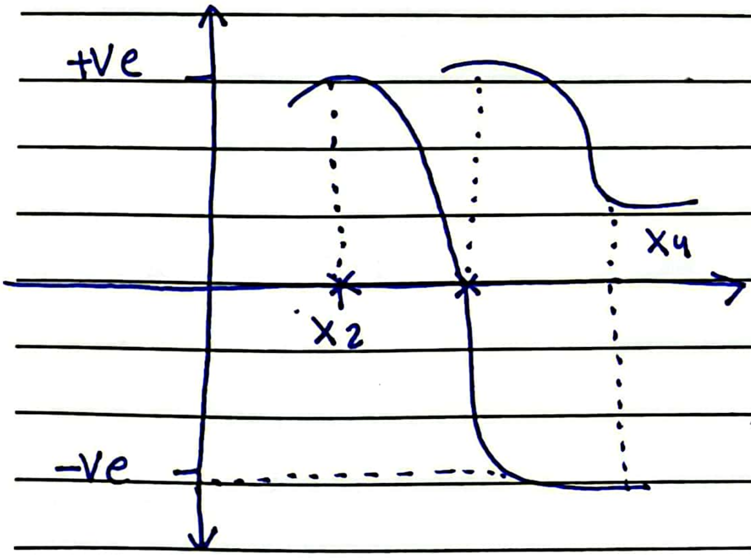
# Roots of the Function

$f(x) = 2x^2 + 5x - 4 = 0$

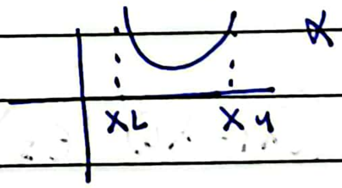


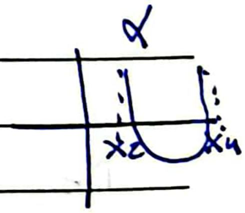

① Brachefing methods
- Bisecfion method
- false position.

② open methods
- fixed point itheration
- newton rephson method
- secound method



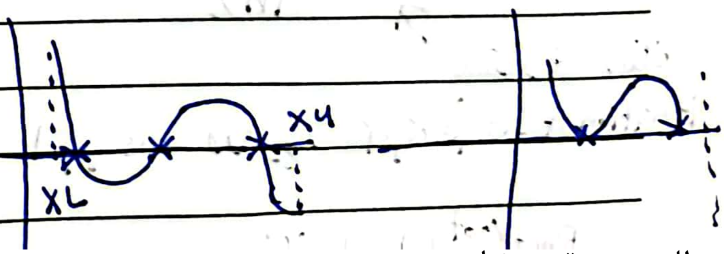

① two intid guses $x_2, x_4$, they should be in bofwear.




② $f(1/4) * f(1/4) < 0$

③ contieoues function.

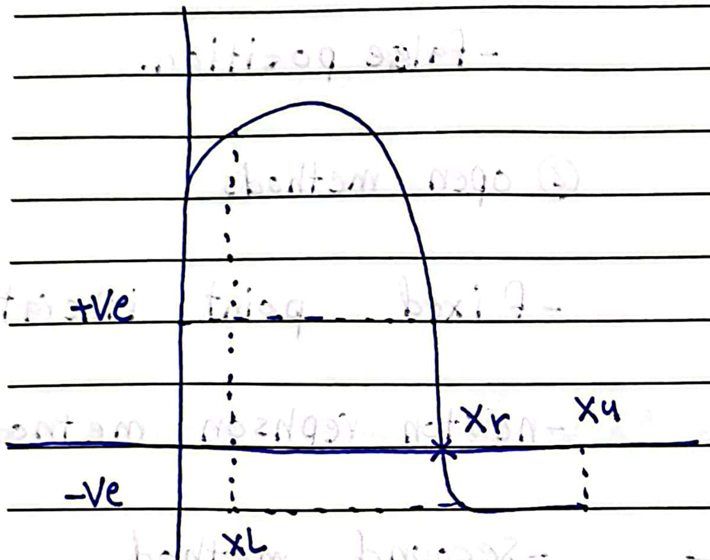


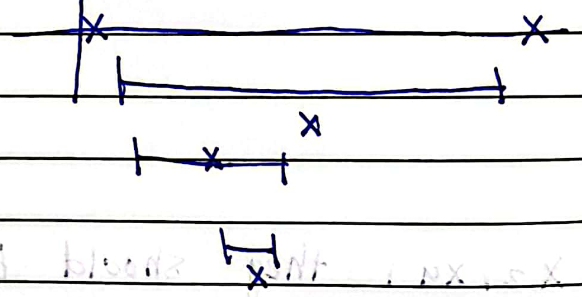
① Roots of the Function

① Bisection method

$\rightarrow f(x_L) * f(x_u) \leq 0$ $\qquad x_r = \frac{x_u + x_L}{2}$

$x_n \qquad f(x_r) \rightarrow 0$ $\qquad x_{r_2} = \frac{x_u + x_L}{2}$



$n = \frac{\ln((x_u - x_L)/E_s)}{\ln 2}$ ← فقط بستخدم بال Bisection

بستخدمه عشان

* تعرف n إلي أكبر عدد ممكن أعرف كم عدد الiteration

عشان أوصل للBest root.

*Ex :- Find an estimate For $\sqrt[4]{25}$ within $\varepsilon_s$ 0.05 From the actual root starting with $[2, 3]$ ($x_2$, $x_4$) using 5 Figure placed.

*Sol :- $x = \sqrt[4]{25}$ $x^4 = 25 \rightsquigarrow x^4 - 25 = 0$ R(T).

$f(x_2) = f(2) = -9$ $-ve$ $f(x_4) = f(3) = 56$ $+ve$.

$i = 1 \rightsquigarrow x_{ri} = \frac{2+3}{2} = 2.5$

$i = 2 \rightsquigarrow f(x_{R1}) = f(2.5) = 39$ $+ve$

$x_{r_2} = \frac{2 + 2.5}{2} = 2.25.$ $\varepsilon_a = 2.25 - 2.5 = 0.25 > 0.05$

| i | XL | Xu | Xr | f(XL) | f(Xu) | f(Xr) | Error |
|---|---|---|---|---|---|---|---|
| | | | 2.5 | -9 | 56 | 14.06 | |
| 1 | 2 | 3 | | | | | |
| | | | 2.25 | -9 | 14.06 | 0.628 | 0.25 |
| 2 | 2 | 2.5 | | | | | |
| | | | 2.125 | -9 | 0.628 | -4.609 | 0.125 |
| 3 | 2 | 2.25 | | | | | |
| | | | 2.1875 | -4.609 | 0.628 | -2.102 | 0.0625 |
| 4 | 2.125 | 2.25 | | | | | |
| 5 | 2.1875 | 2.25 | 2.218 | -2.1022 | 0.628 | -0.765 | 0.03 < 0.0 |
| | | | | | | | Stop |

"بسم الله الرحمن الرحيم"

$*E_s = \frac{X_u - X_L}{2^n}$   $X_u$: X upper

القانون هاد بس لل ← $X_L$: X Lawar

accepected error   n: number of iteration

ولل بايسكشن مشود بس

$n = \frac{\ln\left(\frac{3-2}{0.05}\right)}{\ln(2)} = 4.32$

$n = \frac{\ln\left(\frac{X_u - X_L}{E_1}\right)}{\ln(2)}$

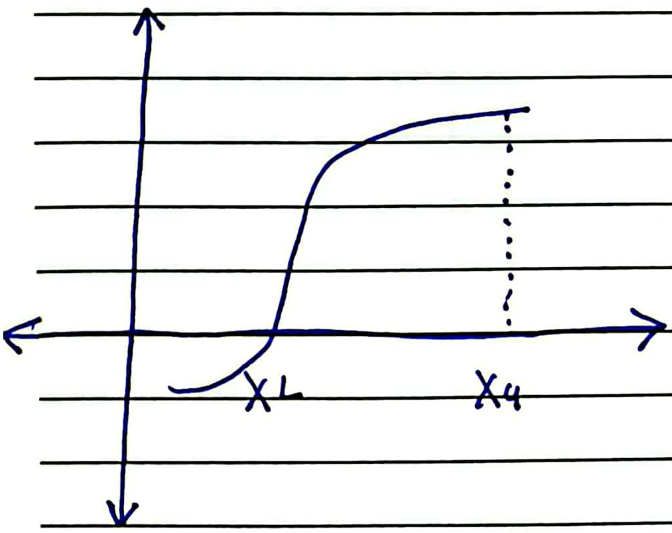
$* E_s = 0.03\%$ , $E_s = 0.05$

$*Ex:- f(x) = \sin(x) - x^2$

$X_L = 0.5$ , $X_u = 1$   $\boxed{E_s = 0.1\%}$

$f(0.5) = 0.229$

$f(1) = -0.159$




$X_u = \frac{0.5+1}{2} = 0.75 \rightarrow f(0.75) = 0.119$

$i = 2$

$X_r = \frac{0.75+1}{2} = 0.875$

$* E_a = \frac{0.875 - 0.75}{0.875} * 100\% = 14.2\%$

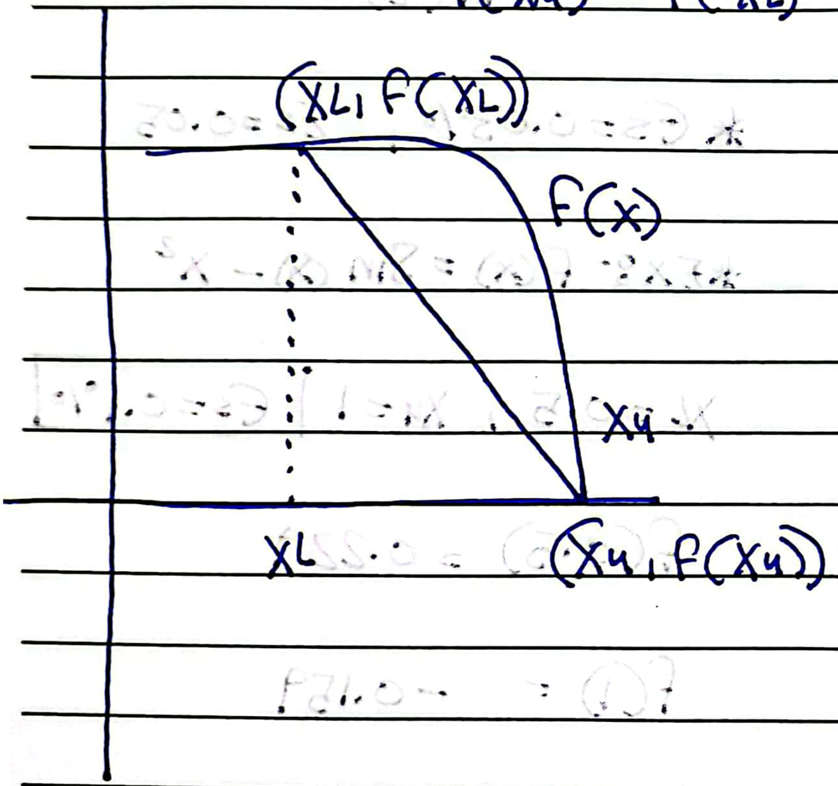
$i=3$ $i=4$ $i=10$

$X_L = 0.975$ $X_u = 0.9063$ $X_r = 0.8764$

* القانون لازم اعوض فيه $E_s$ مش $\epsilon$ إبسيلون.

② false position method

$$X_r = X_u - \frac{f(X_u)(X_u - X_L)}{f(X_u) - f(X_L)}$$




0.795
0.864

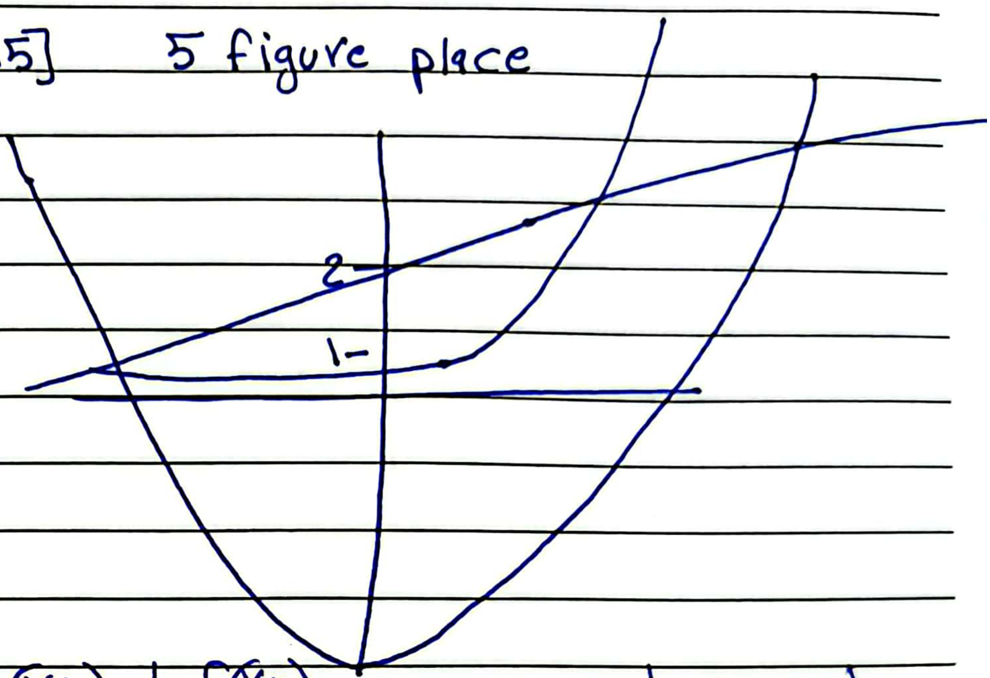
*Ex:- Find the intersection point $f_1(x) = e^x$

$f_2(x) = x+2$ [0, 1.5] 5 figure place

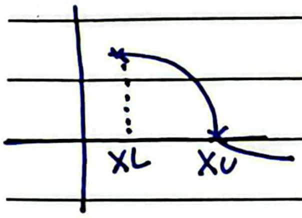
Error < 0.02

$e^x = x+2$

$f(x) = e^x - x - 2 = 0$




| i | XL | Xu | f(XL) | f(Xu) | Xr | f(Xr) | Error |
|---|---|---|---|---|---|---|---|
| | 0 | 1.5 | 1 | 0.981 | | -0.6245 | — |
| 1 | | | | | 0.75+5 | | |
| | 0.75+5 | 1.5 | -0.6245 | 0.981 | | | |
| 2 | | | | | 1.04631 | | |




بالرفع القيمي الـ (False) أنر ع من

الـ (position).

*open methods :-

① Simple Fixed poist iteretion

$f(x) \to$ " " "

$x \to$ "

$\to$ rearrange $f(x)$

$x_{i+1} = g(x_i)$

$f(x) = \sin(x) - x = 0$

$x_{i+1} = \sin(x_i)$

$i=0$

$i=1 \to f(x) = x^2-5x+4=0$

* $x_{i+1} = \sqrt{5x_i-4}$ .... ①  $\frac{5x}{5} = \frac{x^2+4}{5}$ * $x_{i+1} = \frac{x_i^2+4}{5}$ ... ②

* $x^2-5x+4=0 \to x(x-5)+4=0 \to x_{(i+1)} = \frac{-4}{x+5}$ ... ③

$f(x) = \sqrt{\cos(x)} = 0 \to x_{i+1} = \sqrt{\cos(x)} + x_i$

* Ex:- $\sin x - x^2 = 0$  $\boxed{x_0 = 1}$

$0.1\% \, \varepsilon_s$

$x = \sqrt{\sin(x_i)}$

$i=0 \to x_1 = \sqrt{\sin(1)} = 0.9173$

← تحويل (Rad).

$$*E_a = \left| \frac{0.9173-1}{0.9173} \right| * 100\% = 9.01\%$$

$$i=1 \longrightarrow x_2 = \sqrt{\sin(0.9173)} = 0.891$$

$$\left| \frac{0.891 - 0.9178}{0.891} \right| * 100\% = 2.95\%$$

$$i=2 \longrightarrow x_3 = \sqrt{\sin(0.891)} = 0.881$$

$$E_a = \left| \frac{0.891 - 0.881}{0.881} \right| * 100\% = 1.04\%$$

0.8770

$\frac{dy}{dx} \leq 1 \rightarrow$ will converge For any value.

$\frac{dy}{dx} > 1 \rightarrow$ will diverge (ما في حل)

$\frac{dy}{dx} = 1 \rightarrow$ will converge slowly.

$f(x) = \ldots$

$x_{in} = g(x)$.

$x, \frac{dy}{dx} < 1$ will converge , $>1$ diverge, $=1$ converge slowly.

*Ex:- $f(x) = -x^2 + 1.5x + 2.5$

$x_0 = 5$, $\epsilon_s = 0.05\%$

مرفوضة

$x_{i+1} = \frac{x_i^2 - 2.5}{1.8}$ .... ①

$-x^2 + 1.8x + 2.5 = 0$

$x^2 = 1.8x + 2.5$

$\frac{1.8x}{1.8} = \frac{x^2 - 2.5}{1.8}$

$x_{i+1} = \sqrt{1.8x_i + 2.5}$ ... ②

* $\frac{dy}{dx} = \frac{2x_i}{1.8} = \frac{10}{1.8}$

$-x^2 + 1.8x + 2.5 = 0$

$x(1.8 - x) + 2.5 = 0$

صورة مرفوضة $\boxed{5.555 > 1}$

$x = \frac{2.5}{x - 1.8}$ .... ③

$\frac{dy}{dx} \rightarrow \frac{1.8}{2\sqrt{1.8x + 2.5}} = \frac{1.8}{2\sqrt{1.8x + 2.5}} = 0.205$ will converge

$i = 0, x_1 = \sqrt{1.8(5) + 2.5} = 3.391$

$\epsilon_a = \left| \frac{3.391 - 5}{3.391} \right| * 100\% = 47.4\%$

$i = 1, x_i = \sqrt{1.8(3.391) + 2.5} = 2.932$

$\epsilon_a = \left| \frac{2.972 - 3.391}{2.912} \right| * 100\% = 15.6\%$

$i = 2, x_3 = \sqrt{1.8(2.912) + 2.5} = 2.789$

$\epsilon_a = \left| \frac{2.789 - 2.932}{2.769} \right| * 100\% = 5.1\%$

*newton rephson method :-

$$*X_{i+1} = X_i - \frac{f(x_i)}{f'(x_i)}$$

*Ex:- $f(x) = x^2 - 1 \rightsquigarrow X_{i+1} = X_i - \frac{X_i^2 - 1}{2X_i}$

$f(x) = X^2 + 2x + 4$

$$X_{i+1} = X_i - \frac{X_i^2 + 2X_i + 4}{2X_i + 2}$$

$$* X_{i+1} = X_i - \frac{f(x_i)}{f'(x_i)}$$

$$* Ex:- f(x) = \sin x - x^2$$

$$X_0 = 1 \quad , \quad 0.1\% = \epsilon_s$$

$$i=0 \rightsquigarrow X_1 = 1 - \frac{\sin(1) - (1)^2}{\cos(1) - (2*1)} = 0.8913$$

$$\epsilon_a = \left| \frac{0.8913 - 1}{0.8913} \right| * 100\% = 12.2\%$$

$$i=1 \rightsquigarrow X_2 = 0.8915 - \frac{\sin(0.8913) - (0.895)}{\cos(0.8915) - (2*0.8913)} = 0.8770$$

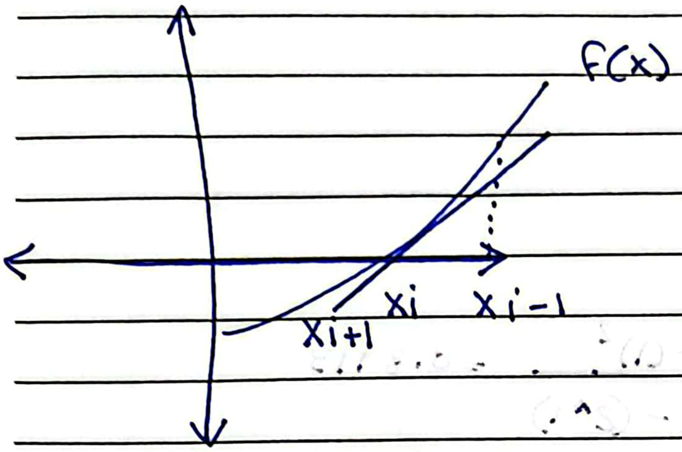
$$\%1.7 = 100\% - \frac{0.8770 - 0.8913}{0.877}$$

$$i=2 \rightsquigarrow X_3 = 0.877 - \frac{\sin(0.877) - (0.877)^2}{\cos(0.877) - (2*0.877)} = 0.8767$$

$$\epsilon_a = \left| \frac{0.8767 - 0.877}{0.8767} \right| * 100\% = 0.034\% < 0.1\%$$

الموضوع:                                                     التاريخ:   /   / ٢٠١

*Secent method :- given $f(x), X_0, X_{i+1}$ :-




$$*X_{i+1} = X_i - \frac{f(X_i)(X_{i+1} - X_i)}{f(X_{i-1}) - f(X_i)}$$

*Ex:- $f(x) = \sin(x) - x^2$

$X_{-1} = 1, X_0 = 2, \epsilon_s = 0.1\%$

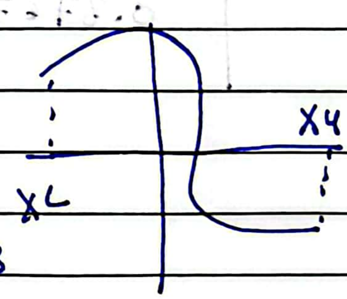
*$i=0 \rightsquigarrow X_1 = 2 -$

$f(2) = \sin(2) - 4 = -3.09$

$f(1) = \sin(1) - 1 = -0.158 = 0.946$

$\epsilon_a = 0.946 \rightsquigarrow \epsilon_a = \left|\frac{0.946 - 2}{0.946}\right| 100\% = 111.4\%$

$*i=1 \rightsquigarrow X_2 = 0.946 - \frac{-0.083(2 - 0.946)}{-3.09 + 0.083}$




$f(0.946) = \sin(0.946) - (0.946)^2 = -0.083$

$f(2) = -3.09$ $*X_2 = 0.9167$ $*\epsilon_a = \left|\frac{0.9167 - 0.946}{0.9167}\right| 100\% = 3.2\%$

*System of non linear equations:-

$10x^3 + 3y^{13} = 23 \rightarrow f_1(x,y)$

$12x^2 + y^3 = 5 \rightarrow f_2(x,y)$

* صعب نستخدم الحذف والتعويض

نلجأ لطريقة newton-Raphson method.

$\frac{df_1}{dx} = 30x^2$

$J$ = Jacabian matrix (مصفوفة الـJ)

$$J = \begin{vmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{vmatrix} \qquad x = \begin{vmatrix} f_1 & \frac{df_1}{dy} \\ f_2 & \frac{df_2}{dy} \end{vmatrix}$$

$$y = \begin{vmatrix} \frac{df_1}{dx} & f_1 \\ \frac{df_2}{dx} & f_2 \end{vmatrix}$$

$$*\ x_{i+1} = x_i - \frac{|x_i|}{|J_i|}$$

$$*\ y_{i+1} = y_i - \frac{|y_i|}{|J_i|}$$

*Ex:- Find the roots $(x_r, y_r)$ For $x^2 + xy = 10$

$y + 3xy^2 = 57$  $\boxed{x_0 = 1,\ y_0 = 2}$

*مهم:- لازم نعيد ترتيبهم بحيث يصير = صفر

$x^2 + xy - 10 = 0$

$y + 3xy^2 - 57 = 0$

$$J = \begin{vmatrix} 2x+y & x \\ 3y^2 & 1+6xy \end{vmatrix} \qquad x = \begin{vmatrix} x^2+xy-10 & x \\ y+3xy^2-57 & 1+6xy \end{vmatrix}$$

$$y = \begin{vmatrix} 2x+y & x^2+xy-10 \\ 3y^2 & y+3xy^2-57 \end{vmatrix}$$

$$* \; i=0 \Rightarrow x_0 = \begin{vmatrix} -7 & 1 \\ -43 & 13 \end{vmatrix} \longrightarrow |x| = (-7 * 13) - (1 * 43) = -48$$

$$* \; y_0 = \begin{vmatrix} 4 & -7 \\ 12 & -43 \end{vmatrix} \longrightarrow |y_0| = -88$$

$$kj_0 = \begin{vmatrix} 4 & 1 \\ 12 & 13 \end{vmatrix} \longrightarrow |J_0| = 40$$

$$x_1 = x_0 - \frac{-48}{40} = 2.2$$

$$y_1 = 2 - \frac{-88}{40} = 4.2$$

* i=1 ~

$$*J_1 = \begin{vmatrix} 8.6 & 2.2 \\ 52.92 & 56.44 \end{vmatrix} \rightarrow 368.96 \qquad *X_1 = \begin{vmatrix} 4.08 & 2.2 \\ 63.624 & 56.44 \end{vmatrix} = 90.3$$

$$*y_1 = \begin{vmatrix} 8.6 & 4.08 \\ 52.92 & 63.624 \end{vmatrix} = 331.25$$

$$*X_2 = 2.2 - \frac{40.3}{368.96} = 1.955 \qquad *y_2 = 4.2 - \frac{33.25}{368.96} = 3.30$$

# يا رب.

# *matrix operations:-

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 4 & 1 \\ 5 & 2 & 0 \end{bmatrix}$$

→ * إسم المصفوفة لازم يكون (A) كابيتال لتي.

3 * 3

↓ rows

↓ colnes

$$*A_{2*2} = \begin{vmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 2 \end{vmatrix} \qquad *B_{2*2} = \begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix}$$

$$*C_{2*2} = A + B, \quad C_{2*2} = \begin{vmatrix} 3 & 6 \\ 2 & 7 \end{vmatrix}$$

$$*A_{2*2} * B_{2*2} = C_{2*2}$$

$$*C_{32} = (1*5) + (2*4) = 13$$

$$C = \begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \end{vmatrix} = \begin{vmatrix} 4 & 14 \\ 6 & 12 \\ 5 & 13 \end{vmatrix}$$

$$A_{3*2} * B_{2*3} = C_{3*3}$$

$$A_{3*3} * A^{-1}_{3*3} = I_{3*3} \quad \text{iden taily matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

*matrix tranypose

* $A_{2*3} = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 3 & 5 \end{vmatrix}$   $X = \begin{vmatrix} 1 \\ 2 \\ 0 \end{vmatrix} \rightsquigarrow X^T = \begin{vmatrix} 1 & 2 & 0 \end{vmatrix}$

* $A_{3*2} = \begin{vmatrix} 0 & 1 \\ 1 & 3 \\ 2 & 5 \end{vmatrix}$

* $2X_1 + X_2 + 4X_3 = 5$

$X_1 - 2X_2 + 3X_3 = 17$

$3X_1 + 4X_2 - 2X_3 = 25$

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & -2 & 3 \\ 3 & 4 & -2 \end{bmatrix} * \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix} = \begin{vmatrix} 5 \\ 17 \\ 25 \end{vmatrix}$$

*rows oper tions =

(1) row swapping        $*2R_1 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \longleftrightarrow \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix} \rightarrow 2R_1 \begin{bmatrix} 4 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$$

*Gaws elimntion *موضوع مهم جداً :-

$4X_1 + 2X_2 + 3X_3 = 5$

$10X_1 + X_2 + 4X_3 = 2$

$5X_1 + 3X_2 + 5X_3 = 1$

$$\begin{array}{ccccc} \text{coff} & & \text{Variables} & & \text{reselty} \\ \begin{bmatrix} 4 & 2 & 3 \\ 10 & 1 & 4 \\ 5 & 3 & 5 \end{bmatrix} & & \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} & = & \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} \\ A & & X & & b \end{array}$$

$\boxed{Ax=b}$

⇒ Augmcuted matrix $[A \vdots b]$.

$$\left[\begin{array}{ccc:c} 4 & 2 & 3 & 5 \\ 10 & 1 & 4 & 2 \\ 5 & 3 & 5 & 1 \end{array}\right]$$

⇒ back ward Fubstitution

$0X_1 + 0X_2 + \boxed{5*3 = 1}$

$0X_1 + \boxed{1X_2} + 4\boxed{X} = ?$

*Ex: $x_1 + x_2 + 6x_3 = 7$

$-x_1 + 2x_2 + 9x_3 = 2$

$x_1 - 2x_2 + 3x_3 = 10$

$$\begin{bmatrix} 1 & 1 & 6 \\ -1 & 2 & 9 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 10 \end{bmatrix}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 6 & 7 \\ -1 & 2 & 9 & 2 \\ 1 & -2 & 3 & 10 \end{array}\right]$$

$-\left(\frac{\text{no tobe zero}}{\text{pivot}}\right) R_1 + R_2 = R_2$

ركيزة

$-1 \quad 2 \quad 9 \quad : \quad 2$

$-\left(\frac{-1}{1}\right) R_1 + R_2 \Rightarrow R_2$

$0 \quad 3 \quad 15 \quad : \quad 9$

$$\left[\begin{array}{ccc:c} 1 & 1 & 6 & 7 \\ 0 & 3 & 15 & 9 \\ 1 & -2 & 3 & 10 \end{array}\right]$$

$1 \quad -2 \quad 3 \quad 10$

$-\left(\frac{1}{1}\right) R_1 + R_3 \Rightarrow R_3$

$0 \quad -3 \quad -3 \;\vdots\; 3$

$$\approx \left[\begin{array}{ccc:c} 1 & 1 & 6 & 7 \\ 0 & 3 & 15 & 9 \\ 0 & -3 & -3 & 3 \end{array}\right]$$

$0 \quad -3 \quad -3 \;\vdots\; 3$

$-\left(\frac{-3}{3}\right) R_2 + R_3 \Rightarrow R_3$

$0 \quad 0 \quad 12 \;\vdots\; 12$

$$\approx \left[\begin{array}{ccc:c} 1 & 1 & 6 & 7 \\ 0 & 3 & 15 & 9 \\ 0 & 0 & 12 & 12 \end{array}\right] \Rightarrow$$

$0 X_1 + 0 X_2 + 12 X_3 = 12$

$\boxed{X_3 = 1}$

$0 X_1 + 3 X_2 + 15(1) = 9$

$\boxed{X_2 = -2}$

$\Rightarrow X_1 + (-2) + 6(1) = 7$

$$\begin{bmatrix} 3 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$x_2 + 6x_3 = 7$

$-x_1 + 2x_2 + 9x_3 = 2$

$x_1 - 2x_2 + 3x_3 = 10$

$$\left[\begin{array}{ccc|c} 0 & 1 & 6 & 7 \\ -1 & 2 & 9 & 2 \\ 1 & -2 & 3 & 10 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} -1 & 2 & 9 & 2 \\ 0 & 1 & 6 & 7 \\ 1 & -2 & 3 & 10 \end{array}\right]$$

① Divot=0

*Ex: $-2x_2 + 3x_3 = 8$

$4x_1 + 6x_2 + 7x_3 = -3$

$2x_1 + x_2 + 6x_3 = 5$

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 6 & 7 & -3 \\ 0 & 2 & 3 & 8 \\ 2 & 1 & 6 & 5 \end{array}\right]$$

$\left(-\frac{2}{4}\right) R_1 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 2 & 1 & 6 & 5 \\ 0 & -2 & 2.5 & 6.5 \end{array}\right]$$

$-\left(\frac{-2}{2}\right) R_2 + R_3 \Rightarrow R_5$

$0 \; -2 \quad 2.5 \; : \; 6.5$

$0 \; -0 \quad 5.5 \; : \; 14.5$

$$\left[\begin{array}{ccc|c} 4 & 6 & 7 & -3 \\ 0 & 2 & 3 & 8 \\ 0 & -2 & 2.5 & 6.5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & 6 & 7 & -5 \\ 0 & 2 & 3 & 8 \\ 0 & 0 & 5.5 & 14.5 \end{array}\right]$$

$0X_1 + 0X_2 + 5.5X_3 = 14.5$

$\boxed{X_5 = 2.65}$

$0X_1 + 2X_2 + 3(2.63) = 8$    $X_2 = 0.045$

$*4X_1 + 6(0.045 + 7(2.63)) = -5.45$

$\boxed{-5.54, 0.045, 2.63}$

*

$X_1 + X_2 + 2X_3 = 5$

$-2X_1 + 3X_2 + 4X_3 = 3$

$3X_1 + 4X_2 + 2X_3 = 6$

## ③ Singular system

$\boxed{Def = 0}$

### Ⓐ systems with infivite soulation

*Ex:
$$\left[\begin{array}{cc|c} 2 & -2 & 4 \\ 4 & -4 & 8 \end{array}\right]$$

$Def = (2 * -4) - (-2 * 4) = 0$

$-\left(\frac{4}{2}\right) R_1 + R_2 \Rightarrow R_2$

$$\begin{array}{cc|c} 4 & -4 & 8 \\ 0 & 0 & 0 \end{array} \rightsquigarrow \left[\begin{array}{cc|c} 2 & -2 & 4 \\ 0 & 0 & 0 \end{array}\right] \quad 0x_1 + 0x_2 =$$

sigular system with infinite soulation.

*Ex:-
$$\left[\begin{array}{cc|c} 2 & -2 & 4 \\ 4 & -4 & 6 \end{array}\right]$$

*$Def = (2 * -4) - (-2 * 4) = 0$

$-\left(\frac{4}{2}\right) R_1 + R_2 = R_2$

$$\left[\begin{array}{cc|c} 4 & -4 & 6 \\ 0 & 0 & -2 \end{array}\right] = \left[\begin{array}{cc|c} 2 & -2 & 4 \\ 0 & 0 & -2 \end{array}\right]$$

# no soultion. $0x_1 + 0x_2 \neq -2$

③ ill condition system

$$\left[\begin{array}{cc|c} 100 & -101 & 200 \\ 200 & -200 & 300 \end{array}\right] \quad X_1 = -48.5,\ X_2 = -50$$

$$\left[\begin{array}{cc|c} 100 & -99 & 200 \\ 200 & -200 & 300 \end{array}\right] \quad X_1 = 51.5,\ X_2 = 50$$

*normelized coelf. matrix

$$\begin{bmatrix} \frac{100}{100} & \frac{-99}{100} \\ \frac{200}{200} & \frac{-200}{200} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -0.99 \\ 1 & -1 \end{bmatrix}$$

*إذا كانت ال det coof matrix ال normel قريبة من الصفر معناته ال system ill coundition =

$*Def = -1 + 0.99 = -0.01 \approx 0$

$$\begin{bmatrix} 3 & 1 & 2 \\ -2 & 6 & 4 \\ 1 & -3 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0.333 & 0.666 \\ -0.333 & 1 & 0.666 \\ \frac{1}{7} & \frac{-3}{7} & 1 \end{bmatrix}$$

$*Def = 1\left(1 + \left(0.666 * \frac{1}{7}\right)\right)$

$-0.333\left(\left(-0.333 * 1\right) - \left(0.666 * \frac{1}{7}\right)\right)$

$+\left(0.666\right)\left(\left(-0.333 * \frac{-3}{7}\right) - \left(1 * \frac{1}{7}\right)\right)$

$\boxed{Def = 1.4}$

الموضوع : الخميس التاريخ ٢٨/٣/٢٠٢٤

*LU Decomposition

$$\overset{A}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}} \Rightarrow \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$$

$Ax=b$

① $A = \underset{\text{Lower}}{L} \cdot \overset{\text{upper}}{U}$

$$\begin{bmatrix} u_{11} & u_{12} & u \\ 0 & u_{22} & 13 \\ 0 & 0 & u_{33} \end{bmatrix} \rightarrow \boxed{Ld = b}$$

$\downarrow$

intermediate vector $\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

③ $Ux = d$ Find x

*Ex:-
$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix}$$

$* \; L_{11}u_{11} + 0 \times 0 + 0 \times 0 = 2$

$\boxed{u_{11} = 2}$

$* \; L_{11}u_{12} + \overset{*}{0}u_{22} + 0 \times 0 = 3$

$\boxed{u_{12} = 3}$

$L_{11}u_{13} + \cancel{\overset{*}{0}u_{23}} + \cancel{0 \overset{*}{} u_{33}} = -2$ $\boxed{u_{13} = -2}$

$\Rightarrow L_{21} * u_{11} + L_{22} * 0 + 0 * 0 = 1$

$L_{21} * 2 = 1 \longrightarrow L_{21} = \frac{1}{2}$

$\Rightarrow L_{21}^{\frac{1}{2}} * L_{12}^{3} + L_{22} * u_{22} + 0 * 0 = 8$

$\boxed{u_{22} = 0.5}$

$\Rightarrow L_{21} * u_{13} + L_{22} * u_{23} + 0 * \cancel{u_{33}}^{0} = 3$ $\boxed{u_{23} = 4}$

$\Rightarrow L_{31} * u_{11} + L_{32} * 0 + L_{33} * 0 = 5$ $\boxed{L_{31} = 2.5}$

$\Rightarrow L_{31} * u_{12} + L_{32} * u_{22} + L_{33} * 0 = 4$ $\boxed{L = -7}$

$\Rightarrow (2.5 * -2) + (-7 * 4) + (1 * u_{33}) = -1$ $\boxed{u_{33} = 32}$

$Ax=b$

$A=LU$

$L^{\vee} d = b^{\vee}$

$\downarrow$ intermediate vectore

$U^{\vee} x = d^{\vee}$

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix} \begin{array}{c} L \\ \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 2.7 & -7 & 1 \end{bmatrix} \end{array} \begin{array}{c} U \\ \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1/2 & 4 \\ 0 & 0 & 32 \end{bmatrix} \end{array}$$

$-(\frac{1}{2}) R_1 + R_2 \Rightarrow R_2$

1 2 3

0 0.5 4

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 5 & 4 & -1 \end{bmatrix}$$

$-(\frac{5}{2}) R_1 + R_3 \Rightarrow R_3$

5 4 -1

0 -3.5 4

$$\rightarrow \begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & -3.5 & 4 \end{bmatrix}$$

$\rightarrow -(\frac{-3.5}{0.5}) R_2 + R_3 \Rightarrow R_3 \rightarrow$ 0 -3.5 4

0 0 12

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ & 1 & 0 \\ & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \rightarrow$$

Ex:

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$* \; 1 d_1 + 0 \cancel{d_2}^{0} + 0 \cancel{d_3}^{0} = 3 \qquad \boxed{d_1 = 2}$

$* \; 1 + d_2 + 0 \cancel{d_3}^{0} = 1 \qquad \boxed{d_2 = 0}$

$* \; 5 - 0 + d_3 = 3 \qquad \boxed{d_3 = -3}$

$d$

$\begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$ inter... vector

$UX = d$

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

$0x_1 + 0x_2 + 32x_3 = -3$

$x_3 = \frac{-3}{32} = -0.09375 \Rightarrow 0x_1 + 0.5x_2 - 4 * 0.09375$

$= 0 \quad \boxed{x_2 = 0.75}$

$2x_1 + (3 * 0.75) + \frac{6}{32} = 2 \quad \boxed{x_1 = -0.21875}$

$A \quad A^{-1} = I$

$$A \qquad\qquad A^{-1}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

$b_1 \quad b_2 \quad b_3 \qquad Ax_2 = b_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow Ax_2 = b_2 \qquad Ax_1 = b_1$$

$Ax_3 = b_3$

*Ex:- calculate the $3^d$ colums of the inverse matrix For

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.7 & -7 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix}$$

$Ld = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0d_1 + 0\cancel{d_2} + 0\cancel{d_3} = 0$$

$\boxed{d_1 = 0}$

$\Rightarrow 0.5 d_1 + d_2 + 0\cancel{d_3} = 0$ $\boxed{d_2 = 0}$

$2 - 5\cancel{(0)} - 7\cancel{(0)} + d_3 = 1$ $\boxed{d_3 = 1}$ $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = d \rightarrow \boxed{ux = d}$

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$0x_1 + 0x_2 + 32x_3 = 1$ $\boxed{x_3 = 0.03125}$

$0x_1 + 0.5x_2 + 4(x_3) = 0$ $\boxed{x_2 = -0.25}$

$2x_1 + 3(-0.25) - 2(0.03125) = 0$ $\boxed{x_1 = 0.40625}$

$$\begin{bmatrix} 0.40625 \\ -0.25 \\ 0.03125 \end{bmatrix} = \begin{bmatrix} -0.15625 \\ 0.25 \\ 0.21875 \end{bmatrix}$$

* (Jacobid and Gauses Sieded):-

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$ (مش داخل بالمير).

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

$$x_{1,i+1} = \frac{b_1 - a_{12}x_{2,i} - a_{13}x_{3,i}}{a_{11}}$$

$$x_{2,i+1} = \frac{b_2 - a_{21}x_{1,i} - a_{23}x_{3,j}}{a_{22}}$$

$$x_{3,i+1} = \frac{b_3 - a_{31}x_{1,i} - a_{32}x_{2,j}}{a_{33}}$$

$\Rightarrow a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$\Rightarrow a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$\Rightarrow a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

(A) Jacobis:-

$$x_{1,i+1} = \frac{b_1 - a_{12}x_{2,i} - a_{3}x_{3,i}}{a_{11}}$$

$$x_{3,i+1} = \frac{b_2 - a_{21}x_{1,i} - a_{23}x_{3,i}}{a_{22}}$$

$$x_{3,i+1} = \frac{b_3 - a_{31}x_{1,i} - a_{32}x_{2,i}}{a_{33}}$$

(B) G. Sieded

$\Rightarrow x_{1,i+1}$

$$\Rightarrow x_{2,i+1} = \frac{b_2 - a_{21}x_{1,i+1} - a_{23}x_{3,i}}{a_{22}}$$

$$\Rightarrow x_{3,i+1} = \frac{b_3 - a_{31}x_{1,i+1} - a_{32}x_{2,i+1}}{a_{33}}$$

*Ex:- $x_1 + 5x_2 + 4x_3 = 12$

$6x_1 + 3x_2 + 2x_3 = 17$

$3x_1 + x_2 + 7x_3 = 1$

*Ex:- use G. Sieded

$$X = \begin{array}{ccc} x_{1,0} & x_{2,0} & x_{3,0} \\ \hline \end{array}$$
$$X = [\,0,\ 0,\ 0\,]$$

$\varepsilon_s = 0.1\%$

$6x_1 - 2x_2 + x_3 = 11$

$-2x_1 + 7x_2 + 2x_3 = 5$

$x_1 + 2x_2 - 5x_3 = -1$

* sol:- $X_{1,i+1} = \frac{11+2X_{2i} - X_{3i}}{6}$

$X_{2,i+1} = \frac{5+2X_{1,i+1} - 2X_{3,i}}{7}$

$X_{3,i+1} = \frac{-1 - X_{1,i+1} - 2X_{2,i+1}}{-5}$

* For $i = 0$

$X_{1,1} = \frac{11+2*0-0}{6} = 1.834$

$X_{2,1} = \frac{5+ 1.834-0}{7} = 1.238$

$X_{3,1} = \frac{-1 - 1.834 - 2 * 1.238}{-5} = 1.062$

hech error = 100% → لأنه العدد ده نفسه بدنا كمان iteration

$i = 1$

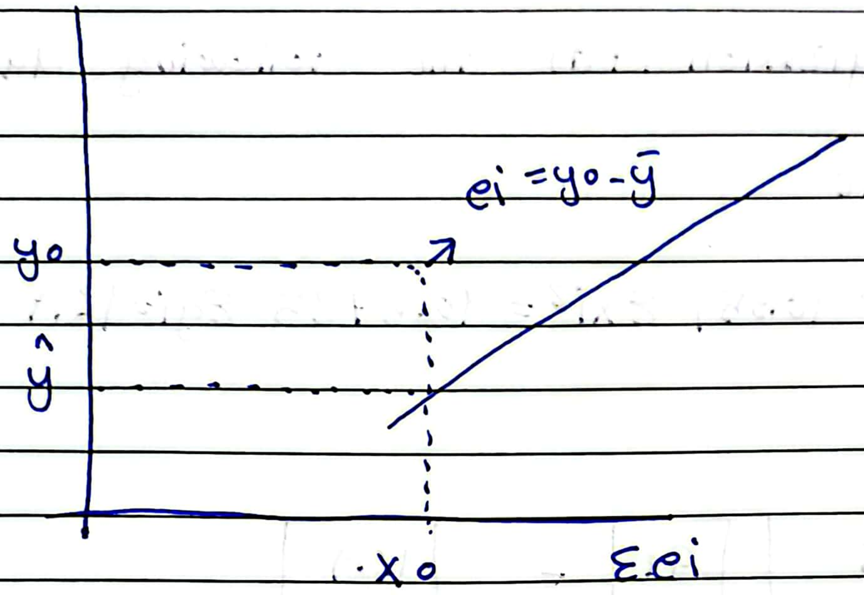
$X_{1,r2} = \frac{11+2(1.238)-1.062}{6} = 2.069$ → error = 11.4%

$X_{2r2} = \frac{5+2(2.069)-2(1.062)}{7} = 1.002$ → %23.65

$X_{3r2} = \frac{-1-2069-2* 1.002}{-5} = \boxed{1.0146}$

* curve Fitting →

Linear regression




$\hat{y} = a_0 + a_1 x$

$\Sigma (e_i)^2$

$P = \frac{\quad}{x}$

$\Sigma e_i$

$Sr = (y_1 - a_0 - a_1 x)^2_{min}$

$y = a_0 + a_0 x$

$a_0 n + a_1 \Sigma x_i = \Sigma y_i \ldots\ldots$ ①  $a_0 \Sigma x_i + a_1 \Sigma x_i^2 = \Sigma x_i y_i \ldots$

number of data psichs

$$\begin{bmatrix} n & \Sigma x_i \\ x_i & \Sigma x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{bmatrix} \leftarrow \text{"حفظ"}$$

Ex: 

| | | | |
|---|---|---|---|
| x | 2 | 3.25 | 5.1 |
| y | 3.5 | 5.6 | 7.8 |

*using Linear regression, f(t) the Following data

int astright line

*Sol:- $n=3$, $\Sigma x_i = 10.35$, $\Sigma x_i^2 = 40.5725$ $\Sigma y_i = 16.9$

$\Sigma x_i y_i = 64.98$.

$$\begin{bmatrix} 3 & 10.35 \\ 10.35 & 40.57 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 16.9 \\ 64.98 \end{bmatrix}$$

$a_0 = 0.8973$, $a_1 = 1.3727$ $y = 0.8978 + 1.3727x$

Correlection coefficent R used to test the

goodness of the groposed ~~fixed~~ Fit $\boxed{R=1}$ exact

R approches to $0 \rightarrow$ poor

$R = 0 \rightarrow$ no relation( indepdent)

$$R = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma b^2 - (\Sigma y)^2}}$$

*
$$R = \frac{(3 \cdot 64.98) - (10.35 \cdot 16.9)}{\sqrt{3 \cdot 40.57 - 107.12}\sqrt{3 \cdot 104.45 - 285.81}}$$

$(\Sigma x_i)^2 = 107.12$ (مثال سابق)

$(\Sigma y_i)^2 = 285.61$

$\boxed{R = 0.995 \quad \text{exc}}$

$y = a_0 + a_1 x \longrightarrow$ Data Linear ization

① power $y = \alpha x^{B}$

$y^* = a_0 + a_1{}^* x$

$\ln y = \ln \alpha x^{B}$

| x | 1 | 2.8 | 5 |
|---|---|---|---|
| y | 4 | 18 | 20 |

$\ln y = \ln \alpha + \ln x^{B}$

$\ln y = \ln \alpha + B \ln x$

| $x^*$ | ln 1 | ln 2 | ln 5 |
|---|---|---|---|
| $y^*$ | ln 4 | ~~ln~~ 18 | ln 20 |

② Exponnatial $y = \alpha e^{Bx}$

$\ln y = \ln \alpha + Bx \quad \ln e \to 1$

$\ln y = \ln \alpha + Bx$

$y^* = a_0 + a_1 x^*$

③ Groth rate

$y = \dfrac{\alpha x}{B + x}$

$\dfrac{1}{y} = \dfrac{B + x}{\alpha x} \qquad y^* = a_0 \quad a_1 x$

*write Linear regression to Find the ridus of $\alpha B$ For the Following relation $y = x e^{Bx^2}$

| x | 2 | 3 | 5 |
|---|---|---|---|
| y | 4.9 | 8.7 | 13.5 |

$y = \alpha e^{B x^2}$

$\ln y = \ln \alpha + B x^2$

| $x^\circ$ | 4 | 9 | 25 |
|---|---|---|---|
| $y^\circ$ | 1.59 | 2.163 | 2.6 |

$y^* = a_0 + a_1 x^\circ$

$$\begin{bmatrix} 3 & 38 \\ 38 & 722 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 63552 \\ 90.89 \end{bmatrix}$$

$$\boxed{\begin{matrix} a_0 = 1.5715 \\ a_1 = 0.04317 \end{matrix}}$$

$a_0 = \ln \alpha$

$1.5713 = \ln \alpha$

$\alpha = e^{1.5713} = \boxed{4.8}\ \alpha$

$y = 4.8 \cdot e^{0.04317 x^2}$

* polynomel at regredition

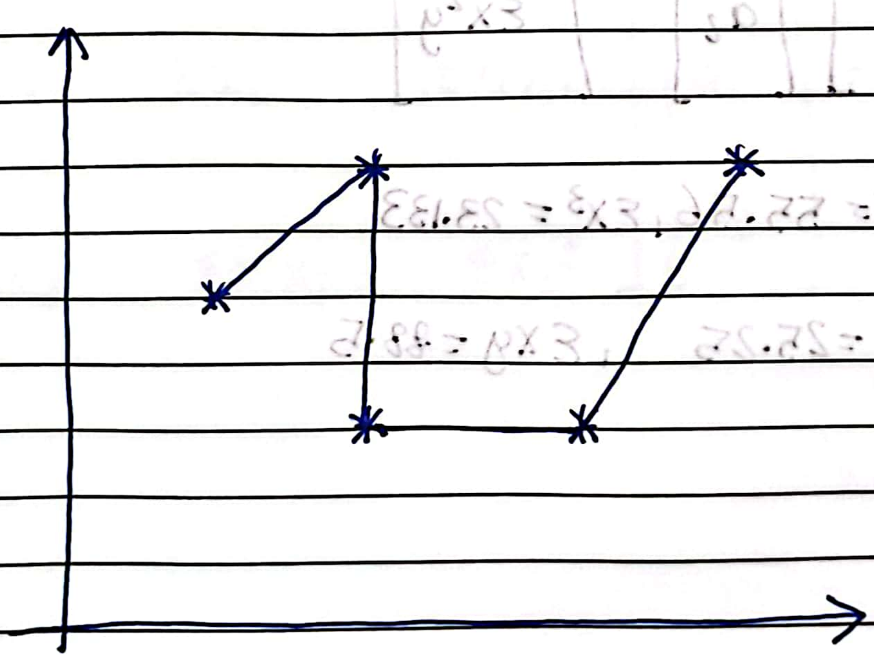
$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m$$

$$\begin{bmatrix} n & \Sigma x & \Sigma x^2 & \Sigma x^m \\ \Sigma x & \Sigma x^2 & \Sigma x^3 & \Sigma x^{m+1} \\ \Sigma x^2 & & & \\ \vdots & & & \\ \Sigma x^m & \Sigma x^{m+1} & \Sigma x^{m+2} & \Sigma x^{2m} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma xy \\ \Sigma x^2 y \\ \vdots \\ \Sigma x^n y \end{bmatrix}$$

$y = 1 \longrightarrow m = 0$

$\boxed{n \geq m+1}$

$y = x - 2 \longrightarrow m = 1$



* Ex:- using regresation analysis the $2^{nd}$ order polynomeal coaffients using the Following data point are?

| | | | | |
|---|---|---|---|---|
| x | 2 | 3.25 | 4 | 5 |
| y | 5 | 9 | 7 | 4.25 |

$y = a_0 + a_1 x + a_2 x^2$

$$\begin{bmatrix} n & \Sigma x & \Sigma x^2 \\ \Sigma x & \Sigma x^2 & \Sigma x^3 \\ \Sigma x^2 & \Sigma x^3 & \Sigma x^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma xy \\ \Sigma x^2 y \end{bmatrix}$$

* $n = 4$, $\Sigma x = 14.25$, $\Sigma x^2 = 55.56$, $\Sigma x^3 = 23.133$

$\Sigma x^4 = 1008.57$, $\Sigma y = 25.25$, $\Sigma xy = 88.5$

$\Sigma x^3 y = 333.31$

$$\begin{bmatrix} 4 & 14.25 & 55.56 \\ 14.25 & 55.56 & 231.33 \\ 55.56 & 231.33 & 1008.57 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 25.25 \\ 88.5 \\ 333.31 \end{bmatrix}$$

$a_0 = -11.59$ , $a_1 = 11.8 \longrightarrow y = -11.59 + 11.8x - 1.73x^2$

$\boxed{a_2 = -1.73}$

* multiple Linear regreastion :-

* $y = a_0 + a_1 X + a_2 X_2 + \dots + a_n X_n$

$$\begin{bmatrix} n & \Sigma X_1 & \Sigma X_2 \\ \Sigma X_1 & \Sigma X_1^2 & \Sigma X_1 X_2 \\ \Sigma X_2 & \Sigma X_1 X_2 & \Sigma X_3^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma X_1 y \\ \Sigma X_2 y \end{bmatrix}$$

*Ex:- Find $(a_0, a_1, a_2)$:-

$y = a_0 + a_1 x_1 + a_2 x_2$

| | | | |
|---|---|---|---|
| $x_1$ | 1 | 1 | 2 |
| $x_2$ | 2 | 3 | 2 |
| $y$ | 2 | 5 | 9 |

$n = 3$, $\Sigma x_1 = 4$, $\Sigma x_2 = 7$, $\Sigma x_1^2 = 6$

$\Sigma x_1 x_2 = 9$, $\Sigma x_2^2 = 17$

$\Sigma y = 16$, $\Sigma x_1 = -5$

$\Sigma x_2 y = 37$.

$$\begin{bmatrix} 3 & 4 & 7 \\ 4 & 6 & 9 \\ 7 & 9 & 17 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 25 \\ 37 \end{bmatrix}$$

*$a_0 = -11$, $a_1 = 7$, $a_2 = 3$, $y = -11 + 7x_1 + 3x_2$

الموضوع : الخميس التاريخ ٢٨ / ٤ / ٢٠٢٤

| x | y |
|---|---|
| 15 | 5% |
| 16 | 32% |
| 17 | 8% |

$$\frac{17-15}{17-16} = \frac{8-5}{8-x}$$

* Interplaton polynomial :-

① Netwon devided difference $\boxed{NDD}$

$$*F_4(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)$$

$$(x-x_1) + \ldots\ldots + b_n(x-x_0)(\ldots\ldots)(x-x_{n-1})$$

| $x_i$ | $f(x_i)$ | 1st $f(x_{i+1}, x_i)$ | 2nd $F(x_{i+2}, x_{i+1}, x_i)$ | 3nd $F(x_{i+3}, \ldots, x_n)$ |
|---|---|---|---|---|
| | $b_0$ | $b_1$ | $b_2$ | $b_3$ |
| $i=0$ $x_0$ | $y_0$ | $\frac{y_1-y_0}{x_1-x_0}$ | $\frac{f(x_2,x_1) - f(x_1,x_0)}{x_2-x_0}$ | |
| $i=1$ $x_1$ | $y_1$ | | | $\frac{f(x_2,x_1,x_0) - f(x_3,x_2,x_1)}{(x_3-x_0)}$ |
| $i=2$ $x_2$ | $y_2$ | $\frac{y_2-y_1}{x_2-x_1}$ | $\frac{f(x_3,x_2) - f(x_2,x_1)}{x_3-x_1}$ | |
| $i=3$ $x_3$ | $y_3$ | $\frac{y_3-y_2}{x_3-x_2}$ | | |

* كلما زاد عدد ال point كل .

ما زاد عدد ال axirce .

*Ex8- using NDD estimate $f_3(2.75)$ using the Following data points.

| x | 0 | 1 | 2.5 | 3 |
|---|---|---|---|---|
| y | 2 | 5 | 9 | 11 |

$$f_3(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

| x | y | 1st | 2nd | 3rd |
|---|---|---|---|---|
| 0 | 2 | $\frac{5-2}{1-0}=3$ | $\frac{2.67-3}{2.5-0}=-0.132$ | $\frac{0.665+0.132}{3-0}=0.265$ |
| 1 | 5 | $\frac{9-5}{2.5-1}=2.67$ | $\frac{4-2.67}{3-1}=0.665$ | |
| 2.5 | 9 | $\frac{11-9}{0.5}=4$ | | |
| 3 | 11 | | | |

$$f_3(x) = 2 + 3(x-0) - 0.132(x)(x-1) + 0.267(x)(x-1)(x-2.5).$$

$$\boxed{f_3(2.75) = 9.935}$$

② Lagrenge Interpolution:-

$* F_n(x) = \sum_{j=0}^{n} L_i f(x_i)$

$* L_i$: $i^{th}$ lagrenge polynomial

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

| x | $x_0$ | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|---|
| y | $y_0$ | $y_1$ | $y_2$ | $y_3$ |

$* i = 0 \rightarrow L_0$

$$L_0 = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

$* i = 2 \rightsquigarrow \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 \ldots\ldots)}$ ما حطينا $(x - x_0)$ لـ

لأنه في شرط $(j \neq i)$:

$*$Ex:- $f_3(2.75)$ = using Lagrange.

لـ المثال السابق

| x | 0 | 1 | 2.5 | 3 |
|---|---|---|---|---|
| y | 2 | 5 | 9 | 11 |

$$* L_0 = \frac{(x-1)(x-2.5)(x-3)}{(0-1)(0-2.5)(0-3)}$$

sub $2.73 = x$

$\boxed{L_0 = 0.0145}$

$i = 1 \rightsquigarrow L_1 \quad \frac{(x-0)(x-2.5)(x-3)}{(1-0)(1-2.5)(1-3)}$

$* L_1 = -0.0572$

$L_2 \rightsquigarrow \frac{(x-0)(x-1)(x-3)}{(2.5)(2.5-1)(2.5-3)} = 0.6416$

$L_3 \rightsquigarrow \frac{(x-0)(x-1)(x-2.5)}{(3-0)(3-1)(3-2.5)} = 0.401$

$f_3(2.75) = (0.0145 * 2) + (-0.0572 * 5) + (0.6416 * 9)$

$+ (0.401 * 11) = 9.9284$

$9.935$

*Derivating :- Equaly Seacted data.

$$* F(x_{i+1}) = \sum_{j=0}^{\infty} \frac{f^{j}(x_j)\,(x_{j+1} - x_j)^{j}}{j!}$$

$x_{j+1} - x_j \rightarrow h$ (size)

A مسافة بين كل نقطتين

| x | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| y | | | | |

$$\rightsquigarrow f(x_{i+1}) = f(x_i) + f'(x_i)h + \sum_{j=2}^{\infty} \boxed{\;}$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} - \boxed{\sum_{j=2}^{\infty} \boxed{\;}}$$

* $O(h)$ Low accuracy
* $(Oh)^2$ medium "
* $(Oh)^4$ high "

* Equally spech data

→ centred		h=constant

→ Forward		$o(h), o(h)^2, o(h)^4$

→ back ward

*Ex:- using Following data

| X | 1.5 | 1.75 | 2 | 2.75 | 2.5 |
|---|---|---|---|---|---|
| f(x) | 2 | 4.5 | 7.25 | 9.15 | 12 |

تكملة

| 2.75 | 3 |
|---|---|
| 14.25 | 15 |

a) Estimete $f''(2.25)$ centred with $o(h)^4$

b) Estimete $f'(2)$ centred $o(h)^2, (h=0.5)$

a) $f''(x_i) = \dfrac{-f(x_{i+2}) + 16 f(x_{i+1}) - 30 f(x_i) + 16 f(x_{i+1}) + f(x_0)}{12 h^2}$

$f''(2.25) = \dfrac{-14.75 + (16 * 12) - 30(9.15) + (16 * 75) - 4.5}{12 * (0.25)^2}$

$= 19.6$		*① إذا ما جديني طريقة بحل كامل (Center).
القانون

*② بالامتحان لما يعطيني أو مارح يحددلي إذا هي لل center/ Forward
رح يعطيك القوانين وانت لازم تفرق بينهم.

*③ ممكن يعطينا الـ (Function) وما يعطي الـ X وال y.

(b) $f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} = \frac{12-2}{2*(0.5)} = 10$

*Ex:- Estimate $f''(0.5)$ For $f(x) = \cos^2(x)$

Forward $O(h)$, $h=0.15$.

| x | 0.5 | 0.65 | 0.8 |
|---|---|---|---|
| y | 0.7701 | 0.6337 | 0.4854 |

$$F''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_{i+1})}{h^2}$$

$$= \frac{0.4854 - 2(0.6337) + 0.7701}{(0.15)^2} \rightarrow = -0.5288.$$

*unequalty → Spaced data

*Ex:- extimate $f'(2.25)$

| X | 1 | 2.25 | 3 |
|---|---|---|---|
| f(x) | 2 | 7 | 9 |

| X | F(x) | 1st | 2st | 3st |
|---|---|---|---|---|
| 1 | 2 | $\frac{4-2}{1.25}=4$ | $\frac{2.667-4}{3-1}$ | $-0.6665$ |
| 2.25 | 7 | $\frac{2}{0.75}=2.667$ | | |
| 3 | 9 | | | |

$*F(x) = 2+4(x-1) - 0.6665(x-1)(x-2.25)$

$*F'(x) = 4 - 0.6665(2x-3.25)$

$*F'(x) \rightsquigarrow f'(2.25) = 3.1668$ ✓

الموضوع :                                    التاريخ   /  /  ٢٠١

(محاضرات آخر إسبوع لمادة النيوميركال)

* $f(x) = 2 + 4(x-1) - 0.6665(x-1)(x-2.25)$

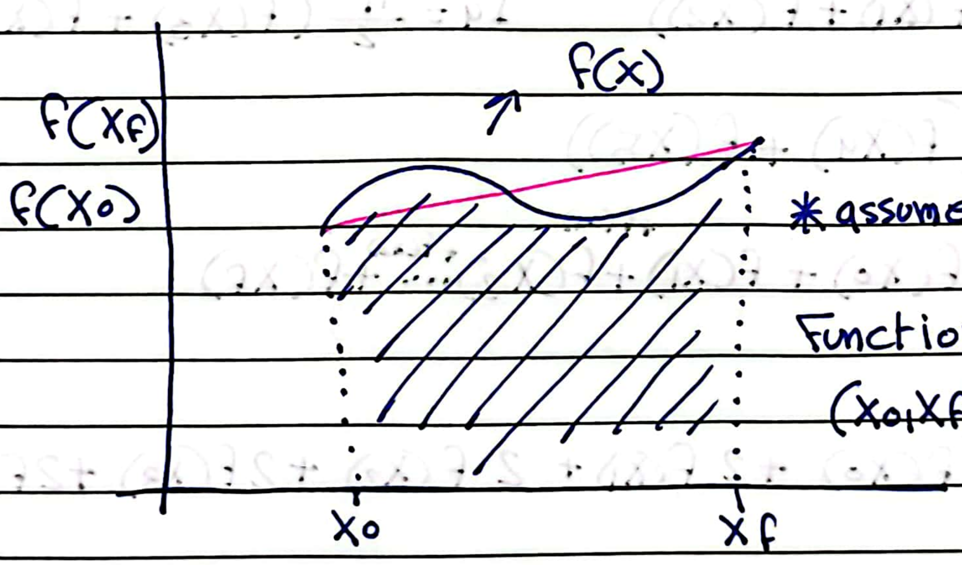
* $f'(x) = 4 - 0.6665(2x - 3.25)$        $\boxed{f(2.25) = 3.166}$

* numvrical integration

→ Equally spaced data → step (ثابت)

① Trapizoidal rale
② sinpross rale → Ⓐ $\frac{1}{3}$ S.R   Ⓑ $\frac{3}{8}$ S.R      $v = \frac{s}{t}$

① Trapizoidal Rule

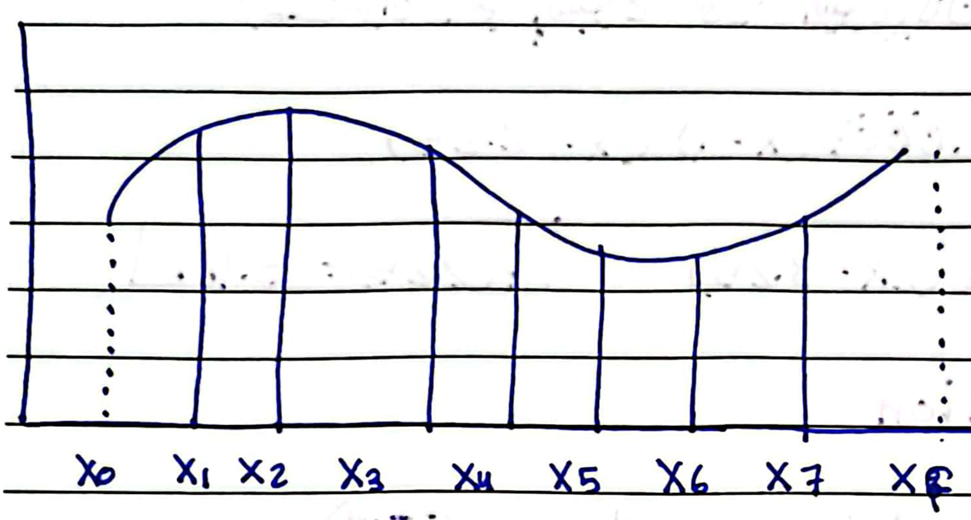



* assume a linear Function between $(x_0, x_f)$

$$I = \int_{x_0}^{x_f} f(x)\,dx \qquad I = \frac{1}{2}\left(f(x_0) + f(x_f)\right) * \boxed{(x_f - x_0)}$$

h (step size)

$$= \frac{h}{2}\left(f(x_0) + f(x_f)\right)$$




$n = 5$

* كلما قل ال error قل ال (Step size) بتزيد ال (accuraty).

* $h = \frac{x_f - x_0}{n}$

n: عدد القطع

* $I_1 = \frac{h}{2}(f(x_0) + f(x_1))$

* $I_2 = \frac{h}{2}(f(x_1) + f(x_2))$

$I_3 = \frac{h}{2}(f(x_2) + f(x_3))$

$I_4 = \frac{h}{2}(f(x_3) + f(x_4))$

* $I_5 = \frac{h}{2}(f(x_4) + f(x_5))$

* $I = \frac{h}{2}(f(x_0) + f(x_1) + f(x_2) \overset{+f(x_1)\ +f(x_2)}{\cdots} + f(x_f))$

* $I = \frac{h}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_5))$

(الصيغة العامة)

باعتبار 5 (Segments)

$= \frac{h}{2}\left(f(x_0) + \left(2\sum^{n-1} f(x_i)\right) + f(x_f)\right)$

(Ex) Find integration For $\int_1^4 e^{2x}\,dx$

$I = \frac{1}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + f(x_3))$

$= \frac{1}{2}(e^2 + 2e^4 + 2e^6 + e^8) = 1952.2$

(exact)

* real value $= 1486.78$ Error $= 465.41$

يكون ← error = 0

* (Constant (lower) linear error يكون اقل)

* لا نستطيع الحكم على accurces في التغير "h".

* Simpsons rule (S.R)

(A) $\frac{1}{3}$ S.R quadradic → 2 secuad order

* Asume a Function

→ ~~Single~~ Single application (n = 2 لازم تكون)

$I = \frac{h}{3}(f(x_i) + 4f(x_1) + f(x_f))$

$x_i$ $x_1$ $x_f$ $h = \frac{x_f - x_i}{2}$

→ multiple application must be → n must be even.

hmin n min = 4 → 5 point

n = even → 4 → even وفوق بس

$I = \frac{h}{3}\left(f(x_0) + 2\sum_{i=2,4,6}^{n-2} f(x_i) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + f(x_f)\right)$

$I = \frac{3}{8}h\,(f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4)$

$+ 3f(x_5) + 2f(x_6) \dots + f(x_f))$

Ⓑ $\frac{3}{8}$ S.R

Assume a $3^{rd}$ order Functions : minium no of Segments $\boxed{n=3}$

→ Single applications

$$I = \frac{3}{8} h \left( f(x_0) + 3f(x_1) + 3f(x_2) + f(x_f) \right)$$

$\boxed{h=3}$

أي رقم فوق أو يساوي = ٤.

→ maltiplue application $n \geq 4$

$$I = \frac{3}{8} h \left( f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4) + 3f(x_5) + 2f(x_6) \dots f(x_f) \right)$$

* مضاعفات ال ٣ مضروب بـ (2)

الباقي مضروب بـ (3).

* Ex : $\int_1^4 e^{2x} dx$    السابق

| | |
|---|---|
| $x_0=1$ | $x_1=2$ |
| $x_2=3$ | $x_3=4$ |

4 points → 3 segment (n)

$x_0=1$   $x_1=2$

$x_2=3$   $x_3=4$

$h=1$

$$I = \frac{3}{8}(1)\left(e^2 + 3e^4 + 3e^6 + e^8\right)$$

$= 1635.9$

Ex: $\int_1^4 f(x)\, dx$

ال step size

| x | ~~0~~ | 1 | 2 | 3 | 4 | ~~5~~ |
|---|---|---|---|---|---|---|
| y | ~~[illegible]~~ | 2 | 4 | 6 | ~~8~~ 7 | ~~10~~ |

ما إذا تغيرت : I

→ Runge Kalb a muthods used to solve the ordiary diff eq

$\frac{dy}{dx} = f(x,y)$ , $y(x_0) = y_0$ $(x_0, y_0)$

① Eulers methods

$y_{i+1} = y_i + (f(x_i, y_i))^* h$ → step size

Ex estimate $y(1)$ using euler method

$\frac{dy}{dx} = \frac{5x^2+1}{y}$ , $y(0) = 1$

ⓐ $h = 1$ ⓑ $h = 0.5$ calculate $E_T$ For a and b

$\frac{dy}{dx} = \frac{5x^2+1}{y}$ → تجميع مع بعض

$y\,dy = 5x^2+1\ dx$ $\frac{y^2}{2} = \frac{5}{3} + x + C$ → بعرف الثابت

$y(0) = 1$ → $\frac{1}{2} = C$

$\frac{y^2}{2} = \frac{5}{3}x^3 + x + \frac{1}{2}$ هون بس بدنا $y(1)$ منخلي $y$ موضع القانون

$y = \sqrt{\frac{10}{3}x^3 + 2x + 1}$

$y(1) = \sqrt{\frac{10}{3} + 2 + 1} = 2.516$ → True variable

تعويض $y(0)=1$ في

ⓐ $h=1$

$i=0 \rightarrow y_1 = y_0 + f(x_0,y_0) * h = 1+1*1=2$

$E_a = \left|\frac{2.516-2}{2.516}\right| * 100\% = 20.5\%$

$1=1+0$ فتعبر $h$ ومن $\rightarrow (1,2)$

$i=1 \rightarrow y_2 = y_1 + f(x_1,y_1) * h$

$y_2 = 2+3*1 = 5$ $\quad E_T = 100\%$ $\quad \rightarrow$ فأنا هون مطلوب $y(1)$ ... بس

إحنا طلعنا $y(2)$ يعني مرتين

ⓑ $h=0.5$

$i=0 \rightarrow y_1 = y(0.5) = \overset{y(0)}{y_0} + f(x_0,y_0) * 0.5 = 1+1*0.5$

~~[illegible]~~ $y(2)$ طالب *مثلاً $= 1.5$

ومعطيني $y(0)$ و $h=1$ يعني 2 iteration *

* $x=x+1$ نزيد

$i=1 \rightarrow y_2 = y(1) = 1.5+1.5*0.5 = 2.25$

$E_A = \left|\frac{2.516-2.25}{2.516}\right| * 100\% = \underline{\underline{10.5\%}}$

* 2nd order R-R.

(a) Hamm's method

$y_{i+1} = y_i + \frac{1}{2}(K_1 + K_2) * h$

$K_1 = f(x_i, y_i)$ , $K_2 = f(x_i + h, y_i + K_1 h)$

(b) mid point method

$y_{i+1} = y_i + K_2 h$ $K_1 = f(x_i, y_i)$

$K_2 = f(x_i + 0.5h, y_i + 0.5 K_1 h)$

(c) Ral ston's method

$y_{i+1} = y_i + \frac{1}{3}(K_1 + 2K_2)h$ $K_1 = f(x_i, y_i)$

$K_2 = f(x_i + \frac{3}{4}h, y_i + \frac{3}{4}K_1 h)$

(ex) estimate using hen's method:-

$\frac{dy}{dx} = \frac{5x^2+1}{y}$ 6 $y(0) = 1$, $h = 0.5$

$K_2 = f(0.5, 1.5)$

$i=0 \longrightarrow y_1 = y(0.5) = y(0) + 0.5(\overset{K_1}{1} + 1.5) * 0.5 = 1.625$

$i=1 \longrightarrow y_2 = y(1) = y(0.5) + 0.5(K_1 + K_2) * 0.5$

$K_1 = 1.38$ $K_2 = (1, 2.315) = 2.59$

$= 2.617$

Q $3^{rd}$ order RK

$$y_{i+1} = y_i + \frac{1}{6}(K_1 + 6K_2 + K_3)*h$$

$$K_1 = f(x_i, y_i)$$

$$K_2 = f(x_i + 0.5h, y_i + 0.5K_1h)$$

$$K_3 = f(x_i + h, y_i - K_1h + K_2h)$$

* $4^{rd}$ order RK

$$y_{i+1} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)*h$$

$K_1 = f(x_i, y_i)$, $K_2 = f(x_i + 0.5h, y_i + 0.5K_1h)$

$K_3 = f(x_i + 0.5h, y_i + 0.5K_2h)$ $K_4 = f(x_i + h, y_i + K_3h)$

*Ex:- $\frac{dy}{dx} = \frac{5x^2+1}{y}$     $y(0)=1$ , $h=1$

using Rk-4     Find $y(1)$:-

*$y_{i+1} = y_i + \frac{h}{6}(h_1 + 2h_2 + 2h_3 + h_4)$

For $i=0$

$k_1 = f(0,1) = 1$ | $h_2 = f(0.5, 1.5) = 1.5$ | $h_3 = f(0.5, 1.75) = 1.285$

$h_4 = f(1, 2.2857) = 2.625$

$y_1 = y(1) = 1 + \frac{1}{6}(1 + 3 + 2^* 1.2657 + 2.625) = 2.5327$

بسبب التقريب. error ≃ 1     exact = 2.516

#(تم بحمد الله تعالى).