ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

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من شرح:

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جزيل الشكر للطالبة:

فرح فحماوي

Subject *Numerical* Date No.

A mathematical modle is represented as a functional relationship of the form:-

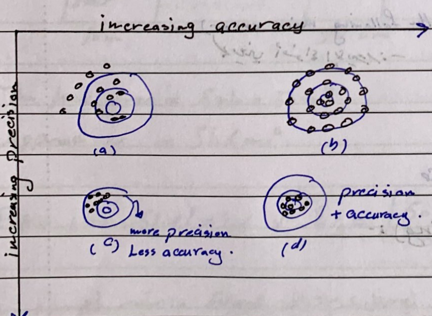
$$\underset{\text{variable}}{\text{Dependent}} = f\left(\underset{\text{variables}}{\text{independent}}, \text{parameters}, \text{forcing function}\right)$$

Ex:- Finding $I = \int (2x) \sin(x^2 - 5)\, dx$

$I = \int \sin(x^2 - 5)\, dx$ → صعب إنه أحلهم في الطرق العادية لذلك نستخدم Numerical methods

Accuracy:- How close is a computer to the truth value

precision:- How close is a " to previously computed.



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* significant Figures

Ex:- 0$\boxed{.0}$560 → significant لا يتغير → يعني عندي صفر 3 significant / ⓵.033 → أول صحيح → 4 significant

Ex:- How many significant figure?

2317 → 4.

0.0035 → 2. (Leading Zero not sig)
Leading zero

1.080 → 4
interior Zero

π → 3.14 → 3

1000 → 1 وبس 1.000 → 4

Neumerical methods (Solution will have the following main error):- يعني أنواع الإيرور:-

1- Round of error:-

A- without chopping (rounding):-

3.4156 → 3.416 → عندي صفر تقريب

B- with chopping (without rounding):-

3.4156 → 3.415
أخذت المنازل بدون تقريب

2- Truncation error:- $\pi \approx 3.14$, $e \approx 2.7$, $g = 9.81$ لا نضع القيمة الحقيقة بل التقريبية

NOTEBOOK

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3- Tylor series (discuss later):-

*

Absolute and Relative Error:-

* Absolute true error (E_t) = | Exact - Approx | (القيمة الحقيقية) (القيمة التقريبية)

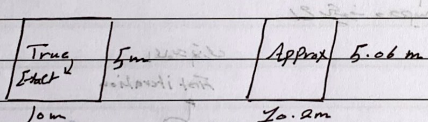
* Relative true error (ε_t) = $\left| \dfrac{\text{Exact} - \text{Approx}}{\text{Exact}} \right| * 100\%$ → $\dfrac{E_t}{\text{Exact}} * 100\%$

Ex:- suppose the exact value is 10,000 and the approximate one is 9,999 find E_t and ε_t :-

sol:- $E_t = | 10,000 - 9,999 | = 1$

$\varepsilon_t = \left| \dfrac{10,000 - 9,999}{10,000} \right| * 100\% = 0.01\%$

Ex:- Find E_t and ε_t for the following area?



نحسب القيمة الحقيقية والقيمة التقريبية

The true area $= 5 * 10 = 50 m^2$

Approx // $= 51.6 m^2$

$E_t = |50 - 51.6| = 1.6$ / $\varepsilon_t = \left| \dfrac{50 - 51.6}{50} \right| * 100\% = 3.2\%$

* طريقة Absolute and Relative error الها سلبيات في حال عدم تواجد Exact Value ما بقدر أحسب Relative true error.

* مشان هيك عندي طريقة ثانية وهي **Relative approximation error** ←

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***Approximation error :-** exact value تستخدم في حال عدم وجود

قوانينها :-

*Absolute approximation error :- $(E_a) = |\text{present value} - \text{previous value}|$.

*Relative approximation error $(\varepsilon_a) = \left|\dfrac{\text{present value} - \text{previous value}}{\text{present value}}\right| * 100\%$

*Previous value = x_i :- الاعادة الأولى

*present value = x_{i+1} → الاعادة الثانية

x_{i+2} → الاعادة الثالثة

وهكذا

وهذه الطريقة تفيدنا بالتحليل العددي لأنه لا يوجد قانون ولا قيمة حقيقية .

How many iterations (تكرار) do we need? (Stopping Criteria) معايير التوقف

when $|\varepsilon_a| < \varepsilon_s$.

ε_s : error acceptable (ا ع يكون معطى في السؤال) الـ

Ex:- using Series Expantion (Maclarin series): $\left(e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}\right)$, (First iteration / الاعادة الأولى)

How many term needed to find approximation value of $(e^{0.5})$ with $\varepsilon_s = 0.05\%$?

Sol:- $2! \xrightarrow[\text{عن}]{\text{عبارة}} 2*1$, $x_0 = 0.5$

\# iterations $\varepsilon_a\ \%$

1 $e^{0.5} = 1$ in 1st iteration no error (ما عندي خطأ لأنه ما في قيمة قديمة)

2 $e^{0.5} = 1 + 0.5 = 1.5$ $\varepsilon_a = \left|\dfrac{1.5-1}{1.5}\right| * 100\% = 33.3$

3 $e^{0.5} = 1 + 0.5 + \dfrac{0.5^2}{2!} = 1.625$ $\varepsilon_a = 7.69$

4 $e^{0.5} = 1 + 0.5 + \dfrac{0.5^2}{2!} + \dfrac{0.5^3}{3!} = 1.645833$ $\varepsilon_a = 1.27$

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5 1.6484375 $\varepsilon_a = 0.158$

6 1.6489791 $\varepsilon_a = 0.0158 < \varepsilon_s$ 0.05% Stop.

Ex:- we used a method to estimate the +ve (positive) root for $F(x) = x^2 - 4x - 5$ and Resulted in the following, calculate:- E_a, ε_a, E_t, ε_t after the 4th iteration

تقاربي

Sol:- $x^2 - 4x - 5 = 0$, $(x-5)=5$, $x+1 = -1$

$E_t = |5 - 4.94| = 0.06$, $\varepsilon_t = \left|\frac{5-4.94}{5}\right| * 100\% = 1.2\%$

$E_a = |4.94 - 4.83| = 0.11$ / $\varepsilon_a = \left|\frac{4.94 - 4.83}{4.94}\right| * 100\% =$

تابع لهذا السؤال

i	x_i
1	3.9
2	4.6
3	4.83
4	4.94

CH4:- Truncation Errors and the Taylor Series.

متوالية

Tylor Series:- $F(x) = \sum_{i=n+1}^{\infty} \frac{(x-x_0)^i f^i(x_0)}{i!}$

Reminder تعبير infinite series, لإيجاد قيمها نستخدم :-

$R_n = F(x)_{exact} - F_n(x)$.

N O T E B O O K

Ex:- $F(x) = \sin(x)$ in a Taylor series and use it to approximate the value of $\sin(2)$, using the value of $f(x)$ and it's derivatives at 0 using 4 significant figure. The value of $f(x)$ and it's 1st three derivative at 0?

إشتقاقات

Sol:- $x = 2$, $x_0 = 0$

Three derivation $\rightarrow n = 0, 1, 2, 3$ ٣ إشتقاقات

* نشتق ٣ مرات

$F^{0}(0) = \sin(0) = 0$

$F'(0) = \cos(0) = 1$ $\longrightarrow F(x) = \sum_{i=0}^{n} \frac{F^{n}(x_0)(x-x_0)^{i}}{i!}$

$F''(0) = -\sin(0) = 0$

$F'''(0) = -\cos(0) = -1$

$$\frac{0 \times (2-0)^0}{0!} + \frac{1 \times (2-0)^1}{1!} + \frac{0 \times (2-0)^2}{2} + \frac{(2-0)^3 \times -1}{3!}$$

$$F(2) = 0 + 2 + 0 - \frac{8}{6} = 0.6667$$

وعشان أحسب الإيرور لازم يكون عندي قيمة exact value بإستخدام الآلة الحاسبة

نعوض في $\sin(2)$ وهي عبارة عن exact.

$\sin(2) = 0.909$, $E_t = |0.909 - 0.6667| = 0.2423$

$$\varepsilon_t = \left|\frac{0.909 - 0.6667}{0.909}\right| * 100\% = 26.65\%$$

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Ex:- $F(x) = \sin(x) + x^2$ in a Taylor series and use it to approximate the value of $F(\frac{\pi}{3})$ using the value of $f(x)$ and it's derivatives at $\frac{\pi}{4}$, using a $\underline{4}$ significant figure calculation, the value of $f(x)$ and it's 1^{st} three derivatives at $\frac{\pi}{4}$?

sol:- $x_0 = \frac{\pi}{4}$, Three derivations :- $n = 0, 1, 2, 3$

إشتقاق ٣ مرات :-

$$F^{0}(\tfrac{\pi}{4}) = \sin(x) + x^2 = \sin(\tfrac{\pi}{4}) + \tfrac{\pi}{4}^2 = 1.324.$$

$$F'(\tfrac{\pi}{4}) = \cos(x) + 2x = \cos(\tfrac{\pi}{4}) + 2 * \tfrac{\pi}{4} = 2.278$$

$$F''(\tfrac{\pi}{4}) = -\sin(x) + 2 = -\sin(\tfrac{\pi}{4}) + 2 = 1.293$$

$$F'''(\tfrac{\pi}{4}) = -\cos(x) = \cos(\tfrac{\pi}{4}) = -0.7070.$$

* وبنعوض هلأ على هذا القانون :-

$$F(x) = \sum_{i=0}^{n} \frac{F^{n}(x_0)(x-x_0)^i}{i!}$$

$$F(\tfrac{\pi}{3}) = \frac{(\frac{\pi}{3} - \frac{\pi}{4})^0 * \overset{F^{0}(x_0)}{1.324}}{0!} + \frac{(\frac{\pi}{3} - \frac{\pi}{4})^1 * \overset{F'(x_0)}{2.278}}{1!}$$

$$+ \frac{(\frac{\pi}{3} - \frac{\pi}{4})^2 * \overset{F''(x_0)}{1.293}}{2!} + \frac{(\frac{\pi}{3} - \frac{\pi}{4})^3 * \overset{F'''(x_0)}{-0.7070}}{3!}$$

$$F(\tfrac{\pi}{3}) = 1.963$$

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Ex:- calculate R_4 for T.s approximation of $F(x) = \cos(x)$ at $x=2$ use $x_0 = 0$?

يعني 4 إشتقاقات

Sol:- إشتقاقات 4 مرات

$F^{0}(0) = \cos(0) = 1$

$F'(0) = -\sin(0) = 0$

$F''(0) = -\cos(0) = -1$

$F'''(0) = \sin(0) = 0$

$F^{4}(0) = \cos(0) = 1$

$$F(2) = \frac{(2-0)^0 * 1}{0!} + \frac{(2-0)^1 * 0}{1!} + \frac{(2-0)^2 * -1}{2!} + \frac{(2-0)^3 * 0}{3!} + \frac{(2-0)^4 * 1}{4!}$$

($x - x_0$)

$F(2) = -0.3333$

$F(2)_{exact} = -0.4161$ → عبارة عن $\cos(2)$

$R_4 = f(2) - f_4(2) = -0.416 - (-0.3333) = -0.0828$ E_t

* Total numerical تعتبر مزيج او مجموع

Total numerical error = Turncation error + Round off error

Ex:- calculate $R_4(1)$ for $\cos(1)$ using $x_0 = 0$, step size $= h = x - x_0$

$$F_4(1) = \frac{(1-0)^0 * 1}{0!} + 0 + \frac{(1-0)^2 * -1}{2!} + 0 + \frac{(1-0)^4 * 1}{4!} = 0.5416$$

مشتقه الرابعة

$F(1)_{exact} = 0.5403$ / $R_4 = f(1)_{exact} - f_4(1) = 0.5403 - 0.5416 = -0.0013$

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Chapter 5:- Roots Equations "Bracketing methods"

إيجاد الجذور

* $F(x) = x^2 - 4x - 5$, $x^2 - 4x - 5 = 0 \rightarrow x = 5, x = -1.$

* $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$

* Ex:- $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$

$\sin x + x = 0$ لكن لا يمكن إيجاد الجذور لذلك نستخدم "numerical methods"

$f(x) = e^x + \tan(x)$

* Bracketing methods:-

1. Bisection:-

$x_r = \frac{x_L + x_u}{2} \rightarrow r:$ root $\leftarrow$ قانون

If:- $F(X_r) * F(X_u) = 0$, $X_r \rightarrow$ is the root.

< 0, Root is located (X_r, X_u)

> 0, " " " (X_L, X_r)

NOTEBOOK

Ex:- Find an estimate for $\sqrt[4]{25}$ within 0.05 from actual root starting within [2, 3] and using five signification figure?

عبارة على شكل معادلة

Sol:- $x = \sqrt[4]{25} \longrightarrow x^4 = 25 \rightarrow x^4 - 25$ $\quad F(x) = x^4 - 25$

$[2, 3]$ ($x_L = 2$, $x_u = 3$) $\quad F(x_L) = 2^4 - 25 = -9$, $F(x_u) = 3^4 - 25 = 56$

$\rightarrow F(x_L) * F(x_u) < 0 \rightarrow$ correct methods

$x_r = \frac{2 + 3}{2} = 2.5$

نغيرها لأنه $f(x_r)$ موجب

نغيرها لأنه تغير اشارة

نقدر خلال التعويض في المعادلة

i	x_L	$f(x_L)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_a %
1	2	-9	3	56	2.5	14.0625	—
2	2	-9	2.5	14.0625	2.25	0.6289	0.25 %
3	2	-9	2.25	0.6289	2.125	-4.609	0.125 %
4	2.125	-4.609	2.25	0.6289	2.1875	-2.1022	0.0625 %
5	2.1875	-2.1022	2.25	0.6289	2.21875	-0.76553	0.03125

st < 0.05

نغير لأنه $f(x)$ سالب

$x_r = 2.21875$

ممكن تيجي في دائرة

(* [illegible] لو في السؤال معطيني maximum error بقدر أعرف عدد الـ iteration)

$* \ln(2^n) \left(\frac{\Delta x}{\varepsilon_s}\right) \xrightarrow{\text{اشتقاق}} \dfrac{\ln\left(\frac{x_u - x_L}{\varepsilon_s}\right)}{\ln(2)} \xrightarrow{\text{تعويض}} n = \dfrac{\ln\left(\frac{3-2}{0.05}\right)}{\ln(2)} = 4.32 \simeq 5$

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Ex:- How many iteration are required in Bisection method to acheive an error of 0.025, if $X_u = 6$, $X_L = 2$?

$$n = \frac{\ln\left(\frac{X_u - X_L}{\varepsilon_s}\right)}{\ln(2)} \quad \varepsilon_s \rightarrow \text{error} \longrightarrow n = \frac{\ln\left(\frac{6-2}{0.025}\right)}{\ln(2)} = 7.321 \simeq 8.$$

تقريب للعدد الأكبر.

* False position methods:- ← درستها أعلى

$$\boxed{X_r = X_u - \frac{F(X_u)(X_u - X_L)}{F(X_u) - F(X_L)}} \rightarrow \text{قانون}$$

if:- $F(X_r) * F(X_u) = 0$, X_r is the Root.

< 0, Root is located within (X_r, X_u)

> 0, " " " " (X_L, X_r)

Ex:- Find the intersection point between the two function $F_1(X) = e^x$, $F_2(x) = x + 2$ starting with $[0, 1.5)$ using 5 figure place, (Error < 0.02)

$[0 \rightarrow X_L$, $1.5 \rightarrow X_u$

Sol:- $e^x = x + 2 \rightarrow e^x - x - 2 = 0 \rightarrow F(x) = e^x - x - 2$

$F(X_L) = F(0) = e^0 - 0 - 2 = -1$

$F(X_u) = F(1.5) = e^{1.5} - 1.5 - 2 = 0.98169$

أول تكرار $\rightarrow X_r = 1.5 - \frac{(0.98169)(1.5 - 0)}{(0.98169) - (-1)} = 0.7569$

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i	x_L	$F(x_L)$	x_u	$f(x_u)$	x_r	$f(x_r)$	error
1	0	-1	1.5	0.98169	0.7569	-0.6252	—
2	0.7569	-0.6252	1.5	0.98169	1.1318	-0.03039	0.3749
3	1.1318	-0.03039	1.5	0.98169	1.1428	-7.264×10^{-3}	0.011 < 0.02

$X_r = 1.428$

Ex:- Find the root of the following equation:-

$F(x) = -2 + 7x - 5x^2 + 6x^3$?

A- by using Bisection

$x_L = 0, \; x_U = 1, \; \varepsilon_s = 10\%$

$F(x_L) = -2 + 7(0) - 5(0)^2 + 6(0)^3 = -2$

$F(x_u) = -2 + 7(1) - 5(1)^2 + 6(1)^3 = 6$

$x_r = \dfrac{0 + 1}{2} = 0.5$

i	x_L	x_u	x_r	$F(x_L)$	$F(x_u)$	$F(x_r)$	ε_a
1	0	1	0.5	-2	6	1	—
2	0	0.5	0.25	-2	1	-0.46875	100%
3	0.25	0.5	0.375	-0.46875	1	0.23828	33.3%
4	0.25	0.375	0.3125	-0.46875	0.23828	-0.117676	20%
5	0.3125	0.375	0.34375	-0.117676	0.23828	0.059143	9.1% < 10%

$X_r = 0.34375$

NOTEBOOK

Ex:- using False Position

$F(x_L) = -2$, $F(x_u) = 6$ / $F(x_L) * F(x_u) < 0$ → correct method

$$x_r = x_u - \frac{F(x_u)(x_u - x_L)}{F(x_u) - F(x_L)} = 1 - \frac{6*(1-0)}{6--2} = 0.25$$

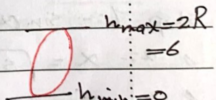
i	x_L	x_u	x_r	$F(x_L)$	$F(x_u)$	$F(x_r)$	ε_a
1	0	1	0.25	-2	6	-0.4688	—
2	0.25	1	0.304	-0.4688	6	-0.166	17.7%
3	0.304	1	0.32	-0.166	6	-0.06	5% < 10% SL

$X_r = 0.32$

Ex:- The volume of water in a spherical tank is given by:- $V = 2\pi h^2 * \left[\frac{3R-h}{3}\right]$, if $V = 30\,m^3$, $R = 3\,m$, use Bisection method to estimate water heigh (h), Solve 4 iterations and calculate E_a and ε_a for each iteration

تعويض قيمة v

Sol:- $30 - \left[2\pi h^2 * \left[\frac{9-h}{3}\right]\right]$



$h_L = 0$, $h_u = 6$, $F(h_L) = 30$, $h_u = -196.19$

$X_r = 3$

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Bisection

i	x_L	x_u	x_r	$F(x_L)$	$F(x_u)$	$F(x_r)$	error	ε_a
1	0	6	3	30	-196.194	-83.097	—	—
2	0	3	1.5	30	-83.097	-5.3429	-1.5	100%
3	0	1.5	0.75	30	-5.3429	20.2806	-0.75	100%
4	0.75	1.5	1.125	20.2806	-5.3429	9.1255	0.375	33.3%

$x_r = 1.125$

* Chapter 6:- Roots Equation "open method".

Ex:- Rearrange

* $\sin(x) - x = 0 \rightarrow x = \sin(x) \rightarrow x_{i+1} = \sin(x_i)$

* $\sqrt{\cos(x)} \longrightarrow \sqrt{\cos(x)} \longrightarrow$ $x = \sqrt{\cos(x)} + x \rightarrow x_{i+1} = \sqrt{\cos(x_i)} + x_{i+1}$

Ex:- ~~F(x)~~ using the fixed point iteration, Determine the Root of the function $\sin(x) - x^2 = 0$, using $x_0 = 1$ as an initial guess and 0.1 % tolerance $\rightarrow \varepsilon_s$

Sol:-

Step 1:- $\sin(x) - x^2 = 0$

〃 2:- $x = \sqrt{\sin(x)}$

〃 3:- $x_{i+1} = \sqrt{\sin(x_i)}$

Radian.

$i = 0 \rightarrow x_{0+1} = \sqrt{\sin(x_0)} \rightarrow x_1 = \sqrt{\sin(1)}$

$x_1 = 0.9173$

$$\varepsilon_a = \left| \frac{0.9173 - 1}{0.9173} \right| * 100\% = 9.0155\% \text{ not} < 0.1\%$$

تتمة الفرع السابق

تكملة المثال السابق

$* \; i=1 \rightarrow x_{1+1} = \sqrt{\sin(x_1)} \rightarrow x_2 \rightarrow \sqrt{\sin(x_1)}$

$x_2 = \sqrt{\sin(0.9173)} = x_2 = 0.8911$

$\varepsilon_a = \left| \dfrac{0.8911 - 0.9173}{0.8911} \right| * 100\% = 2.94\% \text{ not} < 0.1\%$

$* \; i=2 \rightarrow x_{2+1} = \sqrt{\sin(x_2)} \rightarrow x_3 = \sqrt{\sin(x_2)} \rightarrow x_3 = \sqrt{\sin(0.8911)}$

$x_3 = 0.8819$

$\varepsilon_a = \left| \dfrac{0.8819 - 0.8911}{0.8819} \right| * 100\% = 1.043\% \text{ not} < 0.1\%$

* تجد الإيرور تقل مع عدد i يعني معادلتي صحيحة

$* \; i=3 \rightarrow x_{3+1} = \sqrt{\sin(x_3)} \rightarrow x_4 = \sqrt{\sin(x_3)}$

$x_4 = \sqrt{\sin(0.8819)} = x_4 = 0.8786$

$* \; i=4 \rightarrow x_{4+1} = \sqrt{\sin(x_4)} \rightarrow x_5 = \sqrt{\sin(x_4)} \rightarrow x_5 = \sqrt{\sin(0.8786)}$

$x_5 = 0.8774$

$\varepsilon_a = \left| \dfrac{0.8774 - 0.8786}{0.8774} \right| * 100\% = 0.1367\% \text{ not} < 0.1\%$

$* \; i=5 \rightarrow x_{5+1} = \sqrt{\sin(x_5)} \rightarrow x_6 = \sqrt{\sin(x_5)} \rightarrow x_6 = \sqrt{\sin(0.8774)}$

$x_6 = 0.8770$

$\varepsilon_a = \left| \dfrac{0.8770 - 0.8774}{0.8770} \right| * 100\% = 0.045\% < 0.1\%$

يحقق الشرط

$X = 0.8770$

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If:- $\left|\frac{dy}{dx}\right| < 1$, converage for any value.

≈ 1, // slowly.

> 1, only x_0 should be chosen carefully.

Ex:- using the fixed point iteration, Determine the Root of $f(x) = -x^2 + 1.8x + 2.5$, using $x_0 = 5$ as an initial guess perform until $\varepsilon_s = 0.05\%$.

Sol:- $-x^2 + 1.8x + 2.5 = 0$ أول خطوة

① $x = \sqrt{1.8x + 2.5}$, ② $x = \frac{x^2 - 2.5}{1.8}$, ③ $x = \frac{-2.5}{x - 1.8}$

عندي في جذور

نعوض في

$* \; x = \frac{x^2 - 2.5}{1.8} \xrightarrow{\text{إشتقاق}} \frac{2x - 0}{1.8^2} = 3.08 > 1$:- إشتقاق

$* \; x = \sqrt{1.8x + 2.5} \xrightarrow{\text{اشتقاق}} \frac{1.8}{2\sqrt{1.8x + 2.5}} = 0.26 < 1$

نستخدم هذه المعادلة لأنها أقل من واحد

$x_{i+1} = \sqrt{1.8x_i + 2.5}$

$i = 0 \rightarrow x_{0+1} = \sqrt{1.8x_0 + 2.5} \rightarrow x_1 = \sqrt{1.8(5) + 2.5} = x_1 = 3.3911$

$\varepsilon_a = \left|\frac{3.3911 - 5}{3.3911}\right| * 100\% = 47.4\%$ not $< 0.05\%$

$i = 1 \rightarrow x_{1+1} = \sqrt{1.8x_1 + 2.5} \rightarrow x_2 = \sqrt{1.8(3.3911) + 2.5}$

$x_2 = 2.933$, $\varepsilon_a = \left|\frac{2.933 - 3.3911}{2.933}\right| * 100\% = 15.6$ not $< 0.05\%$

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$i=2 \rightarrow x_{2+1} = \sqrt{1.8x_2 + 2.5} \rightarrow x_3 = \sqrt{1.8(2.933) + 2.5} \rightarrow x_3 = 2.789$

$\varepsilon_a = \left| \frac{2.789 - 2.933}{2.789} \right| * 100\% = 5.1$ not $< 0.05\%$

* $i=3 \rightarrow x_{3+1} = \sqrt{1.8x_3 + 2.5} \rightarrow x_4 = \sqrt{1.8(2.789) + 2.5}$

$x_4 = 2.742$ $\varepsilon_a = \left| \frac{2.742 - 2.789}{2.742} \right| * 100\% = 0.0168 < 0.05\%$ ✓

Root $= 2.742$

* Newton Raphson methods :-

Rearrange $\rightarrow x_{i+1} = x_i - \frac{F(x_i)}{F'(x_i)}$

Ex:- $F(x) = x^2 - 1 \longrightarrow x_{i+1} = x_i - \frac{x_i^2 - 1}{2x_i}$

$F(x) = x^2 + 2x + 4 \rightarrow x_i - \frac{x_i^2 + 2x_i + 4}{2x_i + 2}$

عدد الطريقة

Ex:- Using the newton Rasphon methods, determine the Root, $f(x) = \sin(x) - x^2$ using $x_0 = 1$ as an initial guess and 0.1% tolerance?

اشتقاق

تكملة الحل

Sol:- $x_{i+1} = x_i - \frac{\sin(x_i) - x_i^2}{\cos(x_i) - 2x_i}$

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$i=0 \rightarrow x_{0+1} = x_0 - \frac{\sin(x_0) - x_0^2}{\cos(x_0) - 2x_0} \rightarrow x_1 = (1) - \frac{\sin(1) - (1)^2}{\cos(1) - 2(1)}$

$x_1 = 0.8914$

$\varepsilon_a = \left| \frac{0.8914 - 1}{0.8914} \right| * 100\% = 12.18\%$ not $< 0.1\%$

$i=1 \rightarrow x_{1+1} = x_1 - \frac{\sin(x_1) - x_1^2}{\cos(x_1) - 2x_1} \rightarrow x_2 = (0.8914) - \frac{\sin(0.914) - (0.8914)^2}{\cos(0.8814) - 2(0.8914)}$

$x_2 = 0.8870$, $\varepsilon_a = \left| \frac{0.8770 - 0.8914}{0.8770} \right| * 100\% = 1.642\%$ not $< 0.1\%$

$i=2 \rightarrow x_{2+1} = x_2 - \frac{\sin(x_2) - x_1^2}{\cos(x_1) - 2x_1} \rightarrow x_3 = (0.8870) - \frac{\sin(0.8770) - (0.877)^2}{\cos(0.8770) - 2(0.8770)}$

$x_3 = 0.8767$, $\varepsilon_a = \left| \frac{-0.8767 - 0.8770}{0.8767} \right| * 100\% = 0.03\% < 0.1$

* The root $= 0.8767$.

N O T E B O O K

*** Secant methods:-**

$$X(i+1) = x_i - \frac{(F(x_i))(x(i-1) - x_i)}{F(x_i - 1) - F(x_i)}$$ → قانون

هذه الطريقة

Ex:- using the secant methods, determine the Root of the function $f(x) = \sin(x) - x^2$, using $x_{-1} = 1$, $x_0 = 2$ as an initial guess and 0.1% tolerance?

Sol:- $$x_{(i+1)} = x_i - \frac{(F(x_i))(x_{(i-1)} - x_i)}{F(x_i - 1) - F(x_i)}$$

i=0 $$\to x_0 - \frac{(F(x_0))(x_{(0-1)} - x_0)}{F(x_0 - 1) - F(x_0)}$$

$F(x_{-1}) = \sin(1) - 1^2 = -0.1585$

$F(x_0) = \sin(2) - 2^2 = -3.091$

ثم نعوض في المعادلة

$$x(1) = 2 - \frac{(-3.091)(1-2)}{-0.1585-(-3.091)} = 0.946$$

$$\varepsilon_a = \left|\frac{0.946 - 2}{0.946}\right| * 100\% = 111.4\% \text{ not} < 0.1\%$$

$F(x_1) = \sin(0.946) - 0.946^2 = -0.08383$

$F(x_0) = \sin(2) - 2^2 = -3.091$

تكملة →

i=1 $$\to x_1 - \frac{(F(x_1))(x_{(1-1)} - x_1)}{F(x_1 - 1) - F(x_1)}$$ ⇒ [redacted]

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$$x_{(1)} = 0.946 - \frac{(-0.08383)(2-0.946)}{(-3.091)-(-0.08383)} = 0.9166$$

$$\varepsilon_a = \left| \frac{0.9166 - 0.946}{0.9166} \right| * 100\% = 3.208\% \text{ not } < 0.1\%$$

N O T E B O O K

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matrix operation.

Example:-

1) $5x + 3y = 2$
2) $4x + 5y = 13$
3) $3x + 4y = 20$

علشان أحلهم لازم تتحول لـ matrix

$$\begin{bmatrix} 5 & 3 \\ 4 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \\ 20 \end{bmatrix}$$

* $A_{m \times L}$ → the matrix size, $A_{3\times3}$ → مصفوفة ثلاثية

Ex:- $A = \begin{bmatrix} 3 & 4 & 7 \\ 0 & -1 & 10 \\ 4 & 6 & 3 \end{bmatrix}$ Find a_{32}? $\boxed{6}$

الصف الثالث ← 3, 2 → العمود الثاني

جمع و طرح

* Addition and subtraction:- (جمع وطرح المصفوفات)

* To add or subtract, they must be the Same size *

* $A + B = B + A$, $A - B \neq B - A$

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Ex:- Add the matrixes:-

$$\overset{3*3}{\begin{bmatrix} 4 & -3 & 1 \\ 0 & 5 & -2 \\ 5 & -6 & 0 \end{bmatrix}} + \overset{3*3}{\begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix}} = \begin{bmatrix} \overset{4+-1}{3} & \overset{-3+2}{-1} & \overset{1+3}{4} \\ 6 & -2 & 7 \\ 5 & -10 & 8 \end{bmatrix}$$

Ex:- Add the matrix :-

$$A = \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} , B = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 6 & 7 \end{bmatrix}$$

Ex:- subtrac the two matrix:-

$$\begin{bmatrix} 4 & -3 & 1 \\ 0 & 5 & -2 \\ 5 & -6 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix} = \begin{bmatrix} 5 & -5 & -2 \\ -6 & 12 & -11 \\ 5 & -2 & -8 \end{bmatrix}$$

Scalar multiplication:- ضرب المصفوفة في عدد ثابت

المصفوفة

$KA = [ka_{ij}]$

العدد الثابت

نضربها في عدد

Ex:- Find (-1) A where $A = \begin{bmatrix} -1 & 2 & 3 \\ 6 & -1 & 9 \\ 0 & -4 & 8 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -6 & 7 & -9 \\ 0 & 4 & -8 \end{bmatrix}$

N O T E B O O K

* Matrix multiple :- عملية الضرب بين مصفوفتين

وعندي شرط عدد الأعمدة في A = عدد الصفوف في B

$$* \; A_{3*2} * B_{2*4} = C_{3*4}$$

Ex:- $A_{3*2} = \begin{bmatrix} \boxed{1 \quad 5} \\ 6 \quad 2 \\ 4 \quad 1 \end{bmatrix}$, $B_{2*2} = \begin{bmatrix} \boxed{\begin{matrix}2 \\ 4\end{matrix}} & \begin{matrix}1 \\ 3\end{matrix} \end{bmatrix} \rightarrow \begin{bmatrix} 22 & 16 \\ 20 & 12 \\ 12 & 7 \end{bmatrix}$

$1\times2+4\times5=22$

$1*1+\cancel{?}3*5$

$6*1+2*6$

$6*2+2*4$

$$\boxed{A * B \neq B * A}$$

---

* Matrix Inverse A^{-1} :-

$$A_{3*3} * A^{-1}_{3*3} = I_{3*3} \xrightarrow[\text{matrix}]{\text{unity}} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Matrix Transpose :-

$$A_{3*2} \rightarrow A_{2*3}$$

Ex:- $\underbrace{A = \begin{bmatrix} 1 & 5 \\ 6 & 2 \\ 4 & 3 \end{bmatrix}}_{3*2} \rightarrow A^T = \underbrace{\begin{bmatrix} 1 & 6 & 4 \\ 5 & 2 & 3 \end{bmatrix}}_{2*3}$

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Row operation :-

A- Row Swapping (تبديل صفوف)

$$\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix}$$

B- multiapplication by Factor (k)

(ضرب الصف الأول في 3)

$$A_{2*2} = \begin{bmatrix} 2*3 & 1*3 \\ 6 & 6 \end{bmatrix}$$

C- Raw addition:-

$2 * Row\,1 + Row\,2 = R_2$

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 \\ (2*2)+6 & (1*2)+6 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 10 & 5 \end{bmatrix}$$ ✓

(CH9)

* **Gauss Elimination:-** تحويل النظام من صيغة معادلات إلى صيغة ماتريكس.

Ex:-

$$4x_1 + 2x_2 + 3x_3 = 5$$
$$10x_1 + x_2 + 4x_3 = 2$$
$$5x_1 + 3x_2 + 5x_3 = 1$$

$$\begin{array}{c} \\ \left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & b \\ 4 & 2 & 3 & 5 \\ 10 & 1 & 4 & 2 \\ 5 & 3 & 5 & 1 \end{array}\right] \end{array} \rightarrow \text{step 1}$$

$$\downarrow$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \rightarrow \text{step 2}$$

*upper Triangle.

$$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \rightarrow \text{Lower Triangle}$$

$$a_{33} * x_3 = b_3 \rightarrow \text{step 3}$$

$\vdots$

Ex:- Solve the following system using Gaussian :-

$x_1 + x_2 + 6x_3 = 7$

$-x_2 + 2x_2 + 9x_3 = 2$

$x_1 - 2x_2 + 3x_3 = 10$

Sol:- هاي هي الركيزة

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \quad b$$

$$\begin{bmatrix} 1 & 1 & 6 \\ -1 & 2 & 9 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 2 \\ 10 \end{bmatrix}.$$

$* -\left(\frac{-1}{1}\right) * R_1 + R_2$

الركيزة

$* -\left(\frac{1}{1}\right) * R_1 + R_3$

الركيزة

$$\begin{bmatrix} 1 & 1 & 6 \\ 0 & 3 & 15 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 6 \\ 0 & 3 & 15 \\ 0 & -3 & -3 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \\ 3 \end{bmatrix}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \quad b$$

$$\begin{bmatrix} 1 & 1 & 6 \\ 0 & 3 & 15 \\ 0 & 0 & 12 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \\ 3 \end{bmatrix} \qquad * -\left(\frac{-3}{3}\right) * R_2 + R_3$$

تكملة

$12x_3 = 12 \rightarrow x_3 = 1$

$3x_2 + 15x_3 = 9 \rightarrow 3x_2 + 15 = 9 \rightarrow x_2 = -2$

$x_1 + x_2 + 6x_3 = 7 \rightarrow x_1 + -2 + 6*1 = \; x_1 = 3$

$x_3 = 1, \; x_2 = -2, \; x_1 = 3.$

Ex:-

$$x_1 + x_2 + x_3 = 6$$
$$2x_1 + 3x_2 + 4x_3 = 20$$
$$3x_1 - 4x_2 + 2x_3 = 17$$

Sol:-

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & -4 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 20 \\ 17 \end{bmatrix}$$

$-\left(\frac{2}{1}\right) * R_1 + R_2 \rightarrow R_2 \text{ new}.$

$-\left(\frac{3}{1}\right) * R_1 + R_3 \Rightarrow R_3 \text{ new}.$

* upper triangle

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ -1 \end{bmatrix}$$

$-\left(\frac{1}{1}\right) * R_2 + R_3 \rightarrow R_3 \text{ new}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ -1 \end{bmatrix}$$

$-3x_3 = -9 \rightarrow x_3 = 3$

$x_2 + 2x_3 = 9 \rightarrow x_2 = 2$

$x_1 + x_2 + x_3 = 6 \rightarrow x_1 = 1$

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Notes:- $x_2 + 6x_3 = 7$

$-x_1 + 2x_2 + 9x_3 = 2$

$x_1 - 2x_2 + 3x_3 = 10$

$x_1 = 0$ يعني المحورة صفر وطنونا بصير تبديل بين الصفوف وشيمي pitfalls of Gauss

1- pitfalls of Gauss:-

Ex:- $2x_2 + 3x_3 = 8$

$4x_1 + 6x_2 + 7x_3 = -3$

$2x_1 + x_2 + 6x_3 = 5.$

ما بصير تكون صفر.

$$\begin{bmatrix} 0 & 2 & 3 \\ 4 & 6 & 7 \\ 2 & 1 & 6 \end{bmatrix} \begin{bmatrix} 8 \\ -3 \\ 5 \end{bmatrix} \xrightarrow{\text{تبديل صفوف}} \begin{bmatrix} 4 & 6 & 7 \\ 0 & 2 & 3 \\ 2 & 1 & 6 \end{bmatrix} \begin{bmatrix} -3 \\ 8 \\ 6.5 \end{bmatrix}$$

أكمل الحل الحالتين ٨,٨

F.N = $x_3 = 2.64$, $x_2 - 0.045$, $x_1 = -5.43$.

2- Singular system :- Det = 0

no solution or infinite solution.

$$\left[\begin{array}{cc|c} 2 & -2 & 4 \\ 4 & -4 & 8 \end{array}\right]$$

* كيفية حساب الـ Determetaion :-

* $(2 * -4) - (4 * -2) \rightarrow Det = 0$

* singular system.

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*infinite solution $\Rightarrow \left(\frac{-4}{2}\right) R_1 + R_2$.

$$\begin{bmatrix} 2 & -2 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$[\,0 \quad 0 \quad 0\,] \rightarrow$ infinite solution.

B- system with no solution:-.

$$\begin{bmatrix} 2 & -2 \\ 4 & -4 \end{bmatrix} \rightarrow \left(\frac{-4}{2}\right) R_1 + R_2 \rightarrow R_2 \text{ new.}$$

$$\left[\begin{array}{cc:c} 2 & -2 & 4 \\ 4 & -4 & 6 \end{array}\right]$$

$[\,0 \quad 0 \quad -2\,] \rightarrow 0x_1 + 0x_2 = -2$

$0 = -2$? no solution.

3- ill condition system.

$$\begin{array}{cc} x_1 & x_2 \end{array}$$

$$\left[\begin{array}{cc:c} 100 & -101 & 200 \\ 200 & -200 & 300 \end{array}\right] \rightarrow x_1 = -48.5 ,\ x_2 = -50$$

(تغير قيم x_1)

$$\left[\begin{array}{cc:c} 100 & -99 & 200 \\ 200 & -200 & 300 \end{array}\right] \rightarrow x_1 = 51,5 ,\ x_2 = 50$$

* بحسب ال det إذا كان مقاربة للصفر فهذا يعني ع ill condition ، ولو كانت أعلى فهذا يعني إنها well condition

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Ex:-

$$\begin{bmatrix} \frac{100}{100} & \frac{-99}{100} \\ \frac{200}{200} & \frac{-200}{200} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -0.99 \\ 1 & -1 \end{bmatrix}$$

$(-0.99)-(-1) = 0.01 \approx 0$

ill condition

Ex:- is this an ill condition or not?

$$\begin{bmatrix} 3 & 1 & 2 \\ -2 & 6 & 4 \\ 1 & -3 & 7 \end{bmatrix} \rightarrow$$ det حساب كيفية لمصفوفة ثلاثية.

$$\begin{bmatrix} \frac{3}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{-2}{6} & \frac{6}{6} & \frac{4}{6} \\ \frac{1}{7} & \frac{-3}{7} & \frac{7}{7} \end{bmatrix} \rightarrow$$ |العامل|

نسترشيهم

$$\begin{bmatrix} 1 & -0.333 & 0.666 \\ -0.333 & 1 & 0.666 \\ \frac{1}{7} & \frac{-3}{7} & 1 \end{bmatrix} \rightarrow \text{Det} = 1.4 \text{ well condition.}$$

$1 * [(1*1) - (\frac{-3}{7} * 0.666)] - 0.33 * [(-0.333*1) - (0.666 * \frac{1}{7})]$

$- 0.666 [(-0.33 * \frac{-3}{7}) - (1 * \frac{1}{7})]$

N O T E B O O K

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Chapter 10 Lu Decomposition.

$$\left.\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1. \\ 2x_1 - 3x_2 + 4x_3 &= 2. \\ 5x_1 + 2x_2 + 3x_3 &= 9. \end{aligned}\right] \rightarrow$$

تحويلها إلى

$\bar{A}\bar{x} = \bar{b}$

مصفوفة يعني

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$$

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Lower

$$\begin{bmatrix} a_{11} & a_{21} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

upper.

نفرض إنهم قطرياً واحد.

$L_{11} = 1$, $L_{22} = 1$, $L_{33} = 1$

Example: by using LU Decomposition, find the value of x_1, x_2, x_3?

على شكل مصفوفة جاهزة. ←
أي حتكتب بعضي على شكل 3 معادلات.

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}.$$

sol:-

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & 1 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

تكملة الحل →

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$L_{11} = 1$, $L_{22} = 1$, $L_{33} = 1$.

$(U_{11} * L_{11}) + (0 * 0) + (0 * 0) = 2$, $U_{11} = 2$

$(U_{12} * L_{11}) + (U_{22} * 0) + (0 * 0) = 3$, $U_{12} = 3$

$(U_{13} * L_{11}) + (U_{23} * 0) + (U_{33} * 0) = -2$, $U_{13} = -2$.

$(U_{11} * L_{21}) + (0 * L_{22}) + (0 * 0) = 1$, $L_{21} = 0.5$

$(U_{12} * L_{21}) + (U_{22} * L_{22}) + (0 * 0) = 2$, $U_{22} = 0.5$

$(U_{13} * L_{21}) + (U_{23} * L_{22}) + (U_{33} * 0) = 3$, $U_{23} = 4$

$(U_{11} * L_{31}) + (0 * L_{32}) + (0 * L_{33}) = 5$, $L_{31} = 2.5$

$(U_{12} * L_{31}) + (U_{22} * L_{32}) + (0 * L_{33}) = 4$, $L_{32} = -7$

$(U_{13} * L_{31}) + (U_{23} * L_{32}) + (U_{33} * L_{33}) = -1$, $U_{33} = 32$

$$L \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \qquad U \begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix}$$

$U L x = b$ $\qquad$ $U * x = d$

$L * d = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \rightarrow$$

$1 d_1 = 2 \rightarrow d_1 = 2$

$0.5 d_1 + d_2 = 1 \rightarrow d_2 = 0$

$2.5 d_1 - 7 d_2 + d_3 = 2$

N O T E B O O K

$d_3 = -3$

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$$\underbrace{\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}}_{d}$$

$32x_3 = -3 \rightarrow x_3 = 0.09375$.

$0.5x_2 + 4x_3 = 0 \rightarrow x_2 = 0.75$.

$2x_1 + 3x_2 - 2x_3 = 2 \rightarrow x_1 = -2.7182$.

حل نفس السؤال , بس بطريقة ثانية (Gauss) .

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix} \begin{matrix} \rightarrow \frac{-1}{2} * R_1 + R_2 = [0 \quad 0.5 \quad 4] \\ \rightarrow \frac{-5}{2} * R_1 + R_3 = [0 \quad -3.5 \quad 4] \end{matrix}$$

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & -3.5 & 4 \end{bmatrix} \rightarrow -\frac{(-3.5)}{0.5} * R_2 + R_3 = [0 \quad 0 \quad 32]$$

upper

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix} U$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{5}{2} & \frac{-3.5}{0.5} & 1 \end{bmatrix} L$$ ونكمل الحل بطريقة Gauss

Lower

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Calculating matrix Inverse using LU Decomposition

$A * A^{-1} = I \rightarrow$ unity matrix.

Ex:- Calculate the 3^{rd} column of the inverse matrix for

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix} \rightarrow A$$

sol:- $A x_3 = b_3$

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\rightarrow b_3$ لي

LU Decom... استخدام!

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix}$$

* $Ld = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow \begin{aligned} d_1 &= 0 \\ d_2 &= 0 \\ d_3 &= 1 \end{aligned}$$

$\longrightarrow$

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$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$x_3 = 0.03125$, $x_2 = -0.125$, $x_1 = 0.40625$

* لوطلب مني ال 2nd coloum ناخذ هذا العامود $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Jacobi and Gauss seidel iteration. (chapter II).

Example:- solve the following system of linear algebraic equation using Gauss seidal method and starting with the initial guess vector $x_0 = [x_{1,0} = 0, x_{2,0} = 0, x_{3,0} = 0]$ use tolerance 0.1% for all variables.

أكبر معامل

Sol:- $6x_1 - 2x_2 + x_3 = 11$

$-2x_1 + 7x_2 + 2x_3 = 5$

$x_1 + 2x_2 - 5x_3 = -1$

$$x_1^{i+1} = \frac{11 + 2x_{2,i} - x_{3,i}}{6}$$

$$x_2^{i+1} = \frac{5 + 2x_{1,i+1} - 2x_{3,i}}{7}$$

*نعوض بالمعادلات :-

$$x_3^{i+1} = \frac{1 + x_{1,i} + 1 - 2x_{2,i}}{-5}$$

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$x_1 = (11 + 2x_2 - x_3)/6$

$x_2 = (5 + 2x_1 - 2x_3)/7$

$x_3 = (-1 - x_1 - 2x_2)/-5$

$x_1^1 = (11 + 2x_2^0 - x_3^0)/6 = (11 + 0 - 0)/6 = 1.834$

$x_2^1 = (5 + 2x_1^0 - 2x_3^0)/7 = (5 + 0 - 0)/7 = 0.7145$

$x_3^1 = (-1 - x_1^0 - 2x_2^0)/-5 = (-1 - 0 - 0)/-5 = 0.2000$

$$\varepsilon_1 = \left|\frac{x_1^1 - x_1^0}{x_1^1}\right| * 100\% = \left|\frac{1.834 - 0}{1.834}\right| * 100\% = 100\%$$

$$\varepsilon_2 = \left|\frac{x_2^1 - x_2^0}{x_2^1}\right| * 100\% = \left|\frac{0.7145 - 0}{0.7145}\right| * 100\% = 100\%$$

$$\varepsilon_3 = \left|\frac{x_3^1 - x_3^0}{x_3^1}\right| * 100\% = \left|\frac{0.2000 - 0}{0.2000}\right| * 100\% = 100\%$$

$x_1^2 = (11 + 2x_2^1 - x_3^1)/6 = (11 + 2(0.7145) - 0.2000)/6 = 2.039$

$x_2^2 = (5 + 2x_1^1 - 2x_3^1)/7 = (5 + 2(1.834) - 2(0.20000))/7 = 1.182$

$x_3^2 = (-1 - x_1^1 - 2x_2^1)/-5 = (-1 - 1.834 - 2(0.7145))/-5 = 0.856$

تكملة أول . $\longrightarrow$

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chappter 17. (تفيدعنه في عزتقشب) curve fitting

Least squares regression الطريقة الأولى
interpolation الطريقة الثانية.

$$\begin{bmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \Sigma Y_i \\ \Sigma x_i Y_i \end{bmatrix}$$ حفظ هنرة المعادلة

Example:- using linear regression fit the following data to a straight line. (linear rs)

x	2	3.25	5.1
Y	3.5	5.6	7.8

solution $n = 3$

$\Sigma x_i = 2 + 3.5 + 5.1 = 10.35$

$\Sigma x_i^2 = 2^2 + 3.25^2 + 5.1^2 = 40.57$

$\Sigma Y_i = 3.5 + 5.6 + 7.8 = 16.9$

$\Sigma x_i Y_i = 2 * 3.5 + 3.25 * 5.6 + 5.1 * 7.8 = 64.98$.

$$\begin{array}{cc} n & \Sigma x \\ \end{array}$$
$$\begin{bmatrix} 3 & 10.35 \\ 10.35 & 40.57 \end{bmatrix} = \begin{bmatrix} 16.9 \\ 64.98 \end{bmatrix}$$
$$\begin{array}{cc} \Sigma x & \Sigma x^2 \end{array}$$

when $x = 6$ →

$a_0 = 0.897$, $a_1 = 1.372$

$\hat{Y} = 0.897 + 1.372 x_i$

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* Correlation coefficient or factor R.

$$R^2 = \frac{S_t - S_r}{S_t}$$

1- if $R = 1$, (the line will pass through all the given points)

2- as R approaches 1, the fit is referred to as excellent.

3- as R approaches 0, the fit is refered to as poor

4- if $R = 0$, X and Y are independent

$$R = \frac{n\sum XY - \sum X \sum Y}{\sqrt{n \sum X^2 - (\sum X)^2}\,\sqrt{n \sum Y^2 - (\sum Y)^2}}.$$

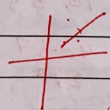
Example:- for previous example

sol.:- $n = 3$

$X \rightarrow y$

$\sum x_i = 10.35$

$\sum x_i^2 = 40.57$



$\sum Y_i = 16.9$

تعامله شغال

$\sum x_i Y_i = 64.98$

$\sum Y_i^2 = 3.5^2 + 5.6^2 + 7.8^2 = 104.45.$

$$\boxed{y = a_0 + a_1 x}$$

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$$R = \frac{n \sum XY - \sum X \sum Y}{\sqrt{n * \sum X^2 - (\sum X^2)}\sqrt{n * \sum Y^2 - (\sum Y)^2}}$$

$$= \frac{3 * 64.98 - (10.35 * 16.9)}{\sqrt{3 * (40.57) - (10.35)^2}\sqrt{3 * (104.45) - (16.9)^2}}$$

$= 0.995 \approx 1$ Excellent.

Data Linearization

Power = $\boxed{Y = a x^{B}} \to a_0 + a_1 x$

$\ln(Y) = \ln(a x^{B})$

$\ln(Y) = \ln(a) + B \ln(x)$

$Y^{\wedge} = a_0 + a_1 x_i$

Example:- using linear regression to find the value of a and B, for the following relation $\boxed{Y = a e^{Bx^2}}$ → linear

X	2	3	5
Y	4.9	8.7	13.5

sol:-

n = 3

$Y = a e^{Bx^2}$

$\ln Y = \ln a + \ln e^{Bx^2}$

$\ln Y = \ln a + B x^2$

x^2	4	9	25
$Y = \ln Y$	1.5892	2.1633	2.6026

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$n = 3 \quad \sum x_i = 38 \, , \sum x_i^2 = 722 \, ,$
$\sum Y_i = 6.3551 \, , \sum x_i Y_i = 90.8915$

$$\begin{bmatrix} 3 & 38 \\ 38 & 722 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 6.3551 \\ 90.8915 \end{bmatrix}$$

$a_0 = 1.5713 \, , \quad a_1 = 0.0431$
$a_0 = \ln a \rightarrow a = e^{a_0} = e^{1.5713} = 4.8129$
$\beta = a_1 = 0.0431$

$$\boxed{Y = 4.8129 \, e^{0.0431 \, x^2}}$$

* Polynomial Regression.

Example :- using regression analysis the 2nd order Polynomial cofficients using the following data points:-

x	2	3.25	4	5
Y	5	9	7	4.25

$Y = a_0 + a_1 x + a_2 x^2$

$$\begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum Y \\ \sum x Y \\ \sum x^2 Y \end{bmatrix}$$

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$n = 4$, $\Sigma x = 14.25$, $\Sigma Y = 25.25$, $\Sigma x^2 = 55.56$,
$\Sigma x^3 = 231.33$, $\Sigma x^4 = 1008.57$, $\Sigma xY = 88.5$, $\Sigma x^2 Y = 333.31$

$$\begin{bmatrix} 4 & 14.25 & 55.56 \\ 14.25 & 55.56 & 231.33 \\ 55.56 & 231.33 & 1008.57 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 25.25 \\ 88.5 \\ 333.31 \end{bmatrix}$$

$a_0 = 11.59$, $a_1 = 11.8$, $a_2 = -1.73$

$Y = -11.59 + 11.8x - 1.73x^2$

$\rightarrow x = 1.5 \rightarrow Y = -11.59 + 11.8(1.5) - (1.73)(1.5)^2$

multiple linear regression.

Example:- calculate a_0, a_1, a_2 for $= Y = a_0 + a_1 x_1 + a_2 x_2$ using the following data points

$\rightarrow x_1$	1	1	2
x_2	2	3	2
Y	2	5	9

$$\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \Sigma Y \\ \Sigma x_1 Y \\ \Sigma x_2 Y \end{bmatrix}$$

$n = 3$, $\Sigma x_1 = 4$, $\Sigma x_2 = 7$, $\Sigma x_1^2 = 6$, $\Sigma x_1 x_2 = 9$

$\Sigma x_1 x_2 = 9$, $\Sigma x_2^2 = 17$, $\Sigma Y = 16$, $\Sigma x_1 Y = 25$, $\Sigma x_2 Y = 37$

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$$\begin{bmatrix} 3 & 4 & 7 \\ 4 & 6 & 9 \\ 7 & 9 & 17 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 25 \\ 37 \end{bmatrix}$$

$a_0 = -11$, $a_1 = 7$, $a_2 = 3$

$y = -11 + 7x_1 + 3x_2$, $x_2 = 2$

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Chapter 18

1- Newton's divided difference. · interpolation.

Example:- using NDD interoplating polynomails estimate $F_3(2.75)$ using the following data points

3rd order

	x_0	x_1	x_2	x_3
x	0	1	2.5	3
y	2	5	9	11

$$F_n(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)$$

$n-1 = 2$

x_i	$F(x_i)$	1^{st}	2^{nd}	
0	2	$\frac{5-2}{1-0} = 3$	$\frac{2.667-3}{2.5-0} = -0.1332$	$\frac{0.6665-(-0.1332)}{3-0} = 0.267$
1	5	$\frac{9-5}{2.5-1} = 2.667$	$\frac{4-2.667}{3-1} = 0.6605$	
2.5	9	$\frac{11-9}{3-2.5} = 4$		
3	11			

$b_0 = 2,\ b_1 = 3,\ b_2 = -0.1332,\ b_3 = 0.267$

$$F_n(x) = \underbrace{2 + 3(x-0)}_{b_0+b_1} \underbrace{- 0.1332(x-0)(x-1)}_{b_2} + \underbrace{0.267(x-0)(x-1)(x-2.5)}_{b_3}$$

$F_3(2.75) = 9.932.$

The second method

2- Lagrange interpolation Polynomial

$$f_n(x) = \sum_{i=0}^{n} L_i(x)\, f(x_i), \quad \text{where} \quad L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

Example:- using the data given in the previous example estimate $f_3(2.75)$ using lagrange interpolating Polynomial

x	0	1	2.5	3
y	2	5	9	11

Sol:-

$$L_0 = \frac{(x-1)(x-2.5)(x-3)}{(0-1)(0-2.5)(0-3)}, \quad x = 2.75$$

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Numerical integration

- Numerical Cote integration Formula

1. Trapezoidal Rule.

Example :- $\int_1^4 e^{2x} \cdot dx$

$x_0 = 1, \; x_1 = 2, \; x_2 = 3, \; x_3 = 4, h,$

$= \frac{1}{2} ((e^2) + 2 \sum_{i=1}^{3-1} F(x_i) + e^8$

$= \frac{1}{2} ((e^2) + 2(e^4 + e^6) + e^8$

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2. simpsons Rule

For signal application.

That mean $\to n = 2$, $h = \frac{x_f - x_0}{n}$

$I_{1/3} = h \, (F(x_0) + 4F(x_1) + F(x_2))$

For multiple application

$I_{1/3} = \frac{h}{3} \; F(x_0) + 2 \sum_{i=2,4,6}^{n-2} \cdots F x_i + 4 \sum_{i=1,3,5}^{n-1} F x_i + F(x_f) +$

Example:-

If the question $\int_1^4 f(x).dx$

x	0	1	2	3	4	5
$F(x)$	0.5	2	4	6	7	10

single mult:-

$n=1$

$\frac{1}{2}[F(x_0) + F(x_f)] * (x_f - x_0)$ $\frac{1}{2}[8+4\cos 0]+[8+\cos\frac{\pi}{2}]$ $(\frac{\pi}{2}-0)$ $=15.7$

* use two segement trap. rule to find the integration of $f(x)=8+4\cos x$ -- between 0 and $\frac{\pi}{2}$

$$\int_0^{\pi/2} 8+4\cos x.dx$$

$$\frac{h}{2}[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i)$$

2x

I
- Trap
  - si $n=1$
  - $n>1$
- sim
  - $\frac{1}{3}$ S
    - s
    - mults
  - $\frac{3}{8}$ S
    - simi
    - M

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use simpsons 1/3 rule to integrate the $f(x) = 0.2 + 2.5x^2$
from 0 to 0.8 $\int_{0}^{0.8} 0.2 + 2.5x^2 \cdot dx$ n=2

$\rightarrow h = \frac{0.8 - 0}{2} = 0.4$

$I_{1/3} = \frac{h}{3} (f(x_0) + 4f(x_1) + f(x_2))$ 0.42

use simpsons 1/3 rule with n=4 to integrate the
$f(x) = 0.2 + 2.5x^2$ from 0 to 0.8

$\int_{0}^{0.8} 0.2 + 2.5x^2 \cdot dx$ $h = \frac{x_n - x_0}{n}$ $\frac{0.8 - 0}{4}$ n>2

$I_{1/3} = \frac{0.2}{3} [f(x_0) + 2 \sum_{i=2}^{n-2} f(x_2) + 4 \sum_{i=1}^{n-1}]$

$0.2 + 2.5(0)^2$

$[f(x_1) + f(x_3)]$

$i=0 \rightarrow x_0 = 0$
$i=1 \rightarrow x_1 = 0.2$
$i=2 \rightarrow x_2 = 0.6$
$i=3 \rightarrow 0.6$
$i=4 \rightarrow 0.8$

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Subject Date No.

numerical Differentiation

Chapter 3

equal step. unequal step

numerical Differentiation.

Equal Spaced Data

Example :- using the following data A- estimate $F''(2.25)$ using centered difference with $\boxed{O(h)^4}$ accuracy and $(h = 0.25)$

↳ error.

				x_i			
x	1.5	1.75	2	$\boxed{2.25}$	2.5	2.75	3
$F(x)$	2	4.5	7.25	9.15	12	14.25	15

Sol:- $$F''(x_i) = \frac{-F(x_i+2) + 16F(x_i+1) - 30F(x_i) + 16F(x_i-1) - F(x_i-2)}{12h^2}$$

$$F''(2.25) = \frac{-1.25 + 16 \times 12 - 30 \times 9.15 + 16 \times 7.25 - 4.5}{12(0.25)^2}$$

$$= 19.6$$

First Derivation

$$F(x_i) = \frac{F(x_i) - F(x_i - 1)}{h}$$

$$f'(x_i) = \frac{3f(x_i) - 4f(x_i - 1) + f(x_i - 2)}{2h}.$$

Second Derivation

$$f''(x_i) = \frac{f(x_i) - 2f(x_i - 1) + f(x_i - 2)}{h^2}$$

$$f''(x_i) = \frac{2f(x_i) - 5f(x_i - 1) + 4f(x_i - 2) - f(x_i - 3)}{h^3}$$

Third Derivation

$$f'''(x_i) = \frac{f(x_i) - 3f(x_i - 1) - f(x_i - 3)}{h^3}$$

$$f'''(x_i) = \frac{5f(x_i) - 18f(x_i - 1) + 24f(x_i - 2) - 14f(x_i - 3) + 3f(x_i - 4)}{2h^3}$$

Fourth Derivation

$$f''''(x_i) - 4f(x_i - 1) + 6f(x_i - 2) - 4f(x_i - 3) + f(x_i - 4)$$
$$\overline{\quad h^4 \quad}$$

$$f''''(x_i) = \frac{3f(x_i) - 14f(x - 1) + 26f(x_i - 2) - 24f(x_i - 3) + 11f(x - 4) - 2f(x_i - 5)}{h^4}$$

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B- find $f'(2)$ if $h = 0.5$, using central diffrance with $O(h)^2$

		x_i	
x	1.5	2	2.5
$F(x)$	2	7.25	12

$$F'(x_i) = \frac{F(x_{i+1}) - F(x_{i-1})}{2h}$$

$$F'(2) = \frac{12-2}{2 * 0.5} = 10$$

Example:-

Find $f''(0.5)$ for $f(x) = \cos^2(x)$, if $h = 0.15$, using forward difference with $O(h)$ error

x	0.5	0.65	0.8
$F(x)$	0.7701	0.6337	0.4854

$$F''(x_i) = \frac{F(x_{i+2}) - 2F(x_{i+1}) + F(x_i)}{h^2}$$

$$F''(x_i) = \frac{0.4854 - 2 * 0.6337 + 0.7701}{(0.15)^2} = -0.528$$

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Using centered Derivative with error oh 2

For $f(x) = 0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x$ to

Find second deriv. $x_i = 0.5$, $h = 0.25$

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

x_{i-1}	$x_i = 0.5$	$x_{i+1} = 0.75$
$(0.1)(0.25)^4$ $-0.15(0.25)^2$		

t (s)	0	1	2	$3 = x_i$
v (m/s)	0.2	1.3	34	1.7

Find the acceleration at time $t = 3$ use uniform spacing of order $h2$

Backward $h = 3 - 2 = 1$

$$f'(v) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2 \times 1}$$

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using equal space Data

Ex:- estimate $F'(2.25)$ using the following data:

x	1	2.25	3
$F(x)$	2	7	9

x_i	$F(x_i)$	1st $F(x_{i+1}, x_i)$	2nd $F(x_{i+2}, x_{i+1})$
1	2	$\frac{7-2}{2.25-1} = 4$	$\frac{2.6667-4}{3-1}$
2.25	7	$\frac{9-7}{3-2.25} = 2.6667$	
3	9		

$F(x) = 2+4(x-1) - 0.66665(x-1)(x-2.25)$

$F'(x) = 4-0.66665(2x-3.25)$

$F'(2.25) = 4-0.66665(2(2.25)-3.25 = 3.166$

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Runge Kutta Methods. Chapter 25.

Euler's Method

$Y_{i+1} = Y_i + (F(X_i, Y_i)) * h$

Example:- Estimate $Y(1)$ using Euler's Method for:-

$$\frac{dy}{dx} = \frac{5x^2+1}{y}, \quad Y(0) = 1$$

A- when $h = 1$

B- " $h = 0.5$ and calculate ε_T for A and B?

sol:- $\frac{dy}{dx} = \frac{5x^2+1}{Y}$

$Ydy = (5x^2+1)\,dx \rightarrow \frac{Y^2}{2} = \frac{5x^3}{3} + X + C$

$Y(0) = 1$

$\frac{(1)^2}{2} = \frac{5(0)^3}{3} + (0) + C \rightarrow C = \frac{1}{2}$

$\frac{(Y)^2}{2} = \frac{5(X)^3}{3} + (X) + 0.5 \rightarrow Y = \sqrt{\frac{10(X)^3}{3} + (2X)+1}$

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$$Y(1) = \sqrt{\frac{10(1)^3}{3} + (2*1) + 1} = 2.5166$$

A- For $h = 1$ $\quad Y_{i+1} = Y_i + (F(X_i, Y_i))*h$

$\rightarrow i = 0$

$Y_{0+1} = Y_1 = Y_0 + F(X_0, Y_0))*h$

$$F(0,1) = \frac{5(0)^2 + 1}{1} = 1$$

$Y_1 = 1 + 1*2 = 2$

$\rightarrow i = 1$

$Y_{1+1} = Y_2 = Y_1 + F(X_1, Y_1))*h$

$$F(1,2) = \frac{5(1)^2 + 1}{2} = 3$$

$Y_1 = 2 + 3*1 = 5.$

$$\varepsilon_t = \left|\frac{2.5166 - 2}{2.5166}\right| * 100\% = 20.52\%$$

B- when $h = 0.5$

$Y_{i+1} = Y_i + (F(X_i, Y_i))*h$

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$\rightarrow i = 0$

$$Y_{0+1} = Y_1 = Y_0 + F(X_0, Y_0)) * h$$

$$F(0, 1) = \frac{5(0)^2 + 1}{1} = 1$$

$$Y_1 = 1 + 1 * 0.5 = 1.5$$

$$\varepsilon_t = \left| \frac{2.5166 - 1.5}{2.5166} \right| * 100\% = 40.39\%$$

$\rightarrow i = 1$

$$Y_{1+1} = Y_2 = Y_1 + F(x_1, y_1)) * h$$

$$F(0.5, 1.5) = \frac{5(0.5)^2 + 1}{1.5} = 1.5$$

$$Y_2 = 1.5 + 1.5 * 0.5 = 2.25$$

$$\varepsilon_t = \left| \frac{2.5166 - 2.25}{2.5166} \right| * 100\% = 10.38\%$$

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2^{nd} order R-K

Improvement of Euler's method

A.- Heun's method

$Y_{i+1} = Y_i + \frac{1}{2}(K_1 + K_2) * h$

$K_1 = F(x_i, Y_i)$

$K_2 = F(X_i + h, Y_i + K_1 h)$

B- mid point method

$Y_{i+1} = Y_i + K_2 * h$

$K_1 = F(X_i, Y_i)$

$K_2 = F(X_i + 0.5h, Y_i, 0.5K_1 h)$

Example:- Estimate Y(1) using Heun's method For:-

$\frac{dy}{dx} = \frac{5x^2 + 1}{Y}$, $Y(0) = 1$, when $h = 0.5$?

$Y_{i+1} = Y_i + \frac{1}{2}(K_1 + K_2) * h$

→ $i = 0$

$K_1 = F(X_i, Y_i) = F(0, 1) = \frac{5(0)^2 + 1}{1} = 1$

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$F(x_i + h, y_i + k_1 h) = F(0.5, 1.5) = \frac{5(0.5)^2 + 1}{1.5} = 1.5$

$Y_{0+1} = Y_1 = 1 + \frac{1}{2}(1 + 1.5) * 0.5 = 1.625$

$\varepsilon_t = \left| \frac{2.5166 - 1.625}{2.5166} \right| * 100\% = 35.42\%$

$\rightarrow \quad i = 1$

$k_1 = F(x_i, y_i) = F(0.5, 1.625) = \frac{5(0.5)^2 + 1}{1.625} = 1.38$

$F(x_i + h, y_i + k_1 h) = F(1, 2.315) = \frac{5(1)^2 + 1}{2.315} = 2.617.$

$Y_{1+1} = Y_2 = 1.625 + \frac{1}{2}(1.38 + 2.617) * 0.5 = 2.617$

$\varepsilon_t = \left| \frac{2.5166 - 2.617}{2.5166} \right| * 100\% = 3.98\%$

3^{rd} order R-K$_3$:-

$Y_{i+1} = Y_i + \frac{1}{6}(k_1 + 6k_2 + k_3)$

$k_1 = F(X_i, Y_i)$

$k_2 = F(x_i + 0.5h, Y_i + 0.5 k_1 h)$

$k_3 = F(X_i + h, Y_i - k_1 h + k_2 h)$

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4^{th} order R-K$_4$ Exact

$Y_{i+1} = Y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = F(X_i, Y_i)$, $K_2 = F(X_i + 0.5h, Y_i + 0.5K_1h)$

$K_3 = F(X_i + 0.5h, Y_i + 0.5K_2h)$, $K_4 = F(X_i + h, Y_i + K_3h)$.

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