Introduction to Machinery Principles Ch. 1

Dr. Feras Alasali

Introduction

1. Electric Machines → mechanical energy to electric energy or vice versa

Mechanical energy	**→**	**Electric energy**	**: GENERATOR**
Electric energy	**→**	**mechanical energy**	**: MOTOR**

2. Almost all practical motors and generators **convert energy** from one form to another **through** the action of a magnetic field.

3. Another related device is the **transformer**. A transformer is a device that converts ac electric energy at one voltage level to ac electric energy at another voltage level.

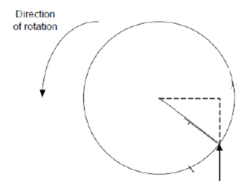
4. Why are electric motors and generators so common?
   Electric power is

- A **clean efficient energy source** that is very **easy to transmit over long distances** and **easy to control.**
- Does not require **constant ventilation** and **fuel**

1.3 Rotational motion

Almost all of electric machines rotate about an axis called a **shaft** of the machine.

- Angular position θ an angle at which the object is oriented with respect to an arbitrary reference point.
  +ve value for anticlockwise rotation
  -ve value for clockwise rotation
  - Angular velocity (speed) ω a rate of change of the angular position.



$$\omega = \frac{d\theta}{dt} \quad rad/s$$

- ω_m angular velocity in radians per second
- f_m angular velocity in revolutions per second
- n_m angular velocity in revolutions per minute

$$f_m = \frac{\omega_m}{2\pi}$$

$$n_m = 60 f_m$$

- Angular acceleration **α** a rate of change of angular velocity

$$\alpha = \frac{d\omega}{dt} \quad rad/s^2$$

- Torque (moment) **τ** is a rotating force.

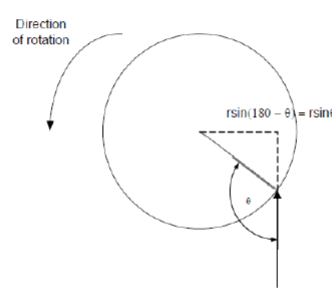
$$\tau = r\ x\ F\ = rF\sin(\theta) \quad Nm$$

Where **F** is an acting force, **r** is the vector pointing from the axis of rotation to the point where the force is applied, **θ** is the angle between two vectors.

- Newton's law of rotation

$$\tau = J\alpha \quad Nm$$

J moment of inertia



1.4 Magnetic Field

Magnetic fields are the fundamental mechanism by which energy is converted from one from to another in motors, generators, and transformers. ***Four basic principles*** describe how magnetic fields are used in these devices

1) A wire carrying a current produces a magnetic field around it.
2) A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil (**transformer action**).
3) A wire carrying a current in the presence of a magnetic field experiences a force induced on it (**motor action**).
4) A wire moving in the presence of a magnetic field gets a voltage induced in it (**generator action**).

Production of magnetic field

Magneto-motive force :

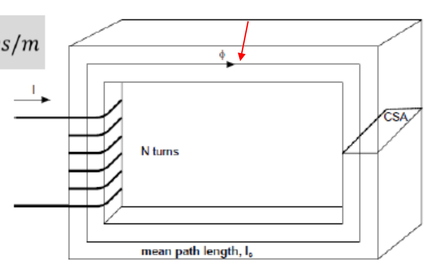
Product of current and number of turns of coil.

$$\mathcal{F} = Ni \qquad turns \cdot Amperes$$

Magnetic field intensity:

is defined as magnetomotive force per unit length

$$H = \frac{Ni}{l_c} \qquad turns \cdot Ampares/m$$



Ampere's Law – the basic law governing the production of a magnetic field by a current

$$\oint H\,dl = I_{net}$$

Magnetic flux density:

$$B = \mu H = \frac{\mu N i}{l_c} \qquad Wb/m^2 \quad \text{or} \quad \text{Tesla (T)}$$

where $\boldsymbol{\mu} = \boldsymbol{\mu_r \mu_o}$ is the magnetic permeability of the material

$\boldsymbol{\mu_o} = 4\pi \ x\ 10^{-7} \quad H/m \ \ is\ the\ permeability\ of\ the\ free\ space$

$\boldsymbol{\mu_r} \ \ is\ the\ relative\ permeability\ of\ the\ material$

$$B = \frac{\Phi}{A} \qquad Wb/m^2 \quad \text{or} \quad \text{Tesla (T)}$$

where Φ is the flux (Wb) and A is the cross section area (m^2)

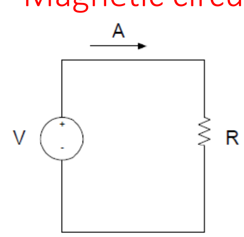
Hence, the total flux in a certain area is

$$\Phi = \int_A B \ dA$$

Is the magnetic flux density is perpendicular to the plane of the area A then,

$$\Phi = BA = \frac{\mu N i A}{l_c}$$

Magnetic circuits



Electric Circuit Analogy

Comparing with electrical circuit

$$V \implies \mathcal{F} = Ni$$

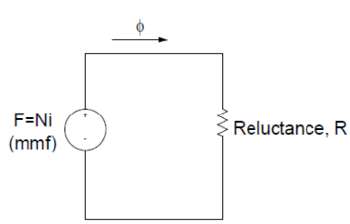
$$I \implies \Phi$$

$$R \ \ resistance \implies \mathcal{R} \ \ relactance$$

Similar to ohm's law

$$V = IR \ \ for\ electrical\ circuit$$

$$\mathcal{F} = \Phi\mathcal{R} \ \ for\ magnetic\ circuit$$



Magnetic Circuit Analogy

From these equations

$$B = \mu H = \frac{\mu N i}{l_c} \qquad Wb/m^2 \quad \text{or} \quad \text{Tesla (T)}$$

$$B = \frac{\Phi}{A} \qquad Wb/m^2 \quad \text{or} \quad \text{Tesla (T)}$$

Then

$$Ni = \mathcal{F} = \Phi \frac{l_c}{\mu A} \Rightarrow \mathcal{F} = \Phi \mathcal{R}$$

Therefore, the reluctance is

$$\mathcal{R} = \frac{l_c}{\mu A}$$

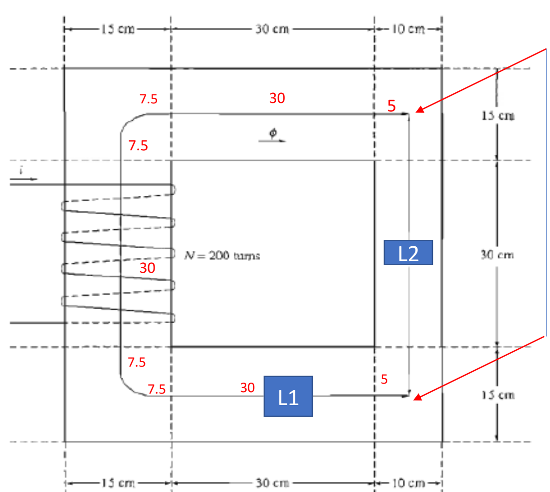
And the same for electrical resistance in series and parallel

Example

A ferromagnetic core is shown. Three sides of this core are of uniform width, while the fourth side is somewhat thinner. The depth of the core (into the page) is 10cm, and the other dimensions are shown in the figure. There is a 200 turn coil wrapped around the left side of the core. Assuming relative permeability μr of 2500, how *much flux will be produced by a 1A Input current*?

$$Ni = \mathcal{F} = \Phi \frac{l_c}{\mu A} \Rightarrow \mathcal{F} = \Phi \mathcal{R}$$

$$\mathcal{R} = \frac{l_c}{\mu A}$$

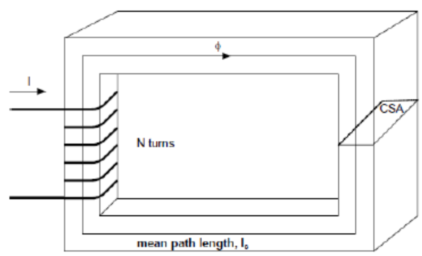


$$\mathcal{R}_1 = \frac{l_{c1}}{\mu A_1} = \frac{130\ x\ 10^{-2}}{2500\ x\ 4\pi x 10^{-7}\ x\ (10x15)x10^{-4}} = 27{,}586.9 \quad \text{A.turns/Wb}$$

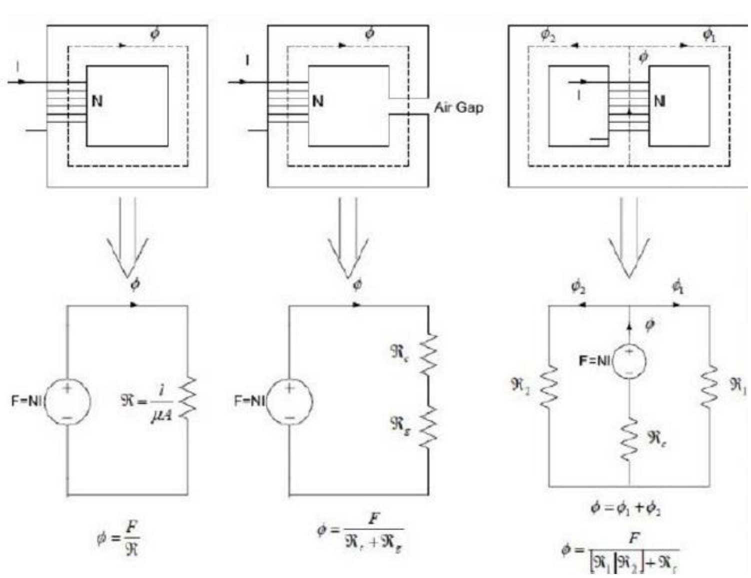
$$\mathcal{R}_2 = \frac{l_{c2}}{\mu A_2} = \frac{45\ x\ 10^{-2}}{2500\ x\ 4\pi x 10^{-7}\ x\ (10x10)x10^{-4}} = 14{,}323.9 \quad \text{A.turns/Wb}$$

$$\mathcal{R}_{eq} = \mathcal{R}_1 + \mathcal{R}_2 = 27{,}586.9 + 14{,}323.9 = 41{,}910.8 \quad \text{A.turns/Wb}$$

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}_{eq}} = \frac{Ni}{\mathcal{R}} = \frac{200\ x\ 1}{41{,}910.8} = 4.77x\ 10^{-3} \quad \text{Wb}$$

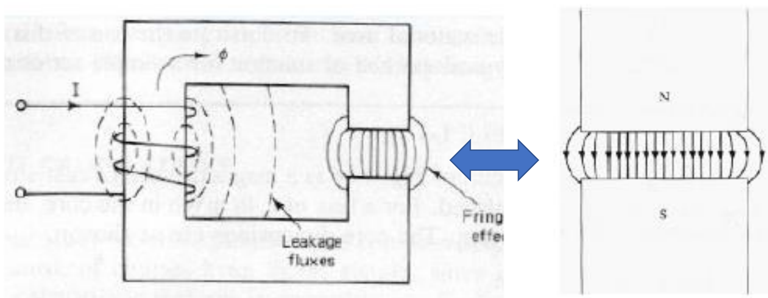


Magnetic Circuit



Approximation in calculation of Magnetic Circuit

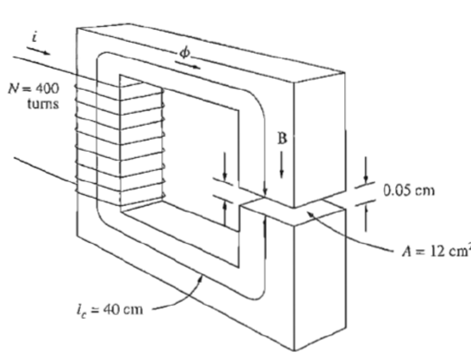
1) All flux is confined within the magnetic core but a leakage flux exists outside the core since permeability of air is non-zero!
2) A mean path length and cross-sectional area are assumed
3) In ferromagnetic materials, the permeability varies with the flux.
4) In air gaps, the cross-sectional area is bigger due to the fringing effect.



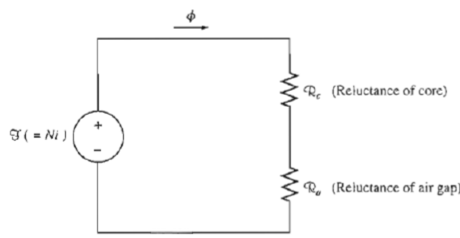
Example

Figure shows a ferromagnetic core whose mean path length is 40cm. There is a small gap of 0.05cm in the structure of the otherwise whole core. The csa of the core is 12cm2, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective csa of the gap by 5%. Given this information, find

1. The total reluctance of the flux path(iron plus air gap)
2. The current required to produce a flux density of 0.5T in the air gap.



The equivalent circuit



$$\mathcal{R}_c = \frac{l_c}{\mu A_c} = \frac{40 \ x \ 10^{-2}}{4000 \ x \ 4\pi x 10^{-7} \ x \ 12 \ x 10^{-4}} = 66{,}300 \quad \text{A.turns/Wb}$$

$$\mathcal{R}_a = \frac{l_a}{\mu_o A_a} = \frac{0.05 \ x \ 10^{-2}}{4\pi x 10^{-7} \ x \ 1.05 x 12 x 10^{-4}} = 316{,}000 \quad \text{A.turns/Wb}$$

$$\mathcal{R}_{eq} = \mathcal{R}_1 + \mathcal{R}_2 = 66{,}300 + 316{,}000 = 382{,}300 \quad \text{A.turns/Wb}$$

$$\mathcal{F} = \mathcal{R}_{eq}\Phi$$

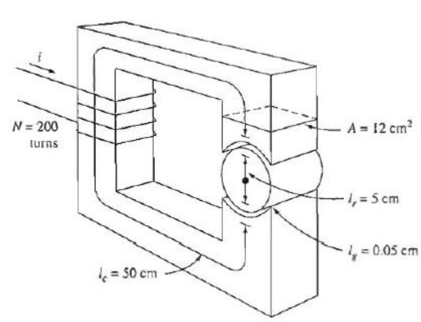
$$Ni = \mathcal{R}_{eq} BA$$

$$i = \frac{BA\mathcal{R}_{eq}}{N} = \frac{(0.5)(0.00126)(382{,}300)}{400} = 0.602 \ A$$

Example

Figure shows a simplified rotor and stator for a de motor. The mean path length of the stator is 50 cm, and its cross-sectional area is 12 cm2, The mean path length of the rotor is 5 cm, and its cross-sectional area also may be assumed to be 12 cm2, Each air gap between the rotor and the stator is 0.05 cm wide, and the cross sectional area of each air gap (including fringing) is 14 cm2, The iron of the core has a relative permeability of 2000, and there are 200 turns of wire on the core.

If the current in the wire is adjusted to be 1 A, what will the resulting flux density in the air gaps be?



$$\mathcal{R}_{eq} = \mathcal{R}_s + \mathcal{R}_{a1} + \mathcal{R}_r + \mathcal{R}_{a1} = 66{,}300 + 316{,}000 = 382{,}300 \quad \text{A.turns/Wb}$$

$$\mathcal{R}_s = \frac{l_s}{\mu A_s} = \frac{0.5}{2000\ x\ 4\pi x 10^{-7}\ x\ 0.0012} = 166{,}000 \quad \text{A.turns/Wb}$$

$$\mathcal{R}_r = \frac{l_r}{\mu A_r} = \frac{0.05}{2000x\ 4\pi x 10^{-7}\ x\ 0.0012} = 16{,}600 \quad \text{A.turns/Wb}$$

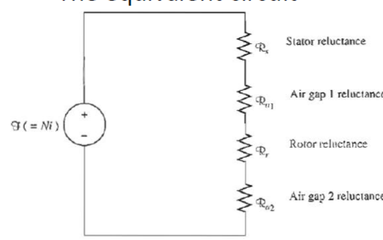
$$\mathcal{R}_a = \frac{l_a}{\mu_o A_a} = \frac{0.0005}{4\pi x 10^{-7}\ x\ 0.0014} = 284{,}000 \quad \text{A.turns/Wb}$$

$$\mathcal{R}_{eq} = \mathcal{R}_s + \mathcal{R}_{a1} + \mathcal{R}_r + \mathcal{R}_{a1} = 166{,}000 + 284{,}000 + 16{,}600 + 284{,}000 = 751{,}000 \quad \text{A.turns/Wb}$$

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}_{eq}} = \frac{Ni}{\mathcal{R}} = \frac{200\ x\ 1}{751{,}000} = 0.00266 \quad \text{Wb}$$

$$B = \frac{\Phi}{A} = \frac{0.00266}{0.0014} = 0.19 \quad \text{T}$$

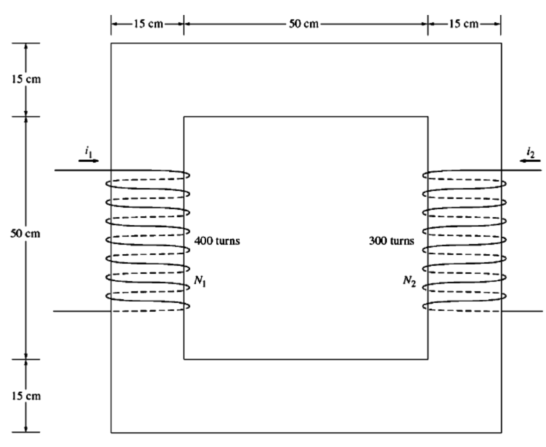
The equivalent circuit



Example

A two-legged core is shown in blow Figure. The winding on the left leg of the core (N1) has 400 turns. and the winding on the right (N2) has 300 turns. The coils are wound in the directions shown in the figure. If the dimensions are as shown.

- what flux would be produced by currents i1 = 0.5 A and i2 = 0.75 A? Assume ur=1000 and constant.



The two coils on this core are would so that their magnetomotive forces are additive, so the total magnetomotive force on this core is

$$\mathcal{F}_{TOT} = N_1 i_1 + N_2 i_2 = (400\ \text{t})(0.5\ \text{A}) + (300\ \text{t})(0.75\ \text{A}) = 425\ \text{A}\cdot\text{t}$$

The total reluctance in the core is

$$\mathcal{R}_{TOT} = \frac{l}{\mu_r \mu_0 A} = \frac{2.60\ \text{m}}{(1000)(4\pi \times 10^{-7}\ \text{H/m})(0.15\ \text{m})(0.15\ \text{m})} = 92.0\ \text{kA}\cdot\text{t/Wb}$$

and the flux in the core is:

$$\phi = \frac{\mathcal{F}_{TOT}}{\mathcal{R}_{TOT}} = \frac{425\ \text{A}\cdot\text{t}}{92.0\ \text{kA}\cdot\text{t/Wb}} = 0.00462\ \text{Wb}$$

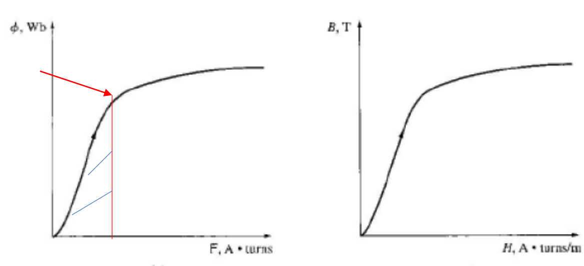
Magnetic behavior of ferromagnetic materials

Magnetic permeability can be defined as:

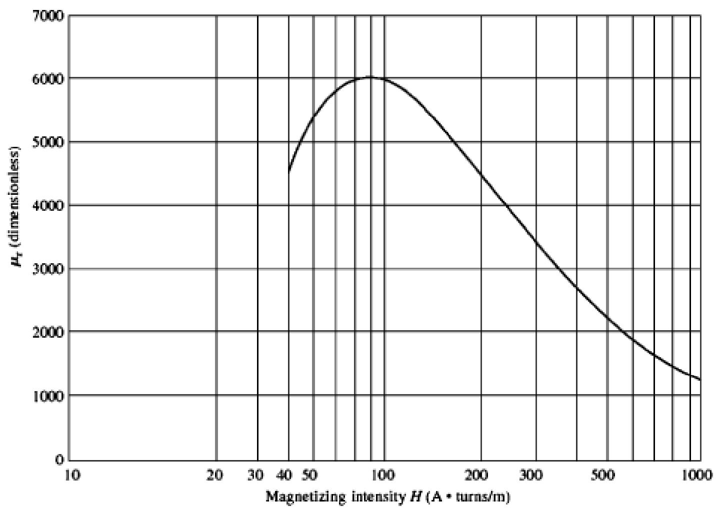
$$\mu = \frac{B}{H} = \frac{\Phi/A}{Ni/l_c} \quad \alpha \quad \frac{\Phi}{\mathcal{F}}$$

- μ is assumed as constant. However, for the ferromagnetic materials (for which permeability can be up to 6000 times the permeability of air), permeability is not a constant,

- Permeability is large and relatively constant in the unsaturated region whereas it drops to low value as the core gets saturated.



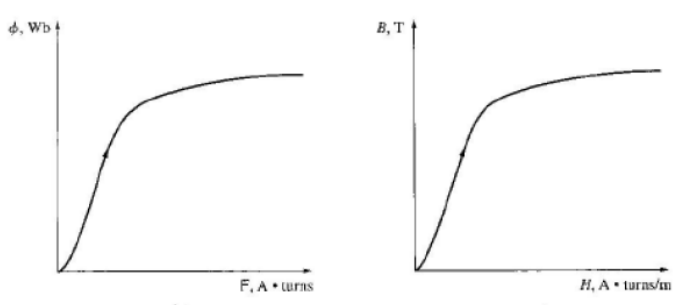
A saturation (magnetization) curve for a DC source



- **Generators and motors operate near the knee of the magnetization curve**

- Generators and motors depend on magnetic **flux** to **produce voltage and torque**, so they need as much flux as possible

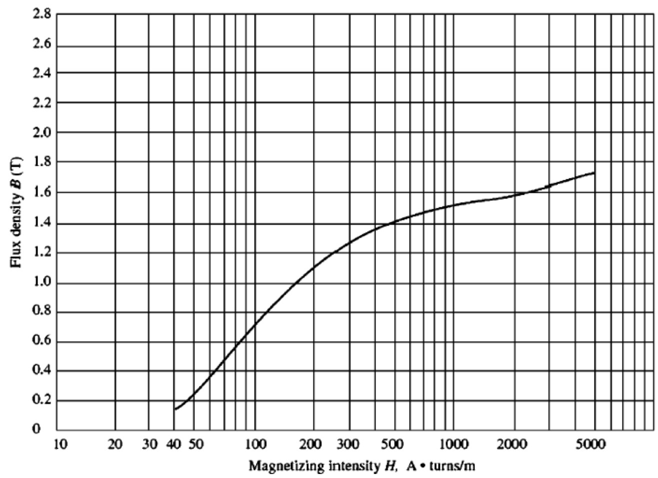
- If the resulting **flux** has to be proportional to the **mmf**, then the core must be operated in the unsaturated region.



Example

A square magnetic core has a mean path length of 55 cm and a cross-sectional area of 150 cm². A 200-turn coil of wire is wrapped around one leg of the core. The core is made of a material having the magnetization curve shown in Figure

(*a*) How much current is required to produce 0.012 Wb of flux in the core?
(*b*) What is the core's relative permeability at that current level?
(*c*) What is its reluctance?



Solution

(*a*) The required flux density in the core is

$$B = \frac{\phi}{A} = \frac{1.012\ \text{Wb}}{0.015\ \text{m}^2} = 0.8\ \text{T}$$

From Figure 1–10c, the required magnetizing intensity is

$$H = 115\ \text{A} \cdot \text{turns/m}$$

From Equation (1–20), the magnetomotive force needed to produce this magnetizing intensity is

$$\mathcal{F} = Ni = Hl_c$$
$$= (115\ \text{A} \cdot \text{turns/m})(0.55\ \text{m}) = 63.25\ \text{A} \cdot \text{turns}$$

so the required current is

$$i = \frac{\mathcal{F}}{N} = \frac{63.25\ \text{A} \cdot \text{turns}}{200\ \text{turns}} = 0.316\ \text{A}$$

(*b*) The core's permeability at this current is

$$\mu = \frac{B}{H} = \frac{0.8\ \text{T}}{115\ \text{A} \cdot \text{turns/m}} = 0.00696\ \text{H/m}$$

Curve

Therefore, the relative permeability is

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.00696\ \text{H/m}}{4\pi \times 10^{-7}\ \text{H/m}} = 5540$$

(*c*) The reluctance of the core is

$$\mathcal{R} = \frac{\mathcal{F}}{\phi} = \frac{63.25\ \text{A} \cdot \text{turns}}{0.012\ \text{Wb}} = 5270\ \text{A} \cdot \text{turns/Wb}$$

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Production of magnetic field

Magneto-motive force :

Product of current and number of turns of coil.

$$\mathcal{F} = Ni \qquad turns \cdot Ampares$$

Magnetic field intensity:

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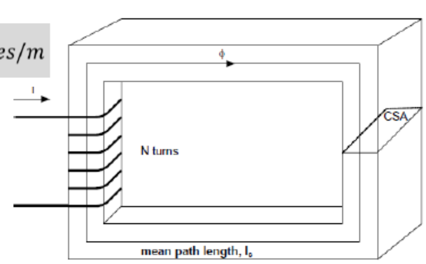
$$\mathrm{H} = \frac{Ni}{l_c} \qquad turns \cdot Ampares/m$$



Magnetic flux density:

$$B = \mu H = \frac{\mu N i}{l_c} \qquad Wb/m^2 \quad \text{or} \quad \text{Tesla (T)}$$

where $\mu = \mu_r \mu_o$ is the magnetic permeability of the material

$\mu_o = 4\pi \times 10^{-7}$ H/m *is the permeability of the free space*

μ_r *is the relative permeability of the material*

$$B = \frac{\Phi}{A} \qquad Wb/m^2 \quad \text{or} \quad \text{Tesla (T)}$$

where Φ is the flux (Wb) and A is the cross section area (m^2)

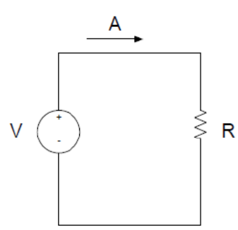
Hence, the total flux in a certain area is

$$\Phi = \int_A B \, dA$$

Is the magnetic flux density is perpendicular to the plane of the area A then,

$$\Phi = BA = \frac{\mu N i A}{l_c}$$

Magnetic circuits



Electric Circuit Analogy

Comparing with electrical circuit

$$V \Longrightarrow \mathcal{F} = Ni$$

$$I \Longrightarrow \Phi$$

$$R \ \ resistance \Longrightarrow \mathcal{R} \ \ relactance$$

Similar to ohm's law

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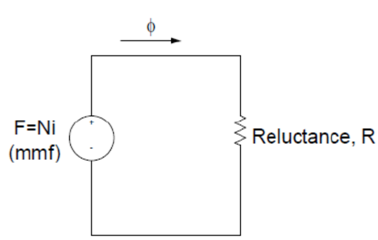

Magnetic Circuit Analogy

Then

$$Ni = \mathcal{F} = \Phi \frac{l_c}{\mu A} \Rightarrow \mathcal{F} = \Phi \mathcal{R}$$

Therefore, the reluctance is

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And the same for electrical resistance in series and parallel

Magnetic behavior of ferromagnetic materials

Magnetic permeability can be defined as:

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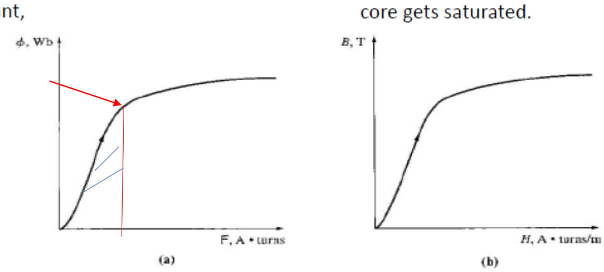

A saturation (magnetization) curve for a DC source

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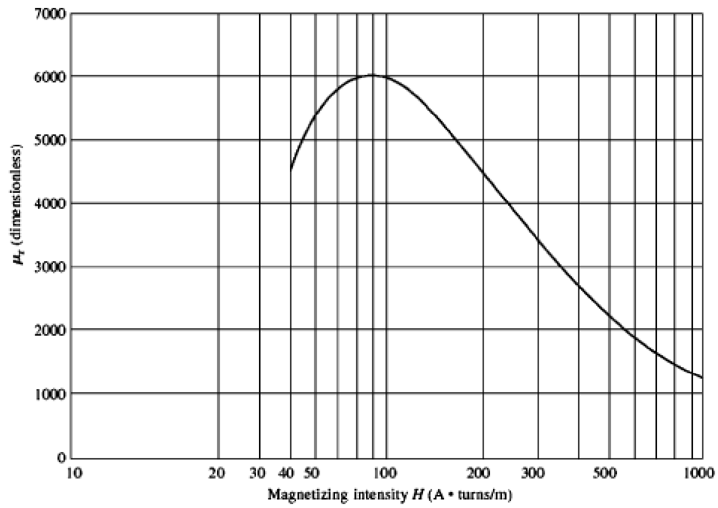

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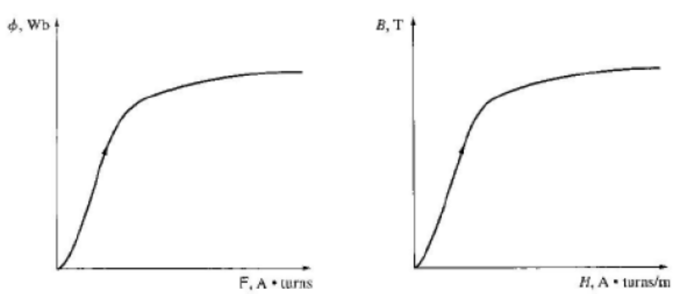

(a) (b)

Example

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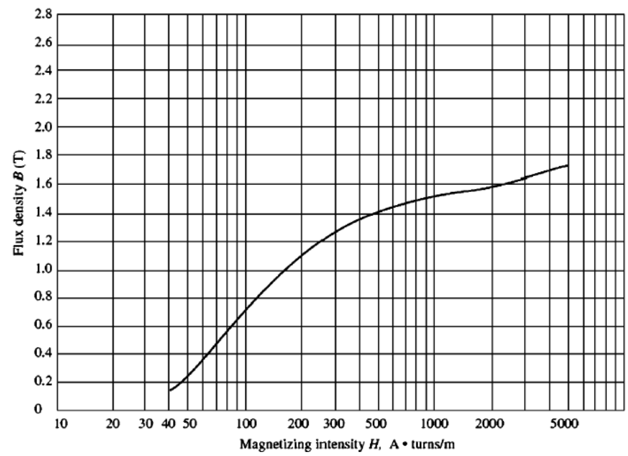

Solution

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so the required current is

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Therefore, the relative permeability is

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(c) The reluctance of the core is

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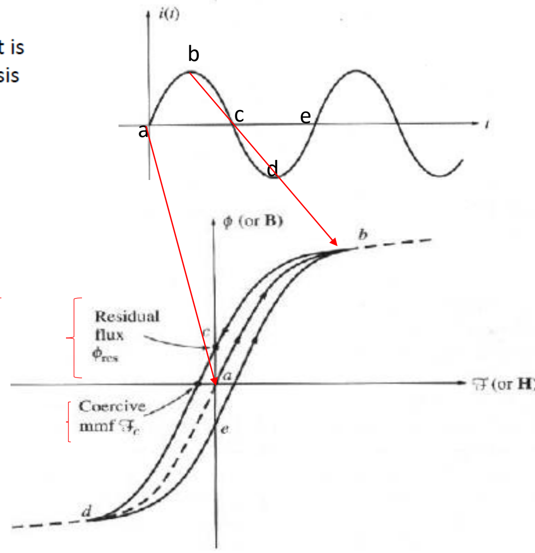
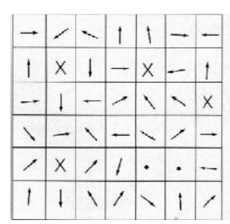
Energy losses in a ferromagnetic core

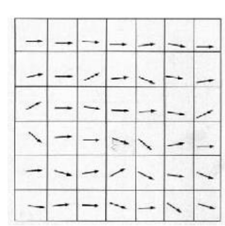
- If instead of a DC, a sinusoidal current is applied to a magnetic core, a hysteresis loop will be observed.
- If a large mmf is applied to a core and then removed, the flux in a core does not go to zero! A magnetic field (or flux), called the residual field (or flux), will be left in the material. To force the flux to zero, an amount of mmf (coercive mmf) is needed.



- Ferromagnetic materials consist of **small domains,** within which magnetic moments of atoms are aligned. However, magnetic moments of domains are oriented randomly

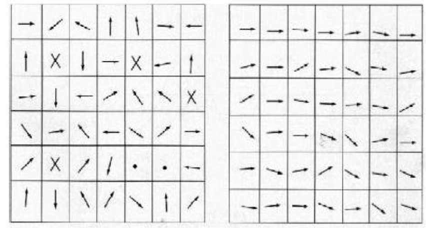


- When an **external magnetic field** is applied, the domains pointing in the direction of that field grow since the atoms at their boundaries physically switch their orientation and align themselves in the direction of magnetic field. This increases magnetic flux in the material which, in turn, causes more atoms to change orientation. As the strength of the external field increases, more domains change orientation until almost all atoms and domains are aligned with the field. Further increase in mmf can cause only the same flux increase as it would be in a vacuum. This is a saturation.



- When the **external field is removed**, the <u>domains do not completely randomize again</u>. Re-aligning the atoms would require energy! Initially, such energy was provided by the external field.

- Atoms can be realigned by an external mmf in other direction, mechanical shock, or heating.



- The **hysteresis loss in** the core <u>is the energy required to re-orient domains during each cycle of AC applied to the core.</u>

- Another type of energy losses is **an eddy currents** loss, which will be examined later.

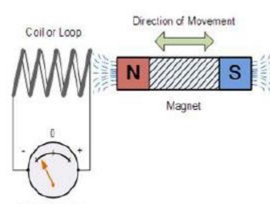
1.5: Faraday's law - Induced Voltage From a Time-Changing Magnetic Field

- If a flux passes through a turn of a coil of wire, a voltage will be induced in that turn which is directly proportional to the rate of change in the flux with respect to time:

$$e_{ind} = -\frac{d\Phi}{dt}$$

For a coil having **N turns**:

$$e_{ind} = -N\frac{d\Phi}{dt}$$



- **e_{ind}** – voltage induced in the coil
- **N** – number of turns of wire in the coil
- **ϕ** – flux passing through the coil

The **"minus sign"** in the equation is a consequence of the **"Lenz's law"**.

"The **direction of the induced voltage** in the coil is such that if the coil terminals were short circuited, it would produce a current that would cause a flux opposing the original flux change."

Assume that the same **flux is passing through each turn of the coil**. If the windings are **closely coupled**, this assumption almost holds. In most cases, a **flux leakage** occurs. Therefore, more accurately:

$$e_i = -\frac{d\Phi_i}{dt}$$

For a coil having **N turns**:

$$e_{ind} = \sum_{i=1}^{N} e_i = \sum_{i=1}^{N} \frac{d\Phi_i}{dt} = \frac{d}{dt}\sum_{i=1}^{N} \Phi_i$$

Where λ- a flux linkage of the coil

$$e_{ind} = \frac{d\lambda}{dt}$$

$$\lambda = \sum_{i=1}^{N} \Phi_i \qquad (Wb \cdot turns)$$

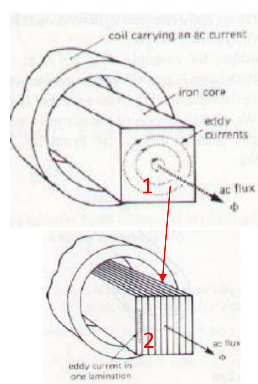
The Eddy Current Losses

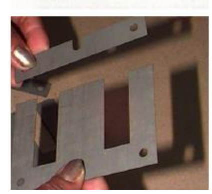
- **Voltages are generated within a ferromagnetic core by a time-changing magnetic flux** same way as they are induced in a wire. These voltages cause currents flowing in the resistive material (ferromagnetic core) called eddy currents. Therefore, energy is dissipated by these currents in the form of heat.

- The amount of energy lost to eddy currents is **proportional to the size of the paths** they travel within the core. Therefore, ferromagnetic cores are **laminated**.

- **Core** consists of a set of **tiny isolated strips**.

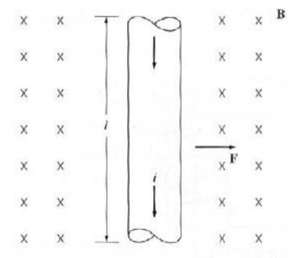
- Eddy current losses are **proportional to the square of the lamination thickness.**





1.6: Production of induced force on a wire (Motor Action)

A second effect of a magnetic field is that it induces a force on a wire carrying a current within the field



$$\vec{F} = i(\vec{l} \, x \, \vec{B})$$

Where:
I is a vector of current,
B is the magnetic flux density vector.

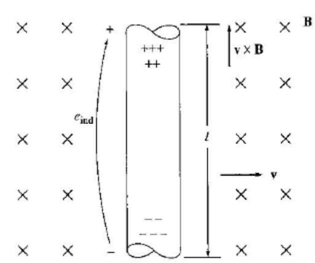
$$F = ilB \sin(\theta)$$

θ is the angle between the length and the magnetic field density

This is a basis for a **motor action**

1.7: Induced voltage on a conductor moving in a magnetic field (Generator Action)

If a wire with the proper orientation moves through a magnetic field, a voltage is induced in it.



$$e_{ind} = (\vec{v} \, x \, \vec{B}) \cdot l$$

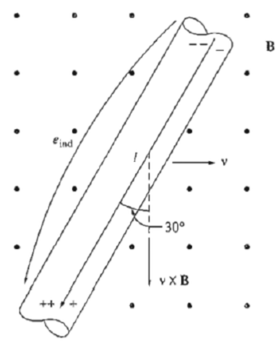
Where
v is the velocity of the wire,
l is its length in the magnetic field,
B – the magnetic flux density

This is a basis for a **Generator action**

Example

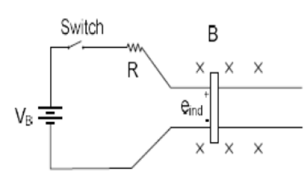
The figure shows a conductor moving with a velocity of 5m/s to the right in the presence of a magnetic field. The flux density is 0.5T into the page, and the wire is 1m length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?

$$
\begin{aligned}
e_{ind} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\
&= (vB \sin 90^\circ)\, l \cos 30^\circ \\
&= (10.0\ \text{m/s})(0.5\ \text{T})(1.0\ \text{m}) \cos 30^\circ \\
&= 4.33\ \text{V}
\end{aligned}
$$



1.8. The Linear DC Machine

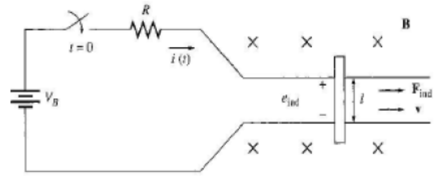
Linear DC machine is the simplest form of DC machine which is easy to understand and it operates according to the same principles and exhibits the same behavior as motors and generators. Consider the following:



1. Production of Force on a current carrying conductor — $\vec{F} = i(\vec{l}\ x\ \vec{B})$
2. Voltage induced on a current carrying conductor moving in a magnetic field — $e_{ind} = \left(\vec{v}\ x\ \vec{B}\right) \cdot l$
3. Kirchoff's voltage law — $V_B - iR - e_{ind} = 0$, $V_B = iR + e_{ind}$
4. Newton's Law for motion — $F_{net} = ma$

Starting the Linear DC Machine

1. To start the machine, the switch is closed



2. Current will flow in the circuit $V_B - iR - e_{ind} = 0$

$$i = \frac{V_B - e_{ind}}{R}$$

3. As the current flows down through the bar, a force will be induced on the bar

$$\vec{F} = i(\vec{l} \, x \, \vec{B})$$
$$F = ilB \sin(\theta)$$
$$F = ilB$$

To the right

4. The bar starts to move, its velocity will increase, and a voltage appears across the bar.

$$e_{ind} = (\vec{v} \, x \, \vec{B}) \cdot l$$

Positive upward

5. Due to the presence of motion and induced potential (e_{ind}), the current flowing in the bar will reduce

$$i \downarrow = \frac{V_B - e_{ind} \uparrow}{R}$$

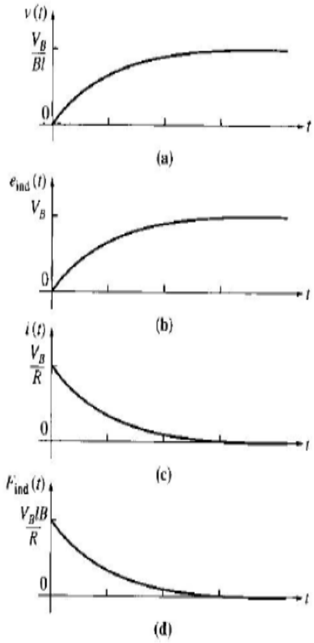
6. The induced force is thus reduced $F \downarrow = i \downarrow lB$

And eventually $F = 0$

At that point, $e_{ind} = V_B$, $i = 0$

And the bar moves at constant no load speed:

$$e_{ind} = V_B = v_{steady\ state} Bl$$
$$v_{steady\ state} = \frac{V_B}{Bl}$$



1. Closing the switch produces a current flow $i = \frac{V_B}{R}$

2. The current flow produces a force on the bar given by $F = ilB$

3. The bar accelerates to the right, producing an induced voltage e_{ind} as it speeds up.

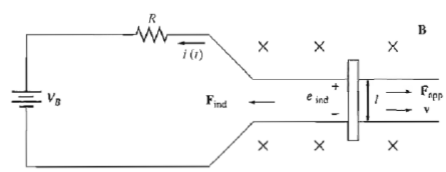
4. This induced voltage reduces the current flow $i\downarrow = \frac{V_B - e_{ind}\uparrow}{R}$

5. The induced force is thus decreased $F\downarrow = i\downarrow lB$

- until eventually $F = 0$
- At that point $e_{ind} = V_B$, $i = 0$
- and the bar moves at a constant no-load speed $v_{steady\ state} = \frac{V_B}{Bl}$

The Linear DC Machine as a Motor

What will happen to the machine if an external load is applied to it in the opposite direction of motion?



1. A force F_{load} is applied opposite to the direction of motion, which causes a net force F_{net} opposite to the direction of motion.

2. The resulting acceleration $a = F_{net}/m$ is negative, so the bar slows down ($v\downarrow$).

3. The voltage $e_{ind} = v\downarrow BI$ falls, and so $i = (V_B - e_{ind}\downarrow)/R$ increases.

4. The induced force $F_{ind} = i\uparrow BI$ increases until $|F_{ind}| = |F_{load}|$ at a lower speed v.

5. An amount of electric power equal to $e_{ind}i$ is now being converted to mechanical power equal to $F_{ind}v$, and the machine is acting as a motor.

What will happen to the machine if an external load is applied to it in the same direction of motion?

1. A force F_{load} is applied in the direction of motion, which causes a net force F_{net} in the direction of motion.

2. The resulting acceleration **a** = F_{net}/m is positive, so the bar speeds up ($v\uparrow$).

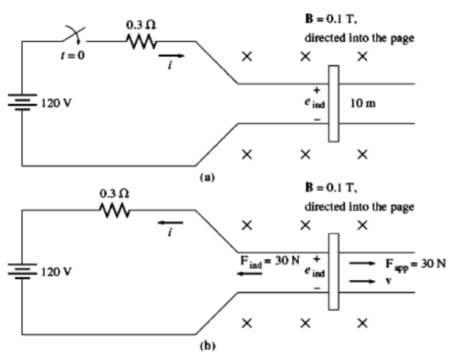
3. The voltage $e_{ind} = v\uparrow BI$ increases, and so **i** = $(e_{ind}\uparrow - V_B)/R$ increases.

4. The induced force $F_{ind} = i\uparrow BI$ increases until $|F_{ind}| = |F_{load}|$ at a higher speed v.

5. An amount of mechanical power equal to $F_{ind}v$ is now being converted to electric power equal to $e_{ind}i$, and the machine is acting as a generator.

Example

The linear dc machine shown in Figure has a battery voltage of 120 V, an internal resistance of 0.3Ω, and a magnetic flux density of 0.1 T.



(a) What is this machine's maximum starting current? What is its steady-state velocity at no load?

When the machine reaches steady state, $F_{ind} = 0$ and $i = 0$. Therefore,

$$V_B = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120\text{ V}}{(0.1\text{ T})(10\text{ m})} = 120\text{ m/s}$$

(c) Now suppose a 30N force pointing to the right were applied to the bar. What would the new steady-state speed be? Is this machine a motor or a generator now?

$$F_{app} = F_{ind} = ilB$$

$$i = \frac{F_{ind}}{lB} = \frac{30\text{ N}}{(10\text{ m})(0.1\text{ T})}$$

$$= 30\text{ A} \quad \text{flowing down through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B - iR$$

$$= 120\text{ V} - (30\text{ A})(0.3\ \Omega) = 111\text{ V}$$

and the final speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{111\text{ V}}{(0.1\text{ T})(10\text{ m})} = 111\text{ m/s}$$

This machine is now acting as a *motor*, converting electric energy from the battery into mechanical energy of motion on the bar.

(b) Suppose that a 30-N force pointing to the left were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming? Explain the difference between these two figures . Is this machine acting as a motor or as a generator?

$$F_{app} = F_{ind} = ilB$$

Therefore,

$$i = \frac{F_{ind}}{lB} = \frac{30\ \text{N}}{(10\text{m})(0.1\ \text{T})}$$

$= 30\ \text{A}$ flowing up through the bar

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B + iR$$

$$= 120\ \text{V} + (30\text{A})(0.3\ \Omega) = 129\ \text{V}$$

and the final steady-state speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{129\ \text{V}}{(0.1\ \text{T})(10\ \text{m})} = 129\ \text{m/s}$$

The bar is *producing* $P = (129\ \text{V})(30\ \text{A}) = 3870\ \text{W}$ of power, and the battery is *consuming* $P = (120\ \text{V})(30\ \text{A}) = 3600\ \text{W}$. The difference between these two numbers is the 270 W of losses in the resistor. This machine is acting as a *generator.*

(d) Assume that the bar is unloaded and that it suddenly runs into a region where the magnetic field is *weakened to 0.08 T*. How fast will the bar go now?

If the bar is initially unloaded, then $e_{ind} = V_B$. If the bar suddenly hits a region of weaker magnetic field, a transient will occur. Once the transient is over, though, e_{ind} will again equal V_B.

This fact can be used to determine the final speed of the bar. The *initial speed* was 120 m/s. The *final speed* is

$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120\ \text{V}}{(0.08\ \text{T})(10\ \text{m})} = 150\ \text{m/s}$$

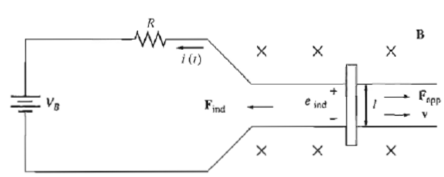
Thus, when the flux in the linear motor weakens, the bar speeds up. The same behavior occurs in real dc motors: When the field flux of a dc motor weakens, it turns faster. Here, again, the linear machine behaves in much the same way as a real dc motor.

Introduction to Machinery Principles
Ch. 1

Dr. Feras Alasali

The Linear DC Machine as a Motor

What will happen to the machine if an external load is applied to it in the opposite direction of motion?



1. A force F_{load} is applied opposite to the direction of motion, which causes a net force F_{net} opposite to the direction of motion.

2. The resulting acceleration $a = F_{net}/m$ is negative, so the bar slows down ($v\downarrow$).

3. The voltage $e_{ind} = v\downarrow BI$ falls, and so $i = (V_B - e_{ind}\downarrow)/R$ increases.

4. The induced force $F_{ind} = i\uparrow BI$ increases until $|F_{ind}| = |F_{load}|$ at a lower speed v.

5. An amount of electric power equal to $e_{ind}i$ is now being converted to mechanical power equal to $F_{ind}v$, and the machine is acting as a motor.

What will happen to the machine if an external load is applied to it in the same direction of motion?

1. A force F_{load} is applied in the direction of motion, which causes a net force F_{net} in the direction of motion.

2. The resulting acceleration $a = F_{net}/m$ is positive, so the bar speeds up ($v\uparrow$).

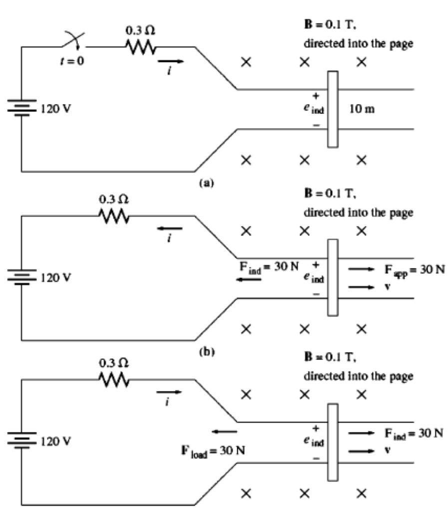
3. The voltage $e_{ind} = v\uparrow BI$ increases, and so $i = (e_{ind}\uparrow - V_B)/R$ increases.

4. The induced force $F_{ind} = i\uparrow BI$ increases until $|F_{ind}| = |F_{load}|$ at a higher speed v.

5. An amount of mechanical power equal to $F_{ind}v$ is now being converted to electric power equal to $e_{ind}i$, and the machine is acting as a generator.

Example

The linear dc machine shown in Figure has a battery voltage of 120 V, an internal resistance of 0.3Ω, and a magnetic flux density of 0.1 T.



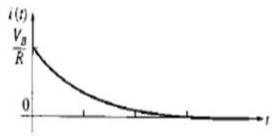
(a) What is this machine's maximum starting current? What is its steady-state velocity at no load?

When the machine reaches steady state, $\mathbf{F}_{ind} = 0$ and $i = 0$. Therefore,

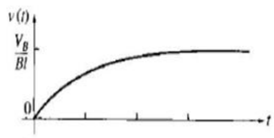
$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 120 \text{ m/s}$$



Closing the switch produces a current flow $i = \frac{V_B}{R}$



The induced force is thus decreased $F\downarrow = i\downarrow lB$

- until eventually $F = 0$

- At that point $e_{ind} = V_B$, $i = 0$

- and the bar moves at a constant no-load speed $v_{steady\ state} = \frac{V_B}{Bl}$

(b) Suppose that a 30-N force pointing to the right were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming? Explain the difference between these two figures . Is this machine acting as a motor or as a generator?

(b) **If a 30-N force to the right is applied to the bar, the final steady state will occur when the induced force $\mathbf{F}_{ind}$ is equal and opposite to the applied force $\mathbf{F}_{app}$, so that the net force on the bar is zero:**

$$F_{app} = F_{ind} = ilB$$

Therefore,

$$i = \frac{F_{ind}}{lB} = \frac{30 \text{ N}}{(10\text{m})(0.1 \text{ T})}$$

$= 30$ A flowing up through the bar

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B + iR$$

$$= 120 \text{ V} + (30\text{A})(0.3\ \Omega) = 129 \text{ V}$$

and the final steady-state speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{129 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 129 \text{ m/s}$$

The bar is *producing* $P = (129 \text{ V})(30 \text{ A}) = 3870$ W of power, and the battery is *consuming* $P = (120 \text{ V})(30 \text{ A}) = 3600$ W. The difference between these two numbers is the 270 W of losses in the resistor. This machine is acting as a *generator.*

(c) Now suppose a 30N force pointing to the left were applied to the bar. What would the new steady-state speed be? Is this machine a motor or a generator now?

(c) This time, the force is applied to the left, and the induced force is to the right. At steady state,

$$F_{app} = F_{ind} = ilB$$

$$i = \frac{F_{ind}}{lB} = \frac{30\text{ N}}{(10\text{ m})(0.1\text{ T})}$$

$$= 30\text{ A} \quad \text{flowing down through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B - iR$$

$$= 120\text{ V} - (30\text{ A})(0.3\ \Omega) = 111\text{ V}$$

and the final speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{111\text{ V}}{(0.1\text{ T})(10\text{ m})} = 111\text{ m/s}$$

This machine is now acting as a *motor*, converting electric energy from the battery into mechanical energy of motion on the bar.

(d) Assume that the bar is unloaded and that it suddenly runs into a region where the magnetic field is weakened to 0.08 T. How fast will the bar go now?

If the bar is initially unloaded, then $e_{ind} = V_B$. If the bar suddenly hits a region of weaker magnetic field, a transient will occur. Once the transient is over, though, e_{ind} will again equal V_B.

This fact can be used to determine the final speed of the bar. The *initial speed* was 120 m/s. The *final speed* is

$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120\text{ V}}{(0.08\text{ T})(10\text{ m})} = 150\text{ m/s}$$

Thus, when the flux in the linear motor weakens, the bar speeds up. The same behavior occurs in real dc motors: When the field flux of a dc motor weakens, it turns faster. Here, again, the linear machine behaves in much the same way as a real dc motor.

Transformers
Ch. 2

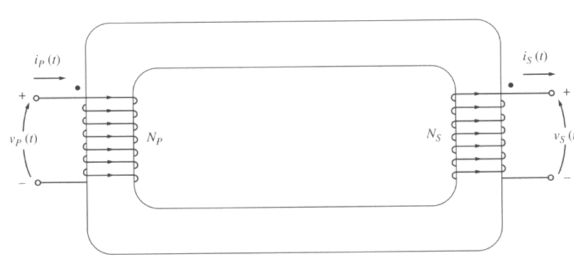
Dr. Feras Alasali

Introduction

A **transformer** is a device that converts one AC voltage to another AC voltage at the same frequency.

- It consists of one or more coil(s) of wire wrapped around a common ferromagnetic core.
- These coils are usually not connected electrically together.
- However, they are connected through the common magnetic flux confined to the core.

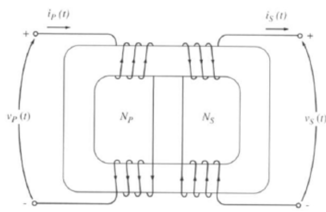
Assuming that the transformer has at least two windings, one of them (**primary**) is connected to a source of AC power; the other (**secondary**) is connected to the loads.



Types and construction

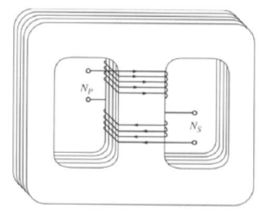
Power transformers

Core form



Windings are wrapped around two sides of a laminated square core.

Shell form



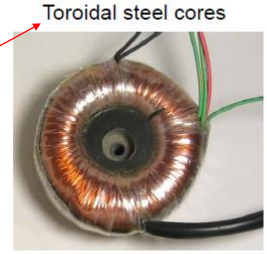
Windings are wrapped around the center leg of a laminated core.

Usually, windings are **wrapped on top of each other** to decrease flux leakage and, therefore, increase efficiency.

Lamination types

Laminated steel cores

Toroidal steel cores



Efficiency of transformers with toroidal cores is usually higher.

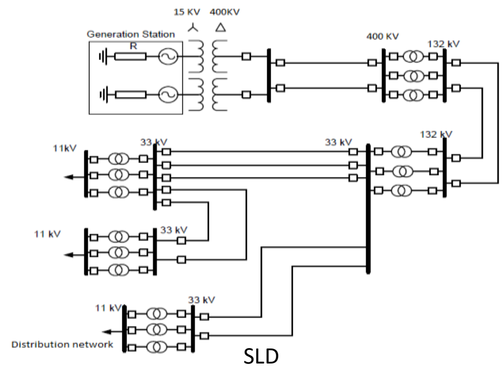
- **Overview of Electrical Power Grid.**

Power transformers used in power distribution systems are sometimes referred as follows:

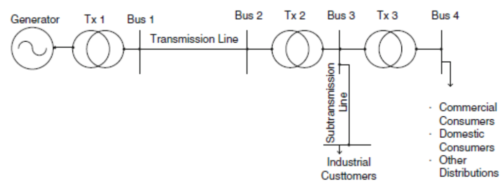
1.Unit transformer: a power transformer connected to the output of a generator and used to step its voltage up to the transmission level (110 kV and higher).

2.Substation transformer: a transformer used at a substation to step the voltage from the transmission level down to the distribution level (2.3 -34.5 kV).

3.Distribution transformer: a transformer converting the distribution voltage down to the final level (110 V, 220 V).



SLD



Ideal transformer

We consider a lossless transformer with an input (primary) winding having N_p turns and a secondary winding of N_s turns.

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

In the phasor notation:

$$\frac{V_p}{V_s} = a$$

$$\frac{I_p}{I_s} = \frac{1}{a}$$

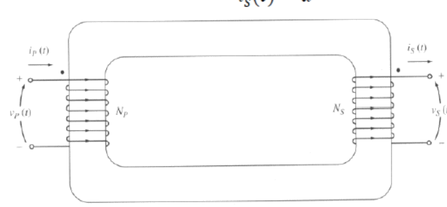
The **phase angles** of primary and secondary voltages **are the same**. The phase angles of primary and secondary currents are the same also. ***The ideal transformer changes magnitudes of voltages and currents but not their angles.***

The relationship between the voltage applied to the primary winding $v_p(t)$ and the voltage produced on the secondary winding $v_s(t)$ is

Here a is the turn ratio of the transformer.

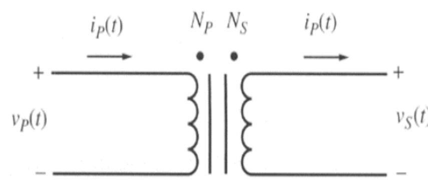
The relationship between the primary $i_p(t)$ and secondary $i_s(t)$ currents is

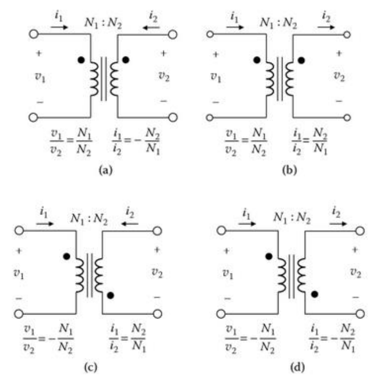
$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$



One winding's terminal is usually marked by a **dot** used to determine the polarity of voltages and currents.

Polarity in electrical terms refers to the positive or negative conductors within a dc circuit, or to the Line and Neutral conductor within an ac circuit. Electrical polarity (positive and negative) is the direction of current flow in an electrical circuit.





If the **voltage is positive at the dotted end of the primary winding** at some moment of time, the **voltage at the dotted end of the secondary winding will also be positive** at the same time instance.

If the **primary current flows into the dotted end of the primary** winding, the **secondary current will flow out of the dotted end of the secondary** winding.

Power in an ideal transformer

Assuming that θ_p and θ_s are the angles between voltages and currents on the primary and secondary windings respectively, the power supplied to the transformer by the primary circuit is:

$$P_{in} = V_p I_p \cos\theta_p$$

The power supplied to the output circuits is

$$P_{out} = V_s I_s \cos\theta_s$$

Since ideal transformers do not affect angles between voltages and currents:

$$\theta_p = \theta_s = \theta$$

Both windings of an ideal transformer have the same power factor.

Since for an ideal transformer the following holds: $V_s = \dfrac{V_p}{a}$ $\quad I_s = aI_p$

Therefore: $$P_{out} = V_s I_s \cos\theta_s = \frac{V_p}{a} aI_p \cos\theta_p = P_{in}$$

The output power of an ideal transformer equals to its input power – to be expected since assumed no loss. Similarly, for reactive and apparent powers:

$$Q_{out} = V_s I_s \sin\theta_s = \frac{V_p}{a} a I_p \sin\theta_p = Q_{in}$$
$$S_{out} = V_s I_s = \frac{V_p}{a} a I_p = S_{in}$$

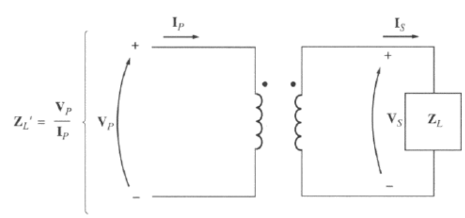
Impedance transformation

The **impedance** is defined as a following ratio of phasors:

$$Z_L = \frac{V_L}{I_L}$$

A transformer changes voltages and currents and, therefore, an apparent impedance of the load that is given by

$$Z_L = \frac{V_S}{I_S}$$



The apparent impedance of the primary circuit is:

$$\grave{Z}_L = \frac{V_P}{I_P}$$

which is

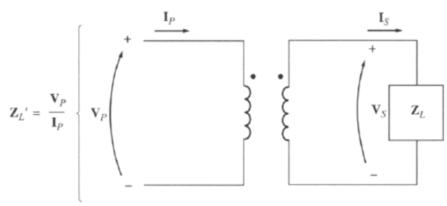
$$\grave{Z}_L = \frac{V_P}{I_P} = \frac{aV_s}{I_S/a} = a^2\frac{V_s}{I_S} = a^2 Z_L$$

It is possible to match magnitudes of impedances (load and a transmission line) by selecting a transformer with the proper turn ratio.

Analysis of circuits containing ideal transformers

- A simple method to analyze a circuit containing an ideal transformer is by replacing the portion of the circuit on one side of the transformer by an equivalent circuit with the same terminal characteristics.
- Next, we exclude the transformer from the circuit and solve it for voltages and currents.
- The solutions obtained for the portion of the circuit that was not replaced will be the correct values of voltages and currents of the original circuit.
- Finally, the voltages and currents on the other side of the transformer (in the original circuit) can be found by considering the transformer s turn ratio.

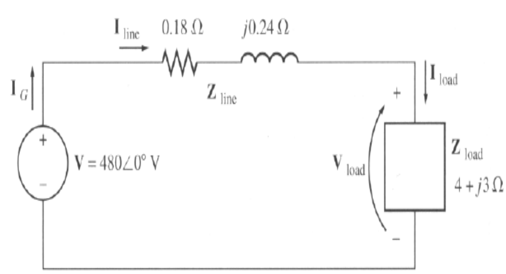
This process is called referring of transformer s sides.



Example

Example 4.1: A single-phase power system consists of a 480-V 60-Hz generator that is connected to the load Z_{load} = 4 + j3 Ω through the transmission line with Z_{line} = 0.18 + j0.24 Ω.

a) What is the voltage at the load? What are the transmission line losses?
b) If a 1:10 step up transformer and a 10:1 step down transformer are placed at the generator and the load ends of the transmission line respectively, what are the new load voltage and the new transmission line losses?

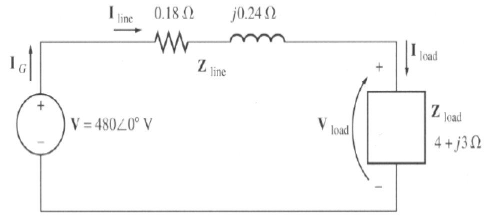


a) $I_G = I_{line} = I_{load} = \dfrac{V}{Z_{line} + Z_{load}}$

$= \dfrac{480\angle 0^o}{0.18 + j0.24 + 4 + j3}$

$= \dfrac{480\angle 0^o}{5.29\angle 37.8^o}$

$= 90.8\angle -37.8^o \quad A$



The load voltage:

$$V_{load} = I_{load} Z_{load} = (90.8\angle -37.8^o)(4 + j3) = (90.8\angle -37.8^o)(5\angle -36.9^o) = 454\angle -0.9^o$$

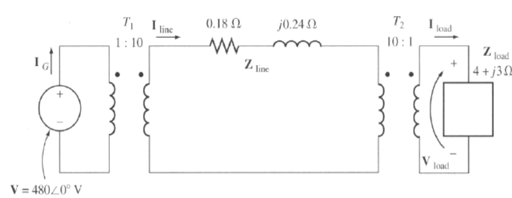
The line losses are:

$$P_{loss} = I_{line}^2 R_{line} = 90.8^2 \cdot 0.18 = 1484\ W$$

b) We will

1) eliminate transformer T_2 by referring the load over to the transmission line's voltage level.

2) Eliminate transformer T_1 by referring the transmission line's



The load impedance when referred to the transmission line (while the transformer T_2 is eliminated) is:

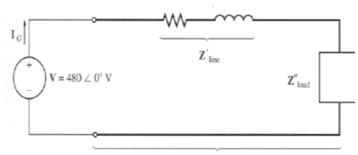
$$\grave{Z}_{load} = a_2{}^2 Z_{load} = \left(\frac{10}{1}\right)^2 (4 + j3)$$
$$= 400 + j300$$

The total impedance on the transmission line level is

$$Z_{equ} = Z_{line} + \grave{Z}_{load} = 400.18 + j300.24$$
$$= 500.3\angle 36.88^o$$

The total impedance is now referred across T_1 to the source's voltage level:

$$\grave{Z}_{equ} = a_1{}^2 Z_{load} = \left(\frac{1}{10}\right)^2 500.3\angle 36.88^o = 5.003\angle 36.88^o$$



Equivalent circuit

The generator's current is

$$I_G = \frac{V}{\hat{Z}_{equ}} = \frac{480\angle 0^o}{5.003\angle} = 95.94\angle - 36.88^o$$

Knowing transformers' turn ratios, we can determine line and load currents:

$$I_{line} = a_1 I_G = 0.1(95.94\angle - 36.88^o) = 9.594\angle - 36.88^o \quad A$$

$$I_{load} = a_2 I_{line} = 10(9.594\angle - 36.88^o) = 95.94\angle - 36.88^o \quad A$$

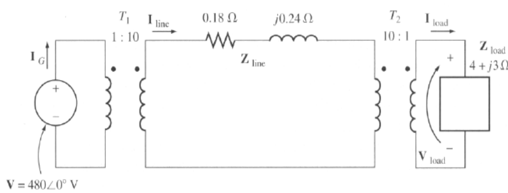
Therefore, the load voltage is:

$$V_{load} = I_{load} Z_{load} = (95.94\angle - 36.88^o)(5\angle - 36.7^o) = 479.7\angle - 0.01^o \quad V$$

The losses in the line are:

$$P_{loss} = I_{line}^2 R_{line} = 9.594^2 \cdot 0.18 = 16.7 \ W$$

Note: transmission line losses are reduced by a factor nearly 90, the load voltage is much closer to the generator's voltage – effects of increasing the line's voltage.



Transformers
Ch. 2

Dr. Feras Alasali

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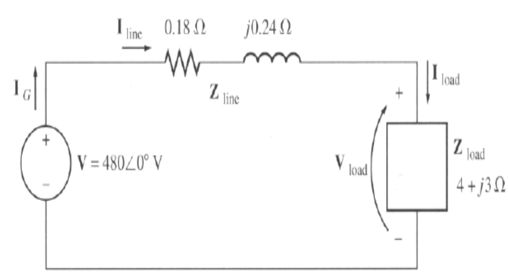

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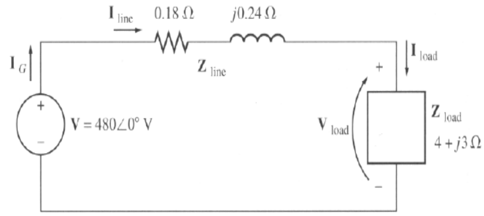

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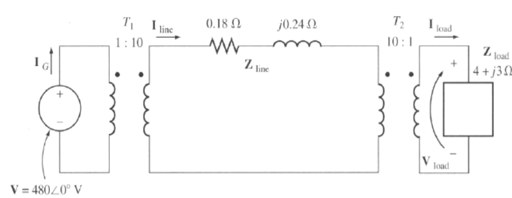

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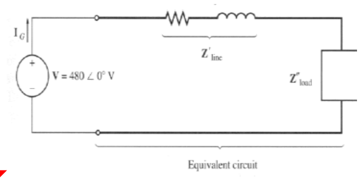

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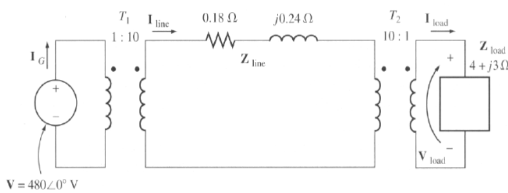
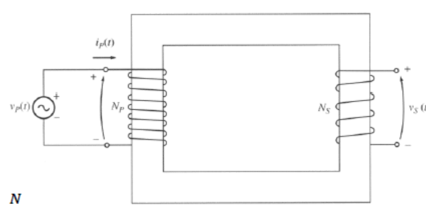


Theory of operation of real single phase transformers

Real transformers approximate ideal ones to some degree.



The basis transformer operation can be derived from Faraday's law:

$$e_{ind} = \frac{d\lambda}{dt}$$

Here λ is the flux linkage in the coil across which the voltage is induced:

$$\lambda = \sum_{i=1}^{N} \phi_i$$

where ϕ_i is the flux passing through the i^{th} turn in a coil – slightly different for different turns. However, we may use an average flux per turn in the coil having N turns:

$$\bar{\phi} = \lambda / N$$

Therefore:

$$e_{ind} = N\frac{d\bar{\phi}}{dt}$$

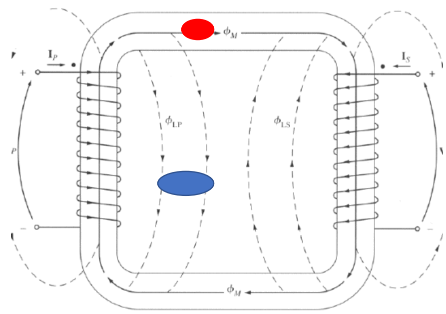
If the source voltage $v_p(t)$ is applied to the primary winding, the average flux in the primary winding will be:

$$\bar{\phi} = \frac{1}{N_p}\int v_p(t)dt$$

A portion of the flux produced in the primary coil passes through the secondary coil (mutual flux); the rest is lost (leakage flux):

$$\overline{\phi_p} = \phi_m + \phi_{Lp}$$

average primary flux mutual flux leakage flux



Similarly, for the secondary coil:

Average secondary flux $$\overline{\phi_s} = \phi_m + \phi_{Ls}$$

From the Faraday's law, the primary coil's voltage is:

$$v_p(t) = N_p\frac{d\bar{\phi}_p}{dt} = N_p\frac{d\phi_m}{dt} + N_p\frac{d\phi_{Lp}}{dt}$$

The secondary coil's voltage is:

$$v_s(t) = N_s\frac{d\bar{\phi}_s}{dt} = N_s\frac{d\phi_m}{dt} + N_s\frac{d\phi_{Ls}}{dt}$$

The primary and secondary voltages due to the mutual flux are:

$$e_p(t) = N_p\frac{d\phi_m}{dt}$$

$$e_s(t) = N_s\frac{d\phi_m}{dt}$$

Combining the last two equations:

$$\frac{e_p(t)}{N_p} = \frac{d\phi_m}{dt} = \frac{e_s(t)}{N_s}$$

Therefore:

$$\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a$$

That is, the ratio of the primary voltage to the secondary voltage both caused by the mutual flux is equal to the turns ratio of the transformer.

For well-designed transformers: $\Phi_m \gg \Phi_{LP} \ and \ \Phi_m \gg \Phi_{LS}$

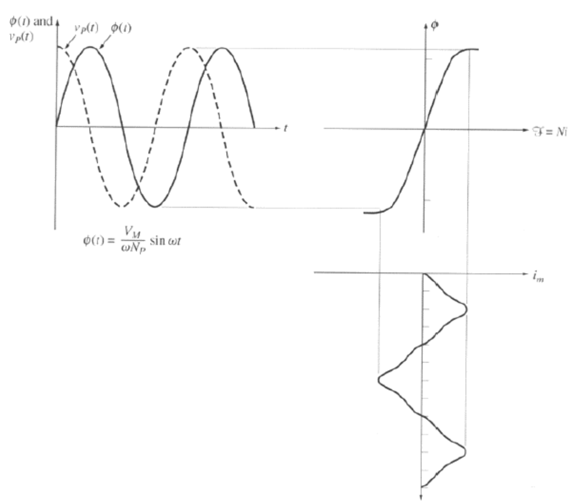
Therefore, the following approximation normally holds:

$$\frac{v_p(t)}{v_s(t)} \approx \frac{N_p}{N_s} \approx a$$

The magnetization current in a real transformer

Even when no load is connected to the secondary coil of the transformer, a current will flow in the primary coil. This current consists of:

1. The magnetization current i_m needed to produce the flux in the core;
2. The core-loss current i_{h+e} hysteresis and eddy current losses.



- If the values of current are comparable to the flux they produce in the core, it is possible to sketch a magnetization current. We observe:

1.Magnetization current is not sinusoidal: there are high frequency components;

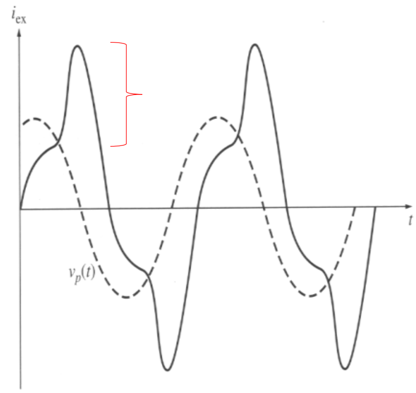
2. Once saturation is reached, a small increase in flux requires a large increase in magnetization current;

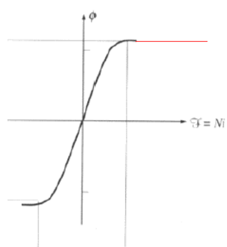
Core-loss current is:

1. Nonlinear due to nonlinear effects of hysteresis;
2. In phase with the voltage.

The total no-load current in the core is called the excitation current of the transformer:

$$i_{ex} = i_m + i_{h+e}$$



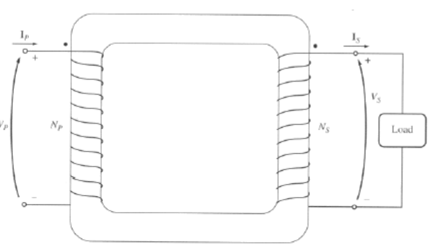


The current ratio on a transformer

- If a load is connected to the secondary coil, there will be a current flowing through it.
- A current flowing into the dotted end of a winding produces a positive magnetomotive force F
- The net magnetomotive force in the core
- where (R) is the reluctance of the transformer core. For well designed transformer cores, the reluctance is very small if the core is not saturated. Therefore:

$$F_p = N_p i_p$$

$$F_s = N_s i_s$$



$$F_{net} = N_p i_p - N_s i_s = \phi \Re$$

$$F_{net} = N_p i_p - N_s i_s \approx 0$$

The transformers equivalent circuit & Transformer losses

To model a real transformer accurately, we need to account for the following losses:

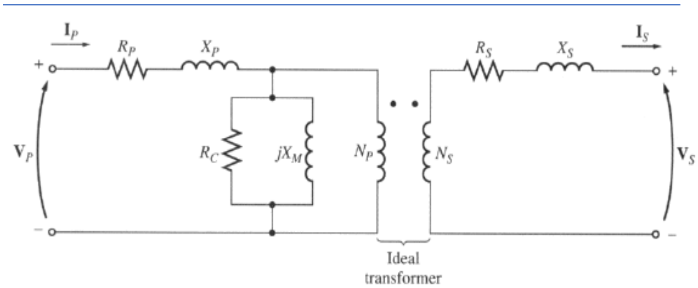
1. Copper losses resistive heating in the windings: I^2R

2. Eddy current losses resistive heating in the core: proportional to the square of voltage applied to the transformer.

3. Hysteresis losses energy needed to rearrange magnetic domains in the core: nonlinear function of the voltage applied to the transformer.

4. Leakage flux that escapes from the core and flux that passes through one winding only.

The exact equivalent circuit of a real transformer



Copper losses are modeled by the resistors R_p and R_s.

The leakage flux can be modeled by primary and secondary inductors.
The magnetization current can be modeled by a reactance X_M connected across the primary voltage source.
The core-loss current can be modeled by a resistance R_C connected across the primary voltage source.
Both currents are nonlinear; therefore, X_M and R_C are just approximations.

Since much of the leakage flux pass through air, and air has a constant reluctance that is much higher than the core reluctance, the primary coil's leakage flux is:

Leakage flux in a primary winding produces the voltage:

$$e_{Lp}(t) = N_p \frac{d\Phi_{LP}}{dt}$$

$$\Phi_{LP} = PN_p i_p$$

Where **P** permeance of flux path

$$e_{Lp}(t) = N_p \frac{d}{dt}(PN_p i_p) = N_p{}^2 P \frac{di_p}{dt}$$

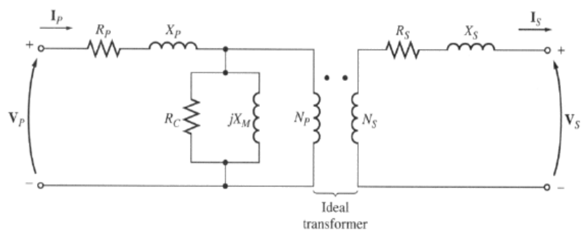
Recognizing that the self-inductance of the primary coil is

$$L = N_p{}^2\, P$$

The induced voltages are:

Primary coil: $e_{Lp}(t) = L_p \frac{di_p}{dt}$

Secondary coil: $e_{Ls}(t) = L_s \frac{di_s}{dt}$



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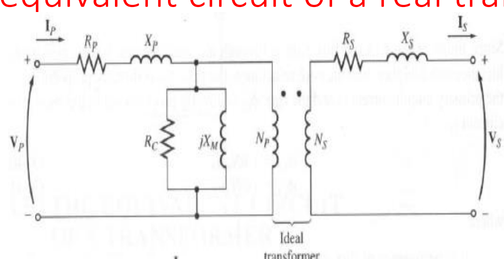
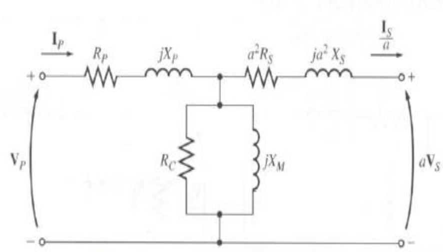
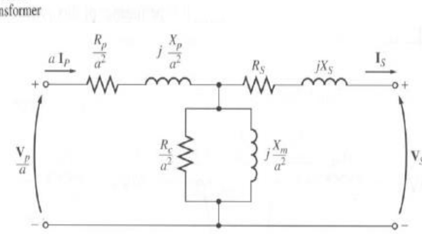


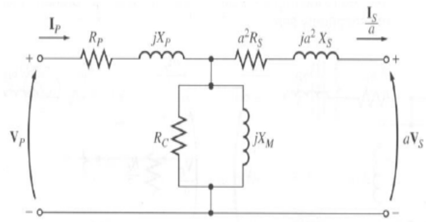


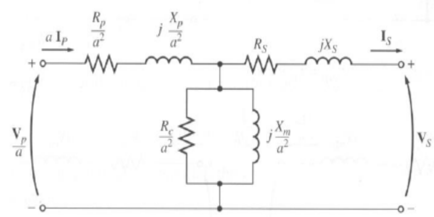
Equivalent circuit of the transformer referred to its primary side.

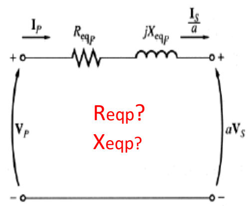


Equivalent circuit of the transformer referred to its secondary side.

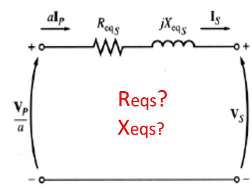
Approximate equivalent circuit of a transformer







Without an excitation branch referred to the primary side.

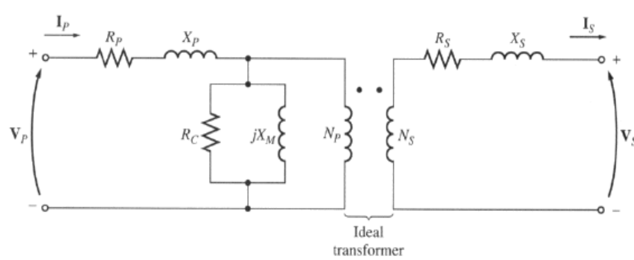


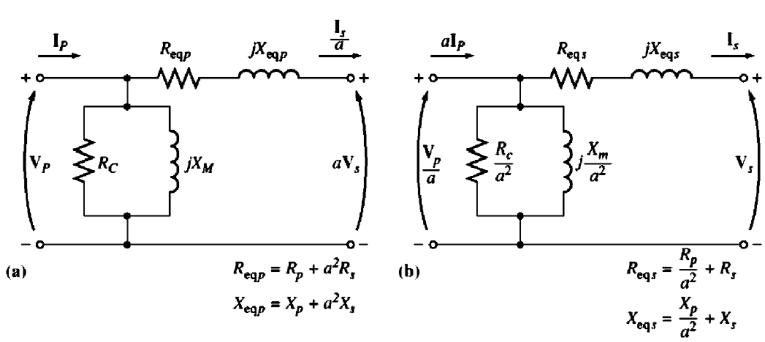
Without an excitation branch referred to the secondary side.

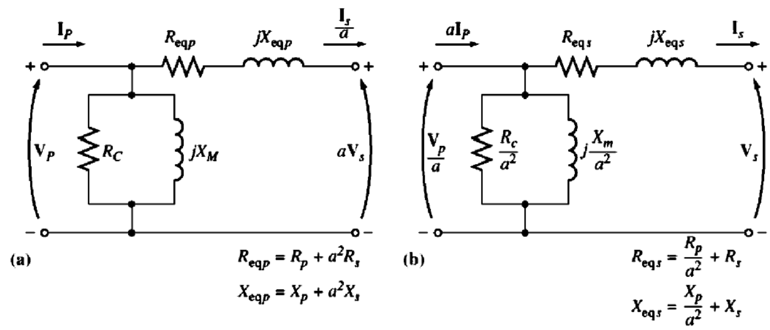
Transformers
Ch. 2

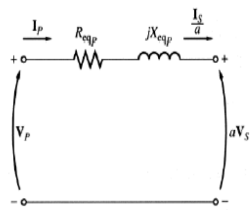
Dr. Feras Alasali

equivalent circuit of a transformer

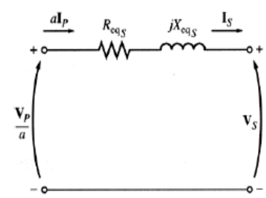








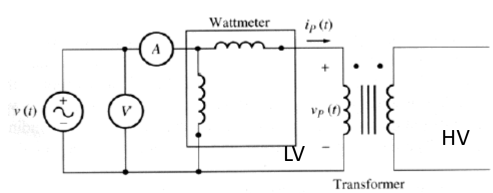
Without an excitation branch referred to the primary side.



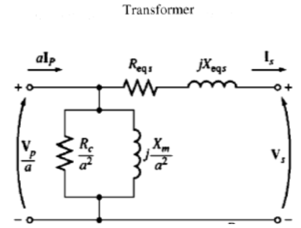
Without an excitation branch referred to the secondary side.

Determining the values of Tr. components

- **The open-circuit test.**



- Full line voltage is applied to the primary side of the transformer. The input voltage, current, and power are measured.
- From this information, the power factor of the input current and the magnitude and the angle of the excitation impedance can be determined.



- To evaluate R_C and X_M, we determine the conductance of the core-loss resistor is:

$$G_C = \frac{1}{R_C}$$

- The susceptance of the magnetizing inductor is:

$$B_M = \frac{1}{X_M}$$

- Since both elements are in parallel, their admittances add. Therefore, the total excitation admittance is:

$$Y_E = G_C - jB_M$$

$$Y_E = \frac{1}{R_c} - j\frac{1}{X_M}$$

- The **magnitude of the excitation admittance** in the open-circuit test is:

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$

- The **angle of the admittance** in the oper circuit test can be found from the circuit power factor (PF):

$$\cos(\theta) = PF = \frac{P_{OC}}{V_{OC}I_{OC}}$$

  - In real transformers, the power factor is always lagging, so the angle of the current always lags the angle of the voltage by θ degrees. The admittance is:

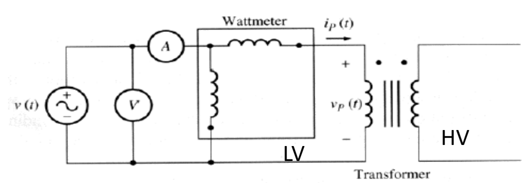
$$|Y_E| = \frac{I_{OC}}{V_{OC}}\angle - \theta$$

 - Therefore, it is possible to determine values of R_C and X_M in the open-circuit test.

$$= \frac{I_{OC}}{V_{OC}}\angle - cos^{-1}PF$$

O.C Test

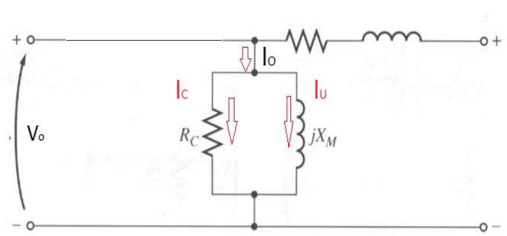
- Vo= rated voltage
- Io= small value
- Wo = core losses (small value)

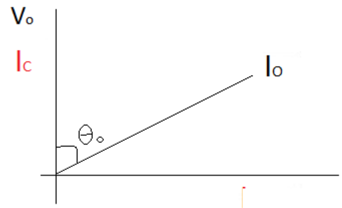


- From O.C test (Vo, Io, Wo)
- Where Wo= (Vo) (Io) (cosϴo)

cosϴo = Wo/(Vo)(Io)

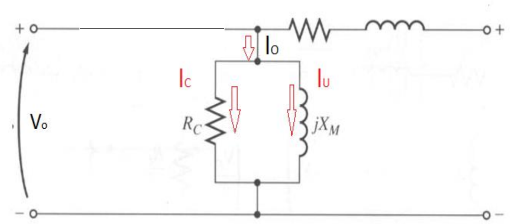
- ϴo is no load PF angle





$I_c = I_o \cos\theta_o$

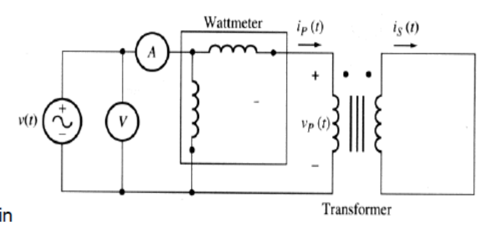
$I_u = I_o \sin\theta_o$



$R_c = R_o = V_o / I_c$

$X_m = X_o = V_o / I_u$

The short circuit test.

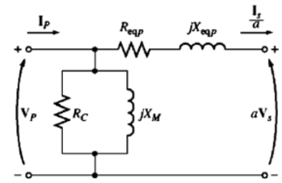


- Fairly low input voltage is applied to the primary side of the transformer. This voltage is adjusted until the current in the secondary winding equals to its rated value.
- The input voltage, current, and power are again measured.
- Since the input voltage is low, the current flowing through the excitation branch is negligible; therefore, all the voltage drop in the transformer is due to the series elements in the circuit. The magnitude of the series impedance referred to the primary side of the transformer is:

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

- The power factor of the current is given by: $\cos(\theta) = PF = \dfrac{P_{SC}}{V_{SC} I_{SC}}$

- Therefore: $$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle\theta$$



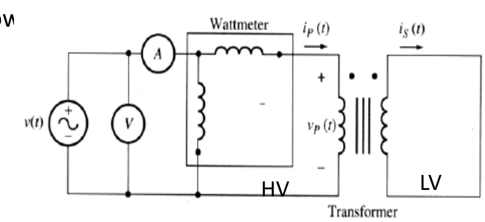
- Since the serial impedance Z_{SE} is equal to $$Z_{SE} = R_{eq} + jX_{eq}$$

- it is possible to determine the total series impedance referred to the primary side of the transformer. However, there is no easy way to split the series impedance into primary and secondary components. $$Z_{SE} = (R_p + a^2R_s) + j(X_p + a^2X_s)$$

- The **same tests can be performed on the secondary** side of the transformer. The results will yield the equivalent circuit impedances referred to the secondary side of the transformer.

S.C Test

- Vsc= short circuit voltage (small value)
- Isc= short circuit current
- Wsc = copper losses (short circuit pow

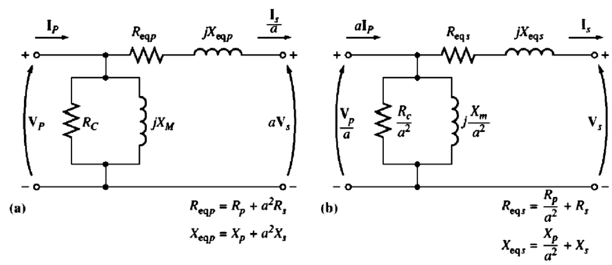


- From S.C test (Vsc, Isc, Wsc)
- Where Wsc= $(I_{sc})^2R_{eq}$
- $R_{seq} = \frac{W_{sc}}{(I_{sc})^2}$
- $Z_{seq} = \frac{V_{sc}}{I_{sc}}$ → $Z_{SE} = (R_p + a^2R_s) + j(X_p + a^2X_s)$
- $X_{seq} = \sqrt{(Z_{seq})^2 - (R_{seq})^2}$

Example

Example: We need to determine the equivalent circuit impedances of a **20 kVA, 8000/240 V, 60 Hz** transformer. The open-circuit and short-circuit tests led to the following data:

V_{OC} = 240 V	V_{SC} = 489 V
I_{OC} = 7.133 A	I_{SC} = 2.5 A
P_{OC} = 400 W	P_{SC} = 240 W



- **The open-circuit test.**

The power factor during the open-circuit test is $\cos(\theta) = PF = \frac{P_{OC}}{V_{OC} I_{OC}} = 0.234\ lagging$

The excitation admittance is $Y_E = \frac{I_{OC}}{V_{OC}} \angle - cos^{-1}PF = 0.0297\angle - cos^{-1} 0.234$

$$= 0.00693 - j0.0288$$

$$Y_E = \frac{1}{R_c} - j\frac{1}{X_M}$$

$$R_C = \frac{1}{0.00693} = 144\ \Omega \qquad X_M = \frac{1}{0.0288} = 34.63\ \Omega$$

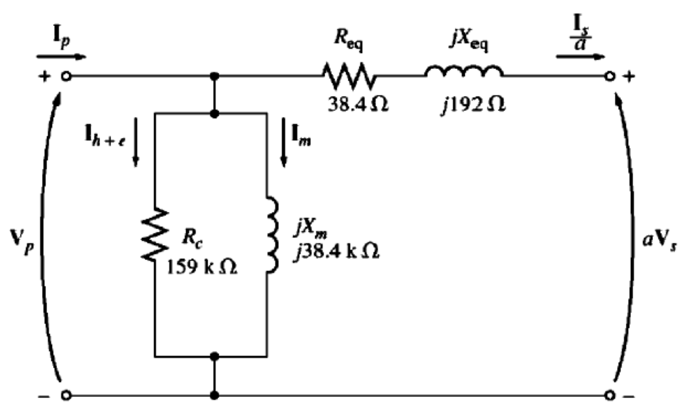
- **The short-circuit test.**

The power factor during the short-circuit test is $\cos(\theta) = PF = \frac{P_{SC}}{V_{SC} I_{SC}} = \frac{240}{(489)(2.5)} = 0.196\, lagging$

The Series impedance is $Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle cos^{-1}PF = \frac{489}{2.5} \angle - cos^{-1} 0.196$

$= 38.4 - j192$

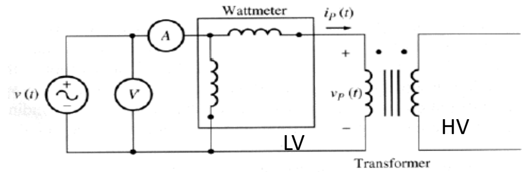
$R_{eq} = 38.4\Omega$ $X_{eq} = 192\Omega$



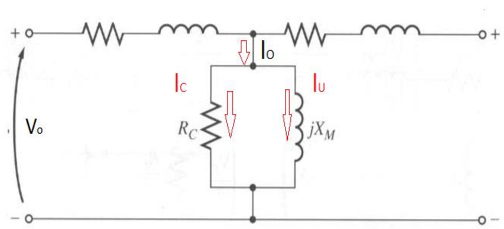
Rc,p = a^2 Rc,s = 33.3^2 (144) = 159 k ohm
Xm,p = a^2 Xm,s = 33.3^2 (34.63) = 38.4 k ohm

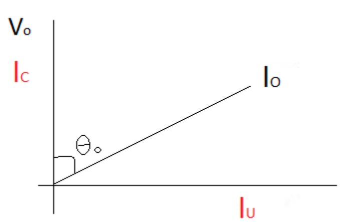
O.C Test

- V_o= rated voltage
- I_o= small value
- W_o = core losses (small value)



- From O.C test (V_o, I_o, W_o)
- Where W_o= (V_o) (I_o) ($\cos\Theta_o$) → $\cos\Theta_o = W_o/(V_o)(I_o)$
- Θ_o is no load PF angle



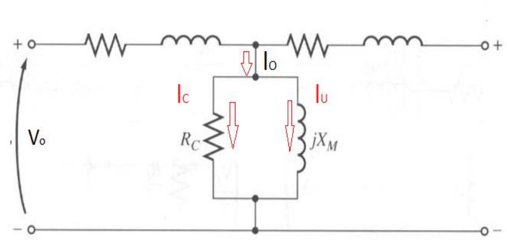


$I_c = I_o \cos\Theta_o$

$I_u = I_o \sin\Theta_o$

$R_c = R_o = V_o / I_c$

$X_m = X_o = V_o / I_u$



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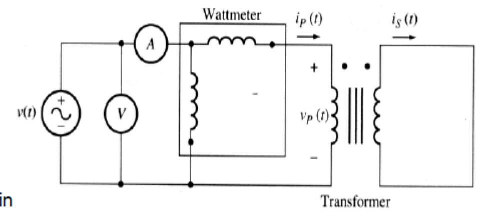
- Fairly low input voltage is applied to the primary side of the transformer. This voltage is adjusted until the current in the secondary winding equals to its rated value.



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- Therefore:

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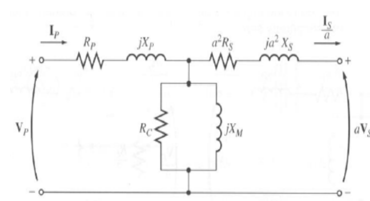

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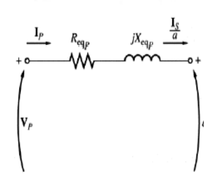

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- it is possible to determine the total series impedance referred to the primary side of the transformer. However, there is no easy way to split the series impedance into primary and secondary components.

$$Z_{SE} = (R_p + a^2R_s) + j(X_p + a^2X_s)$$

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S.C Test

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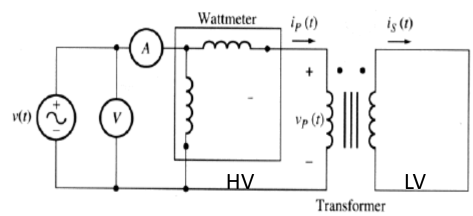
- Wsc = copper losses (short circuit power)

- From S.C test (Vsc, Isc, Wsc)
- Where Wsc= $(I_{sc})^2 R_{eq}$
- $R_{seq} = \frac{W_{sc}}{(I_{sc})^2}$
- $Z_{seq} = \frac{V_{sc}}{I_{sc}}$ ➡ $Z_{SE} = (R_p + a^2 R_s) + j(X_p + a^2 X_s)$
- $X_{seq} = \sqrt{(Z_{seq})^2 - (R_{seq})^2}$



Example

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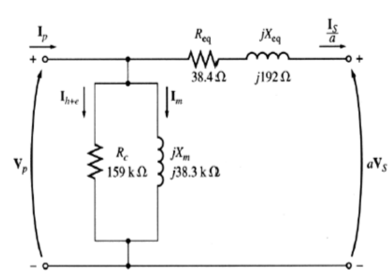

- **The open-circuit test.**

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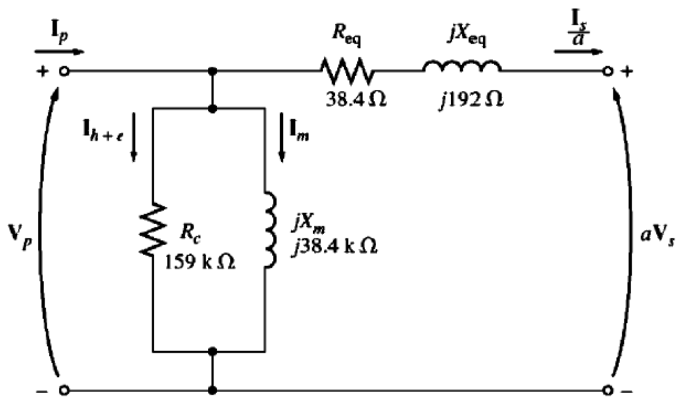
The Series impedance is $Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle cos^{-1} PF = \frac{489}{2.5} \angle - cos^{-1} 0.196$

$$= 38.4 - j192$$

$$R_{eq} = 38.4\Omega \qquad X_{eq} = 192\Omega$$

Quiz



- Rc,p = a^2 Rc,s = 33.3^2 (144) = 159 k ohm
- Xm,p = a^2 Xm,s = 33.3^2 (34.63) = 38.4 k ohm

The per unit system

- Another approach to solve circuits containing transformers is the per-unit system. Impedance and voltage-level conversions are avoided. Also, machine and transformer impedances fall within fairly narrow ranges for each type and construction of device while the per-unit system is employed.

- The voltages, currents, powers, impedances, and other electrical quantities are measured as fractions of some base level instead of conventional units.

$$Quantity\ per\ unit = \frac{actual\ value}{base\ value\ quantity}$$

$$P_{base}, Q_{base}, or\ S_{base} = V_{base} I_{base}$$

- Usually, two base quantities are selected to define a given per-unit system. Often, such quantities are voltage and power (or apparent power). In a 1-phase system:

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^{2}}{S_{base}}$$

$$Y_{base} = \frac{I_{base}}{V_{base}}$$

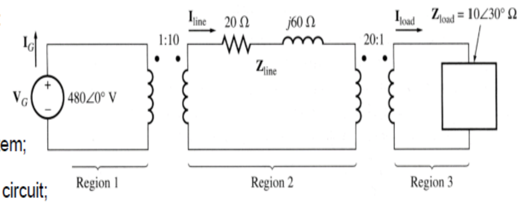
- Ones the base values of P (or S) and V are selected, all other base values can be computed form the above equations.
- In a power system, a base apparent power and voltage are selected at the specific point in the system . Note that a transformer has no effect on the apparent power of the system, since the apparent power into a transformer equals the apparent power out of a transformer. As a result, the base apparent power remains constant everywhere in the power system.
- On the other hand, voltage (and, therefore, a base voltage) changes when it goes through a transformer according to its turn ratio. Therefore, the process of referring quantities to a common voltage level is done automatically in the per unit system.

Example

A simple power system is given by the circuit

The generator is rated at 480 V and 10 kVA.

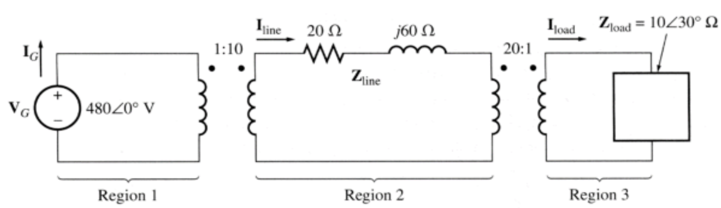
a) Find the base voltage, current, impedance, and apparent power at every points in the system;



b) Convert the system to its per-unit equivalent circuit;

c) Find the power supplied to the load in this system;

e) Find the power lost in the transmission line (Region 2).



a. In the generator region: $V_{base\ 1} = 480\ V$ and $S_{base} = 10\ kVA$

$$I_{base1} = \frac{S_{base1}}{V_{base1}} = \frac{10000}{480} = 20.83A \qquad Z_{base1} = \frac{V_{base1}}{I_{base1}} = \frac{480}{20.83} = 23.04\Omega$$

The turns ratio of the transformer T_1 is $a_1 = 0.1$; therefore, the voltage in the transmission line region is

$$V_{base2} = \frac{V_{base1}}{a_1} = \frac{480}{0.1} = 4800V \qquad S_{base2} = 10000\ VA$$

$$I_{base2} = \frac{S_{base2}}{V_{base2}} = \frac{10000}{4800} = 2.083A \qquad Z_{base2} = \frac{V_{base2}}{I_{base2}} = \frac{4800}{2.083} = 2304\Omega$$

The turns ratio of the transformer T_2 is $a_2 = 20$, therefore, the voltage in the load region is

$$V_{base3} = \frac{V_{base2}}{a_2} = \frac{4800}{20} = 240V \qquad S_{base3} = 10000\ VA$$

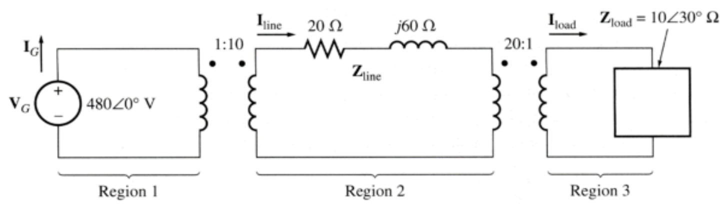
$$I_{base3} = \frac{S_{base3}}{V_{base3}} = \frac{10000}{240} = 41.67A \qquad Z_{base3} = \frac{V_{base3}}{I_{base3}} = \frac{240}{41.67} = 5.76\Omega$$

b. To convert a power system to a per-unit system, each component must be divided by its base value in its region. The generator's per-unit voltage is

$$V_{G,pu} = \frac{V_G}{V_{base1}} = \frac{480\angle 0°}{480} = 1\angle 0°$$

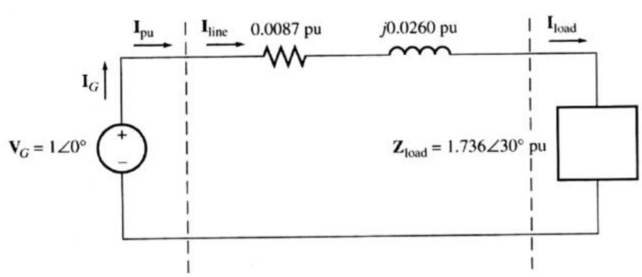
The transmission line's per-unit impedance is

$$Z_{line,pu} = \frac{Z_{line}}{Z_{base2}} = \frac{20 + j60}{2304} = 0.0087 + j0.026pu$$



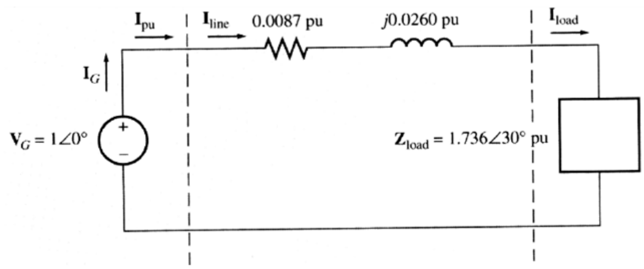
The load's per-unit impedance is

$$Z_{load,pu} = \frac{Z_{load}}{Z_{base3}} = \frac{10\angle 30°}{5.76} = 1.736\angle 30°$$



c. The current flowing in this per-unit power system is

$$I_{pu} = \frac{V_{pu}}{Z_{total,pu}} = \frac{1\angle 0°}{0.0087 + j0.026 + 1.736\angle 30°} = 0.569\angle 30.6° pu$$



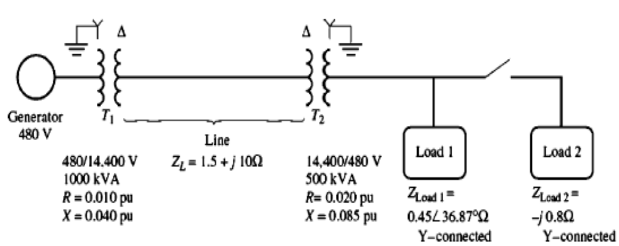
Therefore, the per-unit power on the load is $P_{load,pu} = I_{pu}{}^2 R_{pu} = 0.569^2 \cdot 1.503$ =0.487pu

The actual power on the load is $P_{load} = P_{load,pu} S_{base} = 0.487 \cdot 10000$ =487W

d. The per-unit power lost in the transmission line is $P_{line,pu} = I_{pu}{}^2 R_{line,pu} = 0.569^2 \cdot 0.0087$ =0.00282pu

The actual power lost in the transmission line $P_{line} = P_{line,pu} S_{base} = 0.00282 \cdot 10000$ =28.2W

Problem 2-23 in the book



Notes

- When only one device (transformer or motor) is analyzed, its own ratings are used as the basis for per unit system. When considering a transformer in a per unit system, transformer s characteristics will not vary much over a wide range of voltages and powers.
- For example, the series resistance is usually from 0.02 to 0.1 pu ; the magnetizing reactance is usually from 10 to 40 pu ; the core loss resistance is usually from 50 to 200 pu . Also, the per unit impedances of synchronous and induction machines fall within relatively narrow ranges over quite large size ranges.
- **If more than one transformer is present in a system, the system base voltage and power can be chosen arbitrary. However, the entire system must have the same base power, and the base voltages at various points in the system must be related by the voltage ratios of the transformers**.
- System base quantities are commonly chosen to the base of the largest component in the system.

Notes

Per-unit values given to another base can be converted to the new base either through an intermediate step (converting them to the actual values) or directly as follows:

$$(P,Q,S)_{pu,base2} = (P,Q,S)_{pu,base1}\frac{S_{base2}}{S_{base1}} \qquad V_{pu,base2} = V_{pu,base1}\frac{V_{base2}}{V_{base1}}$$

$$(R,X,Z)_{pu,base2} = (R,X,Z)_{pu,base1}\frac{Z_{base2}}{Z_{base1}} = (R,X,Z)_{pu,base1}\frac{V_{base2}^2 S_{base2}}{V_{base1}^2 S_{base1}}$$

Example 2

Sketch the appropriate per-unit equivalent circuit for the 8000/240 V, 60 Hz, 20 kVA transformer with R_c = 159 kΩ, X_M = 38.4 kΩ, R_{eq} = 38.3 Ω, X_{eq} = 192 Ω.

To convert the transformer to per-unit system, the primary circuit base impedance needs to be found.

$$S_{base1} = 20000\,VA$$

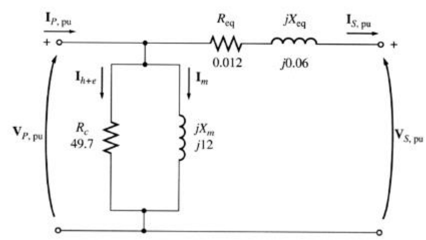
$$V_{base1} = 8000V$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base1}} = \frac{8000}{20000} = 3200\Omega$$

$$Z_{Rc,pu} = \frac{159000}{3200} = 49.7pu$$

$$Z_{XM,pu} = \frac{38400}{3200} = 12pu$$

$$Z_{Eq,pu} = \frac{38.3 + j192}{3200} = 0.012 + j0.06pu$$



Voltage regulation and efficiency

Since a real transformer contains series impedances, the transformer's output voltage varies with the load even if the input voltage is constant. To compare transformers in this respect, the quantity called a full-load voltage regulation (VR) is defined as follows:

$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s.fl}} \cdot 100\% = \frac{V_p/a - V_{s,fl}}{V_{s.fl}} \cdot 100\%$$

In a per-unit system: $$VR = \frac{V_{p,pu} - V_{s,fl,pu}}{V_{s.fl,pu}} \cdot 100\%$$

Where $V_{s,nl}$ and $V_{s,fl}$ are the secondary no load and full load voltages.

Note, the VR of an ideal transformer is zero.

The transformer phasor diagram

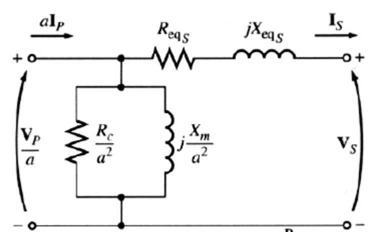
- To determine the VR of a transformer, it is necessary to understand t**<u>he voltage drops within it</u>** . Usually, the effects of the excitation branch on transformer VR can be ignored and, therefore, **<u>only the series impedances</u>** need to be considered. The VR depends **<u>on the magnitude of the impedances and on the current phase angle</u>**.

A phasor diagram is often used in the VR determinations. The phasor voltage V_s is assumed to be at 0^0 and all other voltages and currents are compared to it.

Considering the diagram and by applying the Kirchhoff's voltage law, the primary voltage is:

$$\frac{V_P}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

A transformer phasor diagram is a graphical representation of this equation.



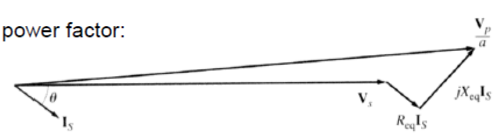
$$R_{eq\,S} = \frac{R_P}{a^2} + R_S$$

$$X_{eq\,S} = \frac{X_P}{a^2} + X_S$$

$$\frac{V_P}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

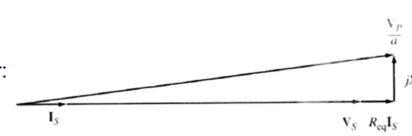
A transformer operating at a lagging power factor:

It is seen that $V_p/a > V_s$, VR > 0



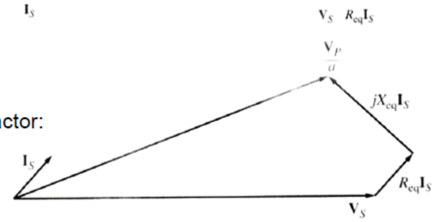
A transformer operating at a unity power factor:

It is seen that $VR > 0$



A transformer operating at a leading power factor:

If the secondary current is leading, the secondary voltage can be higher than the referred primary voltage; VR < 0.



The transformer efficiency

The efficiency of a transformer is defined as: $\eta = \frac{P_{out}}{P_{in}} \cdot 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \cdot 100\%$

Note: the same equation describes the efficiency of motors and generators.

Considering the transformer equivalent circuit, we notice three types of losses:

1. Copper (I^2R) losses – are accounted for by the series resistance
2. Hysteresis losses – are accounted for by the resistor R_c.
3. Eddy current losses – are accounted for by the resistor R_c.

Since the output power is $P_{out} = V_s I_s cos\theta$

The transformer efficiency is $\eta = \frac{V_s I_s cos\theta}{P_{cu} + P_{core} + V_s I_s cos\theta} \cdot 100\%$

Transformers
Ch. 2

Dr. Feras Alasali

The transformer efficiency

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The transformer efficiency is $$\eta = \frac{V_s I_s cos\theta}{P_{cu} + P_{core} + V_s I_s cos\theta} \cdot 100\%$$

The copper losses are: $$P_{cu} = I^2{}_s R_{eq}$$

The core losses are: $$P_{core} = \frac{(V_p/a)^2}{R_c}$$

Example

A 15 kVA, 2300/230 V transformer was tested to by open-circuit and closed-circuit tests. The following data was obtained:

V_{OC} = 230 V	V_{SC} = 47 V
I_{OC} = 2.1 A	I_{SC} = 6.0 A
P_{OC} = 50 W	P_{SC} = 160 W

a. Find the equivalent circuit of this transformer referred to the high-voltage side.

b. Find the equivalent circuit of this transformer referred to the low-voltage side.

c. Calculate the full-load voltage regulation at 0.8 lagging power factor, at 1.0 power factor, and at 0.8 leading power factor.

d. Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.

e. What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

a. The excitation branch values of the equivalent circuit can be determined as:

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = \cos^{-1}\frac{50}{(230)(2.1)} = 84°$$

Ic = $I_{OC}\cos 84°$ ⇨ Rc,s = Voc/Ic = 1050 Ω

Iu = $I_{OC}\sin 84°$ Xm = Voc/Iu = 110 Ω

From the test result , we know that the OC test is done on secondary side (so result referred to secondary)

From the short-circuit test data, the short-circuit impedance angle is

$$\theta_{SC} = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{160}{(47)(6)} = 55.4°$$

The equivalent series impedance is thus

$$Z_{SE} = \frac{V_{SC}}{I_{SC}}\angle\theta_{SC} = \frac{47}{6}\angle 55.4° = 4.45 + j6.45\ \Omega$$

The series elements referred to the primary winding are:

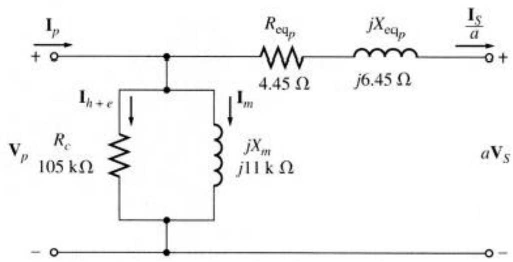
$$R_{eq} = 4.45\Omega$$
$$X_M = 6.45\Omega$$

Something wrong with lag , angle need to be negative

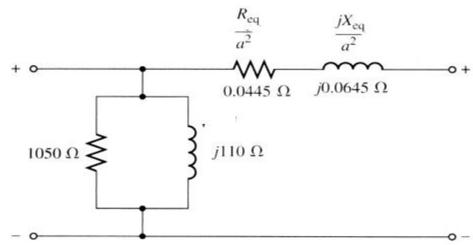
In (a) we need to find equivalent circuit referred to HV

$R_{c,p} = a^2 R_{c,s} = 105k\ ohm$

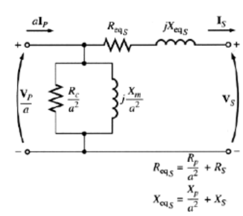
$X_{m,p} = a^2 X_{m,s} = 11k\ ohm$



b) Referred to LV



c. The full-load current on the secondary side of the transformer is



$$\frac{V_P}{a} = V_S + R_{eq} I_S + jX_{eq} I_S$$

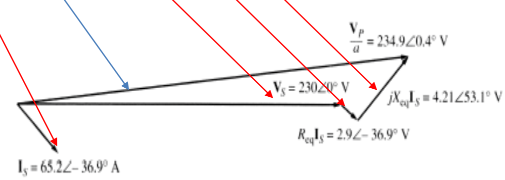
$$I_{S,rated} = \frac{S_{rated}}{V_{S,rated}} = \frac{15000}{230} = 65.2A$$

At PF = 0.8 lagging, current $I_{S,rated} = 65.2\angle - cos^{-1}(0.8) = 65.2\angle - 36.9°$

$$\frac{V_P}{a} = 230\angle 0° + 0.0445(65.2\angle - 36.9°) + j0.0645(65.2\angle - 36.9°)$$
$$= 234.85\angle 0.40°\ V$$

The voltage regulation is $VR = \frac{|V_p/a| - V_{s,fl}}{V_{s,fl}} \cdot 100\%$

$$= \frac{234.85 - 230}{230} \cdot 100\%$$
$$= 2.1\%$$



At PF = 1, current $I_{S,rated} = 65.2\angle - cos^{-1}(1) = 65.2\angle - 0°$

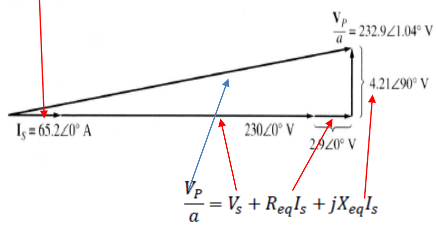
$$\frac{V_P}{a} = 230\angle 0° + 0.0445(65.2\angle 0°) + j0.0645(65.2\angle 0°)$$

$$= 232.94\angle 1.04°\, V$$

The voltage regulation is $VR = \frac{|V_p/a| - V_{s,fl}}{V_{s,fl}} \cdot 100\%$

$$= \frac{232.94 - 230}{230} \cdot 100\%$$

$$= 1.28\%$$



At PF = 0.8 leading, current $I_{S,rated} = 65.2\angle cos^{-1}(0.8) = 65.2\angle 36.9°$

$$\frac{V_P}{a} = 230\angle 0° + 0.0445(65.2\angle 36.9°) + j0.0645(65.2\angle 36.9°)$$

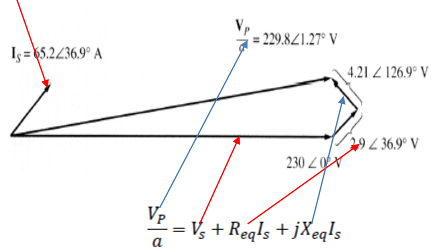
$$= 229.85\angle 1.27°\, V$$

The voltage regulation is

$$VR = \frac{|V_p/a| - V_{s,fl}}{V_{s,fl}} \cdot 100\%$$

$$= \frac{229.85 - 230}{230} \cdot 100\%$$

$$= -0.062\%$$



e. To find the efficiency of the transformer, first calculate its losses.

The efficiency of the transformer is

$$\eta = \frac{V_s I_s cos\theta}{P_{cu} + P_{core} + V_s I_s cos\theta} \cdot 100\%$$

The output power of the transformer at the given Power Factor is:

$$P_{out} = V_s I_s cos\theta = 230 \cdot 65.2 \cdot \cos(36.9°) = 12000W$$

The copper losses are: $P_{cu} = I^2{}_s R_{eq} = 65.2^2 - 0.0445 = 189W$

The core losses are: $P_{core} = \frac{(V_p/a)^2}{R_c} = \frac{234.85^2}{1050} = 52.5W$

The efficiency of the transformer is

$$\eta = \frac{V_s I_s cos\theta}{P_{cu} + P_{core} + V_s I_s cos\theta} \cdot 100\% = 98.03\%$$

Transformer taps and voltage regulation

- The transformer **turns ratio is a fixed (constant)** for the given transformer. However, distribution transformers have a **series of taps in the windings** to permit **small changes in their turns ratio**. Typically, transformers may have **4 taps** in addition to the nominal setting with spacing of **2.5 % of full load** voltage. Therefore, adjustments **up to 5 % above** or below the nominal voltage rating of the transformer are possible

Example: **A 500 kVA**, **13 200/480 V** transformer has **four 2.5 %** taps on its primary winding. What are the transformer's voltage ratios at each tap setting?

+ 5.0% tap	13 860/480 V
+ 2.5% tap	13 530/480 V
Nominal rating	13 200/480 V
- 2.5% tap	12 870/480 V
- 5.0% tap	12 540/480 V

Transformer taps and voltage regulation

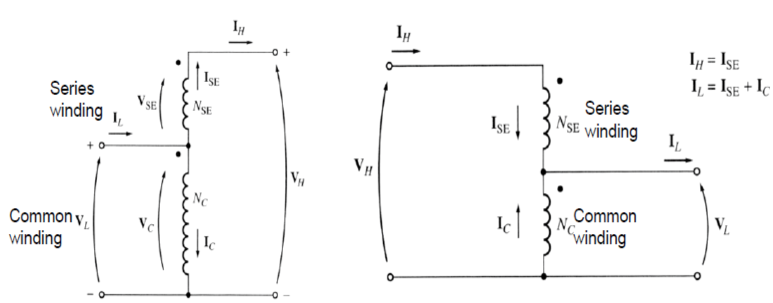
- Taps allow adjustment of the transformer in the field to **accommodate for local voltage variations.**
- Sometimes, transformers are used on a power line, whose voltage varies widely with the load (due to high line impedance, for instance). Normal loads need fairly constant input voltage though.
- One possible solution to this problem is to use a special transformer called a **tap changing under load (TCUL)** transformer or voltage regulator . TCUL is a transformer with the ability to change taps while power is connected to it. A voltage regulator is a TCUL with build in voltage sensing circuitry that automatically changes taps to keep the system voltage constant.
- These **self adjusting** transformers *are very common in modern power systems*.

The autotransformer

It is desirable to change the voltage by a small amount (for instance, when the consumer is far away from the generator and it is needed to raise the voltage to compensate for voltage drops).

In such situations, it would be expensive to wind a transformer with two windings of approximately equal number of turns. An autotransformer (a transformer with only one winding) is used instead.

Diagrams of step-up and step-down autotransformers:

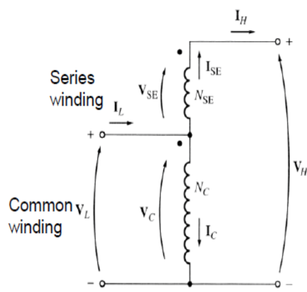


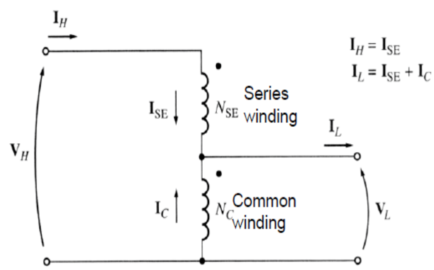
Since the autotransformer's coils are physically connected, a different terminology is used for autotransformers:

The voltage across the common winding is called a common voltage V_C, and the current through this coil is called a common current I_C. The voltage across the series winding is called a series voltage V_{SE}, and the current through that coil is called a series current I_{SE}.

The voltage and current on the low-voltage side are called V_L and I_L; the voltage and current on the high-voltage side are called V_H and I_H.

For the autotransformers:





$$\frac{V_C}{V_{SE}} = \frac{N_C}{N_{SE}}$$

$$N_C I_C = N_{SE} I_{SE}$$

$$V_L = V_C \qquad I_L = I_C + I_{SE}$$

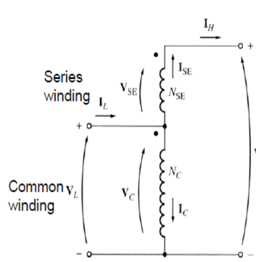
$$V_H = V_C + V_{SE} \qquad I_H = I_{SE}$$

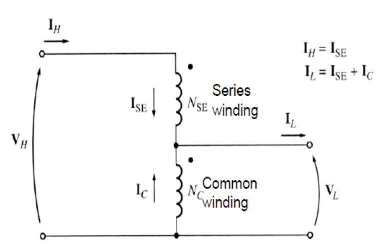
$$V_H = V_C + \frac{N_{SE}}{N_C} V_C = V_L + \frac{N_{SE}}{N_C} V_L$$

$$\boxed{\frac{V_L}{V_H} = \frac{N_C}{N_C + N_{SE}}}$$

$$I_L = I_{SE} + \frac{N_{SE}}{N_C} I_{SE} = I_H + \frac{N_{SE}}{N_C} I_H$$

$$\boxed{\frac{I_L}{I_H} = \frac{N_C + N_{SE}}{N_C}}$$





The apparent power advantage

Not all the power traveling from the primary to the secondary winding of the autotransformer goes through the windings. As a result, an autotransformer can handle much power than the conventional transformer (with the same windings).

Considering a step-up autotransformer, the apparent input and output powers are:

$$S_{in} = V_L I_L$$

$$S_{out} = V_H I_H$$

It is easy to show that

$$S_{in} = S_{out} = S_{IO}$$

where S_{IO} is the input and output apparent powers of the autotransformer.
However, the apparent power in the autotransformer's winding is

$$S_W = V_C I_C = V_{SE} I_{SE}$$

$$S_W = V_L(I_L - I_H) = V_L I_L - V_L I_H$$

$$V_H = V_C + \frac{N_{SE}}{N_C} V_C = V_L + \frac{N_{SE}}{N_C} V_L$$

$$\boxed{\frac{V_L}{V_H} = \frac{N_C}{N_C + N_{SE}}}$$

$$I_L = I_{SE} + \frac{N_{SE}}{N_C} I_{SE} = I_H + \frac{N_{SE}}{N_C} I_H$$

$$\boxed{\frac{I_L}{I_H} = \frac{N_C + N_{SE}}{N_C}}$$

$$S_W = V_L I_L - V_L I_L \frac{N_C}{N_C + N_{SE}} = S_{IO} \frac{N_{SE}}{N_C + N_{SE}}$$

Therefore, the ratio of the apparent power in the primary and secondary of the autotransformer to the apparent power **actually** traveling through its windings is

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}}$$

The last equation described the apparent power rating advantage of an autotransformer over a conventional transformer.

S_W is the apparent power actually passing through the windings. The rest passes from primary to secondary parts without being coupled through the windings.

Note that the smaller the series winding, the greater the advantage!

example

a 5 MVA autotransformer that connects a 110 kV system to a 138 kV system would have a turns ratio (common to series) 110:28. Such an autotransformer would actually have windings rated at:

$$S_W = S_{IO}\frac{N_{SE}}{N_{SE}+N_C} \qquad = 5\cdot\frac{28}{28+110} = 1.015MVA$$

Therefore, the autotransformer would have windings rated at slightly over 1 MVA instead of 5 MVA, which makes is 5 times smaller and, therefore, considerably less expensive.

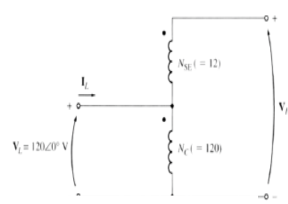
However, the construction of autotransformers is usually slightly different. In particular, the insulation on the smaller coil (the series winding) of the autotransformer is made as strong as the insulation on the larger coil to withstand the full output voltage.

The primary disadvantage of an autotransformer is that there is a direct physical connection between its primary and secondary circuits. Therefore, the electrical isolation of two sides is lost.

Example

A 100 VA, 120/12 V transformer will be connected to form a step-up autotransformer with the primary voltage of 120 V.

a. What will be the secondary voltage?
b. What will be the maximum power rating?
c. What will be the power rating advantage?



a. The secondary voltage: $V_H = \frac{N_{SE}+N_C}{N_c}V_L = 120\cdot\frac{120+12}{120} = 132V$

b. The max series winding current: $I_{SE,max} = \frac{S_{max}}{V_{SE}} = \frac{100}{12} = 8.33A$

c. The power rating advantage:

The secondary apparent power: $S_{out} = V_S I_S = V_H I_H = 132 \cdot 8.33 = 1100VA$

$$\frac{S_{IO}}{S_W} = \frac{1100}{100} = 11$$

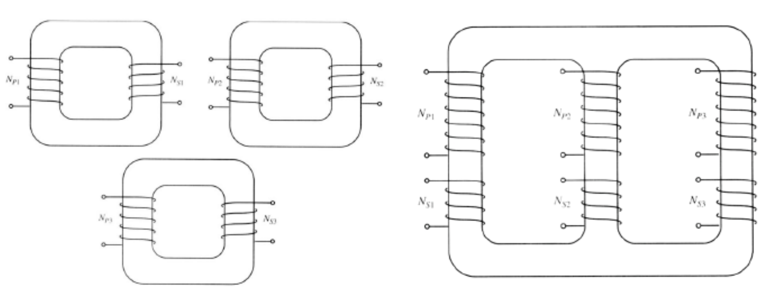
$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} = \frac{120 + 12}{12} = 11$$

Transformers Ch. 2

Dr. Feras Alasali

3 phase transformers

- The majority of the power generation/distribution systems in the world are 3 phase systems. The transformers for such circuits can be constructed either as a systems:
- as a 3 phase bank of independent identical transformers (can be replaced independently) or as a **single transformer** wound on a single 3 legged core (lighter, cheaper, more efficient).
- We assume that any single transformer in a 3 phase transformer (bank) behaves exactly as a single phase transformer. The impedance, voltage regulation, efficiency, and other calculations for 3 phase transformers are done on a per phase basis, using the techniques studied previously for single phase transformers.



Three-phase transformer connections

- Y-Y (Wye-Wye)
- Y-Δ (Wye-Delta)
- Δ-Y (Delta-Wye)
- Δ-Δ (Delta-Delta)

1. Y-Y connection:

The primary voltage on each phase of the transformer is

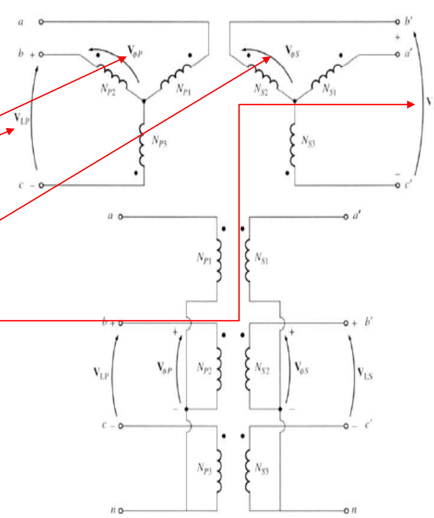
$$V_{\phi P} = \frac{V_{LP}}{\sqrt{3}}$$

The secondary phase voltage is

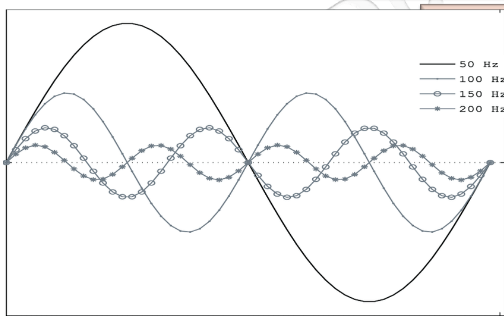
$$V_{LS} = \sqrt{3} V_{\phi S}$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3} V_{\phi P}}{\sqrt{3} V_{\phi S}} = a$$



Advantages	Disadvantages
Requires less turns per winding ie cheaper Phase voltage is $1/\sqrt{3}$ times of line voltage	If the load on the secondary side unbalanced then the shifting of neutral point is possible
Cross section of winding is large *i.e.* stronger to bear stress during short circuit Line current is equal to phase current	The third harmonic present in the alternator voltage may appear on the secondary side. This causes distortion in the secondary phase voltages
Less dielectric strength in insulating materials phase voltage is less	Magnetizing current of transformer has 3rd harmonic component



2. Y-Δ connection:

The primary voltage on each phase of the transformer is

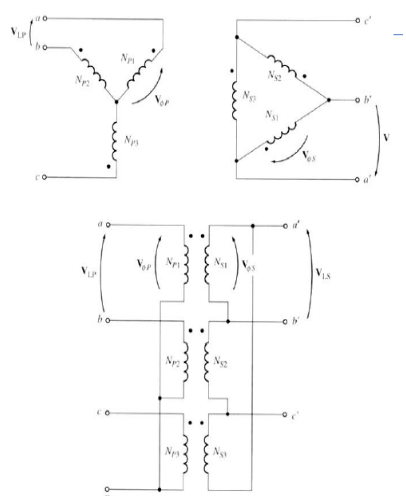
$$V_{\emptyset P} = \frac{V_{LP}}{\sqrt{3}}$$

The secondary phase voltage is

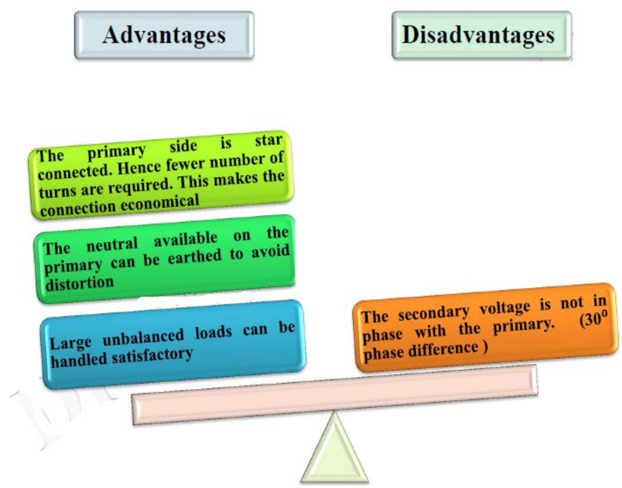
$$V_{LS} = V_{\emptyset S}$$

The overall voltage ratio is

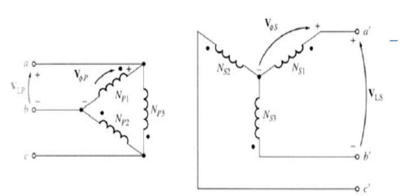
$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\emptyset P}}{V_{\emptyset S}} = \sqrt{3}a$$

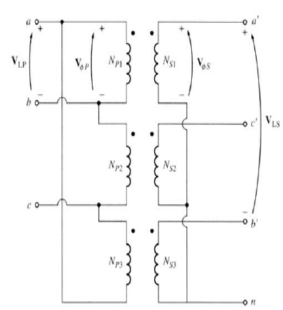


- The Y-Delta connection has no problem with third harmonic components due to circulating currents in Delta . It is also more stable to unbalanced loads since the Delta partially redistributes any imbalance that occurs.
- One problem associated with this connection is that the secondary voltage is shifted by 30 with respect to the primary voltage. This can cause problems when paralleling 3 phase transformers since transformers secondary voltages must be in phase to be paralleled. Therefore, we must pay attention to these shifts.



3. Δ-Y connection:





The primary voltage on each phase of the transformer is

$$V_{\emptyset P} = V_{LP}$$

The secondary phase voltage is

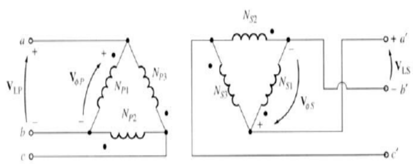
$$V_{LS} = \sqrt{3}V_{\emptyset S}$$

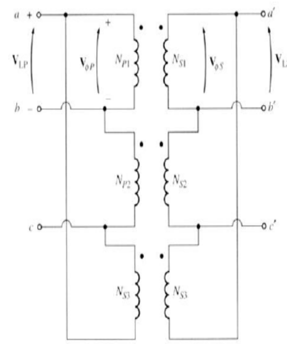
The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\emptyset P}}{\sqrt{3}V_{\emptyset S}} = \frac{a}{\sqrt{3}}$$

The same advantages and the same phase shift as the Y-Δ connection.

4. Δ-Δ connection:





The primary voltage on each phase of the transformer is

$$V_{\emptyset P} = V_{LP}$$

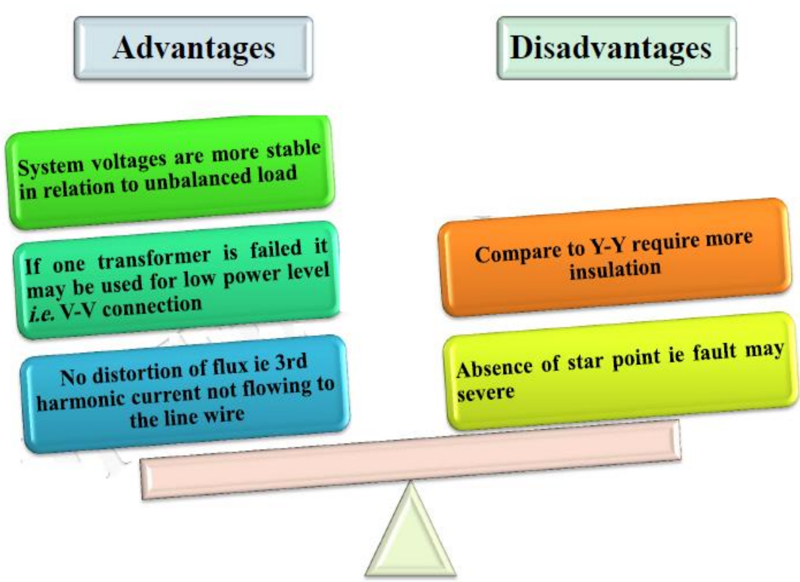
The secondary phase voltage is

$$V_{LS} = V_{\emptyset S}$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\emptyset P}}{V_{\emptyset S}} = a$$

No phase shift, no problems with unbalanced loads or harmonics.



- Where are these transformers used?

- **Delta-Delta** is generally **used** in systems where it need to be carry large currents on low voltages and especially when continuity of service is to be maintained even though one of the phases develops fault
- **Delta-Wye** connected **transformers** are widely **used** in low power distribution with the primary windings providing a three-wire balanced load to the utility company while the secondary windings provide the required 4th-wire neutral or earth connection.
- **Wye- Wye**
- **Wye-Delta**

3 phase transformer: per unit system

The per-unit system applies to the 3-phase transformers as well as to single-phase transformers. If the total base VA value of the transformer bank is S_{base}, the base VA value of one of the transformers will be

$$S_{1\emptyset,base} = \frac{S_{base}}{3}$$

Therefore, the base phase current and impedance of the transformer are

$$I_{\emptyset,base} = \frac{S_{1\emptyset,base}}{V_{\emptyset,base}} = \frac{S_{base}}{3V_{\emptyset,base}}$$

$$Z_{base} = \frac{(V_{\emptyset,base})^2}{S_{1\emptyset,base}} = \frac{3(V_{\emptyset,base})^2}{S_{base}}$$

The line quantities on 3-phase transformer banks can also be represented in per-unit system. If the windings are in Δ:

$$V_{L,base} = V_{\emptyset,base}$$

If the windings are in Y:

$$V_{L,base} = \sqrt{3}V_{\emptyset,base}$$

And the base line current in a 3-phase transformer bank is

$$I_{\emptyset,base} = \frac{S_{1\emptyset,base}}{V_{\emptyset,base}} = \frac{S_{base}}{3V_{\emptyset,base}} \qquad I_{L,base} = \frac{S_{base}}{\sqrt{3}V_{L,base}}$$

The application of the per-unit system to 3-phase transformer problems is similar to its application in single-phase situations. The voltage regulation of the transformer bank is the same.

example

A 50 kVA, 13 800/208 V Δ-Y transformer has a resistance of 1% and a reactance of 7% per unit.

a. What is the transformer's phase impedance referred to the high voltage side?
b. What is the transformer's voltage regulation at full load and 0.8 PF lagging, using the calculated high-side impedance?
c. What is the transformer's voltage regulation under the same conditions, using the per-unit system?

a. The high-voltage side of the transformer has the base voltage 13 800 V and a base apparent power of 50 kVA. Since the primary side is Δ-connected, its phase voltage and the line voltage are the same. The base impedance is:

$$Z_{base} = \frac{3\left(V_{\emptyset,base}\right)^2}{S_{base}} = \frac{3(13800)^2}{50000} = 114226\ \Omega$$

$$Z_{eq} = 0.01 + j\ 0.07\ pu$$

$$Z_{eq} = Z_{base} \text{ x Zpu} = 114.2 + \text{j}800 \text{ ohm}$$

b)

The rated phase voltage on the secondary of the transformer is

$$V_{\phi S} = \frac{208}{\sqrt{3}} = 120V$$

When referred to the primary (high-voltage) side, this voltage becomes

$$V_{\phi S}' = aV_{\phi S} = 13800V$$

Assuming that the transformer secondary winding is working at the rated voltage and current, the resulting primary phase voltage is

$$V_{\phi P} = aV_{\phi S} + R_{eq}I_{\phi} + jX_{eq}I_{\phi}$$

$$V_{\phi P} = 13800\angle 0° + 114.2 \cdot 1.208\angle cos^{-1}(-0.8) + 800 \cdot 1.208\angle cos^{-1}(-0.8) = 14506\angle 2.73°$$

The voltage regulation, therefore, is

$$VR = \frac{|V_{\phi P}| - aV_{\phi S}}{aV_{\phi S}} \cdot 100\% = \frac{14506 - 13800}{13800} \cdot 100\% = 5.1\%$$

Therefore, the base phase current and impedance of the transformer are

$$I_{\emptyset,base} = \frac{S_{1\emptyset,base}}{V_{\emptyset,base}} = \frac{S_{base}}{3V_{\emptyset,base}}$$

c. In the per-unit system, the output voltage is $1\angle 0^0$, and the current is $1\angle cos^{-1}$ (-0.8). Therefore, the input voltage is

$$V_{\phi P} = 1\angle 0° + 0.01 \cdot 1\angle cos^{-1}(0.8) + j0.07 \cdot 1\angle cos^{-1}(0.8) = 1.051\angle 2.73°$$

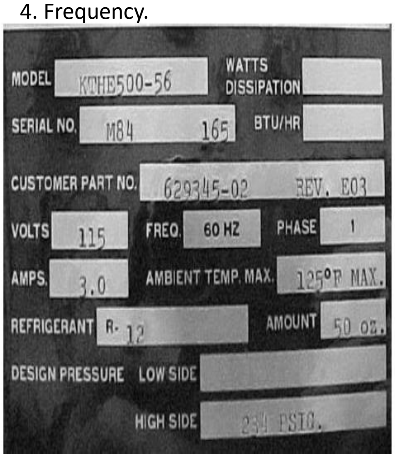
Thus, the voltage regulation in per-unit system will be

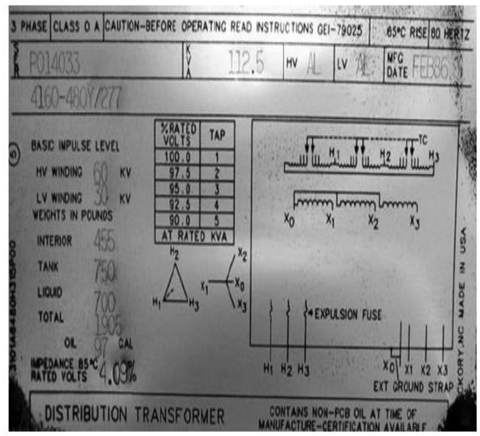
$$VR = \frac{1.051 - 1}{1} \cdot 100\% = 5.1\%$$

The voltage regulation in per-unit system is the same as computed in volts…

Transformer ratings

Transformers have the following major ratings:
1. Apparent power;
2. Voltage;
3. Current;
4. Frequency.





The **voltage** rating is a) used to protect the winding insulation from breakdown; b) related to the magnetization current of the transformer (more important)

An increase in voltage will lead to a proportional increase in flux. However, after some point (in a saturation region), such increase in flux would require an unacceptable increase in magnetization current!

Therefore, the maximum applied voltage (and thus the rated voltage) is set by the maximum acceptable magnetization current in the core.

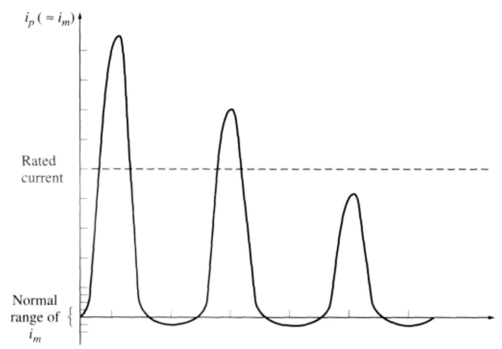
The apparent power rating sets (together with the voltage rating) the current through the windings. The current determines the I^2R losses and, therefore, the heating of the coils. Remember, overheating shortens the life of transformer's insulation!

In addition to apparent power rating for the transformer itself, additional (higher) rating(s) may be specified if a forced cooling is used. Under any circumstances, the temperature of the windings must be limited.

Note, that if the transformer's voltage is reduced (for instance, the transformer is working at a lower frequency), the apparent power rating must be reduced by an equal amount to maintain the constant current.

Transformer ratings: Current inrush

- **Inrush current** is the instantaneous high input **current** drawn by a power supply or electrical equipment at turn-on. This **arises** due to the high initial **currents** required to charge the capacitors and inductors or transformers.
- The maximum flux reached on the first half cycle depends on the phase of the voltage at the instant the voltage is applied.



$v(t) = V_m \sin \omega t$

DC Machinery Fundamentals
Ch. 7 (Edition 5)
Ch. 8 (Edition 4)

Dr. Feras Alasali

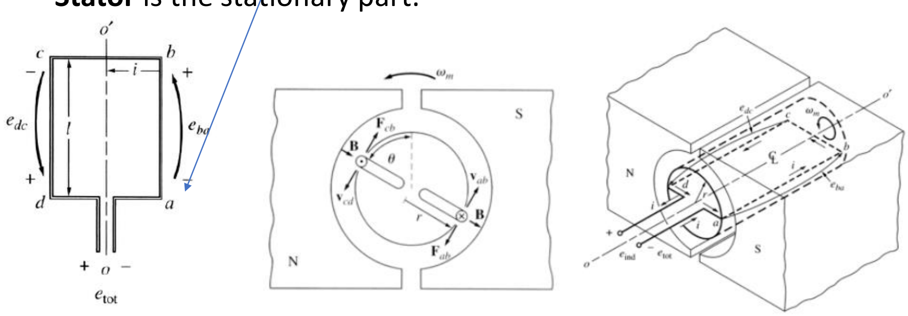
Simple DC Machine

- **1.6: Production of induced force on a wire (Motor Action)**
- **1.7: Induced voltage on a conductor moving in a magnetic field (Generator Action) .**
- **1.8. The Linear DC Machine**

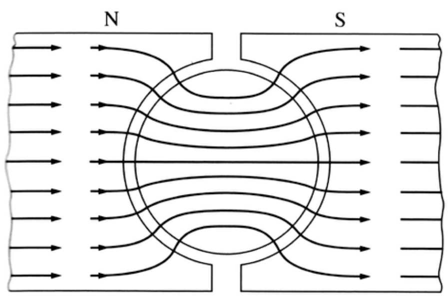
DC Machine

- DC power systems are not very common in the engineering practice. However, DC motors still have many practical applications, such as aircraft, and portable electronics, in speed control application
- An advantage of DC motors is that it is easy to control their speed.
- DC generators are quite rare.
- Most DC machines are similar to AC machines: i.e. they have AC voltages and current within them. DC machines have DC outputs just because they have a mechanism converting AC voltages to DC voltages at their terminals. This mechanism is called a commutator; therefore, DC machines are also called commutating machines.

- The simplest DC rotating machine consists of a single loop of wire rotating about a fixed axis. The magnetic field is supplied by the North and South poles of the magnet.
- **Rotor** is the rotating part;
- **Stator** is the stationary part.



- We notice that there is an air gap between the rotor and stator.
- The reluctance of air is much larger than the reluctance of core. Therefore, the magnetic flux must take the shortest path through the air gap.

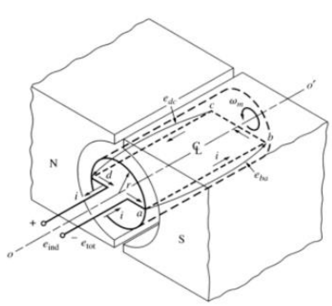


1. Voltage induced in a rotating loop

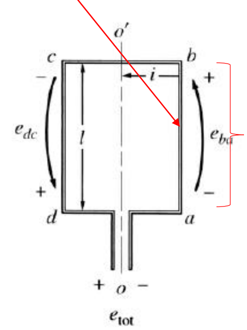
If a rotor of a DC machine is rotated, a voltage will be induced…

The total voltage will be a sum of voltages induced on each segment of the loop.

Voltage on each segment is:



$$e_{ind} = (V\ xB).I$$



The total induced voltage on the loop is: $e_{tot} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$

Segment ab: velocity ***v*** is perpendicular to the magnetic field ***B***, and the vector product *v* x *B* points into the page. Therefore, the voltage is

$$e_{ba} = (VxB).I = \begin{cases} vBl & -\text{ into page under the pole face} \\ 0 & -\text{ beyond the pole edges} \end{cases}$$

Segment bc: In this segment, vector product ***v* x *B*** is perpendicular to ***l***. Therefore, the voltage is **zero**

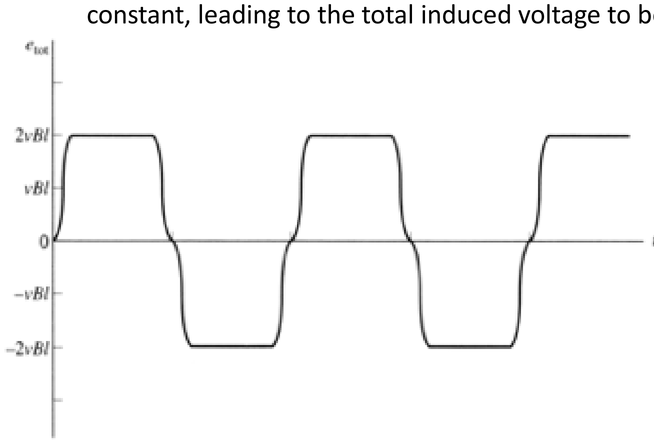
Segment cd: velocity ***v*** is perpendicular to the magnetic field ***B***, and the vector product *v* x *B* points out of the page. Therefore, the voltage is

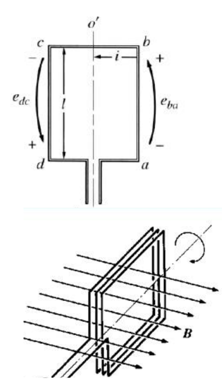
$$e_{dc} = (VxB).I = \begin{cases} vBl & -\text{ out of page under the pole face} \\ 0 & -\text{ beyond the pole edges} \end{cases}$$

Segment da: In this segment, vector product ***v* x *B*** is perpendicular to ***l***. Therefore, the voltage is **zero**

$$e_{tot} = \begin{cases} 2vBl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

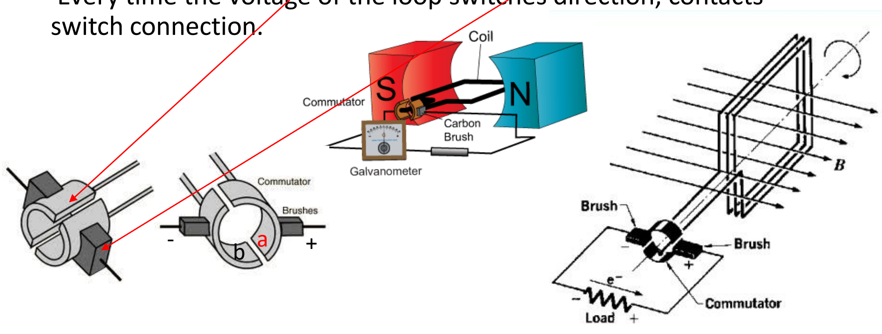
- When the loop rotates through 180 degree, segment ***ab*** is under the north pole fac instead of the south pole face. Therefore, the direction of the voltage on the segment **reverses** but its magnitude remains constant, leading to the total induced voltage to be

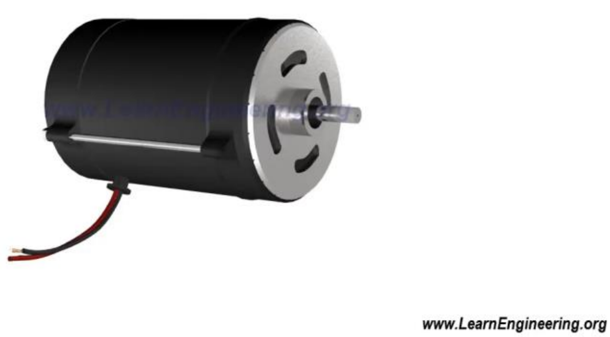




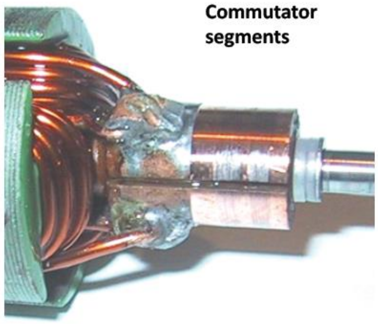
- **2. Getting DC voltage out of a rotating loop**

- A voltage out of the loop is alternatively a constant positive value and a constant negative value.
- One possible way to convert an alternating voltage to a constant voltage is by adding a **Commutator segment/brush circuitry** to the end of the loop.
- Every time the voltage of the loop switches direction, contacts switch connection.





Video



All DC machines have five principal components,

1. Field system (Stator)
2. Armature core
3. Armature winding
4. Commutator
5. Brushes

1- Field system

- The function of the field system **<u>is to produce uniform magnetic field</u>** within which the armature rotates.
- It consists of a number of poles bolted to the inside of circular frame (generally called **yoke**).

2- Armature core -----Rotor (Armature Core)

Armature core is a part of rotor on which conductors (Armature Winding) are wound. It consists of thin laminations.

The main parts of an armature Core are;

Slots :

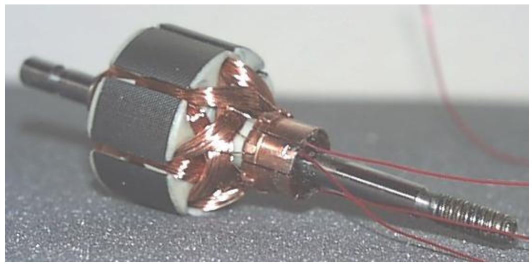
Tooth:

Shaft:

The purpose of this desgine is to reduce the eddy current loss.

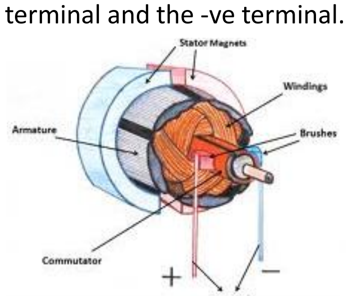
3- Armature winding

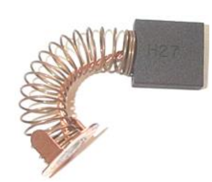
- The armature conductors are connected in series-parallel; the conductors being connected in series so as to increase the voltage and in parallel paths so as to increase the current.



5- Brushes

- The purpose of brushes is to ensure electrical connections between the **rotating commutator** and stationary **external load circuit**.
- The brushes are made of carbon and rest on the commutator.
- imperfect contact with the commutator may produce sparking.
- Multipole machines have as many brushes as they **have poles**. For example, a 4-pole machine has 4 brushes.
- The successive brushes have positive and negative polarities. Brushes having the same polarity are connected together so that we have two terminals viz., the +ve terminal and the -ve terminal.





Example

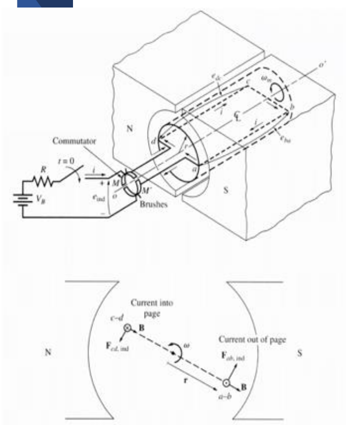


Figure shows a simple rotating loop between curved pole faces connected to a battery and a resistor through a switch. The resistor shown models the total resistance of the battery and the wire in the machine. The physical dimensions and characteristics of this machine are:

- **r = 0.5m** ***l*** **= 1.0m** ***R*** **= 0.3**ohm
- **B = 0.25T** **VB = 120V**

a) What happens when the switch is closed

Start up

$e_{ind} = 0$

↓

$i = \frac{V_B - e_{ind}}{R} = \frac{V_B}{R}$

- This current produces a torque

↓

$\tau_{ind} = \frac{2}{\pi} \phi i$

- torque produces an angular acceleration
- induced voltage is produced

↓

$e_{ind} = \frac{2}{\pi} \phi \omega_m$

e_{ind} ↑

↓

i ↓

↓

$\tau_{ind} = \frac{2}{\pi} \phi i$ ↓

↓

Untill $\tau_{ind} = 0$

↓

$V_B = e_{ind}$

↓

NO LOAD SPEED

$constant \ \omega$

- b) What is the machine's max. starting current?
- What is its steady state angular velocity at no load?

$$i = \frac{V_B}{R} = \frac{120}{0.3} = 400A$$

$$V_B = e_{ind} = \frac{2}{\pi}\omega_m$$

$$\omega_m = \frac{V_B}{2/_{\pi}\emptyset} = \frac{V_B}{2rlB}$$

$$\omega_m = \frac{120V}{2(0.5m)(1.0m)(0.25T)} = 480\ rad/s$$

c) Suppose a load is attached to the loop, and the resulting load torque is 10N.m.

- What would the new steady state speed be?
- How much power is supplied to the shaft of the machine?
- How much power is being supplied by the battery?
- Is this machine a motor or a generator?

If a load torque of **10 Nm** is applied to the shaft of the machine, it will begin to slow down

$$\omega \downarrow$$

$$e_{ind} = \frac{2}{\pi}\emptyset\omega_m \downarrow$$

$$i = \frac{V_B - e_{ind}}{R} = \frac{V_B}{R} \uparrow$$

$$\tau_{ind} = \frac{2}{\pi}\emptyset i \uparrow$$

$Umitl$

$$|\tau_{ind}| = |\tau_{lozad}|$$

$at\ lower\ \omega$

- What would the new steady state speed be?

$$i = \frac{\tau_{ind}}{2rlB} = \frac{10}{2(0.5)(1.0)(0.25)} = 40A$$

$$e_{ind} = 120 - (40)(0.3) = 108\ V$$

$$\omega = \frac{e_{ind}}{2rlB} = \frac{108V}{2(0.5m)(1.0m)(0.25T)} = 432\ rad/s$$

- **How much power is being supplied by the battery?**
- **How much power is supplied to the shaft of the machine?**
- **Is this machine a motor or a generator?**

$$p = \tau\omega_m = (10)(432) = 4320\ W$$

$$p = V_B i = (120)(40) = 4800\ W$$

This machine is operating as a motor, converting electric power to mechanical power.

d) Suppose the machine is again unloaded, and a torque of 7.5 N.m is applied to the shaft in the direction of rotation.

- **What is the new steady state speed? Is this machine now a motor or a generator?**
- If a torque is applied in the direction of motion, the rotor accelerates
- As the speed increases, the internal voltage e_{ind} increases and exceeds VB.
- so the current flows out into the battery . This machine is now a generator

$$i = \frac{\tau_{ind}}{2rlB} = \frac{7.5}{2(0.5)(1.0)(0.25)} = 30A$$

$$e_{ind} = 120 + (30)(0.3) = 129\ V$$

$$\omega = \frac{V_B}{2rlB} = \frac{129V}{2(0.5m)(1.0m)(0.25T)} = 516\ rad/s$$

e) Suppose the machine is running unloaded.

What would the final steady state speed of the rotor be if the flux density were reduced to 0.20 T?

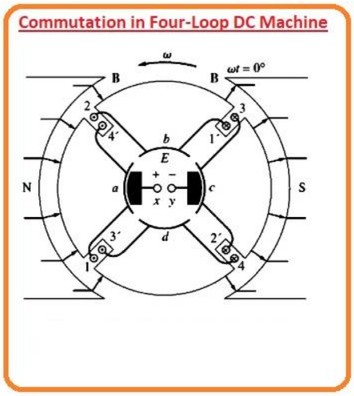
$$\omega = \frac{V_B}{2rlB} = \frac{120V}{2(0.5m)(1.0m)(0.20T)} = 600\ rad/s$$

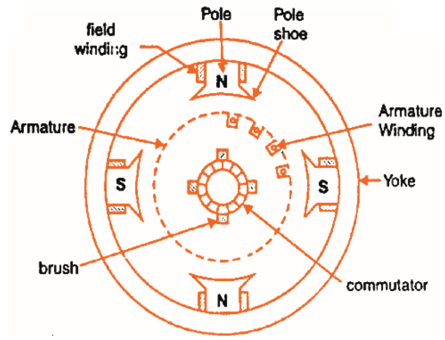
DC Machinery Fundamentals

Dr. Feras Alasali

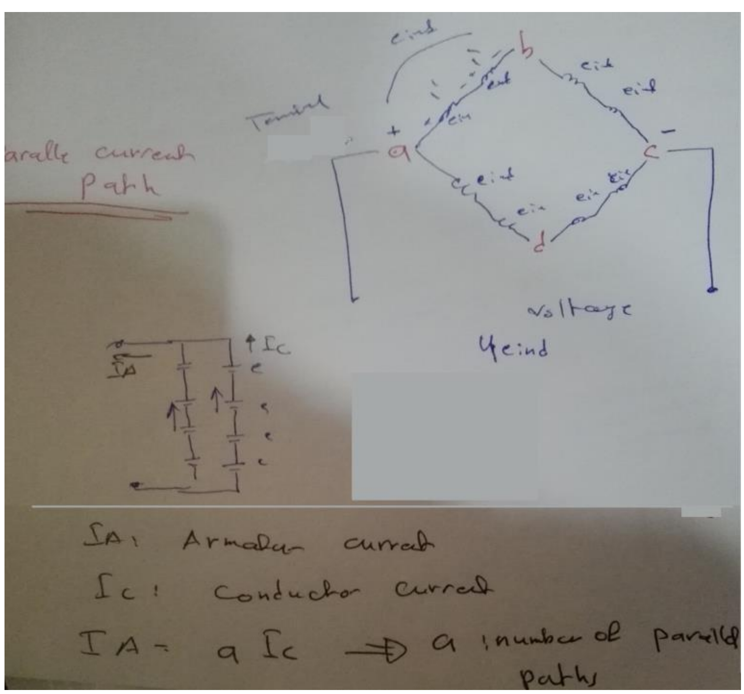
Commutator and Armature Windings

- **These slides cover by the book from section 8.2 to section 8.8 at edition 4 (for edition 5 will be 7.2-7.8).**



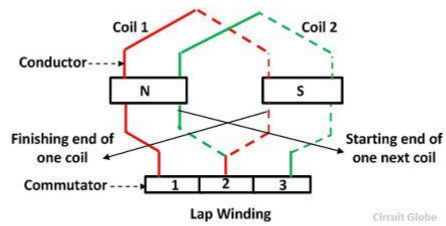


Electrical angle =P\2 (mechanical angle).

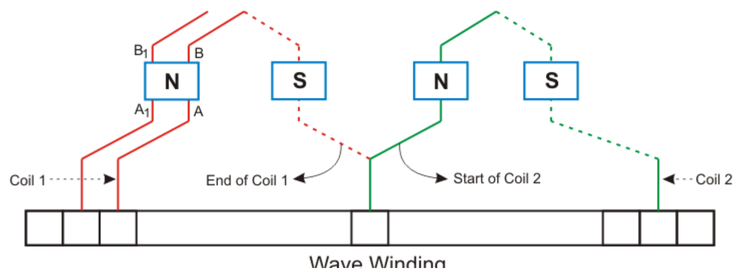


Connections to the commutator segments:

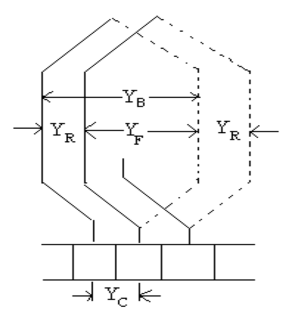
- Once the windings are installed in the rotor slots, they must be connected to the commutator.
- There is mainly two ways to connect the conductors with the Commutator:
- 1- Lap winding



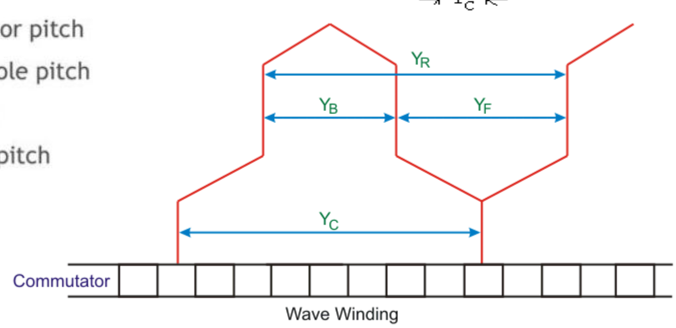
- 2- Wave winding



DC machine winding terms and design



- Z = the number conductors
- P = number of poles
- Y_B = Back pitch
- Y_F = Front pitch
- Y_C = Commutator pitch
- Y_A = Average pole pitch
- Y_P = Pole pitch
- Y_R = Resultant pitch



- **Back pitch (Y_B):** It's the gap measured in terms of armature winding amongst the 2 sides of a coil at the back of the armature shown in the fig. It's represented by Y_B. For instance, if a coil is made by connecting conductor one (higher conductor in a slot) to conductor twelve (bottom conductor in another slot) at the back of the coil, then back pitch is **Y_B=12-1=11 conductors**.
- **Front pitch (Y_F):** It is the gap measured in terms of armature winding between the coil sides hooked up to anyone commutator segment shown in the fig. it's denoted by Y_F. For instance, if coil side twelve and coil side three are connected to an equivalent commutator segments, then front pitch is **Y_F=12-3=9 conductors.**
- **Resultant pitch (Y_R):** It is distance (i.e. measured in terms of armature conductors) between one conductor and the opening of successive conductor to that it's connected shown in fig. It is denoted by Y_R. Therefore, the resultant pitch is that the pure mathematics add of the back and front pitches.

- **Commutator pitch (Y_C):** It's the quantity of commutator segments spanned by every coil of the armature winding.
- For lap winding, Y_C=1
- For wave winding, $Y_C = Y_F + Y_p$

- **Pole Pitch Y_p** : The pole pitch is defined as peripheral distance between center of two adjacent poles in DC machine. This distance is measured in term of armature slots or armature conductor come between two adjacent pole centers.

LAP Winding	Wave Winding
Y_p=Z/P	$Y_p=Y_F$
$Y_F=Y_p$ -1	$Y_p=Y_F$
Y_C=1	**$Y_C=Y_P+Y_F$**
Number of poles = number of brushes	Number of poles = 2 (number of brushes)
Y_A= (Y_B+Y_F)/2	**Y_A= (Y_B+Y_F)/2**
• High current and low voltage • Number of current path (a)= P • Number of brushes = P • Is usually have even number of slots	• High voltage and low current • Number of current path (a) = 2 • Number of brushes = 2 • Is usually have odd number of slots

Example

Simple lap winding Generator with 4-poles and 16 slots, Find the pole pitch (armature pitch) and front pitch?

Y_p=Z/P = 16/4 = 4 slots

$Y_F=Y_p$ -1 = 4-1 =3 slots

Power flow and Losses in DC machine

Unfortunately, not all electrical power is converted to mechanical power by a motor and not all mechanical power is converted to electrical power by a generator…

The efficiency of a DC machine is:

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\%$$

or

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \cdot 100\%$$

There are **five** categories of losses occurring in DC machines.

1. **Electrical or copper losses** – the resistive losses in the armature and field windings of the machine.

Armature loss: $P_A = I^2{}_A R_A$

Field loss: $P_F = I^2{}_F R_F$

Where I_A and I_F are armature and field currents and R_A and R_F are armature and field (winding) resistances usually measured at normal operating temperature.

2. **Brush (drop) losses** – the power lost across the contact potential at the brushes of the machine.

$$P_{BD} = V_{BD} I_A$$

Where I_A is the armature current and V_{BD} is the brush voltage drop. The voltage drop across the set of brushes is approximately constant over a large range of armature currents and it is usually assumed to be about 2 V.

3. **Core losses** – hysteresis losses and eddy current losses.

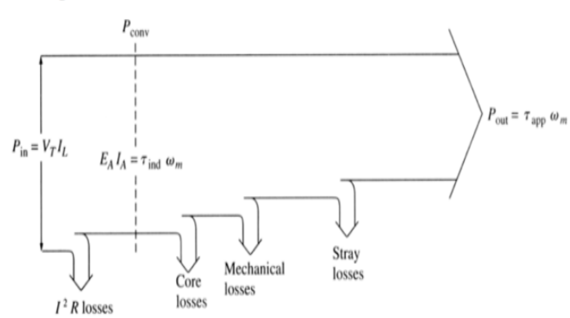
4. **Mechanical losses** – losses associated with mechanical effects: friction (friction of the bearings) and windage (friction between the moving parts of the machine and the air inside the casing). These losses vary as the cube of rotation speed n^3.

5. **Stray (Miscellaneous) losses** – losses that cannot be classified in any of the previous categories. They are usually due to inaccuracies in modeling. For many machines, stray losses are assumed as 1% of full load.

The Power-Flow Diagram

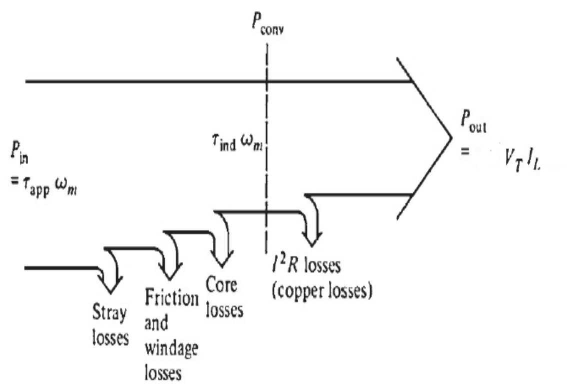
On of the most convenient technique to account for power losses in a machine is the power-flow diagram.

For a DC motor:



Electrical power is input to the machine, and the electrical and brush losses must be subtracted. The remaining power is ideally converted from electrical to mechanical form at the point labeled as P_{conv}.

For a DC generator:



The electrical power that is converted is

$$P_{conv} = E_A I_A$$

And the resulting mechanical power is

$$P_{conv} = \tau_{ind}\omega_m$$

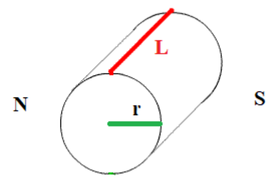
After the power is converted to mechanical form, the stray losses, mechanical losses, and core losses are subtracted, and the remaining mechanical power is output to the load.

Generated (induced) voltage and torque in DC machine

- Area of armature (A) = $2\,\pi\, rL$
- Area of face one pole (Ap) = $2\,\pi\, rL/P$

Where P is the number of poles.

r.L = Ap.P / 2 π



e_{ind} = $V\,\beta\, L$ for single conductor

E_A : Voltage out of the armature of real machine = N. β.$L.V$ / a

Where N is the number of conductor

a is the number of current paths.

V (linear speed) = $\omega.r$ where ω is angular speed

$\omega = 2\pi n\ /60$

Where n is the speed in Revolution per minute [RPM]

- E_A = N. L. ω .r /a But r.L = Ap.P / 2 π
- E_A = N. β. Ap .P. ω / 2 π a But β. Ap =Ø
- E_A = K Ø ω
- Where K is constant K = N.P/ 2 π a
- **Torque induced in Armature**
- F (induced force) = N. I_c. L. β
- Where I_c is the conductor current and I_c = $\frac{I_A}{a}$
- I_A is the armature current and a is the number of current paths.

- Torque $(\tau_{em}) = r.F =$ N. β.L.r.$\frac{I_A}{a}$
- $\tau_{em} = r.F =$ N. β.Ap.P.I_A/ 2 π

But β. Ap =Ø

- τ_{em} = K Ø I_A

Where

K is constant factor = N.P/ 2 πa

I_A is armature current = a I_c

Example

In DC machine, the flux per pole is 3 m wb, and the speed of the machine is 1800 RPM, it has a Lap winding armature with 12 coils (10 turns conductor per coil) . If it has 4 poles and armature draws I_c = 12.5 A. Find the following:

1. EA , τ_{em} and P_{em}
2. If the winding redesigned as wave winding find the EA , τ_{em} and P_{em} and compare it with lap winding connection

For 1:

- a=4 (lap winding a=P)
- I_A **=** a I_c **= 4 (**12.5) = 50 A
- EA = K $\emptyset$ ω = $\frac{N.P}{2\pi\ a}\emptyset\frac{2\pi n}{60}$ = 10.8 V
- τ_{em} = K $\emptyset$ I_A = $\frac{N.P}{2\pi\ a}\emptyset$ a I_c = 2.866 N.m
- P_{em} = E_A I_A = 50 (21.6) = 540 Watt

- **For 2:**
- a=2(wave winding)
- I_A **=** a I_c **= 2 (**12.5) = 25 A
- EA = K $\emptyset$ ω = $\frac{N.P}{2\pi\ a}\emptyset\frac{2\pi n}{60}$ = 21.6 V
- τ_{em} = K $\emptyset$ I_A = $\frac{N.P}{2\pi\ a}\emptyset$ a I_c = 2.866 N.m
- P_{em} = E_A I_A = 25 (43.2) = 540 Watt
- Note: when we change from Lap To Wave Current and volatge will be effected but power and torque will not be effected.

DC Motors and Generators
Ch. 9 -4th edition
Ch.8- 5th edition

Dr. Feras Alasali

Introduction to DC motors

- **DC motors are driven from DC power supply, there are five major types of DC motors in general use:**

1- The separately excited dc motor.

2- The shunt dc motor.

3- The permanent –magnet DC motor

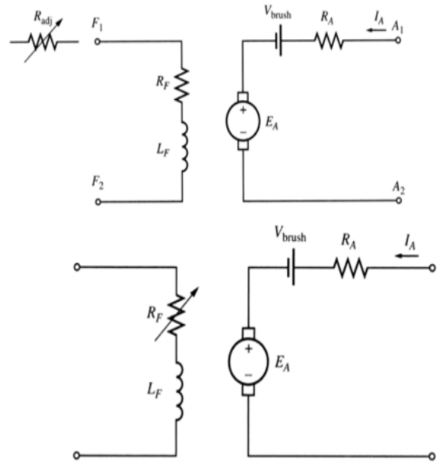
4- The series DC motor.

5- The compounded dc motor

The Equivalent Circuit of a DC Motor

The armature circuit (the entire rotor structure) is represented by an ideal voltage source E_A and a resistor R_A. A battery V_{brush} in the opposite to a current flow in the machine direction indicates brush voltage drop.

The field coils producing the magnetic flux are represented by inductor L_F and resistor R_F. The resistor R_{adj} represents an external variable resistor (sometimes lumped together with the field coil resistance) **used to control the amount of current in the field circuit**.



The internal generated voltage in the machine is

$$E_A = K\phi\omega$$

The induced torque developed by the machine is

$$\tau_{ind} = K\phi I_A$$

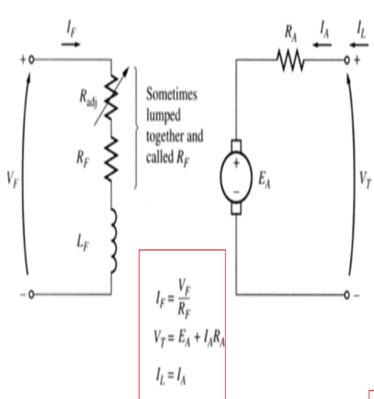
Here ***K*** is the constant **depending on the design of a particular DC machine** (*number and commutation of rotor coils*, etc.) and ϕ is the total flux inside the machine.

Note that for a single rotating loop $K = \pi/2$.

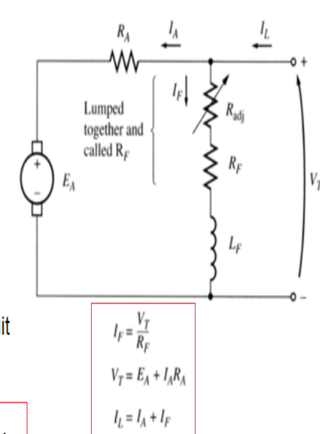
Separately excited and Shunt DC motors

- **Separately DC motor**: is a motor whose filed circuit is supplied from a separate constant voltage power supply.
- **Shunt DC motor**: is a motor whose filed circuit gets its power directly across the armature terminals of motor.
- When the supply voltage to a motor is assumed constant, there is no practical difference in behviour between these two motors.

Separately excited DC motor:
a field circuit is supplied from a separate constant voltage power source.



Shunt DC motor:
a field circuit gets its power from the armature terminals of the motor.



For the armature circuit of these motors:

$$V_T = E_A + I_A R_A$$

The terminal characteristic of *a Shunt motor*:

A **terminal characteristic** of a machine is a plot of the machine's output quantities vs. each other.

For a motor, the output quantities are shaft torque and speed. Therefore, the **terminal characteristic of a motor** is its **output torque vs. speed**.

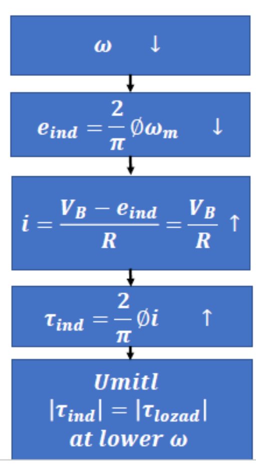
If the load on the shaft increases, the load torque τ_{load} will exceed the induced torque τ_{ind}, and the motor will **slow down**. Slowing down the motor will **decrease its internal generated voltage** (since $E_A = K\phi\omega$), so the **armature current increases** ($I_A = (V_T - E_A)/R_A$).
As the **armature current increases**, the **induced torque in the motor increases** (since $\tau_{ind} = K\phi I_A$), and the induced torque will equal the load torque at a lower speed ω.

This is linked to Ch. simple DC machine and also to Ch 8 (Section 1).

$$\omega = \frac{V_T}{K\emptyset} - \frac{R_A}{(K\emptyset)^2}\tau_{ind}$$

Recall From Ch. 8 :

If a load torque of **10 Nm** is applied to the shaft of the machine, it will begin to slow down.

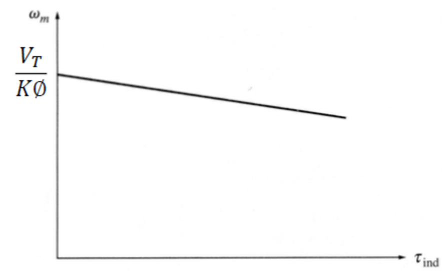


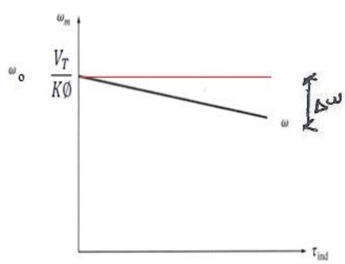
- The terminal characteristic of a Shunt motor

This equation is just a straight line with a negative slope.

$$\omega = \frac{V_T}{K\emptyset} - \frac{R_A}{(K\emptyset)^2}\tau_{ind}$$

The resulting torque –speed characteristic of a shunt DC motor.





$\omega = \omega_o - \Delta\omega$

ω: speed at load; ω_o: no load speed; $\Delta\omega$: drop speed

$$\omega = \frac{V_T}{K\phi} - \frac{T_{ind} R_A}{(K\phi)^2}$$

$\Rightarrow$ $T_{ind} = K\phi I_A \Rightarrow I_A = \frac{T_{ind}}{K\phi}$

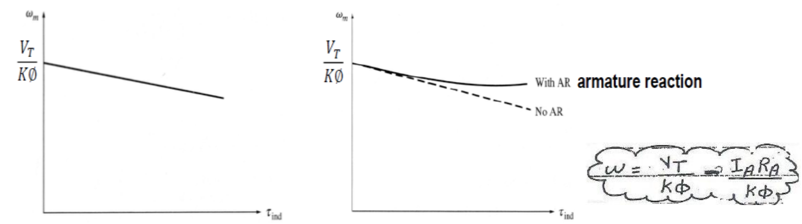
$$\omega = \frac{V_T}{K\phi} - \frac{I_A R_A}{K\phi}$$

In order for motor speed to vary **linearly** with the torque, the other terms in the equation must be constant , otherwise the shape of the curve will be effected. So, the curve can be effected by

1- terminal voltage 2- Armature reaction

- The terminal characteristic of a Shunt motor

Assuming that the terminal voltage and other terms are constant, the motor's speed vary linearly with torque.



However, if a motor has an **armature reaction**, flux-weakening reduces the flux when torque increases. Therefore, the **motor's speed will increase**.

If a shunt DC motor or separately excited has compensating winding so that its flux is constant regardless of load, so there will be no flux-weakening problem in the machine .

A compensation **winding** in a **DC** shunt **motor** is a **winding** in the field pole face plate that carries armature current to reduce stator field distortion. Its purpose is to reduce brush arcing and erosion in **DC motors** that are operated with weak fields, variable heavy loads.

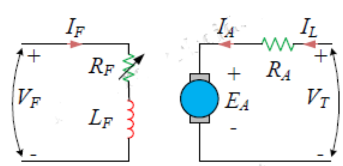
1- Separately Excited DC Motor

$I_A = I_L$

$V_T = E_A + I_A R_A$

$V_F = I_F R_F$

$E_A = k\phi\omega_m$



Where
I_A: is the armature current
I_L: is the load current
E_A: is the internal generated voltage
V_T: is the terminal voltage
I_F: is the field current
V_F: is the field voltage
R_A: is the armature winding resistance
R_F: is the field winding resistance
ϕ: is the flux
ω_m: is the rotor angular speed

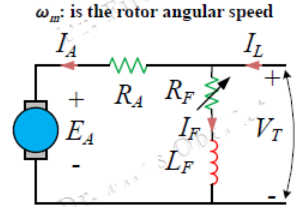
2- Shunt DC Motor

$I_L = I_A + I_f$

$V_T = E_A + I_A R_A$

$V_F = I_F R_F$

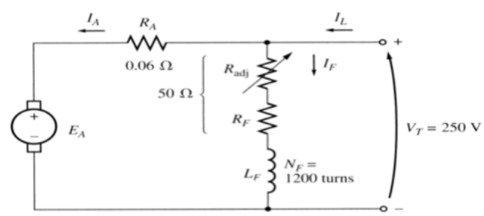
$E_A = k\phi\omega_m$



Shunt motor: terminal characteristic – Example

Example : A 50 hp = (50 x 745.699872 W), 250 V, 1200 rpm DC shunt motor **with** compensating windings has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06 Ω. Its field circuit has a total resistance $R_{adj} + R_F$ of 50 Ω, which produces a no-load speed of 1200 rpm. The shunt field winding has 1200 turns per pole.

a) Find the motor speed when its input current is 100 A.
b) Find the motor speed when its input current is 200 A.
c) Find the motor speed when its input current is 300 A.
d) Plot the motor torque-speed characteristic.



The internal generated voltage of a DC machine (with its speed expressed in rpm):

$$E_A = K\emptyset\omega$$

Since the field current is constant (both field resistance and V_T are constant) and since there are no armature reaction (due to compensating windings), we conclude that the **flux in the motor is constant**. The speed and the internal generated voltages at different loads are related as

$$\frac{E_{A2}}{E_{A1}} = \frac{K\emptyset\omega_2}{K\emptyset\omega_1} = \frac{n_2}{n_1}$$

Therefore:

$$n_2 = \frac{E_{A2}}{E_{A1}} n_1$$

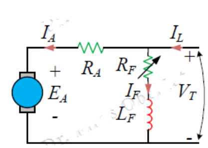
At no load, the armature current is zero and therefore $E_{A1} = V_T = 250$ V.

$I_L = I_A + I_f$

$V_T = E_A + I_A R_A$

$V_F = I_F R_F$

$E_A = k\phi\omega_m$

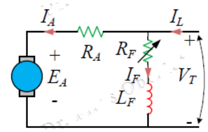


$I_L = I_A + I_f$

$V_T = E_A + I_A R_A$

$V_F = I_F R_F$

$E_A = k\phi\omega_m$



a) Since the input current is 100 A, the armature current is

$$I_A = I_L - I_F = I_L - \frac{V_T}{R_F} = 100 - \frac{250}{50} = 95A$$

Therefore: $$E_A = V_T - I_A R_A = 250 - (95)(0.06) = 244.3V$$

and the resulting motor speed is:

$$n_2 = \frac{E_{A2}}{E_{A1}} n_1 = \frac{244.3}{250} x1200 = 1173rpm$$

b) Similar computations for the input current of 200 A lead to n_2 = 1144 rpm.

c) Similar computations for the input current of 300 A lead to n_2 = 1115 rpm.

d) To plot the output characteristic of the motor, we need to find the torque corresponding to each speed. At no load, the torque is zero.

Since the induced torque at any load is related to the power converted in a DC motor:

$$P_{conv} = E_A I_A = \tau_{ind}\omega$$

The induced torque is:

$$\tau_{ind} = \frac{E_A I_A}{\omega}$$

For the input current of 100 A:

$$\tau_{ind} = \frac{244.3\ x\ 95}{2\pi\ x\ 1173/60} = 190\ N.m$$

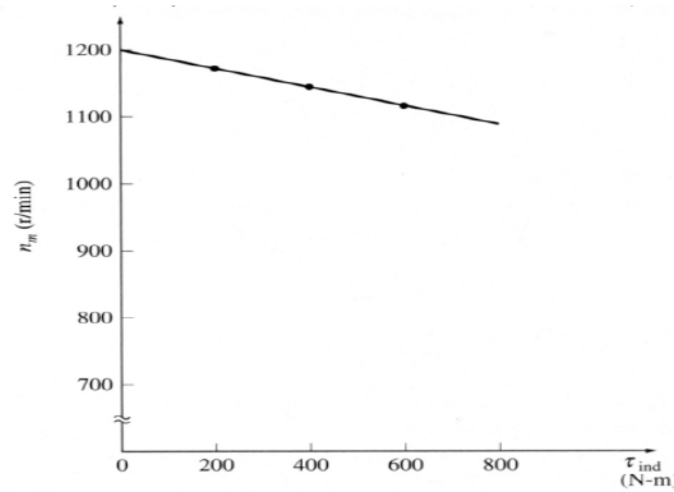
For the input current of 200 A:

$$\tau_{ind} = \frac{238.3\ x\ 195}{2\pi\ x\ 1144/60} = 388\ N.m$$

For the input current of 300 A:

$$\tau_{ind} = \frac{232.3\ x\ 295}{2\pi\ x\ 1115/60} = 587\ N.m$$

The torque-speed characteristic of the motor is:



Nonlinear Analysis of a Shunt motor:

The **flux** ϕ and, therefore the **internal generated voltage** E_A of a DC machine are nonlinear functions of its mmf and must be determined based on the magnetization curve.

Two main contributors to the **mmf** are its **field current** and the **armature reaction** (if present).

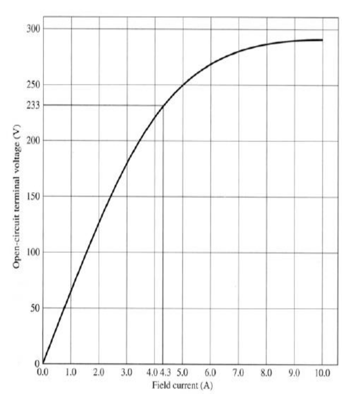
Since the magnetization curve is a plot of the **generated voltage vs. field current**, the effect of changing the field current can be determined directly from the magnetization curve.

If a machine has armature reaction, its flux will reduce with increase in load. The total mmf in this case will be

$$F_{net} = N_F I_F - F_{AR}$$

It is customary to define an **equivalent field current** that would produce the same output voltage as the net (total) mmf in the machine:

$$I_F^* = I_F - \frac{F_{AR}}{N_F}$$



Conducting a nonlinear analysis to determine the internal generated voltage in a DC motor, we may need to account for the fact that a motor can be running at a speed other than the rated one.

The voltage induced in a DC machine is

$$E_A = K\emptyset\omega$$

For a given effective field current, the flux in the machine is constant and the internal generated voltage is directly proportional to speed:

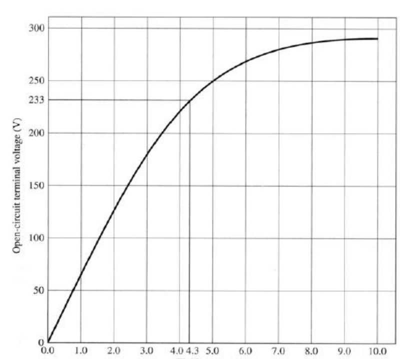
$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

Where **E_{A0} and n_0 represent the reference (rated) values** of voltage and speed, respectively. Therefore, if the reference conditions are known from the magnetization curve and the **actual E_A is computed**, the actual speed can be determined.

Example

Example: **A 50 hp, 250 V, 1200** rpm DC shunt motor **without** compensating windings has an armature resistance (including the brushes and interpoles) of **0.06 Ω**. Its field circuit has a total resistance **R_{adj} + R_F of 50 Ω**, which produces a no-load speed of **1200 rpm**. The shunt field winding has **1200 turns per pole**. The armature reaction produces a demagnetizing **mmf of 840 A-turns at a load current of 200A**. The magnetization curve is shown.

a) Find the motor speed when its input current is 200 A.
b) How does the motor speed compare to the speed of the motor from previous Example (same motor but **with** compensating windings) with an input current of 200 A?
c) Plot the motor torque-speed characteristic.



a) Since the input current is 200 A, the armature current is

$$I_A = I_L - I_F = I_L - \frac{V_T}{R_F} = 200 - \frac{250}{50} = 195A$$

Therefore:

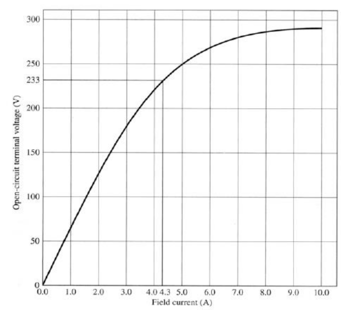
$$E_A = V_T - I_A R_A = 250 - (195)(0.06) = 238.3V$$

At the given current, the demagnetizing mmf due to armature reaction is 840 A-turns, so the effective shunt field current of the motor is

$$I_F^* = I_F - \frac{F_{AR}}{N_F} = 5 - \frac{840}{1200} = 4.3A$$

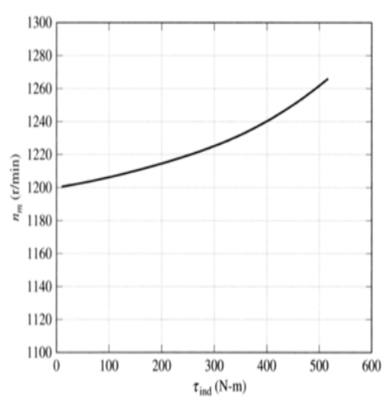
From the magnetization curve, this effective field current will produce an internal voltage of E_{A0} = 238.3 V at a speed of 1200 rpm. For the actual voltage, the speed is

$$n = \frac{E_A}{E_{A0}} n_0 = \frac{238.3}{233} x1200 = 1227rpm$$



b) A speed of a motor with compensating windings was **1144 rpm** when the input current was **200 A**. We notice that the speed of the motor with armature reactance is higher than the speed of the motor without armature reactance. This increase is due to the flux weakening.

c) Assuming that the mmf due to the armature reaction varies linearly with the increase in current, and repeating the same computations for many different load currents, the motor's torque-speed characteristic can be plotted.



Speed Control of Shunt DC motor:

➢How can the speed of a shunt DC motor be controlled?

There are two common methods to control the speed of a shunt DC motor:

1. Adjusting the field resistance *RF* (and thus the field flux)
2. Adjusting the terminal voltage applied to the armature

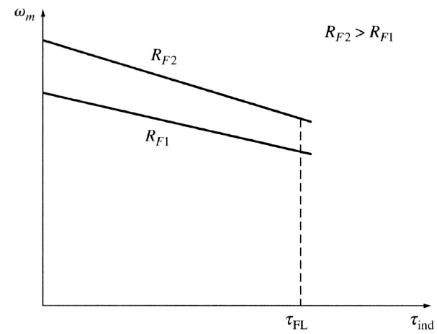
1.Adjusting the field resistance

- Firstly, to understand what happens when the filed resistance of a DC motor is changed, assume that filed resistor increases and observe the response.

1) Increasing field resistance R_F decreases the field current ($I_F = V_T/R_F$);
2) Decreasing field current I_F decreases the flux ϕ;
3) Decreasing flux decreases the internal generated voltage ($E_A = K\phi\omega$);
4) Decreasing E_A increases the armature current ($I_A = (V_T - E_A)/R_A$);
5) Changes in armature current dominate over changes in flux; therefore, increasing I_A increases the induced torque ($\tau_{ind} = K\phi I_A$);
6) Increased induced torque is now larger than the load torque τ_{load} and, therefore, the speed ω increases;
7) Increasing speed increases the internal generated voltage E_A;
8) Increasing E_A decreases the armature current I_A…
9) Decreasing I_A decreases the induced torque until $\tau_{ind} = \tau_{load}$ at a higher speed ω.

The effect of increasing the field resistance within a normal load range: from no load to full load.

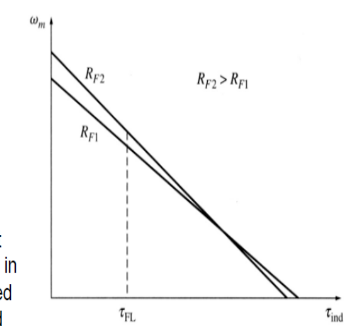
Increase in the field resistance increases the motor speed. Observe also that the slope of the speed-torque curve becomes steeper when field resistance increases.



The effect of increasing the field resistance with over an entire load range: from no-load to stall.

At very slow speeds (overloaded motor), an increase in the field resistance decreases the speed. In this region, the increase in armature current is no longer large enough to compensate for the decrease in flux.

Some small DC motors used in control circuits may operate at speeds close to stall conditions. For such motors, an increase in field resistance may have no effect (or opposite to the expected effect) on the motor speed. The result of speed control by field resistance is not predictable and, thus, this type of control is **not very common.**

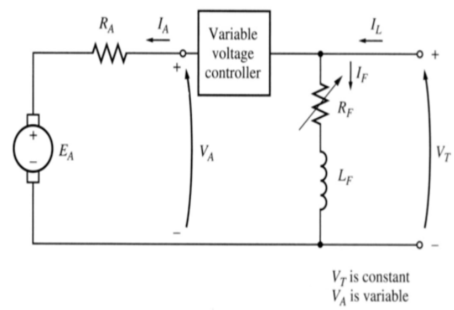


Note: as the flux in the machine decreases, the no load speed of the motor increases, while the slope of the torque-speed curve becomes steeper. Naturally, decreasing RF would reverse the whole process and the speed of the motor would drop.

2. Changing the armature voltage

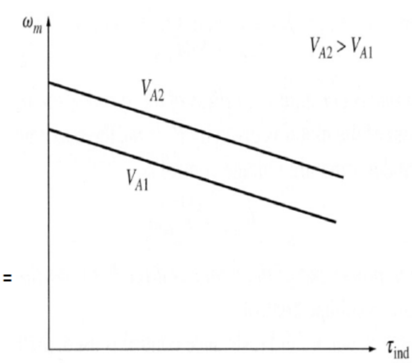
2. Changing the armature voltage

This method implies changing the voltage applied to the armature of the motor without changing the voltage applied to its field. Therefore, the motor must be separately excited to use armature voltage control.



Armature voltage speed control

1) Increasing the armature voltage V_A increases the armature current ($I_A = (V_A - E_A)/R_A$);
2) Increasing armature current I_A increases the induced torque τ_{ind} ($\tau_{ind} = K\phi I_A$);
3) Increased induced torque τ_{ind} is now larger than the load torque τ_{load} and, therefore, the speed ω;
4) Increasing speed increases the internal generated voltage ($E_A = K\phi\omega$);
5) Increasing E_A decreases the armature current I_A…
6) Decreasing I_A decreases the induced torque until $\tau_{ind} = \tau_{load}$ at a higher speed ω.



Increasing the armature voltage of a separately excited DC motor does not change the slope of its torque-speed characteristic.

Note: the no load speed of the motor is shifted by method of speed control but the slope of the curve remains constant.

Speed control of separately excited and shunt DC motors

Adjusting the field resistance R_F

- Increasing R_F causes $I_F = V_T/R_F$ to decrease.
- Decreasing I_F decreases ϕ.
- Decreasing ϕ, lowers E_A=k $\phi\omega_m$.
- Decreasing E_A increases $I_A = (V_T - E_A)/R_A$.
- Increasing I_A increases T_{ind}=$k\phi I_A$, with the change in I_A dominant over the change in flux.
- Increasing T_{ind} makes T_{ind}>T_{load} and the speed ω_m increases.
- Increasing ω_m increases E_A=$k\phi\omega_m$.
- Increasing E_A decreases I_A.
- Decreasing I_A decreases T_{ind} until T_{ind}=T_{load} at a higher speed ω_m.

Adjusting the terminal voltage applied to the armature

- An increase in V_T increases $I_A = (V_T - E_A)/R_A$.
- Increasing I_A increases T_{ind}=$k\phi I_A$
- Increasing T_{ind} makes T_{ind}>T_{load} and the speed ω_m increases.
- Increasing ω_m increases E_A=$k\phi\omega_m$.
- Increasing E_A decreases $I_A = (V_T - E_A)/R_A$.
- Decreasing I_A decreases T_{ind} until T_{ind}=T_{load} at a higher speed ω_m

Inserting a resistor in series with the armature circuit

- resistor is inserted in series with the armature circuit, the effect is to drastically increase the slope of the motor's torque- speed characteristic, making it operate more slowly if loaded.
- The insertion of a resistor is a very wasteful method of speed control, since the losses in the inserted resistor are very large. For this reason, it is rarely used. It will be found only in applications in which the motor spends almost all its time operating at full speed or in applications too inexpensive to justify a better form of speed control.

For the **armature voltage control**, the flux in the motor is constant. Therefore, the maximum torque in the motor will be constant too regardless the motor speed:

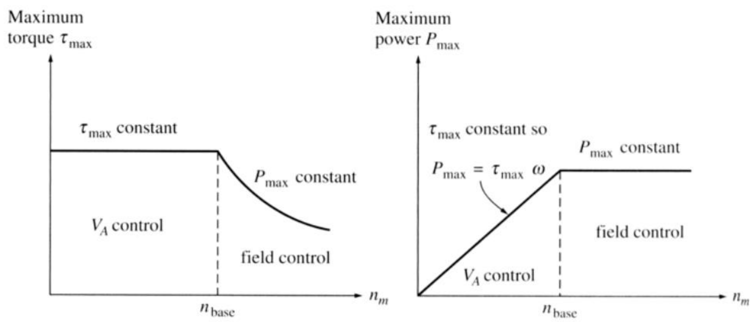
$$\tau_{max} = K\emptyset I_{A,max}$$

Since the maximum power of the motor is

$$P_{max} = \tau_{max}\omega$$

The maximum power out of the motor is directly proportional to its speed.

For the **field resistance control**, the maximum power out of a DC motor is constant, while the maximum torque is reciprocal to the motor speed.



Torque and power limits as functions of motor speed for a shunt (or separately excited) DC motor.

Note: the maximum power out of a DC motor under filed current control is constant , while the maximum torque varies a the reciprocal of motor speed.

If a motor is operated at its rated terminal voltage, power, and field current, it will be running at the rated speed also called a **base speed**.

Field resistance control can be used for speeds above the base speed but not below it. Trying to achieve speeds slower than the base speed by the field circuit control, requires large field currents that may damage the field winding.

Since the armature voltage is limited to its rated value, **no speeds exceeding the base speed can be achieved safely while using the armature voltage control.**

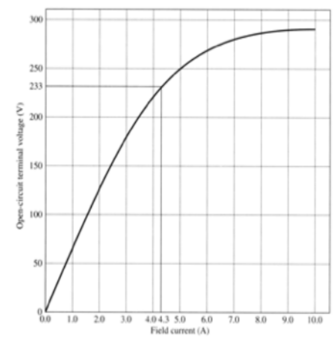
Therefore, armature voltage control can be used to achieve speeds below the base speed, while the field resistance control can be used to achieve speeds above the base speed.

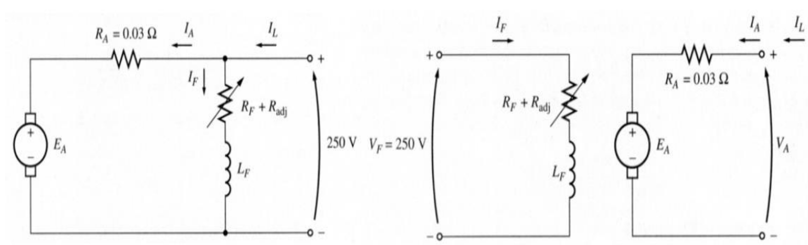
Shunt and separately excited DC motors have excellent speed control characteristic.

Example

Example: A **100 hp**, **250 V**, **1200 rpm** DC shunt motor with an armature resistance of **0.03 Ω** and a field resistance of **41.67 Ω**. The motor has compensating windings, so armature reactance can be ignored. Mechanical and core losses may be ignored also. The motor is driving a load with a line current of **126 A** and an initial speed of **1103 rpm**. Assuming that the armature current is constant and the magnetization curve is

a) What is the motor speed if the field resistance is increased to 50 Ω?
b) Calculate the motor speed as a function of the field resistance, assuming a constant-current load.
c) Assuming that the motor is connected as a separately excited and is initially running with V_A = 250 V, I_A = 120 A and at n = 1103 rpm while supplying a constant-torque load, estimate the motor speed if V_A is reduced to 200 V.





shunt separately-excited

For the given initial line current of 126 A, the initial armature current will be

$$I_{A1} = I_{L1} - I_{f1} = 126 - \frac{250}{41.67} = 120A$$

Therefore, the initial generated voltage for the shunt motor will be

$$E_{A1} = V_T - I_{A1}R_A = 250 - 120 \cdot 0.03 = 246.4A$$

After the field resistance is increased to 50 Ω, the new field current will be

$$I_{F1} = \frac{250}{50} = 5A$$

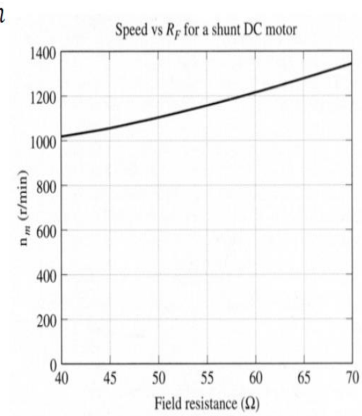
The ratio of the two internal generated voltages is

$$\frac{E_{A2}}{E_{A1}} = \frac{K\emptyset_2\omega_2}{K\emptyset_1\omega_1} = \frac{\emptyset_2 n_2}{\emptyset_1 n_1}$$

The values of E_A on the magnetization curve are directly proportional to the flux. Therefore, the ratio of internal generated voltages equals to the ratio of the fluxes within the machine. From the magnetization curve, at I_F = 5A, E_{A1} = 250V, and at I_F = 6A, E_{A1} = 268V. Thus:

$$n_2 = \frac{\emptyset_1 n_1}{\emptyset_2} = \frac{E_{A2} n_1}{E_{A1}} = \frac{268}{250} 1103 = 1187\ rpm$$

b) A speed vs. R_F characteristic is shown below:



c) For a separately excited motor, the initial generated voltage is

$$E_{A1} = V_{T1} - I_{A1}R_A$$

$$\frac{E_{A2}}{E_{A1}} = \frac{K\emptyset_2\omega_2}{K\emptyset_1\omega_1} = \frac{\emptyset_2 n_2}{\emptyset_1 n_1}$$

and since the flux ϕ is constant $$n_2 = \frac{E_{A2}n_1}{E_{A1}}$$

Since both the torque and the flux are constants, the armature current I_A is also constant. Then

$$n_2 = \frac{E_{A2}n_1}{E_{A1}} = \frac{V_{T2} - I_{A2}R_A}{V_{T1} - I_{A1}R_A}n_1 = \frac{200 - 120 \cdot 0.03}{250 - 120 \cdot 0.03}1103 = 879rpm$$

DC Motors and Generators
Ch. 9 -4th edition
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Dr. Feras Alasali

Speed Control of Shunt DC motor:

- How can the speed of a shunt DC motor be controlled?

There are two common methods to control the speed of a shunt DC motor:

1. Adjusting the field resistance *RF* (and thus the field flux)
2. Adjusting the terminal voltage applied to the armature

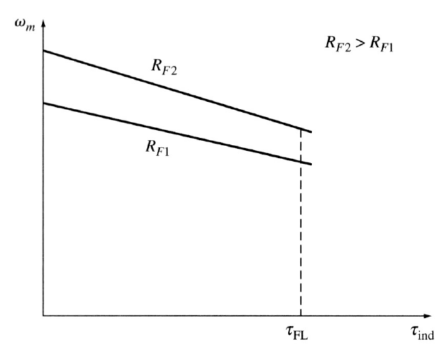
1.Adjusting the field resistance

- Firstly, to understand what happens when the filed resistance of a DC motor is changed, assume that filed resistor increases and observe the response.

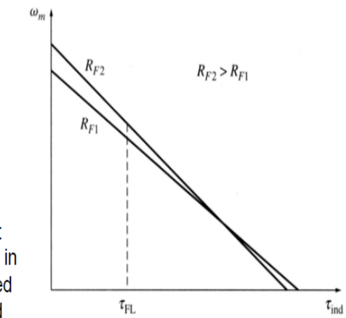
1) Increasing field resistance R_F decreases the field current ($I_F = V_T/R_F$);
2) Decreasing field current I_F decreases the flux ϕ;
3) Decreasing flux decreases the internal generated voltage ($E_A = K\phi\omega$);
4) Decreasing E_A increases the armature current ($I_A = (V_T - E_A)/R_A$);
5) Changes in armature current dominate over changes in flux; therefore, increasing I_A increases the induced torque ($\tau_{ind} = K\phi I_A$);
6) Increased induced torque is now larger than the load torque τ_{load} and, therefore, the speed ω increases;
7) Increasing speed increases the internal generated voltage E_A;
8) Increasing E_A decreases the armature current I_A…
9) Decreasing I_A decreases the induced torque until $\tau_{ind} = \tau_{load}$ at a higher speed ω.

The effect of increasing the field resistance within a normal load range: from no load to full load.

Increase in the field resistance increases the motor speed. Observe also that the slope of the speed-torque curve becomes steeper when field resistance increases.



The effect of increasing the field resistance with over an entire load range: from no-load to stall.



At very slow speeds (overloaded motor), an increase in the field resistance decreases the speed. In this region, the increase in armature current is no longer large enough to compensate for the decrease in flux.

Some small DC motors used in control circuits may operate at speeds close to stall conditions. For such motors, an increase in field resistance may have no effect (or opposite to the expected effect) on the motor speed. The result of speed control by field resistance is not predictable and, thus, this type of control is **not very common.**

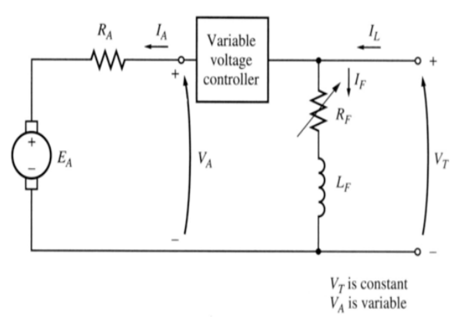
Note: as the flux in the machine decreases, the no load speed of the motor increases, while the slope of the torque-speed curve becomes steeper. Naturally, decreasing RF would reverse the whole process and the speed of the motor would drop.

2. Changing the armature voltage

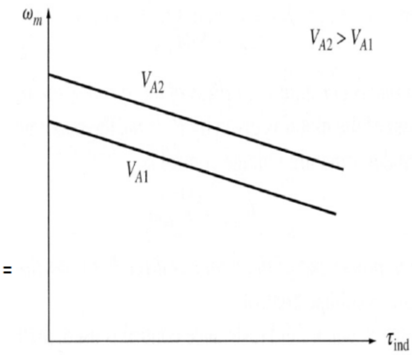
2. Changing the armature voltage

This method implies changing the voltage applied to the armature of the motor without changing the voltage applied to its field. Therefore, the motor must be separately excited to use armature voltage control.

Armature voltage speed control



1) Increasing the armature voltage V_A increases the armature current ($I_A = (V_A - E_A)/R_A$);
2) Increasing armature current I_A increases the induced torque τ_{ind} ($\tau_{ind} = K\phi I_A$);
3) Increased induced torque τ_{ind} is now larger than the load torque τ_{load} and, therefore, the speed ω;
4) Increasing speed increases the internal generated voltage ($E_A = K\phi\omega$);
5) Increasing E_A decreases the armature current I_A…
6) Decreasing I_A decreases the induced torque until $\tau_{ind} = \tau_{load}$ at a higher speed ω.



Increasing the armature voltage of a separately excited DC motor does not change the slope of its torque-speed characteristic.

Note: the no load speed of the motor is shifted by method of speed control but the slope of the curve remains constant.

Speed control of separately excited and shunt DC motors

Adjusting the field resistance R_F

- Increasing R_F causes $I_F = V_T/R_F$ to decrease.
- Decreasing I_F decreases ϕ.
- Decreasing ϕ, lowers $E_A = k\phi\omega_m$.
- Decreasing E_A increases $I_A = (V_T - E_A)/R_A$.
- Increasing I_A increases $T_{ind} = k\phi I_A$, with the change in I_A dominant over the change in flux.
- Increasing T_{ind} makes $T_{ind} > T_{load}$ and the speed ω_m increases.
- Increasing ω_m increases $E_A = k\phi\omega_m$.
- Increasing E_A decreases I_A.
- Decreasing I_A decreases T_{ind} until $T_{ind} = T_{load}$ at a higher speed ω_m.

Adjusting the terminal voltage applied to the armature

- An increase in V_T increases $I_A = (V_T - E_A)/R_A$.
- Increasing I_A increases $T_{ind} = k\phi I_A$
- Increasing T_{ind} makes $T_{ind} > T_{load}$ and the speed ω_m increases.
- Increasing ω_m increases $E_A = k\phi\omega_m$.
- Increasing E_A decreases $I_A = (V_T - E_A)/R_A$.
- Decreasing I_A decreases T_{ind} until $T_{ind} = T_{load}$ at a higher speed ω_m

Inserting a resistor in series with the armature circuit

- resistor is inserted in series with the armature circuit, the effect is to drastically increase the slope of the motor's torque- speed characteristic, making it operate more slowly if loaded.
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Since the maximum power of the motor is

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The maximum power out of the motor is directly proportional to its speed.

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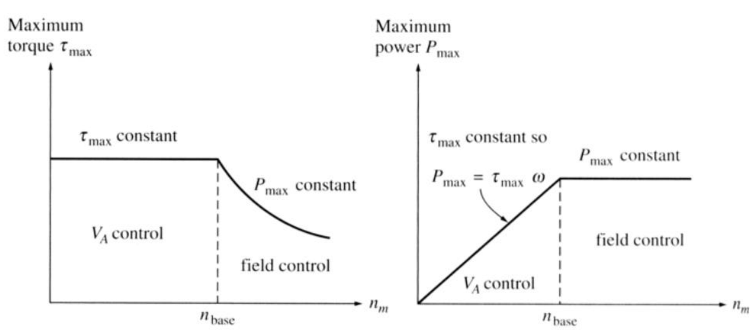

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Shunt and separately excited DC motors have excellent speed control characteristic.

Example

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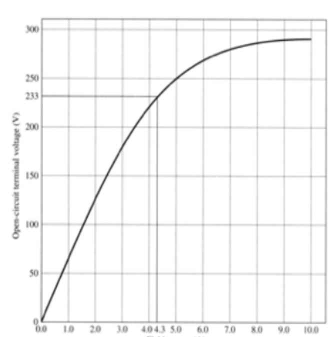
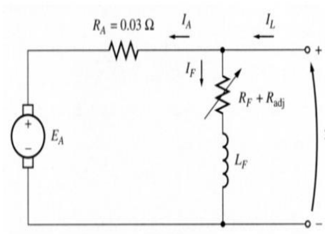
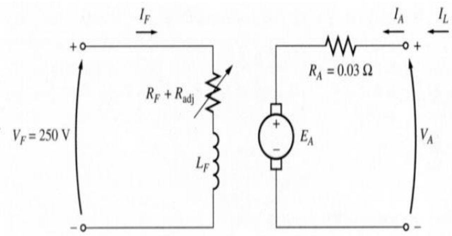




shunt



separately-excited

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Therefore, the initial generated voltage for the shunt motor will be

$$E_{A1} = V_T - I_{A1}R_A = 250 - 120 \cdot 0.03 = 246.4A$$

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$$I_{F1} = \frac{250}{50} = 5A$$

The ratio of the two internal generated voltages is

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The values of E_A on the magnetization curve are directly proportional to the flux. Therefore, the ratio of internal generated voltages equals to the ratio of the fluxes within the machine. From the magnetization curve, at I_F = 5A, E_{A1} = 250V, and at I_F = 6A, E_{A1} = 268V. Thus:

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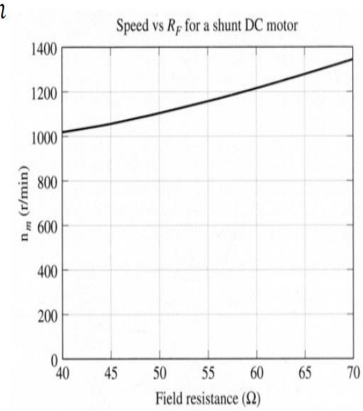


c) For a separately excited motor, the initial generated voltage is

$$E_{A1} = V_{T1} - I_{A1} R_A$$

$$\frac{E_{A2}}{E_{A1}} = \frac{K\emptyset_2 \omega_2}{K\emptyset_1 \omega_1} = \frac{\emptyset_2 n_2}{\emptyset_1 n_1}$$

and since the flux ϕ is constant

$$n_2 = \frac{E_{A2} n_1}{E_{A1}}$$

Since both the torque and the flux are constants, the armature current I_A is also constant. Then

$$n_2 = \frac{E_{A2} n_1}{E_{A1}} = \frac{V_{T2} - I_{A2} R_A}{V_{T1} - I_{A1} R_A} n_1 = \frac{200 - 120 \cdot 0.03}{250 - 120 \cdot 0.03} 1103 = 879 rpm$$

DC Motors and Generators Ch. 9

Dr. Feras Alasali

Types of DC Motors

- DC motors are driven from DC power supply, there are five major types of DC motors in general use:

1- The separately excited dc motor.

2- The shunt dc motor.

3- The permanent –magnet DC motor

4- The series DC motor.

5- The compounded dc motor

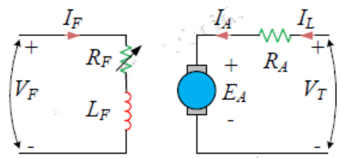
1- Separately Excited DC Motor

$$I_A = I_L$$

$$V_T = E_A + I_A R_A$$

$$V_F = I_F R_F$$

$$E_A = k\phi\omega_m$$



Where
I_A: is the armature current
I_L: is the load current
E_A: is the internal generated voltage
V_T: is the terminal voltage
I_F: is the field current
V_F: is the field voltage
R_A: is the armature winding resistance
R_F: is the field winding resistance
ϕ: is the flux
ω_m: is the rotor angular speed

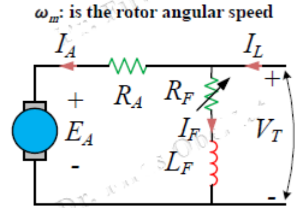
2- Shunt DC Motor

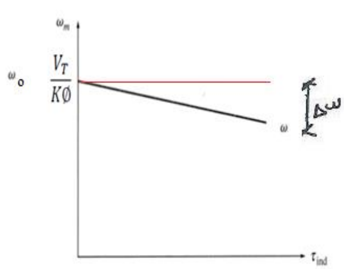
$$I_L = I_A + I_f$$

$$V_T = E_A + I_A R_A$$

$$V_F = I_F R_F$$

$$E_A = k\phi\omega_m$$





$$\omega = \omega_o - \Delta\omega$$

ω: speed at load; ω_o: no load speed; $\Delta\omega$: drop speed

$$\omega = \frac{V_T}{k\phi} - \frac{T_{ind} R_A}{(k\phi)^2}$$

$$T_{ind} = k\phi I_A \Rightarrow I_A = \frac{T_{ind}}{k\phi}$$

$$\omega = \frac{V_T}{k\phi} - \frac{I_A R_A}{k\phi}$$

Speed control of separately excited and shunt DC motors

Adjusting the field resistance R_F

- Increasing R_F causes $I_F = V_T/R_F$ to decrease.
- Decreasing I_F decreases ϕ.
- Decreasing ϕ, lowers $E_A = k\phi\omega_m$.
- Decreasing E_A increases $I_A = (V_T - E_A)/R_A$.
- Increasing I_A increases $T_{ind} = k\phi I_A$, with the change in I_A dominant over the change in flux.
- Increasing T_{ind} makes $T_{ind} > T_{load}$ and the speed ω_m increases.
- Increasing ω_m increases $E_A = k\phi\omega_m$.
- Increasing E_A decreases I_A.
- Decreasing I_A decreases T_{ind} until $T_{ind} = T_{load}$ at a higher speed ω_m.

Adjusting the terminal voltage applied to the armature

- An increase in V_T increases $I_A = (V_T - E_A)/R_A$.
- Increasing I_A increases $T_{ind} = k\phi I_A$
- Increasing T_{ind} makes $T_{ind} > T_{load}$ and the speed ω_m increases.
- Increasing ω_m increases $E_A = k\phi\omega_m$.
- Increasing E_A decreases $I_A = (V_T - E_A)/R_A$.
- Decreasing I_A decreases T_{ind} until $T_{ind} = T_{load}$ at a higher speed ω_m

Inserting a resistor in series with the armature circuit

- resistor is inserted in series with the armature circuit, the effect is to drastically increase the slope of the motor's torque- speed characteristic, making it operate more slowly if loaded.
- The insertion of a resistor is a very wasteful method of speed control, since the losses in the inserted resistor are very large. For this reason, it is rarely used. It will be found only in applications in which the motor spends almost all its time operating at full speed or in applications too inexpensive to justify a better form of speed control.

8.5- The Permanent Magnet DC Motor

A permanent-magnet DC (PMDC) *motor* is a DC motor whose poles are made of permanent magnets. Permanent-magnet dc motors offer a number of benefits compared with shunt dc motors in some applications. Since these motors do not require an external field circuit, they do not have the field circuit copper losses associated with shunt dc motors.

Because no field windings are required, they can be smaller than corresponding shunt DC motors. they are especially common in smaller fractional- and sub fractional-horsepower sizes.

PMDC motors are generally less expensive, smaller in size, simpler, and higher efficiency than corresponding DC motors with separate electromagnetic fields. This makes them a good choice in many DC motor applications. The armatures of PMDC motors are essentially identical to the armatures of motors with separate field circuits, so their costs are similar too. However, the elimination of separate electromagnets on the stator reduces the size of the stator, the cost of the stator, and the losses in the field circuits.

PMDC motors also have disadvantages. Permanent magnets cannot produce as high a flux density as an externally supplied shunt field, so a PMDC motor will have a lower induced torque T_{ind} per ampere of armature current I_A than a shunt motor of the same size and construction. In addition, PMDC motors run the risk of demagnetization.

A permanent-magnet DC motor is basically the same machine as a shunt dc motor, except that *the flux of a PMDC motor is fixed.* Therefore, it is not possible to control the speed of a PMDC motor by varying the field current or flux. The only methods of speed control available for a PMDC motor are armature voltage control and armature resistance control.

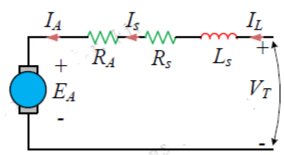
The techniques to analyze a PMDC motor are basically the same as the techniques to analyze a shunt dc motor with the field current held constant.

8.6 The series DC motors

- A series Dc motor is a dc motor whose field windings consists of a relatively few turns connected in series with armature circuit.

this motor has field coil connected in series to the armature winding. For this reason relatively higher current flows through the field coils, and its designed accordingly as mentioned below.

1. The field coils of **DC series motor** are wound with relatively fewer turns as the current through the field is its armature current and hence for required mmf less numbers of turns are required.
2. The wire is heavier, as the diameter is considerable increased to provide minimum electrical resistance to the flow of full armature current.
3. In spite of the above mentioned differences, about having fewer coil turns the running of this DC motor remains unaffected, as the current through the field is reasonably high to produce a field strong enough for generating the required amount of torque.



Since the entire supply current flows through both the armature and field conductor.

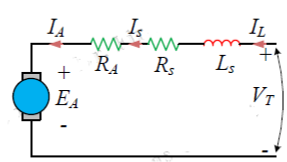
$$Therefore,\ I_{total} = I_{s} = I_a$$

Where, I_s is the series current in the field coil and I_a is the armature current.

$$I_A = I_S = I_L$$

$$V_T = E_A + I_A(R_A + R_S)$$

$$E_A = k\phi\omega_m$$



$$\tau_{ind} = K\emptyset I_A$$

This is the basic voltage equation of a **series wound DC motor**.

Another interesting fact about the **DC series motor** worth noting is that, the field flux like in the case of any other DC motor is proportional to field current.

$$I_s \propto \phi \quad \text{But since here} \quad I_s = I_a = I_{total}$$

$$\phi \propto I_s \propto I_a$$

i.e. the field flux is proportional to the entire armature current or the total supply current. And for this reason, the flux produced in this motor is strong enough to produce sufficient torque, even with the bare minimum number of turns it has in the field coil.

Therefore, the flux in the machine can be given by

$$\phi = c\, I_a$$

Where c is constant of propotionality. The induced torque in machine is thus given by

$$\tau_{ind} = K\emptyset I_A = K\,c\,I_A^2$$

- In other words, the torque in this motor is proportional to the square of armature current.

$$\tau_{ind} = K\,c\,I_A^2$$

- As result , it is easy to see that the series motor gives more torque per ampere than any other dc motor.

- Therefore, is used in application requiring very high torques such as starter motors in cars and elevator.

The terminal characteristic if a Series DC motor Speed and Torque of Series DC Motor

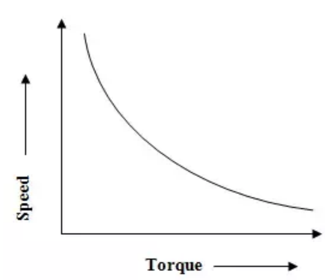
1- A series wound motors has relationship existing between the field current and the amount of torque produced. As in this case relatively higher current flows through the heavy series field winding with thicker diameter.

2- The electromagnetic torque produced here is much higher than normal. This high electromagnetic torque produces motor speed, strong enough to lift heavy load overcoming its initial inertial of rest.

3- Series motors are generally operated for a very small duration, about only a few seconds, just for the purpose of starting. Because if its run for too long, the high series current might burn out the series field coils thus leaving the motor useless.

The resulting torque- speed relationship is

$$\omega_m = \frac{V_T}{\sqrt{K_c}} \frac{1}{\sqrt{\tau_{ind}}} - \frac{R_A + R_c}{K_c}$$



Speed control of series DC motor

Speed control of series DC motors

1) Change the terminal voltage of the motor.
2) Insertion of a series resistor into the motor circuit, but this technique is very wasteful of power and is used only for intermittent periods during the start-up of some motors.

Example

A 250 V series dc motor with compensating winding , the total series (RA+RS) of 0.08 ohm. The series field consists of 25 turns per pole , with the magnetization curve shown in the following figure.

1- Find the speed and induced torque of this motor for when its armature current is 50A.

2- Calculate and plot the torque speed characteristic for this motor

$$E_A = V_T - I_A(R_A + R_S) = 250 \text{ V} - (50\text{A})(0.08\ \Omega) = 246 \text{ V}$$

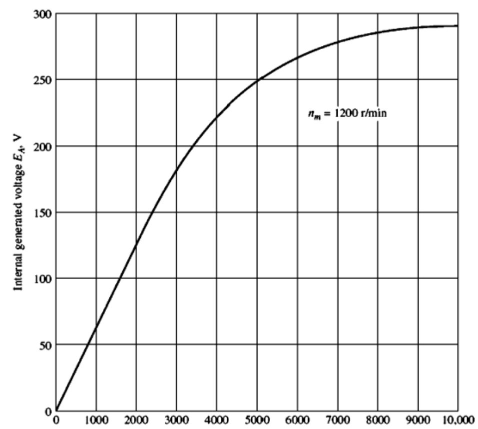
Since $I_A = I_F = 50$ A, the magnetomotive force is

$$\mathscr{F} = NI = (25 \text{ turns})(50 \text{ A}) = 1250 \text{ A} \bullet \text{turns}$$

From the magnetization curve at $\mathscr{F} = 1250$ A • turns, $E_{A0} = 80$ V. To get the correct speed of the motor, remember that,

$$n = \frac{E_A}{E_{A0}} n_0$$

$$= \frac{246 \text{ V}}{80 \text{ V}} 1200 \quad \text{r/min} = 3690 \text{ r/min}$$

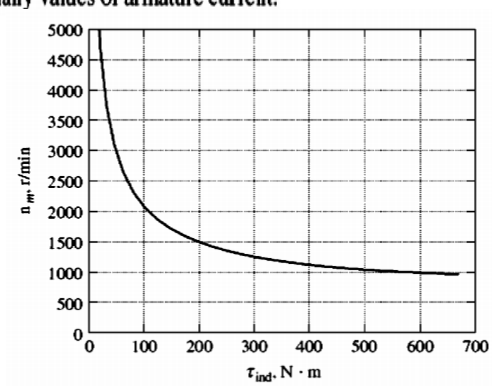


To find the induced torque supplied by the motor at that speed, recall that $P_{conv} = E_A I_A = \tau_{ind}\omega$. Therefore,

$$\tau_{ind} = \frac{E_A I_A}{\omega}$$

$$= \frac{(246 \text{ V})(50 \text{ A})}{(3690 \text{ r/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/r})} = 31.8 \text{ N} \bullet \text{m}$$

(*b*) To calculate the complete torque–speed characteristic, we must repeat the steps in *a* for many values of armature current.



DC Motors and Generators

Dr. Feras Alasali

Types of DC Motors

- **DC motors are driven from DC power supply, there are five major types of DC motors in general use:**

1- The separately excited dc motor.

2- The shunt DC motor.

3- The permanent –magnet DC motor

4- The series DC motor.

5- The compounded dc motor

The Compounded DC motor

- A compound wound motor is self excitation motor and has both series and shunt windings which can be connected ***as short-shunt or long shunt with armature winding***

Briefly, we can say that Dc compound motor is a combination of both a **shunt wound Dc motor** and **series wound Dc motor** which means that compound motor has the advantage of high starting torque and efficient speed regulation so it can be used in industrial applications include: • **Mixers.** • **Drivers.**

Compound Motors

Advantages of Compound Motors

- Quick start and stop of the motor can do.
- Reversing and acceleration of the motor can do fast

Disadvantages of Compound Motors

- Operation and maintenance cost of DC motors are expensive.
- In generally Every DC motor using brushes so the lifetime of such motors is less compared with AC motors.

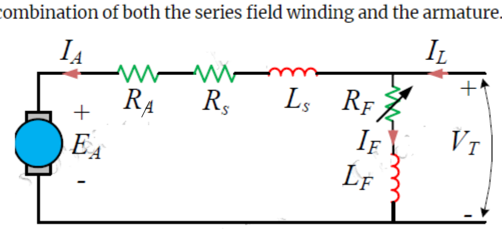
Applications of Compound Motors

- Compound motors are widely used in applications such as elevators/air compressors,

This type of Motor can be connected in two ways:

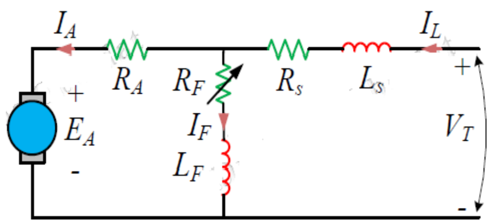
1- For Long Shunt Compound DC Motor

In the long shunt motor, we connect the shunt field winding parallel across the series combination of both the series field winding and the armature.



2- For Short Shunt Compound DC Motor

In short shunt motor, we connect the shunt field winding parallel across only the armature winding and we connect the series field winding to the supply current.

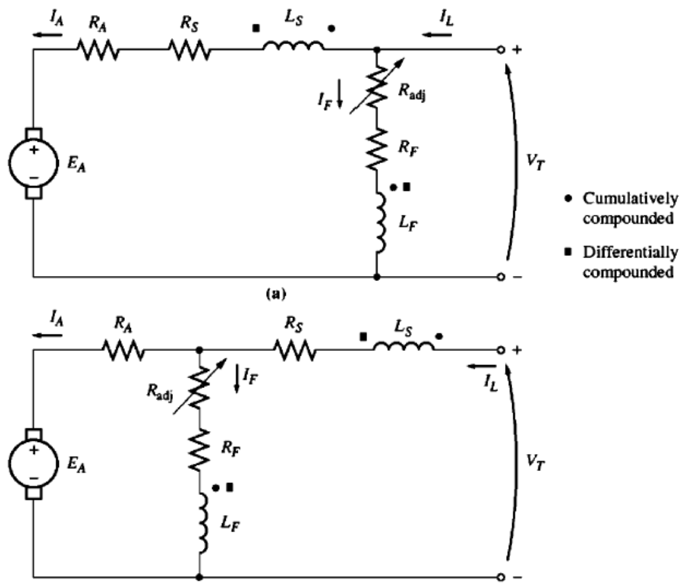


- Sub classification:

- Depending on the polarity of connections of shunt filed winding, field series winding and armature

These motors have both series and shunt windings. If series excitation helps the shunt excitation *i.e.* series flux is in the *same* direction ; then the motor is said to be cummulatively compounded. If on the other hand, series field opposes the shunt field, then the motor is said to be differentially compounded.

Compound Motors



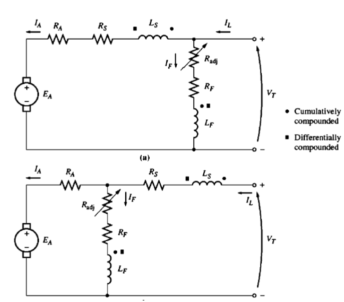
For Long Shunt Cumulatively Compound DC Motor

$$I_L = I_A + I_F$$

$$V_T = E_A + I_A(R_A + R_s)$$

$$V_T = I_F R_F$$

$$E_A = k\phi\omega_m$$



For Short Shunt Cumulatively Compound DC Motor

$$I_L = I_A + I_F$$

$$V_T = E_A + I_A R_A + I_L R_s$$

$$E_A = k\phi\omega_m$$

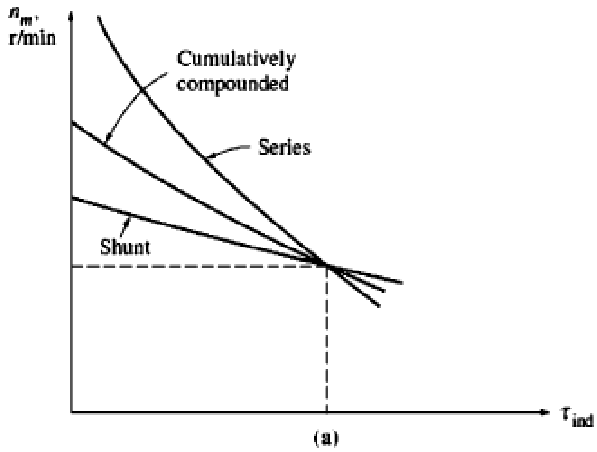
The net magnetomotive force and the effective shunt field current in the compounded motor are given by

$$\mathscr{F}_{net} = \mathscr{F}_F \pm \mathscr{F}_{SE} - \mathscr{F}_{AR}$$

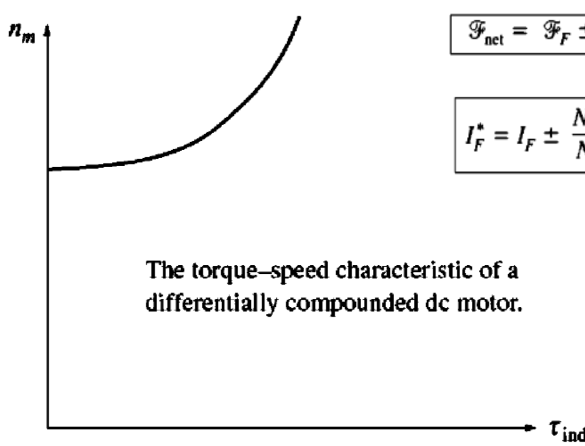
and

$$I_F^* = I_F \pm \frac{N_{SE}}{N_F} I_A - \frac{\mathscr{F}_{AR}}{N_F}$$

- Torque-speed characteristic a cumulatively compounded DC motor



- The torque –speed characteristic of Differentially compounded DC motor



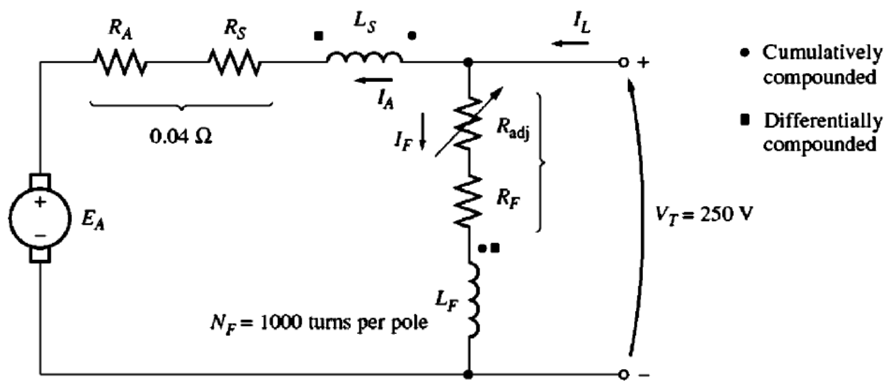
$$\mathcal{F}_{net} = \mathcal{F}_F \pm \mathcal{F}_{SE} - \mathcal{F}_{AR}$$

$$I_F^* = I_F \pm \frac{N_{SE}}{N_F} I_A - \frac{\mathcal{F}_{AR}}{N_F}$$

The torque–speed characteristic of a differentially compounded dc motor.

- The nonlinear Analysis of compounded DC motors

Example A 100-hp, 250-V compounded dc motor with compensating windings has an internal resistance, including the series winding, of 0.04 Ω. There are 1000 turns per pole on the shunt field and 3 turns per pole on the series winding. The machine is shown in Figure 9–27, and its magnetization curve is shown in Figure 9–9. At no load, the field resistor has been adjusted to make the motor run at 1200 r/min. The core, mechanical, and stray losses may be neglected.



The compounded dc motor in Example

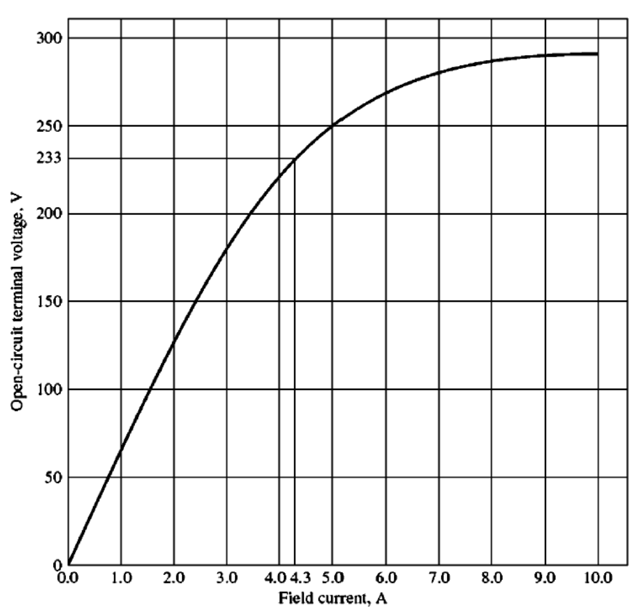


FIGURE 9–9
The magnetization curve of a typical 250-V dc motor, taken at a speed of 1200 r/min.

(a) What is the shunt field current in this machine at no load?
(b) If the motor is cumulatively compounded, find its speed when $I_A = 200$ A.
(c) If the motor is differentially compounded, find its speed when $I_A = 200$ A.

Solution

(a) At no load, the armature current is zero, so the internal generated voltage of the motor must equal V_T, which means that it must be 250 V. From the magnetization curve, a field current of 5 A will produce a voltage E_A of 250 V at 1200 r/min. Therefore, the shunt field current must be 5 A.

$$E_A = V_T - I_A(R_A + R_S)$$

(b) When an armature current of 200 A flows in the motor, the machine's internal generated voltage is

$$E_A = V_T - I_A(R_A + R_S)$$
$$= 250 \text{ V} - (200 \text{ A})(0.04\ \Omega) = 242 \text{ V}$$

The effective field current of this cumulatively compounded motor is

$$I_F^* = I_F + \frac{N_{SE}}{N_F} I_A - \frac{\mathcal{F}_{AR}}{N_F}$$
$$= 5 \text{ A} + \frac{3}{1000} 200 \text{ A} = 5.6 \text{ A}$$

From the magnetization curve, $E_{A0} = 262$ V at speed $n_0 = 1200$ r/min. Therefore, the motor's speed will be

$$n = \frac{E_A}{E_{A0}} n_0$$
$$= \frac{242 \text{ V}}{262 \text{ V}} 1200 \text{ r/min} = 1108 \text{ r/min}$$

(*c*) If the machine is differentially compounded, the effective field current is

$$I_F^* = I_F - \frac{N_{SE}}{N_F} I_A - \frac{\mathcal{F}_{AR}}{N_F}$$

$$= 5\text{ A} - \frac{3}{1000} 200\text{ A} = 4.4\text{ A}$$

From the magnetization curve, $E_{A0} = 236$ V at speed $n_0 = 1200$ r/min. Therefore, the motor's speed will be

$$n = \frac{E_A}{E_{A0}} n_0$$

$$= \frac{242\text{ V}}{236\text{ V}} 1200\text{ r/min} = 1230\text{ r/min}$$

Notice that the speed of the cumulatively compounded motor decreases with load, while the speed of the differentially compounded motor increases with load.

- Speed control in the Cumulatively compounded DC motor

The techniques available for the control of speed in a cumulatively compounded dc motor are the same as those available for a shunt motor:

1. Change the field resistance R_F.
2. Change the armature voltage V_A.
3. Change the armature resistance R_A.

The arguments describing the effects of changing R_F or V_A are very similar to the arguments given earlier for the shunt motor.

Theoretically, the differentially compounded dc motor could be controlled in a similar manner. Since the differentially compounded motor is almost never used, that fact hardly matters.

8.8 DC motor starters/ Protection

In order for a dc motor to function properly on the job, it must have some special control and protection equipment associated with it. The purposes of this equipment are

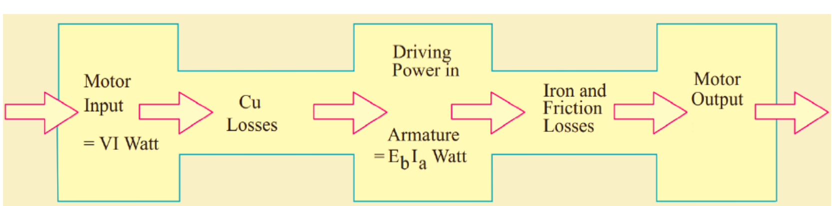
1. To protect the motor against damage due to short circuits in the equipment
2. To protect the motor against damage from long-term overloads
3. To protect the motor against damage from excessive starting currents
4. To provide a convenient manner in which to control the operating speed of the motor

The protection circuit section combines several different devices which together ensure the safe operation of the motor. Some typical safety devices included in this type of drive are

1. *Current-limiting fuses*, to disconnect the motor quickly and safely from the power line in the event of a short circuit within the motor. Current-limiting fuses can interrupt currents of up to several hundred thousand amperes.
2. An *instantaneous static trip*, which shuts down the motor if the armature current exceeds 300 percent of its rated value. If the armature current exceeds the maximum allowed value, the trip circuit activates the fault relay, which deenergizes the run relay, opening the main contactors and disconnecting the motor from the line.
3. An *inverse-time overload trip*, which guards against sustained overcurrent conditions not great enough to trigger the instantaneous static trip but large enough to damage the motor if allowed to continue indefinitely. The term *inverse time* implies that the higher the overcurrent flowing in the motor, the faster the overload acts . For example, an inverse-time trip might take a full minute to trip if the current flow were 150 percent of the rated current of the motor, but take 10 seconds to trip if the current flow were 200 percent of the rated current of the motor.

4. An *undervoltage trip*, which shuts down the motor if the line voltage supplying the motor drops by more than 20 percent.
5. A *field loss trip*, which shuts down the motor if the field circuit is lost.
6. An *overtemperature trip*, which shuts down the motor if it is in danger of overheating.

8.10 DC motors efficiency calculation



To calculate the efficiency of a dc motor, the following losses must be determined:

1. Copper losses
2. Brush drop losses
3. Mechanical losses
4. Core losses
5. Stray losses

Example

Example A 50-hp, 250-V, 1200 r/min shunt dc motor has a rated armature current of 170 A and a rated field current of 5 A. When its rotor is blocked, an armature voltage of 10.2 V (exclusive of brushes) produces 170 A of current flow, and a field voltage of 250 V produces a field current flow of 5 A. The brush voltage drop is assumed to be 2 V. At no load with the terminal voltage equal to 240 V, the armature current is equal to 13.2 A, the field current is 4.8 A, and the motor's speed is 1150 r/min.

(a) How much power is output from this motor at rated conditions?
(b) What is the motor's efficiency?

Solution

The armature resistance of this machine is approximately

$$R_A = \frac{10.2\text{ V}}{170\text{ A}} = 0.06\ \Omega$$

and the field resistance is

$$R_F = \frac{250\text{ V}}{5\text{ A}} = 50\ \Omega$$

Therefore, at full load the armature I^2R losses are

$$P_A = (170\text{ A})^2(0.06\ \Omega) = 1734\text{ W}$$

and the field circuit I^2R losses are

$$P_F = (5\text{ A})^2(50\ \Omega) = 1250\text{ W}$$

The brush losses at full load are given by

$$P_{\text{brush}} = V_{\text{BD}}I_A = (2\text{ V})(170\text{ A}) = 340\text{ W}$$

The rotational losses at full load are essentially equivalent to the rotational losses at no load, since the no-load and full-load speeds of the motor do not differ too greatly. These losses may be ascertained by determining the input power to the armature circuit at no load and assuming that the armature copper and brush drop losses are negligible, meaning that the no-load armature input power is equal to the rotational losses:

$$P_{tot} = P_{core} + P_{mech} = (240\text{ V})(13.2\text{ A}) = 3168\text{ W}$$

(a) The input power of this motor at the rated load is given by

$$P_{in} = V_T I_L = (250\text{ V})(175\text{ A}) = 43{,}750\text{ W}$$

Its output power is given by

$$\begin{aligned} P_{out} &= P_{in} - P_{brush} - P_{cu} - P_{core} - P_{mech} - P_{stray} \\ &= 43{,}750\text{ W} - 340\text{ W} - 1734\text{ W} - 1250\text{ W} - 3168\text{ W} - (0.01)(43{,}750\text{ W}) \\ &= 36{,}820\text{ W} \end{aligned}$$

where the stray losses are taken to be 1 percent of the input power.

(b) The efficiency of this motor at full load is

$$\begin{aligned} \eta &= \frac{P_{out}}{P_{out}} \times 100\% \\ &= \frac{36{,}820\text{ W}}{43{,}750\text{ W}} \times 100\% = 84.2\% \end{aligned}$$

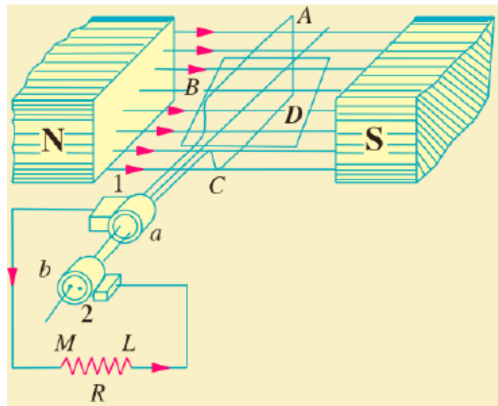
DC Motors and Generators Ch. 9/8

Dr. Feras Alasali

9.11 Introduction To DC Generators

- **1.8 linear DC machine**

- A single turn rectangular copper ABCD rotating about its own axis in a magnetic field provided by either permanent magnet or electromagnet. The two ends of the coil are joined to slip ring ‘a’ and ‘b’ which are insulated from each other and from the central shaft. Two collecting brushes press against the slip rings; their function is to collect the current induced in the coil and to convey it to external load resistance.



Types of DC Generators

Generators are usually classified according to the way in which their fields are excited

❑Separately Excited Generators: are those whose field magnets are energized from an **independent external source of DC current**.

❑Self Excited Generators: are those whose field magnets are energized **by current produced by the generators themselves**. There are three types of self excited generators named according to the manner in which their field coils are connected to the armature.

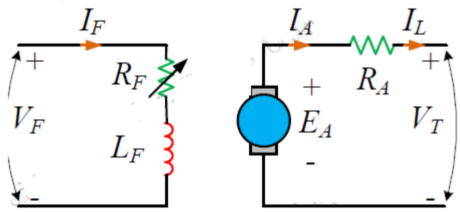
✓Shunt Wound: the field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them.

✓Series Wound: the field windings are joined in series with the armature conductors

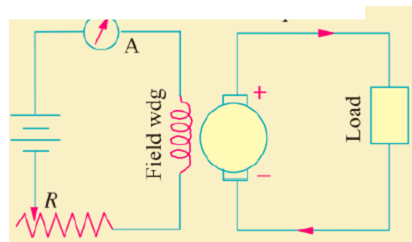
✓Compound Wound: it is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt.

9.12 Separately Excited DC Generator

$$I_A = I_L$$
$$V_T = E_A - I_A R_A$$
$$V_F = I_F R_F$$
$$E_A = k\phi\omega_m$$



Where
I_A: is the armature current
I_L: is the load current
E_A: is the internal generated voltage
V_T: is the terminal voltage
I_F: is the field current
V_F: is the field voltage
R_A: is the armature winding resistance
R_F: is the field winding resistance
ϕ: is the flux
ω_m: is the rotor angular speed



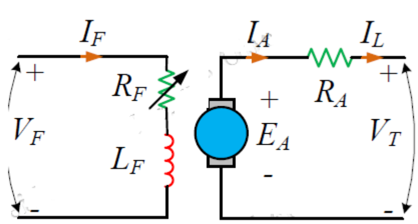
Control of terminal voltage.

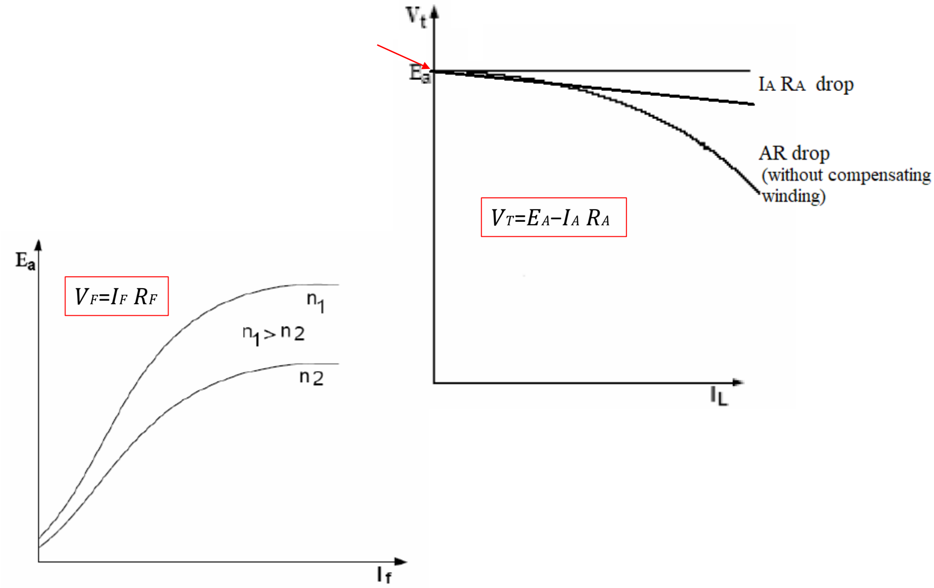
- **The terminal voltage (V_T) can be controlled by:**

1.Change the speed of rotation: If ω increases, then $E_A=k\varphi\omega_m$ increases, so $V_T=E_A-I_A R_A$ increases as well.

2.Change the field current. If R_F is decreased, then the field current increases ($V_F=I_F R_F$). Therefore, the flux in the machine increases. As the flux rises, $EA=k\varphi\omega m$ must rise too, so $V_T=E_A-I_A R_A$ increases.

$$I_A = I_L$$
$$V_T = E_A - I_A R_A$$
$$V_F = I_F R_F$$
$$E_A = k\phi\omega_m$$





Example

Separately Excited DC Generator works with speed = 1200 rpm and Voltage out of armature is 120 V. if we decrease the speed to 1000 rpm , what is the new E_A?

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

- E_A = 120 * (1000/1200) = 100 V

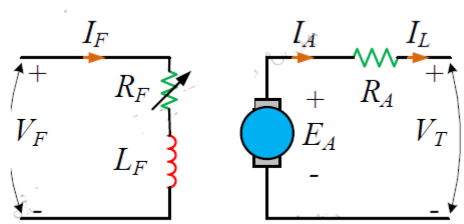
Example

Separately Excited DC Generator with speed = 1200 rpm connected to load with current 200 A and 125 V. If the armature resistance R_A equal to 0.04 ohm , find

1- load resistance and voltage out of the armature?

2- the voltage and current out of armature if the speed is decreased to 1000 rpm, where the load does not change.

$$I_A = I_L$$
$$V_T = E_A - I_A R_A$$
$$V_F = I_F R_F$$
$$E_A = k\phi\omega_m$$



$$E_a = V_L + I_a R_a = 125 + (200 \times 0.04) = 133v$$

$$R_L = \frac{V_L}{I_L} = \frac{125}{200} = 0.625\Omega$$

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

E_A = 133 * (1000/1200) = 111 V

$$I_A = \frac{E_A}{R_a + R_L} = \frac{111}{0.04 + 0.625} \cong 167A$$

Nonlinear Analysis

The internal generated voltage has nonlinear relationship with magnet force, due to AR reaction.

From previous examples (in motors), we can notice that the only way to accurately determine **the output voltage is by using graphical analysis (magnetization curve)** .

$$\mathscr{F}_{\text{net}} = N_F I_F - \mathscr{F}_{\text{AR}}$$

$$I_F^* = I_F - \frac{\mathscr{F}_{\text{AR}}}{N_F}$$

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

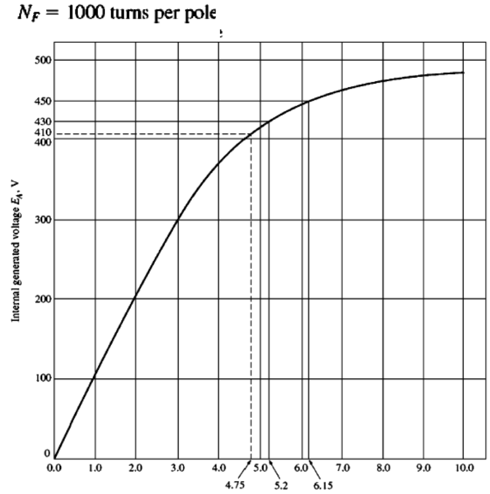
The total force is effected by the AR reaction.

Example A separately excited dc generator is rated at 172 kW, 430 V, 400 A, and 1800 r/min. magnetization curve is shown in Fig- This machine has the following characteristics:

$R_A = 0.05\ \Omega$ $V_F = 430$ V

$R_F = 20\ \Omega$ $N_F = 1000$ turns per pole

$R_{adj} = 0$ to $300\ \Omega$



Note: When the field current is zero, E_A is about 3 V.

(a) If the variable resistor R_{adj} in this generator's field circuit is adjusted to 63 Ω and the generator's prime mover is driving it at 1600 r/min, what is this generator's no-load terminal voltage?

(b) What would its voltage be if a 360-A load were connected to its terminals? Assume that the generator has compensating windings.

(c) What would its voltage be if a 360-A load were connected to its terminals but the generator does not have compensating windings? Assume that its armature reaction at this load is 450 A • turns.

(d) What adjustment could be made to the generator to restore its terminal voltage to the value found in part a?

(e) How much field current would be needed to restore the terminal voltage to its no-load value? (Assume that the machine has compensating windings.) What is the required value for the resistor R_{adj} to accomplish this?

(a) If the generator's total field circuit resistance is $R_F + R_{adj} = 83\ \Omega$

then the field current in the machine is $I_F = \frac{V_F}{R_F} = \frac{430\text{ V}}{83\ \Omega} = 5.2\text{ A}$

From the machine's magnetization curve, this much current would produce a voltage $E_{A0} = 430$ V at a speed of 1800 r/min. Since this generator is actually turning at $n_m = 1600$ r/min, its internal generated voltage E_A will be

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

$$E_A = \frac{1600\text{ r/min}}{1800\text{ r/min}} 430\text{ V} = 382\text{ V}$$

Since $V_T = E_A$ at no-load conditions, the output voltage of the generator is $V_T = 382$ V.

(b) If a 360-A load were connected to this generator's terminals, the terminal voltage of the generator would be

$$V_T = E_A - I_A R_A = 382\text{ V} - (360\text{ A})(0.05\ \Omega) = 364\text{ V}$$

(c) If a 360-A load were connected to this generator's terminals and the generator had 450 A • turns of armature reaction, the effective field current would be

$$I_F^* = I_F - \frac{\mathscr{F}_{AR}}{N_F} = 5.2\text{ A} - \frac{450\text{ A} \bullet \text{turns}}{1000\text{ turns}} = 4.75\text{ A}$$

From the magnetization curve, $E_{A0} = 410$ V, so the internal generated voltage at 1600 r/min would be

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

$$E_A = \frac{1600\text{ r/min}}{1800\text{ r/min}} 410\text{ V} = 364\text{ V}$$

Therefore, the terminal voltage of the generator would be

$$V_T = E_A - I_A R_A = 364\text{ V} - (360\text{ A})(0.05\ \Omega) = 346\text{ V}$$

It is lower than before due to the armature reaction.

(d) The voltage at the terminals of the generator has fallen, so to restore it to its original value, the voltage of the generator must be increased. This requires an increase in E_A, which implies that R_{adj} must be decreased to increase the field current of the generator.

(e) For the terminal voltage to go back up to 382 V, the required value of E_A is

$$E_A = V_T + I_A R_A = 382\text{ V} + (360\text{ A})(0.05\ \Omega) = 400\text{ V}$$

To get a voltage E_A of 400 V at n_m = 1600 r/min, the equivalent voltage at 1800 r/min would be

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

$$E_{A0} = \frac{1800\text{ r/min}}{1600\text{ r/min}} 400\text{ V} = 450\text{ V}$$

From the magnetization curve, this voltage would require a field current of I_F = 6.15 A. The field circuit resistance would have to be

$$R_F + R_{adj} = \frac{V_F}{I_F}$$

$$20\ \Omega + R_{adj} = \frac{430\text{ V}}{6.15\text{ A}} = 69.9\ \Omega$$

$$R_{adj} = 49.9\ \Omega \approx 50\ \Omega$$

Notice:

Notice that, for the same field current and load current, the generator with armature reaction had a lower output voltage than the generator without armature reaction. The armature reaction in this generator is exaggerated to illustrate its effects—it is a good deal smaller in well-designed modern machines.

DC Motors and Generators

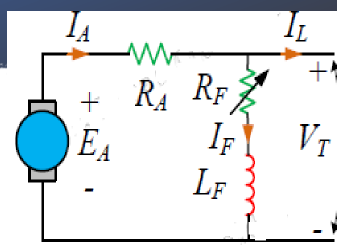
Dr. Feras Alasali

Types of DC Generators

Generators are usually classified according to the way in which their fields are excited

- ❑**Separately Excited Generators: are those whose field magnets are energized from an independent external source of DC current.**
- ❑**Self Excited Generators: are those whose field magnets are energized by current produced by the generators themselves. There are three types of self excited generators named according to the manner in which their field coils are connected to the armature.**
- ✓**Shunt Wound: the field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them.**
- ✓**Series Wound: the field windings are joined in series with the armature conductors**
- ✓**Compound Wound: it is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt.**

8.13 The shunt DC Generators

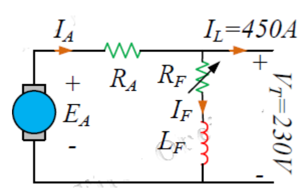


$I_A = I_F + I_L$
$V_T = E_A - I_A R_A$
$V_F = I_F R_F = V_T$
$E_A = k\varphi\omega_m$

The terminal voltage can be controlled by:
1. Change the speed of rotation: If ω increases, then E_A=$k\varphi\omega_m$ increases, so $V_T = E_A - I_A R_A$ increases as well.
2. Change the field current. If R_F is decreased. then the field current increases ($V_F = I_F R_F$). Therefore, the flux in the machine increases. As the flux rises, E_A=$k\varphi\omega_m$ must rise too, so V_T increases.

Example

Example: A shunt DC generator delivers 450A at 230V and the resistance of the shunt field and armature are 50Ω and 0.3 Ω respectively. Calculate emf.



$I_F = 230/50 = 4.6A$
$I_A = I_F + I_L = 4.6 + 450 = 454.6A$
$E_A = V_T + I_A R_A = 230 + 454.6 \times 0.3 = 243.6V$

Example

Example: A shunt DC generator, if $E_A = V_T = 140$ V, rated speed is 1500 rpm, what is the E_A if the speed decrease to 1200 rpm. <u>Assume the filed current is constant</u>.

$E_A = k\varphi\omega_m$

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

$E_A = 140 * (1200/1500) = 112$ V

Example

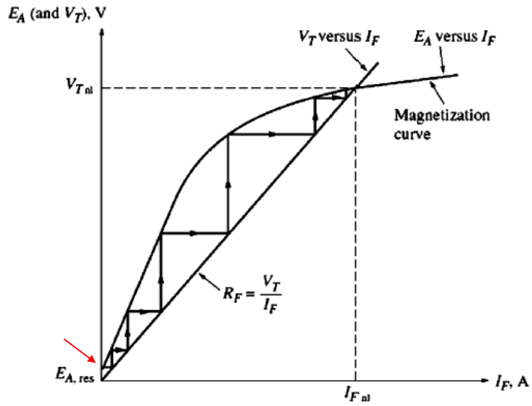
Example: A shunt DC generator, if $E_A = V_T = 140$ V, the filed current is 5 A, what is the E_A if the filed current decrease BY <u>1 A</u>.

$E_A = k\varphi\omega_m$

$$\frac{E_A}{E_{A0}} = \frac{\phi}{\phi_0}$$

$E_A = 140 * (4/5) = 112$ V

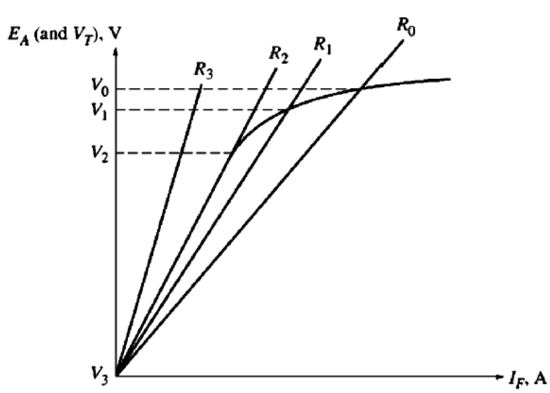
Voltage building in a shunt Generator



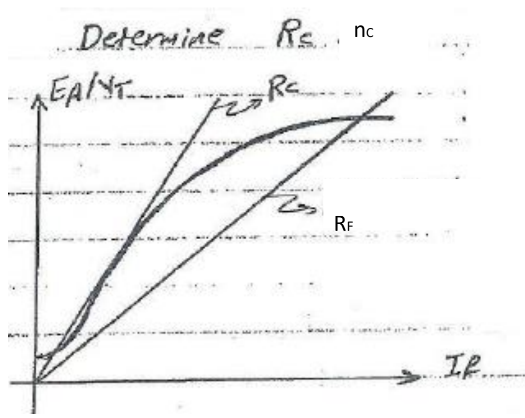
Voltage buildup on starting in a shunt dc generator.

$$E_A = K\phi_{res}\omega$$

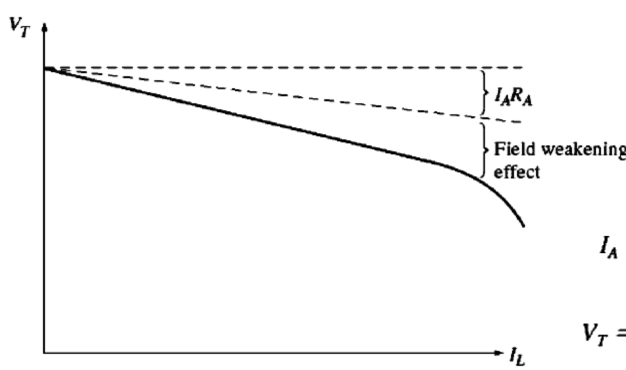
What if a shunt generator is started and no voltage builds up? What could be wrong? There are several possible causes for the voltage to fail to build up during starting.



The effect of shunt field resistance on no-load terminal voltage in a dc generator. If $R_F > R_2$ (the critical resistance), then the generator's voltage will never build up.



The terminal characteristic of a shunt DC Generator

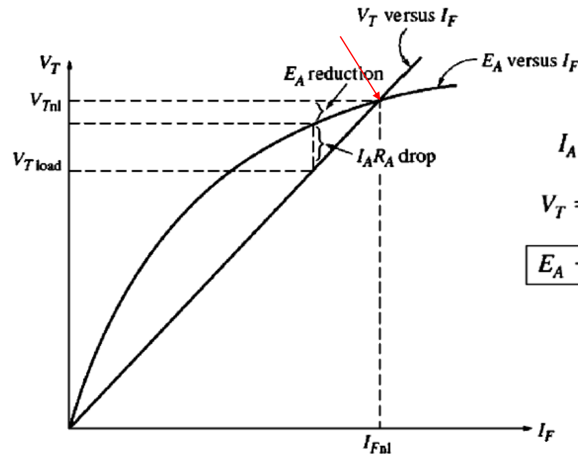


The terminal characteristic of a shunt dc generator.

$$I_A = I_F + I_L \uparrow$$

$$V_T = E_A - I_A \uparrow R_A$$

Analysis of Shunt DC generators



Graphical analysis of a shunt dc generator with compensating windings.

$$I_A = I_F + I_L \uparrow$$

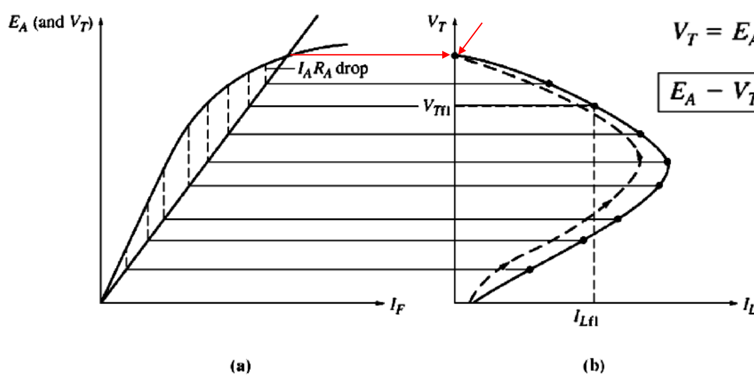
$$V_T = E_A - I_A R_A$$

$$\boxed{E_A - V_T = I_A R_A}$$

$$I_A = I_F + I_L \uparrow$$

$$V_T = E_A - I_A R_A$$

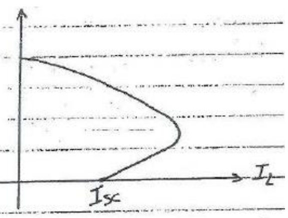
$$\boxed{E_A - V_T = I_A R_A}$$



Graphical derivation of the terminal characteristic of a shunt dc generator.

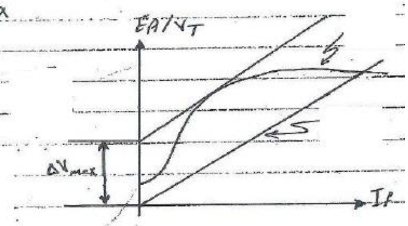
Determine I_{sc} (SSSC) ← steady state short circuit current

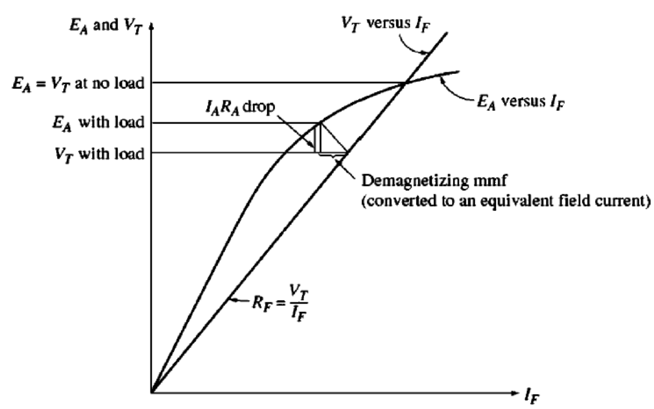
$$I_{sc} = \frac{E_{Rez}}{R_A}$$



Determine I_{max}

$$I_{max} = \frac{\Delta V_{max}}{R_A}$$



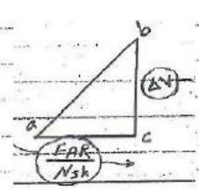


Graphical analysis of a shunt dc generator with armature reaction.

$$F_{net} = F_{sh} - F_{AR}$$

$$I_{net} N_{sh} = I_{sh} N_{sh} - F_{AR}$$

$$I_{net} = I_{sh} - \frac{F_{AR}}{N_{sh}}$$

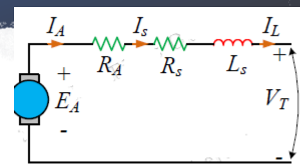


8.13 The Series DC Generators

$I_A = I_S = I_L$

$V_T = E_A - I_A(R_A + R_S)$

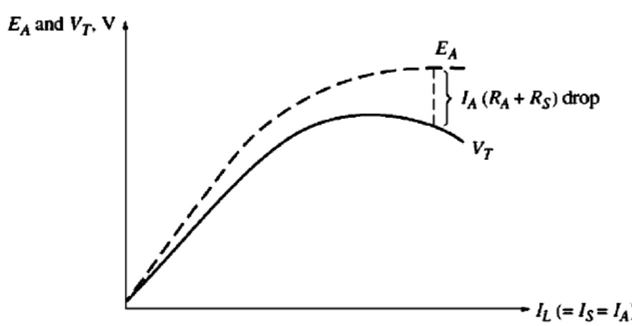
$E_A = k\varphi\omega_m$



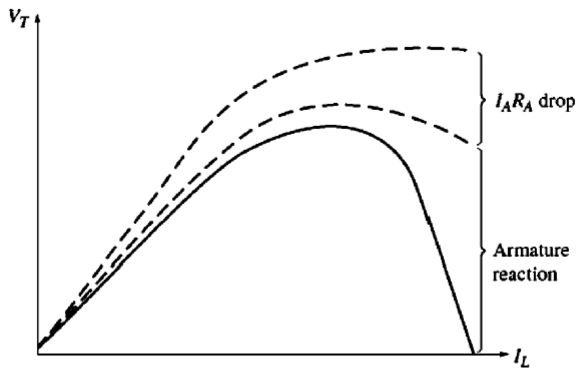
- At no load, there is no field current, so V_T is reduced to a small level given by the residual flux in the machine.
- As the load increases, the field current rises, so E_A rises rapidly. The

$I_A(R_A + R_S)$ drop goes up too, but at first the increase in E_A goes up more rapidly than the $I_A(R_A + R_S)$ drop rises, so V_T increases. After a while, the machine approaches saturation, and E_A becomes almost constant. At that point, the resistive drop is the predominant effect, and V_T starts to fall.

The terminal characteristic of a series Generator.



Derivation of the terminal characteristic for a series dc generator.



A series generator terminal characteristic with large armature reaction effects, suitable for electric welders.

DC Motors and Generators

Dr. Feras Alasali

Types of DC Generators

Generators are usually classified according to the way in which their fields are excited

- ❑Separately Excited Generators: are those whose field magnets are energized from an independent external source of DC current.
- ❑Self Excited Generators: are those whose field magnets are energized by current produced by the generators themselves. There are three types of self excited generators named according to the manner in which their field coils are connected to the armature.
- ✓Shunt Wound: the field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them.
- ✓Series Wound: the field windings are joined in series with the armature conductors
- ✓Compound Wound: it is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt.

8.15 The Compound DC Generator

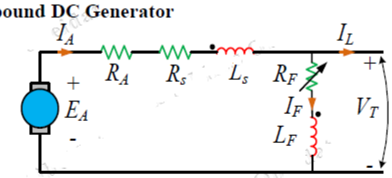
For Long Shunt Cumulatively Compound DC Generator

$I_A = I_F + I_L$

$V_T = E_A - I_A(R_A + R_s)$

$V_T = I_F R_F$

$E_A = k\phi\omega_m$

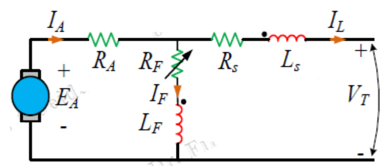


For Short Shunt Cumulatively Compound DC Generator

$I_A = I_F + I_L$

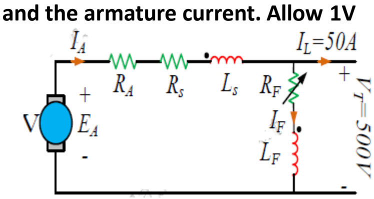
$V_T = E_A - I_A R_A - I_L R_s$

$E_A = k\phi\omega_m$



Example

A long shunt compound DC generator delivers a load current of 50A at 500V and has armature, series field and shunt field resistances of 0.05Ω, 0.03Ω and 250Ω respectively. Calculate the generated voltage and the armature current. Allow 1V per brush for contact drop.



$I_F = 500/250 = 2A$

$I_A = I_F + I_L = 2 + 50 = 52A$

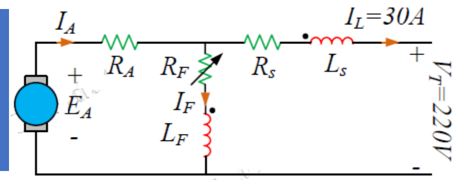
Voltage drop across series winding=$I_A R_s$=52×0.03=1.56V

Armature voltage drop=$I_A R_A$=52×0.05=2.6V

Drop at brushes=2×1=2V

$E_A = V_T + I_A R_A + series\ drop + brushes\ drop$ = 500 + 2.6 + 1.56 + 2 = 506.16V

Example



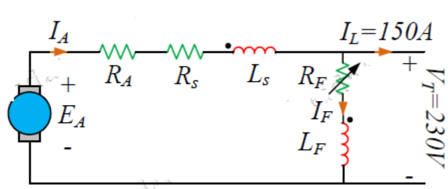
A short shunt compound DC generator delivers a load current of 30A at 220V and has armature, series field and shunt field resistances of 0.05Ω, 0.3Ω and 200Ω respectively. Calculate the induced emf and the armature current. Allow 1V per brush for contact drop

Voltage drop across series winding=$I_L R_S$ =30×0.3=9V
Voltage across shunt winding=220+9=229V

I_F=229/200=1.145A
Armature voltage drop=$I_A R_A$ =31.145×0.05=1.56V
Drop at brushes=2×1=2V
$E_A = V_T + I_A R_A + series\ drop + brushe\ drop$=220+9+1.56+2=232.56V

Example

Example: A long shunt compound DC generator delivers a load current of 150A at 230V and has armature, series field and shunt field resistances of 0.032Ω, 0.015Ω and 92Ω respectively. Calculate (i) induced emf (ii) total power generated and (iii) distribution of this power.



(i) $I_F = 230/92 = 2.5A$

$I_A = I_F + I_L = 2.5 + 150 = 152.5A$

Voltage drop across series winding= $I_A R_S$=152.5 ×0.015=2.2875V
Armature voltage drop=$I_A R_A$=152.5 ×0.032=4.88V

$E_A = V_T + I_A R_A + I_A R_S = 230 + 2.2875 + 4.88 = 237.1675V$

(ii) Total power generated by the armature= $E_A I_A$= 237.1675×152.5=**36168.04375**W

(iii) Power lost in armature=$I_A R_A$=152.52×0.032=744.2W

Power dissipated in shunt winding=$V_T I_F$=230×2.5=575W

Power dissipated in series winding=$I_A R_S$=152.52×0.015=348.84375W
Power delivered to the load=$V_T I_L$=230×150=34500W

Total power generated by the armature=744.2 + 575 + 348.843 + 34500=**36168.04375**W

The Terminal Characteristic of Cumulatively Compound DC Generator

$$V_T = E_A - I_A(R_A + R_s)$$

$$\mathscr{F}_{net} = \mathscr{F}_F + \mathscr{F}_{SE} - \mathscr{F}_{AR}$$

$$V_T = E_A - I_A R_A - I_L R_s$$

$$I_F^* = I_F + \frac{N_{SE}}{N_F} I_A - \frac{\mathscr{F}_{AR}}{N_F}$$

The terminal voltage Cumulatively Compound DC Generator can be controlled by:

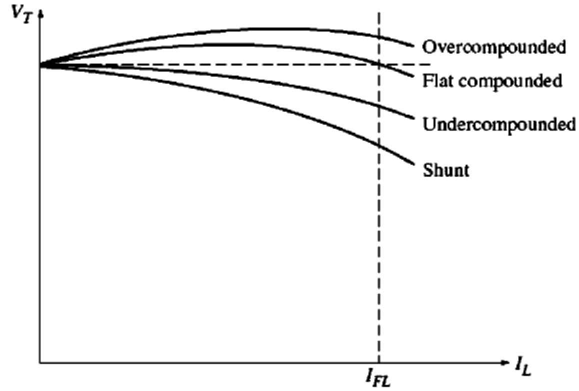
- **1. Change the speed of rotation: If ω increases, then $E_A=k\varphi\omega_m$ increases, so $V_T = E_A - I_A R_A$ increases as well.**
- **2. Change the field current. If R_F is decreased. then the field current increases ($V_F = I_F R_F$). Therefore, the flux in the machine increases. As the flux rises, $E_A=k\varphi\,\omega_m$ must rise too, so V_T increases.**

$$V_T = E_A - I_A(R_A + R_s)$$

$$V_T = E_A - I_A R_A - I_L R_s$$

$$\mathcal{F}_{net} = \mathcal{F}_F + \mathcal{F}_{SE} - \mathcal{F}_{AR}$$

$$I_F^* = I_F + \frac{N_{SE}}{N_F} I_A - \frac{\mathcal{F}_{AR}}{N_F}$$



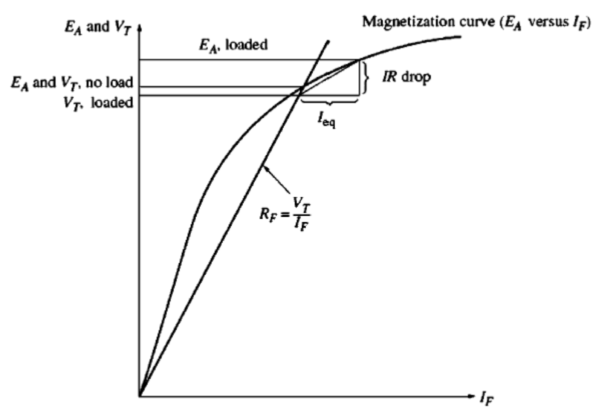
Terminal characteristics of cumulatively compounded dc generators.

Analysis of cumulatively compounded DC Generator

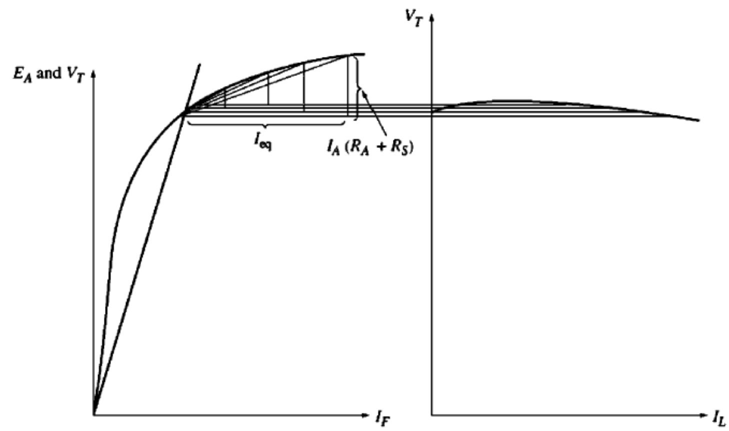
$$I_{eq} = \frac{N_{SE}}{N_F} I_A - \frac{\mathcal{F}_{AR}}{N_F}$$

$$I_F^* = I_F + I_{eq}$$

$$(R_F = V_T / R_F)$$



Graphical analysis of a cumulatively compounded dc generator.



Graphical derivation of the terminal characteristic of a cumulatively compounded dc generator.

8.16 The Differentially compounded DC generator

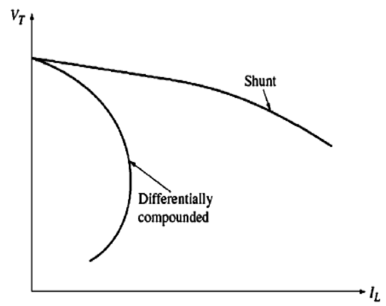
$$\mathscr{F}_{net} = \mathscr{F}_F - \mathscr{F}_{SE} - \mathscr{F}_{AR}$$

$$\mathscr{F}_{net} = N_F I_F - N_{SE} I_A - \mathscr{F}_{AR}$$

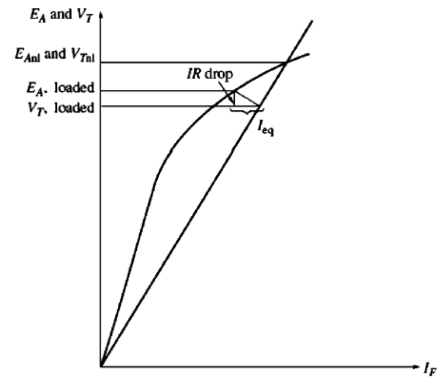
$$I_{eq} = -\frac{N_{SE}}{N_F} I_A - \frac{\mathscr{F}_{AR}}{N_F}$$

$$I_F^* = I_F + I_{eq}$$

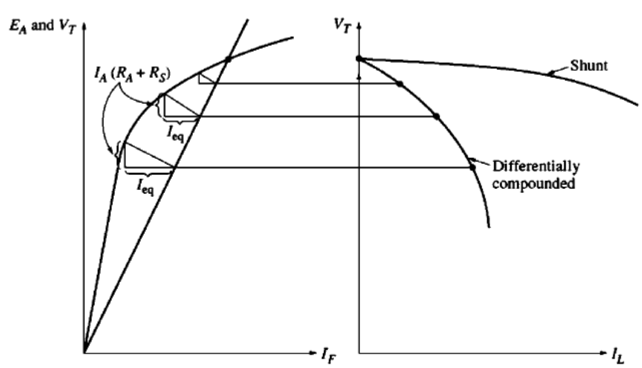
$$I_F^* = I_F - \frac{N_{SE}}{N_F} I_A - \frac{\mathscr{F}_{AR}}{N_F}$$



The terminal characteristic of a differentially compounded dc generator.

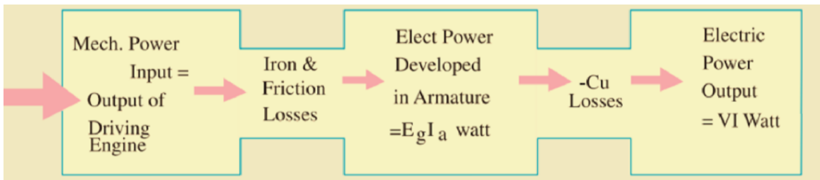


Graphical analysis of a differentially compounded dc generator.



Graphical derivation of the terminal characteristic of a differentially compounded dc generator.

Power Stages and Efficiency



Example

A <u>**shunt generator**</u> **delivers 195A at terminal voltage of 250V. The armature resistance and shunt field resistance are 0.02Ω and 50Ω respectively. The iron and friction losses equal 950W. Find (a) emf generated (b) Cu losses (c) output of the prime motor (d) commercial, mechanical and electrical efficiencies.**

(a) $I_F=250/50=5A$

$I_A=I_F+I_L=5+195=200A$

$E_A=V_T+I_AR_A=250+200\times0.02=254V$

(b) $Armature\ Cu\ loss=IA^2R_A=2002\times0.02=800W$

$Shunt\ Cu\ loss=IF^2\ R_F=52\times50=1250W$

<u>$Total\ Cu\ loss$</u>$=800+1250=2050W$

(c) Stray losses=950W ---------Total losses=950+2050=3000W

$Generator\ output=V_TI_L=250\times195=48750W$

$Output\ of\ the\ prime\ motor=Generator\ input$

$Generator\ input = Generator\ output + total\ losses =$
48750+3000=51750W $Output\ of\ the\ prime\ motor$=51750$W$

(c) $Generated\ electrical\ power (E_A I_A) = Generator\ input - stray\ loss$

$Generated\ electrical\ power (E_A I_A) = 51750 - 950 = 50800W$

$$\eta_m = \frac{E_A I_A}{Output\ of\ driving\ engine} \times 100\% = \frac{50800}{51750} \times 100\% = 98.2\%$$

$$\eta_e = \frac{VI_L}{E_A I_A} = \frac{48750}{50800} \times 100\% = 95.9\%$$

$$\eta_c = \frac{VI_L}{Output\ of\ driving\ engine} \times 100\% = \frac{48750}{51750} \times 100\% = 94.2\%$$

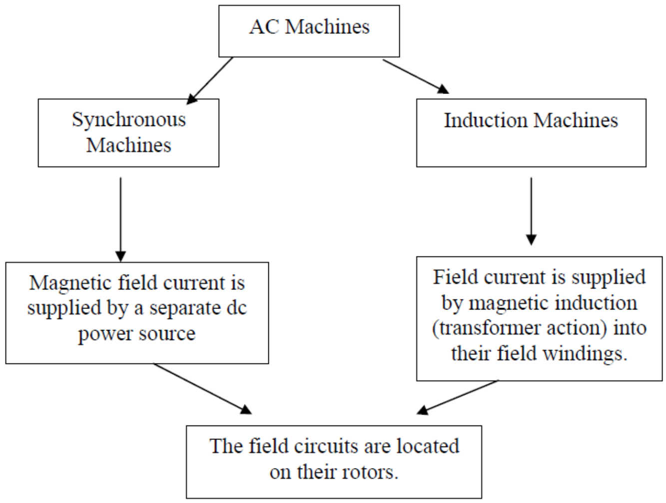
AC Machinery Fundamentals
Ch. 3 / Fifth Edition

Dr. Feras Alasali

Introduction

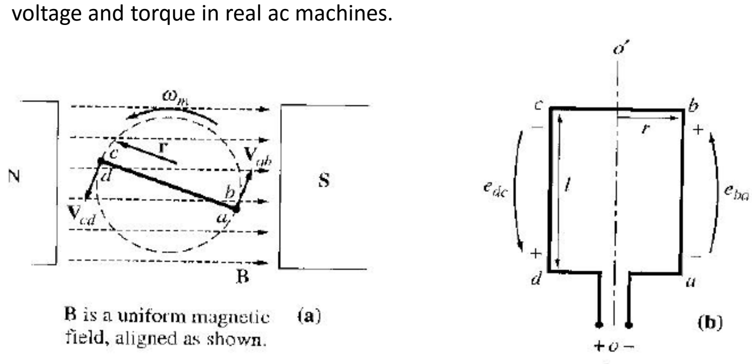
AC machines are AC motors and AC generators and There are two types of AC machines:

- **Synchronous machines** : the magnetic field current is supplied by a separate DC power source;
- **Induction machines:** the magnetic field current is supplied by magnetic induction (transformer action) into their field windings.
- The field circuits of most AC machines are located on their rotors.
- Every AC (or DC) motor or generator has two parts: rotating part (rotor) and a stationary part (stator).



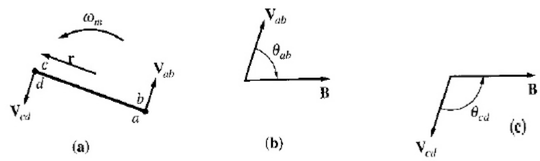
A simple loop in a uniform magnetic field

- The figure below shows a simple rotating loop in a uniform magnetic field. (a) is the front view and (b) is the view of the coil. The rotating part is called the rotor, and the stationary part is called the stator.
- This case in not representative of real ac machines (flux in real ac machines is not constant in either magnitude or direction). However, the factors that control the voltage and torque on the loop are the same as the factors that control the voltage and torque in real ac machines.



B is a uniform magnetic field, aligned as shown.

- If the rotor (loop) is rotated, a voltage will be induced in the wire loop. To determine the magnitude and shape, examine the phasors below·



- To determine the total voltage induced e_{tot} on the loop, examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by equation

$$e_{ind} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot l \qquad e_{ind} = \phi_{max} \omega \sin \omega t$$

- **The Torque Induced in a Current-Carrying Loop**

The force on each segment of the loop is given by:

$$\mathbf{F} = i\,(\boldsymbol{l} \times \boldsymbol{B})$$

$$\tau = rF \sin\theta$$

3.2 The rotating magnetic field

The basic idea of an electric motor is to generate two magnetic fields: rotor magnetic field and stator magnetic field , then torque will be induced in the rotor which will cause the rotor to turn and align itself with stator

The fundamental principle of AC machine operation is to make a 3-phase set of currents, each of equal magnitude and with a phase difference of 120°, to flow in a 3-phase winding. In this situation, a constant magnitude rotating field will be generated.

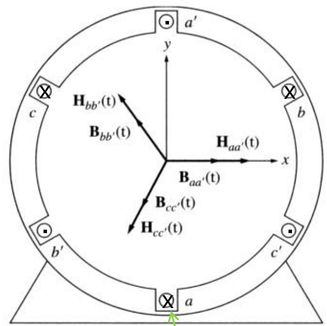
The 3-phase winding consists of 3 separate windings spaced 120° apart around the surface of the machine.

The rotating magnetic field

Consider a simple 3-phase stator containing three coils, each 120^0 apart. Such a winding will produce only one north and one south magnetic pole; therefore, this motor would be called a two-pole motor.

Assume that the currents in three coils are:

$$\begin{cases} i_{aa'}(t) = I_M \sin \omega t \\ i_{bb'}(t) = I_M \sin\left(\omega t - 120^0\right) \\ i_{cc'}(t) = I_M \sin\left(\omega t - 240^0\right) \end{cases}$$



The directions of currents are indicated.

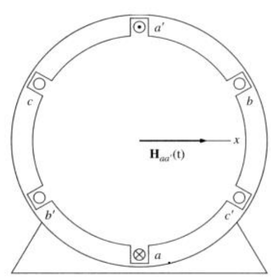
Therefore, the current through the coil *aa'* produces the magnetic field intensity

$$H_{aa'}(t) = H_M \sin \omega t \angle 0°$$

The rotating magnetic field

where the magnitude of the magnetic field intensity is changing over time, while 0^0 is the spatial angle of the magnetic field intensity vector. The direction of the field can be determined by the right-hand rule.

Note, that while the magnitude of the magnetic field intensity $H_{aa'}$ varies sinusoidally over time, its direction is always constant. Similarly, the magnetic fields through two other coils are



$$H_{bb'}(t) = H_M \sin\left(\omega t - 120°\right) \angle 120°$$
$$H_{cc'}(t) = H_M \sin\left(\omega t - 240°\right) \angle 240°$$

The magnetic flux densities resulting from these magnetic field intensities can be found from

$$B = \mu H$$

The rotating magnetic field

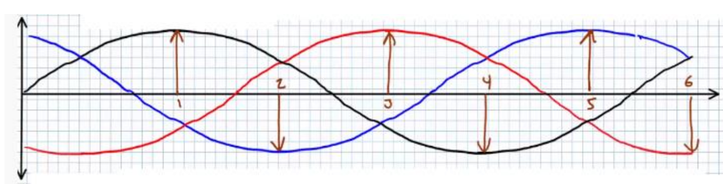
$$B_{aa'}(t) = \mu H_M \sin \omega t \angle 0°$$
$$B_{bb'}(t) = \mu H_M \sin\left(\omega t - 120°\right) \angle 120°$$
$$B_{cc'}(t) = \mu H_M \sin\left(\omega t - 240°\right) \angle 240°$$

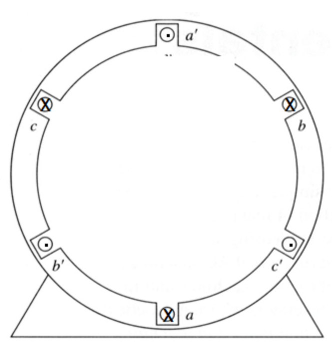
At the time $t = 0$ ($\omega t = 0$) :

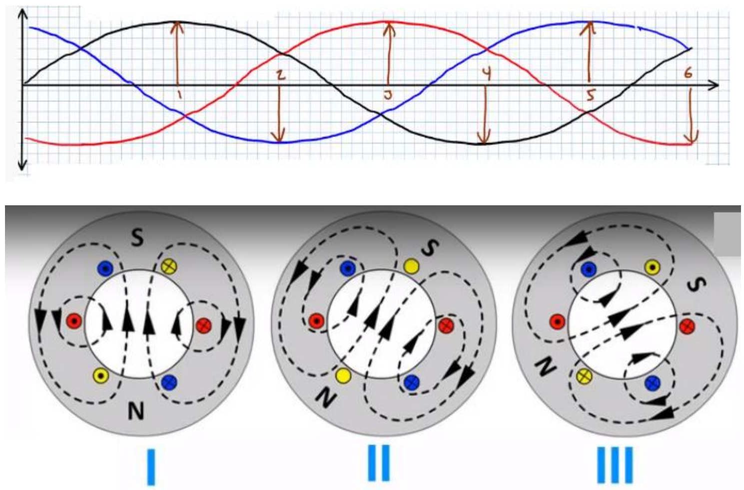
$$B_{aa'}(t) = 0$$
$$B_{bb'}(t) = \mu H_M \sin\left(-120°\right) \angle 120°$$
$$B_{cc'}(t) = \mu H_M \sin\left(-240°\right) \angle 240°$$

The total magnetic field from all three coils added together will be

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'} = 0 + \left(-\frac{\sqrt{3}}{2}\mu H_M\right)\angle 120° + \left(\frac{\sqrt{3}}{2}\mu H_M\right)\angle 240° = 1.5\mu H_M \angle -90°$$





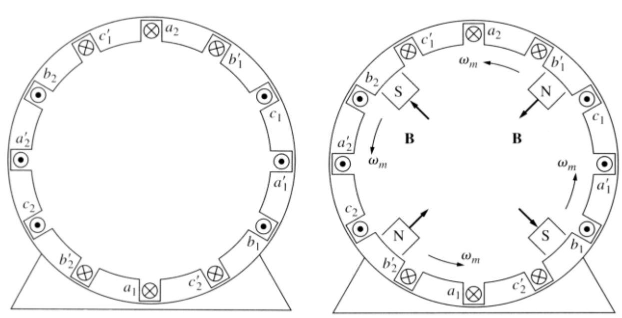


https://www.youtube.com/watch?v=8XF-11MQGQ0

Relationship between electrical frequency and speed of field rotation

The relationship between the electrical angle θ_e (current's phase change) and the mechanical angle θ_m (at which the magnetic field rotates) in this situation is:

$$\theta_e = 2\theta_m$$



Therefore, for a four-pole stator:

$$\left.\begin{aligned} f_e\,[Hz] &= 2\,f_m\,[rps] \\ \omega_e[rad/s] &= 2\omega_m\,[rad/s] \end{aligned}\right\}\ four\ \ poles$$

Relationship between electrical frequency and speed of field rotation

For an AC machine with *P* poles in its stator:

$$\theta_e = \frac{P}{2}\theta_m$$

$$f_e = \frac{P}{2} f_m$$

$$\omega_e = \frac{P}{2}\omega_m$$

Relating the electrical frequency to the motors speed in *rpm*:

$$f_e = \frac{P}{120} n_m$$

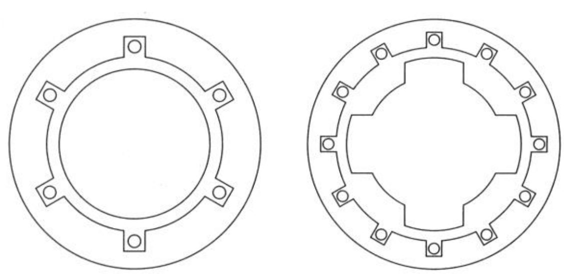
AC Machinery Fundamentals
Ch. 3 / Fifth Edition

Dr. Feras Alasali

3.3 Magnetomotive force and flux distribution on an AC machine

- In the previous discussion, we assumed that the flux produced by a stator inside an AC machine. However, in real machines, there is a ferromagnetic rotor in the center with a small gap between a rotor and a stator.

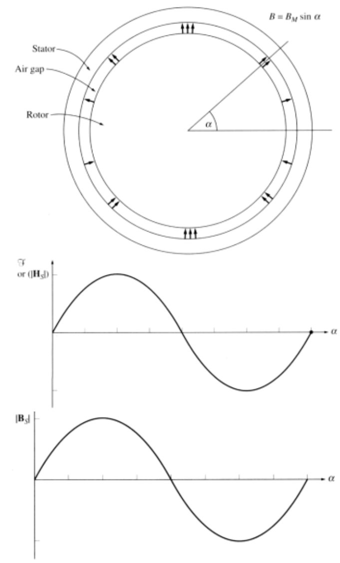
A rotor can be cylindrical (such machines are said to have non-salient poles), or it may have pole faces projecting out from it (salient poles). We will restrict our discussion to non-salient pole machines only (cylindrical rotors).



Magnetomotive force and flux distribution on an AC machine

The reluctance of the air gap is much higher than the reluctance of either the rotor or the stator; therefore, the flux density vector *B* takes the shortest path across the air gap: it will be perpendicular to both surfaces of rotor and stator.

To produce a sinusoidal voltage in this machine, the magnitude of the flux density vector *B* must vary sinusoidally along the surface of the air gap. Therefore, the magnetic field intensity (and the mmf) will vary sinusoidally along the air gap surface.



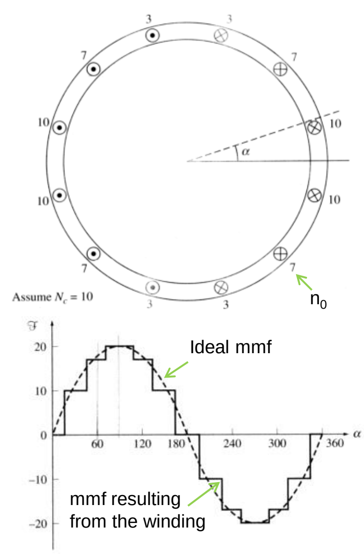
Magnetomotive force and flux distribution on an AC machine

One obvious way to achieve a sinusoidal variation of mmf along the air gap surface **would be to distribute the turns of the winding that produces the mmf in closely spaced slots along the air gap surface and vary the number of conductors in each slot sinusoidally**, according to:

$$n_c = N_c \cos \alpha$$

where N_c is the number of conductors at the angle of 0^0 and α is the angle along the surface.

However, in practice, only a finite number of slots and integer numbers of conductors are possible. As a result, real mmf will approximate the ideal mmf if this approach is taken.



Induced voltage in AC machines

Just as a 3-phase set of currents in a stator can produce a rotating magnetic field, a rotating magnetic field can produce a ***3-phase set of voltages in the coils a AC machine***.

The induced voltage in a single coil on a two-pole stator

Assume that a rotor with a sinusoidally distributed magnetic field rotates in the center of a stationary coil.

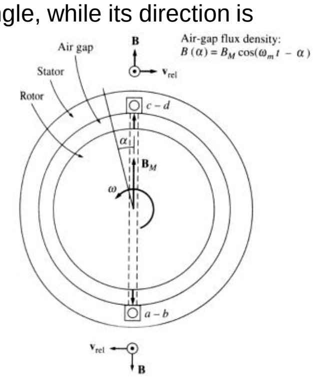
We further assume that the magnitude of the flux density B in the air gap between the rotor and the stator varies sinusoidally with mechanical angle, while its direction is always radially outward.

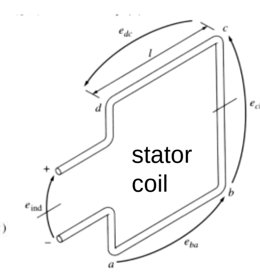
Note, that this is an ideal flux distribution.

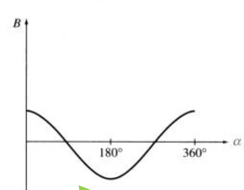
The magnitude of the flux density vector at a point around the rotor is

$$B = B_M \cos\alpha$$

Where α is the angle from the direction of peak flux intensity.







Flux density in a gap

The induced voltage in a single coil on a two-pole stator

Since the rotor is rotating within the stator at an angular velocity ω_m, the magnitude of the flux density vector at any angle α around the stator is

$$B = B_M \cos(\omega t - \alpha)$$

The voltage induced in a wire is

$$e_{ind} = (v \times B) \cdot l$$

Here *v* is the velocity of the wire relative to the magnetic field
B is the magnetic flux density vector
l is the length of conductor in the magnetic field

However, this equation was derived for a moving wire in a stationary magnetic field. In our situation, the wire is stationary and the magnetic field rotates. Therefore, the equation needs to be modified: we need to change reference such way that the field appears as stationary.

The induced voltage in a single coil on a two-pole stator

The total voltage induced in the coil is a sum of the voltages induced in each of its four sides. These voltages are:

1. Segment *ab*: α = 180^0; assuming that *B* is radially outward from the rotor, the angle between *v* and *B* is 90^0, so

$$e_{ba} = (v \times B) \cdot I = -vB_M l \cos(\omega_m t - 180°)$$

2. Segment *bc*: the voltage will be zero since the vectors (*v* x *B*) and *l* are perpendicular.

$$e_{cb} = (v \times B) \cdot I = 0$$

3. Segment *cd*: α = 0^0; assuming that *B* is radially outward from the rotor, the angle between *v* and *B* is 90^0, so

$$e_{dc} = (v \times B) \cdot I = vB_M l \cos(\omega_m t)$$

4. Segment *da*: the voltage will be zero since the vectors (*v* x *B*) and *l* are perpendicular.

$$e_{ad} = (v \times B) \cdot I = 0$$

The induced voltage in a single coil on a two-pole stator

Therefore, the total voltage on the coil is:

$$e_{ind} = e_{ba} + e_{dc} = -vB_M l\cos(\omega_m t - 180°) + vB_M l\cos\omega_m t$$
$$= \{\cos\theta = -\cos(\theta)\} = 2vB_M l\cos\omega_m t$$

Since the velocity of the end conductor is $v = r\omega_m$

Then: $e_{ind} = 2rlB_M\omega_m\cos\omega_m t$

The flux passing through a coil is $\phi = 2rlB_M$

Therefore: $e_{ind} = \phi\omega_m\cos\omega_m t$

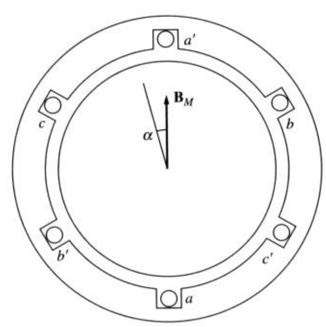
Finally, if the stator coil has N_C turns of wire, the total induced voltage in the coil:

$$e_{ind} = N_C\phi\omega_m\cos\omega_m t$$

The induced voltage in a 3-phase set of coils

In three coils, each of N_C turns, placed around the rotor magnetic field, the induced in each coil will have the same magnitude and phases differing by 120^0:

$$e_{aa'}(t) = N_C\phi\omega_m\sin\omega_m t$$
$$e_{bb'}(t) = N_C\phi\omega_m\sin(\omega_m t - 120°)$$
$$e_{cc'}(t) = N_C\phi\omega_m\sin(\omega_m t - 240°)$$



A 3-phase set of currents can generate a uniform rotating magnetic field in a machine stator, and a uniform rotating magnetic field can generate a 3-phase set of voltages in such stator.

The rms voltage in a 3-phase stator

The peak voltage in any phase of a 3-phase stator is:

$$E_{\max} = N_C \phi \omega_m$$

For a 2-pole stator: $\omega_m = \omega_e = \omega = 2\pi f$

Thus: $E_{\max} = 2\pi N_C \phi f$

The rms voltage in any phase of a 2-pole 3-phase stator is:

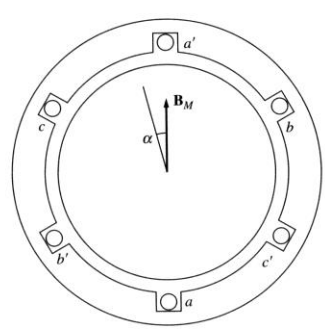
$$E_A = \frac{2\pi}{\sqrt{2}} N_C \phi f = \sqrt{2}\pi N_C \phi f$$

The rms voltage at the terminals will depend on the type of stator connection: if the stator is Y-connected, the terminal voltage will be $\sqrt{3}E_A$. For the delta connection, it will be just E_A.

Induced voltage: Example

Example 6.1: The peak flux density of the rotor magnetic field in a simple 2-pole 3-phase generator is 0.2 T; the mechanical speed of rotation is 3600 rpm; the stator diameter is 0.5 m; the length of its coil is 0.3 m and each coil consists of 15 turns of wire. The machine is Y-connected.

a) What are the 3-phase voltages of the generator as a function of time?
b) What is the rms phase voltage of the generator?
c) What is the rms terminal voltage of the generator?



The flux in this machine is given by

$$\phi = 2rlB = dlB = 0.5 \cdot 0.3 \cdot 0.2 = 0.03\,Wb$$

The rotor speed is

$$\omega = \frac{3600 \cdot 2\pi}{60} = 377\,\frac{rad}{s}$$

Induced voltage: Example

a) The magnitude of the peak phase voltage is

$$E_{\max} = N_C \phi \omega = 15 \cdot 0.03 \cdot 377 = 169.7\ V$$

and the three phase voltages are:

$$e_{aa'}(t) = 169.7 \sin(377t)$$
$$e_{bb'}(t) = 169.7 \sin(377t - 120°)$$
$$e_{cc'}(t) = 169.7 \sin(377t - 240°)$$

b) The rms voltage of the generator is

$$E_A = \frac{E_{\max}}{\sqrt{2}} = \frac{169.7}{\sqrt{2}} = 120\ V$$

c) For a Y-connected generator, its terminal voltage is

$$V_T = \sqrt{3} \cdot 120 = 208\ V$$

Induced torque in an AC machine

In an AC machine under normal operating conditions two magnetic fields are present: a field from the rotor and a field from the stator circuits. The interaction of these magnetic fields produces the torque in the machine.

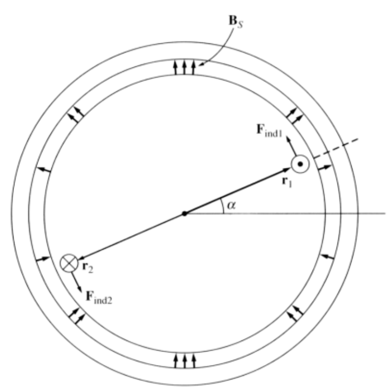
Assuming a sinusoidal stator flux distribution peaking in the upward direction

(where B_S is the magnitude of the peak flux density) and a single coil of wire mounted on the rotor, the induced force on the first conductor (on the right)

$$F = i(\mathbf{l} \times \mathbf{B}) = ilB_S \sin\alpha$$

The torque on this conductor is (counter-clockwise)

$$\tau_{ind,1} = \mathbf{r} \times \mathbf{F} = rilB_S \sin\alpha$$



$B_S(\alpha) = B_S \sin\alpha$

Induced torque in an AC machine

The induced force on the second conductor (on the left) is

$$F = i(\mathbf{l} \times \mathbf{B}) = ilB_S \sin\alpha$$

The torque on this conductor is (counter-clockwise)

$$\tau_{ind,2} = \mathbf{r} \times \mathbf{F} = rilB_S \sin\alpha$$

Therefore, the torque on the rotor loop is

$$\tau_{ind} = 2rilB_S \sin\alpha$$

Winding insulation in AC machines

Winding insulation is of critical importance. If insulation of a motor or generator breaks down, the machine shorts out and the repair is expensive and sometimes even impossible. Most insulation failures are due to overheating.

To limit windings temperature, the maximum power that can be supplied by the machine must be limited in addition to the proper ventilation.

The life expectancy of a motor with a given type of insulation is halved for each 10^0C rise above the rated winding temperature.

AC machine power flows and losses

The efficiency of an AC machine is defined as

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\%$$

Since the difference between the input and output powers of a machine is due to the losses occurring inside it, the efficiency is

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \cdot 100\%$$

AC machine power losses

Losses occurring in an AC machine can be divided into four categories:

1. Electrical or Copper losses

These losses are resistive heating losses that occur in the stator (armature) winding and in the rotor (field) winding of the machine. For a 3-phase machine, the stator copper losses and synchronous rotor copper losses are:

$$P_{SCL} = 3I_A^2 R_A$$
$$P_{RCL} = 3I_F^2 R_F$$

Where I_A and I_F are currents flowing in each armature phase and in the field winding respectively. R_A and R_F are resistances of each armature phase and of the field winding respectively. These resistances are usually measured at normal operating temperature.

AC machine power losses

2 -Core losses are the hysteresis losses and eddy current losses. They vary as B^2 (flux density) and as $n^{1.5}$ (speed of rotation of the magnetic field).

3 -mechanical losses: friction (friction of the bearings) and windage (friction between the moving parts of the machine and the air inside the casing). These losses are often lumped together and called the no-load rotational loss of the machine. They vary as the cube of rotation speed n^3.

4- Stray losses that cannot be classified in any of the previous categories. They are usually due to inaccuracies in modeling. For many machines, stray losses are assumed as 1% of full load.

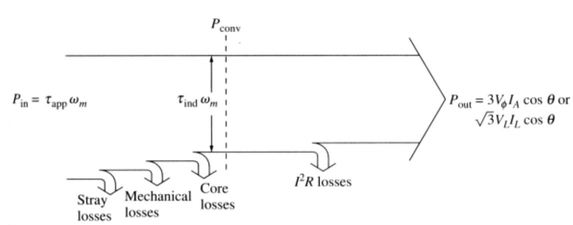
The power-flow diagram

On of the most convenient technique to account for power losses in a machine is the power-flow diagram.

AC generator:

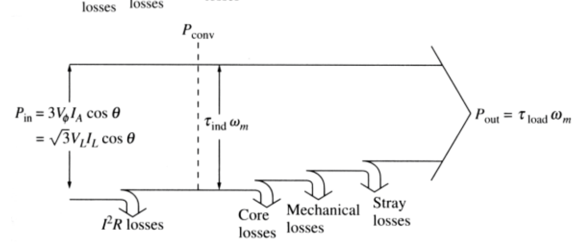
The mechanical power is input, and then all losses but cupper are subtracted. The remaining power P_{conv} is ideally converted to electricity:

$$P_{conv} = \tau_{ind}\omega_m$$



AC motor:

Power-flow diagram is simply reversed.



Voltage regulation

Voltage regulation (VR) is a commonly used figure of merit for generators:

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \cdot 100\%$$

Here V_{nl} and V_{fl} are the no-load full-load terminal voltages of the generator. VR is a rough measure of the generator's voltage-current characteristic. A small VR (desirable) implies that the generator's output voltage is more constant for various loads.

Speed regulation

Speed regulation (SR) is a commonly used figure of merit for motors:

$$SR = \frac{n_{nl} - n_{fl}}{n_{fl}} \cdot 100\%$$

$$SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \cdot 100\%$$

Here n_{nl} and n_{fl} are the no-load full-load speeds of the motor. SR is a rough measure of the motor's torque-speed characteristic. A positive SR implies that a motor's speed drops with increasing load. The magnitude of SR reflects a steepness of the motor's speed-torque curve.

Suggestion problems from our Book

1. The simple loop is rotating in a uniform magnetic field shown in Figure has the following characteristics:

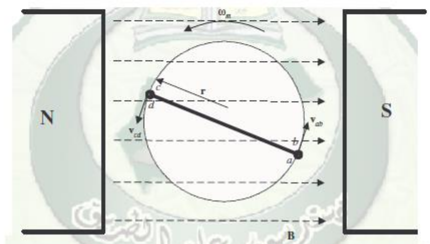
   B = 0.5 T to the right $r = 0.1$ m

   $l = 0.5$ m $\omega = 103$ rad/s

   (a) Calculate the voltage $e_{tot}(t)$ induced in this rotating loop.

   (b) Suppose that a 5 Ω resistor is connected as a load across the terminals of the loop. Calculate the current that would flow through the resistor.

   (c) Calculate the magnitude and direction of the induced torque on the loop for the conditions in *(b)*.



3. A three-phase four-pole winding is installed in 12 slots on a stator. There are 40 turns of wire in each slot of the windings. All coils in each phase are connected in series, and the three phases are connected in Δ. The flux per pole in the machine is 0.060 Wb, and the speed of rotation of the magnetic field is 1800 r/min.

   (a) What is the frequency of the voltage produced in this winding?

   (b) What are the resulting phase and terminal voltages of this stator?

4. A three-phase Y-connected 50-Hz two-pole synchronous machine has a stator with 2000 turns of wire per phase. What rotor flux would be required to produce a terminal (line-to-line) voltage of 6 kV?

Induction Motors Ch. 6 / Fifth Edition

Dr. Feras Alasali

Introduction

- Conversion of electrical power into mechanical power takes place in the rotating part of an electrical motor.
- Machines are called induction machines because the rotor voltage (which produces the rotor current and the rotor magnetic field) is *induced* in the rotor windings rather than being physically connected by wires.
- In AC motors, the rotor does not receive electrical power but conduction by induction in the same way as the secondary of 2-winding transformer receives its power from the primary winding.

Advantages of 3 phase induction motor

- Generally easy to build and cheaper than corresponding dc or synchronous motors
- Induction motor is robust
- The motor is driven by the rotational magnetic field produced by 3 phase currents, hence no commutator or blush is required
- Maintenance is relatively easy and at low cost
- Satisfactory efficiency and reasonable power factor
- A manageable torque-speed curve
- Stable operation under load
- Range in size from few Watts to several MW

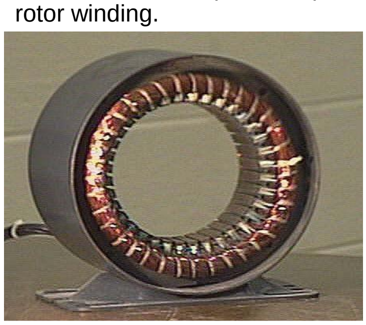
Disadvantages of 3 phase induction motor

- Induction motor has low inherent starting torque
- Draw large starting currents, typically 6-8 x their full load values
- Speeds not easily controlled as DC motors
- Operate with a poor lagging power factor when lightly loaded

6.1 Induction machine construction

An induction motor has two main parts:

- A stator – consisting of a steel frame that supports a hollow, cylindrical core of stacked laminations. Slots on the internal circumference of the stator house the stator winding.
- A rotor – also composed of punched laminations, with rotor slots for the rotor winding.



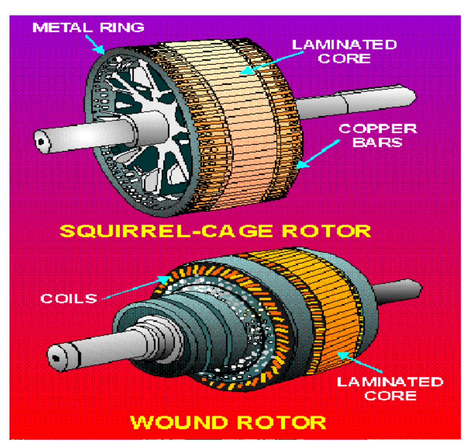
Stator

- The stator of induction motor is made up of a number of stampings, which are slotted to receive the windings.
- The stator carries 3-phase winding and is fed from a 3-phase supply.
- It is wound for definite number of poles, the exact number of poles being determined by the requirements of speed.
- When the stator winding supplied with 3-phase current, produce magnetic flux, which is of constant magnitude but which revolves (or rotates) at synchronous speed. This revolving magnetic flux induces an emf in the rotor by mutual induction.
- Spreading the coil in this manner creates a sinusoidal flux distribution per pole, which improves performance and makes the motor less noisy (sound and electrically).

Rotors

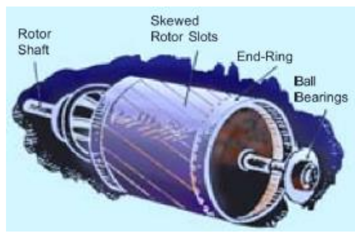
- There are two different types of induction motor rotors which can be placed inside the stator. One is called a *cage rotor,* while the other is called a *wound rotor.*

- Squirrel-cage windings, which produce a *squirrel-cage induction motor* (most common). *Almost 90% of the three-phase AC Induction motors are of this type.*
- Conventional 3-phase windings made of insulated wire, which produce a *wound-rotor induction motor .*



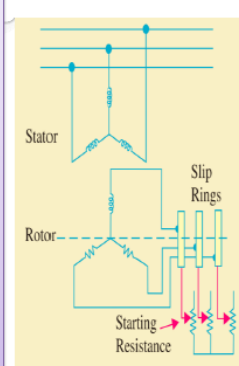
Squirrel-cage Rotor

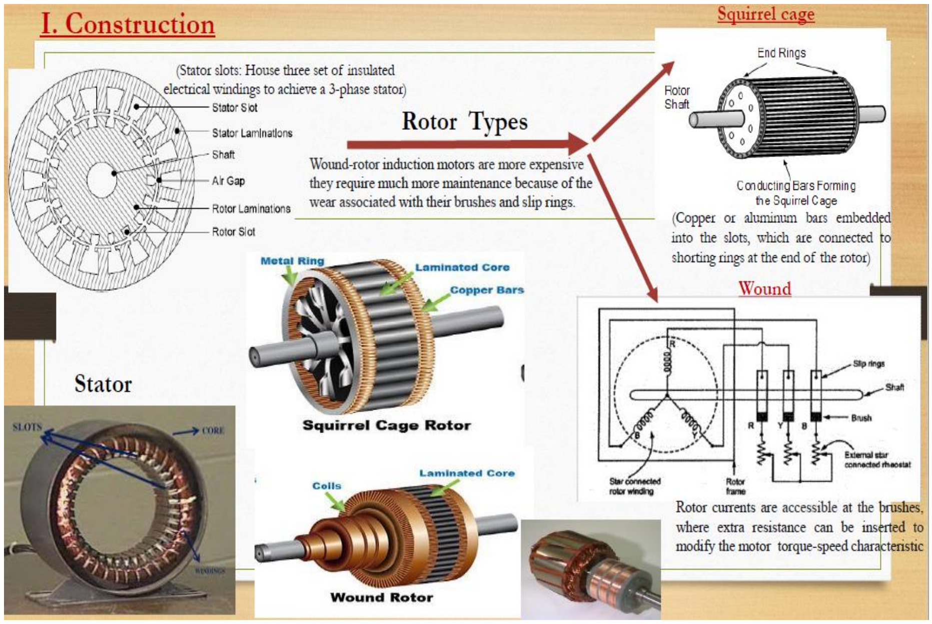
- Most of the induction motors are squirrel cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible.
- The rotor consists of cylindrical laminated core with parallel slots for carrying the rotor conductors which are not wires but consist of heavy bars of copper, aluminums or alloys. One bar is placed in each slot. The rotor bars are brazed or electrically welded or bolted to two heavy and stout short circuiting end ring.



Phase-wound Rotor

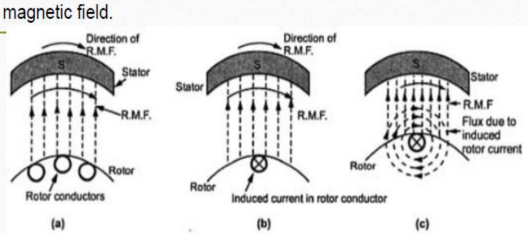
- A wound rotor has a complete set of three-phase windings that are similar to the windings on the stator. The three phases of the rotor windings are usually Y-connected, and the ends of the three rotor wires are tied to slip rings on the rotor's shaft. The rotor windings are shorted through brushes riding on the slip rings. These three brushes are further connected externally to 3-phase star connected rheostat. This make possible the introduction of additional resistance in the rotor circuit during starting period for increasing the starting torque of the motor.
- Wound-rotor induction motors are more expensive than cage induction motors, and they require much more maintenance because of the wear associated with their brushes and slip rings. As a result, wound-rotor induction motors are rarely used.

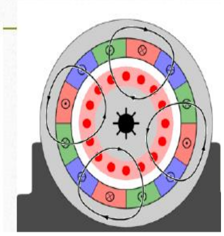




6.1 Basic induction motor concepts

(a) When the motor is excited with three-phase supply, three-phase stator winding produce a rotating magnetic field at synchronous speed.
(b) The stator's magnetic field is therefore changing or rotating relative to the rotor. Hence, according to the principle of Faraday's laws of electromagnetic induction, a voltage is induced at the rotor. Thus, when the rotor is short-circuited or closed through an external impedance, a current is induced in the induction motor's rotor.
(c) Finally, a flux will be produced in rotor due to induced rotor current forcing the rotor to rotate in the same direction of the stator rotating magnetic field.

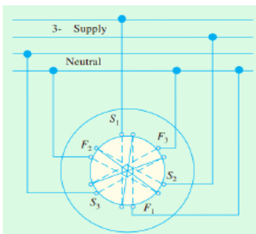




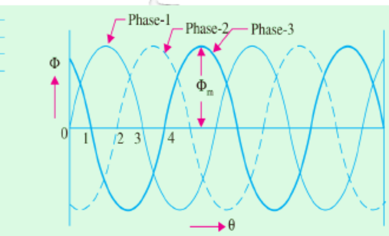
Production of Rotating Field

➢ **When stationary coils wound for three phase are supplied by three phase supply, a uniformly rotating (or revolving) magnetic flux of constant value is produced.**

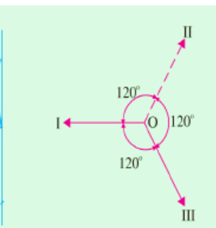
➢ **When three phase winding displaced in space by 120°, are fed by three phase current displaced in time by 120°, they produce a resultant magnetic flux which rotates in space as if actual magnetic poles were being rotated mechanically.**



3-phase, 2-poles stator having three identical windings places 120° space degree



The flux due to three phase windings



Positive direction of fluxes

Why Does the Rotor Rotate

- When 3-phase stator windings are fed by 3-phase supply, a magnetic flux of constant magnitude, but rotating at synchronous speed, is set up.
- The flux passes through the air gap, sweeps past the rotor surface and so cuts the rotor conductors which, as yet, are stationary.
- Due to relative speed between the rotating flux and the stationary conductors an e.m.f. is induced in the conductors According to faraday's law.
- The frequency of the induced e.m.f. is the same as the supply frequency.
- The e.m.f. magnitude is proportional to the relative velocity between the flux and the conductors, and its direction is given by Fleming' right hand rule.
- Since the rotor bars or conductors form a closed circuit, rotor current is produced whose direction, as given by Lenz's law, is such as to oppose the very cause producing it.
- The cause which produces the rotor current is the relative velocity between the rotating flux of the stator and the stationary rotor conductors.
- To reduce the relative speed, the rotor starts running in the same direction as that of the flux and tries to catch up with the rotating flux.

Relationship between electrical frequency and speed of field rotation

For an AC machine with *P* poles in its stator:

$$\theta_e = \frac{P}{2}\theta_m$$

$$f_e = \frac{P}{2} f_m$$

$$\omega_e = \frac{P}{2}\omega_m$$

Relating the electrical frequency to the motors speed in *rpm*:

$$f_e = \frac{P}{120} n_m$$

The speed of the magnetic field's rotation is given by

$$n_{sync} = \frac{120 f_e}{P}$$

where f_e is the system frequency in hertz and P is the number of poles in the machine. This rotating magnetic field $\mathbf{B}_S$ passes over the rotor bars and induces a voltage in them.

The voltage induced in a given rotor bar is given by the equation

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$

where $\mathbf{v}$ = velocity of the bar *relative to the magnetic field*
$\mathbf{B}$ = magnetic flux density vector
$\mathbf{l}$ = length of conductor in the magnetic field

The Concept of Rotor Slip

- The voltage induced in a rotor bar of an induction motor depends on the speed of the rotor *relative to the magnetic fields.* Since the behavior of an induction motor depends on the rotor's voltage and current, it is often more logical to talk about this relative speed. Two terms are commonly used to de fine the relative motion of the rotor and the magnetic field s. One is *slip speed,* de fined as the difference between synchronous speed and rotor speed:

$$n_{sync} = \frac{120 f}{P}$$

- So, the induction motor will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *slip speed*

$$n_{slip} = n_{sync} - n_m$$

Where n_{slip} = slip speed
n_{sync} = speed of the magnetic field
n_m = mechanical shaft speed of the motor

The Slip

$$s = \frac{n_{sync} - n_m}{n_{sync}}$$

$$s = \frac{n_{slip}}{n_{sync}} (\times 100\%)$$

Where *s* is the *slip*

Notice that : if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above by 100. Notice that the slip is a ratio and doesn't have units.

This equation can also be expressed in terms of angular velocity ω (radians per second) as

$$s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} (\times 100\%)$$

$$n_m = (1 - s)n_{sync}$$

$$\omega_m = (1 - s)\omega_{sync}$$

These equations are useful in the derivation of induction motor torque and power relationships.

The Electrical Frequency on the Rotor

- An induction motor works by inducing voltages and currents in the rotor of the machine, and for that reason it has sometimes been called a *rotating transformer.* Like a transformer, the primary (stator) induces a voltage in the secondary (rotor), but *unlike* a transfomer, the secondary frequency is not necessarily the same as the primary frequency.

If the rotor of a motor is locked so that it cannot move, then the rotor will have the same frequency as the stator. On the other hand, if the rotor turns at synchronous speed, the frequency on the rotor will be zero.

At $n_m = 0$ r/min, the rotor frequency $f_{re} = f_{se}$, and the slip $s = 1$. At $n_m = n_{sync}$, the rotor frequency $f_{re} = 0$ Hz, and the slip $s = 0$. For any speed in between, the rotor frequency is directly proportional to the *difference* between the speed of the magnetic field n_{sync} and the speed of the rotor n_m.

the rotor frequency can be expressed as

$$f_{re} = s f_{se}$$

where, f_{re} and f_{se} are the rotor and stator electrical frequencies respectively

Since, $s = \frac{n_{sync} - n_m}{n_{sync}}$ and $n_{sync} = 120 f_{se} / P$ Hence, $f_{re} = \frac{P}{120}(n_{sync} - n_m)$

Example

A 6-pole induction motor is excited by a 3-phase, 60 Hz source. If the full-load speed is 1140 r/min, calculate the slip.

Solution:

The synchronous speed of the motor is:

$$n_s = 120\,f/p = 120 \times 60/6$$
$$= 1200 \text{ r/min}$$

The difference between the synchronous speed and rotor speed is the slip speed:

$$n_s - n = 1200 - 1140 = 60 \text{ r/min}$$
$$s = (n_s - n)/n_s = 60/1200$$
$$= 0.05 \text{ or } 5\%$$

Example

A 208-V, 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent.

(a) What is the synchronous speed of this motor?

(b) What is the rotor speed of this motor at the rated load?

(c) What is the rotor frequency of this motor at the rated load?

(d) What is the shaft torque of this motor at the rated load?

Solution

(a) The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_e}{P}$$
$$= \frac{120(60 \text{ Hz})}{4 \text{ poles}} = 1800 \text{ r/min}$$

(b) The rotor speed of the motor is given by

$$n_m = (1 - s)n_{sync}$$
$$= (1 - 0.95)(1800 \text{ r/min}) = 1710 \text{ r/min}$$

0.05

(c) The rotor frequency of this motor is given by

$$f_r = sf_e = (0.05)(60 \text{ Hz}) = 3 \text{ Hz}$$

Alternatively, the frequency can be found from Equation :

$$f_r = \frac{P}{120}(n_{sync} - n_m)$$
$$= \frac{4}{120}(1800 \text{ r/min} - 1710 \text{ r/min}) = 3 \text{ Hz}$$

(d) The shaft load torque is given by

$$\tau_{load} = \frac{P_{out}}{\omega_m}$$
$$= \frac{(10 \text{ hp})(746 \text{ W/hp})}{(1710 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min}/60 \text{ s})} = 41.7 \text{ N} \cdot \text{m}$$

The shaft load torque in English units is given by

$$\tau_{load} = \frac{5252P}{n}$$

where τ is in pound-feet, P is in horsepower, and n_m is in revolutions per minute. Therefore,

$$\tau_{load} = \frac{5252(10 \text{ hp})}{1710 \text{ r/min}} = 30.7 \text{ lb} \cdot \text{ft}$$

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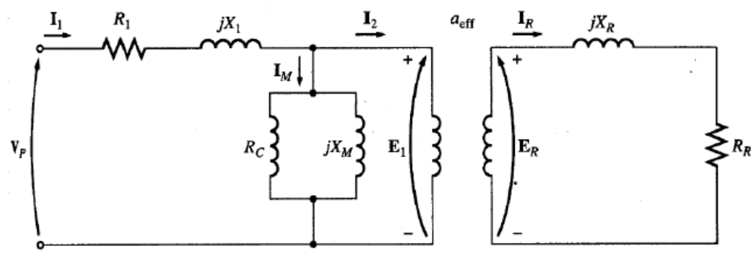
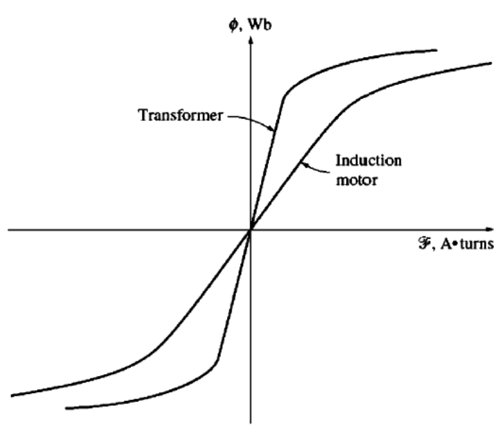
6.3 The equivalent circuit of an induction motor

- An induction motor relies for its operation on the induction of voltages and currents in its rotor circuit from the stator circuit (transformer action). Hence, the equivalent circuit of an induction motor will out to be very similar to the equivalent circuit of a transformer.
- An induction motor is called a *singly excited* machine since power is supplied to only the stator circuit. Because an induction motor does not have an independent field circuit, its mode l will not contain an internal voltage source such as the internal generated voltage E in a synchronous machine

The Transformer Model of an Induction Motor

The transformer model or an induction motor. with rotor and stator connected by an ideal transformer of turns ratio





The magnetization curve of an induction motor compared to that of a transformer.

The Rotor Circuit Model

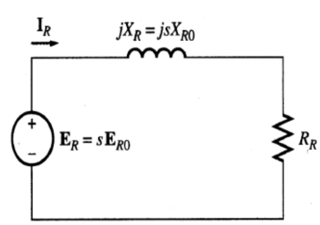
- In an induction motor, when the voltage is applied to the stator windings, a voltage is induced in the rotor windings of the machine.
- In general, *the greater the relative motion between the rotor and the stator magnetic fields, the greater the resulting rotor voltage and rotor frequency.*
- The largest relative motion occurs when the rotor is stationary, called the *locked-rotor* or *blocked-rotor* condition, so the largest voltage and rotor frequency arc induced in the rotor at that condition.
- The smallest voltage (0 V) and frequency (0 Hz) occur when the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion.
- The magnitude and frequency of the voltage induced in the rotor at any speed between these extremes is *directly proportional to the slip of the rotor,* Therefore, if the magnitude of the induced rotor voltage at locked-rotor conditions is called *ERO.*

The magnitude of the induced voltage at any slip will be given by the equation

$$E_R = sE_{R0}$$

$$f_r = sf_e$$

- Then, we can draw the rotor equivalent circuit as follows



$$X_R = \omega_r L_R = 2\pi f_r L_R$$

$$f_r = sf_e$$

$$X_R = 2\pi s f_e L_R = s(2\pi f_e L_R) = sX_{R0}$$

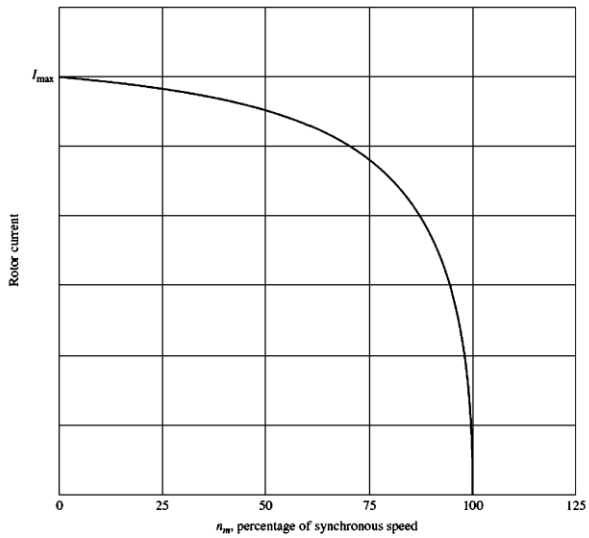
$$I_R = \frac{E_R}{R_R + jX_R}$$

where X_{R0} is the blocked-rotor rotor reactance.

$$\boxed{I_R = \frac{E_R}{R_R + jsX_{R0}}}$$

$$\boxed{I_R = \frac{E_{R0}}{R_R/s + jX_{R0}}}$$

- Notice that it is possible to treat all of the rotor effects due to varying rotor speed as being caused by a *varying impedance* supplied with power from a constant-voltage source ERO. The equivalent rotor impedance from this point of view is



$$Z_{R,eq} = R_R/s + jX_{R0}$$

The Final Equivalent Circuit

- To produce the final per-phase equivalent circuit for an induction motor, it is necessary to refer the rotor part of the model over to the stator side. The rotor circuit model that will be referred to the stator side is the model shown in

$$\mathbf{V}_P = \mathbf{V}'_S = a\mathbf{V}_S$$

$$\mathbf{E}_1 = \mathbf{E}'_R = a_{eff}\mathbf{E}_{R0}$$

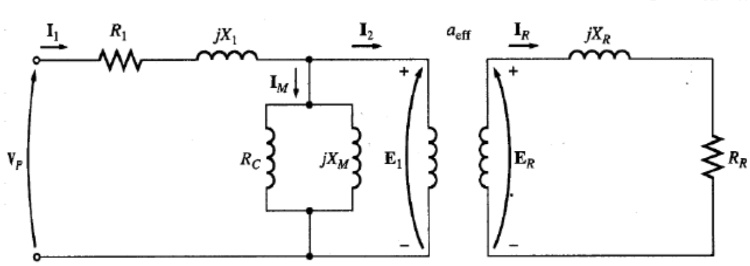
$$\mathbf{I}_P = \mathbf{I}'_S = \frac{\mathbf{I}_S}{a}$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_R}{a_{eff}}$$

$$Z'_S = a^2 Z_S$$

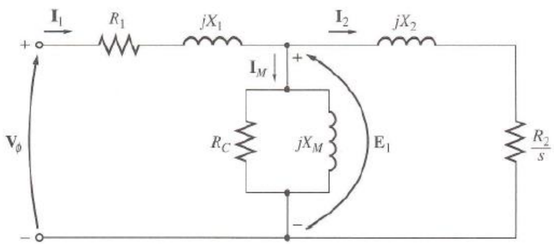
$$Z_2 = a_{eff}^2\left(\frac{R_R}{s} + jX_{R0}\right)$$

$$R_2 = a_{eff}^2 R_R \qquad X_2 = a_{eff}^2 X_{R0}$$

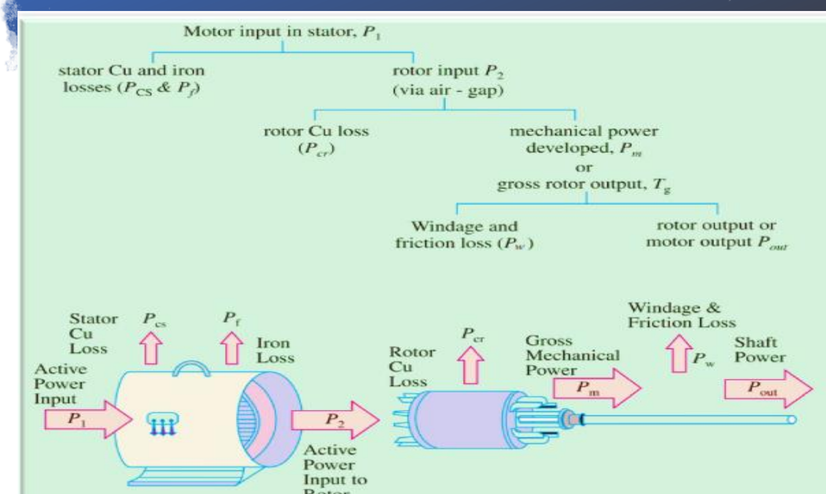


Per-phase equivalent circuit

- Motor Slip $$s = \frac{n_s - n_m}{n_s}$$
- R_1 and R_2: stator and rotor winding resistances
- X_1 and X_2: stator and rotor winding leakage reactances
- X_m: magnetizing reactance
- R_c: core loss resistance
- Rotor winding parameters are referred to the stator side



6.4 Power and torque in induction motors

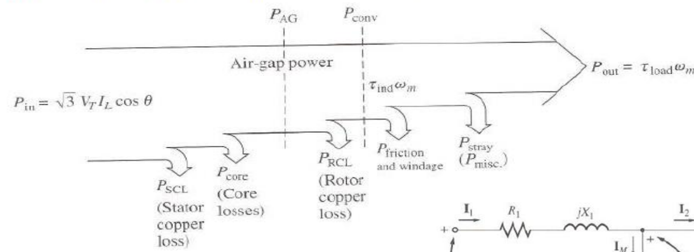


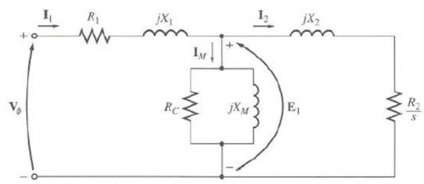
$$P_{SCL} = 3I_1^2 R_1$$
$$p_{core} = 3E_1^2 / R_C$$
$$P_{AG} = 3I_2^2 (R_2 / S)$$
$$P_{RCL} = 3I_2^2 R_2$$
$$P_{conv} = P_{AG} - P_{RCL} = 3I_2^2 [R_2(1-S)/S]$$

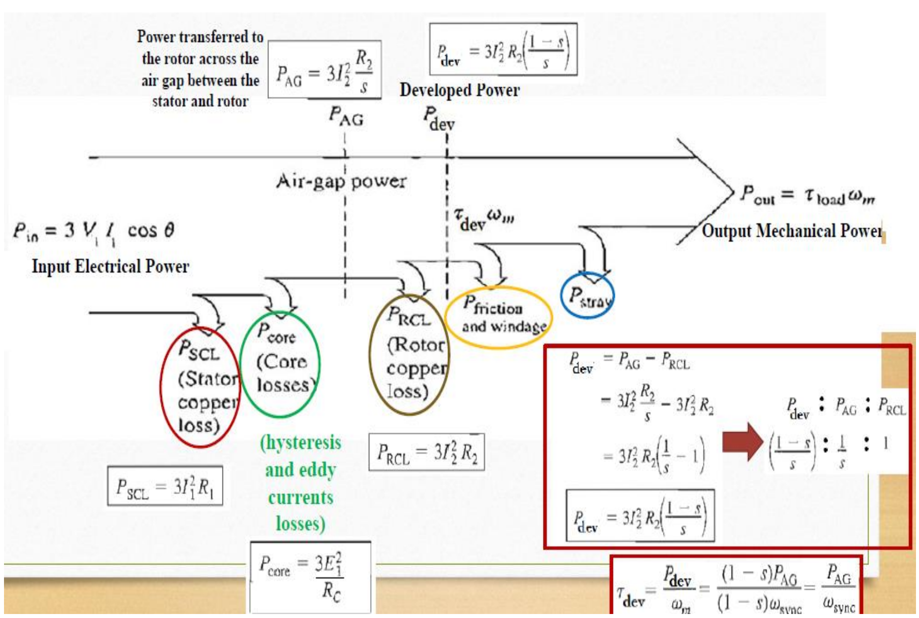
$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{eq}}$$

where

$$Z_{eq} = R_1 + jX_1 + \frac{1}{G_C - jB_M + \frac{1}{V_2/s + jX_2}}$$







- **The supply power is:**

$$Pin = \sqrt{3}V_L I_L \cos\theta$$

- **The motor efficiency:**

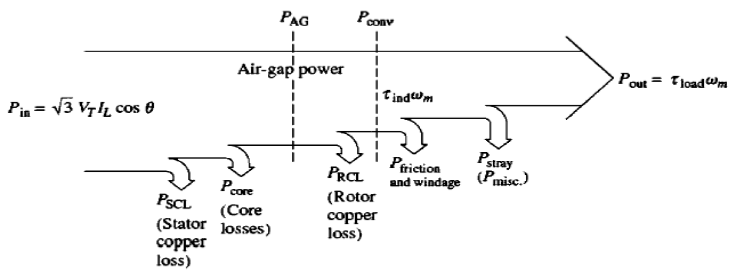
$$\eta = \frac{P_{out}}{P_{in}}$$

- **Motor torque:**

$$T_{out} = \frac{P_{out}}{\omega} \qquad T_{conv} = \frac{P_{Conv}}{\omega}$$

- **Rotor cu losses, gap power and developed power:**

$$P_{Cu2} = sP_{Gap} \qquad P_{conv} = P_{Gap} - P_{Cu2} = (1-s)P_{Gap}$$



The power-flow diagram of an induction motor.

Example

A 3-phase induction motor having a synchronous speed of 1200 r/min draws 80 kW from a 3-phase feeder. The copper losses and iron losses in the stator amount to 5 kW. If the motor runs at 1152 r/min, calculate:

a. the active power transmitted to the rotor;
b. the rotor I^2R losses;
c. the mechanical power developed;
d. the mechanical power delivered to the load, knowing that the windage and friction losses are equal to 2 kW;
e. the efficiency of the motor.

Solution:

a. Active power to the rotor is:

$P_r = P_e - P_{js} - P_f$

$= 80 - 5 = 75$ kW

b. 1 The slip is:

$s = (n_s - n)/n_s = (1200 - 1152)/1200$

$= 48/1200 = 0.04$

b. 2 Rotor I^2R losses are:

$P_{jr} = sP_r = 0.04 \times 75 = 3$ kW

c. The mechanical power developed is:

$P_m = P_r - I^2R$ losses in rotor

$= 75 - 3 = 72$ kW

d. The mechanical power P_L delivered to the load is slightly less than P_m owing to the friction and windage losses.

$P_L = P_m - P_v = 72 - 2 = 70$ kW

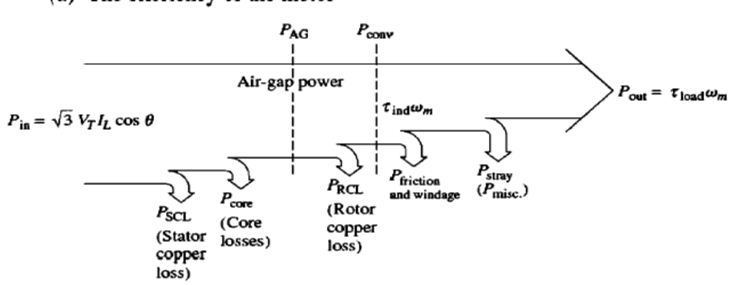
e. $\eta = P_L/P_e = 70/80$

$= 0.875$ or 87.5%

Example:

A 480-V, 60-Hz, 50-hp, three-phase induction motor is drawing 60 A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

(a) The air-gap power P_{AG}
(b) The power converted P_{conv}
(c) The output power P_{out}
(d) The efficiency of the motor



The power-flow diagram of an induction motor.

(a)

$$P_{in} = \sqrt{3}V_T I_L \cos\theta$$
$$= \sqrt{3}(480\text{ V})(60\text{ A})(0.85) = 42.4\text{ kW}$$

From the power-flow diagram, the air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$
$$= 42.4\text{ kW} - 2\text{ kW} - 1.8\text{ kW} = 38.6\text{ kW}$$

(b) From the power-flow diagram, the power converted from electrical to mechanical form is

$$P_{conv} = P_{AG} - P_{RCL}$$
$$= 38.6\text{ kW} - 700\text{ W} = 37.9\text{ kW}$$

(c) From the power-flow diagram, the output power is given by

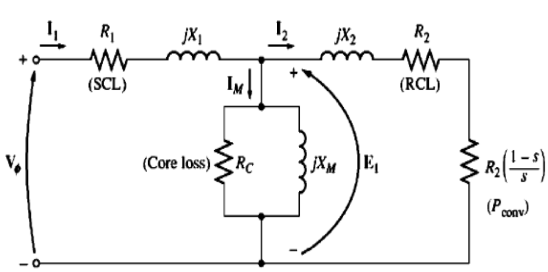
$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$
$$= 37.9\text{ kW} - 600\text{ W} - 0\text{ W} = 37.3\text{ kW}$$

(d) Therefore, the induction motor's efficiency is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$
$$= \frac{37.3\text{ kW}}{42.4\text{ kW}} \times 100\% = 88\%$$

Separating the Rotor Copper Losses and the Power Converted in an Induction Motor 's Equivalent Circuit

- Part of the power coming across the air gap in an induction motor is consumed in the rotor copper losses, and part of it is converted to mechanical power to drive the motor's shaft. It is possible to separate the two uses of the air-gap power and to indicate them separately on the motor equivalent circuit.



The per-phase equivalent circuit with rotor losses and P_{core} separated.

$$R_{conv} = \frac{R_2}{s} - R_2 = R_2\left(\frac{1}{s} - 1\right)$$

$$\boxed{R_{conv} = R_2\left(\frac{1-s}{s}\right)}$$

Example

A 460-V, 25-hp, 60-Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

(a) Speed
(b) Stator current
(c) Power factor
(d) P_{conv} and P_{out}
(e) τ_{ind} and τ_{load}
(f) Efficiency

(a) The synchronous speed is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

The rotor's mechanical shaft speed is

$$n_m = (1 - s)n_{sync}$$
$$= (1 - 0.022)(1800\text{ r/min}) = 1760\text{ r/min}$$

(b)

$$Z_2 = \frac{R_2}{s} + jX_2$$
$$= \frac{0.332}{0.022} + j0.464$$
$$= 15.09 + j0.464\ \Omega = 15.10\angle 1.76°\ \Omega$$

The combined magnetization plus rotor impedance is given by

$$Z_f = \frac{1}{1/jX_M + 1/Z_2}$$
$$= \frac{1}{-j0.038 + 0.0662\angle -1.76°}$$
$$= \frac{1}{0.0773\angle -31.1°} = 12.94\angle 31.1°\ \Omega$$

Therefore, the total impedance is

$$Z_{tot} = Z_{stat} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1°\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6°\ \Omega$$

The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{\text{tot}}}$$

$$= \frac{266\angle 0° \text{ V}}{14.07\angle 33.6° \ \Omega} = 18.88\angle -33.6° \text{ A}$$

(c) The power motor power factor is

$$\text{PF} = \cos 33.6° = 0.833 \qquad \text{lagging}$$

(d) The input power to this motor is

$$P_{\text{in}} = \sqrt{3} V_T I_L \cos\theta$$

$$= \sqrt{3}(460 \text{ V})(18.88 \text{ A})(0.833) = 12{,}530 \text{ W}$$

The stator copper losses in this machine are

$$P_{\text{SCL}} = 3I_1^2 R_1$$

$$= 3(18.88 \text{ A})^2(0.641\ \Omega) = 685 \text{ W}$$

The air-gap power is given by

$$P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} = 12{,}530 \text{ W} - 685 \text{ W} = 11{,}845 \text{ W}$$

$$P_{\text{conv}} = (1 - s)P_{\text{AG}} = (1 - 0.022)(11{,}845 \text{ W}) = 11{,}585 \text{ W}$$

The power P_{out} is given by

$$P_{\text{out}} = P_{\text{conv}} - P_{\text{rot}} = 11{,}585 \text{ W} - 1100 \text{ W} = 10{,}485 \text{ W}$$

$$= 10{,}485 \text{ W}\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 14.1 \text{ hp}$$

(e) The induced torque is given by

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

$$= \frac{11{,}845 \text{ W}}{188.5 \text{ rad/s}} = 62.8 \text{ N} \cdot \text{m}$$

and the output torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{10{,}485 \text{ W}}{184.4 \text{ rad/s}} = 56.9 \text{ N} \cdot \text{m}$$

(f) The motor's efficiency at this operating condition is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{10{,}485 \text{ W}}{12{,}530 \text{ W}} \times 100\% = 83.7\%$$

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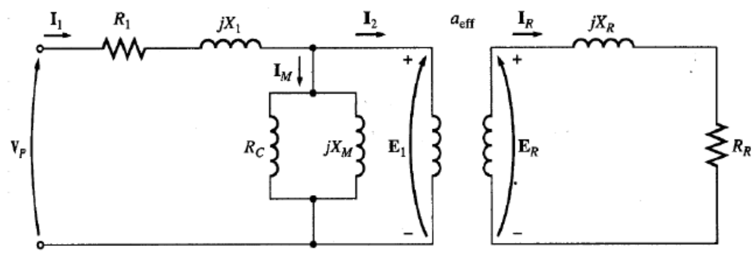

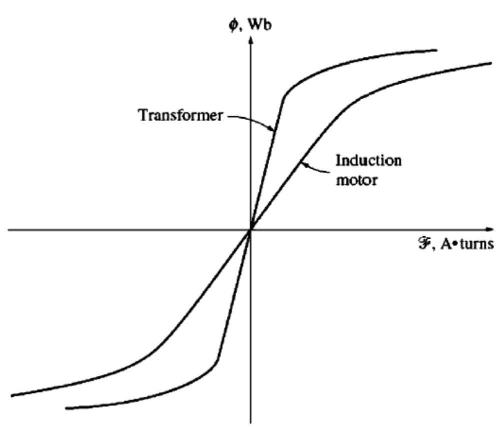

The magnetization curve of an induction motor compared to that of a transformer.

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The magnitude of the induced voltage at any slip will be given by the equation

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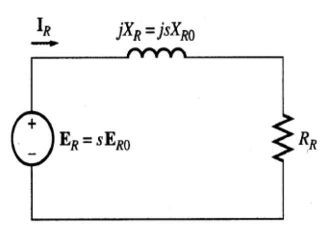
$$f_r = sf_e$$

- Then, we can draw the rotor equivalent circuit as follows



$$X_R = \omega_r L_R = 2\pi f_r L_R$$

$$f_r = sf_e$$

$$X_R = 2\pi s f_e L_R = s(2\pi f_e L_R) = sX_{R0}$$

$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jX_R}$$

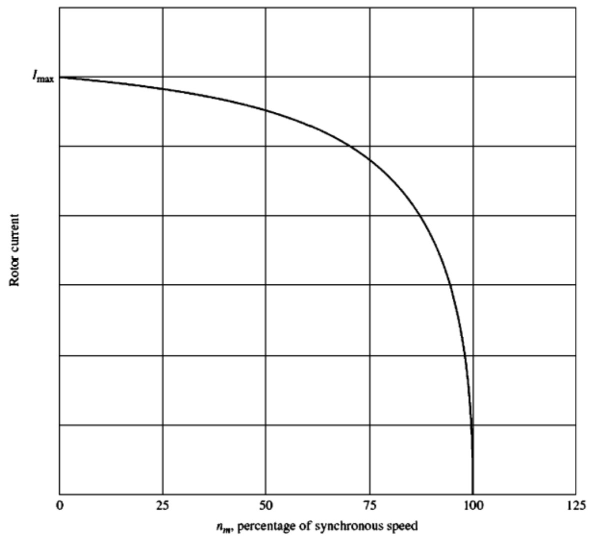
where X_{R0} is the blocked-rotor rotor reactance.

$$\boxed{\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jsX_{R0}}}$$

$$\boxed{\mathbf{I}_R = \frac{\mathbf{E}_{R0}}{R_R/s + jX_{R0}}}$$

- Notice that it is possible to treat all of the rotor effects due to varying rotor speed as being caused by a *varying impedance* supplied with power from a constant-voltage source ERO. The equivalent rotor impedance from this point of view is

$$Z_{R,eq} = R_R/s + jX_{R0}$$



The Final Equivalent Circuit

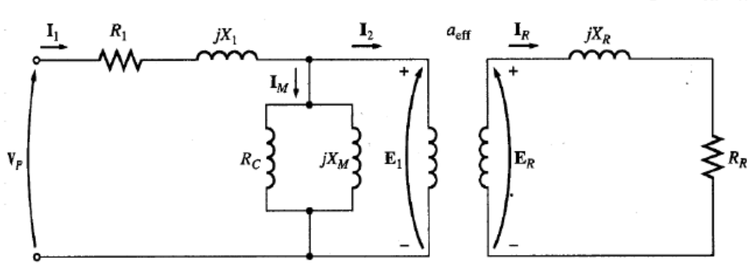
- To produce the final per-phase equivalent circuit for an induction motor, it is necessary to refer the rotor part of the model over to the stator side. The rotor circuit model that will be referred to the stator side is the model shown in

$$V_P = V'_S = aV_S \qquad E_1 = E'_R = a_{eff} E_{R0}$$

$$I_P = I'_S = \frac{I_S}{a} \qquad I_2 = \frac{I_R}{a_{eff}}$$

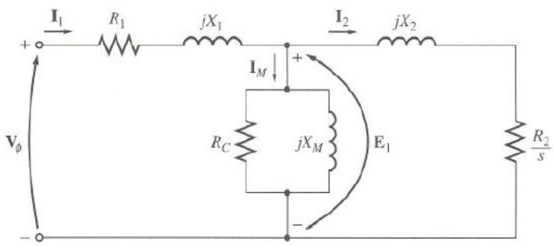
$$Z'_S = a^2 Z_S \qquad Z_2 = a_{eff}^2\left(\frac{R_R}{s} + jX_{R0}\right)$$

$$R_2 = a_{eff}^2 R_R \qquad X_2 = a_{eff}^2 X_{R0}$$

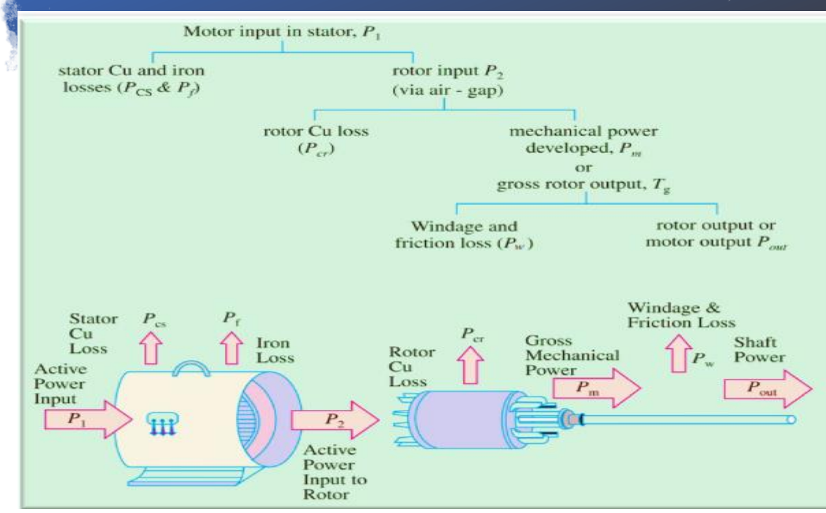


Per-phase equivalent circuit

- Motor Slip $$s = \frac{n_s - n_m}{n_s}$$
- R_1 and R_2: stator and rotor winding resistances
- X_1 and X_2: stator and rotor winding leakage reactances
- X_m: magnetizing reactance
- R_c: core loss resistance
- Rotor winding parameters are referred to the stator side



6.4 Power and torque in induction motors

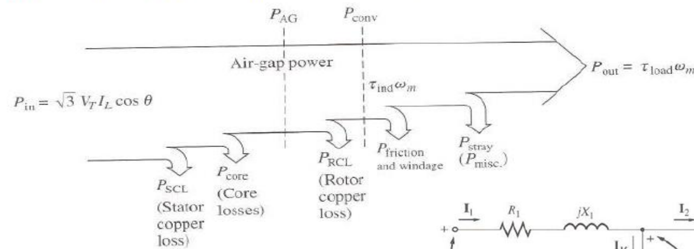


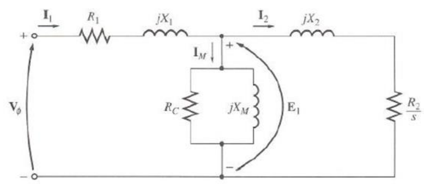
$$P_{SCL} = 3I_1^2 R_1$$
$$P_{core} = 3E_1^2 / R_C$$
$$P_{AG} = 3I_2^2 (R_2 / S)$$
$$P_{RCL} = 3I_2^2 R_2$$
$$P_{conv} = P_{AG} - P_{RCL} = 3I_2^2 [R_2(1-S)/S]$$

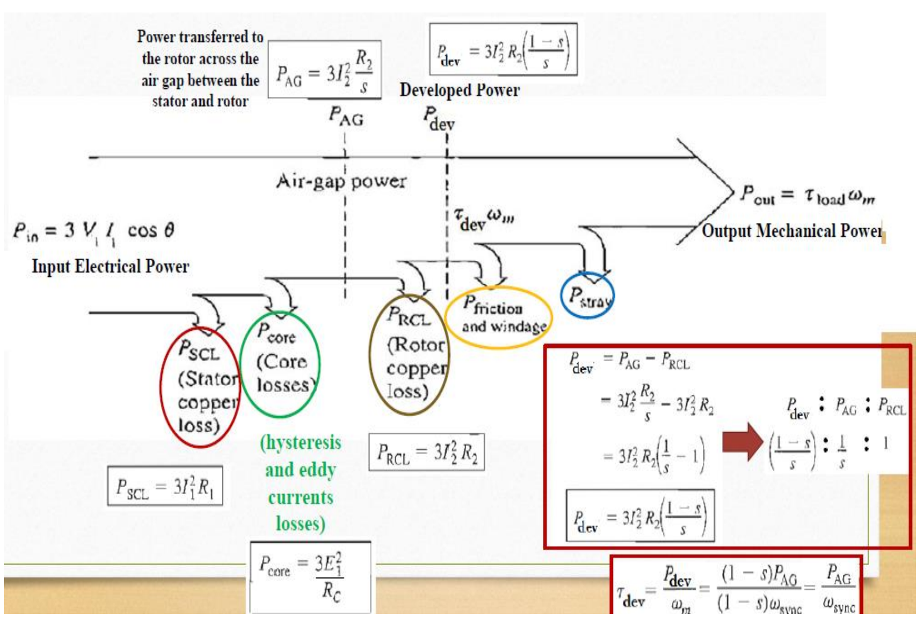
$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{eq}}$$

where

$$Z_{eq} = R_1 + jX_1 + \cfrac{1}{G_C - jB_M + \cfrac{1}{V_2/s + jX_2}}$$







- **The supply power is:**

$$Pin = \sqrt{3}V_L I_L \cos\theta$$

- **The motor efficiency:**

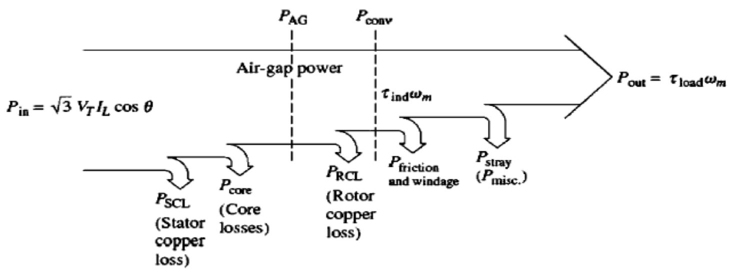
$$\eta = \frac{P_{out}}{P_{in}}$$

- **Motor torque:**

$$T_{out} = \frac{P_{out}}{\omega} \qquad T_{conv} = \frac{P_{Conv}}{\omega}$$

- **Rotor cu losses, gap power and developed power:**

$$P_{Cu2} = sP_{Gap} \qquad P_{conv} = P_{Gap} - P_{Cu2} = (1-s)P_{Gap}$$



The power-flow diagram of an induction motor.

Example

A 3-phase induction motor having a synchronous speed of 1200 r/min draws 80 kW from a 3-phase feeder. The copper losses and iron losses in the stator amount to 5 kW. If the motor runs at 1152 r/min, calculate:

a. the active power transmitted to the rotor;

b. the rotor I^2R losses;

c. the mechanical power developed;

d. the mechanical power delivered to the load, knowing that the windage and friction losses are equal to 2 kW;

e. the efficiency of the motor.

Solution:

a. Active power to the rotor is:

$$P_r = P_e - P_{js} - P_f$$
$$= 80 - 5 = 75 \text{ kW}$$

b. 1 The slip is:

$$s = (n_s - n)/n_s = (1200 - 1152)/1200$$
$$= 48/1200 = 0.04$$

b. 2 Rotor I^2R losses are:

$$P_{jr} = sP_r = 0.04 \times 75 = 3 \text{ kW}$$

c. The mechanical power developed is:

$$P_m = P_r - I^2R \text{ losses in rotor}$$
$$= 75 - 3 = 72 \text{ kW}$$

d. The mechanical power P_L delivered to the load is slightly less than P_m owing to the friction and windage losses.

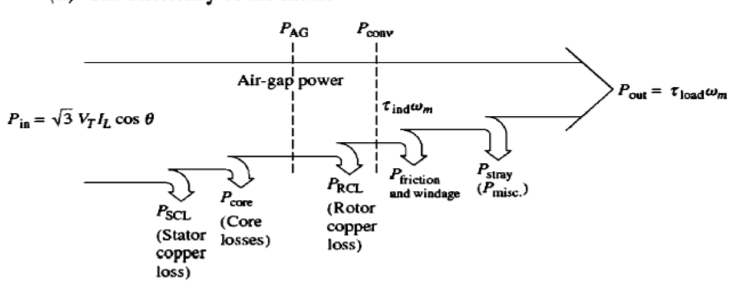
$$P_L = P_m - P_v = 72 - 2 = 70 \text{ kW}$$

e. $$\eta = P_L/P_e = 70/80$$
$$= 0.875 \text{ or } 87.5\%$$

Example:

A 480-V, 60-Hz, 50-hp, three-phase induction motor is drawing 60 A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

(*a*) The air-gap power P_{AG}
(*b*) The power converted P_{conv}
(*c*) The output power P_{out}
(*d*) The efficiency of the motor



The power-flow diagram of an induction motor.

(a)

$$P_{in} = \sqrt{3} V_T I_L \cos\theta$$
$$= \sqrt{3}(480\ \text{V})(60\ \text{A})(0.85) = 42.4\ \text{kW}$$

From the power-flow diagram, the air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$
$$= 42.4\ \text{kW} - 2\ \text{kW} - 1.8\ \text{kW} = 38.6\ \text{kW}$$

(b) From the power-flow diagram, the power converted from electrical to mechanical form is

$$P_{conv} = P_{AG} - P_{RCL}$$
$$= 38.6\ \text{kW} - 700\ \text{W} = 37.9\ \text{kW}$$

(c) From the power-flow diagram, the output power is given by

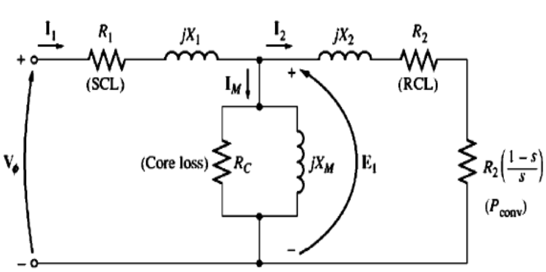
$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$
$$= 37.9\ \text{kW} - 600\ \text{W} - 0\ \text{W} = 37.3\ \text{kW}$$

(d) Therefore, the induction motor's efficiency is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$
$$= \frac{37.3\ \text{kW}}{42.4\ \text{kW}} \times 100\% = 88\%$$

Separating the Rotor Copper Losses and the Power Converted in an Induction Motor 's Equivalent Circuit

- Part of the power coming across the air gap in an induction motor is consumed in the rotor copper losses, and part of it is converted to mechanical power to drive the motor's shaft. It is possible to separate the two uses of the air-gap power and to indicate them separately on the motor equivalent circuit.



The per-phase equivalent circuit with rotor losses and P_{core} separated.

$$R_{conv} = \frac{R_2}{s} - R_2 = R_2\left(\frac{1}{s} - 1\right)$$

$$\boxed{R_{conv} = R_2\left(\frac{1-s}{s}\right)}$$

Example

A 460-V, 25-hp, 60-Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

(a) Speed
(b) Stator current
(c) Power factor
(d) P_{conv} and P_{out}
(e) τ_{ind} and τ_{load}
(f) Efficiency

(a) The synchronous speed is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

The rotor's mechanical shaft speed is

$$n_m = (1 - s)n_{sync}$$
$$= (1 - 0.022)(1800\text{ r/min}) = 1760\text{ r/min}$$

(b)

$$Z_2 = \frac{R_2}{s} + jX_2$$
$$= \frac{0.332}{0.022} + j0.464$$
$$= 15.09 + j0.464\ \Omega = 15.10\angle 1.76°\ \Omega$$

The combined magnetization plus rotor impedance is given by

$$Z_f = \frac{1}{1/jX_M + 1/Z_2}$$
$$= \frac{1}{-j0.038 + 0.0662\angle -1.76°}$$
$$= \frac{1}{0.0773\angle -31.1°} = 12.94\angle 31.1°\ \Omega$$

Therefore, the total impedance is

$$Z_{tot} = Z_{stat} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1°\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6°\ \Omega$$

The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{\text{tot}}}$$

$$= \frac{266\angle 0^\circ \text{ V}}{14.07\angle 33.6^\circ\ \Omega} = 18.88\angle -33.6^\circ \text{ A}$$

(c) The power motor power factor is

$$\text{PF} = \cos 33.6^\circ = 0.833 \qquad \text{lagging}$$

(d) The input power to this motor is

$$P_{\text{in}} = \sqrt{3} V_T I_L \cos\theta$$

$$= \sqrt{3}(460 \text{ V})(18.88 \text{ A})(0.833) = 12{,}530 \text{ W}$$

The stator copper losses in this machine are

$$P_{\text{SCL}} = 3 I_1^2 R_1$$

$$= 3(18.88 \text{ A})^2(0.641\ \Omega) = 685 \text{ W}$$

The air-gap power is given by

$$P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} = 12{,}530 \text{ W} - 685 \text{ W} = 11{,}845 \text{ W}$$

$$P_{\text{conv}} = (1 - s)P_{\text{AG}} = (1 - 0.022)(11{,}845 \text{ W}) = 11{,}585 \text{ W}$$

The power P_{out} is given by

$$P_{\text{out}} = P_{\text{conv}} - P_{\text{rot}} = 11{,}585 \text{ W} - 1100 \text{ W} = 10{,}485 \text{ W}$$

$$= 10{,}485 \text{ W}\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 14.1 \text{ hp}$$

(e) The induced torque is given by

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

$$= \frac{11{,}845 \text{ W}}{188.5 \text{ rad/s}} = 62.8 \text{ N} \cdot \text{m}$$

and the output torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{10{,}485 \text{ W}}{184.4 \text{ rad/s}} = 56.9 \text{ N} \cdot \text{m}$$

(f) The motor's efficiency at this operating condition is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{10{,}485 \text{ W}}{12{,}530 \text{ W}} \times 100\% = 83.7\%$$

Induction Motors Ch. 6 / Fifth Edition

Dr. Feras Alasali

6.5 Induction motor Torque –speed characteristics

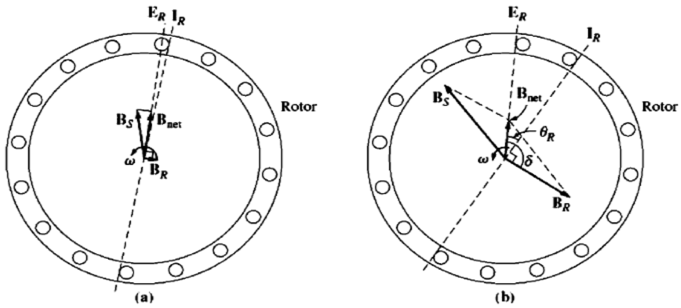
How does the torque of an induction motor change as the load changes?

How much does the speed of an induction motor drop as its shaft load increases?

To find out the answers to these and similar questions, it is necessary to clearly understand the relationships among the motor's torque, speed, and power.

The torque- speed relationship will be examined from the physical viewpoint of the motor's magnetic field behavior. Then, a general equation for torque as a function of slip will be derived from the induction motor equivalent circuit.

Induced Torque from a Physical Standpoint



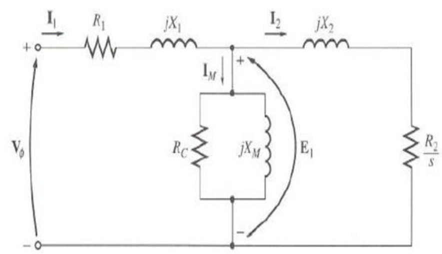
(a) The magnetic fields in an induction motor under light loads. (b) The magnetic fields in an induction motor under heavy loads.

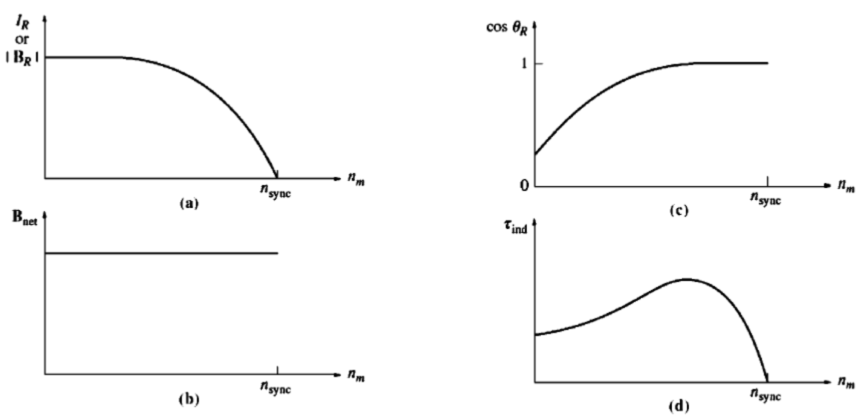
The induced torque, which keeps the rotor turning, is given by the equation

$$\tau_{ind} = k\mathbf{B}_R \times \mathbf{B}_{net}$$

Its magnitude is given by

$$\tau_{ind} = kB_R B_{net} \sin \delta$$





Graphical development of an induction motor torque–speed characteristic. (a) Plot of rotor current (and thus $|\mathbf{B}_R|$) versus speed for an induction motor; (b) plot of net magnetic field versus speed for the motor; (c) plot of rotor power factor versus speed for the motor; (d) the resulting torque–speed characteristic.

1. B_R. The rotor magnetic field is directly proportional to the current flowing in the rotor, as long as the rotor is unsaturated. The current flow in the rotor increases with increasing slip (decreasing speed);
This current flow was plotted in Figure a

2. B_{net}. The net magnetic field in the motor is proportional to E_1 and therefore is approximately constant (E_1 actually decreases with increasing current flow, but this effect is small compared to the other two, and it will be ignored in this graphical development). The curve for B_{net} versus speed is shown in b

3. sin δ. The angle δ between the net and rotor magnetic fields can be expressed in a very useful way. Look at Figure b. In this figure, it is clear that *the angle δ is just equal to the power-factor angle of the rotor plus* 90°:

$$\delta = \theta_R + 90°$$

Therefore, $\sin \delta = \sin(\theta_R + 90°) = \cos \theta_R$. This term is the power factor of the rotor. The rotor power-factor angle can be calculated from the equation

$$\theta_R = \tan^{-1}\frac{X_R}{R_R} = \tan^{-1}\frac{sX_{R0}}{R_R}$$

The resulting rotor power factor is given by

$$\text{PF}_R = \cos\theta_R$$

$$\boxed{\text{PF}_R = \cos\left(\tan^{-1}\frac{sX_{R0}}{R_R}\right)}$$

A plot of rotor power factor versus speed is shown in Figure c.

This characteristic curve can be di vided roughly into three regions.

- The first region is the *low-slip region* of the curve. In the low-slip region, the motor slip increases approximately linearly with increased load, and the rotor mechanical speed decreases approximately linearly with load. In this region of operation, the rotor reactance is negligible, so the rotor power factor is approximately unity. In normal operation, an induction motor has a linear speed droop.
- The second region on the induction motor's curve can be called the *moderate-slip region* . In the moderate-slip region, the rotor frequency is higher than before, and the rotor reactance is on the same order of magnitude as the rotor resistance. In this region, the rotor current no longer increases as rapidly as before, and the power factor starts to drop. The peak torque (the *pullout torque)* of the motor occurs at the point where, for an incremental increase in load, the increase in the rotor current is exactly balanced by the decrease in the rotor power factor.
- The third region on the induction motor's curve is called the *high-slip region*. In the high-slip region, the induced torque actually decreases with increased load, since the increase in rotor current is completely overshadowed by the decrease in rotor power factor.

The Derivation of the Induction Motor Induced-Torque Equation

- It is possible to use the equivalent circuit of an induction motor and the power flow diagram for the motor to derive a general expression for induced torque as a function of speed. The induced torque in an induction motor is given by Equation:

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m}$$

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

Simplified per-phase equivalent circuit

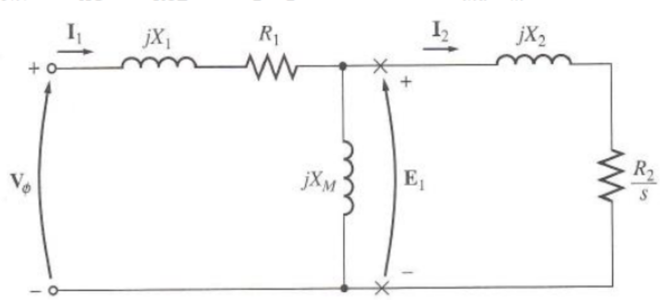
Core loss is embedded with friction, windage and stray-load loss

$$P_{SCL} = 3I_1^2 R_1$$

$$P_{RCL} = 3I_2^2 R_2$$

$$P_{AG} = 3I_2^2 (R_2 / S) = \tau_{ind}\omega_{sync}$$

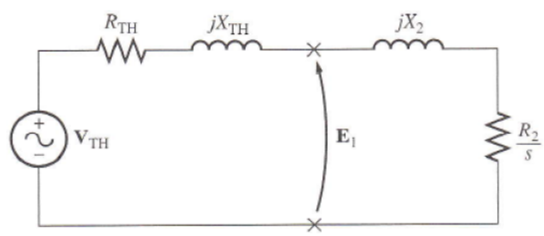
$$P_{conv} = P_{AG} - P_{RCL} = 3I_2^2 R_2 (1-S)/S = \tau_{ind}\omega_m$$



Thevenin equivalent circuit and torque-slip equation

$$V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}, \quad X_{TH} \approx X_1, \; R_{TH} \approx R_1 (\frac{X_M}{X_1 + X_M})^2$$

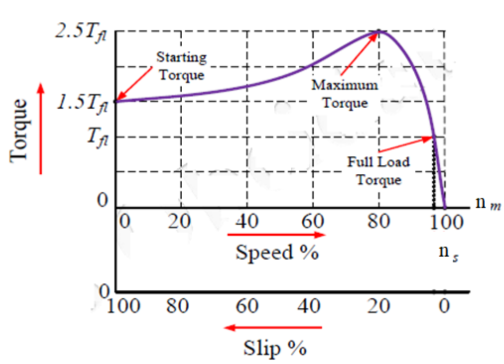
$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{3V_{TH}^2 R_2 / S}{\omega_{sync} \left[(R_{TH} + R_2/S)^2 + (X_{TH} + X_2)^2 \right]}$$



Torque-Speed Curve

The torque speed (slip) curve for an induction motor gives us the information about the variation of torque with the slip.

When the rotor stationary (at standstill) $n_m = 0$ rpm, the rotor frequency $f_r = f_s$ and the slip $s=1$. At $n_m = n_s$, the rotor frequency $f_r = 0$ Hz, and the slip $s=0$.



- **At full load, the motor runs at speed of n_m. When mechanical load increases, motor speed decreases tell the motor torque again becomes equal to the load torque.**
- **As long as the two torques are in balance, the motor will run at constant (but lower) speed.**
- **If the load torque exceeds the induction motor maximum torque, the motor will suddenly stop.**

Notes: the Induction Motor Torque- Speed Curve

1. The induced torque of the motor is zero at synchronous speed.

2. The torque- speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.

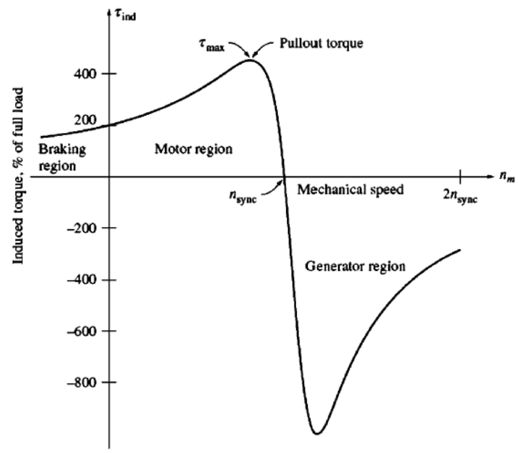
3. There is a maximum possible torque that cannot be exceeded. This torque, called the *pullout torque* or *breakdown torque,* is 2 to 3 times the rated full load torque of the motor.

4. The starting torque on the motor is slightly larger than its full-load torque. So this motor will start carrying any load that it can supply at full power.

5. The torque on the motor for a given slip varies as the square of the applied voltage. This fact is useful in one form of induction motor speed control.

6. If the rotor of the induction motor is driven faster than synchronous speed, then the direction of the induced torque in the machine reverses and the machine becomes a generator, converting mechanical power to electric power.

7. If the motor is turning backward relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor. The act of switching two phases in order to stop the motor very rapidly is called plugging.



Induction motor torque–speed characteristic curve, showing the extended operating ranges (braking region and generator region).

Maximum Torque in Induction Motor

- Since the induced torque is equal to *PAG/w* the maximum possible torque occurs when the air-gap power is maximum.
- Since the air-gap power is equal to the power consumed in the resistor *R2 /s,* the *maximum induced torque will occur when the power consumed by that resistor is maximum.*
- the maximum power transfer theorem states that maximum power transfer to the load resistor *R2l s* will occur when the *magnitude* of that impedance is equal to the *magnitude* of the source impedance

$$Z_{source} = R_{TH} + jX_{TH} + jX_2$$

so the maximum power transfer occurs when

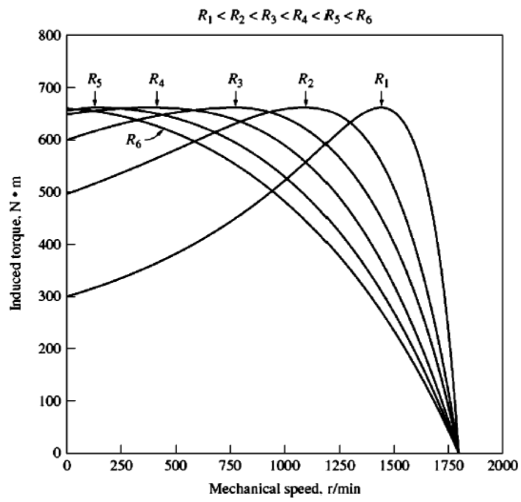
$$\frac{R_2}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}$$

we see that *the slip at pullout torque is given by*

$$\boxed{s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}}$$

$$\boxed{\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}}$$

Effect of varying rotor resistance (by adding external resistance to wound rotor)



Example

A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min. Neglecting the rotational losses.

(a) What is the motor's slip?

(b) What is the induced torque in the motor in Nom under these conditions?

(c) What will the operating speed of the motor be if its torque is doubled?

(d) How much power will be supplied by the motor when the torque is doubled?

(a) The synchronous speed of this motor is

$$n_{sync} = \frac{120f_e}{P} = \frac{120(50\text{ Hz})}{2\text{ poles}} = 3000\text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{sync} - n_m}{n_{sync}}(\times 100\%)$$

$$= \frac{3000\text{ r/min} - 2950\text{ r/min}}{3000\text{ r/min}}(\times 100\%)$$

$$= 0.0167 \text{ or } 1.67\%$$

(b) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

$$= \frac{15\text{ kW}}{(2950\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})}$$

(c) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

$$n_m = (1 - s)n_{sync} = (1 - 0.0333)(3000\text{ r/min}) = 2900\text{ r/min}$$

(d) The power supplied by the motor is given by

$$P_{conv} = \tau_{ind}\omega_m = (97.2\text{ N}\cdot\text{m})(2900\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s}) \quad = 29.5\text{ kW}$$

Example

A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

R1=0.641Ω X1=1.106 Ω R2=0.332Ω X2=0.464Ω Xm=26.3Ω

(a) What is the maxim run torque of this motor? At what speed and slip does itoccur?

(b) What is the starting torque of this motor?

(c) When the rotor resistance is doubled. what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

The Thevenin voltage of this motor is

$$V_{TH} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$= \frac{(266\text{ V})(26.3\ \Omega)}{\sqrt{(0.641\ \Omega)^2 + (1.106\ \Omega + 26.3\ \Omega)^2}} = 255.2\text{ V}$$

The Thevenin resistance is

$$R_{TH} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2$$

$$\approx (0.641\ \Omega)\left(\frac{26.3\ \Omega}{1.106\ \Omega + 26.3\ \Omega}\right)^2 = 0.590\ \Omega$$

The Thevenin reactance is

$$X_{TH} \approx X_1 = 1.106\ \Omega$$

(a) The slip at which maximum torque occurs is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$= \frac{0.332\ \Omega}{\sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}} = 0.198$$

This corresponds to a mechanical speed of

$$n_m = (1 - s)n_{sync} = (1 - 0.198)(1800\text{ r/min}) = 1444\text{ r/min}$$

The torque at this speed is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}$$

$$= \frac{3(255.2\text{ V})^2}{2(188.5\text{ rad/s})[0.590\ \Omega + \sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}]}$$

$$= 229\text{ N}\bullet\text{m}$$

(*b*) The starting torque of this motor is found by setting $s = 1$

$$\tau_{start} = \frac{3V_{TH}^2 R_2}{\omega_{sync}[(R_{TH} + R_2)^2 + (X_{TH} + X_2)^2]}$$

$$= \frac{3(255.2\ V)^2(0.332\ \Omega)}{(188.5\ rad/s)[(0.590\ \Omega + 0.332\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 104\ N \bullet m$$

(*c*) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$s_{max} = 0.396$$

and the speed at maximum torque is

$$n_m = (1 - s)n_{sync} = (1 - 0.396)(1800\ r/min) = 1087\ r/min$$

The maximum torque is still

$$\tau_{max} = 229\ N \bullet m$$

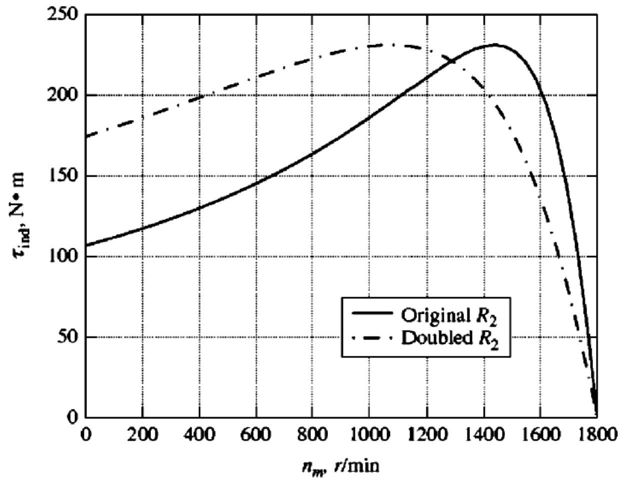
The starting torque is now

$$\tau_{start} = \frac{3(255.2\ V)^2(0.664\ \Omega)}{(188.5\ rad/s)[(0.590\ \Omega + 0.664\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]} = 170\ N \bullet m$$

6.6 Variations in Induction Motor Torque Speed Characteristics

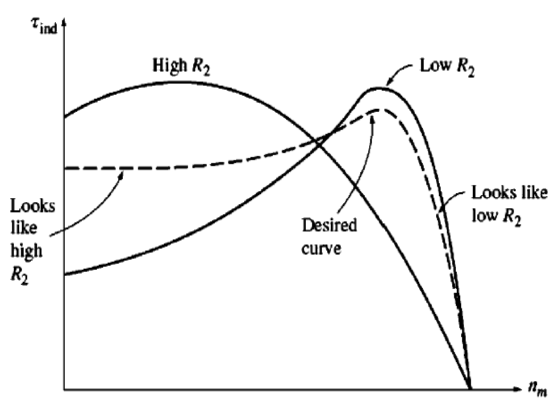
- Section 6.5 contained the derivation of the torque-speed characteristic for an induction motor. In fact, several characteristic curves were shown, depending on the rotor resistance.
- If a rotor is designed with high resistance, then the motor 's starting torque is quite high, but the slip is also quite high at normal operating conditions. Recall that *P conv*= (I - *s)*PAG, so *the higher the slip, the smaller the fraction of air-gap power actually converted to mechanical form,* and thus the lower the motor's efficiency.
- A motor with high rotor resistance has a good starting torque but poor efficiency at normal operating conditions.
- On the other hand, a motor with low rotor resistance has a low starting torque and high starting current, but its efficiency at normal operating conditions is quite high.

Torque-speed characteristics for the motor of Example 6- 5.



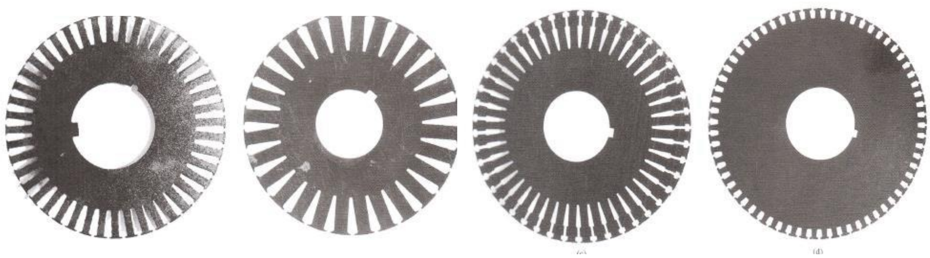
An induction motor designer is forced to compromise between the conflicting requirements of high starting torque and good efficiency

- One possible solution to this difficulty is using a wound-rotor induction motor and insert extra resistance into the rotor during starting.
- The extra resistance could be completely removed for better efficiency during normal operation. Unfortunately, wound-rotor motors are more expensive, need more maintenance, and require a more complex automatic control circuit than cage rotor motors.
- It would be nice to figure out some way to add extra rotor resistance at starting and to remove it during normal running without slip rings and *without operator or control circuit intervention.*



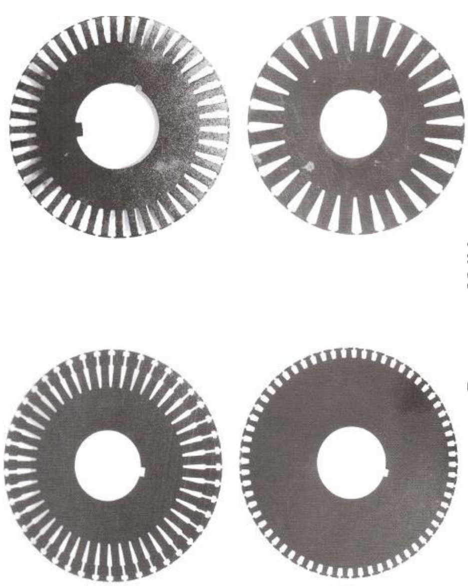
Control of Motor Characteristics by Cage Rotor Design

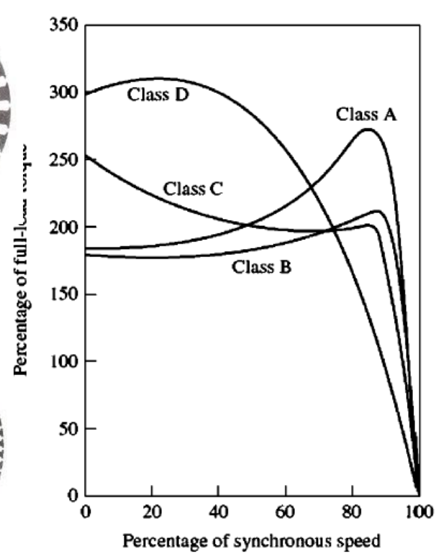
- Recall that leakage reactance is the reactance due to the rotor flux lines that do not also couple with the stator windings



Laminations from typical cage induction motor rotors. showing the cross section of the rotor bars: (a) class A- large bars near the surface; (b) class B- large, deep rotor bars. (c) class C--double-cage rotor design; (d) class D-small bars near the surface.

Typical torque-speed curves for different rotor designs





6.7 Trends in induction motor design

Several techniques are used to improve the efficiency of these motors compared to the traditional standard-efficiency designs.

I. More copper is used in the stator windings to reduce copper losses.

2. The rotor and stator core length is increased to reduce the magnetic flux density in the air gap of the machine.

3. More steel is used in the stator of the machine, al lowing a greater amount of heat transfer out of the motor and reducing its operating temperature. The rotor's an is then redesigned to reduce windage losses.

4. The steel used in the stator is a special high-grade electrical steel with low hysteresis losses.

5. The steel is made of an especially thin gauge (i.e. , the laminations are very close together), and the steel has a very high internal resistivity. Both effects tend to reduce the eddy current losses in the motor.

6. The rotor is carefully machined to produce a uniform air gap, reducing the stray load losses in the motor.

6.8 Starting Induction Motors

- Induction motors do not present the types of starting problems that synchronous motors do. In many cases, induction motors can be started by simply connecting them to the power line. However, there are sometimes good reasons for not doing this. For example, the starting current required may cause such a dip in the power system voltage that *across-the-line starting* is not acceptable.
- For wound-rotor induction motors, starting can be achieved at relatively low currents by inserting extra resistance in the rotor circuit during starting.
- For cage induction motors, the starting current can vary widely depending primarily on the motor's rated power and on the effective rotor resistance at starting conditions.

- To estimate the rotor current at starting, the cage rotor motors have a code letter to limits the amount of this current .These limits are expressed in terms of the starting apparent power of the motor as a function of its horsepower rating.
- *Sstart*= (rated horse power)(code letter factor)
- A table containing the starting kilo voltamperes per horsepower for each code letter.

Nominal code letter	Locked rotor, kVA/hp	Nominal code letter	Locked rotor, kVA/hp
A	0–3.15	L	9.00–10.00
B	3.15–3.55	M	10.00–11.00
C	3.55–4.00	N	11.20–12.50
D	4.00–4.50	P	12.50–14.00

- To determine the starting current for an induction motor, read the rated voltage, horsepower, and code letter from its nameplate. Then the starting apparent power for the motor will be

$$I_L = \frac{S_{start}}{\sqrt{3}V_T}$$

Example:

- What is the starting Current of a 15-hp, 208-V, code-letter-F, three phase induction motor?

Sstart= (15 hp)(5.6) = 84 kVA

$$I_L = \frac{S_{start}}{\sqrt{3}V_T}$$

$$= \frac{84 \text{ kVA}}{\sqrt{3}(208 \text{ V})} = 233 \text{ A}$$

- A magnetic motor starter circuit of this sort has several but It -in protective features:

I. Short-circuit protection

2. Overload protection

3. Undervoltage protection

- There are really only two techniques by which the speed of an induction motor can be controlled.
- One is to vary the synchronous speed, which is the speed of the stator and rotor magnetic field, since the rotor speed always remains near n_{sync}.
- The other technique is to vary the slip of the motor for a give n load.

$$n_{sync} = \frac{120 f_e}{P}$$

Induction Motor Speed Control by Pole Changing

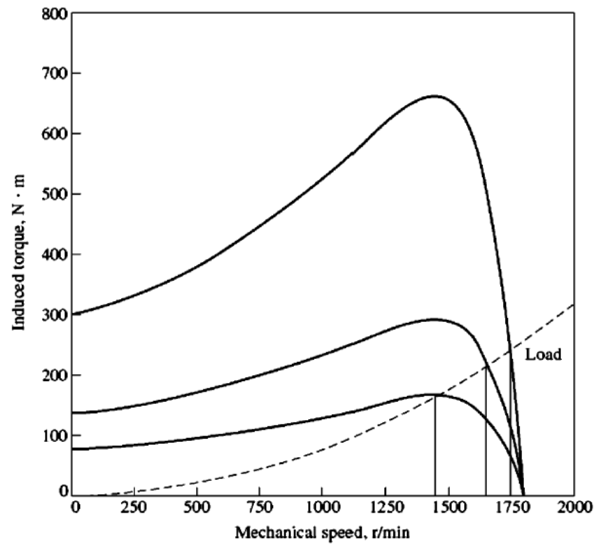
There are two major approaches to changing the number of poles in an induction motor:

- I. The method of consequent poles: is quite an old method for speed control , having been originally developed in 1897. It relies on the fact that the number of poles in the stator windings of an induction motor can easily be changed by a factor of 2: I with only simple changes in coil connections. When the motor is reconnected from two-pole to four-pole operation, the resulting maximum torque of the induction motor can be the same as before. The major disadvantage of the consequent-pole method of changing speed is that the speeds *must* be in a ratio of 2: I.
- 2. Multiple stator windings: For example, a motor might be wound with a four-pole and a six-pole set of stator windings, and its synchronous speed on a 6O-Hz system could be switched from 1800 to 1200 *r/min* simply by supplying power to the other set of windings. Unfortunately, multiple stator winding increase the expense of the motor and are therefore used only when absolutel necessary.

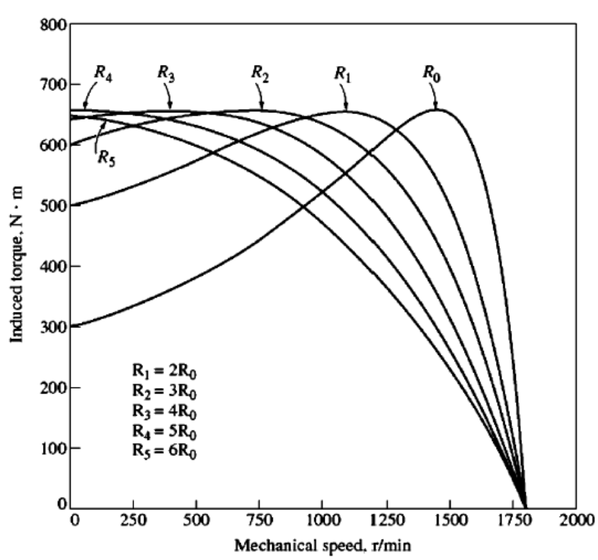
Speed Control by Changing the Line Frequency

- I f the electrical frequency applied to the stator of an induction motor is changed, the rate of rotation of its magnetic fields will change in direct proportion to the change in electrical frequency, and the no-load point on the torque-speed characteristic curve will change with it.
- The synchronous speed of the motor at rated conditions is known as the *base speed.* By using variable frequency control, it is possible to adjust the speed of the motor either above or below base speed.
- A properly designed variable-frequency induction motor drive can be *very* flexible. It can control the speed of an induction motor over a range from as little as 5 percent of base speed up to about twice base speed.
- However, it is important to maintain certain voltage and torque limits on the motor as the frequency is varied, to ensure safe operation.
- When running at speeds below the base speed of the motor, it is necessary to reduce the terminal voltage applied to the stator for proper operation

Speed Control by Changing the Line Voltage



Speed Control by Changing the Rotor resistance

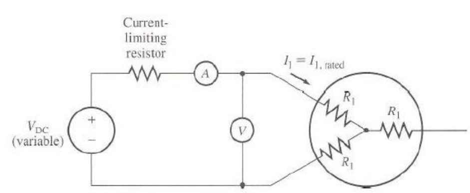


- The equivalent circuit of an induction motor is a very useful tool for determining the motor's response to changes in load.
- However, if a model is to be used for a real machine, it is necessary to determine what the element values are that go into the model.
- How can R1, *R2* , X1, *X2,* and *XM* be determined for a real motor?

Determining motor circuit parameters

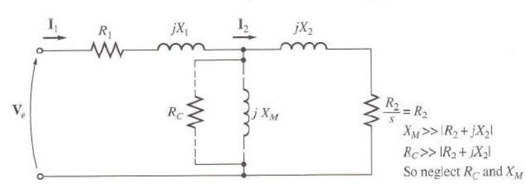
- **DC Test** (or use Ohm-meter)

  Measuring V, I → R1



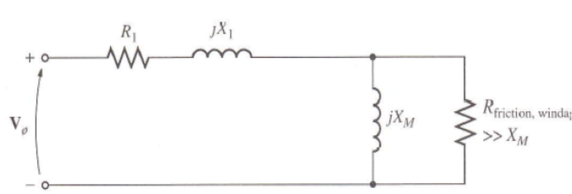
- **Locked-rotor Test**

  Measuring V_ϕ, I1, P, Q
  → R1+R2, X1+X2



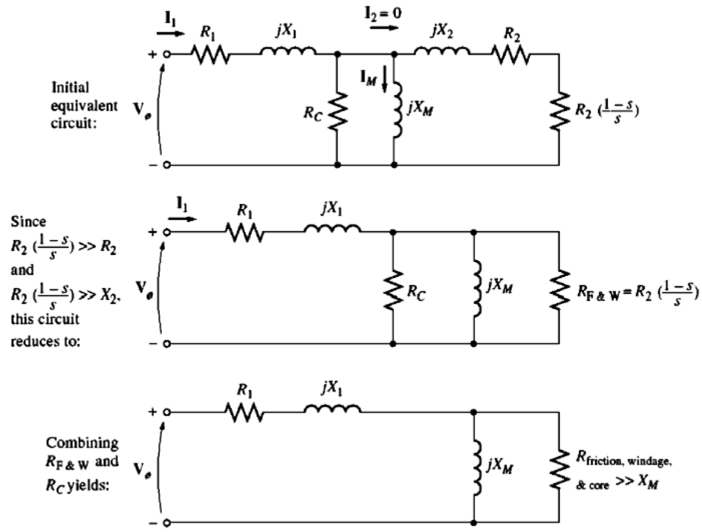
- **No-load Test**

  Measuring Vϕ, I1, P, Q
  → R1, X1+Xm



The No-Load test

- The no-load test of an induction motor measures the rotational losses of the motor and provides information about its magnetization current.



- In this motor at no- load conditions, the input power measured by the meters must equal the losses in the motor. The rotor copper losses are negligible .

$$\boxed{P_{SCL} = 3I_1^2 R_1}$$

so the input power must equal

$$P_{in} = P_{SCL} + P_{core} + P_{F\&W} + P_{misc}$$
$$= 3I_1^2 R_1 + P_{rot}$$

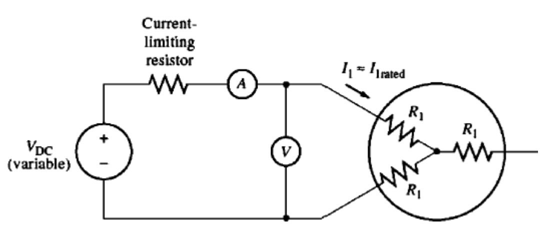
where P_{rot} is the rotational losses of the motor:

$$P_{rot} = P_{core} + P_{F\&W} + P_{misc}$$

$$\boxed{|Z_{eq}| = \frac{V_\phi}{I_{1,nl}} \approx X_1 + X_M}$$

The DC test for stator resistance

- The rotor resistance *R2* plays an extremely critical role in the operation of an in duction motor. Among other things, R1 determines the shape of the torque-speed curve, determining the speed at which the pullout torque occurs.
- To find the rotor resistance R2 accurately, it is necessary to know R1 so that it can be subtracted from the total.
- This test for R1 independent of R1 , X1 and X2. This test is called the dc test. Basically, a dc volt age is applied to the stator windings of an induction motor. Because the current is dc, there is no induced voltage in the rotor circuit and no resulting rotor current now. Also, the reactance of the motor is zero at direct current.
- Therefore, the only quantity limiting current n ow in the motor is the stator resistance, and that resistance can be determined.

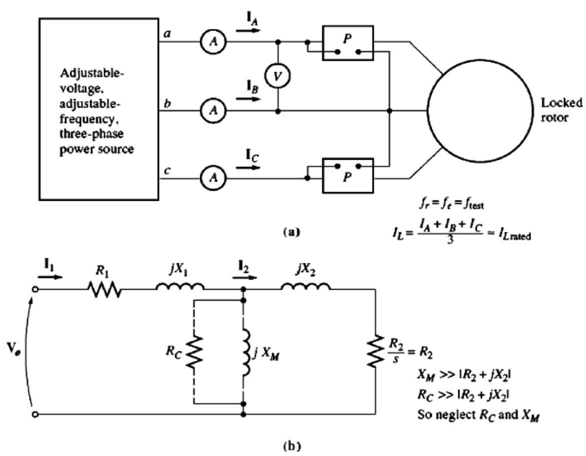


$$2R_1 = \frac{V_{DC}}{I_{DC}}$$

$$\boxed{R_1 = \frac{V_{DC}}{2I_{DC}}}$$

The Locked-Rotor Test

- The third test that can be performed on an induction motor to determine its circuit parameters is called the *locked-rotor test,* or sometimes the *blocked-rotor test.*
- This test corresponds to the short-circuit test on a transformer. **In** this test, the rotor is locked or blocked so that it *cannot* move, a voltage is applied to the motor, and the resulting voltage, current, and power are measured



$$P = \sqrt{3} V_T I_L \cos \theta$$

so the locked-rotor power factor can be found as

$$\text{PF} = \cos \theta = \frac{P_{in}}{\sqrt{3} V_T I_L}$$

and the impedance angle θ is just equal to $\cos^{-1}$ PF.

The magnitude of the total impedance in the motor circuit at this time is

$$|Z_{LR}| = \frac{V_\phi}{I_1} = \frac{V_T}{\sqrt{3} I_L}$$

and the angle of the total impedance is θ. Therefore,

$$Z_{LR} = R_{LR} + jX'_{LR}$$
$$= |Z_{LR}| \cos \theta + j|Z_{LR}| \sin \theta$$

The locked-rotor resistance R_{LR} is equal to

$$R_{LR} = R_1 + R_2$$

while the locked-rotor reactance X'_{LR} is equal to

$$X'_{LR} = X'_1 + X'_2$$

where X'_1 and X'_2 are the stator and rotor reactances *at the test frequency*, respectively. The rotor resistance R_2 can now be found as

$$R_2 = R_{LR} - R_1$$

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2$$

	X_1 and X_2 as functions of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	0.5 X_{LR}	0.5 X_{LR}
Design A	0.5 X_{LR}	0.5 X_{LR}
Design B	0.4 X_{LR}	0.6 X_{LR}
Design C	0.3 X_{LR}	0.7 X_{LR}
Design D	0.5 X_{LR}	0.5 X_{LR}

Example:

The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, design A, Y-connected induction motor having a rated current of 28 A.

DC test:

$V_{DC} = 13.6\text{ V}$ $\quad I_{DC} = 28.0\text{ A}$

No-load test:

$V_T = 208\text{ V}$ $\quad f = 60\text{ Hz}$

$I_A = 8.12\text{ A}$ $\quad P_{in} = 420\text{ W}$

$I_B = 8.20\text{ A}$

$I_C = 8.18\text{ A}$

Locked-rotor test:

$V_T = 25\text{ V}$ $\quad f = 15\text{ Hz}$

$I_A = 28.1\text{ A}$ $\quad P_{in} = 920\text{ W}$

$I_B = 28.0\text{ A}$

$I_C = 27.6\text{ A}$

(a) Sketch the per-phase equivalent circuit for this motor.

(b) Find the slip at the pullout torque, and find the value of the pullout torque itself.

(a) From the dc test,

$$R_1 = \frac{V_{DC}}{2I_{DC}} = \frac{13.6\text{ V}}{2(28.0\text{ A})} = 0.243\ \Omega$$

From the no-load test,

$$I_{L,av} = \frac{8.12\text{ A} + 8.20\text{ A} + 8.18\text{ A}}{3} = 8.17\text{ A}$$

$$V_{\phi,nl} = \frac{208\text{ V}}{\sqrt{3}} = 120\text{ V}$$

Therefore,

$$|Z_{nl}| = \frac{120\text{ V}}{8.17\text{ A}} = 14.7\ \Omega = X_1 + X_M$$

When X_1 is known, X_M can be found. The stator copper losses are

$$P_{SCL} = 3I_1^2 R_1 = 3(8.17\text{ A})^2(0.243\ \Omega) = 48.7\text{ W}$$

Therefore, the no-load rotational losses are

$$P_{rot} = P_{in,nl} - P_{SCL,nl}$$
$$= 420\text{ W} - 48.7\text{ W} = 371.3\text{ W}$$

From the locked-rotor test,

$$I_{L,av} = \frac{28.1\text{ A} + 28.0\text{ A} + 27.6\text{ A}}{3} = 27.9\text{ A}$$

The locked-rotor impedance is

$$|Z_{LR}| = \frac{V_\phi}{I_A} = \frac{V_T}{\sqrt{3}I_A} = \frac{25\text{ V}}{\sqrt{3}(27.9\text{ A})} = 0.517\ \Omega$$

and the impedance angle θ is

$$\theta = \cos^{-1} \frac{P_{in}}{\sqrt{3} V_T I_L}$$

$$= \cos^{-1} \frac{920 \text{ W}}{\sqrt{3}(25 \text{ V})(27.9 \text{ A})}$$

$$= \cos^{-1} 0.762 = 40.4°$$

Therefore, $R_{LR} = 0.517 \cos 40.4° = 0.394\ \Omega = R_1 + R_2$. Since $R_1 = 0.243\ \Omega$, R_2 must be $0.151\ \Omega$. The reactance at 15 Hz is

$$X'_{LR} = 0.517 \sin 40.4° = 0.335\ \Omega$$

The equivalent reactance at 60 Hz is

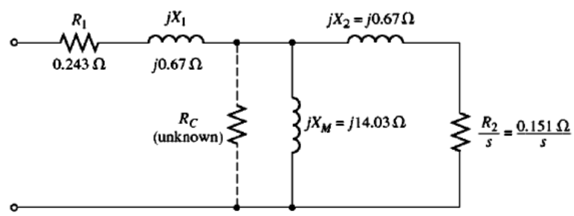
$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = \left(\frac{60 \text{ Hz}}{15 \text{ Hz}}\right) 0.335\ \Omega = 1.34\ \Omega$$

For design class A induction motors, this reactance is assumed to be divided equally between the rotor and stator, so

$$X_1 = X_2 = 0.67\ \Omega$$

$$X_M = |Z_{nl}| - X_1 = 14.7\ \Omega - 0.67\ \Omega = 14.03\ \Omega$$

The final per-phase equivalent circuit is shown in Figure



Motor per-phase equivalent circuit

(b) For this equivalent circuit, the Thevenin equivalents are

$$V_{TH} = 114.6 \text{ V} \qquad R_{TH} = 0.221\ \Omega \qquad X_{TH} = 0.67\ \Omega$$

Therefore, the slip at the pullout torque is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$= \frac{0.151\ \Omega}{\sqrt{(0.243\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}} = 0.111 = 11.1\%$$

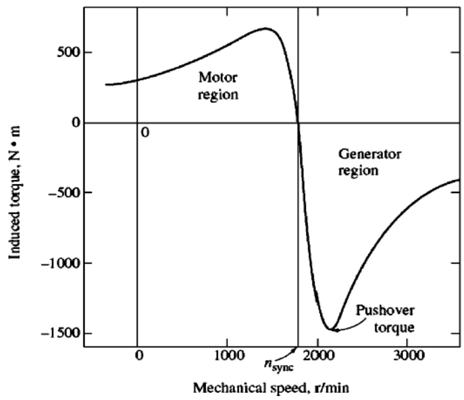
The maximum torque of this motor is given by

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X^2)}]}$$

$$= \frac{3(114.6 \text{ V})^2}{2(188.5 \text{ rad/s})[0.221\ \Omega + \sqrt{(0.221\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}]}$$

$$= 66.2 \text{ N} \bullet \text{m}$$

- The torque-speed characteristic curve shows that if an induction motor is dri at a speed *greater* than nsync by an external prime mover, the direction of inducted torque will reverse and it will act as a generator. As the torque appl to its shaft by the prime mover increases, the amount of power produced by the induction generator increases.
- There is a maximum possible inducec torque in the generator mode of operation. This torque is known as the *pushover torque* of the generator. If a prime mover applies a torque greater than the pushover torque to the shaft an induction generator, the generator will overspeed.



Motor Specifications

Specifications: EFM4104T

208V AMPS:	76
BEARING-DRIVE-END:	6311
BEARING-OPP-DRIVE-END:	6309
CUSTOMER-PART-NUMBER:	--
DESIGN CODE:	B
DOE-CODE:	010A
FL EFFICIENCY:	93.6
ENCLOSURE:	TEFC
FRAME:	286T
GREASE:	POLYREX EM
HERTZ:	60
CATALOG NUMBER:	EFM4104T
SPEC. NUMBER:	10C156Y758G1
INSULATION-CLASS:	F
KVA-CODE:	G
MAX. SPACE HEATER TEMP.:	--
SPEED [rpm]:	1770
OUTPUT [hp]:	30
PHASE:	3
POWER-FACTOR:	83
RATING:	40C AMB-CONT
SERIAL-NUMBER:	--
SERVICE FACTOR:	1.15
SPACE-HEATER-AMPS:	--
SPACE-HEATER-VOLTS:	--
VOLTAGE:	230/460
FL AMPS:	72/36

Suggestion Problems

- 6.4
- 6.5
- 6.7, 6.8, 6.10
- 6.14, 6.15
- 6.18
- 6.19 , 6.28