



تقدم لجنة EiCoM الاكاديمية

دفتر لمادة:

آلات كهربائية (1)

من شرح:

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Machines (1)

NO.

Ch. 1 - Introduction to machinery Principles

1.3 : Rotational motion

1- Angular Position (θ) → angle at which the object is oriented (degree, radian)

2- Angular velocity (ω) → Ratio change of angular Position
$$\omega_m = \frac{d\theta}{dt} \quad \text{rad/s}$$

$$f_m = \frac{\omega_m}{2\pi} \quad \text{rad/s}$$

$$N_m = 60 f_m \quad \text{rev/min} \quad \text{or} \quad \text{RPM}$$

3- Acceleration (α) → Rate change of angular velocity
$$\alpha = \frac{d\omega}{dt} \quad \text{rad/s}^2$$

4- Torque (τ) → twisting force

$$\tau = F \cdot x \quad \text{N.m} \quad \times \quad x: \text{distance between}$$

$\tau = F r \sin\theta$ force and the center of rotation



5- Work (W) $\rightarrow W = \int \tau \cdot d\theta \rightarrow W = \tau \theta$ Joule

6- Power (P) $\rightarrow P = \frac{dW}{dt} = \frac{d(\tau \theta)}{dt} = \tau \frac{d\theta}{dt}$
 $P = \tau \omega$ Watt

1.4 : Magnetic field

4 basic principles :

for
Transformer

1) Current carrying wire produced a magnetic field around

2) Time changing magnetic field produce an Induced voltage (E_{ind})

for
Motor

3) Current carrying wire in presence of magnetic field produced Induced force (Input \rightarrow Electric, output \rightarrow mechanic)

for
generator

4) Moving wire in presence of a magnetic field induced voltage (E_{ind}) (Input \rightarrow mechanic, output \rightarrow Electric)

Production of magnetic field

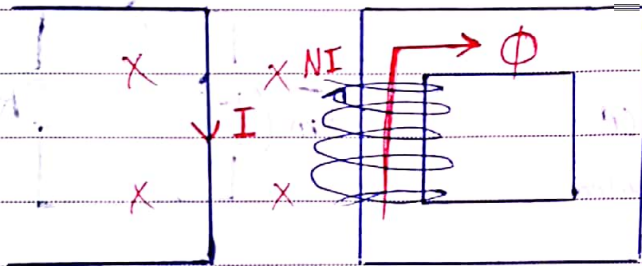
* Ampere's law

$$I = \int H \cdot dl$$

H : magnetic flux intensity

I : Electric current

L : Path



Iron Core

~~$\int H \cdot dl$~~
 بزرگه عن اللان
 بزار ال Φ
 Ferro بزار ال Φ

$$NI = HL$$

$$\rightarrow H = \frac{NI}{L} \quad \left(\frac{\text{AmPer} \cdot \text{Turns}}{m} \right)$$

$$B = \mu H$$

B : magnetic flux density

μ : permeability (core)

$$B = \frac{NI}{L} \mu \quad (\text{web} / m^2) (\text{Tesla})$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$\mu_r = \frac{\mu}{\mu_0}$$

(2000-2600) for Iron

الطوبه اذات في الجبل

تصنع في الجبل

$$\Phi = \text{Total flux} = B \cdot A \quad (\text{weber}) \rightarrow \Phi = \int B \cdot dA$$

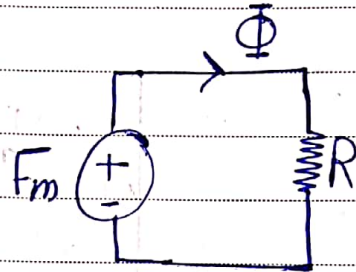
A : cross sectional area of the core

$$\Phi = BA = \mu HA = \mu \frac{NI}{L} A$$

$$\Phi = \frac{\mu A}{L} \overset{\text{mmf}}{NI} \quad (\text{magnetomotive force})$$

$$\text{mmf} = NI$$

R : Reluctance
of a coil, of a wire

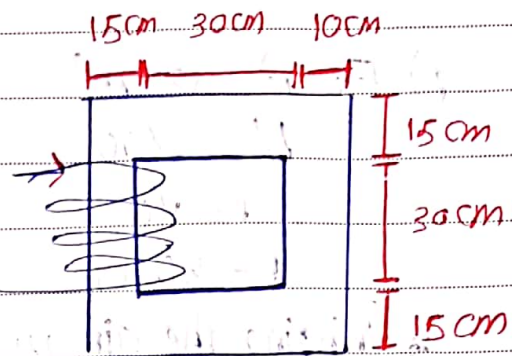


$$R = \frac{L}{\mu A} \quad (\text{A. turns/wb}) \quad , \quad F = \Phi R$$

$$R = \frac{1}{\mu} \quad , \quad \mu : \text{Permeance}$$

$$\Phi = \frac{F}{R}$$

Ex $N = 200$
 depth = 10 cm
 $\mu_r = 2500$, $I = 1A$
 find Φ ?!



Sol. $R_1 = \frac{L_1}{\mu A_1}$

$$L_1 = 5 + 30 + 7.5 + 7.5 \quad F = NI \quad \oplus$$

$$+ 30 + 7.5 + 5 + 7.5$$

$$L_1 = 130 \text{ cm}$$

$$\mu = \mu_r \mu_0 = 4\pi \times 10^{-7} \times 2500$$

$$A = 15 \times 10^{-4} \text{ m}^2$$

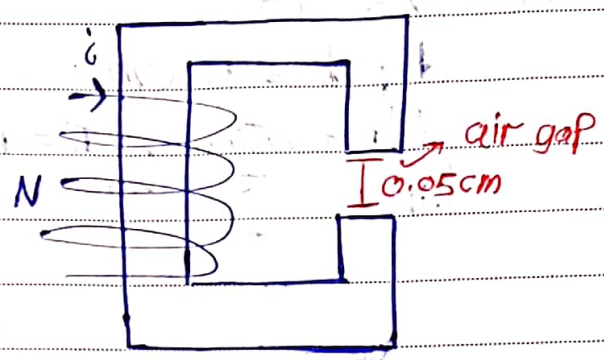
$$R_1 = \frac{130 \text{ cm}}{4\pi \times 10^{-7} \times 2500 \times 150 \times 10^{-4}}$$

$$R_2 = \frac{L_2}{\mu A_2} = \frac{7.5 + 30 + 7.5}{\mu \times 10 \times 10^{-4}}$$

$$R_{eq} = R_1 + R_2 = 41900 \text{ Ampere-turns/weber}$$

$$\Phi = \frac{F}{R} = \frac{NI}{R_{eq}} = 0.0048 \text{ weber}$$

$\text{Ex } A = 12 \text{ cm}^2$
 $N = 400$
 $\mu_r = 4000$
 $L = 40 \text{ cm}$

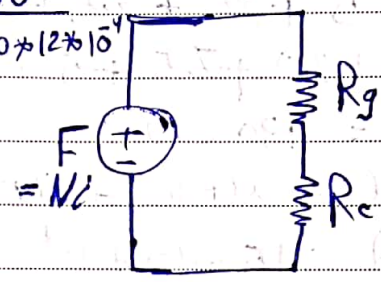


* fringing the air gap increasing Area by 5%.

- find R, i to give $B = 0.5 \text{ Tesla}$ in the air gap?

Sol. $R_c = \frac{L}{\mu A} = \frac{40 \times 10^{-2}}{4\pi \times 10^{-7} \times 4000 \times 12 \times 10^{-4}}$

$R_c = 66300 \text{ A.Turns/wb}$



$R_g = \frac{L}{\mu_0 \mu_r A_g}$

$A_g = 12 + \frac{5}{100} \times 12 = (1.05)(12)$

$R_g = \frac{0.05 \times 10^{-2}}{4\pi \times 10^{-7} (1.05)(12 \times 10^{-2})} = 316000 \text{ A.Turns/wb}$

$F = Ni = \Phi R_{eq}$

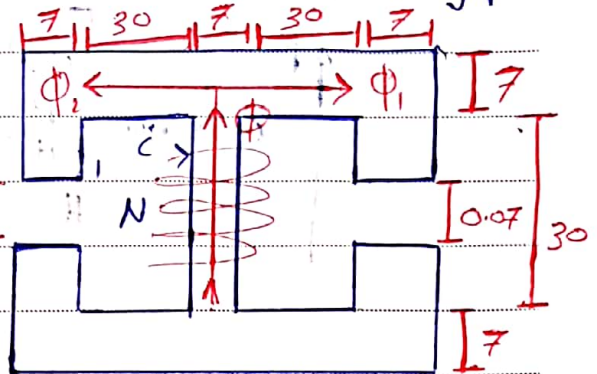
$\Phi = B A_{gap} \rightarrow i = \frac{B A_{gap} R_{eq}}{N} = 0.6 \text{ A}$

Core جي دائرہ میں Air gap کے دائرہ میں Φ کے ساتھ i کی رفتار ~~...~~

* errors in calculation :

- 1] leakage fluxes : fluxes leave core to the air
- 2] μ is no linear with the current (not constant)
- 3] Corner area : R not accurate
- 4] fringing in the air gap that increase Area of the gap

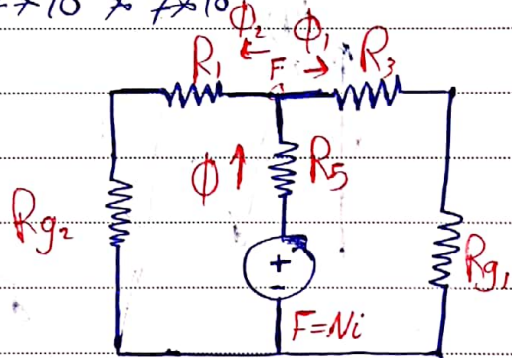
Ex $N=300, i=1A$
 $\mu_r=2000, \text{depth}=7cm$
 fringing increase Area 5%.



Sol. $R_3 = \frac{3.5 + 30 + 3.5 + 3.5 + (30 - 0.07) + 3.5 + 3.5 + 30 + 3.5}{4\pi \times 10^{-7} \times 2000 \times 7 \times 10^{-2} \times 7 \times 10^{-2}} \times 10^2$

$R_3 = 90.1 \text{ KA.Turns/wb}$
 $R_3 = R_1$

$R_5 = \frac{(35 + 30 + 3.5) \times 10^{-2}}{\mu_0 \mu_r (7 \times 7 \times 10^{-4})}$



$R_{g1} = \frac{0.07 \times 10^{-2}}{4\pi \times 10^{-7} (1.05) (7 \times 7 \times 10^{-4})} = 108.3 \text{ K}$

$R_{g2} = \frac{0.05 \times 10^{-2}}{(\mu_0) (1.05) (7 \times 7 \times 10^{-2})} = 77 \text{ K A}$

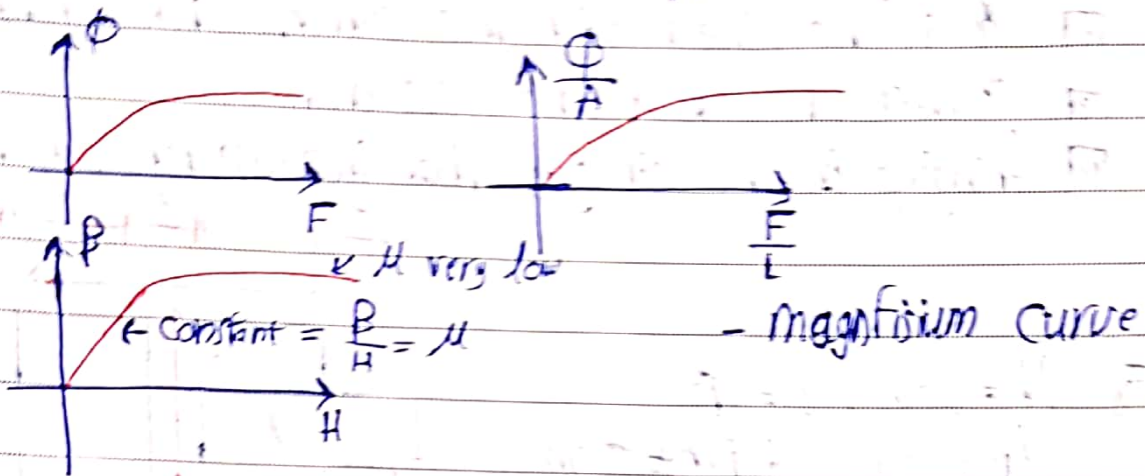
$\Phi = \frac{F}{R_{eq}}, R_{eq} = (R_1 + R_{g2}) \parallel (R_3 + R_{g1}) + R_5 = 120.8 \text{ K}$
 $\Phi = 2.5 \text{ m}$

$\Phi_1 = \frac{R_1 + R_{g2}}{R_1 + R_{g2} + R_3 + R_{g1}} \cdot \Phi, \Phi_2 = \Phi - \Phi_1 = 0.00193$

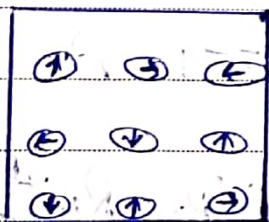
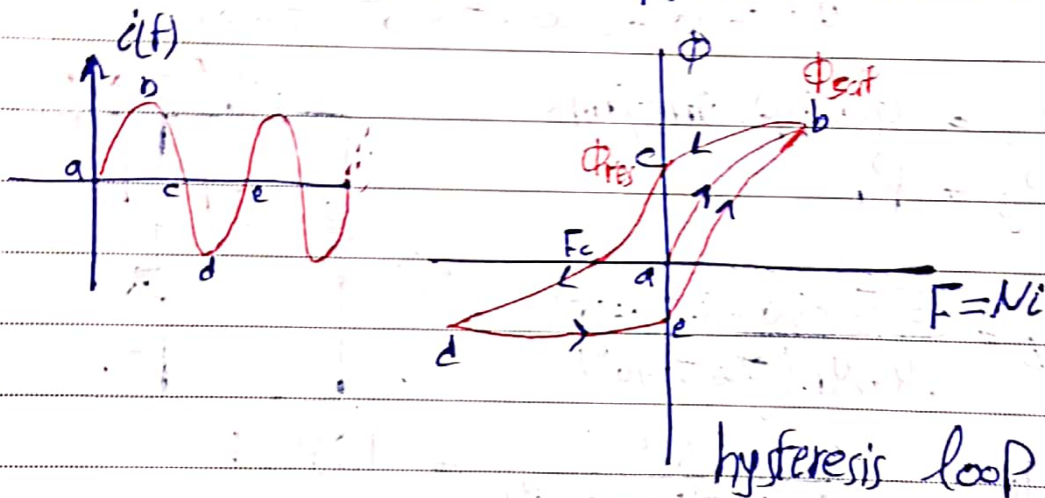
or $\frac{F}{R_1 + R_{g2}} + \frac{F}{R_3 + R_{g1}} + \frac{F - Ni}{R_5} = 0, \Phi = \frac{Ni - F}{R_5}$

* Behavior of ferromagnetic core :

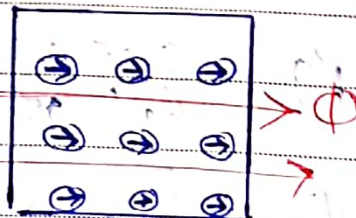
- if an DC current is applied to the core :



- if an AC current is applied to the core :



Randomly $\rightarrow \Phi = 0$



not randomly when Φ is applied

* if an ac current applied produce external flux
 → some domains will oriented toward
 the external flux Randomly

* from a-b, flux increase from a-b
 at Point b → all domains oriented
 toward the external flux → Φ_{set}

* if current decreased from b-c, some domains (not ^{all} of them)
 will reoriented randomly toward the new flux
 until $i = 0$ or in other words:

i decrease from b-c → ϕ decrease from b-c
 but with different path → at Point c, $i = 0$, $\phi \neq 0$

* Φ_{res} : Residual flux = flux remain in the
 core when $i = 0$
 - core → permanent magnet

* at Point i_c : ~~minimum~~ Coercive force → to force the flux
 to be zero

** why hysteresis ?!

→ ferromagnetic material (Iron, steel, nickel)

There are a small regions called domains,

in each region there is a magnetic field,

the direction of the domain flux is randomly oriented

→ $\Phi = 0$

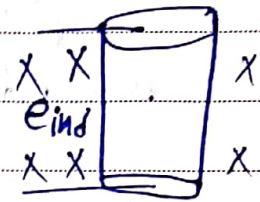
* hysteresis loss : energy required to accomplish the reorientation of domains during each cycle of an AC current

• accomplish : انجام

2 Changing flux \rightarrow e_{ind}

* Faraday's law

$$e_{ind} = -N \frac{d\phi}{dt}$$

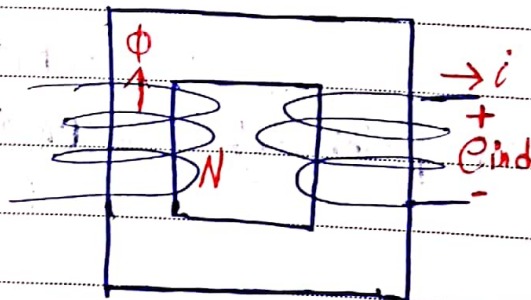


$$e_i = \frac{d\phi_i}{dt} \rightarrow e_{ind} = \sum_{i=1}^N e_i$$

$$e_{ind} = \sum_{i=1}^N \frac{d\phi_i}{dt} = \frac{d}{dt} \sum_{i=1}^N \phi_i$$

$$\lambda = \sum_{i=1}^N \phi_i \rightarrow \text{flux linkage}$$

$$e_{ind} = \frac{d\lambda}{dt}$$



* Time changing fluxes produce induced voltage e_{ind} in the core as in the coil

* these voltages (e_{ind}) ~~cause~~ ^{caused} a swirl of currents in the core like eddies

* eddy currents \rightarrow flow in the core

Resistance \rightarrow caused power loss \rightarrow heating the core

$$P_{loss} = I_{ed}^2 R \text{ (heating loss)}$$

* to reduce \rightarrow eddy current:

1) core broken into strips (laminations)

to decrease the path of currents

$e_{ind} \downarrow$ $i \downarrow$ $P \downarrow$

2) Put insulating material between strips

3) Add silicon material to the core

to increase the core resistivity

$R \uparrow$, $i \downarrow$ $P_{loss} \downarrow$

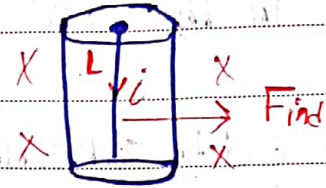
* core losses \rightarrow eddy currents & hysteresis loss

1.6 Production of an induced force in the current carrying wire in presence of a flux

$$F_{ind} = i L \times B$$

$$= i L B \sin \theta$$

if $\theta = 90 \rightarrow F_{ind} = i L B$



* Principle of motor $P_{Ele} \Rightarrow P_{mech}$

$$E_a I_a = \tau \omega$$

1.7 moving wire in presence of a mag. field produce an e_{ind}

$$e_{ind} = (v \times B) \cdot L$$

$$= v B \sin \theta L \cos \phi$$

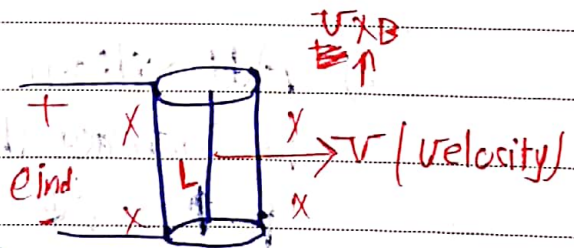
$$v, B$$

$$\theta = 90$$

$$v \times B, L$$

$$\phi = 0$$

So $e_{ind} = v B L$



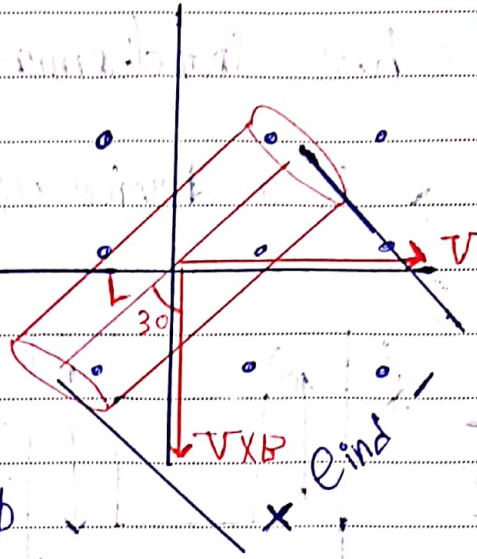
* Principle of generator $P_{mech} \Rightarrow P_{Ele}$

$$v = 10 \text{ m/s}$$

$$B = 0.5 \text{ T (out of page)}$$

$$L = 1 \text{ m}$$

find ϵ_{ind} ?!



Sol. $\epsilon_{\text{ind}} = v \times B \cdot L$

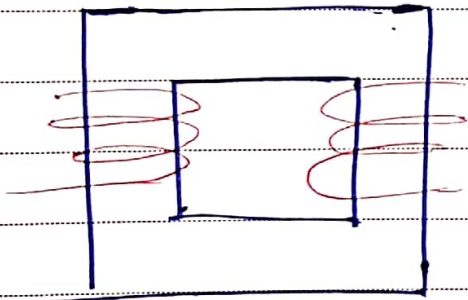
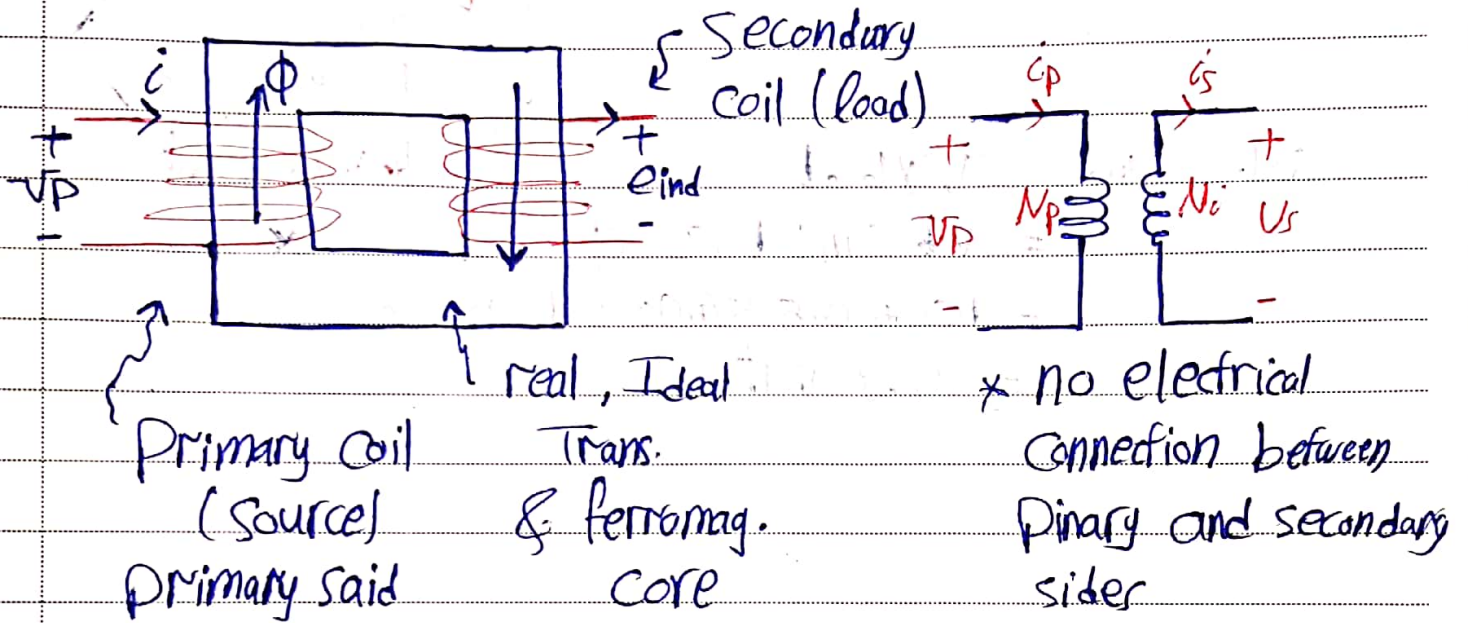
$$= v B \sin \theta L \cos \phi$$

$$= 10 \times 0.5 \sin 90^\circ \times 1 \cos 30^\circ$$

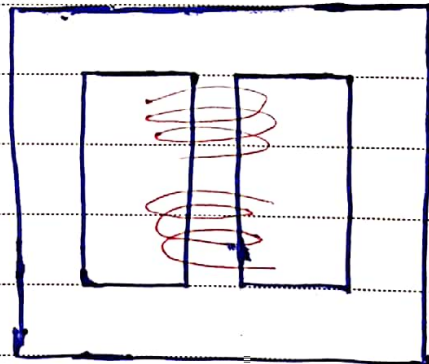
$$= 4.3 \text{ Volt}$$

Ch.2 Transformer

→ is a device used to change voltage level!



Core form



Shell form

$$\boxed{V_P = \frac{N_P}{N_S} V_S} \quad , \quad \frac{N_P}{N_S} = a \quad \text{: Turns Ratio}$$

$$\rightarrow V_P = a V_S$$

$$* \quad \underline{V_S} > \underline{V_P} \rightarrow \text{Step up Transformer}$$

$$a < 1 \quad \text{: } N_P < N_S$$

$$* \quad \underline{V_S} < \underline{V_P} \rightarrow \text{Step down Transformer}$$

$$a > 1 \quad \text{: } N_P > N_S$$

$$* \quad \underline{V_S} = \underline{V_P} \rightarrow \text{Isolation Transformer}$$

$$a = 1 \quad \text{: } N_P = N_S$$

$$N_P I_P = N_S I_S \rightarrow \boxed{I_P = \frac{I_S}{a} = \frac{N_S}{N_P} I_S}$$

$$\boxed{Z_P = \frac{V_P}{I_P} = \frac{a V_S}{I_S / a} = a^2 \frac{V_S}{I_S} = a^2 Z_S}$$

$$\boxed{Z_S = \frac{V_S}{I_S}}$$

$$\boxed{Z_P = a^2 Z_S}$$

Active Power (average)

Reactive Power (Q)

$$P_{in} = V_P I_P \cos \theta$$

$$Q_{in} = V_P I_P \sin \theta$$

$$P_o = V_S I_S \cos \theta$$

$$Q_o = V_S I_S \sin \theta$$

$$P_o = \frac{V_P}{a} \cdot a I_P \cos \theta$$

$$P_o = V_P I_P \cos \theta = P_{in}$$

$$\boxed{Q_o = Q_{in}}$$

$$\boxed{P_o = P_{in}}$$

$$S_{in} = |S| \text{ (apparent Power)}$$

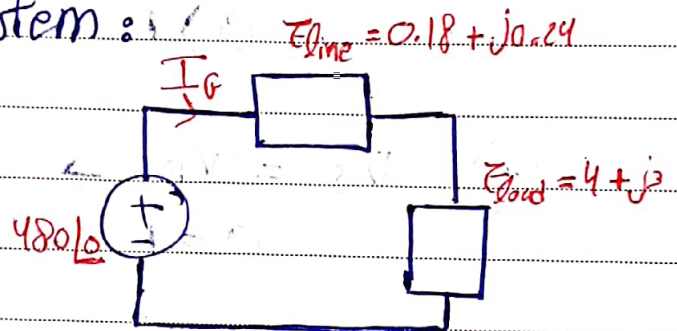
$$S_{in} = V_p I_p$$

$$S_o = V_s I_s$$

$$S_o = S_{in}$$

Ex Single Phase Power system:

find: 1) load voltage
2) transmission line losses



Sol. 1) $V_L = \frac{4 + j3}{4 + j3 + 0.18 + j0.24} \cdot 480 \angle 0$ Transmission line

$$V_L = 454 \angle -0.9 \text{ Volt}$$

2) losses $\rightarrow P_{loss} = P_{line}$
 $I_0 = \frac{480 \angle 0}{E_{line} + E_{load}} = 90.8 \angle -37.8 \text{ A}$

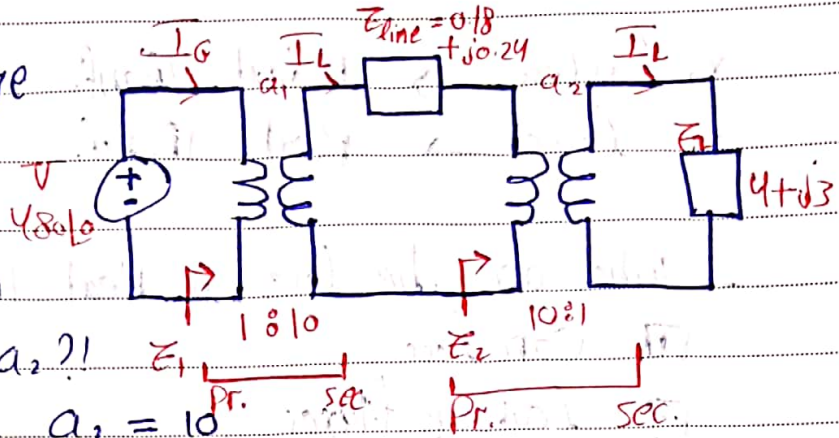
$$P_{loss} = |I_0|^2 R_{line} = (90.8)^2 \cdot 0.18 = 1484 \text{ W}$$

$$\eta = \frac{P_o}{P_{in}} = \frac{P_{load}}{P_{source}}$$

$$P_{in} = P_{source} = P_{load} + P_{line}$$

$$P_{source} = (480)(90.8)$$

find: 1) load voltage
2) power loss



Sol: $E_2 = E_{load}$, a_2 ?
 $a_1 = \frac{1}{10}$, $a_2 = 10$

راج بقول ال P_{loss}

$$E_2 = 400 + j300$$

بوجود ال Trans.

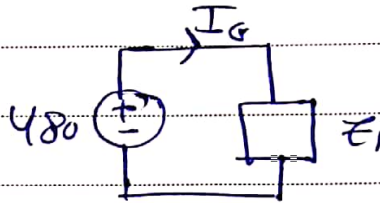
$$E_1 = a_1^2 (E_{line} + E_2)$$

Imped. ال زائد ال

$$E_1 = 4.0018 + j3.0024$$

لا يزيد ال (voltage drop) ال load

$$I_G = \frac{V_G}{E_1} = \frac{480}{4.0018 + j3.0024}$$



$$I_G = 95.94 \angle -36.88 \text{ A}$$

$$I_{line} = a_1 I_{a1} = 95.94 \angle -36.88$$

$$I_{load} = I_{line} a_2 = 95.94 \angle -36.88$$

$$\text{load voltage } V_L = I_{load} E_{load} = 479.7 \angle 0.01$$

$$P_{loss} = |I_{line}|^2 (Z_{line})$$

* lossless \rightarrow Ideal Transf.

* losses \rightarrow Real Transf.

- what are the losses in Real Transf. ?

- 1] Core loss
- 2] leakage fluxes
- 3] copper loss

* lossless \rightarrow Ideal Transf.

* losses \rightarrow Real Transf.

what are the losses in Real Transf. ?

1) Core loss \rightarrow hysteresis loss & heating loss

2) leakage fluxes

3) copper loss

2.5 equivalent of Real Transformer

1) Copper loss : Resistive heating of the primary and the secondary coils (windings)

R_p R_s

2) leakage fluxes losses : leave the core to the air
 - Not all the flux produced by the primary coil pass through the secondary

$$\Phi_p = \Phi_m + \Phi_{lp}$$

Φ_p : Primary Flux

Φ_m : mutual flux in the core

Φ_{lp} : Primary leakage flux
 (linking primary & secondary)

$$v_p(t) = N_p \frac{d\phi_p}{dt} = N_p \frac{d}{dt} [\phi_m + \phi_{lp}]$$

$$v_p(t) = N_p \frac{d\phi_m}{dt} + N_p \frac{d\phi_{lp}}{dt}$$

$$v_p(t) = e_p(t) + e_{lp}(t) \quad \text{for Real Trans.}$$

* for Ideal Transf. $\phi_m \gg \phi_{lp}$

$$v_p(t) = e_p(t) = N_p \frac{d\phi_m}{dt}$$

$$\frac{v_p}{v_s} = \frac{N_p}{N_s} = a$$

* for the secondary side:

$$\phi_s = \phi_m + \phi_{ls}$$

$$v_s(t) = N_s \frac{d\phi_s}{dt} = N_s \frac{d\phi_m}{dt} + N_s \frac{d\phi_{ls}}{dt}$$

$$v_s(t) = e_s(t) + e_{ls}(t) \quad \text{for Real Trans.}$$

$$v_s(t) = N_s \frac{d\phi_m}{dt} \quad \text{for Ideal } \phi_m \gg \phi_{ls}$$

$$v_s(t) = e_s(t)$$

$$* e_{lp}(t) = N_p \frac{d\phi_{lp}}{dt}$$

$$* e_{ls}(t) = N_s \frac{d\phi_{ls}}{dt}$$

$$\phi = \frac{F}{R} \rightarrow \phi_{lp} = \frac{N_p i_p}{R_{air}}$$

$$\phi = \frac{F}{R} \rightarrow \phi_{ls} = \frac{N_s i_s}{R_{air}}$$

$$e_{lp}(t) = \frac{N_p^2}{R} \frac{di_p}{dt}$$

$$e_{ls}(t) = \frac{N_s^2}{R} \frac{di_s}{dt} = L_s \frac{di_s}{dt}$$

* L_p : Primary

leakage Inductance

* leakage fluxes are modelled as L_p, L_s
series with primary & secondary coils Resistance

* Input Current consist of Two components :

$i_{ex} = i_m + i_{hte}$, i_{ex} : excitation current
 in the primary side \rightarrow i_m : Magnetization current

$i_{hte} \rightarrow$ Core loss
 Resistive heat
 so R_c

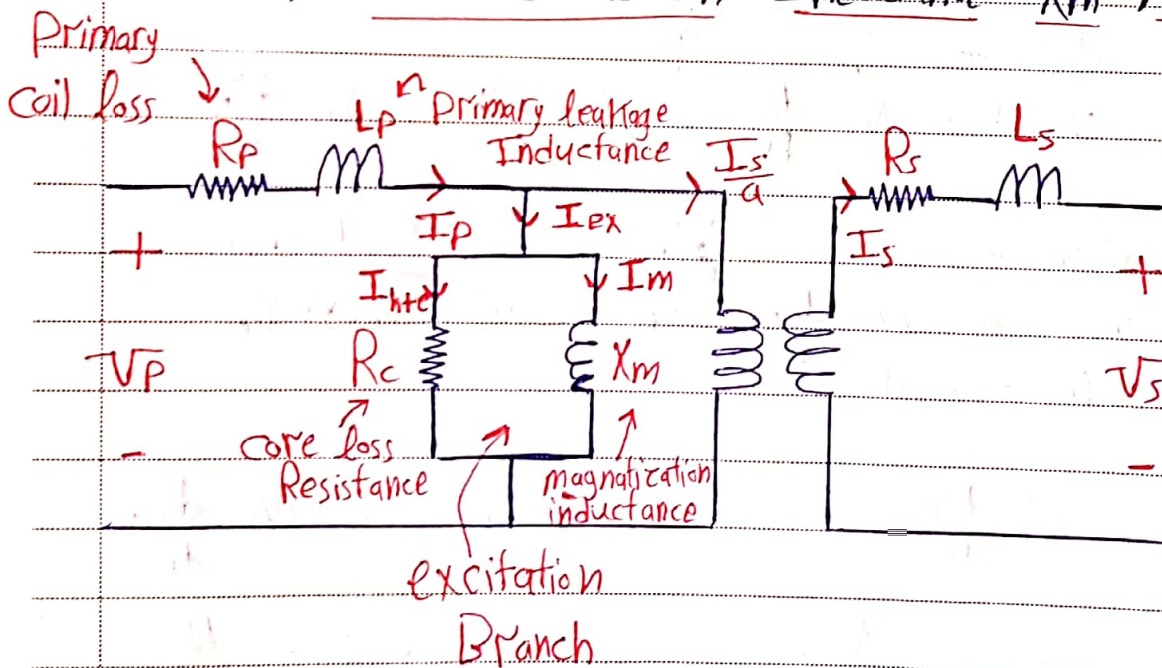
i_{hte} : current required to
 make hysteresis and
 eddy current

$V_p(f) = N_p \cdot \frac{d\Phi_m}{dt}$, Φ_m lag V_p by 90°

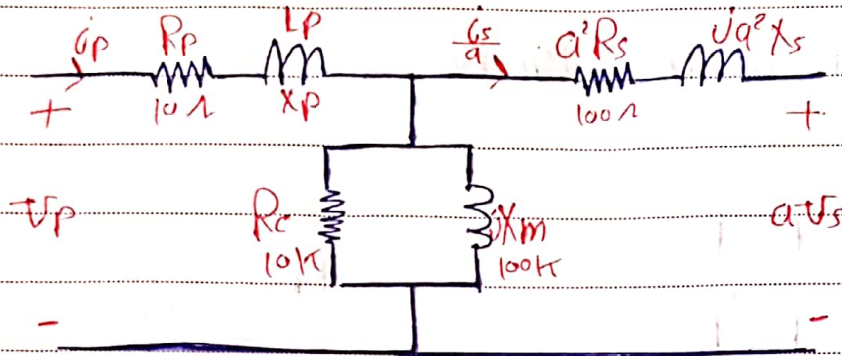
Φ_m in phase I_m

I_m lag V_p by 90°

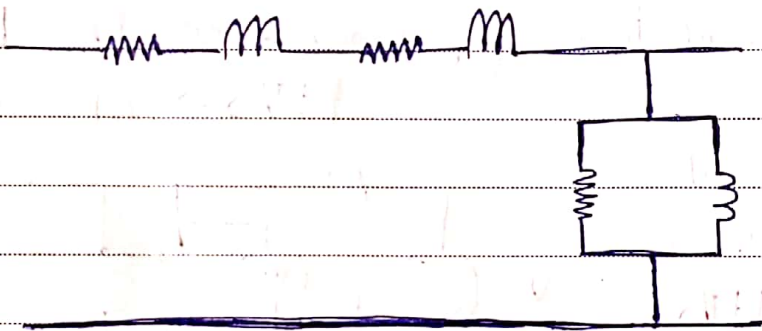
\rightarrow Modeled as an Inductance X_m , L_m



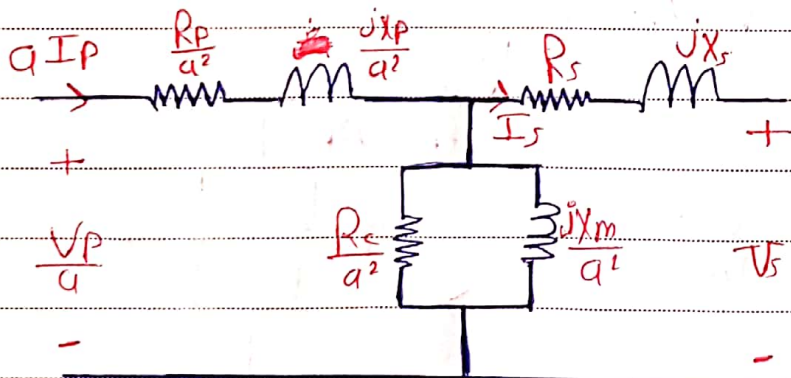
* eq. Referred to Primary side :



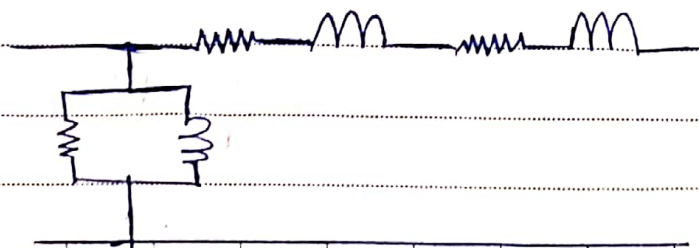
OR
↓



* eq. Referred to secondary :

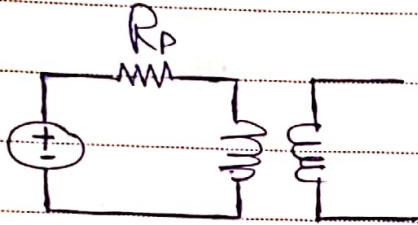


OR
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* To find Transformer Parameters :

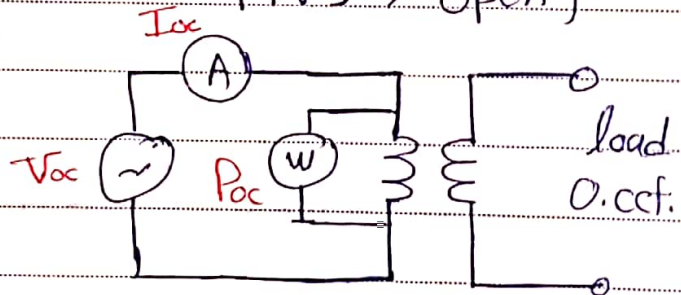
1) DC Test on the Primary



$$R_p = \frac{V_{oc}}{I_{oc}}$$

2) Open circuit test (Performed on the LV side
HTS → open)

$\frac{100V}{1000V}$
 $\frac{V_p}{V_s}$
 LV side HTS



- 1) V_{oc} 2) I_{oc} 3) P_{oc}

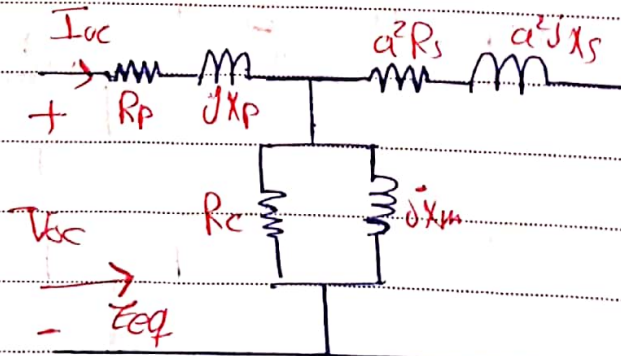
$$Z_{eq} = R_p + jX_p + R_c // jX_m$$

$R_p \ll R_c$

$$Z_{eq} = R_c // jX_m$$

$$V_{oc} = \frac{1}{\frac{1}{R_c} - \frac{j}{X_m}}$$

$$V_{oc} = \frac{I_{oc}}{V_{oc}} \angle -\theta$$

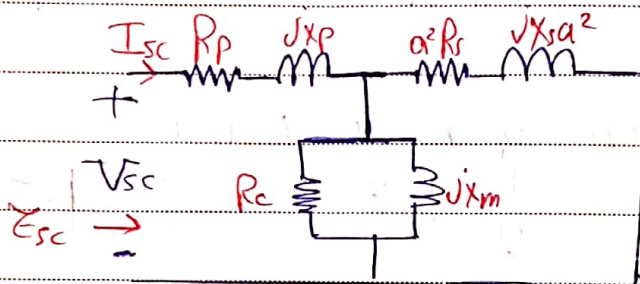
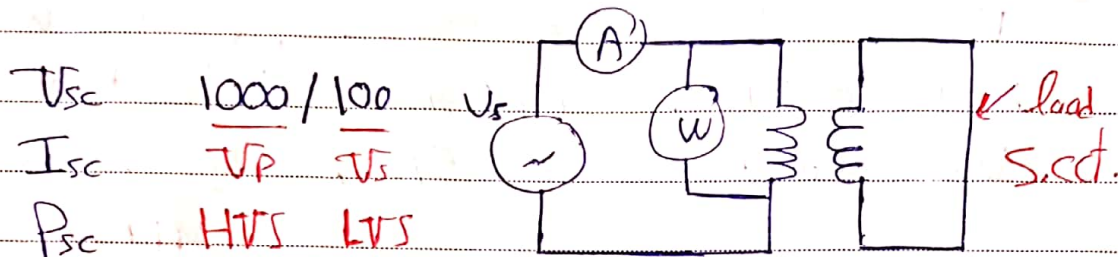


$$P_{oc} = V_{oc} I_{oc} \cos \theta$$

$$\theta = \cos^{-1} \frac{P_{oc}}{V_{oc} I_{oc}}$$

$$Z_{eq} = \frac{V_{oc}}{I_{oc}} \angle (\theta - \phi)$$

3] Short circuit test { Performed on HTS }
 → LVS shorted.



$$Z_{sc} = R_c // jX_m // (a^2 R_s + j a^2 X_s) + R_p + jX_p$$

$R_c, X_m \gg R_s, X_s$

$$P_{sc} = R_p + jX_p + a^2 R_s + j a^2 X_s$$

$$P_{scp} = \underbrace{(R_p + a^2 R_s)}_{R_{eqp}} + j \underbrace{(X_p + a^2 X_s)}_{X_{eqp}}$$

$$P_{scp} = R_{eqp} + j X_{eqp}$$

$$P_{scs} = \underbrace{R_s + \frac{R_p}{a^2}}_{R_{eqs}} + j \underbrace{X_s + \frac{X_p}{a^2}}_{X_{eqs}}$$

$$R_s + \frac{R_p}{a^2} \quad X_s + \frac{X_p}{a^2}$$

$$Z_{sc} = \frac{V_{sc}}{I_{sc}} \angle \theta \rightarrow Z_{sc} = \frac{V_{sc}}{I_{sc}} \cos \theta + j \frac{V_{sc}}{I_{sc}} \sin \theta$$

$$\theta = \cos^{-1} \left(\frac{P_{sc}}{I_{sc} V_{sc}} \right) \quad P_{sc} = V_{sc} I_{sc} \cos \theta$$

Ex 20 kVA , 8000 / 240 , 60 Hz
 Transformer V_P (HVS) V_S (LVS)

Open circuit test (on secondary, LVS)

$$V_{oc} = 240 \text{ V} , I_{oc} = 7.133 \text{ A} , P_{oc} = \underline{400 \text{ W}}$$

Short circuit test (on primary, HVS) P_{core}

$$V_{sc} = 489 \text{ V} , I_{sc} = 2.5 \text{ A} , P_{sc} = \underline{240 \text{ W}}$$

!! Copper loss

find eq. circuit of need Trans. referred to primary?!

Sol. 1) from open circuit test:

$$Y_{oc} = \frac{I_{oc}}{V_{oc}} \angle -\theta = \frac{7.133}{240} \angle -76.5$$

$$\theta = \cos^{-1} \frac{P_{oc}}{V_{oc} I_{oc}} = 76.5$$

$$Y_{oc} = 0.00693 - j0.0288 = \frac{1}{R_s} - \frac{j}{X_{ms}}$$

$$R_s = 144 \Omega \quad X_{ms} = 34.62 \Omega$$

$$a = \frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{8000}{240} = 33.33$$

$$Z_P = a^2 Z_s \rightarrow R_c = a^2 R_s = 159 \text{ k}\Omega$$

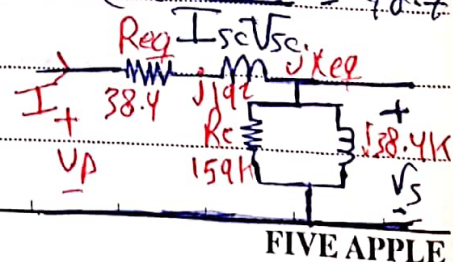
$$X_m = a^2 X_{ms} = 38.4 \text{ k}\Omega$$

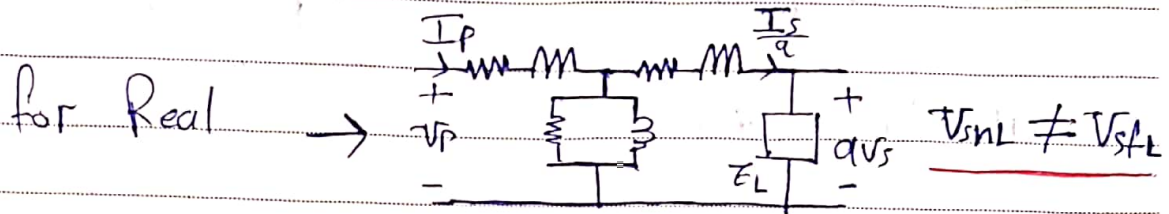
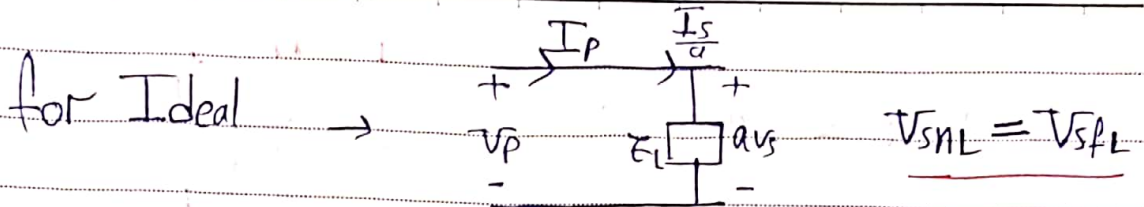
e) from short circuit test:

$$Z_{sc} = \frac{V_{sc}}{I_{sc}} \angle \theta = \frac{489}{2.5} \angle 78.7 , \theta = \cos^{-1} \frac{P_{sc}}{I_{sc} V_{sc}} = 78.7$$

$$Z_{sc} = 38.4 + j192$$

$$R_p + a^2 R_s \quad \leftarrow \quad \rightarrow \quad X_p + a^2 X_s$$





2.7 Voltage Regulation V_R

V_R : quantity that compare the output voltage at ~~load~~ load with output voltage at no load for Trans.

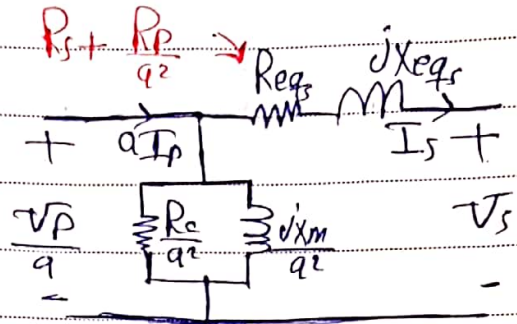
$$V_R = \frac{V_{sNL} - V_{sFL}}{V_{sFL}} \times 100\%$$

for Ideal : $V_R = 0\% \rightarrow V_{sNL} = V_{sFL}$

for Real : $V_{sNL} \neq V_{sFL}$
 $V_{sNL} = \frac{V_p}{a}$

$$V_{sNL} = \frac{VP}{a}$$

$V_s \rightarrow V_{sFL} \rightarrow$ Rating Value



□ for lagging pf

$$(R_{eqs} + jX_{eqs}) I_s + V_s - \frac{VP}{a} = 0$$

$$\frac{VP}{a} = V_{sNL} = (R_{eqs} + jX_{eqs}) I_s + V_{sFL}$$

$V_{sFL} \rightarrow V_{sRated}$

$$S \cos \phi = V_s I_s \cos \phi$$

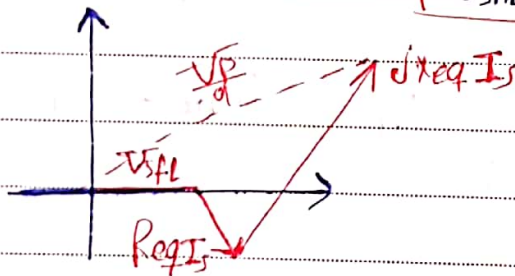
$$I_{sRated} = \frac{S}{V_s} \cos \phi, \quad \theta = \cos^{-1} pf$$

$$\frac{S \cos \phi}{V_s \cos \phi} = I_s \cos \phi$$

$$\frac{S \cos \phi}{V_s} = I_s \cos \phi$$

$$VR = \frac{V_{sNL} - V_{sFL}}{V_{sFL}} \times 100\%$$

$VR > 0\%$
 $V_{sNL} > V_{sFL}$



□ for Unity Power factor

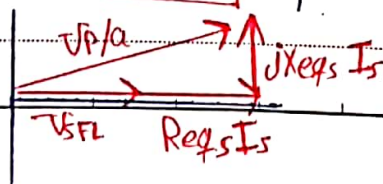
$$\frac{VP}{a} = (R_{eqs} + jX_{eqs}) I_s + V_{sFL}$$

$V_{sFL} \rightarrow V_{sRated}$

$$I_{sRated} = \frac{S}{V_{sFL}}$$

$V_{sNL} > V_{sFL} \rightarrow VR > 0$

$$VR = \frac{V_{sNL} - V_{sFL}}{V_{sFL}} > 0$$



③ for leading pf

$$\frac{V_p}{a} = (R_{eq_s} + jX_{eq_s}) I_s + V_{sfl}$$

$V_{sfl} \rightarrow V_{Rating}$

$$I_s = \frac{S}{V_{Rated}} \cos \theta$$

$$V_{sNL} < V_{sfl} \rightarrow V_R < 0$$

* efficiency $\eta = \frac{P_o}{P_{in}} \times 100\%$

$$P_o = \underbrace{V_s}_{\text{Full load } V_{sfl} \text{ (rated value)}} I_s \cos \theta, \quad P_{in} = P_o + P_{loss}$$

$$I_s = \frac{S}{V_{rated}}$$

$$P_{loss} = P_{copper} + P_{core}$$

$$P_{copper} = |I_s|^2 R_{eq_s}$$

$$P_{core} = \frac{\left(\frac{V_p}{a}\right)^2}{R_c} = \frac{V_p^2 a^2}{a^2 R_c} \Rightarrow P_{core} = \frac{V_p^2}{R_c}$$

Ex 15kVA , 2300 / 230 V
 Open ckt. test (LVS): $V_{oc} = 230 V$

$I_{oc} = 2.1 A$ $P_{oc} = 50 W$

Short ckt. test (HVS):

$V_{sc} = 47 V$ $I_{sc} = 6 A$ $P_{sc} = 160 W$

- find :
- eq. ckt. Referred to LVS, HVS
 - VR at 0.8 lag, 0.8 lead, Unity PF
 - % at 0.8 lagging PF

Sol. □ for O.ckt.T :

$Y_{oc} = \frac{I_{oc}}{V_{oc}} \angle -\theta = \frac{1}{R_c} - \frac{j}{X_m}$, $\theta = \cos^{-1} \frac{P_{oc}}{V_{oc} I_{oc}} = 84^\circ$
 $\alpha = 10$

$R_{cs} = 1050 \Omega \rightarrow R_c = \alpha^2 R_{cs} = 105 k\Omega$

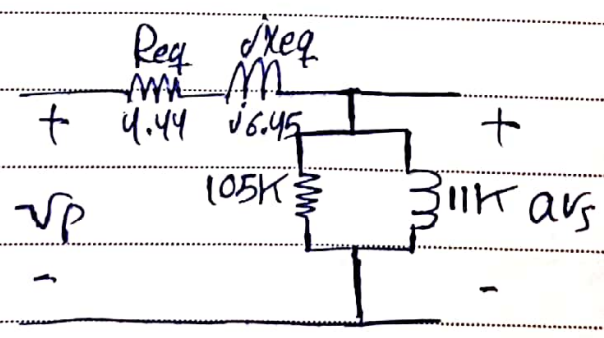
$X_{ms} = 110 \Omega \rightarrow X_m = \alpha^2 X_{ms} = 11 k\Omega$

for S.ckt.T :

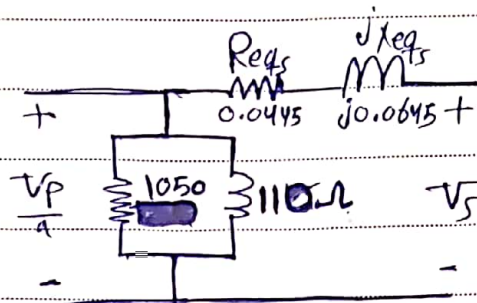
$Z_{sc} = \frac{V_{sc}}{I_{sc}} \angle \theta$, $\theta = \cos^{-1} \frac{P_{sc}}{V_{sc} I_{sc}}$

$Z_{sc} = R_{eqp} + jX_{eqp}$, $R_{eqp} = 4.4$
 $X_{eqp} = 6.45$

A) eq. referred to Primary :



2) eq. referred to secondary :



$$2) \quad VR = \frac{V_{SNL} - V_{SFL}}{V_{SFL}} \times 100\%$$

$$\frac{V_p}{a} = (R_{eq_s} + jX_{eq_s}) I_s + V_{SFL}, \quad V_{SFL} = 230V$$

$$I_s = \frac{S}{V_s} = \frac{15k}{230} = 65.2 A$$

A) $Pf = 0.8$ lagging :

$$\theta = \cos^{-1}(0.8) = +36.87$$

$$I_s = 65.2 \angle 36.87^\circ$$

$$\frac{V_p}{a} = 234.85 \angle 0.4^\circ \rightarrow VR = 2.1\%$$

B) $Pf = 0.8$ leading :

$$\theta = \cos^{-1}(0.8) = -36.87$$

$$I_s = 65.2 \angle -36.87^\circ$$

$$\frac{V_p}{a} = 229.85 \angle 11.24^\circ V, \quad VR = -0.062\%$$

C) $Pf = 1$ unity :

$$\theta = \cos^{-1}(1) = 0$$

$$I_s = 65.2 \angle 0^\circ$$

$$\frac{V_p}{a} = 233 \angle 0^\circ, \quad VR = 1.28\%$$

3] at 0.8 lagging, η ?

$$\eta = \frac{P_o}{P_{in}} \times 100\%$$

$$P_o = V_s I_s \cos \phi = (230)(65.2)(0.8) \rightarrow P_o = 12 \text{ kW}$$

151 of \approx \leftarrow
Trans

$$P_{in} = P_o + P_{loss}$$

$$P_{loss} = P_{core} + P_{copper}$$

$$P_{core} = \frac{(V_p)^2}{R_{cs}} = \frac{(V_{sNL})^2}{R_{cs}} = 52.5 \text{ W} \approx P_{oc}$$

$$P_{copper} = I_s^2 R_{eqs} = (65.2)^2 (0.044) = 189 \text{ W}$$

$\approx P_{sc}$

$$P_{loss} = 52.5 + 189 = 241.5$$

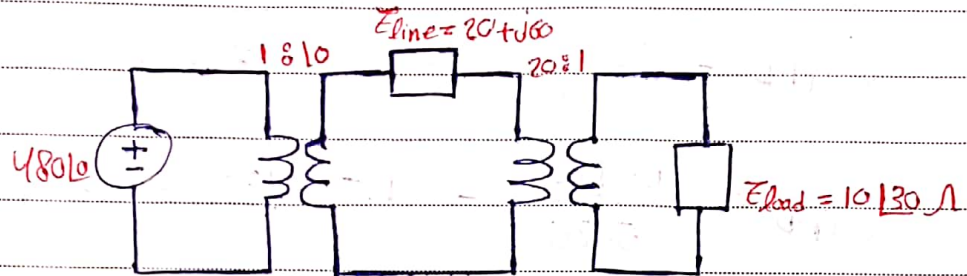
$$\eta = \frac{12 \text{ kW}}{12 \text{ kW} + 241.5} \times 100 = 98.03\%$$

2.6 Per unit

→ electrical quantity is measured as a fraction of base level

$$\text{Quantity per unit} = \frac{\text{Actual value}}{\text{Base value}}$$

Ex



given : $S = 10 \text{ KVA}$ $a_1 = \frac{1}{10}$ $a_2 = 20$
 find : P_{load} , P_{loss} Using PU ?!

$$S_{\text{base 1}} = 10 \text{ KVA} \quad I_{b1} = \frac{S_{\text{base 1}}}{V_{\text{base 1}}} = 20.83 \text{ A}$$

$$V_{\text{base 1}} = 480 \text{ V}$$

$$Z_{b1} = \frac{V_{\text{base 1}}}{I_{\text{base 1}}} = 23.04 \Omega$$

$$S_{b2} = 10 \text{ KVA}$$

$$V_{b2} = 4800 \quad \left\{ V_{b1} = a V_{b2} \right\}$$

$$I_{b2} = \frac{S_{b2}}{V_{b2}} = 2.083 \text{ A}$$

$$Z_{b2} = \frac{V_{b2}}{I_{b2}} = 2304 \Omega$$

$$\text{or } Z_{b2} = \frac{Z_{b1}}{a^2}$$

$$S_{b3} = 10 \text{ kVA}$$

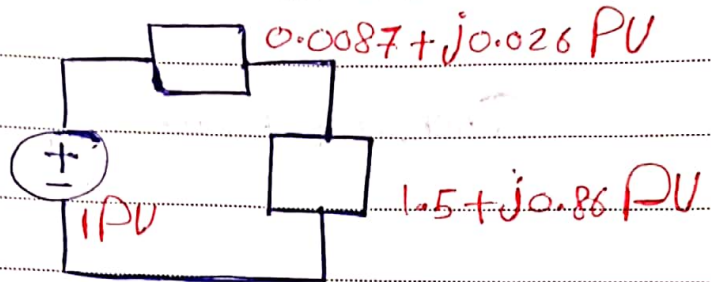
$$V_{b3} = \frac{V_{b2}}{a_2} = 240 \text{ V}$$

$$I_{b3} = \frac{S_{b3}}{V_{b3}} = 41.67$$

$$Z_{b3} = \frac{V_{b2}}{I_{b3}} = 5.76 \Omega$$

$$Q_{\text{PU}} = \frac{\text{Actual}}{\text{Base}}$$

$$V_{\text{QPU}} = \frac{480}{480} = 1 \text{ PU}$$



$$Z_{\text{line QPU}} = \frac{20 + j60}{2304} = 0.0087 + j0.026 \text{ PU}$$

$$Z_{\text{load QPU}} = \frac{10 \angle 30}{5.76} = 1.736 \angle 30 = 1.5 + j0.86 \text{ PU}$$

$$I_{\text{QPU}} = \frac{V_{\text{QPU}}}{(Z_{\text{line}} + Z_{\text{load}})_{\text{QPU}}} = \frac{1}{0.0087 + j0.026 + 1.5 + j0.86} = 0.57 \angle -30.6 \text{ PU}$$

$$P_{\text{loss PU}} = |I_{\text{PU}}|^2 R_{\text{line}} = (0.57)^2 (0.0087) = 0.00282 \text{ PU}$$

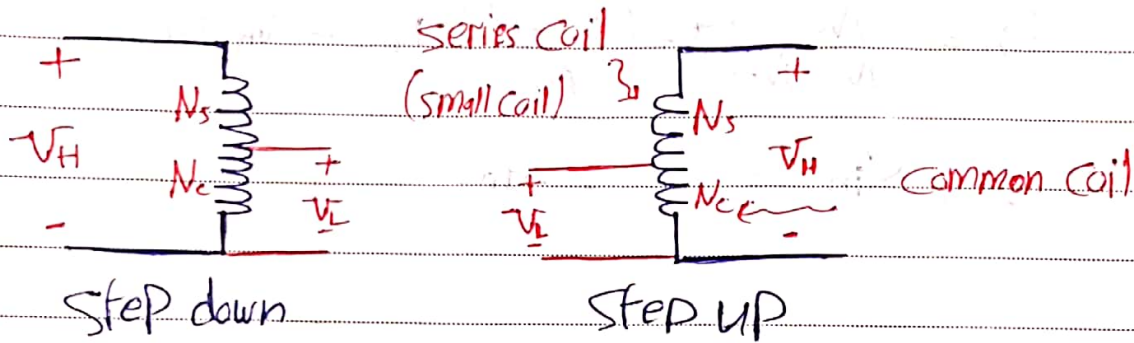
$$P_{\text{load PU}} = |I_{\text{PU}}|^2 R_{\text{load}} = 0.487 \text{ PU}$$

$$\boxed{*} P_{\text{loss}} = P_{\text{loss PU}} * P_{\text{base}} = 0.00282 * 10 \text{ k} = 28.2 \text{ W}$$

$$\boxed{*} P_{\text{load}} = P_{\text{load PU}} * P_{\text{base}} = 0.487 * 10 \text{ k} = 4870 \text{ W}$$

2.9 Auto Transformer

→ To change the voltage level by small amount (110-120)V



electrical connection between primary & secondary

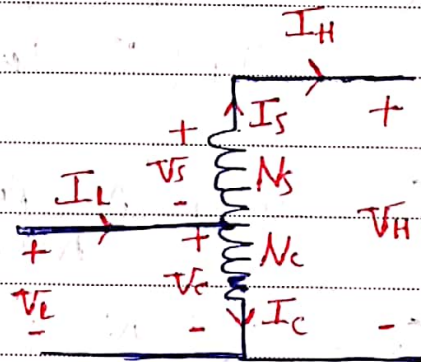
$$\ast V_L = V_C$$

$$V_H = V_S + V_C$$

$$\frac{V_C}{V_S} = \frac{N_c}{N_s}$$

$$V_H = V_L + \frac{N_s}{N_c} V_C$$

$$V_H = \frac{N_c + N_s}{N_c} V_L \rightarrow \boxed{\frac{V_L}{V_H} = \frac{N_c}{N_c + N_s}}$$



$$\ast I_H = I_S$$

$$I_L = I_C + I_S$$

$$N_c I_C = N_s I_S$$

$$I_L = I_C + I_S$$

$$= \frac{N_s}{N_c} I_S + I_S$$

$$I_L = \left(\frac{N_s + N_c}{N_c} \right) I_H$$

$$\boxed{\frac{I_L}{I_H} = \frac{N_s + N_c}{N_c}}$$

* Input output Power :

$$S_{in} = V_L I_L$$

$$S_o = V_H I_H$$

$$S_{in} = \left(\frac{N_c}{N_c + N_s} \right) V_H \left(\frac{N_s + N_c}{N_c} \right) I_H = V_H I_H$$

$$\underline{S_o} \quad \boxed{S_{in} = S_o = S_{io}}$$

* Winding power :

$$\begin{aligned} S_w &= V_c I_c = V_s I_s = V_L (I_L - I_s) \\ &= V_L (I_L - I_H) = V_L \left(I_L - \frac{N_c}{N_c + N_s} I_L \right) \end{aligned}$$

$$S_w = V_L I_L \left(\frac{N_c + N_s - N_c}{N_c + N_s} \right) = V_L I_L \left(\frac{N_s}{N_c + N_s} \right)$$

$$\boxed{S_w = S_{io} \left(\frac{N_s}{N_c + N_s} \right)}$$

$$\underline{S_{io} > S_w}$$

$$\frac{V_L}{V_H} = \frac{N_c}{N_c + N_s}$$

$$\boxed{S_{io} = S_w \left(\frac{N_c + N_s}{N_s} \right)}$$

$$S_w = S_{io} \left(\frac{N_s}{N_s + N_c} \right)$$

* Advantages :

- 1) Auto transformer handle much more power than conventional transformer \rightarrow reasons:
 - Ⓐ electrical connection (much more power rating)
 - Ⓑ low impedance
- 2) Smaller than conventional transformer
- 3) less expensive

* Disadvantages :

- 1) No electrical isolation
- 2) Z_{eq} (Impedance) small than conventional transformer

$$Z_{eq} = \frac{N_s}{N_s + N_c} Z_{eq} \leftarrow \text{prove!}$$

Ex 5000 KVA, Auto transformer connecting
 $\overset{S_{IO}}{110\text{KV}}$ to $\overset{V_L}{138\text{KV}}$
 find power rating for winding ?!

Sol. $S_w = \frac{N_s}{N_s + N_c} S_{IO} = 1015 \text{ KVA}$

$$\frac{V_L}{V_H} = \frac{110\text{KV}}{138\text{KV}} = \frac{N_c}{N_c + N_s}, \quad N_c = 110$$

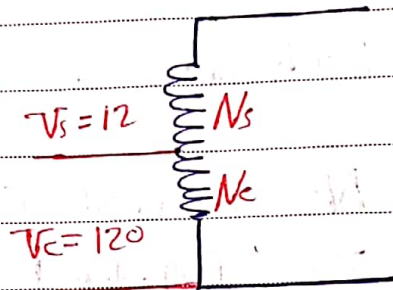
$$N_s = 28$$

Ex 100 VA , 120 V , 12 V Transformer connected as an Auto transformer (step up)

- 1) find secondary voltage ?!
- 2) Power rating ?!

Sol. 1) $\frac{V_P}{V_S} = \frac{N_P}{N_S}$

$$\frac{N_P}{N_S} = \frac{120}{12} = \frac{N_C}{N_S}$$



$$V_H = \frac{N_C + N_S}{N_C} \times \frac{V_L}{120}$$

$$V_H = 132 \text{ V}$$

$$2) S_{IO} = \frac{N_C + N_S}{N_S} S_w = \frac{120 + 12}{12} \times 100$$

$$S_{IO} = 1100 \text{ VA}$$

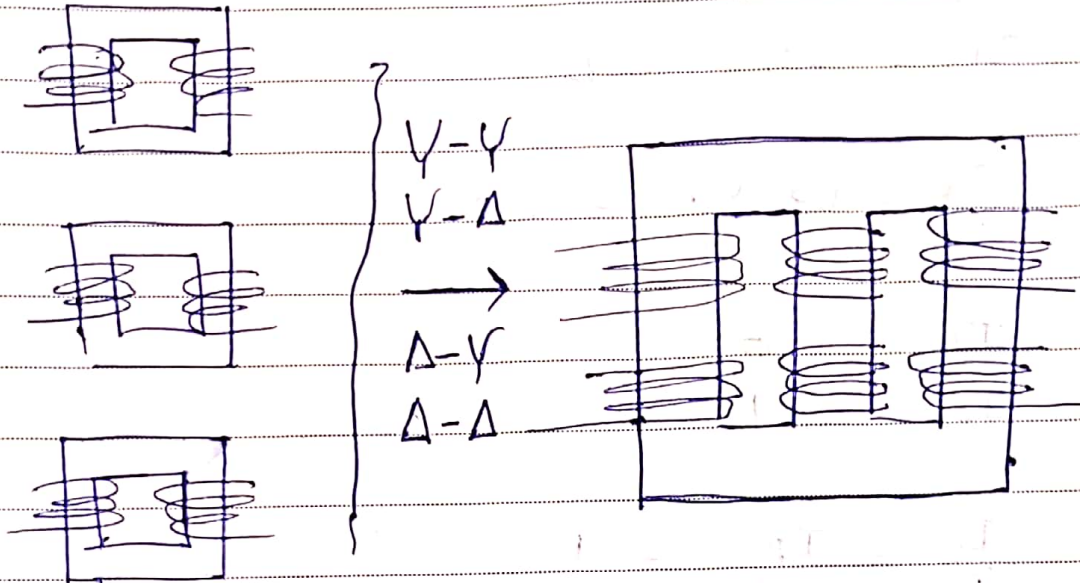
× اذى اىلىك من اوتلىك لىكلىك اتىكلىك : Auto transformer

- 1) S_{IO}
- 2) V_L
- 3) V_H

× اذى اىلىك من اوتلىك لىكلىك اتىكلىك : Transformer

- 1) S_w
- 2) V_S
- 3) V_P

2.10 3-Phase Transformer



Y-Y

$$V_{LP} = \sqrt{3} V_{\phi P} \rightarrow Y$$

$$V_{LS} = \sqrt{3} V_{\phi S} \rightarrow Y$$

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = \frac{N_P}{N_S} = a$$

$$\frac{V_{LP}}{V_{LS}} = a$$

$$I_L = I_{\phi}$$

$$\frac{I_{LP}}{I_{LS}} = \frac{1}{a} = \frac{I_{\phi P}}{I_{\phi S}}$$

Y-Δ

$$V_{LP} = \sqrt{3} V_{\phi P} \dots Y$$

$$V_{LS} = V_{\phi S} \dots \Delta$$

$$\frac{V_{LP}}{V_{LS}} = \sqrt{3} \frac{V_{\phi P}}{V_{\phi S}} = \sqrt{3} a$$

$$I_{LP} = I_{\phi P} \dots Y$$

$$I_{LS} = \sqrt{3} I_{\phi S}$$

$$\frac{I_{LP}}{I_{LS}} = \frac{I_{\phi P}}{\sqrt{3} I_{\phi S}} = \frac{1}{\sqrt{3} a}$$

NO.

سوالوں کے مسائل اور مسائل کے سوالوں کے مسائل

Suggested Problem : 1, 2, 4, 6, 8, 14

16, 23

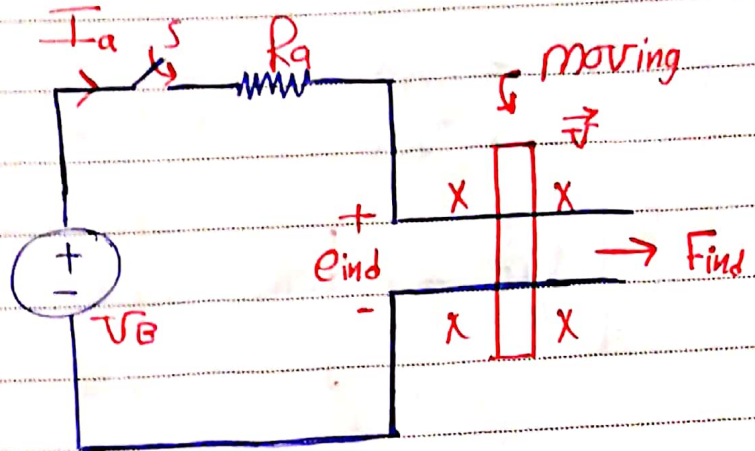
+ 3, 4, 5, 6, 8, 9

DC Master

1.8 linear DC machines

S → Closed

$$I_{\text{starting}} = \frac{V_B}{R_a}$$



⊗ Current carrying wire
in presence of mag. field :

field → $F_{\text{ind}} = I_a L B$ to the right

⊗ wire move to the right → velocity (\vec{v})

⊗ moving wire in presence of mag. field :

$$\rightarrow e_{\text{ind}} \rightarrow e_{\text{ind}} = \vec{v} \times B L$$

$$\downarrow I_a = \frac{V_B - e_{\text{ind}}}{R_a}, \quad \begin{array}{l} e_{\text{ind}} \text{ increase} \\ I_a \text{ will decrease} \end{array}$$

$$\downarrow F_{\text{ind}} = \downarrow I_a L B, \quad \begin{array}{l} I_a \text{ decrease} \\ F_{\text{ind}} \text{ will decrease} \end{array}$$

$$e_{\text{ind}} \uparrow \text{ until } \underline{e_{\text{ind}} = V_B} \rightarrow I_a = \frac{V_B - e_{\text{ind}}}{R_a} = 0$$

$$\rightarrow F_{\text{ind}} = I_a L B = 0$$

∴ bar move with constant speed

$$V_B = e_{\text{ind}} = \vec{v} \times B L$$

$$\vec{v}_{\text{ss}} = \frac{V_B}{B L}$$

*) Principle of motor with no load :

$$I_a \uparrow \rightarrow F_{ind} \uparrow \rightarrow \vec{v} \uparrow \rightarrow e_{ind} \uparrow \rightarrow I_a \downarrow \rightarrow F_{ind} \downarrow$$

after long time $\rightarrow \underline{V_B = e_{ind}}$

$$I_a = 0, \quad F_{ind} = 0$$

$$\vec{v} = \vec{v}_{sc} = \frac{V_B}{BL}$$

*) With load (Motor with load) :

$F_{load} \rightarrow$ to the left opposing direction of motion

$$F_{load} \uparrow \rightarrow \text{velocity } \vec{v} \downarrow \rightarrow \underline{e_{ind} \downarrow < V_B}$$

$$\rightarrow \uparrow I_a = \frac{V_B - e_{ind} \downarrow}{R_a} \rightarrow \uparrow F_{ind} = I_a LB \text{ to right}$$

until $F_{ind} = F_{load} \rightarrow$ bar move with constant velocity
 $\vec{v} < \vec{v}_{sc}$

*) Principle of generator : Mech. Power \rightarrow Ele. Power

$$\boxed{F \vec{v} = e_{ind} I_a}$$

F_{app} with the direction of motion

$F_{app} \rightarrow$ to right

$$\text{Velocity } \vec{v} \uparrow \rightarrow \uparrow e_{ind} = V_B L \rightarrow \underline{e_{ind} > V_B}$$

$$I_a = \frac{e_{ind} - V_B}{R_a} \rightarrow \text{Current in opposite direction}$$

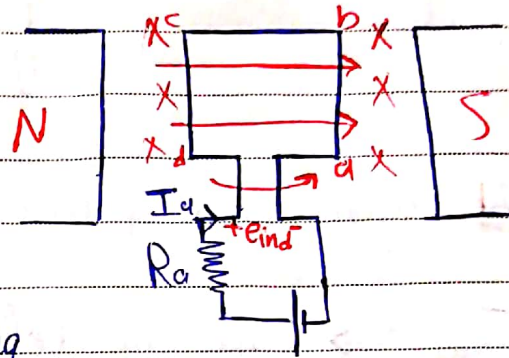
$$I_a \uparrow \rightarrow \uparrow F_{ind} = I_a LB, \text{ until } F_{ind} = F_{app}$$

Ch. 7 DC machines fundamental

DC , AC
 comutator & AC → DC

DC :

- Simple rotating loop around fixed axis (shaft)
- Rotating part (loop) → Rotor consist of armature winding in which voltage induced

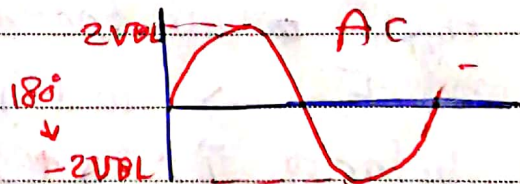


- Stationary part (fixed) → stator consist of field winding is produce the flux

$E_{ind} = V \theta \sin \theta L \cos \phi$

- bars ab, cd → $E_{ind} \rightarrow$ bar $\perp \Phi$
 bc, ad → $E_{ind} = 0 \rightarrow$ bar $\parallel \Phi$

$E_{ind} = 2 \vec{V} \theta L$



$E_a = 2 \vec{V} \theta L$, $\vec{V} = \omega w$, $\theta = \frac{\Phi}{A}$

$E_a = 2 \frac{\Phi}{A} \omega w L \rightarrow E_a = \frac{2 \omega L}{A} \Phi w$

→ $E_a = K \Phi \omega$, $\omega = \frac{2\pi n}{60}$
 → $E_a = K \Phi n$

$$e_{ind} = \frac{z}{a} \vec{v} \cdot \vec{B} L, \quad z: \# \text{ of conductors}$$

$a: \# \text{ of current paths}$

$$\rightarrow e_{ind} = \frac{z}{a} \vec{v} \cdot \vec{B} L$$

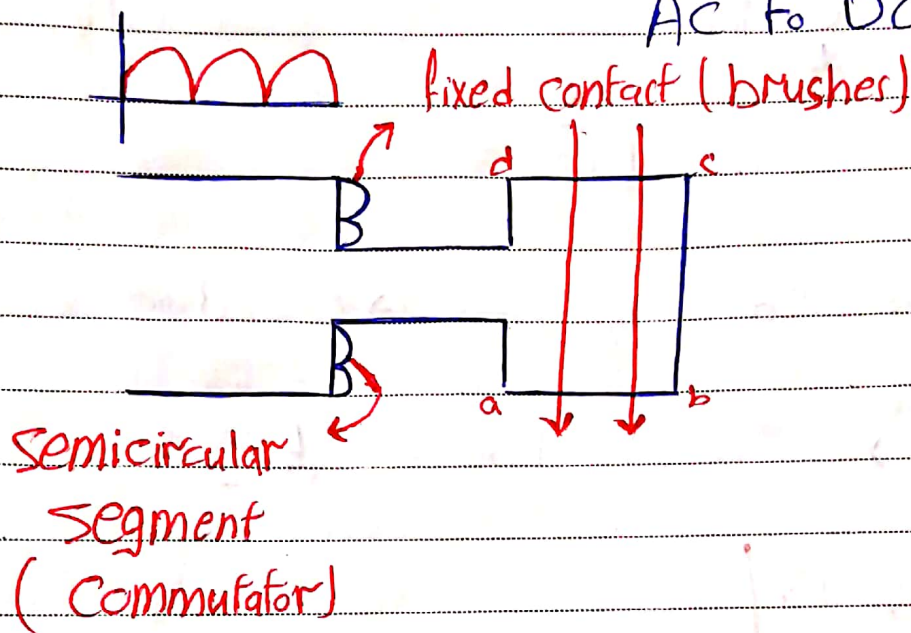
Torque $\tau = N F \sin \theta$

$$\tau \rightarrow ab, cd, \quad \tau = 0 \rightarrow bc, da$$

$$F = iLB \rightarrow \tau = 2rcaLB = \frac{2rL}{A} \Phi Ia$$

$$\tau = K \Phi Ia$$

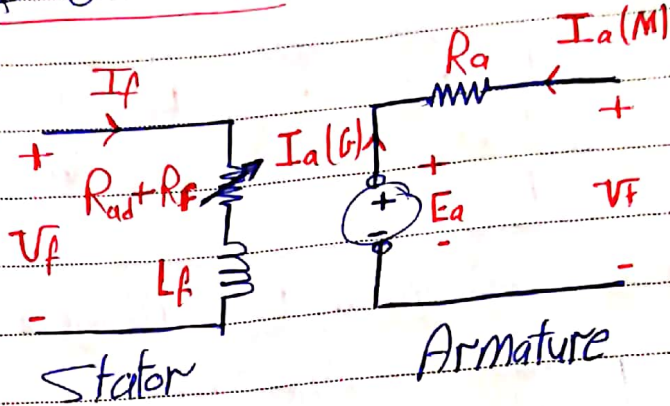
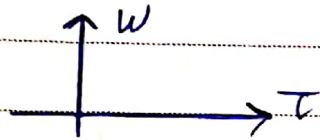
Commutation Process : To convert output of DC machines from AC to DC



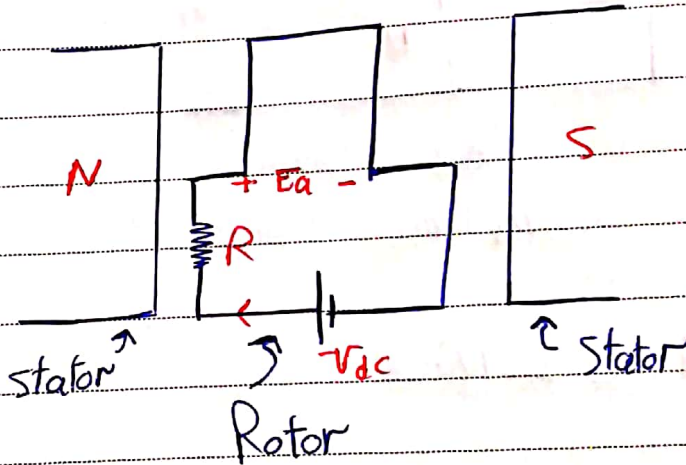
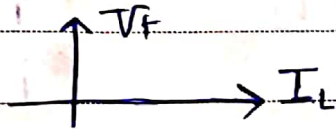
- Two semicircular conducting segment (commutator) added to the end of the loop
- Two fixed contact (brushes) are set up at an angle, such that when the voltage at the loop zero \rightarrow the contact short circuit
- output from the contact always built up in the same direction

Ch. 8 DC Motors & Generators

* for Motor :

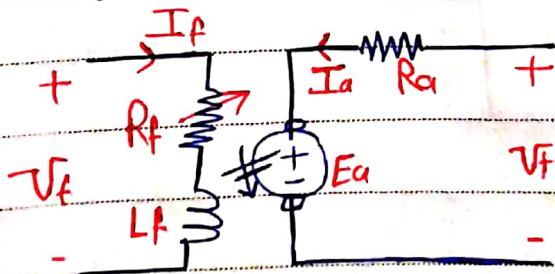


* for generator :

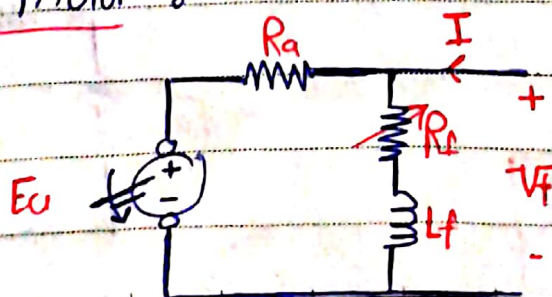


Motors types :

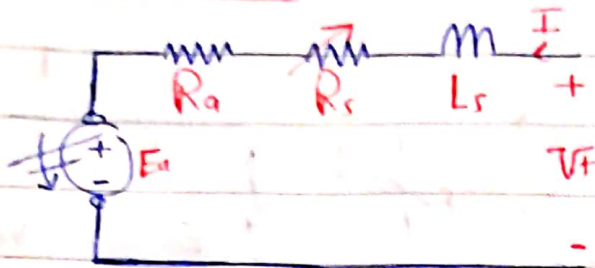
(1) Separately excited Motor :



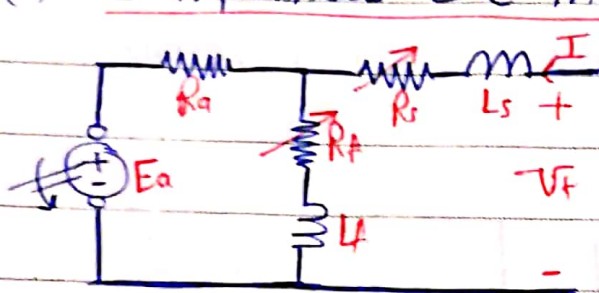
(2) Shunt Motor :



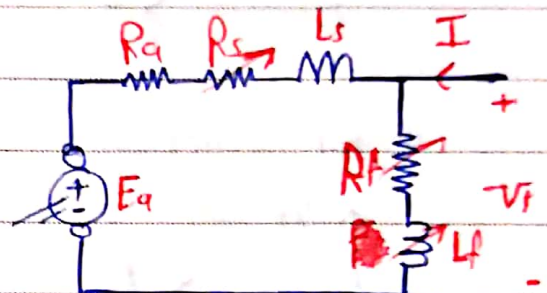
(3) Series Motor :



(4) Compounded DC Motor :



Short Shunt
Compounded Motor



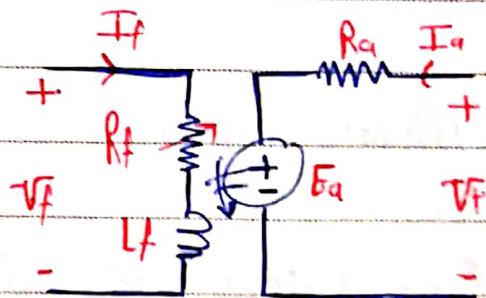
Long Shunt
Compounded Motor

* S.E.M :

$$I_f = \frac{V_f}{R_f} \rightarrow \text{disc (DC)}$$

$$V_f = E_a + I_a R_a$$

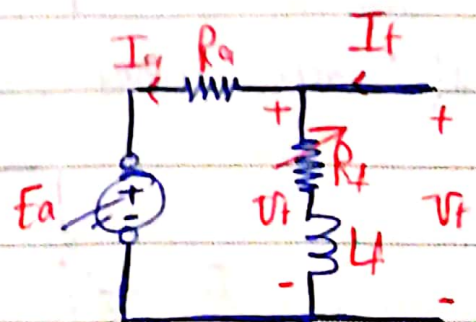
$$E_a = K \Phi n$$



* Shunt Motor :

$$I_f = \frac{V_f}{R_f}$$

$$V_f = E_a + I_a R_a$$

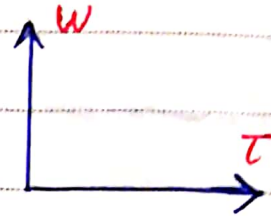


Terminal Characteristic

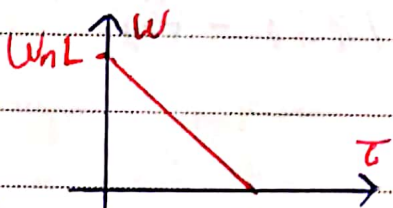
$$V_F = E_a + I_a R_a$$

$$E_a = K\phi\omega, \quad \tau_{ind} = K\phi I_a$$

$$V_F = K\phi\omega + \frac{R_a}{K\phi} \tau_{ind}$$



$$\omega = \frac{V_F}{K\phi} - \frac{R_a}{(K\phi)^2} \tau_{ind}$$



for shunt & S.E.M Motors

$\tau_{ind} = 0 \rightarrow$ no load

$\omega = \omega_{NL} \rightarrow \tau_{ind} = 0,$

$V_F = E_a,$

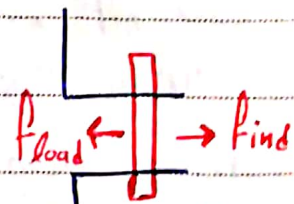
$I_a = 0$

- with load :

$$\tau_{load} > \tau_{ind}$$

$$\rightarrow \omega \downarrow \rightarrow E_a = K\phi\omega$$

$$\rightarrow I_a \uparrow = \frac{V_F - E_a}{R_a} \rightarrow \tau_{ind} \uparrow = K\phi I_a$$



until $\tau_{ind} = \tau_{load}$ steady speed

Armature Reaction : (AR)

→ With load $I_a \uparrow$, will produce its own flux in the armature

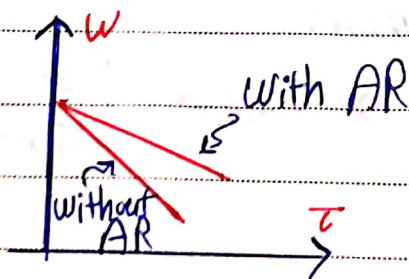
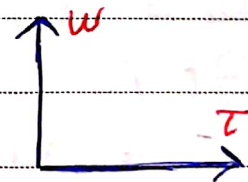
so, it will distort original flux from field

$\Phi \rightarrow$ flux weakening $\Phi_f \downarrow \rightarrow W \uparrow \rightarrow$ See (4)

- AR Distortion of the flux in a DC machines when load is add

- at no load $\rightarrow I_a = 0 \rightarrow$ No AR distortion

Terminal characteristics :



To Solve the Problem of AR
→ Add compensating winding

Ex 50 hp, 250 V, 1200 r/min, DC Shunt Motor

* with compensating winding

No load speed = 1200 r/min

$R_a = 0.06 \Omega$, $R_f + R_{adj} = 50 \Omega$, $N = 1200$ turns
Pole

find speed & torque at $I_L = 100 A, 200 A, 300 A$

Sol. 1) at $I_L = 100 A$

$$E_a = V_f - I_a R_a$$

$$I_a = I_L - I_f$$

$$I_a = 100 - \frac{250}{50} = 95 A$$

$$E_a = 250 - 95 \times 0.06 = 244.3 V$$

$$E_a = K \phi n \rightarrow 244.3 = K \phi n \dots \textcircled{1}$$

* At no load $\rightarrow \tau_{ind} = 0$, $I_a = 0$, $E_{a0} = V_f$

$$E_{a0} = V_f = 250 = K \phi n_0 \rightarrow 250 = K \phi (1200) \dots \textcircled{2}$$

$$\textcircled{1} \div \textcircled{2} \rightarrow n = 1173 \text{ r/min}$$

$$P_{conv} = E_a I_a = \tau_{ind} \omega \rightarrow \tau_{ind} = \frac{(244.3)(95)}{1173 \times \frac{2\pi}{60}} = 190 \text{ N.m}$$

2) at $I_L = 200 A$

$$E_a = V_f - I_a R_a, \quad I_a = I_L - I_f = 200 - 5 = 195 A$$

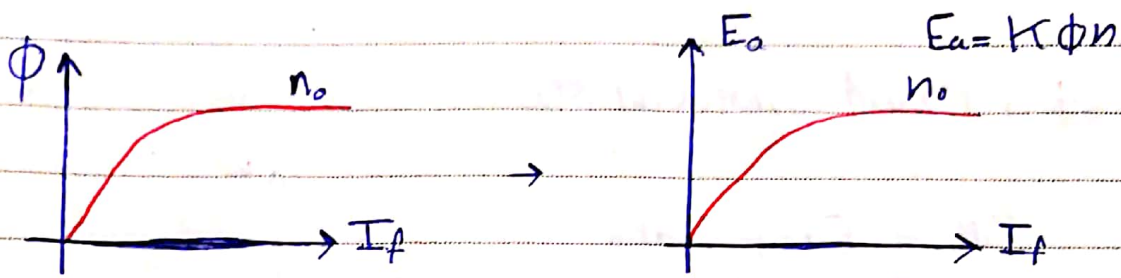
$$E_a = 238.3 V = K \phi n \dots \textcircled{1}$$

* At no load $\rightarrow E_{a0} = V_f = 250 = K \phi (1200) \dots \textcircled{2}$

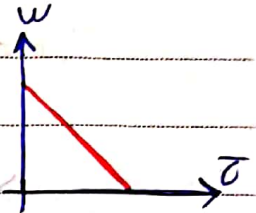
$$\textcircled{1} \div \textcircled{2} \rightarrow n = 1144 \text{ r/min}$$

$$I_L \uparrow \rightarrow I_a \uparrow \rightarrow \text{load} \uparrow \rightarrow \omega \downarrow \rightarrow \tau \uparrow$$

$$E_a I_a = \tau \omega \rightarrow \tau = \frac{(238.3)(195)}{\frac{2\pi}{60} \cdot 1144} = 388 \text{ N.m}$$



* load \uparrow $I_a \uparrow$ $\phi \downarrow$ $\omega = \frac{1}{\phi} \uparrow$



$$* F = NI \rightarrow F_F = N_F I_F, F_{AR}$$

$$F = F_F - F_{AR} \rightarrow N_F I_F^* = N_F I_F - F_{AR}$$

$$I_F^* = I_F - \frac{F_{AR}}{N_F}$$

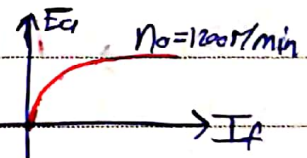
Ex 50 hp, 250 V, 1200 r/min, DC Shunt Motor

* without compensating windings

$$R_a = 0.06 \Omega, R_f = 50 \Omega, N_f = 1200 \text{ turns}$$

$$\rightarrow F_{AR} = 840 \text{ A.Turns at } I_L = 200 \text{ A}$$

* find motor speed at $I_L = 200 \text{ A}$



Sol. $E_a = V_f - I_a R_a$

$$I_a = I_L - I_f = 200 - 5 = 195 \text{ A}$$

$$E_a = 250 - 195 \times 0.06 = 238.3 \text{ V} = K \phi n \dots \textcircled{1}$$

$$I_f^* = I_f - \frac{F_{AR}}{N_f} = 5 - \frac{840}{1200} = 4.3 \text{ A}$$

$$\rightarrow \text{from curve at } I_f^* = 4.3 \text{ A} \rightarrow E_{a0} = 233 \text{ V}$$

$$E_{a0} = 233 = K \phi (1200) \dots \textcircled{2}$$

$$\textcircled{1} \div \textcircled{2} \rightarrow n = 1227 \text{ r/min with AR}$$

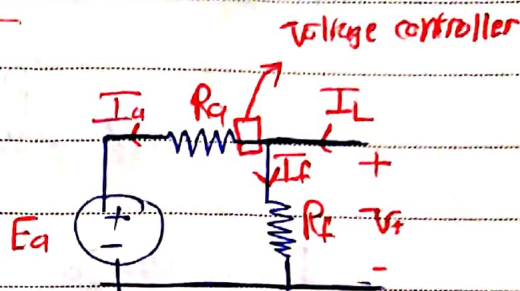
Speed control of shunt motor :

(1) change R_f :

$R_f \uparrow \rightarrow I_f = \frac{V_f}{R_f} \downarrow \rightarrow \Phi \downarrow$

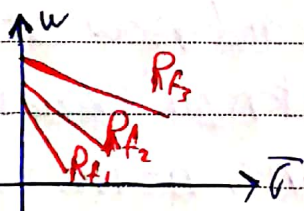
$\rightarrow \downarrow E_a = K \Phi \omega \rightarrow \uparrow I_a = \frac{V - E_a}{R_a}$

$\rightarrow \uparrow \tau_{ind} = K \Phi I_a \uparrow \rightarrow \omega \uparrow$



$E_a = K \Phi_1 n_1$
 $E_a = K \Phi_2 n_2$

* if $R_f \rightarrow \infty$: open ckt. the field $\rightarrow \omega = \infty$
 \rightarrow Runaway the motor (damage)

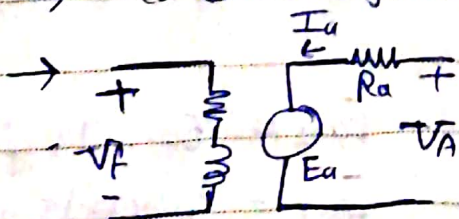


$R_{f3} > R_{f2} > R_{f1}$

(2) change Armature voltage :

$I_a = \frac{V_A - E_a}{R_a}$ so $I_a \uparrow \rightarrow \tau \uparrow \rightarrow \omega \uparrow$

\rightarrow Used Voltage controller / SEG



(3) Insert variable Resistance with Armature



$$I_a = \frac{V_f - E_c}{R_a} \quad \text{if } R_a \uparrow \rightarrow I_a \downarrow \rightarrow \tau \downarrow \rightarrow \omega \downarrow$$

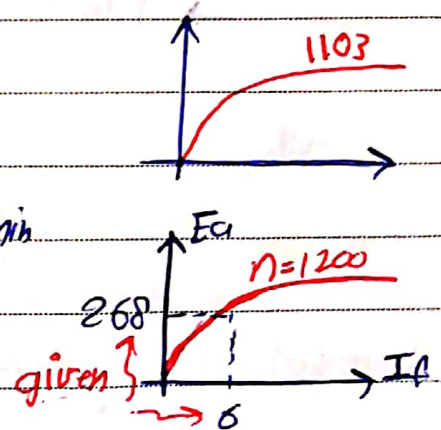
* Rarely used

Ex 100 hp, 250 V, 1200 r/min, Shunt motor
 $R_a = 0.03 \Omega$, $R_f = 41.67 \Omega$, $I_L = 126$, initial speed
 with compensating windings = 1103 r/min

Assume I_a constant

* if this motor is connected now as
 S.E.M with constant torque (load)

$V_A = 250$ V, $I_a = 120$ A, $n = 1103$ r/min
 find motor speed at $V_A = 200$ V?!



Sol. if $V_A = 250$ V:

$$E_{a1} = V_A - I_a R_a$$

$$E_{a1} = 246.4 \text{ V} = K \Phi (1103) \dots \textcircled{1}$$

$$\textcircled{1} \div \textcircled{2} \rightarrow n = 879 \text{ r/min}$$

So $V_A \downarrow \rightarrow n \downarrow$

if $V_A = 200$ V

$$E_{a2} = V_A - I_a R_a$$

$$E_{a2} = 196.4 \text{ V} = K \Phi n_2 \dots \textcircled{2}$$

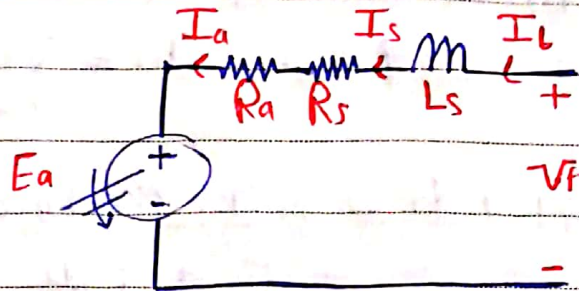
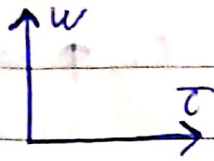
$$E_{a02} = 250 = K \Phi_2 (1200)$$

$$E_{a01} = 268 = K \Phi_1 (1200)$$

$$\frac{E_{a01}}{E_{a02}} = \frac{268}{250} = \frac{\Phi_1}{\Phi_2}$$

* I_a constant (Torque constant)

8.6 Series Motors



$$I_L = I_s = I_a$$

$$V_F = E_a + I_a(R_a + R_s)$$

$$\phi \propto I_a \rightarrow \phi = C I_a$$

$$\tau_{ind} = K \phi I_a \rightarrow \tau_{ind} = K C I_a^2 \rightarrow I_a = \sqrt{\frac{\tau_{ind}}{K C}}$$

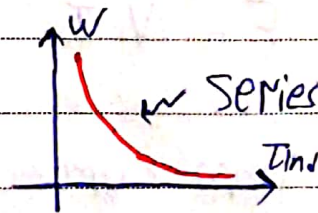
$$I_a = \sqrt{\frac{\tau_{ind}}{K C}}$$

$$E_a = K \phi \omega \rightarrow E_a = K C I_a \omega \rightarrow E_a = K C \sqrt{\frac{\tau_{ind}}{K C}} \omega$$

$$E_a = \sqrt{K C \tau_{ind}} \omega$$

$$V_F = \sqrt{K C \tau_{ind}} \omega + \sqrt{\frac{\tau_{ind}}{K C}} (R_a + R_s)$$

$$\omega = \frac{V_F}{\sqrt{K C \tau_{ind}}} - \frac{(R_a + R_s)}{K C}$$



* at no load $\rightarrow \tau_{ind} = 0$ so $\omega = \infty$
 $I_a = 0$

* don't unload series motor \rightarrow it will damage motor
 (run away) $\rightarrow \tau_{ind} = 0, \omega = \infty$

* high starting torque : suitable for load

* Speed control of series Motor :

1) Change V_f :

$$V_f \uparrow \rightarrow I_a = \frac{V_f - E_a}{R_a + R_s} \uparrow \rightarrow \tau_{ind} \uparrow \rightarrow \omega \uparrow$$

2) Add a series resistance with armature :

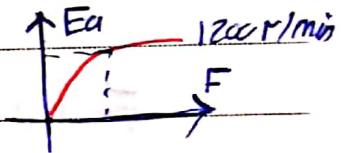
$$R_a + R_s + R_{add}$$

$$\rightarrow I_a \downarrow \rightarrow \tau_{ind} \downarrow \rightarrow \omega \downarrow$$

Ex 250V, Series Motor with compensating windings

$$R_a + R_s = 0.08 \Omega, N_s = 25 \text{ Turns}$$

find speed & torque at $I_L = 50 \text{ A}$



Sol. $E_a = V_f - I_a (R_a + R_s)$, $V_f = 250 \text{ V}$, $I_a = I_L = 50 \text{ A}$
 $E_a = 246 \text{ V} = k \phi n$

$$F = N_s I_s = 25 \times 50 = 1250 \text{ A.Turns}$$

from curve at $F = 1250 \rightarrow E_{a0} = 80 = k \phi (1200)$

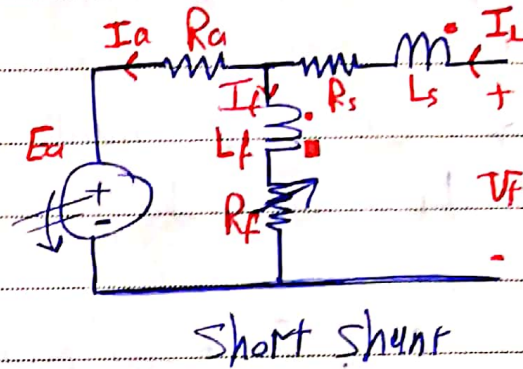
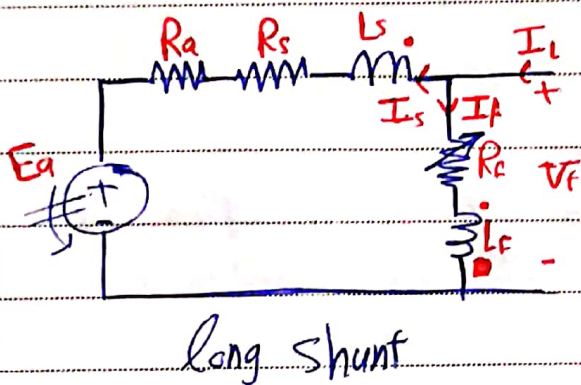
$$\frac{80}{246} = \frac{k \phi (1200)}{k \phi n} \rightarrow n = 3690 \text{ r/min}$$

$$E_a I_a = \tau \omega \rightarrow \tau = \frac{(50)(246)}{3690 \times \frac{2\pi}{60}}$$

$$\tau = 31.8 \text{ N.m}$$

8.7 Compounded DC Motor

- Combines the best features of series and Shunt Motor



- if the two currents I_s, I_a enter dof

$$\rightarrow F \uparrow, \Phi \uparrow$$

$$F_{net}^* = F_f + F_s - F_{AR} \rightarrow \text{Cumulatively Compounded DC Motor}$$

- if one of the currents enter dof and the other leave dof

$$\rightarrow F \downarrow, \Phi \downarrow$$

$$F_{net}^* = F_f - F_s - F_{AR} \rightarrow \text{Differentially compounded DC Motor}$$

$$F_{net}^* = F_f \pm F_s - F_{AR}$$

$$N_f I_f^* = N_f I_a \pm N_s I_s - F_{AR}$$

$$I_f^* = I_a \pm \frac{N_s}{N_f} I_s - \frac{F_{AR}}{N_f}$$

for short shunt

*

$$I_L = I_s$$

$$V_f = I_s R_s + I_a R_f$$

$$V_f = I_a R_f = I_a R_a + E_a$$

$$V_f = I_L R_s + I_a R_a + E_a$$

for long shunt

*

$$I_L = I_f + I_a, \quad I_f = \frac{V_f}{R_f}, \quad I_a = I_s$$

$$V_f = E_a + I_a (R_a + R_s)$$

* Commutatively Properties:

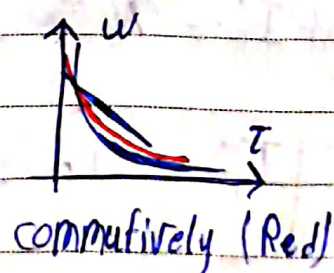
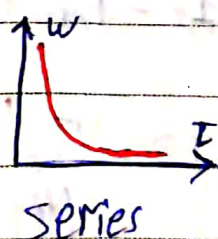
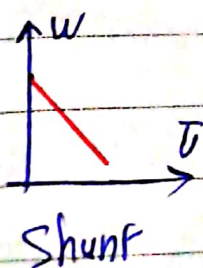
- has higher starting torque than shunt motor and lower than series motor

- at small loads $\rightarrow I_a$ small, I_L

\Rightarrow series flux small behaves as shunt motor

- at large loads $\rightarrow I_a \uparrow$

\Rightarrow series flux has large effect than shunt flux behaves as series motor

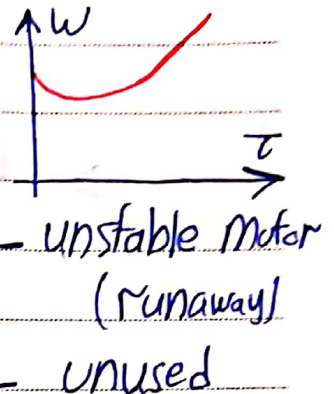


* Differentially Properties :

load $\uparrow \rightarrow I_a \uparrow, I_L \uparrow, \Phi_s \uparrow$

$\rightarrow F_s \uparrow \rightarrow \Phi \downarrow \rightarrow E_a = (K \Phi \omega) \downarrow$

$\rightarrow I_a \uparrow \rightarrow \tau \uparrow \rightarrow \omega \uparrow$

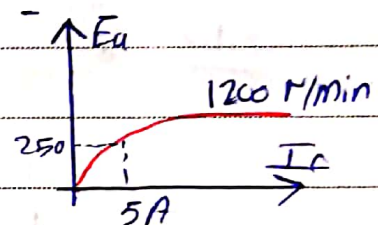


Ex 100 hp, 250 V compounded DC Motor with compensating winding $R_a + R_s = 0.04 \Omega$, $N_F = 1000$, $N_S = 3$ long shunt

□ at no load if R_a adjust to give

$$N_0 = 1200 \text{ r/min}$$

find I_F at no load ?!



\rightarrow at no load $E_{a0} = V_f$, $I_a = 0 \rightarrow E_{a0} = V_f = 250 \text{ V}$
from curve $E_{a0} = 250 \rightarrow I_f = 5 \text{ A}$

□ if Motor is cumulatively compounded

find the speed when $I_a = 200 \text{ A}$

$$\rightarrow E_a = V_f - I_a(R_a + R_s) \rightarrow E_a = 242 \text{ V} = K \Phi \omega$$

$$F_{\text{net}} = F_f + F_s \rightarrow I_f^* = I_f + \frac{N_s}{N_f} I_s$$

$$I_s = I_a = 200 \rightarrow \text{for long} \rightarrow I_f^* = 5.6 \text{ A}$$

from curve at $I_f^* = 5.6 \rightarrow E_{a0} = 262$ at $N_0 = 1200$

$$\frac{262}{242} = \frac{K \Phi (1200)}{K \Phi \omega} \rightarrow \omega = 1108 \text{ r/min}$$

3] if the motor is differentially compounded
at $I_a = 200 \text{ A}$, find speed ?

$$\rightarrow E_a = V_f - I_a(R_a + R_s) \rightarrow E_a = 242 = K\phi n$$

$$I_f^* = I_f - \frac{N_s}{N_f} I_a = 4.4 \text{ A}$$

at $I_f^* = 4.4 \text{ A}$ from curve $\rightarrow E_{a0} = 236 \text{ V}$

$$236 = K\phi(1200) \rightarrow \frac{242}{236} = \frac{K\phi n}{K\phi(1200)}$$

$$\rightarrow n = 1230 \text{ r/min}$$

8.10 Efficiency calculation

$$\eta = \frac{P_o}{P_{in}} \times 100\%$$

$$P_{in} = V_f I_L$$

$$P_o = P_{in} - P_{loss}$$

* losses :

(1) Copper loss \rightarrow loss R_a, R_f

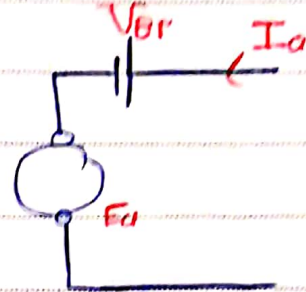
$$P_{R_f} = I_f^2 R_f, \quad P_{R_a} = I_a^2 R_a$$

$$V_f \text{ rated} \rightarrow \text{find } I_a \rightarrow R_f = \frac{V_f}{I_f}$$

- to find R_a is locked rotor test (Blocked the Rotor)
 $\rightarrow E_a = 0 \rightarrow R_a = \frac{V_f}{I_a} \times I_a = \frac{V_f - E_a}{I_a}$

(2) Brushes loss

$$P_{BR} = V_{BR} I_a$$



(3) Rotational loss

$$P_{rot} = P_{mech} + P_{core}$$

* no load test : $I_a = 0$, $P_{Ra} = 0$, $P_{BR} = 0$

$$P_{rot} = P_{in} - P_{pp}$$

(4) Stray loss $\rightarrow P_{stray} = 1\% P_{in}$

Ex 50 hp, 250 V, 1200 r/min, Shunt Motor

$$I_{a\text{rated}} = 170 \text{ A}, \quad I_{f\text{rated}} = 5 \text{ A}$$

* When motor is blocked: $V_A = 10.2 \text{ V}$, $I_a = 170 \text{ A}$

* and if field voltage 250 V produce 5 A field current
the brush voltage = 2 V, $P_{\text{stray}} = 1\%$

* at no load with $V_f = 240 \text{ V}$, $I_a = 13.2 \text{ A}$

$$I_f = 4.8 \text{ A}, \quad n = 1150 \text{ r/min}$$

→ Calculate η ?!

Sol. From blocked rotor test ($E_a = 0$)

$$\rightarrow R_a = \frac{V_A}{I_a} = \frac{10.2}{170} = 0.06 \Omega$$

$$R_f = \frac{V_f}{I_f} = \frac{250}{5} = 50 \Omega$$

$$P_d = P_{in} - P_{cu} - P_{br}$$

$$P_d = E_a I_a$$

↳ with load

$$P_{cu} = P_{Rf} + P_{Ra} = (I_f)^2 R_f + (I_a)^2 R_a$$

$$= 1250 + 1734 = 2984 \text{ W}$$

$$P_{br} = V_{br} I_a = (2)(170) = 340 \text{ W}$$

From no load test: $P_{\text{rot}} = P_{\text{mech}} + P_{\text{core}}$
 $= P_{in} - P_{Rf}$

$$P_{in_{nl}} = V_f I_L = V_f (I_a + I_f), \quad P_{Rf} = I_f^2 R_f$$

$$P_{\text{rot}} = (240)(13.2 + 4.8) - (4.8)^2(50) = 3168 \text{ W}$$

or $P_{\text{rot}} = E_a I_a = (240)(13.2) = 3168 \text{ W}$

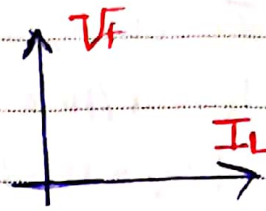
$$P_{\text{stray}} = 1\% P_{in} \rightarrow P_{in} = V_f I_L = (250)(170 + 5) = 43750 \text{ W}$$

$$P_o = P_{in} - P_{\text{loss}} = P_{in} - P_{cu} - P_{\text{stray}} - P_{br} - P_{Rf} = 36820 \text{ W}$$

$$\eta = \frac{P_o}{P_{in}} \times 100\% = 84.2\%$$

8.11 DC Generators

* $P_m (\text{mech}) \rightarrow P_o (\text{Elec.})$



* Prime Mover

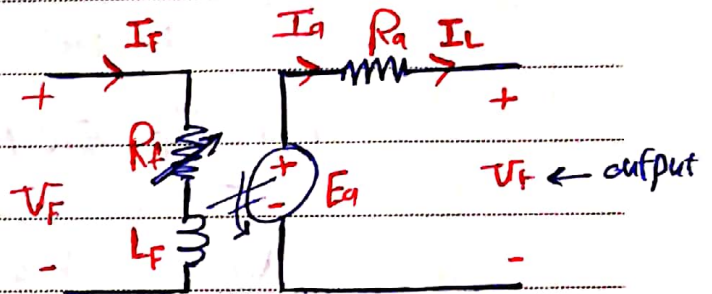
II Separately excited DC generator

$$I_L = I_a$$

$$I_F = \frac{V_F}{R_F}$$

$$V_F = E_a - I_a R_a$$

$$V_F = E_a - I_L R_a$$



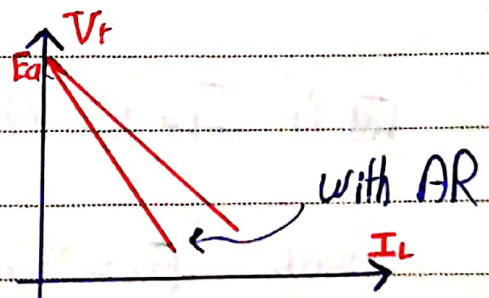
Load $\uparrow \rightarrow I_L \uparrow \rightarrow I_a \uparrow \rightarrow V_F \downarrow$

$\rightarrow \Phi \downarrow \rightarrow E_a \downarrow$

* with AR problem

* at no load $I_L = I_a = 0$

$$V_F = E_a$$



* V_F Controller : Change $E_a = k\Phi n \rightarrow E_a \uparrow \rightarrow V_F \uparrow$
to change E_a :

1) Change ω : $\omega \uparrow \rightarrow E_a \uparrow \rightarrow V_F \uparrow$ (limited)

2) Change Φ : $\Phi \uparrow \rightarrow E_a \uparrow \rightarrow V_F \uparrow$

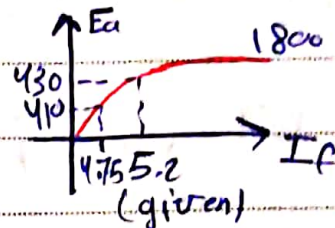
3) Change R_F : $R_F \downarrow \rightarrow I_F \uparrow \rightarrow \Phi \uparrow \rightarrow E_a \uparrow \rightarrow V_F \uparrow$

Ex 172 kW, 430 V, 400 A, $N_F = 1000$

1800 r/min, DC S.E.G

if $R_{adj} = 63 \Omega$, $R_F = 20$

Prime mover speed = 1600 r/min



□ What is the no load terminal voltage of no load?

Sol. $V_F = E_a = K\phi(1600)$

$$I_F = \frac{V_F}{R_F + R_{adj}} = \frac{430}{63 + 20} = 5.2 \text{ A}$$

At $I_F = 5.2 \text{ A}$ from curve

$$E_{a0} = 430 = K\phi(1800)$$

$$\frac{V_F}{430} = \frac{K\phi(1600)}{K\phi(1800)} = 382 \text{ V} = V_F \text{ at no load}$$

□ if $I_L = 360 \text{ A}$, find V_F with compensating winding?

Sol. $V_F = E_a - I_L R_a = 382 - (360)(0.05) = 364 \text{ V} \downarrow$ with load

□ if $I_L = 360 \text{ A}$, $F_{AR} = 450$, find V_F ?

Sol. $E_a = V_F = K\phi(1600)$

$$I_F^* = I_F - \frac{F_{AR}}{N_F} \rightarrow I_F^* = 4.75 \text{ A}$$

From curve $I_F = 4.75 \rightarrow E_{a0} = 410 \text{ V} = K\phi(1800)$

$$V_F = 364 \text{ V} = E_a \text{ at no load}$$

$$V_F = E_a - I_L R_a = 364 - (360)(0.05) = 346 \text{ V} \downarrow \text{ with AR}$$

□ adjustment to restore $V_F = V_{F_{no\ load}} = 382\text{ V}$
 $I_L = 360\text{ A}$

$$V_F = 382 = E_a - I_a R_a$$

$$E_a = 400 = K \Phi (1600)$$

$$E_{a_0} = K \Phi (1800)$$

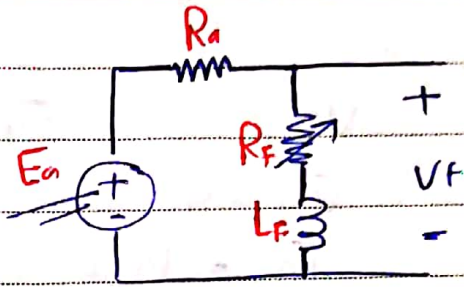
$$E_{a_0} = 450\text{ V from curve}$$

$$I_F = 6.15\text{ A} = \frac{V_F}{R_F + R_{adj}} \rightarrow 6.15 = \frac{430}{20 + R_{adj}}$$

$$R_{adj} = 50\ \Omega$$

8.13 Shunt Generator

☑ supplies its own field current



☑ At no load $I_a = 0$
 $\rightarrow V_f = E_a = K \phi_{res} \omega$

☑ if the prime mover start turns the shaft,
 Voltage built up E_a depends on ϕ_{res}

$$E_a = K \phi_{res} \omega = V_f \uparrow$$

$$I_f = \frac{V_f \uparrow}{R_f} \rightarrow \phi \uparrow > \phi_{res}$$

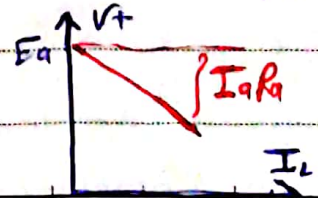
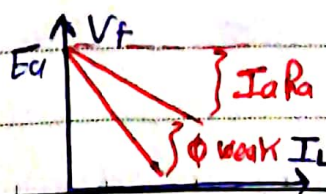
$$E_a \uparrow \rightarrow V_f \uparrow \rightarrow I_f \uparrow \rightarrow \phi \uparrow \rightarrow E_a \uparrow \rightarrow V_f \uparrow \rightarrow \phi_{sat}$$

$$V_{fmax} = E_a = K \phi \omega$$

$$I_a = I_L + I_f, \quad I_f = \frac{V_f}{R_f}$$

$$V_f = \downarrow E_a - I_a R_a \uparrow$$

load \uparrow , $I_L \uparrow$, $I_a \uparrow$, $I_a R_a \uparrow$, $V_f \downarrow$,
 $I_f \downarrow$, $\phi \downarrow$, $E_a = K \phi \omega \downarrow \rightarrow V_f \downarrow \downarrow$



❌ if the Shunt generator started and no voltage built up ($V_F = 0$) :

$$\square \Phi_{res} = 0 \quad \& \quad E_a = K\Phi_{res}\omega = V_F = 0$$

→ flashing the field & disconnect field terminal and connected it to an external source to get Φ_{res} in the field

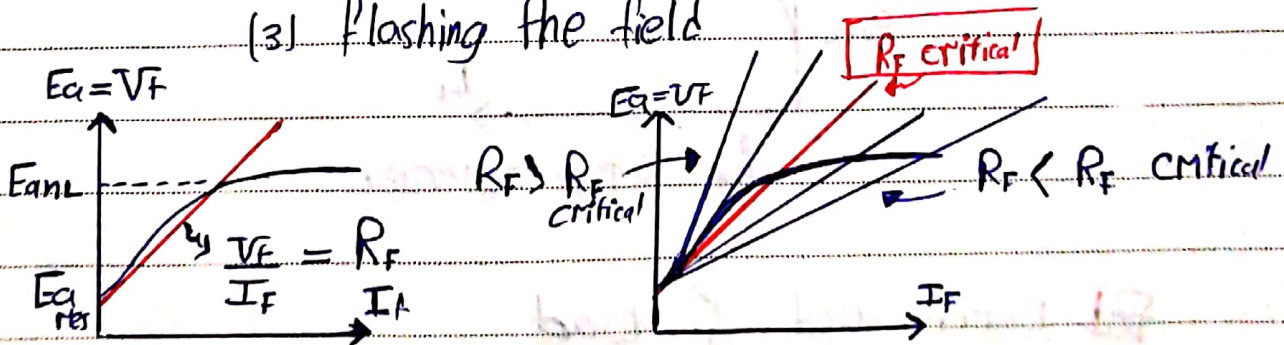
❑ Direction of rotation is reversed &

→ Direction of field connection is reversed → flux opposing $\Phi_{res} \rightarrow \Phi \downarrow$

Solutions (1) Reverse field connection

(2) Reverse direction of rotation

(3) Flashing the field



$R_F \downarrow \rightarrow \omega \downarrow \rightarrow R_F$ adjusted to values greater than critical value

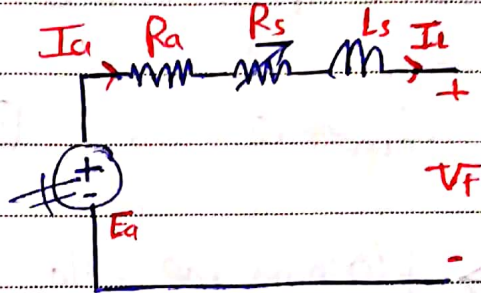
* V_F controlled : $V_F = E_a - I_a R$, $E_a = K\Phi\omega$

1) $\omega \uparrow \rightarrow E_a \uparrow \rightarrow V_F \uparrow$

2) $R_F \downarrow \rightarrow I_F \uparrow \rightarrow \Phi \uparrow \rightarrow E_a \uparrow \rightarrow V_F \uparrow$

8.14 Series Generator

* at no load $I_a = 0$
 $\rightarrow V_f = E_a = K\phi_{res} \omega$

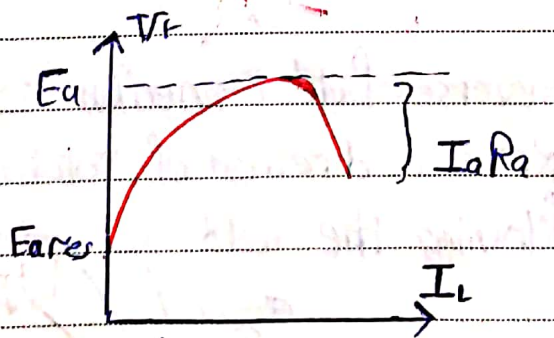


* at load $I_L \uparrow \rightarrow I_a \uparrow$
 $\rightarrow \phi \uparrow \rightarrow E_a = K\phi \omega \uparrow$

$V_f \uparrow \rightarrow \phi \uparrow \rightarrow \phi_{sat}$

E_a constant \rightarrow load $\uparrow \rightarrow I_a \uparrow \rightarrow V_f \downarrow$

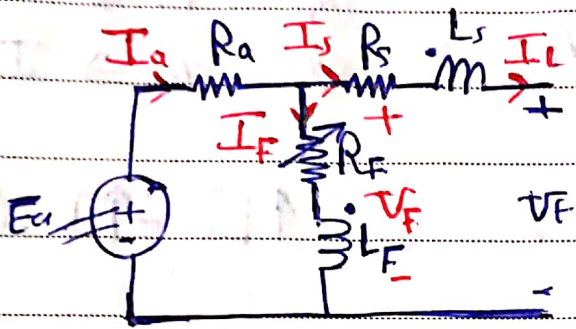
$V_f = \uparrow E_a - \uparrow I_a(R_a + R_s)$ but $I_a(R_a + R_s)$ less increasing than E_a



for series generator

* Rarely used & bad

8.15 Commutatively Compounded DC Generator
(Series + Shunt field)



Short Shunt

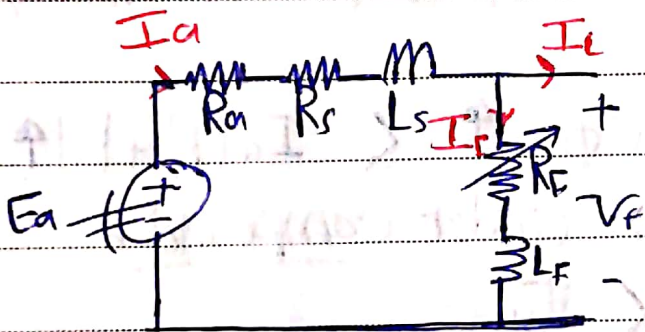
$$I_a = I_L + I_F$$

$$I_s = I_L$$

$$V_F = E_a - I_a R_a - I_L R_s$$

$$I_F = \frac{V_F}{R_F}, \quad V_F = I_L R_s + V_F$$

$$= E_a - I_a R_a$$



Long Shunt

$$I_a = I_s$$

$$I_F = \frac{V_F}{R_F}$$

$$I_a = I_L + I_F$$

$$V_F = E_a - I_a (R_a + R_s)$$

* Commutatively, Differential :

$$F^* = F_F \pm F_S - F_{AR} = N_F I_F \pm N_S I_S - F_{AR}$$

$$I_F^* = I_F \pm \frac{N_S}{N_F} I_S - \frac{F_{AR}}{N_F}$$

* Short $\rightarrow I_L = I_S$

* long $\rightarrow I_a = I_S$

* Terminal for commutatively : $V_f = E_a - I_a(R_a + R_s)$

if load $\uparrow \rightarrow I_a \uparrow \rightarrow I_L \uparrow \rightarrow V_f \downarrow$

$I_a \uparrow \rightarrow \phi \uparrow \rightarrow E_a = K\phi\omega \uparrow \rightarrow V_f \uparrow$

$I_a \uparrow = I_S \uparrow \rightarrow F^* = (F_F + F_S) \uparrow - F_{AR} \rightarrow \phi \uparrow$

(1) N_S small $\&$ $E_a = K\phi\omega \uparrow < I_a(R_a + R_s) \uparrow$

$\therefore V_f \downarrow \rightsquigarrow$ under compounded

$$\underline{V_{FL} < V_{NL}}$$

(2) More series N_S $\&$ $E_a = K\phi\omega \uparrow > I_a(R_a + R_s) \uparrow$

$\therefore V_f \uparrow$ until reach $\phi_{sat} \rightarrow E_a$ constant

$\rightarrow I_a(R_a + R_s) \uparrow > E_a$ constant

$\therefore V_f \downarrow \rightsquigarrow$ flat compounded

$$\underline{V_{FL} \approx V_{NL}}$$

(3) Large N_S $\&$ $E_a = K\phi\omega \uparrow > I_a(R_a + R_s) \uparrow$

$\therefore V_f \uparrow \rightsquigarrow$ over compounded

$$\underline{V_{FL} > V_{NL}}$$

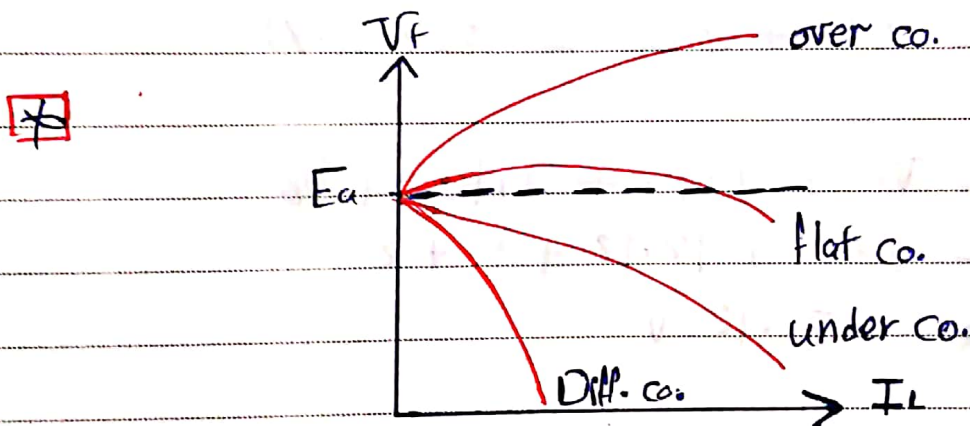
8.16 Differentially compounded

load $\uparrow \rightarrow I_a \uparrow, I_L \uparrow \rightarrow I_a(R_a + R_s) \uparrow \rightarrow V_f \downarrow$
 by the equ. $V_f = E_a - I_a(R_a + R_s)$

$$\rightarrow F^* = F_f - F_r - F_{AR}$$

$$I_F^* = I_f - \frac{N_s}{N_f} I_s - \frac{F_{AR}}{N_f} \rightarrow \Phi \downarrow \rightarrow E_a = k\Phi\omega \downarrow$$

$$\rightarrow V_f = \downarrow E_a - \uparrow I_a(R_a + R_s)$$



Commutatively & Diff. compounded diagram
 [Relationship between V_f, I_L]

* Terminal Voltage controlled [diff. + comm.]

$$V_f = E_a - \underline{I_a(R_a)}$$

(armature resistance loss)

\rightarrow Change $E_a = k\Phi\omega$:

① $\omega \uparrow \rightarrow E_a \uparrow \rightarrow V_f \uparrow$

② $\Phi \uparrow \rightarrow E_a \uparrow \rightarrow V_f \uparrow$

by $R_f \downarrow \rightarrow I_f \uparrow \rightarrow \Phi \uparrow$

Ex 50kW, 250V, short shunt compounded G.

$$R_s = 0.04, R_a = 0.06, R_f = 125, V_B = 2V$$

Find E_a ?!

Sol.
$$I_L = \frac{P_o}{V_f} = \frac{50k}{250} = 200A$$

$$I_a = I_L + I_f$$

$$I_f R_f = R_s I_L + V_f$$

$$I_f = \frac{8 + 250}{125} = 2.06A$$

$$I_a = 200 + 2.06 = 202.06A$$

$$E_a = V_f + I_a R_a + I_L R_s + V_B$$

$$= 250 + 12.12 + 8 + 2$$

$$E_a = 272.12V$$

"1 slip 50"

(5)

Chapter "3": Construction of AC machine.

⇒ There are two classes of AC machines:
* AC machine (Rotor + stator), Field winding located in Rotor.

① Synchronous machine:

- need DC external source, to produce rotor magnetic field (I_a)

- electrical freq synchronised (locked), to Mech speed ~~freq~~

② Induction machine (Motor): IM:-

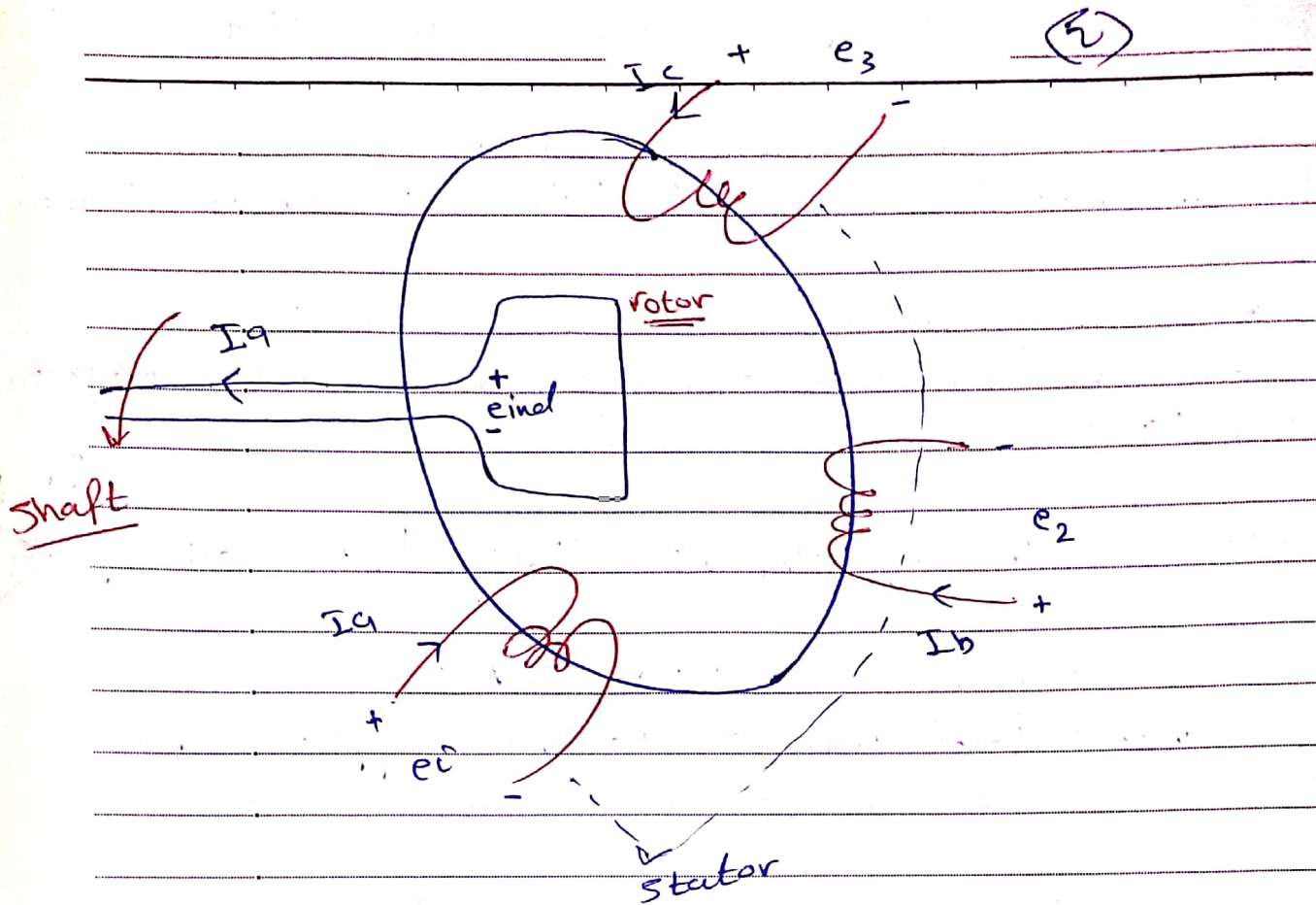
- Magnetic field current (I_a), is supplied by Magnetic Induction ⇒ No external source.

⇒ named "Rotating Transformer"

* Induction Motor:-

(1) stator :- has coils supplied with 3-phase currents
⇒ produce "Rotating Magnetic field" (RMF)

(2) Rotor: attached to o/p shaft, RMF producing induced voltage (e_{ind}), in the Rotor, without using external source. ⇒ "Action of Transformer" ← to p "



1] 3 ϕ currents equal magnitude with 120° phase shift, applied to stator.

2] each current produce magnetic field:

$$\phi_t = \phi_1 + \phi_2 + \phi_3 \Rightarrow 120^\circ \text{ phase}$$

3] Rotating Magnetic field (RMF), rotate with constant speed (n_s) \Rightarrow synchronous speed. (RMFs)

$$n_s = \frac{120 f_s}{P}$$

f: electrical freq

P: # of poles.
 n_s : synchronous speed.

4] RMF \Rightarrow produces induced voltage in the rotor bar given $I_a \rightarrow e_{ind} = vBL$
 \Rightarrow producing B_r (rotor field)

5] Interaction of the (RMF), and the rotor flux, generate a force $\rightarrow F = iLB$, given a torque.

$$T = B_r \times B_s$$

6] Rotor speed \rightarrow mechanical speed
 $n_m < n_s \Rightarrow$ for normal motor.

7] direction of rotor rotation, is the same as RMF.

$s_{slip} = n_s - n_m$ (difference between synch, and Mechanical speed)

$$s = \frac{n_s - n_m}{n_s} \times 100\% \quad (\text{also})$$

- Rotor freq = $f_r = s f_s = f_s \left(\frac{n_s - n_m}{n_s} \right)$
(e_{ind}, I_a) \leftarrow

$$f_r = f_s \cdot \frac{(n_s - n_m)}{120 f_s} \cdot p =$$

$$f_r = \frac{p}{120} (n_s - n_m)$$

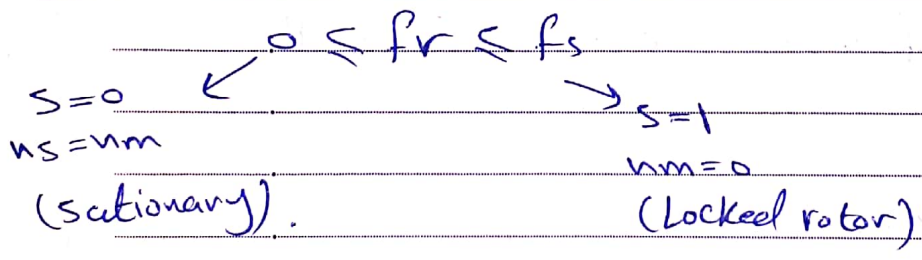
(u)

1) if the rotor run at synch speed $n_m = n_s$
 $s = 0 \Rightarrow f_r = s f_s = 0 \Rightarrow e_{ind} \min \quad I_r = 0$

\Rightarrow Rotor stationary relative to (RMF).
"slow down speed"

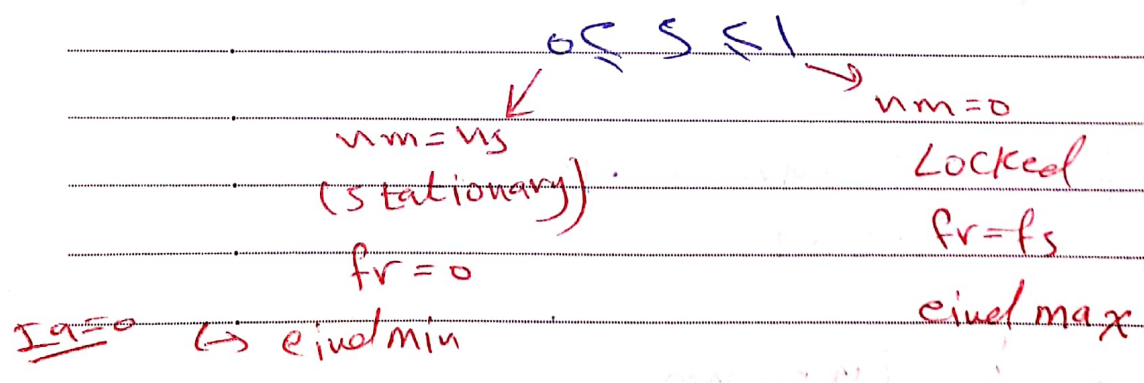
2) if mech speed = zero,
 $s = \frac{n_s - n_m}{n_s} = 1, f_r = f_s \Rightarrow$ Not moved
"the rotor locked", (Blocked rotor).

$s = 1, f_r = f_s, e_{ind} \max, I_r \max, E_{max}$.

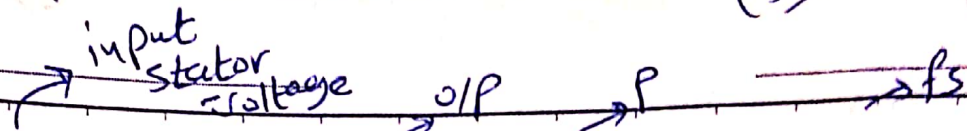


3) $n_m < n_s \Rightarrow$ Steady Induction Motor.
4) $n_m > n_s \Rightarrow$ generator (IG).

$\Rightarrow s = \frac{n_s - n_m}{n_s} \Rightarrow n_m = n_s(1 - s)$



(5)



Ex: 208 V, 10 hp, 4 poles, 60 Hz, Y connected (IM), has full load slip 5%

Find:

① synchronous speed

$$n_s = \frac{120 \cdot f_s}{P} = \frac{120 \times 60}{4} = \boxed{1800 \text{ r/min}}$$

② rotor speed:

$$s = \frac{n_s - n_m}{n_s} \times 100\% \Rightarrow n_m = 1800 (0,95) = \boxed{1710 \text{ r/min}}$$

③ Rotor freq $\Rightarrow f_r = s f_s = 0,05 \cdot 60 = 3 \text{ Hz}$

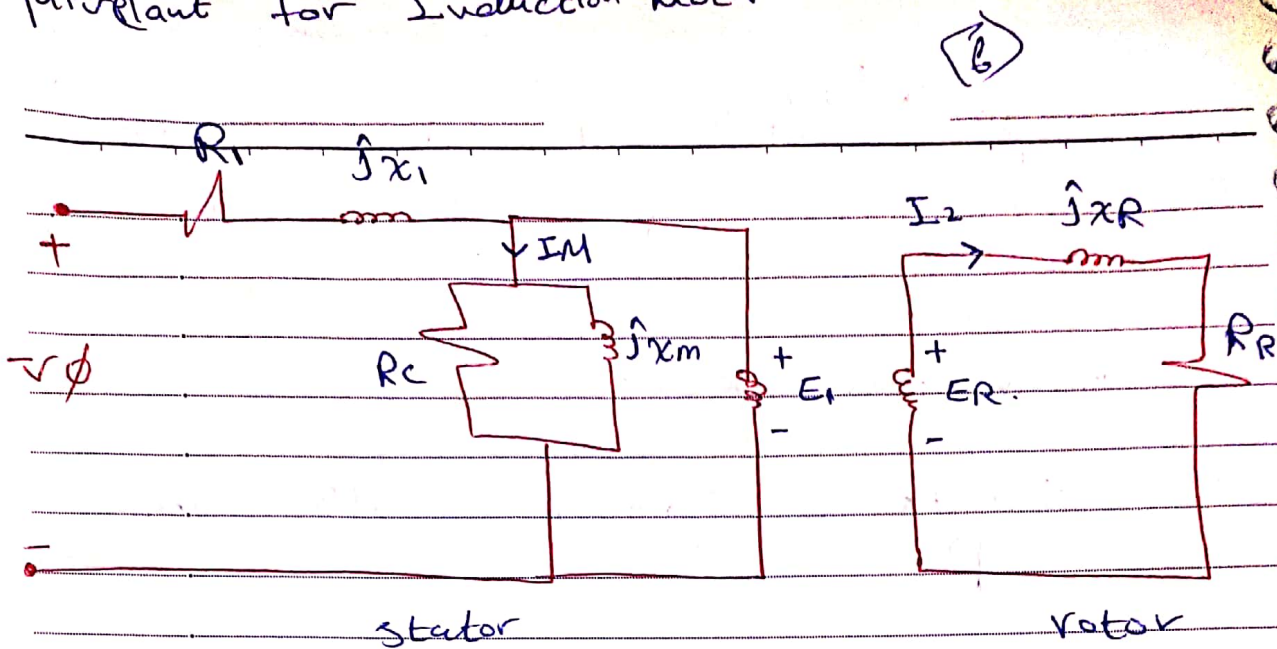
④ what's the shaft torque?

$$\text{Eq. } I_a = T \cdot \omega = P_o$$

$$P_o = T \cdot \omega \Rightarrow T = \frac{10 \times 746}{\frac{2\pi}{60} \times 1710} = \boxed{41,7 \text{ Nm}}$$

Remember (IM) \approx Rotating Transformer.

Equivalent circuit for Induction Motor:



R_1 : stator copper resistance of ^{stator} rotor

X_1 : stator reactance

R_c : Core resistance (air)

X_m : Magnetization reactance

R_2 : Rotor copper resistance

X_2 : Rotor reactance = $\omega_r \cdot L_R = 2\pi f_r \cdot L_R$

$$= 2\pi \cdot L_R (s f_s) = s \cdot \underbrace{2\pi f_s \cdot L_R}_{X_{R0}}$$

X_{R0}

$$X_R = s X_{R0}$$

* X_{R0} : rotor reactance $\Rightarrow s=1, f_r=f_s, n_m=0$. Blocked rotor

* $E_R = s E_{R0}$

E_R : Rotor voltage (Eind in rotor)

* E_{R0} : Locked rotor induced voltage

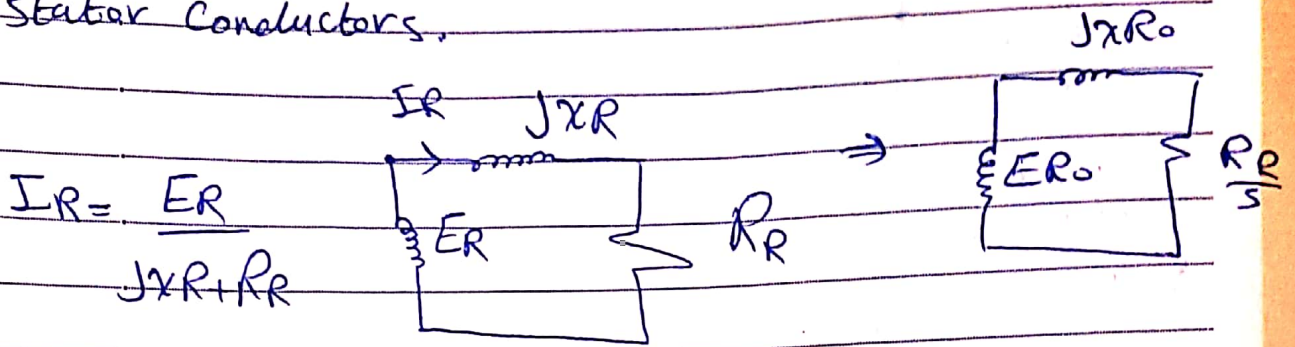


$n_m = \frac{v_s}{\omega_s}, s=0, E_R = 0$ No

(7)

When $s=1 \Rightarrow E_R = E_{R0} \text{ max}$ ($nm=0$)

a_{eff} = Ratio between Rotor Conductors and Stator Conductors.



$$I_R = \frac{s E_{R0}}{R_R + j s X_{R0}} = \frac{E_{R0}}{\frac{R_R}{s} + j X_{R0}} = I_R$$

$$Z_{eq} = \frac{R_R}{s} + j X_{R0} \Rightarrow$$

if $s \uparrow, Z_{eq} \downarrow \rightarrow I_R \uparrow$

$s \uparrow, I_R \uparrow$

$s = n_s - nm \rightarrow nm \downarrow$
 $s \uparrow \rightarrow I_R \uparrow$

$s \downarrow \rightarrow Z_{eq} \uparrow \rightarrow I_R \downarrow$

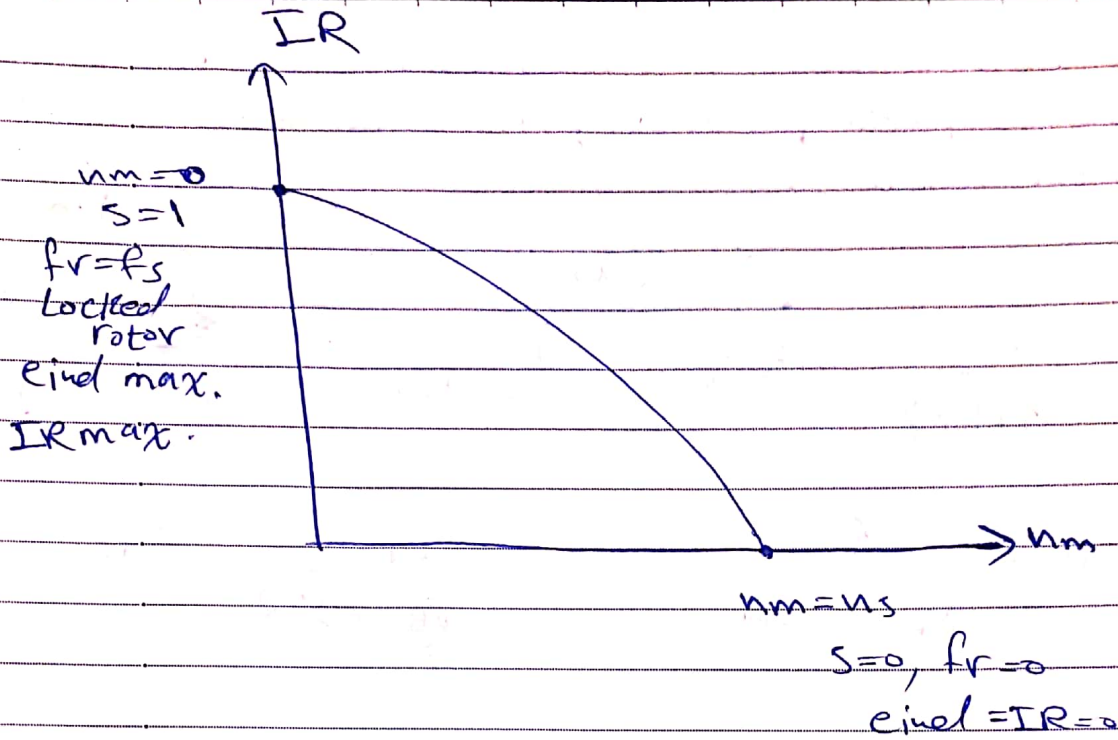
$\hookrightarrow s = n_s - nm \Rightarrow nm \uparrow \rightarrow s \downarrow, I_R \downarrow$

Analysis: ω go slip $\downarrow \Rightarrow \omega \downarrow$ (*)

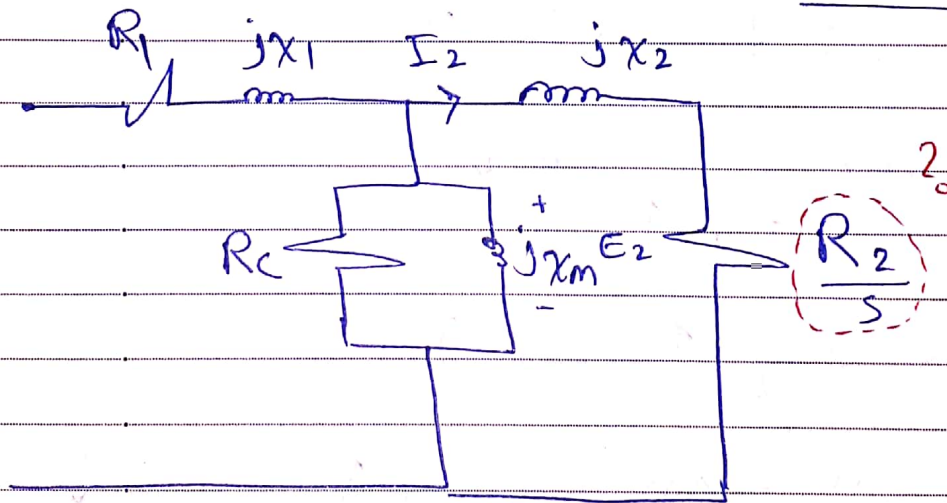
$$I_R = \frac{E_R}{jX_R + R_R} = \frac{s E_{R0}}{R_R + j s X_{R0}} = \frac{E_{R0}}{\frac{R_R}{s} + j X_{R0}} = I_R$$

$$n_m \propto \frac{1}{IR}$$

(8)



⇒ equivalent cct. Referred to stator



$$X_2 = a^2 \cdot X_{R0} \quad \text{---} \rightarrow \quad ? \quad \text{سبب دوسرو}$$

$$R_2 = a^2 \cdot R_R$$

$$E_2 = a E_{R0}$$

$$I_2 = \frac{IR}{a}$$

{ E_R, E_I, E_L

[سبب دوسرو پر دوسرو]
 بہت ہی آسان

9

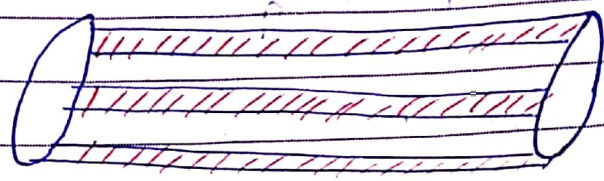
"2 Siplas"

veis

or low

* Rotor types :-

① Cage Rotor (Squirrel Rotor), "No need for brushes"
- Consist of parallel or series of AL, Cu bars all around the rotor shorted together by rings.



⇒ Class A: → Long bars near surface.

$$x_1 = 0,5 x_{eq}$$

$$x_2 = 0,5 x_{eq}$$

(S → Low.
I → Normal, start)
L → high)

⇒ Class B: → Long deep bars.

$$x_1 = 0,4 x_{eq}$$

$$x_2 = 0,6 x_{eq}$$

(S → Low
I → Normal
L → Low)

⇒ Class C: double Cage rotor design

$$x_1 = 0,3 x_{eq}$$

$$x_2 = 0,7 x_{eq}$$

(S → Low
I → Low
L → high)

(expensive)

⇒ Class D: small bars near surface.

$$x_1 = 0,5 x_{eq}$$

$$x_2 = 0,5 x_{eq}$$

(S → high
I → Low
L → very high)

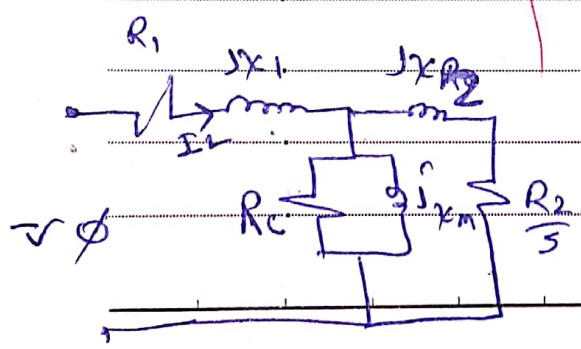
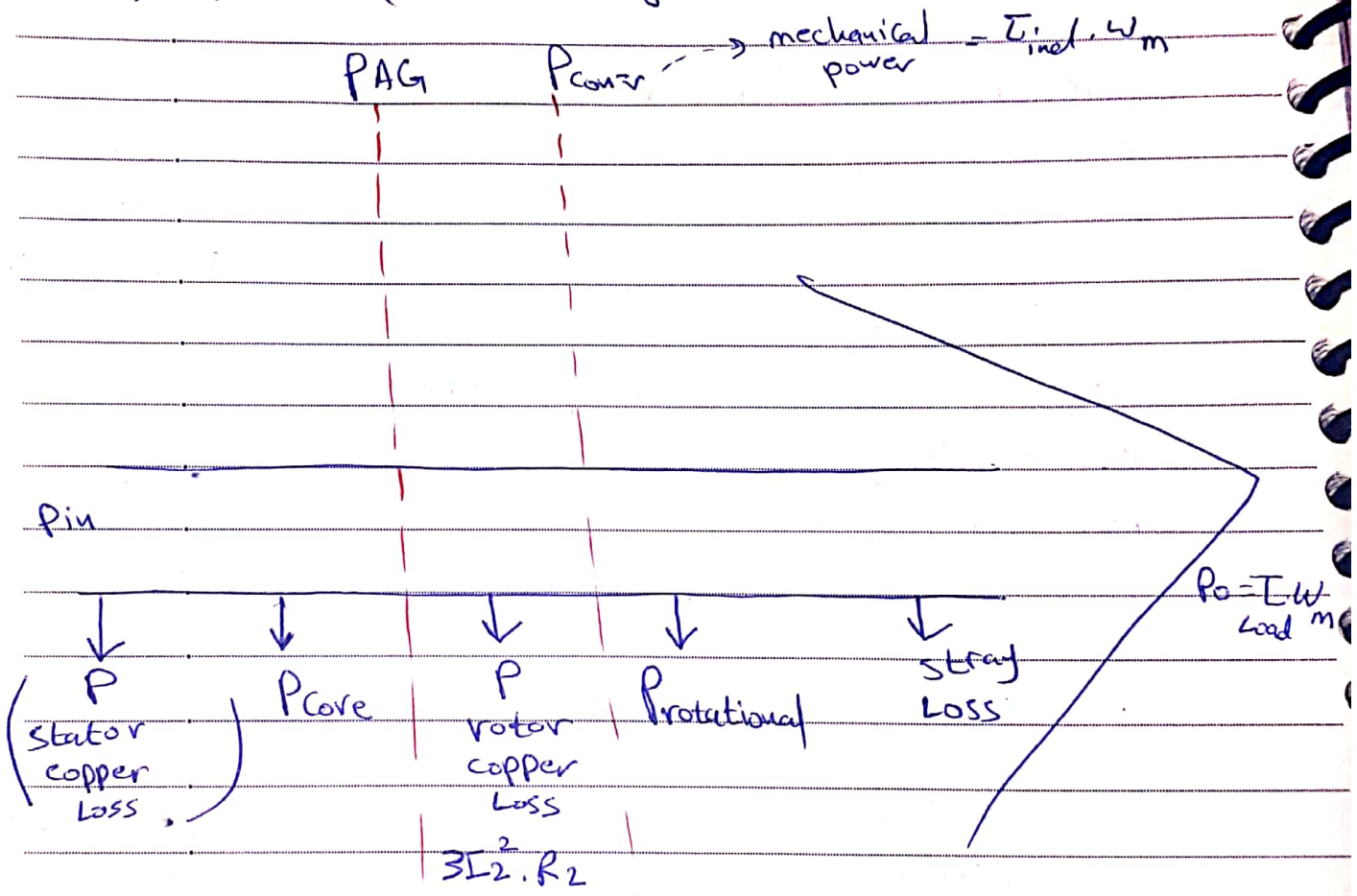
ماتر لفافه

2] Wound Rotor:- (Rarely used):-
has winding in rotor (coils), has rings and brushes connecting to the end of coils.

(expensive, need maintenance)



(6.4) power and torque :-



(11)

$$\Rightarrow P_{in} = 3 \cdot V_L \cdot I_L \cdot \cos \theta$$

$$P_{scu} \rightarrow \text{stator copper loss} = 3 \cdot I_1^2 \cdot R_1$$

$$P_{core} = 3 \frac{E_1^2}{R_c} \quad , \quad E_1 = E_2 = a E_R$$

- P_{AG} = air gap power: power crossing air gap from stator to rotor

$$P_{AG} = P_{in} - P_{scu} - P_{core} \Rightarrow$$

$$P_{rcu} \rightarrow \text{Rotor copper loss} = 3 \cdot I_2^2 \cdot R_2$$

$$\Rightarrow \frac{P_{rcu}}{s} = P_{AG} \Rightarrow s P_{AG} = P_{rcu}$$

$$\Rightarrow P_{conv} = T_{ind} \cdot \omega_m = P_{AG} - P_{rcu}$$

$$\Rightarrow \textcircled{1} P_{conv} = P_{AG} - s P_{AG} = (1-s) \cdot P_{AG}$$

OR \Rightarrow

$$\textcircled{2} P_{conv} = P_{AG} - P_{rcu} = \frac{P_{rcu}}{s} - P_{rcu} = \left(\frac{1-s}{s} \right) P_{rcu}$$

$$\Rightarrow P_{conv} = T_{ind} \cdot \omega_m \Rightarrow T_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s) \cdot P_{AG}}{(1-s) \cdot \omega_s}$$

$$\Rightarrow T_{ind} = \frac{P_{AG}}{\omega_s} = \frac{P_{conv}}{\omega_m}$$

$$\Rightarrow P_o = P_{conv} - P_{rot} - P_{stray}$$

$$P_o = I_{Load} \cdot V_m, \quad \eta = \frac{P_o}{P_{in}} \times 100\%$$

Ex: 480V, 60 Hz, 50 hp, 3φ IM, I_L = 60A at 0,85 Lagging PF.

if you know that: P_{scu} = 2kwatt, P_{fr} = P_{rot} = 600w
P_{core} = 1800w, P_{cu} = 700 watt.

then: find:

① Airgap power:

Ans
$$12400 \text{ W} = \frac{1}{\sqrt{3}} \times 73400 \text{ W}$$

$$P_{in} = 3 \cdot V_{\phi} \cdot I_L \cdot \cos\theta = \frac{3}{\sqrt{3}} \cdot 480 \cdot 60 \cdot 0,85 \text{ W}$$

$$P_{AG} = P_{in} - P_{scu} - P_{core} = 38,6 \text{ kwatt}$$

② find P_{conversion}: (Converted)

$$P_{conv} = P_{AG} - P_{cu} = 38,6 \text{ k} - 700 = 37,9 \text{ kW}$$

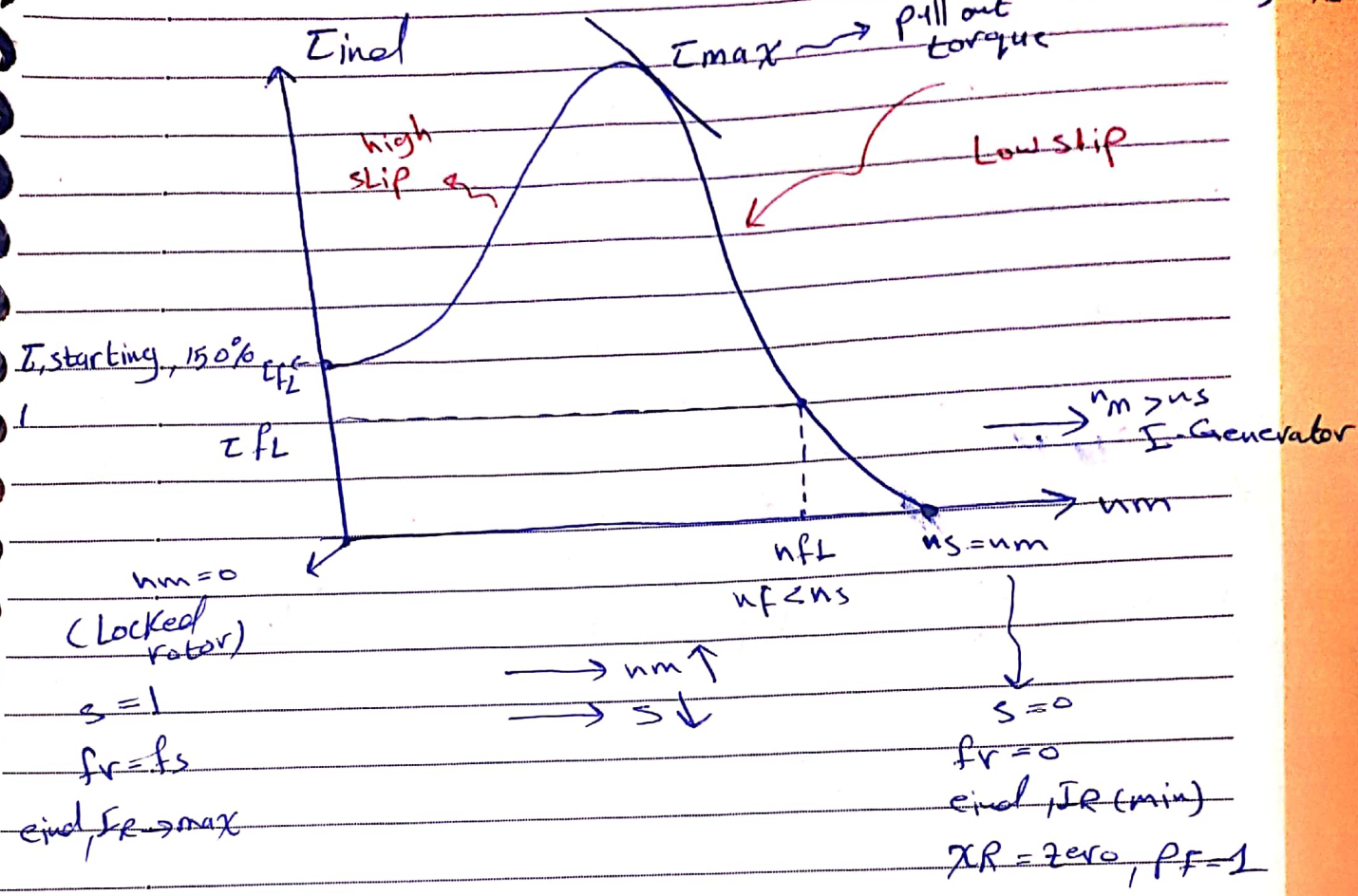
③ P_{out} put:

$$P_o = P_{conv} - P_{rotational} - P_{stray} =$$

$$37,9 \text{ k} - 600 = 37,3 \text{ kW}$$

$$\eta = \frac{P_o}{P_{in}} \times 100\% = \frac{37,3 \text{ k}}{38,6 \text{ k} + 12,4 \text{ k}} = 87,97 \approx 88\%$$

(6.5) Torque-speed: $\rightarrow P_{\text{conv}} = T_{\text{ind}} \cdot \omega_m$ (200 → 250/10) $\cdot \tau_{FL}$



Load $\uparrow \rightarrow \omega_m \downarrow \rightarrow s \uparrow \rightarrow I_R \uparrow, I \uparrow$
 $\omega_m \uparrow, s \downarrow \rightarrow I_R \downarrow$

(6.4)

* Speed control of IM :-

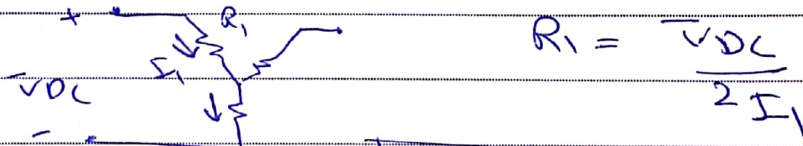
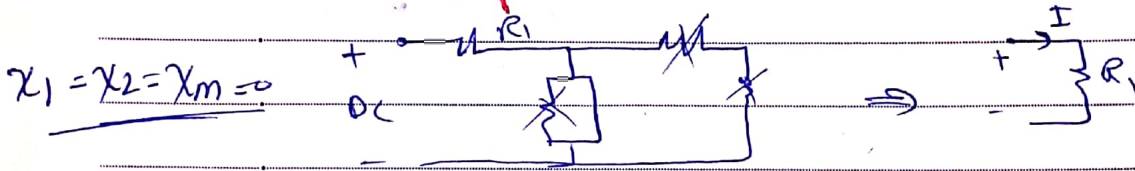
$n_m = (1-s) \cdot n_s$

→ Change $n_s = 120 \cdot \frac{f_s}{P}$
 Elec, freq → Poles

- 1] Change f_s (electrical frequency) (Job 11)
- 2] Change number of poles

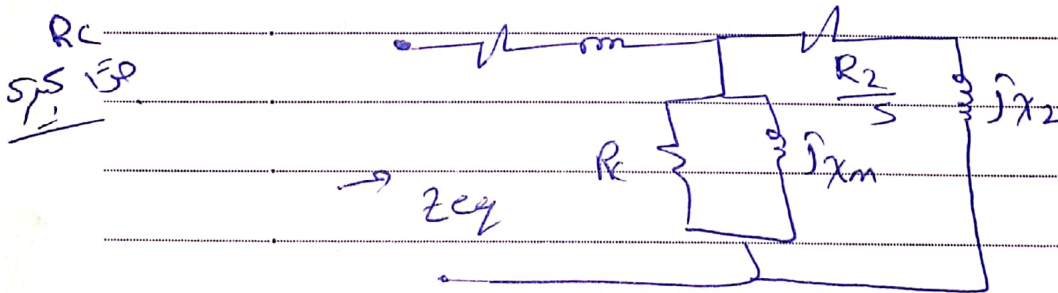
(model parameters of induction Motor:)

1] DC test ⇒ Applied DC source voltage to the stator input ⇒ to find R_1



$R_1 = \frac{V_{DC}}{2 I_1}$

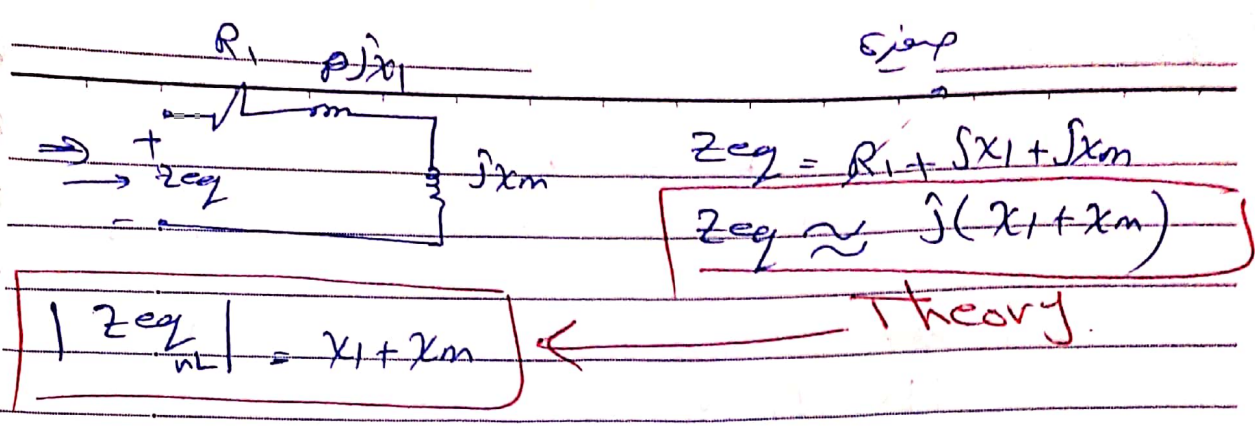
2] No Load test: ⇒ $s \approx 0$ ⇒ 0,001



$R_c \parallel jX_m \parallel \left(\frac{R_2}{s} + jX_2 \right) + (R_1 + jX_1)$
 مقارنه لمدولان جزا قلال (بمقصد)
 كان سو

⇒ $R_c \parallel jX_m \left(\frac{R_2}{s} + jX_2 \right) \approx jX_m$

جزا قلال
 parallel
 في كل واحد عقا وة
 في قلا



(*) Practical: $|Z_{NL}| = \frac{V\phi}{I_a} = \frac{V_L}{\sqrt{3} I_L}$ ← average current

$\Delta \Rightarrow \frac{V_L}{\sqrt{3}} = V\phi, I_L = I\phi$
 $\Delta \Rightarrow V_L = \sqrt{3} V\phi, \frac{I_L}{\sqrt{3}} = I\phi$

\Rightarrow @ no load $\Rightarrow P_{in} = P_o - P_{loss}$

$P_{loss} = P_{scu} + P_{core} + P_{friction} + P_{rcu} \rightarrow \approx 0$
 (Note: $P_{scu}, P_{core}, P_{friction}$ are grouped under P_{rot})

$P_{rot} = P_{in} - P_{scu} \rightarrow 3 \cdot R_1 I_{avg}^2$

[3] Locked Rotor test :- (Blocked the rotor).
 $\omega_m = 0, s = 1, f_{rot} = f_s$

$Z_{eq} = R_1 + jX_1 + (R_c \parallel jX_m) \parallel (R_2 + jX_2)$
 (Note: $s=1$ is indicated for the rotor branch)

$Z_{eq} = R_1 + jX_1 + R_2 + jX_2$

$\Rightarrow Z_{eq} = R_{eq} + jX_{eq}$ ← Theory

Practical

$\Rightarrow I_{avg} \Rightarrow I_a, I_b, I_c$

$\sqrt{\phi}$ or \sqrt{L} (ω) \leftarrow , P_{in}

$(f_{test}) < f_s$

$\Rightarrow f_{test} \leq 0,25 f_s$

$\Rightarrow Z_{eq} = \frac{\sqrt{\phi}}{I_{avg}} = \frac{\sqrt{L}}{\sqrt{3} \cdot I_{avg}} \angle \theta$

$Z_{eq} = R_{eq} + jX_{eq}$

$P_{in} = \sqrt{3} \cdot \sqrt{L} \cdot I_L \cdot PF = 3 \sqrt{\phi} \cdot I_L \cdot PF$

$\theta = \cos^{-1} \left(\frac{P_{in}}{\sqrt{3} \cdot \sqrt{L} \cdot I_L} \right) \rightarrow I_{avg}$

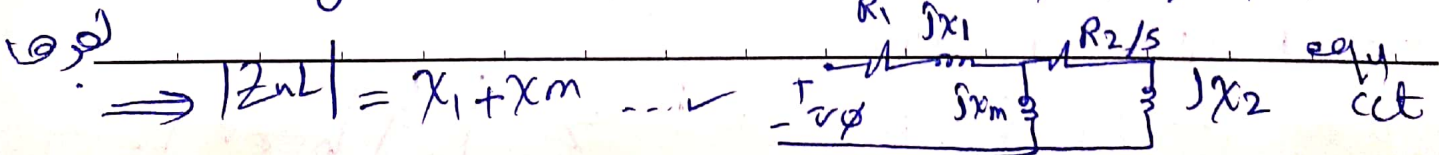
$\Rightarrow R_{1+R2} = \frac{\sqrt{\phi}}{I_{avg}} \cdot \cos \theta, X_{eq} = \frac{\sqrt{\phi}}{I_{avg}} \sin \theta$ } from test
Practical

$\Rightarrow X_{eq} = \frac{f_s}{f_{test}} \cdot X_{eq}'$

$X_{eq} = X_1 + X_2$

$\Rightarrow \frac{f_s}{f_{test}} \cdot 2\pi f_{test} L_{eq} = 2\pi f_s L_{eq} = X_{eq}$

⊗ ω \Rightarrow Class A $\Rightarrow 0,5 X_{eq} = X_1$
 $0,5 X_{eq} = X_2$



Ex: 7.5 hp, 4 poles, 208 V, 60 Hz, Class A,
 Y connected, IM, rated current $I = 28 \text{ A}$

Dc test, $V_{DC} = 13.6 \text{ V}$, $I_{DC} = 28 \text{ A}$

\Rightarrow no load test, $V_L = 208 \text{ V}$, $f = 60 \text{ Hz}$

$I_a = 8.12 \text{ A}$, $I_b = 8.12 \text{ A}$, $I_c = 8.18 \text{ A}$

$P_{in} = 420 \text{ watt}$

Locked rotor test

$\Rightarrow V_L = 25 \text{ V}$, $f = 15 \text{ Hz}$

$P_{in} = 420 \text{ W}$, $I_a = 28.1 \text{ A}$

$I_b = 28 \text{ A}$

$I_c = 27.6 \text{ A}$

Final eqn for IM.

Sol: ① Dc test $\Rightarrow R_1 = \frac{V_{DC}}{2 I_{DC}} = \frac{13.6}{56} = \boxed{0.243 \Omega}$

\Rightarrow no load test

$$|Z_{nl}| = X_1 + X_m, \quad |Z_{nl}| = \frac{V_L}{I_{avg} \sqrt{3}} = \frac{V_L}{I_{avg} \sqrt{3}}$$

$$I_{avg} = (I_a + I_b + I_c) / 3 = 8.16 \text{ A} \approx 8.17 \text{ A}$$

$$\Rightarrow |Z_{nl}| = \frac{208}{\sqrt{3} \times 8.17} = \boxed{14.7 = X_1 + X_m}$$

\Rightarrow if you need prot:

$$P_{rot} = P_{in} - P_{scu} = 420 - (3 \cdot R_1 \cdot I_{avg}^2) = \boxed{371.3 \text{ W}}$$

40,4°

3) Locked Rotor:- $Z_{eq} = R_{eq} + jX_{eq}$

$$\theta = \cos^{-1} \left(\frac{P_{in}}{\sqrt{3} \cdot V_L \cdot I_{avg}} \right) = \cos^{-1} \left(\frac{920}{\sqrt{3} \cdot 25 \cdot 27,4} \right) = 40,4^\circ$$

$$R_{eq} = \frac{V_L}{\sqrt{3} \cdot I_{avg}} \cdot \cos 40,4^\circ = 0,3939 \Omega = R_1 + R_2$$

$$\Rightarrow R_2 = 0,150 \Omega$$

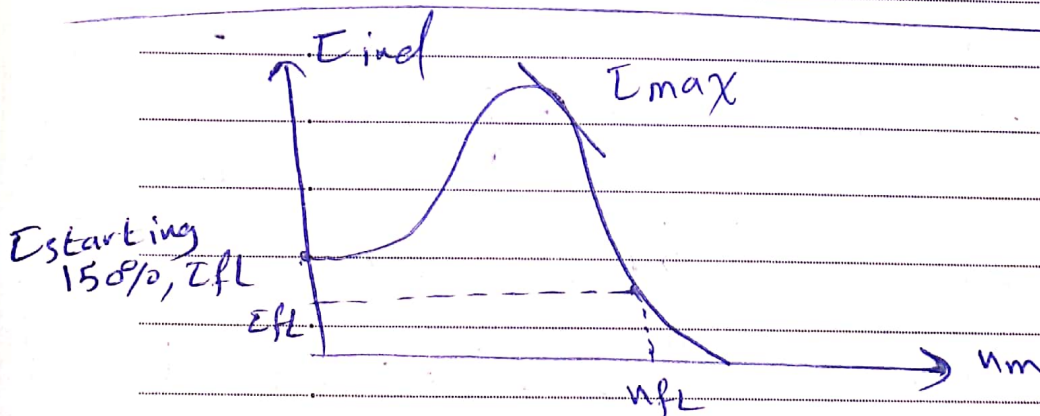
$$\Rightarrow X_{eq}' = \frac{V_L}{\sqrt{3} \cdot I_{avg}} \sin 40,4^\circ = 0,335 = X_1 + X_2 = X_{eq}'$$

~~Class A $\Rightarrow X_1 = X_2 = 0,335 / 2 = 0,1675$~~

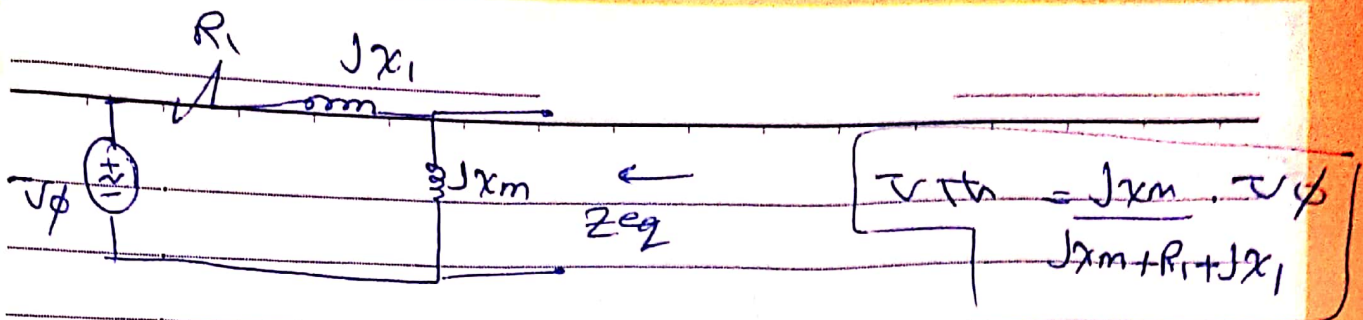
$$X_{eq} = X_{eq}' \cdot \frac{f_s}{f_{test}} = 0,335 \cdot \frac{60}{15} = 1,34 \Omega$$

$$\text{Class A} \Rightarrow X_1 = X_2 = 0,5 \cdot (1,34) = 0,67 \Omega$$

$$\Rightarrow X_m = |Z_{uL}| - X_1 = 14,7 - 0,67 = 14,03 \Omega$$



$$E_{ind} = \frac{P_{conv}}{w_m} = \frac{P_{AG}}{w_s}$$

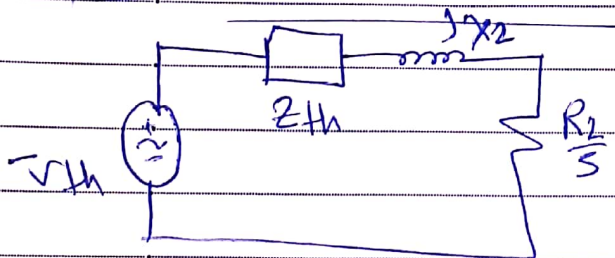


$$|V_{Th}| = \frac{X_m \cdot V_\phi}{\sqrt{R_1^2 + (X_1 + X_m)^2}}, \quad Z_{Th} = (R_1 + jX_1) \parallel jX_m$$

$$Z_{Th} = \frac{jX_m (R_1 + jX_1)}{jX_1 + jX_m + R_1} = \frac{R_1 - j(X_1 + X_m)}{R_1 + j(X_1 + X_m)}$$

$$Z_{Th} = R_1 \left(\frac{X_m}{X_m + X_1} \right)^2 + j \left(\frac{X_m \cdot X_1}{X_m + X_1} \right) \rightarrow X_m \gg X_1$$

$$Z_{Th} = R_{Th} + jX_{Th}$$



$$P_{AG} = 3 I_2^2 \cdot \frac{R_2}{5}$$

$$Z_L = Z_{Th}^*, \quad R_L = |Z_{Th}| \Rightarrow |Z_{Th}| = \frac{R_2}{5}$$

$$\frac{R_2}{5} = \sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}$$

$$S_{max} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}}$$

$$I_{inel} = \frac{P_{AG}}{\omega S} = \frac{3 I_2^2 \cdot R_2 / 5}{\omega S}$$

$$I_2 = \frac{V_{Th}}{Z_{Th} + jX_2 + R_2/5} \Rightarrow |I_2| = \frac{|V_{Th}|}{\sqrt{(R_{Th} + R_2/5)^2 + (X_{Th} + X_2)^2}}$$

$$I_{max} = \frac{3 \cdot V_{th}^2}{2 \omega_s (R_{th} + \sqrt{R_{th}^2 + (X_{th} + X_2)^2})}$$

(incl)

Ex: 460V, 25hp, 60Hz, 4 poles, Y IM
 Slip = 2,2%

$$\Rightarrow R_1 = 0,64 \Omega, R_2 = 0,33 \Omega \rightarrow \text{referred to stator}$$

$$X_1 = 2,206, X_2 = 0,464, X_m = 26,3$$

$R_{rot} = 1100W$, final s-

1) Speed?

$$n_s = \frac{120 \cdot f_s}{p} = 1800 \text{ r/min.}$$

$$\omega_s = \frac{2\pi}{60} \cdot n_s = 188,5 \text{ rad/s}$$

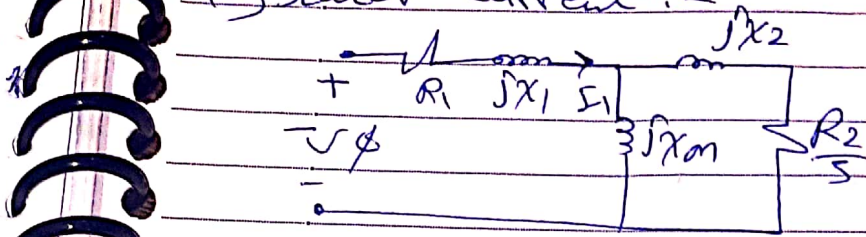
$$\Rightarrow n_m = (1-s) \cdot n_s = 1760 \text{ r/min}, \omega_m = 184,4 \text{ rad/s}$$

2) stator current:

$$|Z_{th}| = \sqrt{X_1^2 + X_m^2} = \sqrt{2,206^2 + 26,3^2} = \frac{V_{th}}{\sqrt{3} \cdot I_{\phi}}$$

$$I_{\phi} = \frac{460}{\sqrt{3} \cdot (27,406)}$$

[2] stator current: -



$$Z_{eq} = \left((jX_2 + \frac{R_2}{s}) \parallel jX_m \right) + R_1 + jX_1$$

$$Z_{eq} = 14 \angle -33,6^\circ \Omega$$

$$I_1 = \frac{\sqrt{3} \cdot V_L}{\sqrt{3} \cdot Z_{eq}} = 18,88 \angle -33,6^\circ \text{ A}$$

[3] PF \Rightarrow PF = $\cos(-33,6) = 0,833$ Lagging
 $\sqrt{3} \phi_I > 0$

[4] $P_{converted}$, P_o ?

$$P_{in} = \sqrt{3} \cdot V_L \cdot I_1 \cdot PF = 12,53 \text{ kW}$$

$$P_{scu} = 3 \cdot R_1 \cdot I_1^2 = 3 \times (18,88)^2 \cdot 0,64 = 685 \text{ watt}$$

$$P_{AG} = P_{in} - P_{scu} = 12,53 \text{ kW} - 685 = 11,845 \text{ kWatt}$$

$$P_{conv} = P_{AG} - P_{scu} = P_{AG} - s P_{AG} = (1-s) P_{AG}$$

$$= 11,845 \text{ kW} (1 - 0,022) = 11,584 \text{ kW}$$

$$\begin{aligned} \boxed{5} \quad P_0 &= P_{\text{conv}} - P_{\text{rotational}} \\ &= 11,584 - 1100 = 10,485 \text{ kWatt} \end{aligned}$$

$$P_0 = \frac{10,485 \text{ K}}{746} = \boxed{14,1 \text{ hp}}$$

$\boxed{6}$ $T_{\text{ind}} ?!$

$$T_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{P_{\text{AG}}}{\omega_s} = \frac{11,845 \text{ K}}{188,5} = \boxed{62,83 \text{ N}\cdot\text{m}}$$

$\boxed{7}$ $T_{\text{load}} ?!$

$$T_{\text{load}} = \frac{P_0}{\omega_m} = \frac{10,485 \text{ K}}{184,4} \approx \boxed{56,9 \text{ N}\cdot\text{m}}$$

$\boxed{8}$ $\eta ?!$ $\eta = \frac{P_0}{P_{\text{in}}} \times 100\%$

$$\eta = \frac{10,485}{12,53} \times 100\% = \approx \boxed{83,7\%}$$

5,16 watt
Gutami