Introduction

By

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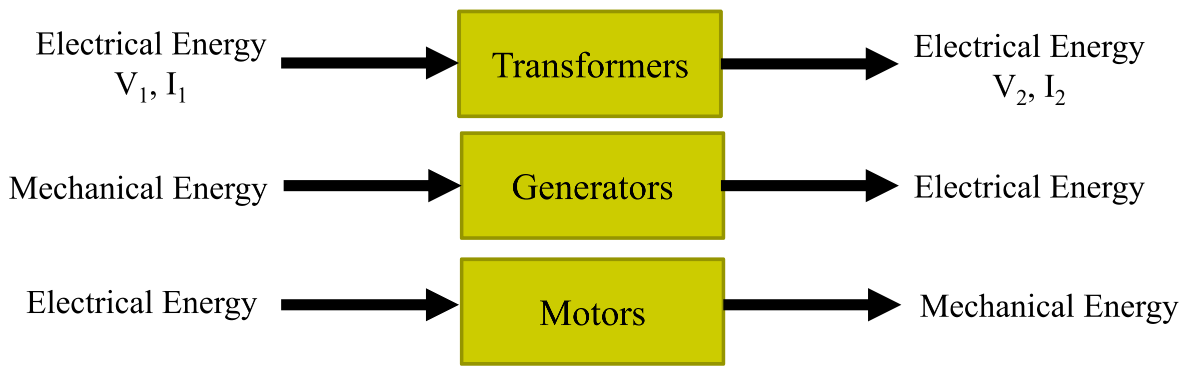
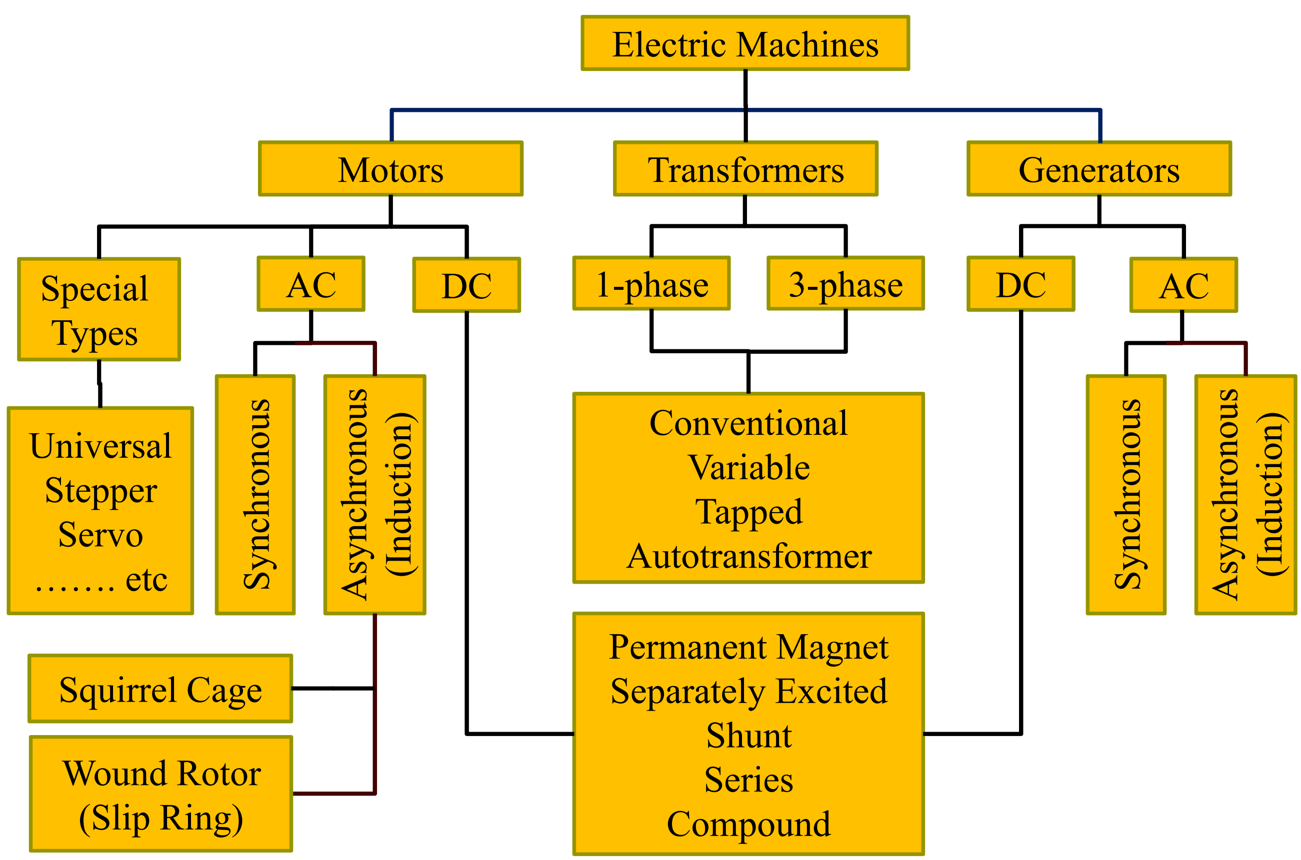
Introduction

- An **Electric Machine** is a general term for **electric** motors and **electric** generators and other electromagnetic **machines**.
- They are electromechanical energy converters: an **electric** motor converts **electricity** to mechanical power while an **electric** generator converts mechanical power to **electricity**.
- Electric machines, in the form of generators, produce virtually all electric power on Earth, and in the form of electric motors consume approximately 60% of all electric power produced.

Introduction



- The common variable in all machines is the magnetic field (Φ).
- Without a magnetic field, machines CANNOT work.



Introduction

- In rotary machines, speed is measured in radian per second [r/s] or revolution per minute [rpm] such that:

$$1\ [\text{rpm}] = 1\left[\frac{revolution}{minute}\right] = 1\left[\frac{2\pi\ [rad]}{60\ [sec]}\right] = \frac{\pi}{30}\ [\text{r/s}]$$

Example: Convert 50 [r/s] to [rpm]

$$1\ [\text{r/s}] = \frac{30}{\pi}\ [\text{rpm}] \Rightarrow 50\ [\text{r/s}] = 50 \times \frac{30}{\pi}\ [\text{rpm}]$$

$$= \frac{1500}{\pi}\ [\text{rpm}]$$

$$\cong 477.5\ [\text{rpm}]$$

Introduction

- In all motors and generators, electric power is measured in **Watts** [W].
- On the other hand, mechanical power is measured either in **Watts** [W] or **Hours Power** [hp] so that:

1 [hp] = 746 [W]

Example: Convert 5 [kW] to [hp]

$$5\ [\text{kW}] = 5000\ [\text{W}] = 5000 \times \frac{1}{746}\ [\text{hp}]$$

$$\cong 6.7\ [\text{hp}]$$

Introduction

- It should be noted that the mechanical power can be computed from the torque and speed as:

Power [W] = Torque [N.m] × Speed [r/s]

- Moreover, electrical power can be computed as:

DC Machines ⇒ Power [W] = Current [A] × Voltage [V]

AC Machines – 1θ ⇒

Active Power [W] = Current [A] × Voltage [V] × cos(θ)

AC Machines – 3θ ⇒

Active Power [W] = $\sqrt{3}$ × Line Current [A] × Line Voltage [V] × cos(θ)

Active Power [W] = 3 × Phase Current [A] × Phase Voltage [V] × cos(θ)

Introduction – Magnetic Circuits

- As mentioned before, machines cannot work without magnetic fields. Hence, studying magnetic circuits is an important topic to understand the basic principles of electric machines operation and characteristics.
- Magnetic fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers.

Introduction – Magnetic Circuits

- Four basic principles describe how magnetic fields are used in these devices:
  1. A current-carrying wire produces a magnetic field in the area around it.
  2. A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil. **(This is the basis of transformer action.)**
  3. A current-carrying wire in the presence of a magnetic field has a force induced on it. **(This is the basis of motor action.)**
  4. A moving wire in the presence of a magnetic field has a voltage induced in it. **(This is the basis of generator action.)**

Introduction – Magnetic Circuits

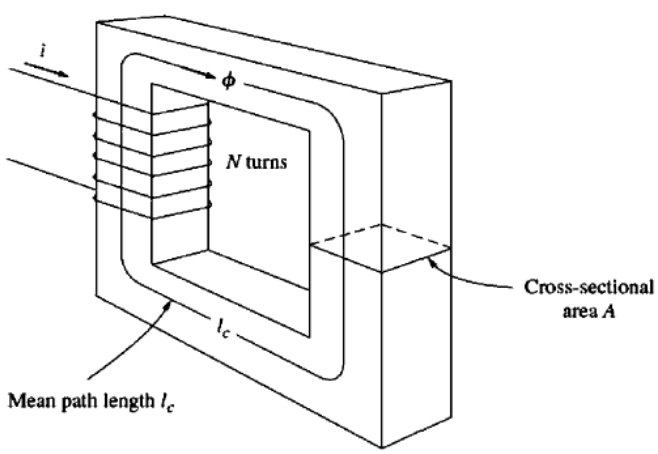


FIGURE 1–3
A simple magnetic core.

The total flux in the core due to the current ***i*** in the winding is

$$\phi = BA = \mu \frac{NiA}{l_c}$$

where ϕ is measured in **Webers** [Web] and B is measured in **Webers per square meters** [Web/m^2] known as **Tesla** [T]

Introduction – Magnetic Circuits

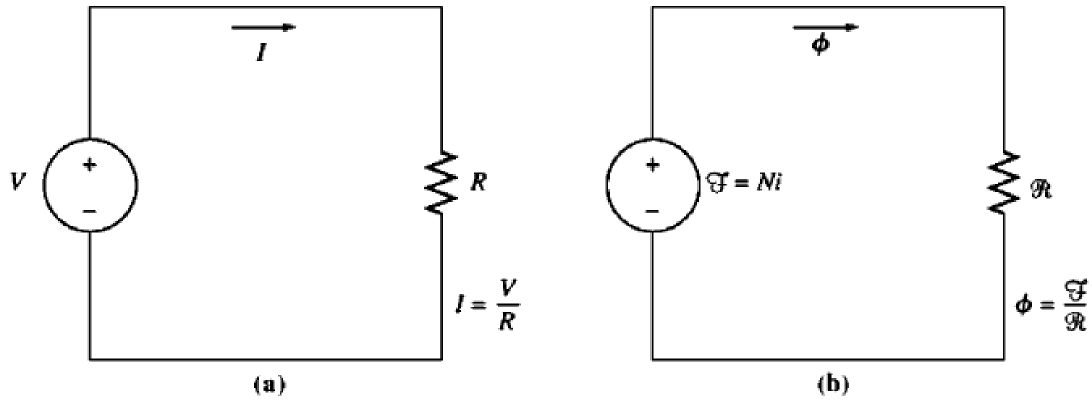
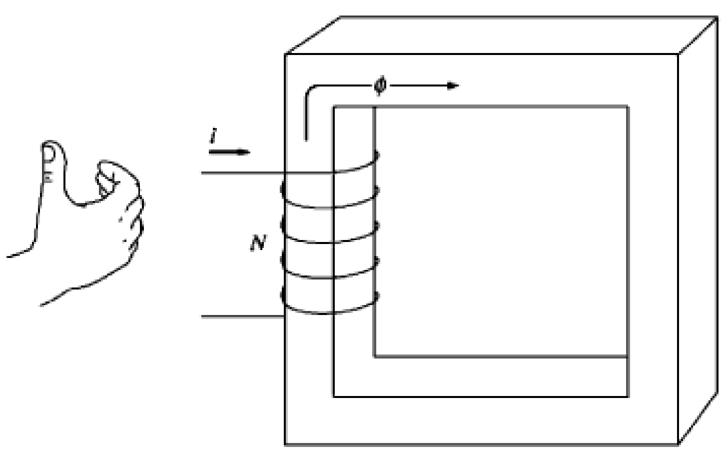


FIGURE 1–4
(a) A simple electric circuit. (b) The magnetic circuit analog to a transformer core.

The magnetic circuit model of magnetic behavior is often used in the design of electric machines and transformers to simplify the otherwise quite complex design process.

Introduction – Magnetic Circuits



The reluctance of a magnetic circuit is the counterpart of electrical resistance, and its units are ampere-turns per weber

F = magnetomotive force of circuit
ϕ = flux of circuit
$\mathcal{R}$ = reluctance of circuit
μ = magnetic permeability of material
H = magnetic field intensity

FIGURE 1–5
Determining the polarity of a magnetomotive force source in a magnetic circuit.

$$\phi = BA = \mu \text{ H A}$$

$$\phi = \mu\left(\frac{Ni}{l_c}\right)\text{A} = Ni\left(\frac{\mu A}{l_c}\right)$$

$$\phi = \mathcal{F}\left(\frac{\mu A}{l_c}\right) = \frac{\mathcal{F}}{\mathfrak{R}}$$

Introduction – Magnetic Circuits

- Calculations of the flux in a core performed by using the magnetic circuit concepts are always approximations. At best, they are accurate to within about 5 percent of the real answer.
- Reasons for this inherent inaccuracy are:

1. The magnetic circuit concept assumes that all flux is confined within a magnetic core. Unfortunately, this is not quite true. The permeability of a ferromagnetic core is 2000 to 6000 times that of air, but a small fraction of the flux escapes from the core into the surrounding low-permeability air. This flux outside the core is called leakage flux, and it plays a very important role in electric machine design.

Introduction – Magnetic Circuits

2. The calculation of reluctance assumes a certain mean path length and cross sectional area for the core. These assumptions are not really very good, especially at corners.

3. In ferromagnetic materials, the permeability varies with the amount of flux already in the material. This nonlinear effect is described in detail. It adds yet another source of error to magnetic circuit analysis, since the reluctances used in magnetic circuit calculations depend on the permeability of the material.

4. If there are air gaps in the flux path in a core, the effective cross sectional area of the air gap will be larger than the cross-sectional area of the iron core on either side. The extra effective area is caused by the "fringing effect" of the magnetic field at the air gap.

Introduction – Magnetic Circuits

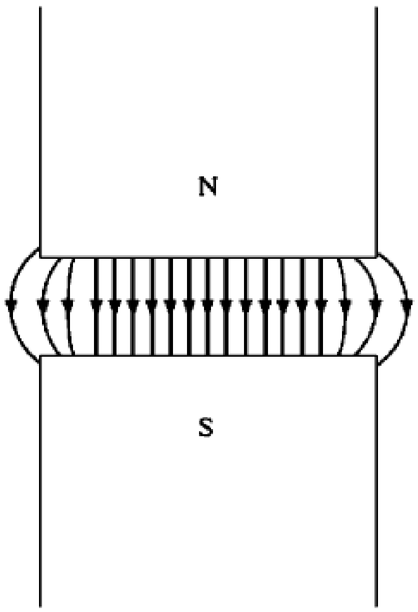


FIGURE 1–6
The fringing effect of a magnetic field at an air gap. Note the increased cross-sectional area of the air gap compared with the cross-sectional area of the metal.

Introduction –
Magnetic Behavior of Ferromagnetic Material

- It was explained that the permeability of ferromagnetic materials is very high, up to 6000 times the permeability of free space.
- The permeability was assumed to be constant regardless of the magnetomotive force applied to the material.
- Although permeability is constant in free space, this most certainly is not true for iron and other ferromagnetic materials.
- Recall

$$\mathbf{B} = \mu \mathbf{H}$$

Introduction –

Magnetic Behavior of Ferromagnetic Material

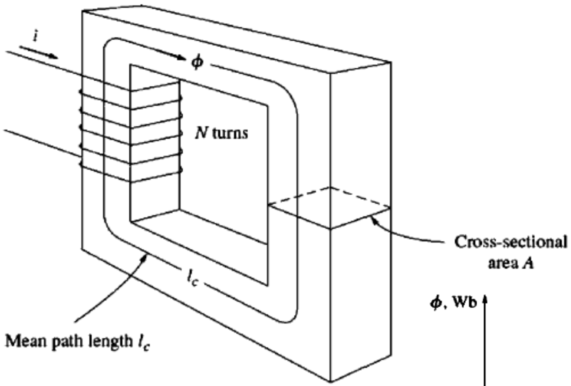


FIGURE 1–3
A simple magnetic core.

This type of plot is called a saturation curve or a magnetization curve. The region of this figure in which the curve flattens out is called the saturation region, and the core is said to be saturated. In contrast, the region where the flux changes very rapidly is called the unsaturated region of the curve, and the core is said to be unsaturated.

The transition region between the unsaturated region and the saturated region is sometimes called the knee of the curve.

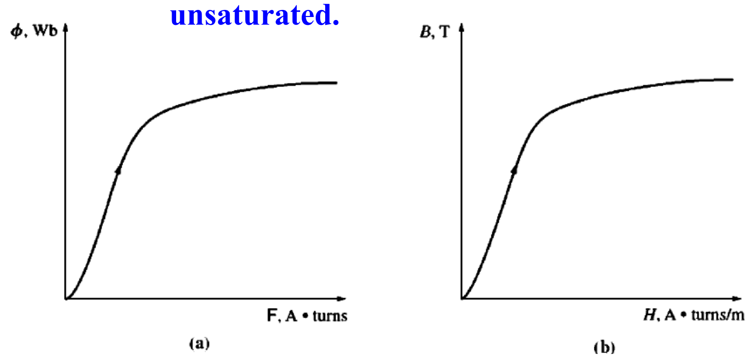


FIGURE 1–10
(a) Sketch of a dc magnetization curve for a ferromagnetic core. (b) The magnetization curve expressed in terms of flux density and magnetizing intensity.

Introduction –

Magnetic Behavior of Ferromagnetic Material

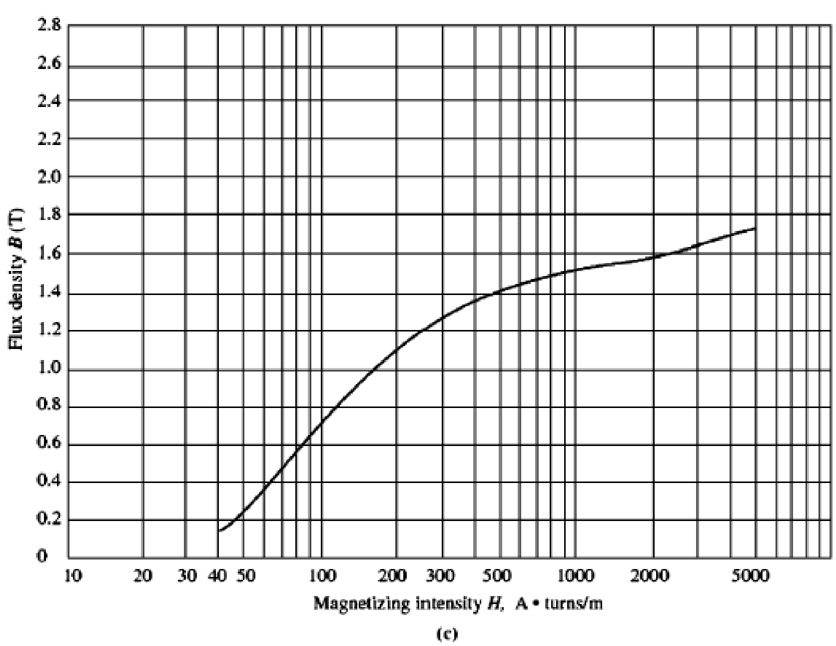


FIGURE 1–10
(c) A detailed magnetization curve for a typical piece of steel.

The advantage of using a ferromagnetic material for cores in electric machines and transformers is that one gets many times more flux for a given magnetomotive force with iron than with air. However, if the resulting flux has to be proportional, or nearly so, to the applied magnetomotive force, then the core must be operated in the unsaturated region of the magnetization curve.

Introduction –

Magnetic Behavior of Ferromagnetic Material

- Since real generators and motors depend on magnetic flux to produce voltage and torque, they are designed to produce as much flux as possible.
- As a result, most real machines operate near the knee of the magnetization curve, and the flux in their cores is not linearly related to the magnetomotive force producing it.
- This nonlinearity accounts for many of the strange behaviors of machines

Introduction – Linear DC Machine

A linear dc machine is about the simplest and easiest-to-understand version of a dc machine, yet it operates according to the same principles and exhibits the same behavior as real generators and motors. It thus serves as a good starting point in the study of machines.

Introduction – Linear DC Machine

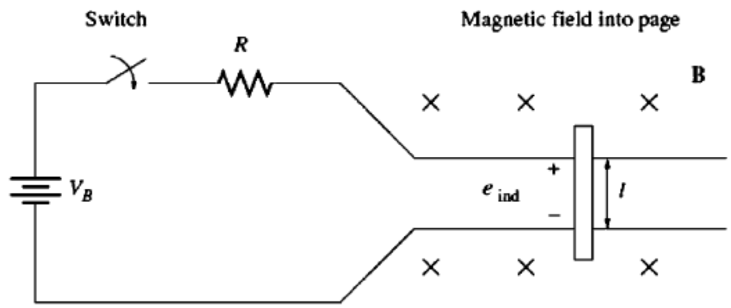


FIGURE 1–19
A linear dc machine. The magnetic field points into the page.

A linear dc machine consists of a battery and a resistance connected through a switch to a pair of smooth, frictionless rails. Along the bed of this "railroad track" is a constant uniform-density magnetic field directed into the page. A bar of conducting metal is lying across the tracks.

Introduction – Linear DC Machine

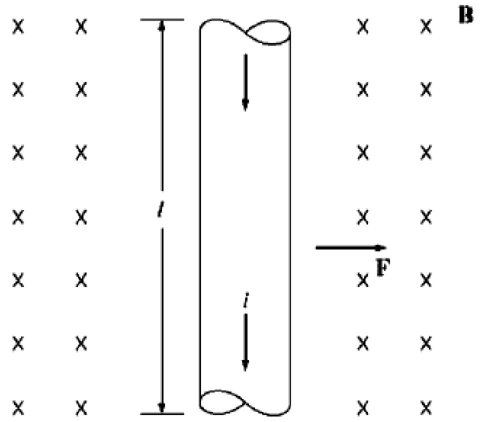
Linear DC machines' behavior can be determined from an application of four basic equations to the machine:

1. The equation for the force on a wire in the presence of a magnetic field:

$$\boxed{\mathbf{F} = i(\mathbf{l} \times \mathbf{B})} \qquad (1\text{–}43)$$

where $\mathbf{F}$ = force on wire
i = magnitude of current in wire
$\mathbf{l}$ = length of wire, with direction of $\mathbf{l}$ defined to be in the direction of current flow
$\mathbf{B}$ = magnetic flux density vector

Introduction – Linear DC Machine



$$F = ilB \sin \theta \qquad (1\text{–}44)$$

FIGURE 1–16
A current-carrying wire in the presence of a magnetic field.

where

i = magnitude of current in wire

l = length of wire, with direction of **l** defined to be in the direction of current flow

B = magnetic flux density vector

Introduction – Linear DC Machine

2. The equation for the voltage induced on a wire moving in a magnetic field:

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45)$$

where e_{ind} = voltage induced in wire

v = velocity of the wire

B = magnetic flux density vector

l = length of conductor in the magnetic field

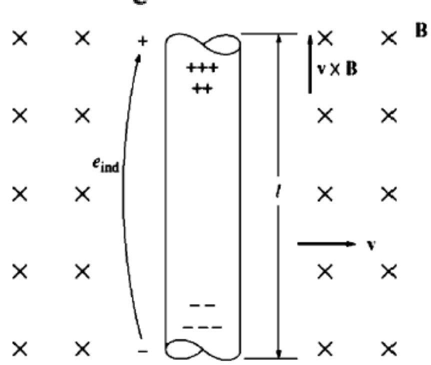
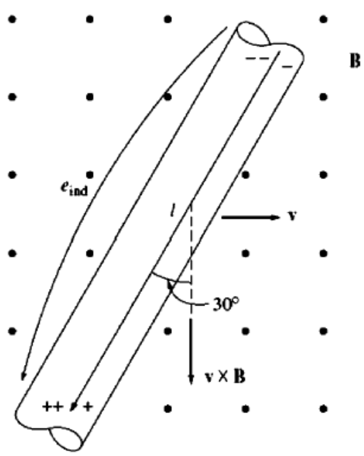


FIGURE 1–17
A conductor moving in the presence of a magnetic field.

Introduction – Linear DC Machine

Example 1.5 Figure 1-18 shows a conductor moving with a velocity of 10 m/s to the right in a magnetic field. The flux density is 0.5 T, out of the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?



$$\begin{aligned} e_{ind} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45) \\ &= (vB \sin 90^\circ)\, l \cos 30^\circ \\ &= (10.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \cos 30^\circ \\ &= 4.33 \text{ V} \end{aligned}$$

FIGURE 1–18
The conductor of Example 1–9.

Introduction – Linear DC Machine

3. Kirchhoff's voltage law for this machine. From Figure 1–19 this law gives

$$V_B - iR - e_{ind} = 0$$

$$\boxed{V_B = e_{ind} + iR = 0} \qquad (1\text{–}46)$$

4. Newton's law for the bar across the tracks:

$$\boxed{F_{net} = ma} \qquad (1\text{–}7)$$

Introduction – Linear DC Machine

1. Closing the switch produces a current flow $i = \frac{V_B}{R}$

2. The current flow produces an induced force on the bar given by $F_{ind} = ilB$

3. The bar accelerates due to the induced force as $a = F_{ind}/m = \Delta v/\Delta t$

4. The bar moves to the right, producing an induced voltage $e_{ind} = vBl$ as it speeds up

5. This induced voltage reduces the current flow $i = (V_B - e_{ind}\uparrow)/R$

6. The induced force is thus decreased $F_{ind} = i\downarrow lB$ until eventually $F_{ind} = 0$. At that point, $e_{ind} = V_B$, $i = 0$, and the bar moves at a constant no-load speed $v_{ss} = V_B/Bl$ where $a = 0$ (note that $F_{ind} = 0$)

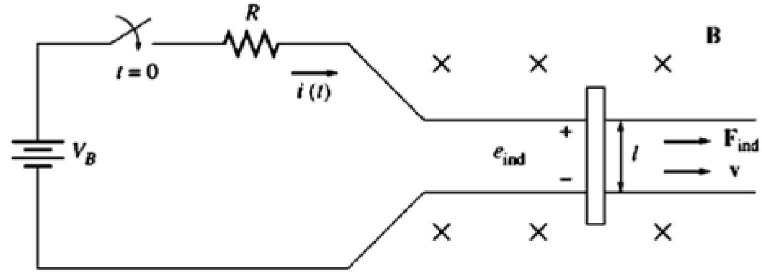


FIGURE 1–20
Starting a linear dc machine.

Introduction – Linear DC Machine

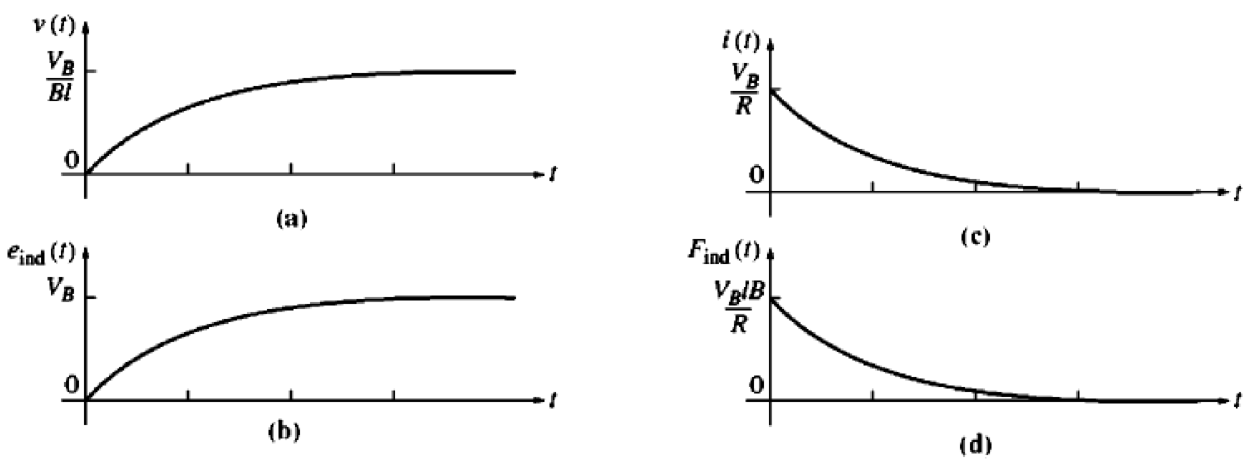


FIGURE 1–21
The linear dc machine on starting.
(a) Velocity $v(t)$ as a function of time;
(b) induced voltage $e_{ind}(t)$; (c) current $i(t)$;
(d) induced force $F_{ind}(t)$.

Introduction – Linear DC Machine

If a force F_{Load} is applied to the bar in the **OPPOSITE** direction of motion, hence the net force is $F_{net} = F_{Load} - F_{ind}$ and the following events occur:

1. The bar decelerates due to the load force as $a = F_{net}/m = \Delta v / \Delta t$ since $F_{net} < 0$ note that $F_{Load} < Find$

2. The effect of this net force will reduce $e_{ind} = v \downarrow Bl$ as ithe bar slows down

3. This induced voltage increases the current flow $i = (V_B - e_{ind} \downarrow) / R$

4. The induced force is thus increased $F = i \uparrow lB$ until $|F_{ind}| = |F_{Load}|$ at a lower speed v

5. An amount of electric power equal to $e_{ind}\, i$ is now being converted to mechanical power equal to $F_{ind}\, v$, and **the machine is acting as a motor**.

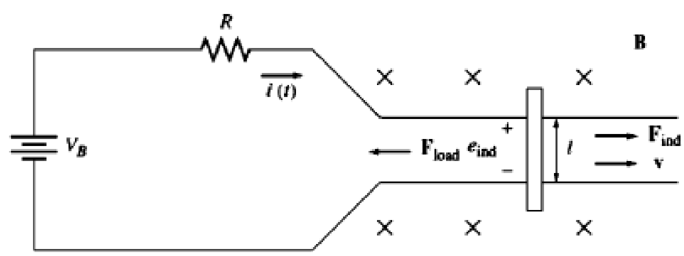


FIGURE 1–22
The linear dc machine as a motor.

Introduction – Linear DC Machine

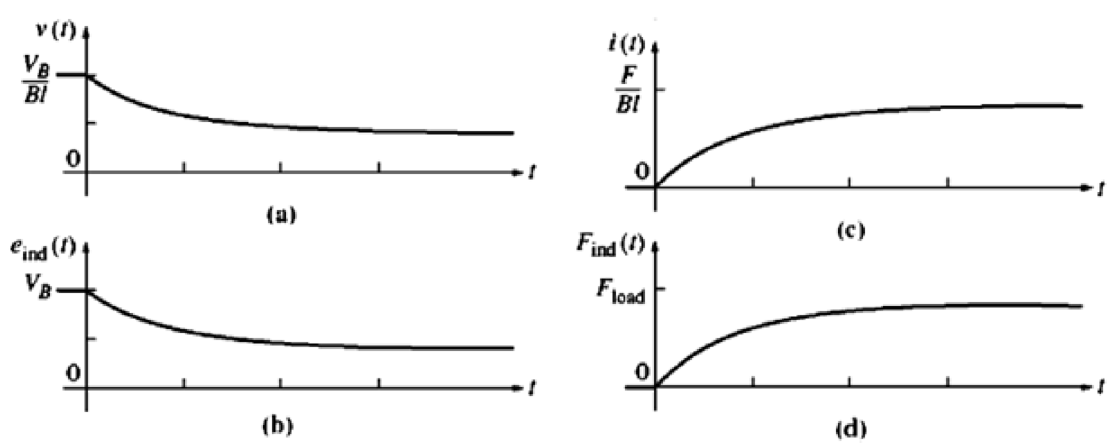


FIGURE 1–23
The linear dc machine operating at no-load conditions and then loaded as a motor.
(a) Velocity $v(t)$ as a function of time;
(b) induced voltage $e_{ind}(t)$; (c) current $i(t)$;
(d) induced force $F_{ind}(t)$.

Introduction – Linear DC Machine

If a force F_{app} is applied to the bar in the **SAME** direction of motion, hence the net force is $F_{net} = F_{app}$ and the following events occur:

1. The bar acelerates due to the load force as $a = F_{net}/m = \Delta v/\Delta t$ since $F_{net} > 0$

2. The effect of this net force will increase $e_{ind} = v\uparrow Bl$ as the bar speeds up

3. This induced voltage reverses the direction of the current flow $i = (V_B - e_{ind}\uparrow)/R$ since ($V_B < e_{ind}$)

4. The induced force is thus reverses its direction due to the reversed current $-F_{ind} = -ilB$ until $|F_{ind}| = |F_{app}|$. Note that $F_{net} = F_{app} - F_{ind}$ in this case.

5. An amount of mechanical power equal to $F_{app}\,v$ is now being converted To electric power equal to $e_{ind}\,i$, and **the machine is acting as a generator**.

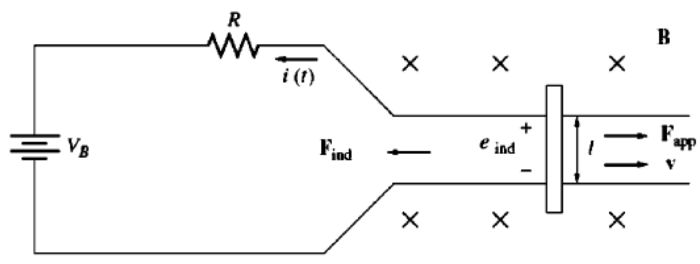


FIGURE 1–24
The linear dc machine as a generator.

Introduction – Linear DC Machine

Example 1.10 The linear dc machine shown in Figure 1-27a has a battery voltage of 120 V. An internal resistance of 0.3 n. and a magnetic flux density of 0.1 T. (a) What is this machine's maximum starting current? What is its steady-state velocity at no load? (b) Suppose that a 30-N force pointing to the right were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming? Explain the difference between these two figures. Is this machine acting as a motor or as a generator? (c) Now suppose a 30-N force pointing to the left were applied to the bar. What would the new steady-state speed be? Is this machine a motor or a generator now? (d) Assume that a force pointing to the left is applied to the bar. Calculate speed of the bar as a function of the force for values from 0 N to 50 N in IO-N steps. Plot the velocity of the bar versus the applied force. (e) Assume that the bar is unloaded and that it suddenly nuns into a region where the magnetic field is weakened to 0.08 T. How fast will the bar go now?

Introduction – Linear DC Machine

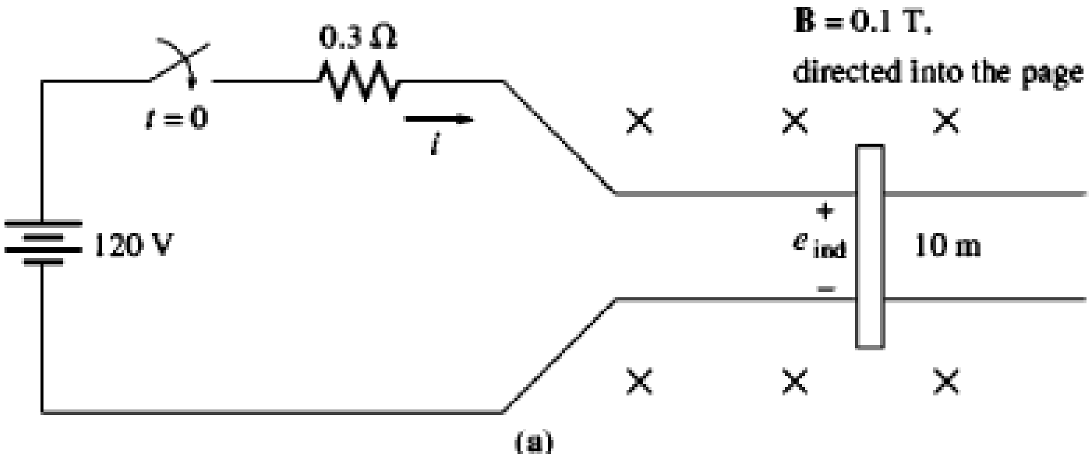


FIGURE 1–27
The linear dc machine of Example 1–10. (a) Starting conditions;

Introduction – Linear DC Machine

(a) At starting conditions, the velocity of the bar is 0, so $e_{ind} = 0$. Therefore,

$$i = \frac{V_B - e_{ind}}{R} = \frac{120\text{ V} - 0\text{ V}}{0.3\ \Omega} = 400\text{ A}$$

When the machine reaches steady state, $\mathbf{F}_{ind} = 0$ and $i = 0$. Therefore,

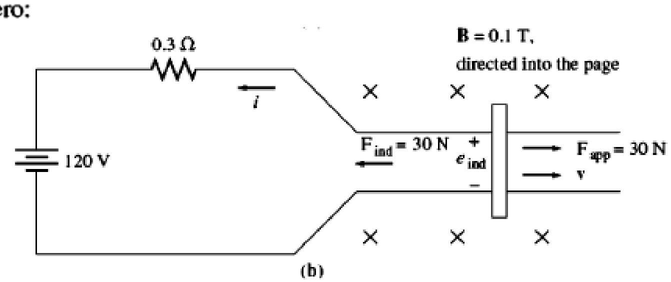
$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120\text{ V}}{(0.1\text{ T})(10\text{ m})} = 120\text{ m/s}$$

(b) Refer to Figure 1–27b. If a 30-N force to the right is applied to the bar, the final steady state will occur when the induced force $\mathbf{F}_{ind}$ is equal and opposite to the applied force $\mathbf{F}_{app}$, so that the net force on the bar is zero:

$$F_{app} = F_{ind} = ilB$$



Introduction – Linear DC Machine

Therefore,

$$i = \frac{F_{ind}}{lB} = \frac{30\ \text{N}}{(10\text{m})(0.1\ \text{T})}$$
$$= 30\ \text{A} \quad \text{flowing up through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B + iR$$
$$= 120\ \text{V} + (30\text{A})(0.3\ \Omega) = 129\ \text{V}$$

and the final steady-state speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$
$$= \frac{129\ \text{V}}{(0.1\ \text{T})(10\ \text{m})} = 129\ \text{m/s}$$

The bar is *producing* $P = (129\ \text{V})(30\ \text{A}) = 3870$ W of power, and the battery is *consuming* $P = (120\ \text{V})(30\ \text{A}) = 3600$ W. The difference between these two numbers is the 270 W of losses in the resistor. This machine is acting as a *generator.*

Introduction – Linear DC Machine

(c) Refer to Figure 1–25c. This time, the force is applied to the left, and the induced force is to the right. At steady state,

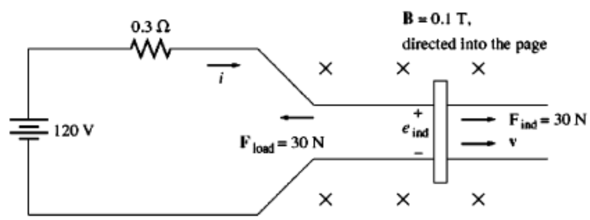
$$F_{app} = F_{ind} = ilB$$
$$i = \frac{F_{ind}}{lB} = \frac{30\ \text{N}}{(10\ \text{m})(0.1\ \text{T})}$$
$$= 30\ \text{A} \quad \text{flowing down through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B - iR$$
$$= 120\ \text{V} - (30\ \text{A})(0.3\ \Omega) = 111\ \text{V}$$

and the final speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$
$$= \frac{111\ \text{V}}{(0.1\ \text{T})(10\ \text{m})} = 111\ \text{m/s}$$



(c)

This machine is now acting as a *motor,* converting electric energy from the battery into mechanical energy of motion on the bar.

Introduction – Linear DC Machine

(*d*) This task is ideally suited for MATLAB. We can take advantage of MATLAB's vectorized calculations to determine the velocity of the bar for each value of force. The MATLAB code to perform this calculation is just a version of the

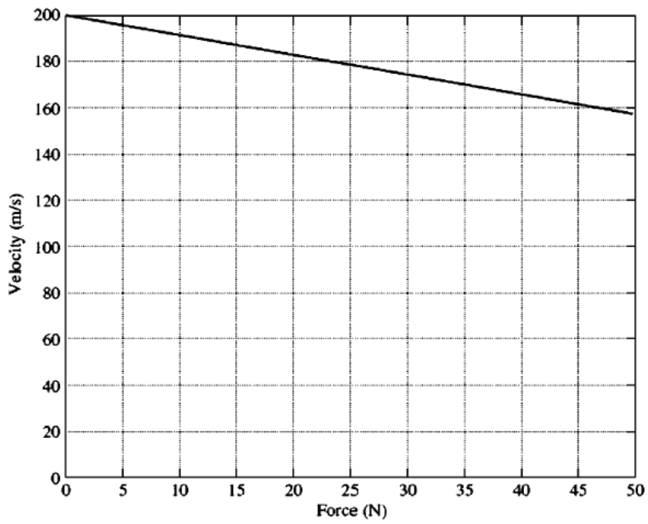


FIGURE 1–28
Plot of velocity versus force for a linear dc machine.

Introduction – Linear DC Machine

(*e*) If the bar is initially unloaded, then $e_{ind} = V_B$. If the bar suddenly hits a region of weaker magnetic field, a transient will occur. Once the transient is over, though, e_{ind} will again equal V_B.

This fact can be used to determine the final speed of the bar. The *initial speed* was 120 m/s. The *final speed* is

$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120\text{ V}}{(0.08\text{ T})(10\text{ m})} = 150\text{ m/s}$$

Thus, when the flux in the linear motor weakens, the bar speeds up. The same behavior occurs in real dc motors: When the field flux of a dc motor weakens, it turns faster. Here, again, the linear machine behaves in much the same way as a real dc motor.

DC Machines

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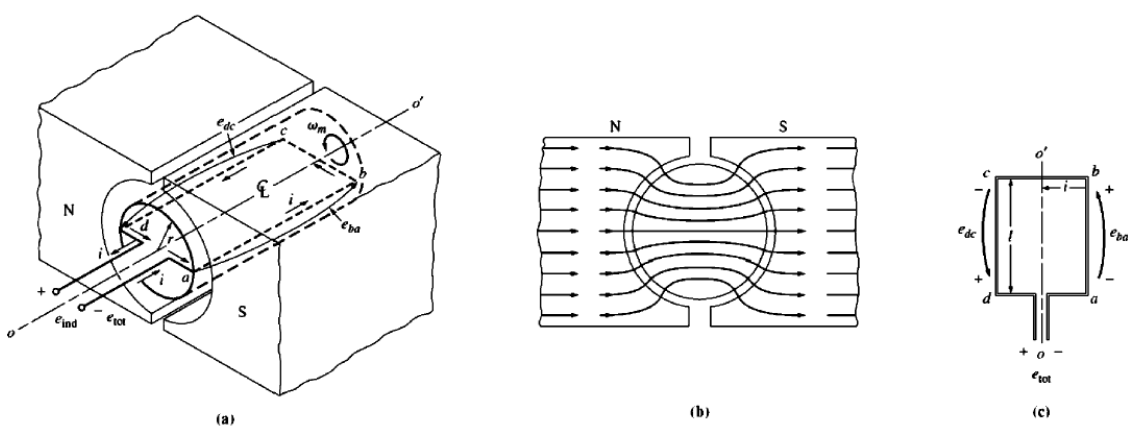
DC Machines – Definition

- DC machines are generators that convert mechanical energy to DC electric energy and motors that convert DC electric energy to mechanical energy.
- Most DC machines are like AC machines in that they have AC voltages and currents within them. DC machines have a DC output only because a mechanism exists that converts the internal AC voltages to DC voltages at their terminals.
- Since this mechanism is called a **commutator**, DC machinery is also known as **commutating machinery**.

DC Machines – Simple Rotating Loop

- The simplest possible rotating dc machine consists of a single loop of wire rotating about a fixed axis.
- The rotating part of this machine is called the **rotor**, and the stationary part is called the **stator**.
- The magnetic field for the machine is supplied by the magnetic north and south poles on the stator.
- Notice that the loop of rotor wire lies in a slot carved in a ferromagnetic core.
- The magnetic field is perpendicular to the rotor surface everywhere under the pole faces.
- The magnetic flux is uniformly distributed (constant) everywhere under the pole faces.

DC Machines – Simple Rotating Loop



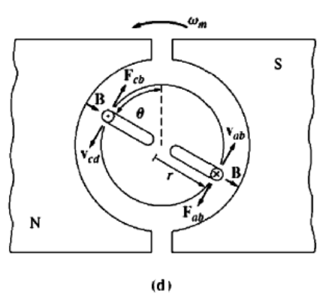


FIGURE 8–1
A simple rotating loop between curved pole faces. (a) Perspective view; (b) view of field lines; (c) top view; (d) front view.

DC Machines – Simple Rotating Loop

- If the rotor of this machine is rotated, a voltage will be induced in the wire loop.
- To determine the total voltage e_{tot} on the loop, we need to examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by

$$e_{ba} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = \begin{cases} vBl & \text{positive into page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases} \quad (8\text{–}1)$$

$$e_{cb} = 0 \quad (8\text{–}2)$$

$$e_{dc} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = \begin{cases} vBl & \text{positive out of page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases} \quad (8\text{–}3)$$

$$e_{ad} = 0 \quad (8\text{–}4)$$

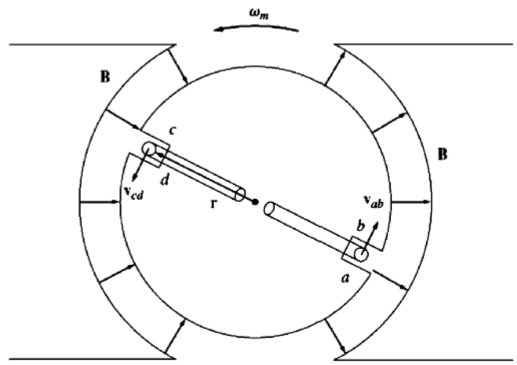


FIGURE 8–2
Derivation of an equation for the voltages induced in the loop.

DC Machines – Simple Rotating Loop

The total induced voltage on the loop e_{ind} is given by

$$e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$$\boxed{e_{ind} = \begin{cases} 2vBl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}} \quad (8\text{–}5)$$

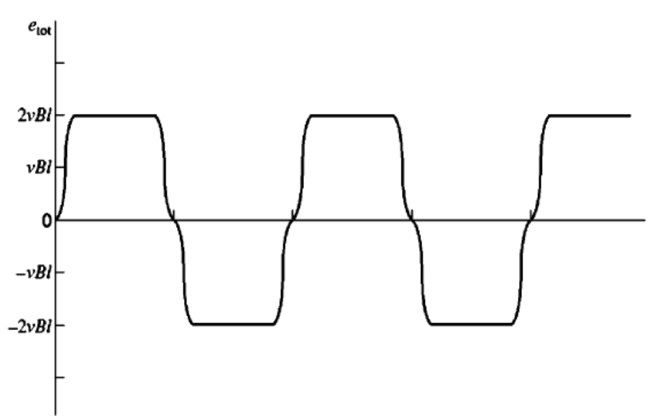


FIGURE 8–3
The output voltage of the loop.

DC Machines – Simple Rotating Loop

- Given $v = r\omega$, $\phi = A_p B$, and pole surface area of $A_p = \pi r l$, an alternative expression can be obtained.

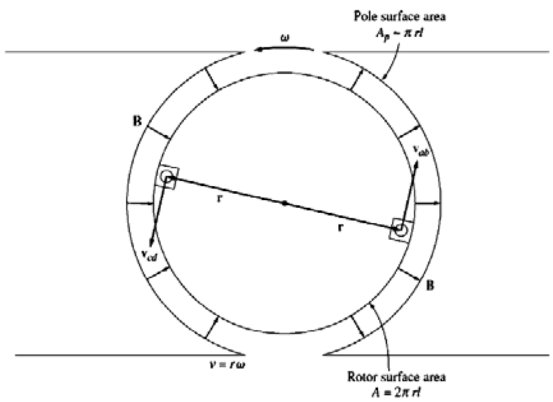


FIGURE 8–4
Derivation of an alternative form of the induced voltage equation.

$$e_{ind} = \begin{cases} 2r\omega Bl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$e_{ind} = \begin{cases} \frac{2}{\pi} A_p B\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$\boxed{e_{ind} = \begin{cases} \frac{2}{\pi} \phi\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}} \qquad (8\text{–}6)$$

DC Machines – Simple Rotating Loop

- The voltage generated in the machine is equal to the product of the flux inside the machine and the speed of rotation of the machine, multiplied by a constant representing the mechanical construction of the machine.
- In general, the voltage in any real machine will depend on the same three factors:

1. The flux in the machine
2. The speed of rotation
3. A constant representing the construction of the machine

DC Machines – Simple Rotating Loop

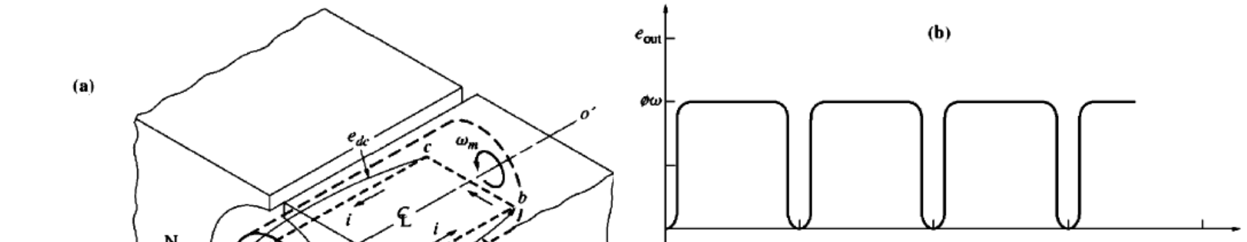


FIGURE 8–5
Producing a dc output from the machine with a commutator and brushes. (a) Perspective view; (b) the resulting output voltage.

In order to produce a DC voltage, two semicircular conducting segments are added to the end of the loop, and two fixed contacts are set up at an angle such that at the instant when the voltage in the loop is zero, the contacts short-circuit the two segments. This connection-switching process is known as **commutation**. The rotating semicircular segments are called **commutator segments**, and the fixed contacts are called **brushes**.

DC Machines – Simple Rotating Loop

Let's suppose that a battery is connected to the machine and see how the torque is developed inside it.

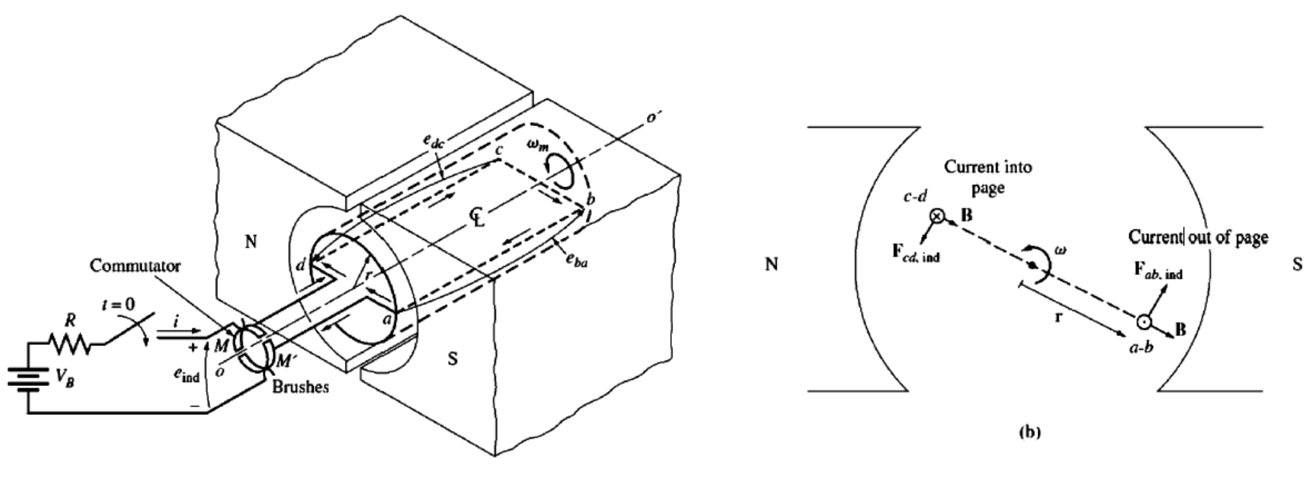


FIGURE 8–6
Derivation of an equation for the induced torque in the loop. Note that the iron core is not shown in part b for clarity.

DC Machines – Simple Rotating Loop

$$\mathbf{F}_{ab} = i(\mathbf{l} \times \mathbf{B})$$
$$= ilB \quad \text{tangent to direction of motion} \qquad (8\text{–}7)$$

The torque on the rotor caused by this force is

$$\tau_{ab} = rF \sin\theta$$
$$= r(ilB)\sin 90^\circ$$
$$= rilB \quad \text{CCW} \qquad (8\text{–}8)$$

$$\mathbf{F}_{bc} = i(\mathbf{l} \times \mathbf{B})$$
$$= 0 \quad \text{since } \mathbf{l} \text{ is parallel to } \mathbf{B} \qquad (8\text{–}9)$$

Therefore,

$$\tau_{bc} = 0 \qquad (8\text{–}10)$$

$$\mathbf{F}_{cd} = i(\mathbf{l} \times \mathbf{B})$$
$$= ilB \quad \text{tangent to direction of motion} \qquad (8\text{–}11)$$

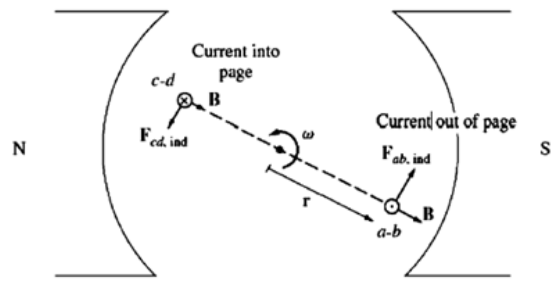
The torque on the rotor caused by this force is

$$\tau_{cd} = rF \sin\theta$$
$$= r(ilB)\sin 90^\circ$$
$$= rilB \quad \text{CCW} \qquad (8\text{–}12)$$

$$\mathbf{F}_{da} = i(\mathbf{l} \times \mathbf{B})$$
$$= 0 \quad \text{since } \mathbf{l} \text{ is parallel to } \mathbf{B} \qquad (8\text{–}13)$$

Therefore,

$$\tau_{da} = 0 \qquad (8\text{–}14)$$



DC Machines – Simple Rotating Loop

The resulting total induced torque on the loop is given by

$$\tau_{\text{ind}} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$$

$$\tau_{\text{ind}} = \begin{cases} 2rilB & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \qquad (8\text{–}15)$$

By using the facts that $A_P \approx \pi rl$ and $\phi = A_P B$, the torque expression can be reduced to

$$\tau_{\text{ind}} = \begin{cases} \dfrac{2}{\pi}\phi i & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \qquad (8\text{–}16)$$

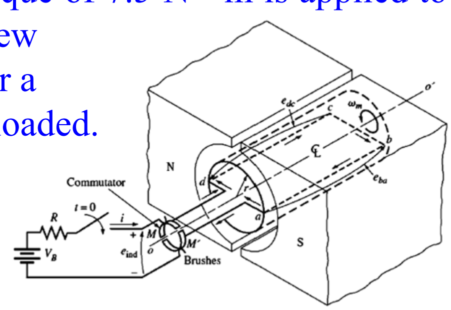
DC Machines – Simple Rotating Loop

- The torque produced in the machine is the product of the flux in the machine and the current in the machine, times some quantity representing the mechanical construction of the machine (the percentage of the rotor covered by pole faces).
- In general, the torque in any real machine will depend on the same three factors:

1. The flux in the machine
2. The current in the machine
3. A constant representing the construction of the machine

DC Machines – Simple Rotating Loop

Example 8.1 Given the system shown in Figure 8-6. The physical dimensions and characteristics of this machine are r = 0.5m, R = 0.3Ω, V_B = 120V, l = 1.0m, B = 0.25T (a) What happens when the switch is closed? (b) What is the machine's maximum starting current? What is its steady-state angular velocity at no load? (c) Suppose a load is attached to the loop, and the resulting load torque is 10 N· m. What would the new steady-state speed be? How much power is supplied to the shaft of the machine? How much power is being supplied by the battery? Is this machine a motor or a generator? (d) Suppose the machine is again unloaded, and a torque of 7.5 N • m is applied to the shaft in the direction of rotation. What is the new steady-state speed? Is this machine now a motor or a generator? (e) Suppose the machine is running unloaded. What would the final steady-state speed of the rotor be if the flux density were reduced to 0.20 T?



DC Machines – Simple Rotating Loop

(a) When the switch in Figure 8–6 is closed, a current will flow in the loop. Since the loop is initially stationary, $e_{ind} = 0$. Therefore, the current will be given by

$$i = \frac{V_B - e_{\text{ind}}}{R} = \frac{V_B}{R}$$

This current flows through the rotor loop, producing a torque

$$\tau_{\text{ind}} = \frac{2}{\pi}\phi i \qquad \text{CCW}$$

This induced torque produces an angular acceleration in a counterclockwise direction, so the rotor of the machine begins to turn. But as the rotor begins to turn, an induced voltage is produced in the motor, given by

$$e_{\text{ind}} = \frac{2}{\pi}\phi\omega$$

so the current i falls. As the current falls, $\tau_{\text{ind}} = (2/\pi)\phi i\downarrow$ decreases, and the machine winds up in steady state with $\tau_{\text{ind}} = 0$, and the battery voltage $V_B = e_{\text{ind}}$.

This is the same sort of starting behavior seen earlier in the linear dc machine.

DC Machines – Simple Rotating Loop

(b) At *starting conditions*, the machine's current is

$$i = \frac{V_B}{R} = \frac{120\text{ V}}{0.3\ \Omega} = 400\text{ A}$$

At *no-load steady-state conditions*, the induced torque τ_{ind} must be zero. But $\tau_{\text{ind}} = 0$ implies that current i must equal zero, since $\tau_{\text{ind}} = (2/\pi)\phi i$, and the flux is nonzero. The fact that $i = 0$ A means that the battery voltage $V_B = e_{\text{ind}}$. Therefore, the speed of the rotor is

$$V_B = e_{\text{ind}} = \frac{2}{\pi}\phi\omega$$

$$\omega = \frac{V_B}{(2/\pi)\phi} = \frac{V_B}{2rlB}$$

$$= \frac{120\text{ V}}{2(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 480\text{ rad/s}$$

DC Machines – Simple Rotating Loop

(*c*) If a load torque of 10 N • m is applied to the shaft of the machine, it will begin to slow down. But as ω decreases, $e_{ind} = (2/\pi)\phi\,\omega\downarrow$ decreases and the rotor current increases $[i = (V_B - e_{ind}\downarrow)/R]$. As the rotor current increases, $|\tau_{ind}|$ increases too, until $|\tau_{ind}| = |\tau_{load}|$ at a lower speed ω.

At steady state, $|\tau_{load}| = |\tau_{ind}| = (2/\pi)\phi\, i$. Therefore,

By Kirchhoff's voltage law, $e_{ind} = V_B - iR$, so

$$e_{ind} = 120\text{ V} - (40\text{ A})(0.3\ \Omega) = 108\text{ V}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{108\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 432\text{ rad/s}$$

The power supplied to the shaft is

$$P = \tau\omega$$

$$= (10\text{ N}\bullet\text{m})(432\text{ rad/s}) = 4320\text{ W}$$

The power out of the battery is

$$P = V_B i = (120\text{ V})(40\text{ A}) = 4800\text{ W}$$

This machine is operating as a *motor,* converting electric power to mechanical power.

DC Machines – Simple Rotating Loop

(*d*) If a torque is applied in the direction of motion, the rotor accelerates. As the speed increases, the internal voltage e_{ind} increases and exceeds V_B, so the current flows out of the top of the bar and into the battery. This machine is now a *generator*. This current causes an induced torque opposite to the direction of motion. The induced torque opposes the external applied torque, and eventually $|\tau_{load}| = |\tau_{ind}|$ at a higher speed ω.

The current in the rotor will be

$$i = \frac{\tau_{ind}}{(2/\pi)\phi} = \frac{\tau_{ind}}{2rlB}$$

$$= \frac{7.5\text{ N}\bullet\text{m}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 30\text{ A}$$

The induced voltage e_{ind} is

$$e_{ind} = V_B + iR$$

$$= 120\text{ V} + (30\text{ A})(0.3\ \Omega)$$

$$= 129\text{ V}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{129\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 516\text{ rad/s}$$

DC Machines – Simple Rotating Loop

(e) Since the machine is initially unloaded at the original conditions, the speed ω = 480 rad/s. If the flux decreases, there is a transient. However, after the transient is over, the machine must again have zero torque, since there is still no load on its shaft. If $\tau_{\text{ind}} = 0$, then the current in the rotor must be zero, and $V_B = e_{\text{ind}}$. The shaft speed is thus

$$\omega = \frac{e_{\text{ind}}}{(2/\pi)\phi} = \frac{e_{\text{ind}}}{2rlB}$$

$$= \frac{120\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.20\text{ T})} = 600\text{ rad/s}$$

Notice that when the flux in the machine is decreased, its speed increases. This is the same behavior seen in the linear machine and the same way that real dc motors behave.

DC Machines – Operation

- **DC Motor, How it works?**
  https://www.youtube.com/watch?v=LAtPHANEfQo&t=1s

DC Machines – Commutation Problems

- **Commutation** is the process of converting the AC voltages and currents in the rotor of a DC machine to DC voltages and currents at its terminals.
- It is the most critical part of the design and operation of any DC machine.
- The commutation process is not as simple in practice as it seems in theory, because two major effects occur in the real world to disturb it:

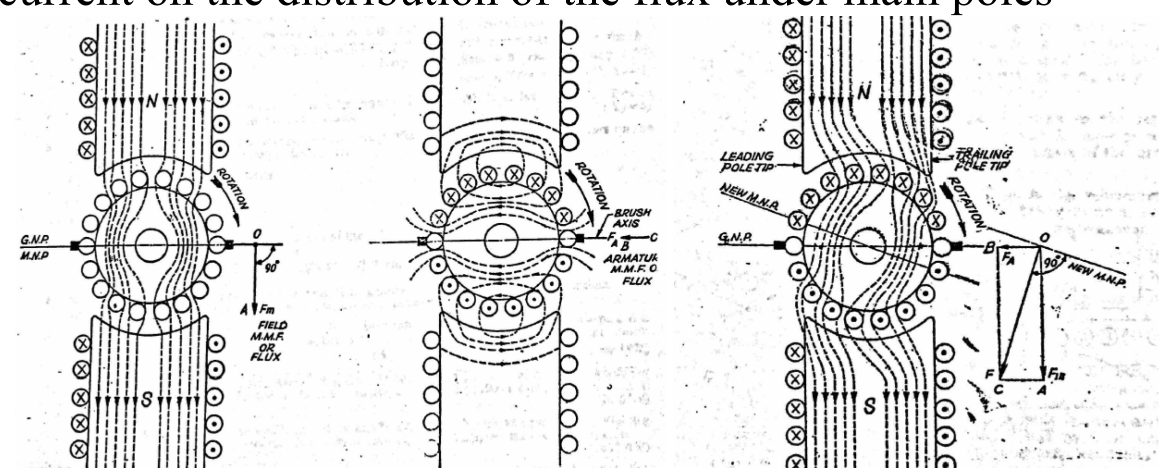
1. Armature reaction

2. $L\frac{di}{dt}$ **voltages**

DC Machines – Commutation Problems

Armature Reaction

It is the effect of the magnetic field set up by the armature current on the distribution of the flux under main poles



Distribution of Flux Due to Main Poles When Generator is Supplying No Load

Distribution of Flux Due To Armature Current Carrying Conductors While Field Coils Carrying No Current

Distribution of Resultant Flux Established From Simultaneous Action of Field and Armature Current

DC Machines – Commutation Problems

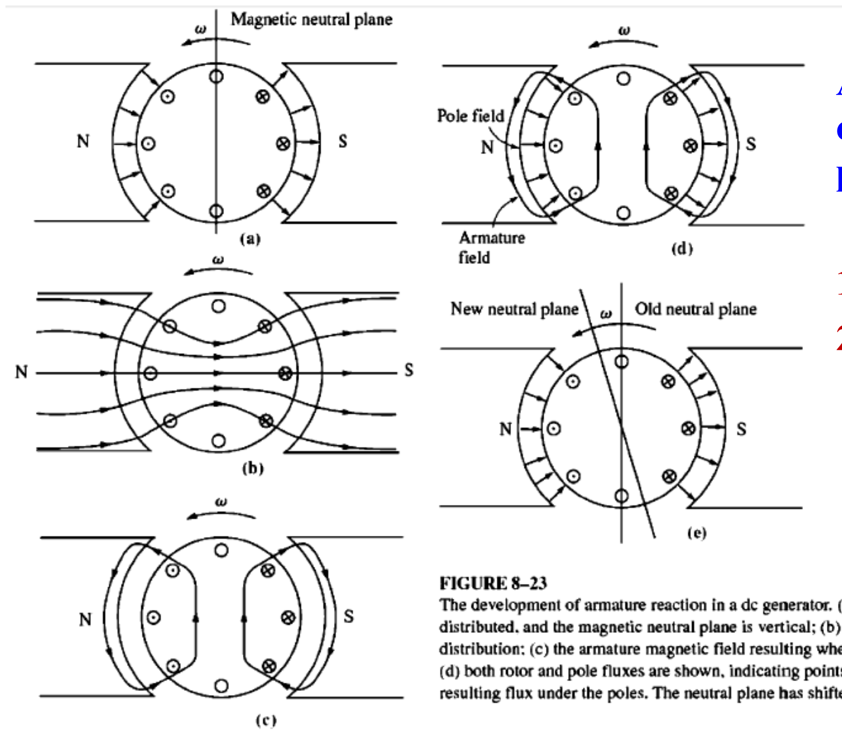


FIGURE 8–23
The development of armature reaction in a dc generator. (a) Initially the pole flux is uniformly distributed, and the magnetic neutral plane is vertical; (b) the effect of the air gap on the pole flux distribution; (c) the armature magnetic field resulting when a load is connected to the machine; (d) both rotor and pole fluxes are shown, indicating points where they add and subtract; (e) the resulting flux under the poles. The neutral plane has shifted in the direction of motion.

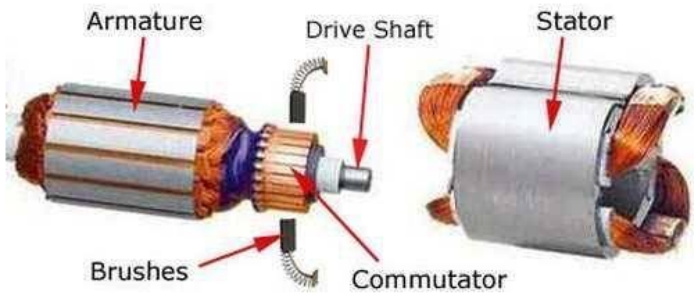
Armature Reaction causes two serious problems:

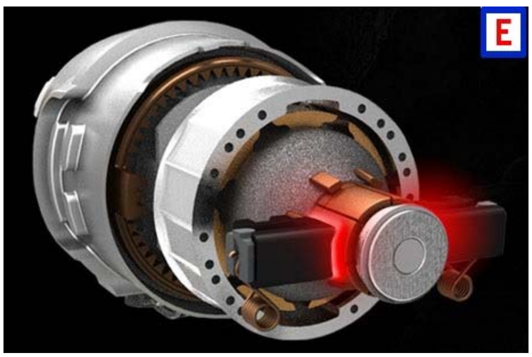
1. **Neutral Plane Shift**
2. **Flux weakening**

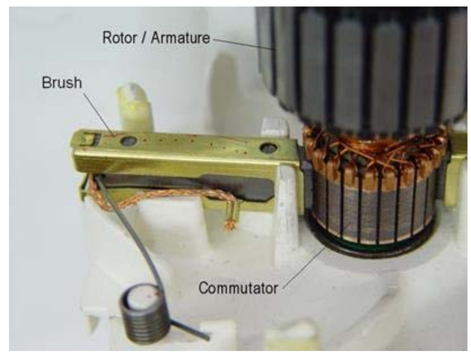
DC Machines – Commutation Problems

- **Magnetic neutral plane** is defined as the plane within the machine where the velocity of the rotor wires is exactly parallel to the magnetic flux lines, so that e_{ind} in the conductors in the plane is exactly zero.
- In general, the neutral-plane shifts in the direction of motion for a generator and opposite to the direction of motion for a motor.
- Furthermore, the amount of the shift depends on the amount of rotor current and hence on the load of the machine.
- The end result is **arcing and sparking at the brushes** and this is a very serious problem

DC Machines – Commutation Problems







DC Machines – Commutation Problems

- **Arcing and sparking at the brushes** lead to:
  - **(1)** drastically reduced brush life
  - **(2)** pitting (making holes) of the commutator segments
  - **(3)** greatly increased maintenance costs

- Notice that this problem cannot be fixed even by placing the brushes over the full-load neutral plane, because then they would spark at no load.

DC Machines – Commutation Problems

- In extreme cases, the neutral-plane shift can even lead to **flashover** in the commutator segments near the brushes.
- The air near the brushes in a machine is normally ionized as a result of the sparking on the brushes.
- **Flashover** occurs when the voltage of adjacent commutator segments gets large enough to sustain an arc in the ionized air above them.
- If flashover occurs, the resulting arc can even melt the commutator's surface.

DC Machines – Commutation Problems

- The effect of flux weakening is simply to reduce the voltage supplied by the generator for any given load.
- In motors, this effect increases the speed and then probably the load, resulting in more flux weakening.
- It is possible for some motors to reach a runaway condition.
- as a result of flux weakening, where the speed of the motor just keeps increasing until the machine is disconnected from the power line or until it destroys itself.

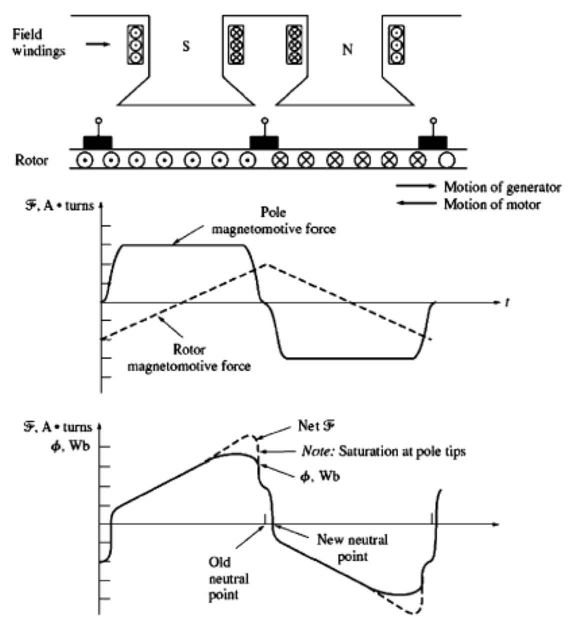


FIGURE 8–25
The flux and magnetomotive force under the pole faces in a dc machine. At those points where the magnetomotive forces subtract, the flux closely follows the net magnetomotive force in the iron; but at those points where the magnetomotive forces add, saturation limits the total flux present. Note also that the neutral point of the rotor has shifted.

DC Machines – Commutation Problems

- $L\frac{di}{dt}$ **voltage** that occurs in commutator segments being shorted out by the brushes, sometimes called **inductive kick**
- This voltage is relatively high and naturally causes sparking at the brushes of the machine, resulting in the same arcing problems that the neutral-plane shift causes.

DC Machines – Commutation Problems

Three approaches have been developed to partially or completely correct the problems of armature reaction and $L\frac{di}{dt}$ voltages:

(1) Brush shifting

(2) Commutating poles or interpoles

(3) Compensating windings

DC Machines – Commutation Problems

- **Shifting brushes** may stop the sparking but it actually aggravated the flux-weakening effect of the armature reaction in the machine. In addition, the neutral plane moves with every change in load, and the shift direction reverses when the machine goes from motor operation to generator operation.
- By about 1910, the brush-shifting approach to controlling sparking was already obsolete. Today, brush-shifting is only used in very small machines that always run as motors.

DC Machines – Commutation Problems

- If the voltage in the wires undergoing commutation can be made zero, then there will be no sparking at the brushes.
- To accomplish this, small poles, called **commutating poles or interpoles**, are placed midway between the main poles.

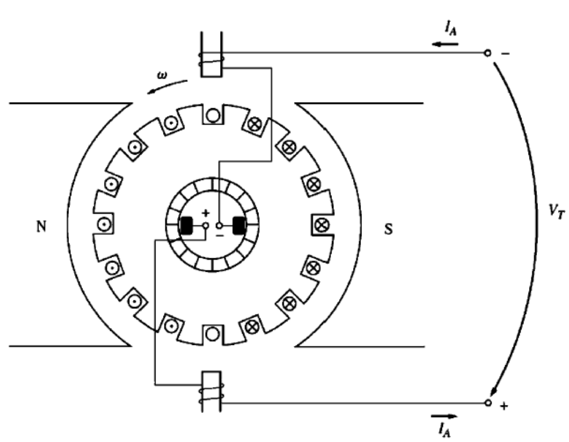


FIGURE 8–28
A dc machine with interpoles.

DC Machines – Commutation Problems

- These commutating poles are located directly over the conductors being commutated. By providing a flux from the commutating poles, the voltage in the coils undergoing commutation can be exactly canceled. If the cancellation is exact, then there will be no sparking at the brushes.
- The use of commutating poles or interpoles is very common, because they correct the sparking problems of dc machines at a fairly low cost.
- They are almost always found in any dc machine of 1 hp or larger.
- It is important to realize, though, that they do nothing for the flux distribution under the pole faces, so the flux-weakening problem is still present.

DC Machines – Commutation Problems

- For very heavy, severe duty cycle motors, the flux-weakening problem can be very serious.
- To completely cancel armature reaction and thus eliminate both neutral-plane shift and flux weakening, **compensating windings** are placed in slots carved in the faces of the poles parallel to the rotor conductors, to cancel the distorting effect of armature reaction.
- These windings are connected in series with the rotor windings, so that whenever the load changes in the rotor, the current in the compensating windings changes, too.

DC Machines – Commutation Problems

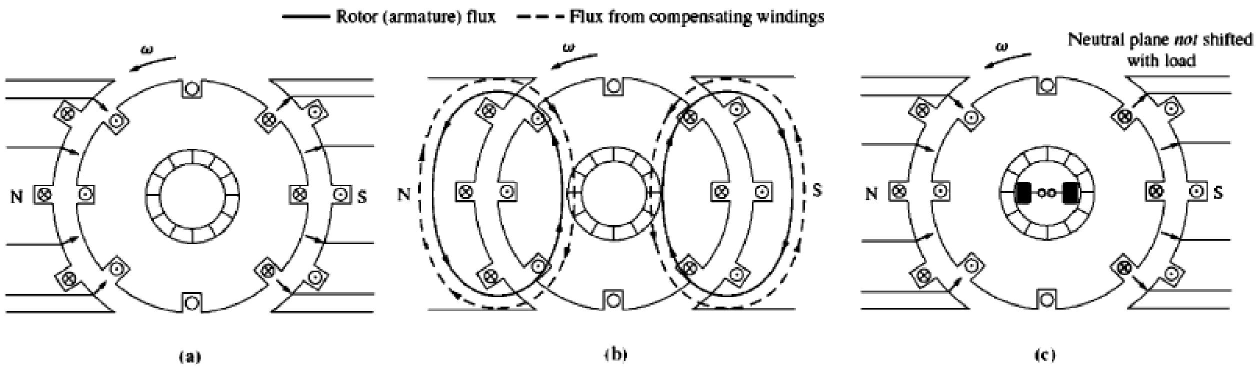


FIGURE 8–30
The effect of compensating windings in a dc machine. (a) The pole flux in the machine; (b) the fluxes from the armature and compensating windings. Notice that they are equal and opposite; (c) the net flux in the machine, which is just the original pole flux.

DC Machines – Commutation Problems

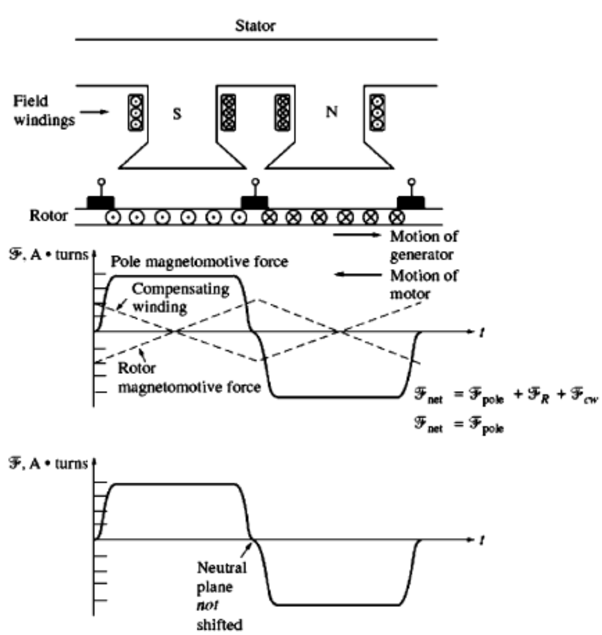


FIGURE 8–31
The flux and magnetomotive forces in a dc machine with compensating windings.

The major disadvantage of compensating windings is that they are expensive, since they must be machined into the faces of the poles. Any motor that uses them must also have interpoles, since compensating windings do not cancel $L\frac{di}{dt}$ effects. Because of the expense of having both compensating windings and interpoles on such a machine, these windings are used only where the extremely severe nature of a motor's duty demands them.

DC Machines – Power Flow and Losses

- In all DC machines, there is always some loss associated with the operation.
- The losses that occur in DC machines can be divided into five basic categories:

1. Electrical or copper losses (I^2R losses)
2. Brush losses
3. Core losses
4. Mechanical losses
5. Stray load losses

DC Machines – Power Flow and Losses

- **Electrical or copper losses** are the losses that occur in the armature and field windings of the machine.
- The copper losses for the armature and field windings are given by

Armature loss:	$P_A = I_A^2 R_A$	(8–52)
Field loss:	$P_F = I_F^2 R_F$	(8–53)

where P_A = armature loss
P_F = field circuit loss
I_A = armature current
I_F = field current
R_A = armature resistance
R_F = field resistance

DC Machines – Power Flow and Losses

- **Brush losses** is the power lost across the contact potential at the brushes of the machine. It is give n by the equation

$$P_{BD} = V_{BD} I_A \quad (8\text{–}54)$$

where P_{BD} = brush drop loss
V_{BD} = brush voltage drop
I_A = armature current

- The reason that the brush losses are calculated in this manner is that the voltage drop across a set of brushes is approximately constant over a large range of armature currents. Unless otherwise specified. the brush voltage drop is usually assumed to be about 2 V.

DC Machines – Power Flow and Losses

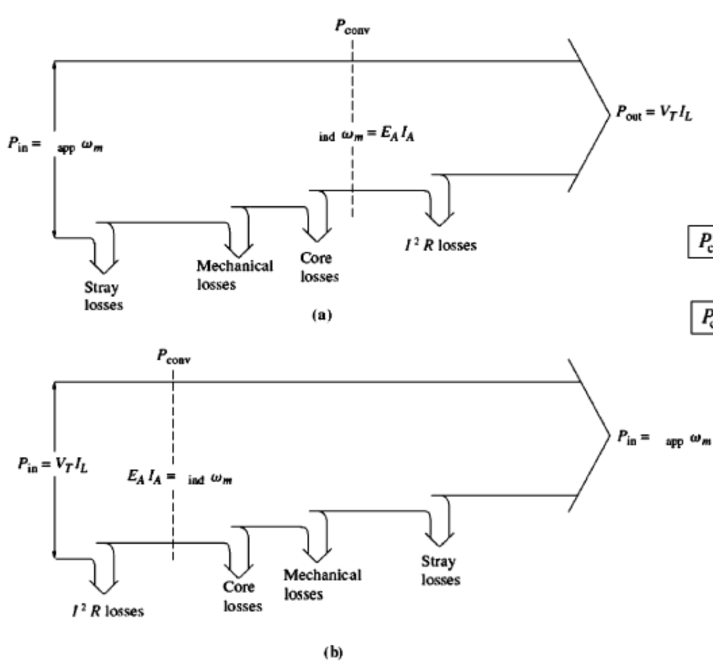
- **Core losses** are the hysteresis losses and eddy current losses occurring in the metal of the motor.
- These losses vary as the square of the flux density (B^2) and for the rotor, as the 1.5th power of the speed of rotation ($n^{1.5}$).
- **Mechanical losses** are associated with mechanical effects and there are two basic types:
  1. **Friction losses** are losses caused by the friction of the bearings in the machine.
  2. **Windage losses** are caused by the friction between the moving parts of the machine and the air inside the motor's casing.
- These losses vary as the cube of the speed (n^3).

DC Machines – Power Flow and Losses

- **Stray losses (or miscellaneous losses)** are losses that cannot be placed in one of the previous categories. No matter how carefully losses are accounted for, some always escape inclusion in one of the above categories.

- All such losses are lumped into stray losses. For most machines, stray losses are taken by convention to be 1% of full load input power.

DC Machines – Power Flow and Losses



$$P_{conv} = \tau_{ind}\omega_m \quad (8\text{–}55)$$

$$P_{conv} = E_A I_A \quad (8\text{–}56)$$

FIGURE 8–39
Power-flow diagrams for dc machine: (a) generator; (b) motor.

DC Machines – Power Flow and Losses

The efficiency of a DC machine is defined by the equation

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{in} - Ploss}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + Ploss} \times 100\%$$

DC Machines – Simple Rotating Loop

Example 9.8 A 50 hp, 250 V, 1200 rpm shunt dc motor has a rated armature current of 170 A and a rated field current of 5 A. When its rotor is blocked, an armature voltage of 10.2 V (exclusive of brushes) produces 170 A of current flow and a field voltage of 250 V produces a field current flow of 5 A. The brush voltage drop is assumed to be 2 V. At no load with the terminal voltage equal to 240 V. the armature current is equal to 13.2 A. the field current is 4.8 A. and the motor's speed is 1150 rpm.

(a) How much power is output from this motor at rated conditions?
(b) What is the motor's efficiency?

DC Machines – Simple Rotating Loop

The armature resistance of this machine is approximately

$$R_A = \frac{10.2 \text{ V}}{170 \text{ A}} = 0.06 \ \Omega$$

and the field resistance is

$$R_F = \frac{250 \text{ V}}{5 \text{ A}} = 50 \ \Omega$$

Therefore, at full load the armature I^2R losses are

$$P_A = (170 \text{ A})^2(0.06 \ \Omega) = 1734 \text{ W}$$

and the field circuit I^2R losses are

$$P_F = (5 \text{ A})^2(50 \ \Omega) = 1250 \text{ W}$$

The brush losses at full load are given by

$$P_{\text{brush}} = V_{\text{BD}} I_A = (2 \text{ V})(170 \text{ A}) = 340 \text{ W}$$

DC Machines – Simple Rotating Loop

The rotational losses at full load are essentially equivalent to the rotational losses at no load, since the no-load and full-load speeds of the motor do not differ too greatly. These losses may be ascertained by determining the input power to the armature circuit at no load and assuming that the armature copper and brush drop losses are negligible, meaning that the no-load armature input power is equal to the rotational losses:

$$P_{\text{rot}} = P_{\text{core}} + P_{\text{mech}} = (240 \text{ V})(13.2 \text{ A}) = 3168 \text{ W}$$

(a) The input power of this motor at the rated load is given by

$$P_{\text{in}} = V_T I_L = (250 \text{ V})(175 \text{ A}) = 43{,}750 \text{ W}$$

Its output power is given by

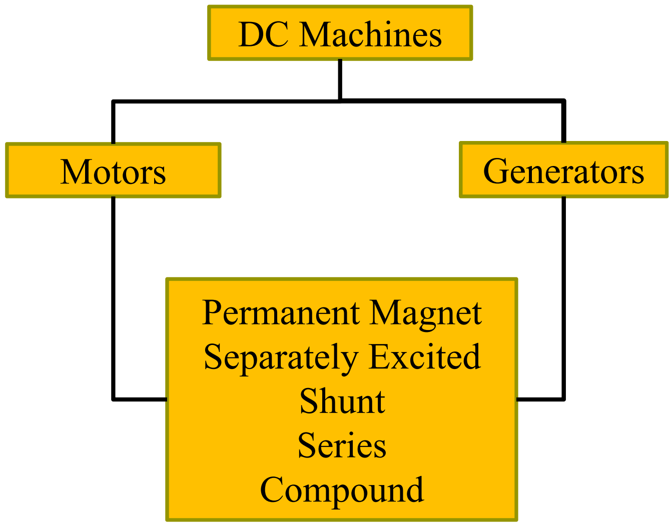
$$\begin{aligned} P_{\text{out}} &= P_{\text{in}} - P_{\text{brush}} - P_{\text{cu}} - P_{\text{core}} - P_{\text{mech}} - P_{\text{stray}} \\ &= 43{,}750 \text{ W} - 340 \text{ W} - 1734 \text{ W} - 1250 \text{ W} - 3168 \text{ W} - (0.01)(43{,}750 \text{ W}) \\ &= 36{,}820 \text{ W} \end{aligned}$$

where the stray losses are taken to be 1 percent of the input power.

(b) The efficiency of this motor at full load is

$$\begin{aligned} \eta &= \frac{P_{\text{out}}}{P_{\text{out}}} \times 100\% \\ &= \frac{36{,}820 \text{ W}}{43{,}750 \text{ W}} \times 100\% = 84.2\% \end{aligned}$$

DC Machines – Types



These various types of DC machines differ in their output (voltage-current) characteristics if they are generators or (speed-torque) characteristics if they are motors, and therefore in the applications to which they are suited.

DC Machines – Types

- In a **separately excited machine**, the field flux is derived from a separate power source.
- In a **shunt machine**, the field flux is derived by connecting the field circuit directly across the terminals of the armature.
- In a **series machine**, the field flux is produced by connecting the field circuit in series with the armature.
- In a **cumulatively compounded machine**, both a shunt and a series field are present, and their effects are additive.
- In a **differentially compounded machine**, both a shunt and a series field are present, but their effects are subtractive .

DC Machines – Types

- **Types of DC Motors - Classification of DC Motors**
  https://www.youtube.com/watch?v=uth3248pzqM

DC Machines – Generators

- DC generators are compared by their voltages, power ratings, efficiencies, and voltage regulations.
- Voltage regulation (VR) is defined by the equation

$$\boldsymbol{VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%}$$

- $\boldsymbol{V_{nl}}$ is the no-load terminal voltage of the generator and $\boldsymbol{V_{fl}}$ is the full- load terminal voltage of the generator
- It is a rough measure of the shape of the generator's voltage-current characteristic. A positive voltage regulation means a drooping characteristic, and a negative voltage regulation means a rising characteristic.

DC Machines – Generators

- All generators are driven by a source of mechanical power, which is usually called the **prime mover** of the generator.
- A prime mover for a DC generator may be a steam turbine, a diesel engine, or even an electric motor.
- Since the speed of the prime mover affects the output voltage of a generator, and since prime movers can vary widely in their speed characteristics, it is customary to compare the voltage regulation and output characteristics of different generators, assuming constant speed prime movers.
- DC generators are quite rare in modern power systems. Even DC power systems such as those in automobiles now use AC generators plus rectifiers to produce DC power.

DC Machines – Generators

Separately Excited DC Generators

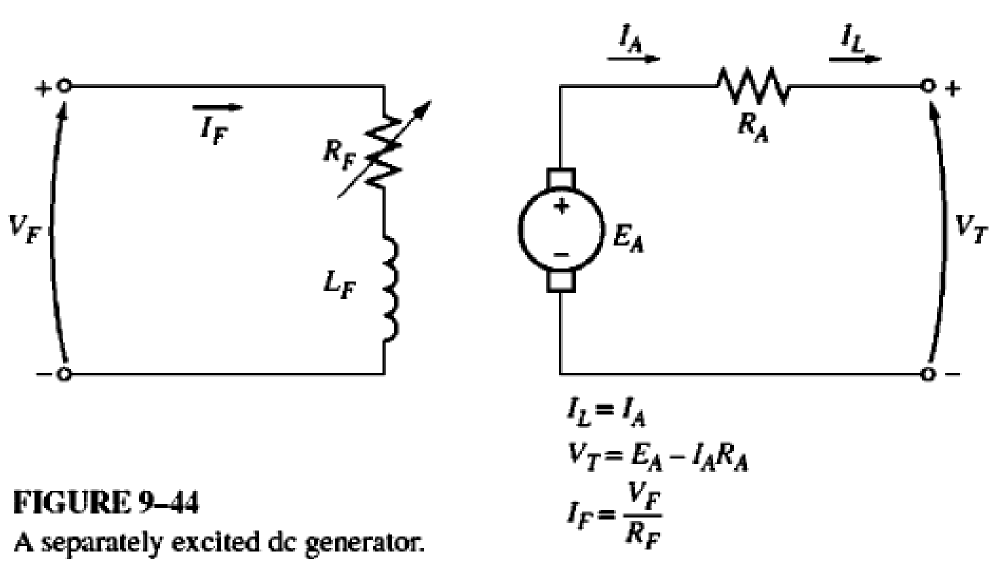


FIGURE 9–44
A separately excited dc generator.

For a constant speed $$V_T = E_A - I_A R_A \quad (9\text{–}41)$$

DC Machines – Generators

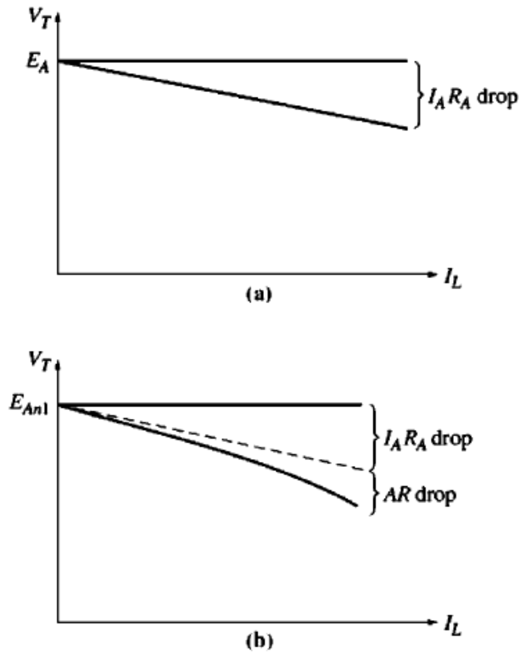


FIGURE 9–45
The terminal characteristic of a separately excited dc generator (a) with and (b) without compensating windings.

When the load supplied by the generator is increased, I_L (and therefore I_A) increases. As the armature current increases, the $I_A R_A$ drop increases, so the terminal voltage of the generator falls. This terminal characteristic is not always entirely accurate. In generators without compensating windings, an increase in I_A causes an increase in armature reaction, and armature reaction causes flux weakening. This flux Weakening causes a decrease in $E_A = K\phi \downarrow \omega$ which further decreases the terminal voltage of the generator.

DC Machines – Generators

- Since the internal generated voltage E_A is given by the equation $E_A = K\phi\downarrow\omega$, there are two possible ways to control the voltage of this generator:

1. **Change the speed:** If ω increases, then $E_A = K\phi\omega\uparrow$ increases, so $V_T = E_A\uparrow - I_A R_A$ increases too.
2. **Change the filed current:** If R_F is decreased, then the field current increases ($I_F = V_F/R_F\downarrow$). Therefore, the flux ϕ in the machine increases. As the flux rises, $E_A = K\phi\uparrow\omega$ must rise too, so $V_T = E_A\uparrow - I_A R_A$ increases.

- In many applications, the speed ran ge of the prime mover is quite limited, so the terminal voltage is most commonly controlled by changing the field current.

DC Machines – Generators

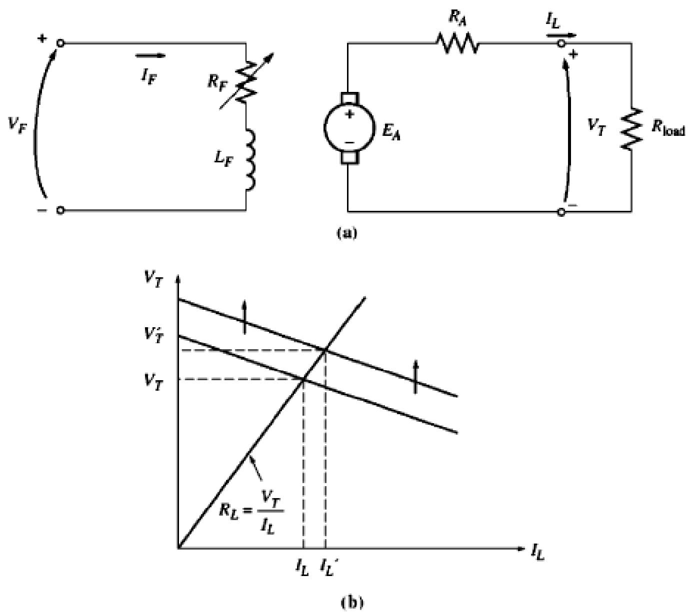


FIGURE 9–46
(a) A separately excited dc generator with a resistive load. (b) The effect of a decrease in field resistance on the output voltage of the generator.

DC Machines – Generators

- Because the internal generated voltage of a generator is a nonlinear function of its magnetomotive force, it is not possible to calculate simply the value of E_A to be expected from a given field current.
- The magnetization curve of the generator must be used to accurately calculate its output voltage for a given input voltage.
- In addition, if a machine has armature reaction, its flux will be reduced with each increase in load, causing E_A to decrease.
- The only way to accurately determine the output voltage in a machine with armature reaction is to use graphical analysis.

DC Machines – Generators

- The total magnetomotive force in a separately excited generator is the field circuit magnetomotive force less the magnetomotive force due to armature reaction (AR):

$$\mathcal{F}_{net} = N_F I_F - \mathcal{F}_{AR}$$

- As with DC motors, it is customary to define an **equivalent field current** that would produce the same output voltage as the combination of all the magnetomotive forces in the machine.
- The resulting voltage E_{A0} can then be determined by locating that equivalent field current on the magnetization curve.

DC Machines – Generators

- The equivalent field current of a separately excited DC generator is given by

$$I_F^* = I_F - \frac{\mathcal{F}_{AR}}{N_F}$$

- Also, the difference between the speed of the magnetization curve and the real speed of the generator must be taken into account:

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

DC Machines – Generators

Example 9.9 A separately excited dc generator is rated at 172kW, 430V, 400A, and 1800 r/min. It is shown in Figure 9-47 and its magnetization curve is shown in Figure 9-48. This machine has the following characteristics: R_A=0.05 *Ω*, R_F=20*Ω*, R_{adj}=0 to 300*Ω*, V_F = 430V, N_F = 1000 turns per pole (a) If the variable resistor R_{adj} in this generator's field circuit is adjusted to 63*Ω* and the generator's prime mover is driving it at 1600 r/min, what is this generator's no-load terminal voltage? (b) What would its voltage be if a 360-A load were connected to its terminals? Assume that the generator has compensating windings. (c) What would its voltage be if a 360-A load were connected to its terminals but the generator does not have compensating windings? Assume that its armature reaction at this load is 450 A·turns. (d) What adjustment could be made to the generator to restore its terminal voltage to the value found in part a? (e) How much field current would be needed to restore the terminal voltage to its no-load value? (Assume that the machine has compensating windings.) What is the required value for the resistor R_{adj} to accomplish this?

DC Machines – Generators

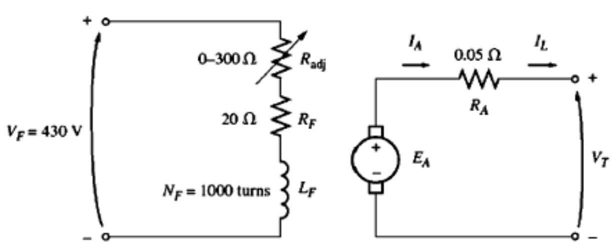
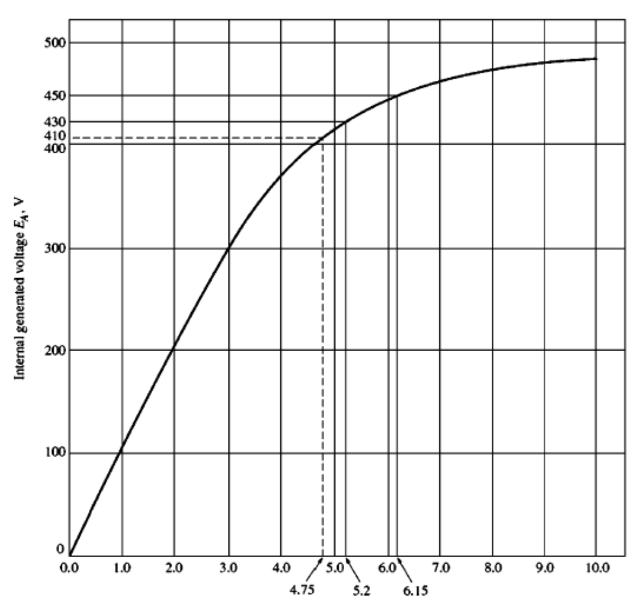


FIGURE 9–47
The separately excited dc generator in Example 9–9.



Note: When the field current is zero, E_A is about 3 V.

FIGURE 9–48
The magnetization curve for the generator in Example 9–9.

DC Machines – Generators

(a) If the generator's total field circuit resistance is

$$R_F + R_{adj} = 83\ \Omega$$

then the field current in the machine is

$$I_F = \frac{V_F}{R_F} = \frac{430\text{ V}}{83\ \Omega} = 5.2\text{ A}$$

From the machine's magnetization curve, this much current would produce a voltage $E_{A0} = 430$ V at a speed of 1800 r/min. Since this generator is actually turning at $n_m = 1600$ r/min, its internal generated voltage E_A will be

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0} \tag{9–13}$$

$$E_A = \frac{1600\text{ r/min}}{1800\text{ r/min}}430\text{ V} = 382\text{ V}$$

Since $V_T = E_A$ at no-load conditions, the output voltage of the generator is $V_T =$ 382 V.

DC Machines – Generators

(b) If a 360-A load were connected to this generator's terminals, the terminal voltage of the generator would be

$$V_T = E_A - I_A R_A = 382\text{ V} - (360\text{ A})(0.05\ \Omega) = 364\text{ V}$$

(c) If a 360-A load were connected to this generator's terminals and the generator had 450 A • turns of armature reaction, the effective field current would be

$$I_F^* = I_F - \frac{\mathscr{F}_{AR}}{N_F} = 5.2\text{ A} - \frac{450\text{ A • turns}}{1000\text{ turns}} = 4.75\text{ A}$$

From the magnetization curve, $E_{A0} = 410$ V, so the internal generated voltage at 1600 r/min would be

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0} \tag{9–13}$$

$$E_A = \frac{1600\text{ r/min}}{1800\text{ r/min}}410\text{ V} = 364\text{ V}$$

Therefore, the terminal voltage of the generator would be

$$V_T = E_A - I_A R_A = 364\text{ V} - (360\text{ A})(0.05\ \Omega) = 346\text{ V}$$

It is lower than before due to the armature reaction.

DC Machines – Generators

(*d*) The voltage at the terminals of the generator has fallen, so to restore it to its original value, the voltage of the generator must be increased. This requires an increase in E_A, which implies that R_{adj} must be decreased to increase the field current of the generator.

(*e*) For the terminal voltage to go back up to 382 V, the required value of E_A is

$$E_A = V_T + I_A R_A = 382\text{ V} + (360\text{ A})(0.05\ \Omega) = 400\text{ V}$$

To get a voltage E_A of 400 V at $n_m = 1600$ r/min, the equivalent voltage at 1800 r/min would be

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0} \tag{9–13}$$

$$E_{A0} = \frac{1800\text{ r/min}}{1600\text{ r/min}} 400\text{ V} = 450\text{ V}$$

From the magnetization curve, this voltage would require a field current of $I_F =$ 6.15 A. The field circuit resistance would have to be

$$R_F + R_{adj} = \frac{V_F}{I_F}$$

$$20\ \Omega + R_{adj} = \frac{430\text{ V}}{6.15\text{ A}} = 69.9\ \Omega$$

$$R_{adj} = 49.9\ \Omega \approx 50\ \Omega$$

DC Machines – Motors

- Today, induction motors with solid-state drive packages are the preferred choice over DC motors for most speed control applications. However, there are still some applications where dc motors are preferred.
- DC motors are often compared by their speed regulations.
- The **speed regulation** (SR) of a motor is defined by

$$SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \times 100\%$$

$$SR = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

DC Machines – Motors

- It is a rough measure of the shape of a motor's speed–torque characteristic. A positive speed regulation means that a motor's speed drops with increasing load, and a negative speed regulation means a motor's speed increases with increasing load.
- The magnitude of the speed regulation tells approximately how steep the slope of the torque- speed curve is.
- It is a rough measure of the shape of a motor's torque-speed characteristic. A positive speed regulation means that a motor's speed drops with increasing load, and a negative speed regulation means a motor's speed increases with increasing load.

DC Machines – Motors

- The magnitude of the speed regulation tells approximately how steep the slope of the torque- speed curve is.
- It should be noted that in order to get the maximum possible power per pound of weight out of a machine, most motors and generators are designed to operate near the saturation point on the magnetization curve (at the knee of the curve).
- This implies that a fairly large increase in field current is often necessary to get a small increase in E_A when operation is near full load.

DC Machines – Motors

Separately Excited and Shunt DC Motors

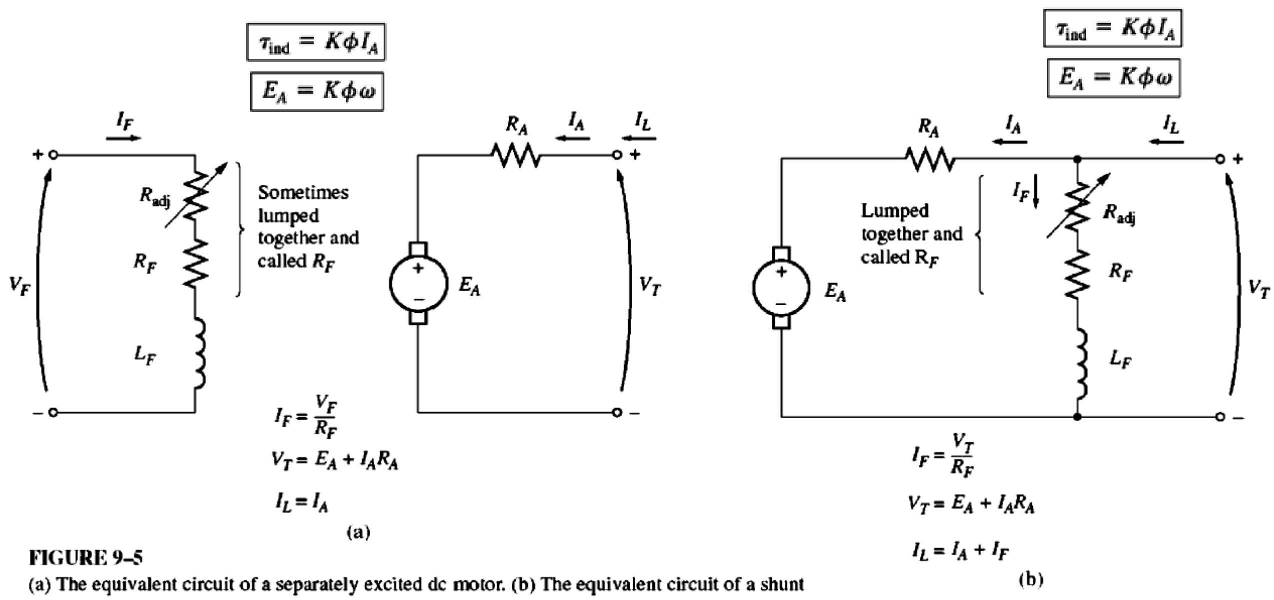


FIGURE 9–5
(a) The equivalent circuit of a separately excited dc motor. (b) The equivalent circuit of a shunt dc motor.

DC Machines – Motors

- The output characteristic of a separately excited and shunt DC motors can be derived from the induced voltage and torque equations of the motor plus Kirchhoff's voltage law as

$$V_T = E_A + I_A R_A$$

$$V_T = K\phi\omega + I_A R_A$$

$$V_T = K\phi\omega + \frac{\tau_{ind}}{K\phi} R_A \quad \text{where} \quad \tau_{ind} = I_A K\phi$$

$$\Rightarrow \quad \omega = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}$$

DC Machines – Motors

It is important to realize that, in order for the speed of the motor to vary linearly with torque, the other terms in this expression must be constant as the load changes.

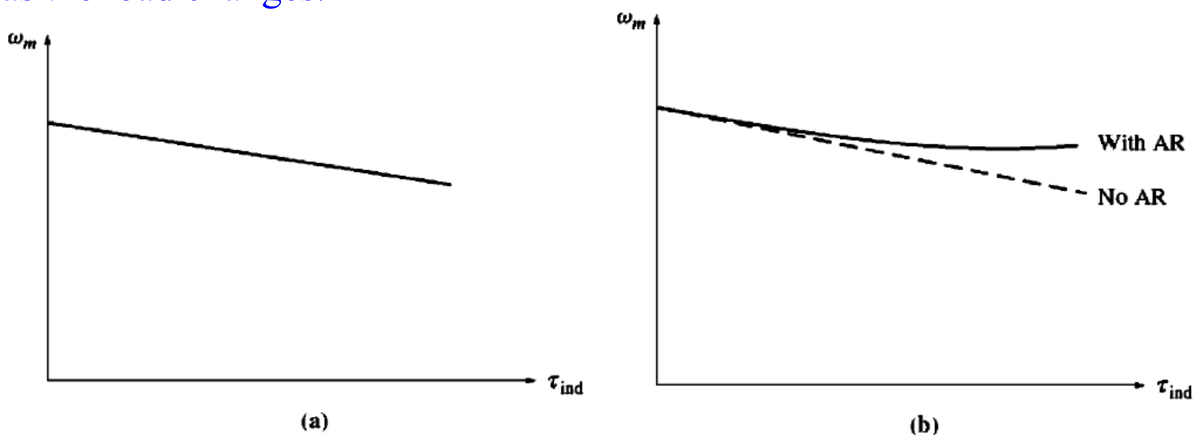


FIGURE 9–6
(a) Torque–speed characteristic of a shunt or separately excited dc motor with compensating windings to eliminate armature reaction. (b) Torque–speed characteristic of the motor with armature reaction present.

DC Machines – Motors

Example 9.1 A 50-hp, 250-V, 1200 r/min dc shunt motor with compensating windings has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06Ω. Its field circuit has a total resistance $R_{adj}+R_F$ of 50Ω, which produces a no-load speed of 1200 r/min. There are 1200 turns per pole on the shunt field winding (see Figure 9-7).

(a) Find the speed of this motor when its input current is 100 A.

(b) Find the speed of this motor when its input current is 200 A.

(c) Find the speed of this motor when its input current is 300 A.

(d) Plot the to torque-speed characteristic of this motor.

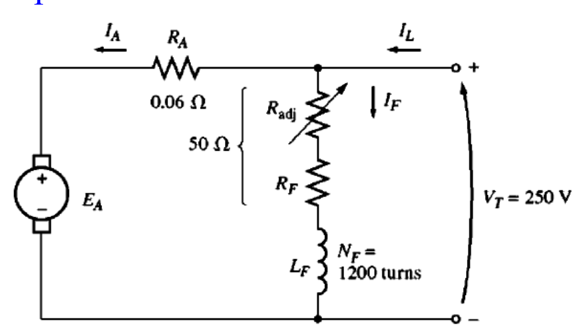


FIGURE 9–7
The shunt motor in Example 9–1.

DC Machines – Motors

The internal generated voltage of a dc machine with its speed expressed in revolutions per minute is given by

$$E_A = K'\phi n \qquad (8\text{–}41)$$

Since the field current in the machine is constant (because V_T and the field resistance are both constant), and since there are no armature reaction effects, *the flux in this motor is constant*. The relationship between the speeds and internal generated voltages of the motor at two different load conditions is thus

$$\frac{E_{A2}}{E_{A1}} = \frac{K'\phi n_2}{K'\phi n_1} \qquad (9\text{–}8)$$

The constant K' cancels, since it is a constant for any given machine, and the flux ϕ cancels as described above. Therefore,

$$n_2 = \frac{E_{A2}}{E_{A1}} n_1 \qquad (9\text{–}9)$$

DC Machines – Motors

(a) If $I_L = 100$ A, then the armature current in the motor is

$$I_A = I_L - I_F = I_L - \frac{V_T}{R_F}$$
$$= 100\text{ A} - \frac{250\text{ V}}{50\ \Omega} = 95\text{ A}$$

Therefore, E_A at this load will be

$$E_A = V_T - I_A R_A$$
$$= 250\text{ V} - (95\text{ A})(0.06\ \Omega) = 244.3\text{ V}$$

The resulting speed of the motor is

$$n_2 = \frac{E_{A2}}{E_{A1}} n_1 = \frac{244.3\text{ V}}{250\text{ V}} 1200\text{ r/min} = 1173\text{ r/min}$$

DC Machines – Motors

(b) If $I_L = 200$ A, then the armature current in the motor is

$$I_A = 200 \text{ A} - \frac{250 \text{ V}}{50 \, \Omega} = 195 \text{ A}$$

Therefore, E_A at this load will be

$$\begin{aligned} E_A &= V_T - I_A R_A \\ &= 250 \text{ V} - (195 \text{ A})(0.06 \, \Omega) = 238.3 \text{ V} \end{aligned}$$

The resulting speed of the motor is

$$n_2 = \frac{E_{A2}}{E_{A1}} n_1 = \frac{238.3 \text{ V}}{250 \text{ V}} 1200 \text{ r/min} = 1144 \text{ r/min}$$

DC Machines – Motors

(c) If $I_L = 300$ A, then the armature current in the motor is

$$\begin{aligned} I_A &= I_L - I_F = I_L - \frac{V_T}{R_F} \\ &= 300 \text{ A} - \frac{250 \text{ V}}{50 \, \Omega} = 295 \text{ A} \end{aligned}$$

Therefore, E_A at this load will be

$$\begin{aligned} E_A &= V_T - I_A R_A \\ &= 250 \text{ V} - (295 \text{ A})(0.06 \, \Omega) = 232.3 \text{ V} \end{aligned}$$

The resulting speed of the motor is

$$n_2 = \frac{E_{A2}}{E_{A1}} n_1 = \frac{232.3 \text{ V}}{250 \text{ V}} 1200 \text{ r/min} = 1115 \text{ r/min}$$

DC Machines – Motors

(d) To plot the output characteristic of this motor, it is necessary to find the torque corresponding to each value of speed. At no load, the induced torque τ_{ind} is clearly zero. The induced torque for any other load can be found from the fact

$$\boxed{P_{conv} = E_A I_A = \tau_{ind}\,\omega} \qquad (8\text{–}55, 8\text{–}56)$$

From this equation, the induced torque in a motor is

$$\tau_{ind} = \frac{E_A I_A}{\omega} \qquad (9\text{–}10)$$

Therefore, the induced torque when $I_L = 100$ A is

$$\tau_{ind} = \frac{(244.3\text{ V})(95\text{ A})}{(1173\text{ r/min})(1\text{ min/60s})(2\pi\text{ rad/r})} = 190\text{ N}\bullet\text{m}$$

The induced torque when $I_L = 200$ A is

$$\tau_{ind} = \frac{(238.3\text{ V})(95\text{ A})}{(1144\text{ r/min})(1\text{ min/60s})(2\pi\text{ rad/r})} = 388\text{ N}\bullet\text{m}$$

The induced torque when $I_L = 300$ A is

$$\tau_{ind} = \frac{(232.3\text{ V})(295\text{ A})}{(1115\text{ r/min})(1\text{ min/60s})(2\pi\text{ rad/r})} = 587\text{ N}\bullet\text{m}$$

The resulting torque–speed characteristic for this motor is plotted in Figure 9–8.

DC Machines – Motors

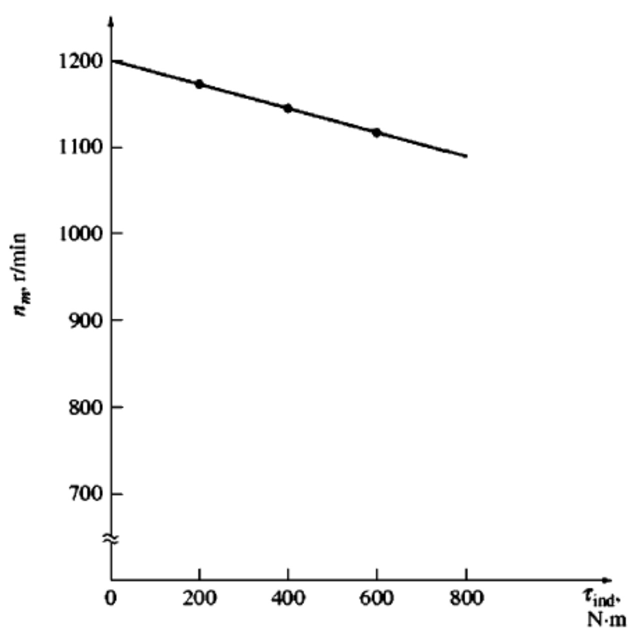


FIGURE 9–8
The torque–speed characteristic of the motor in Example 9–1.

DC Machines – Motors

Example 9.2 A 50-hp, 250-V, 1200 r/min dc shunt motor without compensating windings has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06Ω. Its field circuit has a total resistance $R_{adj}+R_F$ of 50Ω, which produces a no-load speed of 1200 r/min. There are 1200 turns per pole on the shunt field winding and the armature reaction produces a demagnetizing magnetomotive force of 840 A•turns at a load current of 200 A. The magnetization curve of this machine is shown in Figure 9-9.

(a) Find the speed of this motor when its input current is 200 A.
(b) This motor is essentially identical to the one in Example 9- 1 except for the absence of compensating windings. How does its speed compare to that of the previous motor at a load current of 200 A?
(c) Calculate and plot the torque-speed characteristic for this motor.

DC Machines – Motors

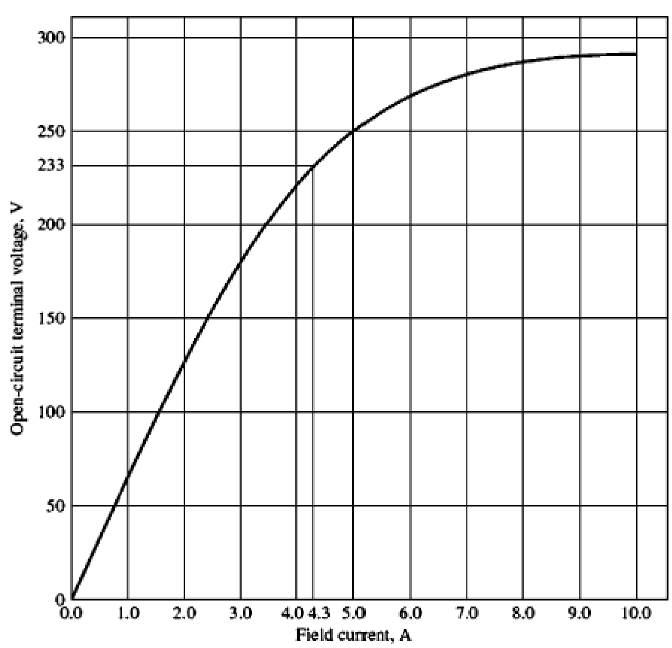


FIGURE 9–9
The magnetization curve of a typical 250-V dc motor, taken at a speed of 1200 r/min.

DC Machines – Motors

(*a*) If $I_L = 200$ A, then the armature current of the motor is

$$I_A = I_L - I_F = I_L - \frac{V_T}{R_F}$$
$$= 200\text{ A} - \frac{250\text{ V}}{50\ \Omega} = 195\text{ A}$$

Therefore, the internal generated voltage of the machine is

$$E_A = V_T - I_A R_A$$
$$= 250\text{ V} - (195\text{ A})(0.06\ \Omega) = 238.3\text{ V}$$

At $I_L = 200$ A, the demagnetizing magnetomotive force due to armature reaction is 840 A • turns, so the effective shunt field current of the motor is

$$I_F^* = I_F - \frac{\mathscr{F}_{AR}}{N_F} \qquad (9\text{–}12)$$
$$= 5.0\text{ A} - \frac{840\text{ A} \cdot \text{turns}}{1200\text{ turns}} = 4.3\text{ A}$$

DC Machines – Motors

From the magnetization curve, this effective field current would produce an internal generated voltage E_{A0} of 233 V at a speed n_0 of 1200 r/min.

We know that the internal generated voltage E_{A0} would be 233 V at a speed of 1200 r/min. Since the actual internal generated voltage E_A is 238.3 V, the actual operating speed of the motor must be

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0} \qquad (9\text{–}13)$$

$$n = \frac{E_A}{E_{A0}} n_0 = \frac{238.3\text{ V}}{233\text{ V}}(1200\text{ r/min}) = 1227\text{ r/min}$$

DC Machines – Motors

(*b*) At 200 A of load in Example 9–1, the motor's speed was $n = 1144$ r/min. In this example, the motor's speed is 1227 r/min. *Notice that the speed of the motor with armature reaction is higher than the speed of the motor with no armature reaction*. This relative increase in speed is due to the flux weakening in the machine with armature reaction.

(*c*)

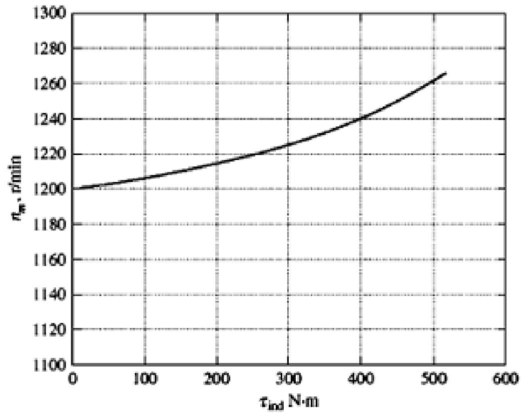


FIGURE 9–10
The torque–speed characteristic of the motor with armature reaction in Example 9–2.

DC Machines – Motors

There are two common ways in which the speed of a separately excited and shunt DC motors can be controlled:

1. Adjusting the field resistance R_F (and thus the field flux)
2. Adjusting the terminal voltage applied to the armature.
3. Inserting a resistor in series with the armature circuit (the less common method of speed control).

DC Machines – Motors

Changing the Field Resistor Control Method

1. Increasing R_F causes I_F (= $V_F/R_F\uparrow$) to decrease.
2. Decreasing I_F decreases ϕ.
3. Decreasing ϕ lowers E_A (= $K\phi\downarrow\omega$).
4. Decreasing E_A increases I_A (= $(V_T - E_A\downarrow)/R_A$)
5. Increasing I_A increases τ_{ind} (= $K\phi\downarrow I_A\Uparrow$), with the change in I_A dominant over the change in flux.
6. Increasing τ_{ind} makes $\tau_{ind} > \tau_{load}$, and the speed ω increases.
7. Increasing ω increases E_A (= $K\phi\omega\uparrow$) again.
8. Increasing E_A decreases I_A
9. Decreasing I_A decreases τ_{ind} until τ_{ind} = τ_{load} at a higher speed ω

DC Machines – Motors

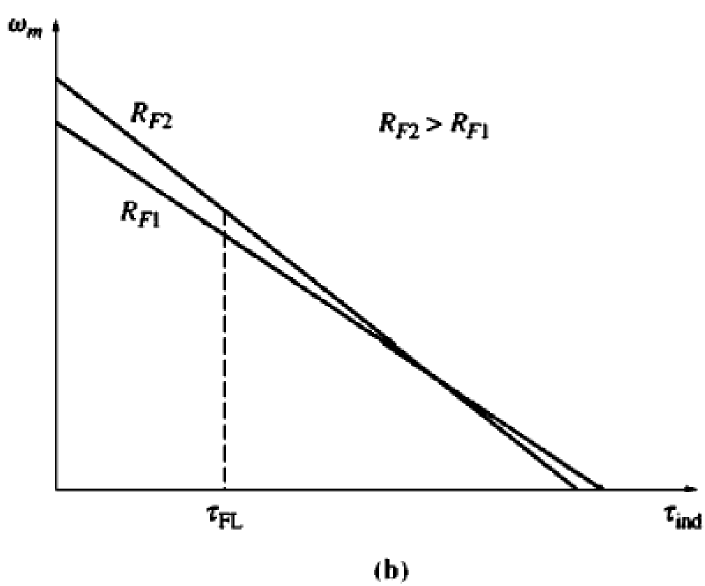


FIGURE 9–12
The effect of field resistance speed control on a shunt motor's torque–speed characteristic:
(b) over the entire range from no-load to stall conditions.

DC Machines – Motors

Changing the Armature Voltage Control Method

1. An increasing V_T increases I_A ($= (V_T\uparrow - E_A)/R_A$)
2. Increasing I_A increases τ_{ind} ($= K\phi I_A\uparrow$)
3. Increasing τ_{ind} makes $\tau_{ind} > \tau_{load}$ and the speed ω increases
4. Increasing ω increases E_A ($= K\phi\omega\uparrow$)
5. Increasing E_A decreases I_A ($= (V_T - E_A\uparrow)/R_A$)
6. Decreasing I_A decreases τ_{ind} until $\tau_{ind} = \tau_{load}$ at a higher speed ω

DC Machines – Motors

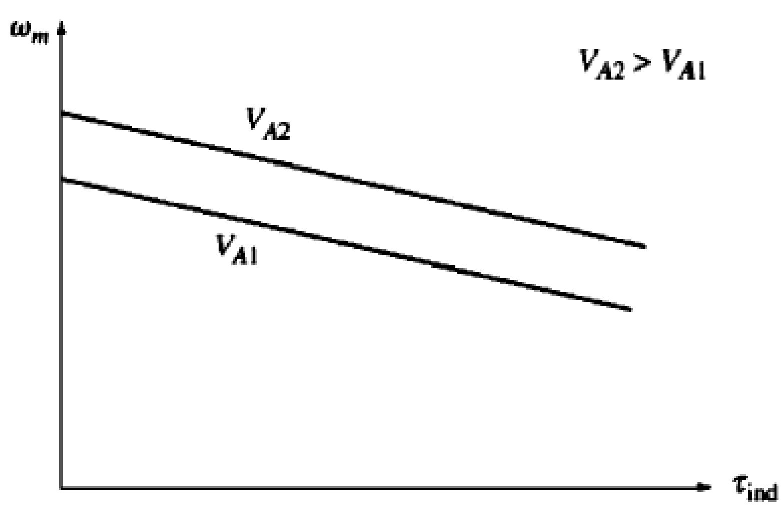


FIGURE 9–14
The effect of armature voltage speed control on a shunt motor's torque–speed characteristic.

DC Machines – Motors

Inserting a Resistor in Series with the Armature Circuit

1. An increasing R_A decreases I_A $(= (V_T - E_A)/R_A\uparrow)$
2. Decreasing I_A decreases τ_{ind} $(= K\phi I_A\downarrow)$
3. Decreasing τ_{ind} makes $\tau_{ind} < \tau_{load}$ and the speed ω decreases
4. Decreasing ω decreases E_A $(= K\phi\omega\downarrow)$
5. Decreasing E_A increases I_A $(= (V_T - E_A\downarrow)/R_A)$
6. Increasing I_A increases τ_{ind} until $\tau_{ind} = \tau_{load}$ at a lower speed ω

DC Machines – Motors

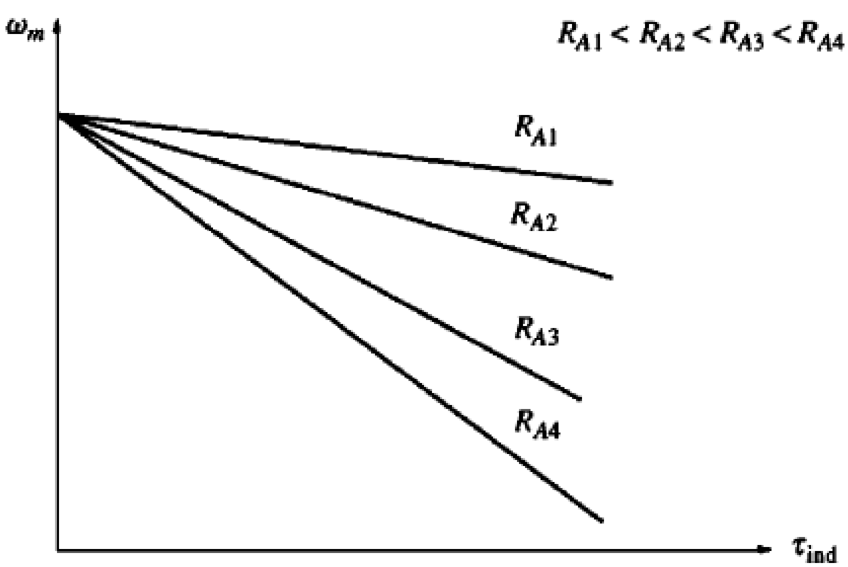


FIGURE 9–15
The effect of armature resistance speed control on a shunt motor's torque–speed characteristic.

DC Machines – Motors

If a motor is operating at its rated terminal voltage, power, and field current, then it will be running at rated speed, also known as **base speed**.

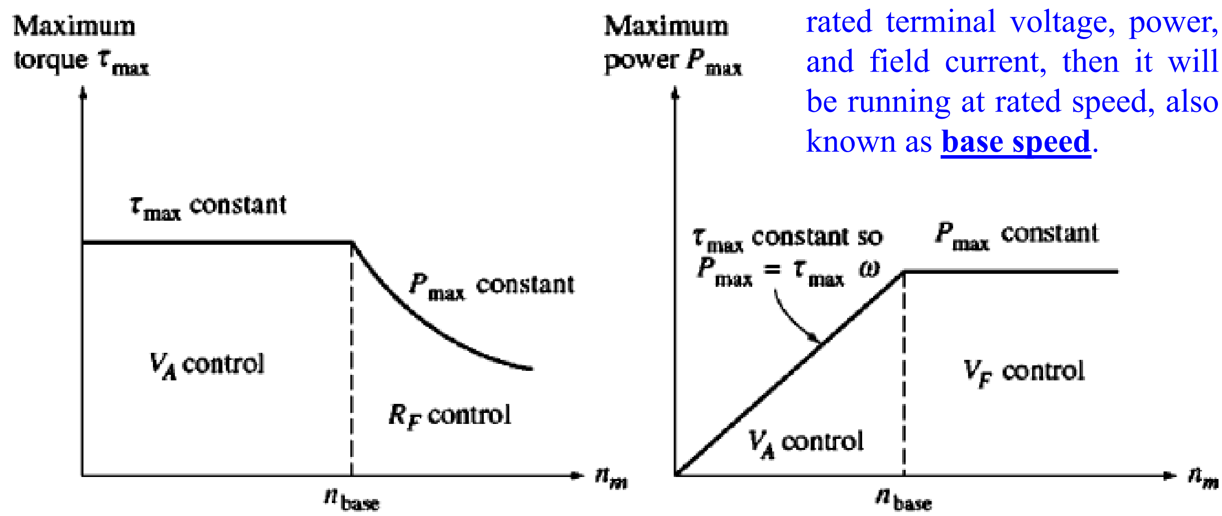


FIGURE 9–16
Power and torque limits as a function of speed for a shunt motor under armature volt and field resistance control.

DC Machines – Motors

Example 9.3 Figure 9-17a shows a 100-hp, 250-V, 1200 r/min shunt dc motor with an armature resistance of 0.03Ω and a field resistance of 41.67 Ω. The motor has compensating windings. so armature reaction can be ignored. Mechanical and core losses may be assumed to be negligible for the purposes of this problem. The motor is assumed to be driving a load with a line current of 126 A and an initial speed of 1103 r/min. To simplify the problem assume that the amount of armature current drawn by the motor remains constant.

(a) If the machine's magnetization curve is shown in Figure 9-9. what is the motor's speed if the field resistance is raised to 50 Ω

(b) Calculate and plot the speed of this motor as a function of the field resistance R_F assuming a constant-current load.

DC Machines – Motors

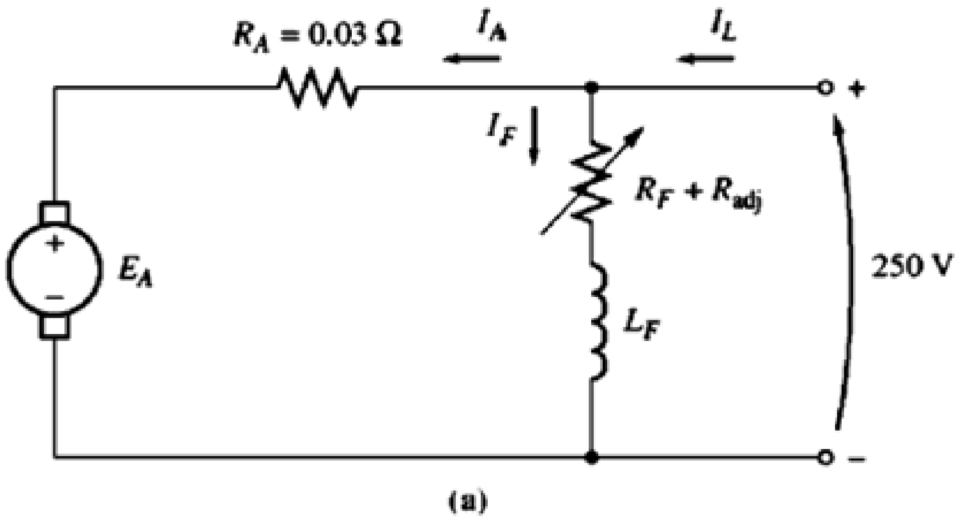


FIGURE 9–17
(a) The shunt motor in Example 9–3.

DC Machines – Motors

(*a*) The motor has an initial line current of 126 A, so the initial armature current is

$$I_{A1} = I_{L1} - I_{F1} = 126\text{ A} - \frac{150\text{ V}}{41.67\ \Omega} = 120\text{ A}$$

Therefore, the internal generated voltage is

$$E_{A1} = V_T - I_{A1}R_A = 250\text{ V} - (120\text{ A})(0.03\ \Omega)$$
$$= 246.4\text{ V}$$

After the field resistance is increased to 50 Ω, the field current will become

$$I_{F2} = \frac{V_T}{R_F} = \frac{250\text{ V}}{50\ \Omega} = 5\text{ A}$$

The ratio of the internal generated voltage at one speed to the internal generated voltage at another speed is given by the ratio of Equation (8–41) at the two speeds:

$$\frac{E_{A2}}{E_{A1}} = \frac{K'\phi_2 n_2}{K'\phi_1 n_1} \qquad (9\text{–}16)$$

DC Machines – Motors

Because the armature current is assumed constant, $E_{A1} = E_{A2}$, and this equation reduces to

$$1 = \frac{\phi_2 n_2}{\phi_1 n_1}$$

or

$$n_2 = \frac{\phi_1}{\phi_2} n_1 \tag{9–17}$$

A magnetization curve is a plot of E_A versus I_F for a given speed. Since the values of E_A on the curve are directly proportional to the flux, the ratio of the internal generated voltages read off the curve is equal to the ratio of the fluxes within the machine. At $I_F = 5$ A, $E_{A0} = 250$ V, while at $I_F = 6$ A, $E_{A0} = 268$ V. Therefore, the ratio of fluxes is given by

$$\frac{\phi_1}{\phi_2} = \frac{268\text{ V}}{250\text{ V}} = 1.076$$

and the new speed of the motor is

$$n_2 = \frac{\phi_1}{\phi_2} n_1 = (1.076)(1103\text{ r/min}) = 1187\text{ r/min}$$

DC Machines – Motors

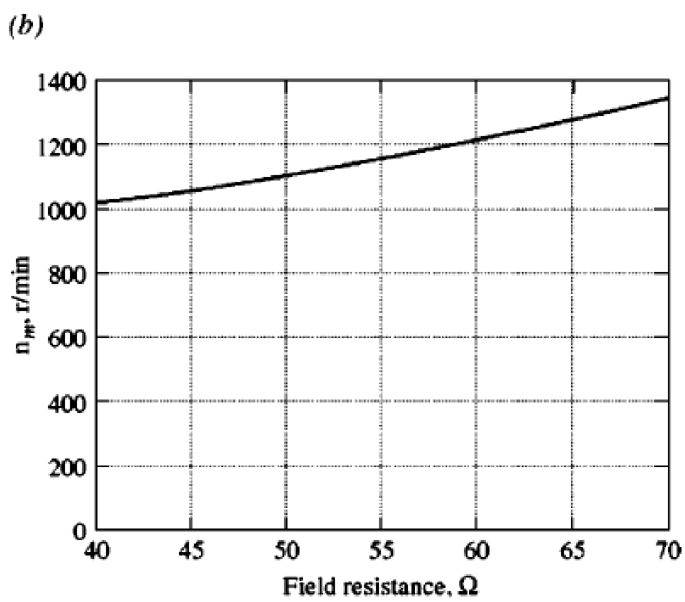


FIGURE 9–18
Plot of speed versus field resistance for the shunt dc motor of Example 9–3.

DC Machines – Motors

Example 9.4 The motor in Example 9-3 is now connected separately excited, as shown in Figure 9-17b. The motor is initially running with V_A = 250V, I_A = 120A, and n = 1103 r/min, while supplying a constant-torque load. What will the speed of this motor be if V_A is reduced to 200V?

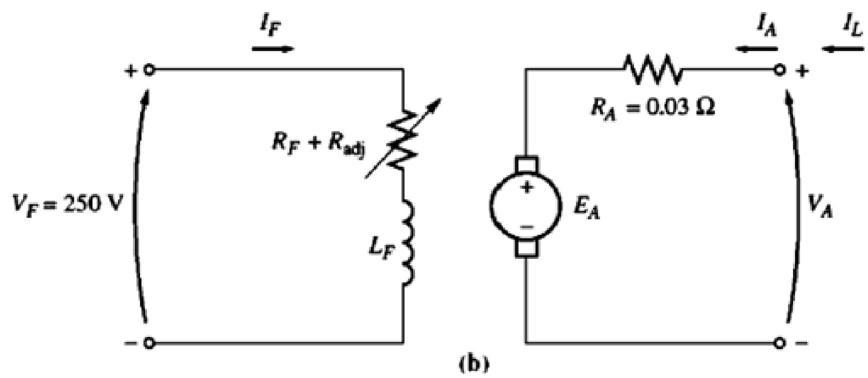


FIGURE 9–17
(b) The separately excited dc motor in Example 9–4.

DC Machines – Motors

The motor has an initial line current of 120 A and an armature voltage V_A of 250 V, so the internal generated voltage E_A is

$$E_A = V_T - I_A R_A = 250\text{ V} - (120\text{ A})(0.03\ \Omega) = 246.4\text{ V}$$

By applying Equation (9–16) and realizing that the flux ϕ is constant, the motor's speed can be expressed as

$$\frac{E_{A2}}{E_{A1}} = \frac{K'\phi_2 n_2}{K'\phi_1 n_1} \qquad (9\text{–}16)$$

$$= \frac{n_2}{n_1}$$

$$n_2 = \frac{E_{A2}}{E_{A1}} n_1$$

To find E_{A2} use Kirchhoff's voltage law:

$$E_{A2} = V_T - I_{A2} R_A$$

Since the torque is constant and the flux is constant, I_A is constant. This yields a voltage of

$$E_{A2} = 200\text{ V} - (120\text{ A})(0.03\ \Omega) = 196.4\text{ V}$$

The final speed of the motor is thus

$$n_2 = \frac{E_{A2}}{E_{A1}} n_1 = \frac{196.4\text{ V}}{246.4\text{ V}} 1103\text{ r/min} = 879\text{ r/min}$$

DC Machines – Motors

Example (A) A 230V dc shunt motor drives a load at 900RPM and drawing a current of 30A. The resistance of armature circuit is 0.4Ω. The torque of the load is proportional to the speed. Calculate the resistance to be connected in series with the armature to reduce the speed to 600RPM. Ignore armature reaction.

Case I:

Supply voltage: $V_T = 230$ V

Armature current: $I_{a1} = 30$ A (since no information is provided about the field circuit, otherwise this current should be considered as the total current withdrawn from the source, I_T)

Speed: $n_1 = 900$ RPM

Back emf: $E_{b1} = V_T - I_{a1}R_a = 230 - 30 \times 0.4 = 218$ V

$E_{b1} = k_b \phi n_1 = k_b \phi 900$

DC Machines – Motors

Case II:

Supply voltage: $V_T = 230$ V

Armature current: $I_{a2} = ?$

Speed: $n_2 = 600$ RPM

Back emf: $E_{b2} = V_T - I_{a2}(R_a + R) = 230 - I_{a2}(0.4 + R)$

$E_{b2} = k_b \phi n_2 = k_b \phi 600$

Since the load is proportional to the speed ($\tau \propto n$) and ($\tau \propto I_a$) then ($n \propto I_a$).

Hence, $\frac{I_{a2}\phi}{I_{a1}\phi} = \frac{n_2}{n_1}$ or $I_{a2} = I_{a1} \times \frac{n_2}{n_1} = 30 \times \frac{600}{900} = 20A$

$\Rightarrow \quad E_{b2} = V_T - I_{a2}(0.4 + R) = 230 - 20(0.4 + R) = 222 - 20R$

Since $\frac{n_2}{n_1} = \frac{E_{b2}}{E_{b1}}$ (magnetic flux is constant) $\Rightarrow \frac{222 - 20R}{198} = \frac{600}{900}$ or $R = 3.833\Omega$

DC Machines – Motors

Example (B) A 250V dc shunt motor has an armature current of 20A when running at 1000RPM against full load torque. The armature resistance is 0.5Ω. What resistance must be inserted in series with the armature to reduce the speed to 500RPM at the same load torque, and what will be the speed if the load torque is halved with this inserted resistance?

Case I:

Supply voltage: $V_T = 250\text{ V}$

Armature current: $I_{a1} = 20\text{ A}$

Speed: $n_1 = 1000\text{ RPM}$

Back emf: $E_{b1} = V_T - I_{a1}R_a = 250 - 20\times 0.5 = 240\text{ V}$

$E_{b1} = k_b\phi n_1 = k_b\phi 1000$

DC Machines – Motors

Case II:

Supply voltage: $V_T = 250\text{ V}$

Armature current: $I_{a2} = 20\text{ A}$ (since the load torque remains constant at full load)

Speed: $n_2 = 500\text{ RPM}$

Back emf: $E_{b2} = V_T - I_{a2}(R_a + R) = 250 - 20(0.5 + R) = 240 - 20R$

$E_{b2} = k_b\phi n_2 = k_b\phi 500$

Since $\frac{n_2}{n_1} = \frac{E_{b2}}{E_{b1}}$ (magnetic flux is constant) $\Rightarrow$ $\frac{240 - 20R}{240} = \frac{500}{1000}$ or $R = 6\Omega$

Case III

Supply voltage: $V_T = 250\text{ V}$

Armature current: $I_{a3} = 10\text{ A}$ (since the load torque is halved: $\tau_{Load} = k_m I_{a1}\phi$ and $\frac{1}{2}\tau_{Load} = k_m I_{a3}\phi$ $\Rightarrow$ $\frac{1}{2} = \frac{I_{a3}}{I_{a1}}$)

Speed: $n_3 = ?$

Back emf: $E_{b3} = V_T - I_{a3}(R_a + R) = 250 - 10(0.5 + 6) = 185\text{ V}$

$E_{b3} = k_b\phi n_3$

Since $\frac{n_3}{n_1} = \frac{E_{b3}}{E_{b1}}$ (magnetic flux is constant) $\Rightarrow$ $\frac{185}{240} = \frac{n_3}{1000}$ or $n_3 = 771\text{ RPM}$

DC Machines – Motors

Example (C) A 250V dc shunt motor has an armature resistance of 0.5Ω and a field resistance of 250Ω. When driving a constant load at 600RPM, the armature takes 20A. If it is desired to raise the speed from 600RPM to 800RPM, what resistance must be inserted in the shunt field circuit so that the change of speed occurs? Assume the magnetizing curve is a straight line.

Case I:

Supply voltage: $V_T = 250\text{ V}$

Armature current: $I_{a1} = 20\text{ A}$

Speed: $n_1 = 600\text{ RPM}$

Back emf: $E_{b1} = V_T - I_{a1}R_a = 250 - 20 \times 0.5 = 240\text{ V}$

$E_{b1} = k_b \phi_1 n_1 = k_b \phi_1 600$

DC Machines – Motors

Case II:

Now let the magnetic flux and armature current be ϕ_2 and I_{a2}, respectively.

Supply voltage: $V_T = 250\text{ V}$

Armature current: $I_{a2} = ?$

Speed: $n_2 = 800\text{ RPM}$

Back emf: $E_{b2} = V_T - I_{a2}R_a = 250 - 0.5 I_{a2}$

$E_{b2} = k_b \phi_2 n_2 = k_b \phi_2 800$

Since the load is constant

$\therefore \tau_{Load} = k_m I_{a2} \phi_2 = k_m I_{a1} \phi_1$ or $I_{a2} = I_{a1} \dfrac{\phi_1}{\phi_2} = 20 \dfrac{\phi_1}{\phi_2}$

and $\dfrac{n_2}{n_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\phi_1}{\phi_2}$ or $\dfrac{800}{600} = \dfrac{250 - 0.5 I_{a2}}{240} \times \dfrac{\phi_1}{\phi_2}$ or $320 = \left(250 - 10\dfrac{\phi_1}{\phi_2}\right) \times \dfrac{\phi_1}{\phi_2}$

since $I_{a2} = 20\dfrac{\phi_1}{\phi_2}$ $\Rightarrow$ $\left(\dfrac{\phi_1}{\phi_2}\right)^2 - 25\dfrac{\phi_1}{\phi_2} + 32 = 0$ $\Rightarrow$ we have a second order equation

$\Rightarrow \dfrac{\phi_1}{\phi_2} = \dfrac{25 \pm \sqrt{25^2 - 4 \times 32}}{2} = 23.65$ or 1.35

DC Machines – Motors

The value 23.65 is rejected as it will not give the required increase in speed. Since magnetizing curve is a straight line

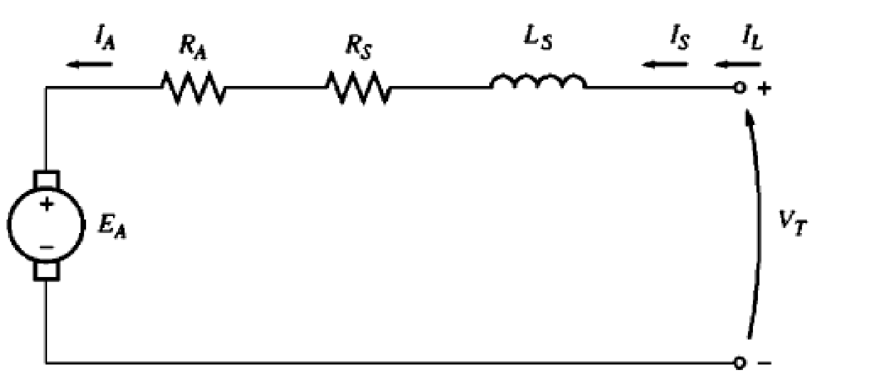
$\therefore \frac{\phi_1}{\phi_2} = \frac{I_{sh1}}{I_{sh2}} = \frac{V_T / R_{sh1}}{V_T / R_{sh2}}$ since in general $V_T = I_{sh} R_{sh}$ and $\phi = c\, I_{sh}$ where $c = const.$

$\Rightarrow \frac{R_{sh2}}{R_{sh1}} = \frac{\phi_1}{\phi_2} = 1.352$ or $R_{sh2} = 1.353 \times R_{sh1} = 1.353 \times 250 = 338\Omega$

Hence, the resistance required to be inserted in the shunt field is $R = R_{sh2} - R_{sh1} = 338 - 250 = 88\Omega$.

DC Machines – Motors

Series DC Motors



$\tau_{ind} = K\phi I_A$

$E_A = K\phi\omega$

$I_A = I_S = I_L$

$V_T = E_A + I_A (R_A + R_S)$

FIGURE 9–20
The equivalent circuit of a series dc motor.

DC Machines – Motors

- The output characteristic of series DC motors can be derived from the induced voltage and torque equations of the motor plus Kirchhoff's voltage law such as

$$\phi = cI_S = cI_A \text{ since } I_A = I_S \text{ then } \phi = cI_A$$

$$\text{Recall } \tau_{ind} = I_A K\phi = cKI_A^2 \quad \Rightarrow \quad \tau_{ind} \propto I_A^2$$

$$\text{Note that } I_A = \sqrt{\frac{\tau_{ind}}{cK}} \text{ and } \phi = c\sqrt{\frac{\tau_{ind}}{cK}} = \sqrt{\frac{c\tau_{ind}}{K}}$$

$$V_T = E_A + I_A(R_A+R_S) = K\phi\omega + I_A(R_A+R_S)$$

DC Machines – Motors

- The output characteristic of series DC motors can be derived from the induced voltage and torque equations of the motor plus Kirchhoff's voltage law such as

$$V_T = K\sqrt{\frac{c\tau_{ind}}{K}}\omega + \sqrt{\frac{\tau_{ind}}{cK}}(R_A+R_S)$$

$$\Rightarrow \quad \omega = \frac{V_T}{\sqrt{cK}}\frac{1}{\sqrt{\tau_{ind}}} - \frac{(RA+RS)}{cK}$$

DC Machines – Motors

When the torque on this motor goes to zero, its speed goes to infinity. In practice, the torque can never go entirely to zero because of the mechanical, core, and stray losses that must be overcome. However, if no other load is connected to the motor, it can turn fast enough to seriously damage itself. Never completely unload a series motor, and never connect one to a load by a belt or other mechanism that could break. If that were to happen and the motor were to become unloaded while running, the results could be serious.

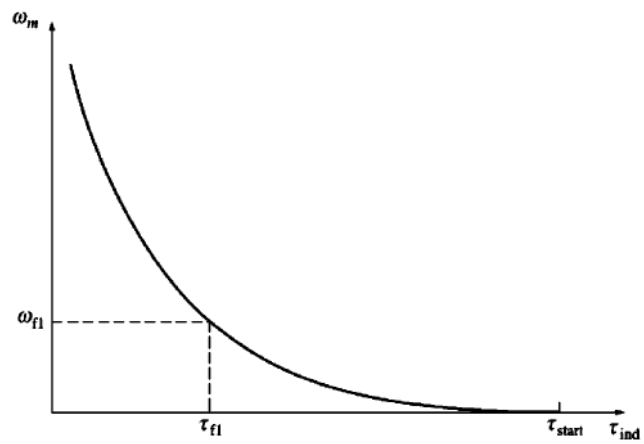


FIGURE 9–21
The torque–speed characteristic of a series dc motor.

DC Machines – Motors

Example 9.5 A 250-V series dc motor with compensating windings. and a total series resistance R_A+R_S of 0.08Ω. The series field consists of 25 turns per pole. with the magnetization curve shown in Figure 9-22.

(a) Find the speed and induced torque of this motor for when its armature current is 50A.

(b) Calculate and plot the torque-speed characteristic for this motor.

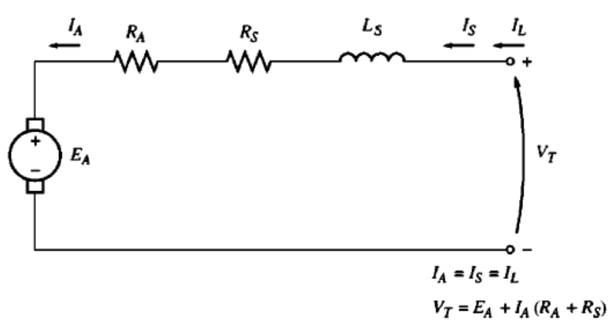


FIGURE 9–20
The equivalent circuit of a series dc motor.

DC Machines – Motors

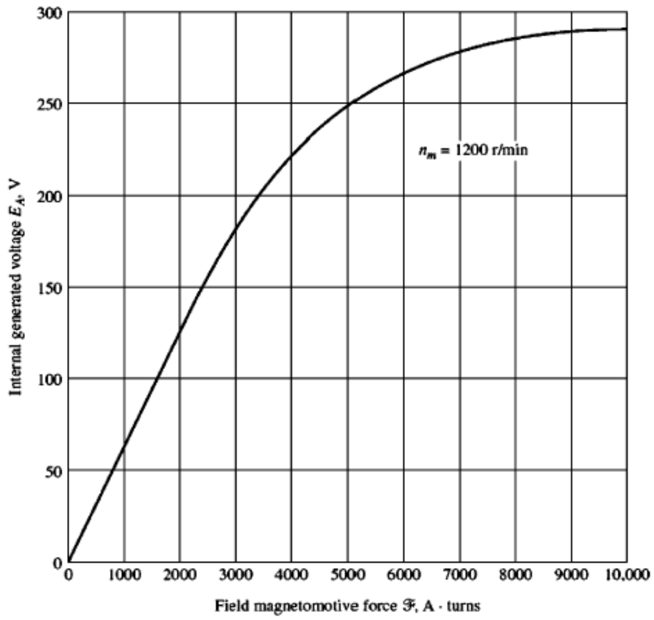


FIGURE 9–22
The magnetization curve of the motor in Example 9–5. This curve was taken at speed n_m = 1200 r/min.

DC Machines – Motors

(*a*) To analyze the behavior of a series motor with saturation, pick points along the operating curve and find the torque and speed for each point. Notice that the magnetization curve is given in units of magnetomotive force (ampere-turns) versus E_A for a speed of 1200 r/min, so calculated E_A values must be compared to the equivalent values at 1200 r/min to determine the actual motor speed.

For I_A = 50 A,

$$E_A = V_T - I_A(R_A + R_S) = 250\ \text{V} - (50\text{A})(0.08\ \Omega) = 246\ \text{V}$$

Since $I_A = I_F$ = 50 A, the magnetomotive force is

$$\mathscr{F} = NI = (25\ \text{turns})(50\ \text{A}) = 1250\ \text{A} \cdot \text{turns}$$

From the magnetization curve at $\mathscr{F}$ = 1250 A • turns, E_{A0} = 80 V. To get the correct speed of the motor, remember that, from Equation (9–13),

$$n = \frac{E_A}{E_{A0}} n_0$$

$$= \frac{246\ \text{V}}{80\ \text{V}} 120\ \text{r/min} = 3690\ \text{r/min}$$

To find the induced torque supplied by the motor at that speed, recall that $P_{conv} = E_A I_A = \tau_{ind}\omega$. Therefore,

DC Machines – Motors

$$\tau_{ind} = \frac{E_A I_A}{\omega}$$

$$= \frac{(246\ \text{V})(50\ \text{A})}{(3690\ \text{r/min})(1\ \text{min}/60\ \text{s})(2\pi\ \text{rad/r})} = 31.8\ \text{N} \cdot \text{m}$$

(b) To calculate the complete torque–speed characteristic, we must repeat the steps in *a* for many values of armature current. A MATLAB M-file that calculates the torque–speed characteristics of the series dc motor is shown below. Note that the magnetization curve used by this program works in terms of field magnetomotive force instead of effective field current.

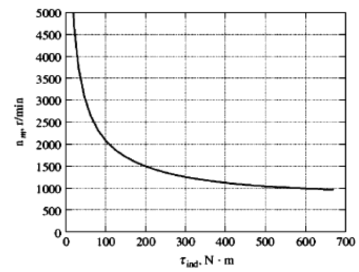


FIGURE 9–23
The torque–speed characteristic of the series dc motor in Example 9–5.

DC Machines – Motors

Example (D) A 200V dc series motor runs at 500RPM when taking a current of 25A. The armature resistance is 0.5Ω and the field resistance is 0.3Ω. If the current remains constant, calculate the resistance necessary to reduce the speed to 250RPM.

Case I:

Supply voltage: $V_T = 200\ \text{V}$

Armature current: $I_{a1} = 25\ \text{A}$

Speed: $n_1 = 500\ \text{RPM}$

Back emf: $E_{b1} = V_T - I_a\left(R_a + R_s\right) = 200 - 25 \times \left(0.5 + 0.3\right) = 180\ \text{V}$

$E_{b1} = k_b \phi n_1 = k_b \phi 500$

DC Machines – Motors

Case II:

Supply voltage: $V_T = 200\,\text{V}$

Armature current: $I_a = 25\,\text{A}$ (since the current remains constant)

Speed: $n_2 = 250\,\text{RPM}$

Back emf: $E_{b2} = V_T - I_a(R_a + R_s + R_{ins}) = 200 - 25(0.5 + 0.3 + R_{ins}) = 180 - 25R_{ins}$

$E_{b2} = k_b \phi n_2 = k_b \phi 250$

Since $\frac{n_2}{n_1} = \frac{E_{b2}}{E_{b1}}$ (magnetic flux is constant) $\Rightarrow$ $\frac{180 - 25R_{ins}}{180} = \frac{250}{500}$ or $R_{ins} = 3.6\,\Omega$

DC Machines – Motors

Example (E) A 200V dc series motor runs at 500RPM when taking a current of 25A. The armature resistance is 0.5Ω and the field resistance is 0.3Ω. If the current remains constant, calculate the resistance necessary to reduce the speed to 250RPM.

Case I:

Supply voltage: $V_T = 200\,\text{V}$

Armature current: $I_{a1} = 40\,\text{A}$

Speed: $n_1 = 1000\,\text{RPM}$

Back emf: $E_{b1} = V_T - I_{a1}(R_a + R_s) = 200 - 40 \times (0.06 + 0.04) = 196\,\text{V}$

$E_{b1} = k_b \phi_1 n_1 = k_b \phi_1 1000$

DC Machines – Motors

Case II:

Supply voltage: $V_T = 200\,\text{V}$

Armature current: $I_{a2} = 75\,\text{A}$ (since the current remains constant)

Speed: $n_2 = ?\ \text{RPM}$

Back emf: $E_{b2} = V_T - I_{a2}\left(R_a + R_s\right) = 200 - 75\left(0.06 + 0.04\right) = 192.5$

$E_{b2} = k_b \phi_2 n_2$

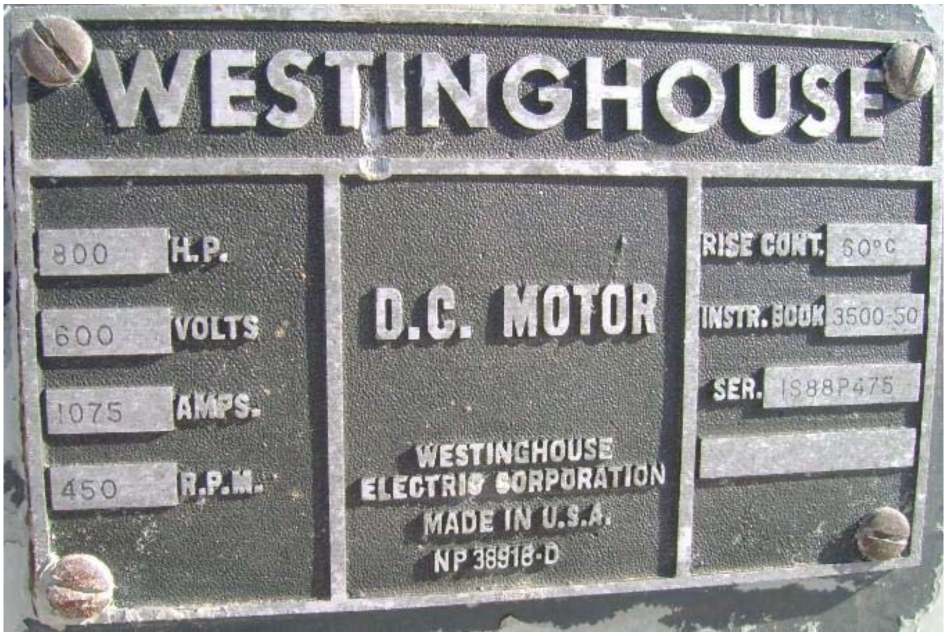
Since the flux increases by 10% then $\phi_2 = 1.1\phi_1$

and $\dfrac{n_2}{n_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\phi_1}{\phi_2}$ or $\dfrac{n_2}{1000} = \dfrac{192.5}{196} \times \dfrac{1}{1.1}$

Hence, the new speed is $n_2 = 894$ RPM.

DC Motors Nameplate/Tag

DC Motors Nameplate/Tag



DC Motors Useful Measurements

- **How to measure DC motor specs**
  https://www.youtube.com/watch?v=roINUVVpEbs

Transformers

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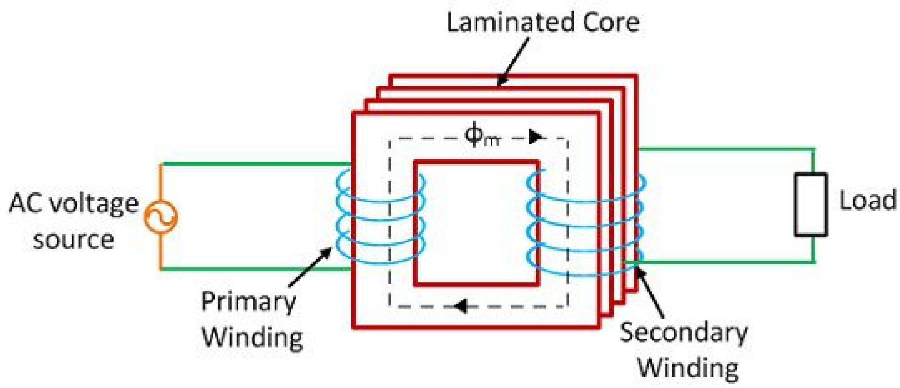
Introduction

- A **transformer** is a device that changes AC electric power at one voltage level to AC electric power at another volt age level through the action of a magnetic field.
- It consists of two or more coils of wire wrapped around a common ferromagnetic core.
- These coils are (usually) not directly connected. The only connection between the coils is the common magnetic flux present within the core.
- One of the transformer windings is connected to a source of AC electric power, and the second transformer winding supplies electric power to loads.

Introduction



- The power cannot be transmitted far with DC power systems. They are inefficient. Hence, The invention of the transformer and the concurrent development of AC power sources eliminated forever the restrictions on the range and power level of power systems.

Introduction

- If a transformer steps up the voltage level of a circuit, it must decrease the current to keep the power into the device equal to the power out of it.
- Therefore, AC electric power can be generated at one central location, its voltage stepped up for transmission over long distances at very low losses, and its voltage stepped down again for final use.
- Since the transmission losses in the lines of a power system are proportional to the square of the current in the lines, raising the transmission voltage and reducing the resulting transmission currents by a factor of 10 with transformers reduces power transmission losses by a factor of 100.

Introduction

- Without the transformer, it would simply not be possible to use electric power in many of the ways it is used today.
- In a modern power system, electric power is generated at voltages of 12 to 25 kV.
- Transformers step up the voltage to between 110 kV and nearly 1000 kV for transmission over long distances at very low losses.
- Transformers then step down the voltage to the 12 to 34.5 kV range for local distribution and finally permit the power to be used safely in homes, offices, and factories at voltages as low as 120 V.

Introduction

- Power transformers are constructed on one of two types of cores. One type of construction consists of a simple rectangular laminated piece of steel with the transformer windings wrapped around two sides of the rectangle. This type of construction is known as **core form.**

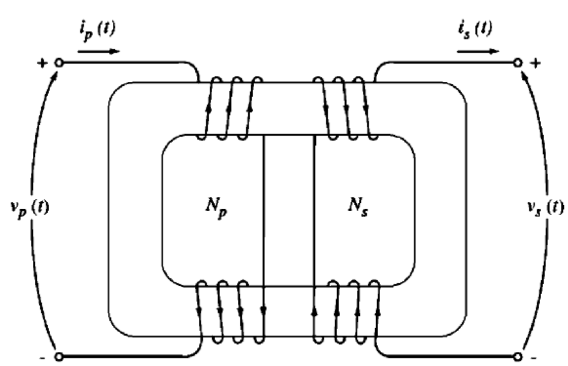


FIGURE 2–2
Core-form transformer construction.

Introduction

- The other type consists of a three-legged laminated core with the windings wrapped around the center leg. This type of construction is known as **shell form**.

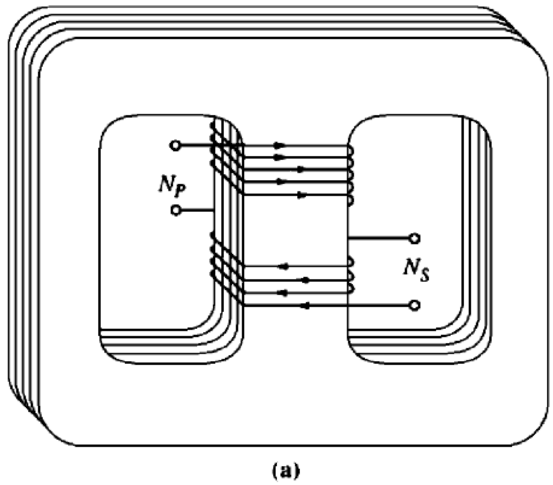


FIGURE 2–3
(a) Shell-form transformer construction

Introduction

- The primary and secondary windings in a physical transformer are wrapped one on top of the other with the low-voltage winding innermost.
- Such an arrangement serves two purposes:

1. It simplifies the problem of insulating the high-voltage winding from the core.

2. It results in much less leakage flux than would be the case if the two windings were separated by a distance on the core.

- In either case, the core is constructed of thin laminations electrically isolated from each other in order to minimize eddy currents.

The Ideal Transformer

- An **ideal transformer** is a lossless device with an input winding and an output winding.

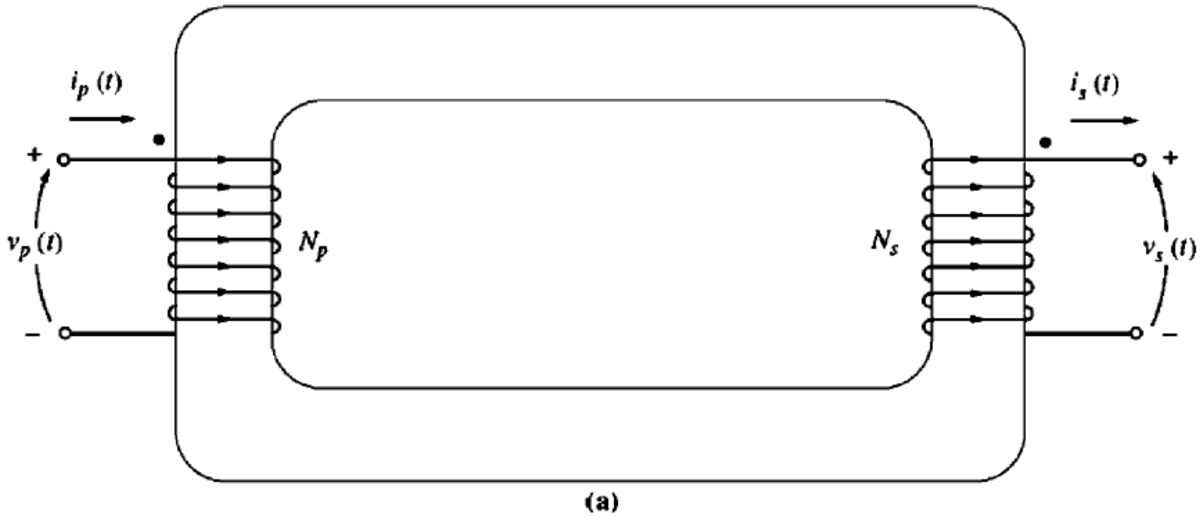
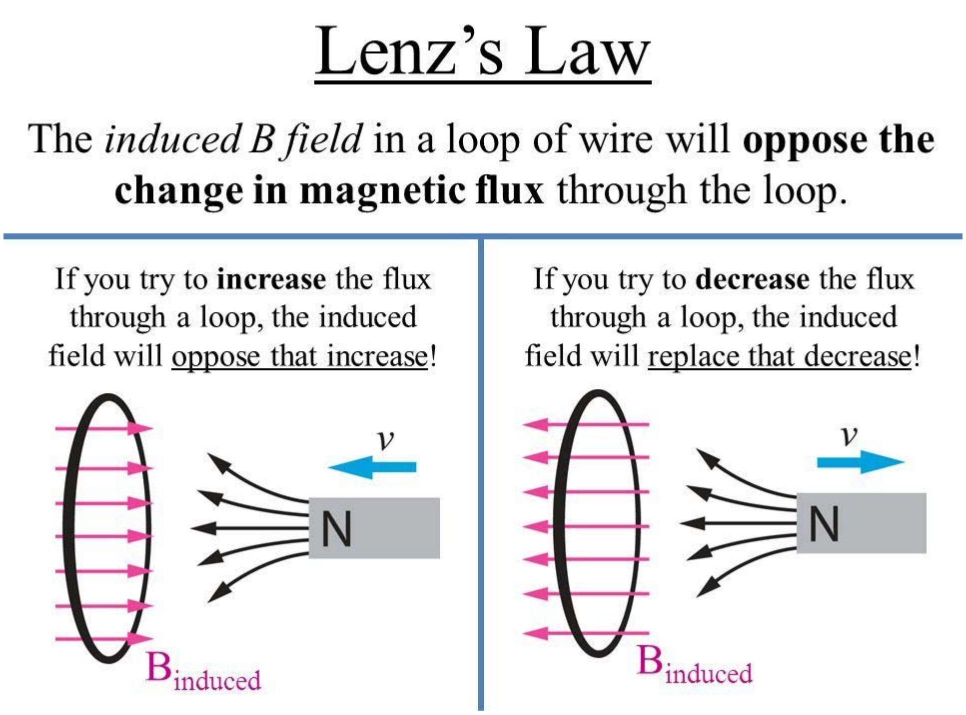
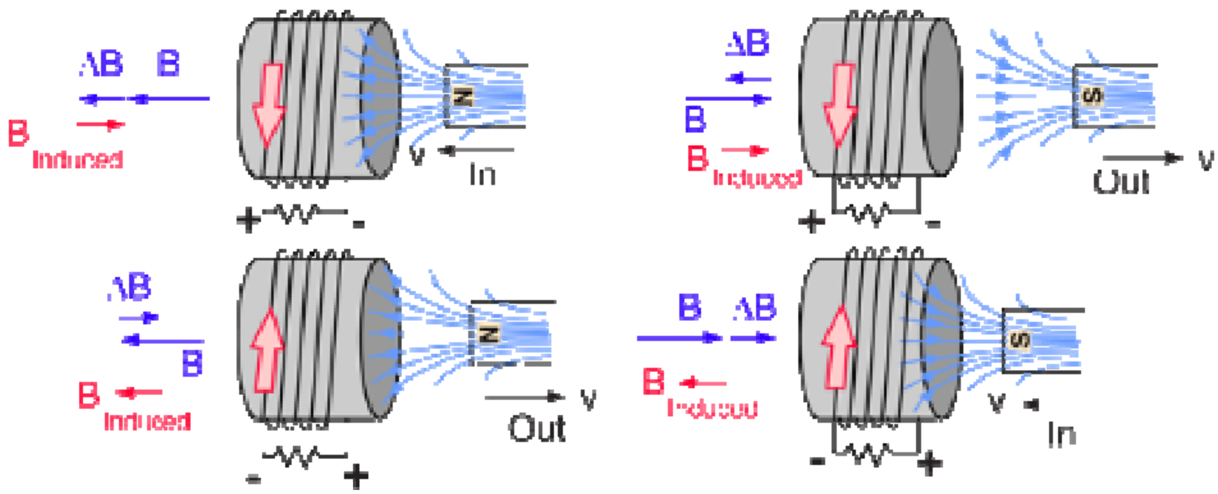


FIGURE 2–4
(a) Sketch of an ideal transformer.

The Ideal Transformer

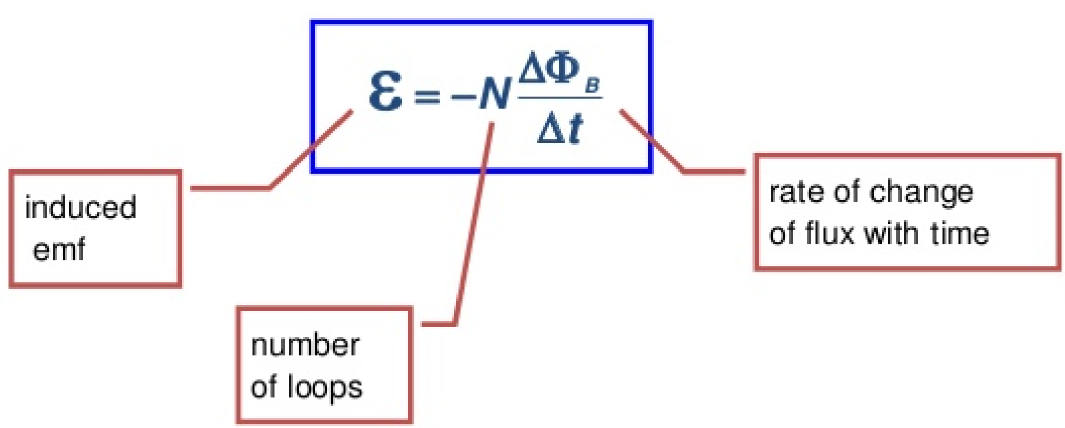


The Ideal Transformer



The Ideal Transformer

Faraday's Law of Induction



Minus sign from Lenz's Law:

The Ideal Transformer

- Since the magetomotive forces are equal on the primary and secondary windings

$$\mathcal{F}_p = \phi\mathcal{R} = \mathcal{F}_s$$

$$N_p i_p = N_s i_s$$

$$\frac{N_p}{N_s} = \frac{i_s}{i_p} = a$$

where a is the **transformer turns ratio**

- From Faraday's laws

V is proportional to N

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = a$$

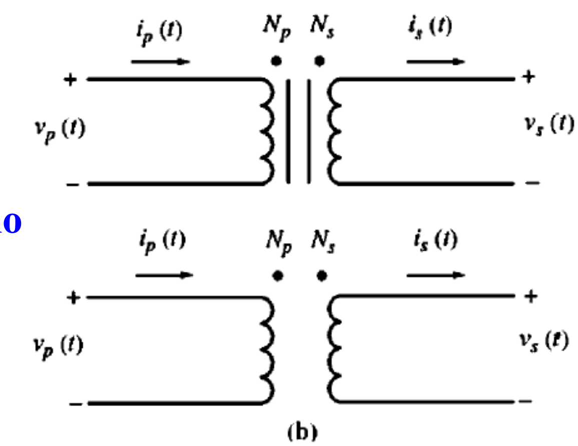


FIGURE 2–4
(b) Schematic symbols of a transformer.

The Ideal Transformer

- Note that

$$V_p i_p = V_s i_s$$
$$S_{in} = S_{out}$$

- Moreover

$$P_{in} = V_p i_p cos(\theta_p)$$
$$P_{out} = Vs i_s cos(\theta_s)$$

- Since $V_p i_p = V_s i_s$ and since voltage and current angles are unaffected by an ideal transformer then $\theta_p = \theta_s$
- The primary and secondary windings of an ideal transformer have the same power factor.
- Then $P_{in} = P_{out}$ and them $Q_{in} = Q_{out}$

The Ideal Transformer

- **What is a Transformer And How Do They Work?**
  https://www.youtube.com/watch?v=Cx4_7lIjoBA&t=26s

The Ideal Transformer

- The impedance of a device or an element is defined as the ratio of the phasor voltage across it to the phasor current flowing through it

$$\mathbf{Z_L} = \frac{\boldsymbol{V_L}}{\boldsymbol{I_L}}$$

- One of the interesting properties of a transformer is that, since it changes voltage and current levels, it changes the ratio between voltage and current and hence the apparent impedance of an element.

The Ideal Transformer

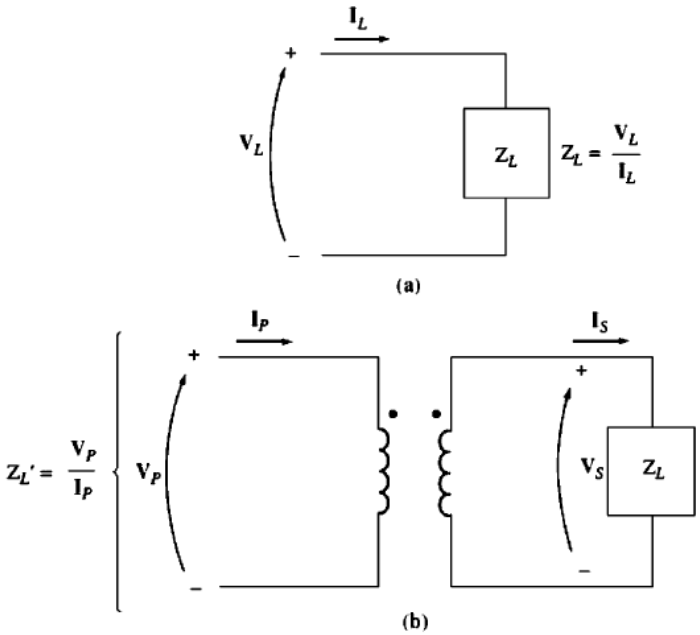


FIGURE 2–5
(a) Definition of impedance. (b) Impedance scaling through a transformer.

With a transformer, it is possible to match the magnitude of a load impedance to a source impedance simply by picking the proper turns ratio.

$$Z_L' = \frac{\mathbf{V}_P}{\mathbf{I}_P} = \frac{a\mathbf{V}_S}{\mathbf{I}_S/a} = a^2\frac{\mathbf{V}_S}{\mathbf{I}_S}$$

$$\boxed{Z_L' = a^2 Z_L} \qquad (2\text{–}15)$$

The Ideal Transformer

Example 2.1 A single-phase power system consists of a 480-V 60-Hz generator supplying a load Z_{Load} = 4 + j3 Ω through a transmission line of impedance Z_{Line} = 0.18 + j0.24 Ω. Answer the following questions about this system.

(a) If the power system is exactly as described above, what will the voltage at the load be? What will the transmission line losses be?

(b) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line. What will the load voltage be now? What will the transmission line losses be now?

The Ideal Transformer

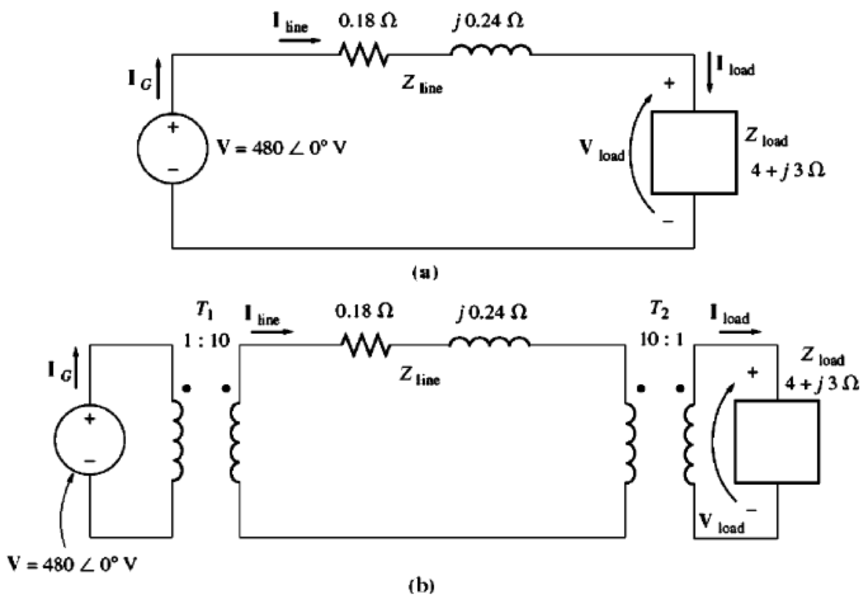


FIGURE 2–6
The power system of Example 2–1 (a) without and (b) with transformers at the ends of the transmission line.

The Ideal Transformer

(*a*) Figure 2–6a shows the power system without transformers. Here $\mathbf{I}_G = \mathbf{I}_{\text{line}} = \mathbf{I}_{\text{load}}$. The line current in this system is given by

$$\begin{aligned} \mathbf{I}_{\text{line}} &= \frac{\mathbf{V}}{\mathbf{Z}_{\text{line}} + \mathbf{Z}_{\text{load}}} \\ &= \frac{480\angle 0^\circ \text{ V}}{(0.18\ \Omega + j0.24\ \Omega) + (4\ \Omega + j3\ \Omega)} \\ &= \frac{480\angle 0^\circ}{4.18 + j3.24} = \frac{480\angle 0^\circ}{5.29\angle 37.8^\circ} \\ &= 90.8\angle -37.8^\circ \text{ A} \end{aligned}$$

Therefore the load voltage is

$$\begin{aligned} \mathbf{V}_{\text{load}} &= \mathbf{I}_{\text{line}}\mathbf{Z}_{\text{load}} \\ &= (90.8\angle -37.8^\circ \text{ A})(4\ \Omega + j3\ \Omega) \\ &= (90.8\angle -37.8^\circ \text{ A})(5\angle 36.9^\circ\ \Omega) \\ &= 454\angle -0.9^\circ \text{ V} \end{aligned}$$

and the line losses are

$$\begin{aligned} P_{\text{loss}} &= (I_{\text{line}})^2 R_{\text{line}} \\ &= (90.8 \text{ A})^2 (0.18\ \Omega) = 1484 \text{ W} \end{aligned}$$

The Ideal Transformer

(*b*) Figure 2–6b shows the power system with the transformers. To analyze this system, it is necessary to convert it to a common voltage level. This is done in two steps:

1. Eliminate transformer T_2 by referring the load over to the transmission line's voltage level.
2. Eliminate transformer T_1 by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side.

The value of the load's impedance when reflected to the transmission system's voltage is

$$\begin{aligned} Z'_{load} &= a^2 Z_{load} \\ &= \left(\frac{10}{1}\right)^2 (4\ \Omega + j3\ \Omega) \\ &= 400\ \Omega + j300\ \Omega \end{aligned}$$

The total impedance at the transmission line level is now

$$\begin{aligned} Z_{eq} &= Z_{line} + Z'_{load} \\ &= 400.18 + j300.24\ \Omega = 500.3 \angle 36.88°\ \Omega \end{aligned}$$

The Ideal Transformer

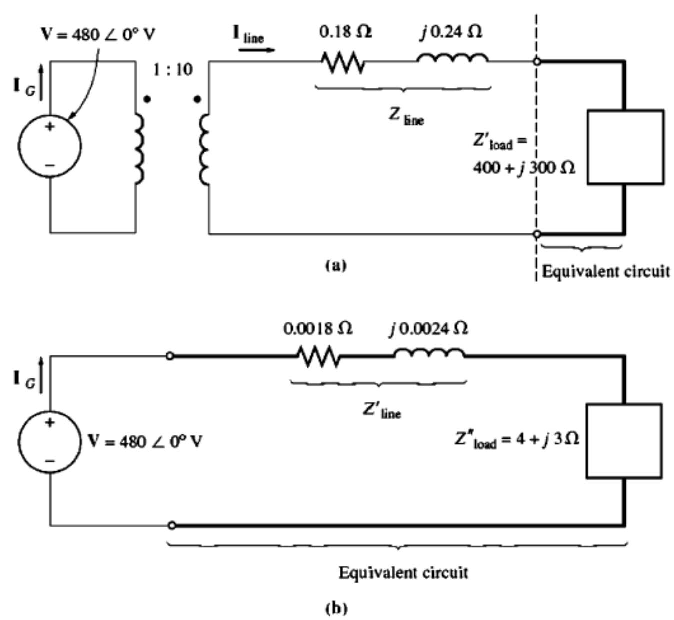


FIGURE 2–7
(a) System with the load referred to the transmission system voltage level. (b) System with the load and transmission line referred to the generator's voltage level.

The Ideal Transformer

This equivalent circuit is shown in Figure 2–7a. The total impedance at the transmission line level ($Z_{line} + Z'_{load}$) is now reflected across T_1 to the source's voltage level:

$$\begin{aligned}
\mathbf{Z}'_{eq} &= a^2\,\mathbf{Z}_{eq} \\
&= a^2(\mathbf{Z}_{line} + \mathbf{Z}'_{load}) \\
&= \left(\frac{1}{10}\right)^2(0.18\ \Omega + j0.24\ \Omega + 400\ \Omega + j300\ \Omega) \\
&= (0.0018\ \Omega + j0.0024\ \Omega + 4\ \Omega + j3\ \Omega) \\
&= 5.003\ \angle 36.88°\ \Omega
\end{aligned}$$

Notice that $\mathbf{Z}''_{load} = 4 + j3\ \Omega$ and $\mathbf{Z}'_{line} = 0.0018 + j0.0024\ \Omega$. The resulting equivalent circuit is shown in Figure 2–7*b*. The generator's current is

$$\mathbf{I}_G = \frac{480\angle 0°\ \text{V}}{5.003\angle 36.88°\ \Omega} = 95.94\angle -36.88°\ \text{A}$$

Knowing the current $\mathbf{I}_G$, we can now work back and find $\mathbf{I}_{line}$ and $\mathbf{I}_{load}$. Working back through T_1, we get

The Ideal Transformer

$$\begin{aligned}
N_{P1}\mathbf{I}_G &= N_{S1}\mathbf{I}_{line} \\
\mathbf{I}_{line} &= \frac{N_{P1}}{N_{S1}}\mathbf{I}_G \\
&= \frac{1}{10}(95.94\angle -36.88°\ \text{A}) = 9.594\angle -36.88°\ \text{A}
\end{aligned}$$

Working back through T_2 gives

$$\begin{aligned}
N_{P2}\mathbf{I}_{line} &= N_{S2}\mathbf{I}_{load} \\
\mathbf{I}_{load} &= \frac{N_{P2}}{N_{S2}}\mathbf{I}_{line} \\
&= \frac{10}{1}(9.594\angle -36.88°\ \text{A}) = 95.94\angle -36.88°\ \text{A}
\end{aligned}$$

It is now possible to answer the questions originally asked. The load voltage is given by

$$\begin{aligned}
\mathbf{V}_{load} &= \mathbf{I}_{load}\,\mathbf{Z}_{load} \\
&= (95.94\angle -36.88°\ \text{A})(5\angle 36.87°\ \Omega) \\
&= 479.7\angle -0.01°\ \text{V}
\end{aligned}$$

and the line losses are given by

$$\begin{aligned}
P_{loss} &= (I_{line})^2 R_{line} \\
&= (9.594\ \text{A})^2\,(0.18\ \Omega) = 16.7\ \text{W}
\end{aligned}$$

The Ideal Transformer

- Notice that raising the transmission voltage of the power system reduced transmission losses by a factor of nearly 90%.
- Also, the voltage at the load dropped much less in the system with transformers compared to the system without transformers.
- This simple example dramatically illustrates the advantages of using higher voltage transmission lines as well as the extreme importance of transformers in modern power systems.

Operation of Real Transformers

- The ideal transformers described earlier can of course never actually be made
- Not all the flux produced in the primary coil also passes through the secondary coil. Some of the flux lines leave the iron core and pass through the air instead.
- The portion of the flux that goes through one of the transformer coils but not the other one is called leakage flux.

Operation of Real Transformers

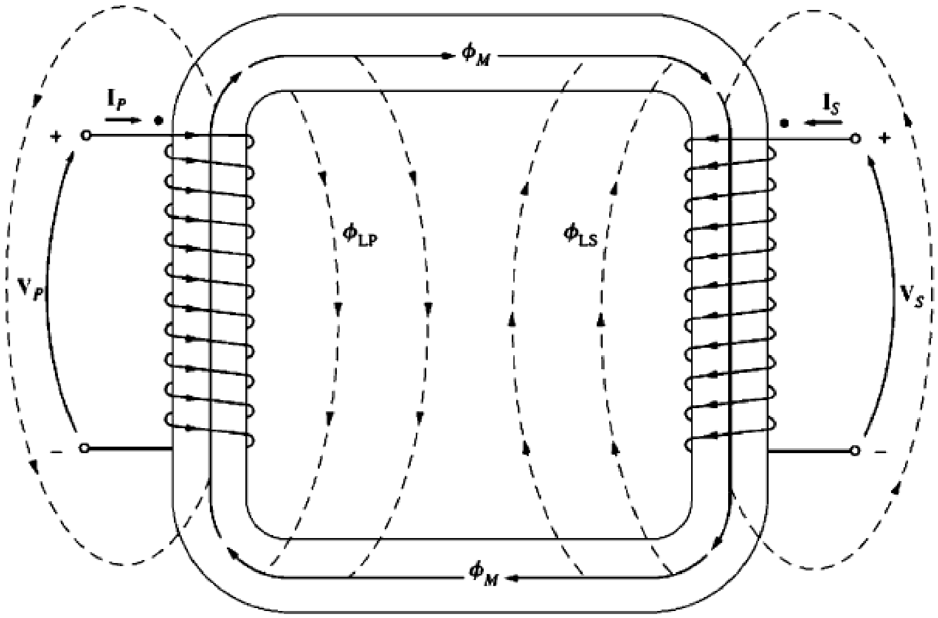


FIGURE 2–10
Mutual and leakage fluxes in a transformer core.

Operation of Real Transformers

- The flux in the primary coil of the transformer can thus be divided into two components: a **mutual flux**, which remains in the core and links both windings, and a small **leakage flux**, which passes through the primary winding but returns through the air, bypassing the secondary winding:

$$\boxed{\bar{\phi}_P = \phi_M + \phi_{LP}} \qquad (2\text{–}19)$$

where $\bar{\phi}_P$ = total average primary flux
ϕ_M = flux component linking both primary and secondary coils
ϕ_{LP} = primary leakage flux

Operation of Real Transformers

- There is a similar division of flux in the secondary winding between mutual flux and leakage flux which passes through the secondary winding but returns through the air, bypassing the primary winding:

$$\boxed{\bar{\phi}_S = \phi_M + \phi_{LS}} \qquad (2\text{–}20)$$

where $\bar{\phi}_S$ = total average secondary flux
ϕ_M = flux component linking both primary and secondary coils
ϕ_{LS} = secondary leakage flux

Operation of Real Transformer

- Since in a well-designed transformer $\phi_M >> \phi_{LP}$ and $\phi_M >> \phi_{LS}$ the ratio of the total voltage on the primary of a transformer to the total voltage on the secondary of a transformer is approximately
- The smaller the leakage fluxes of the transformer are, the closer the total transformer voltage ratio approximates that of the ideal transformer

$$\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a \qquad (2\text{–}28)$$

Operation of Real Transformer

- When an AC power source is connected to a transformer, a current flows in its primary circuit, even when the secondary circuit is open circuited.
- This current is the current required to produce flux in a real ferromagnetic core.
- It consists of two components:

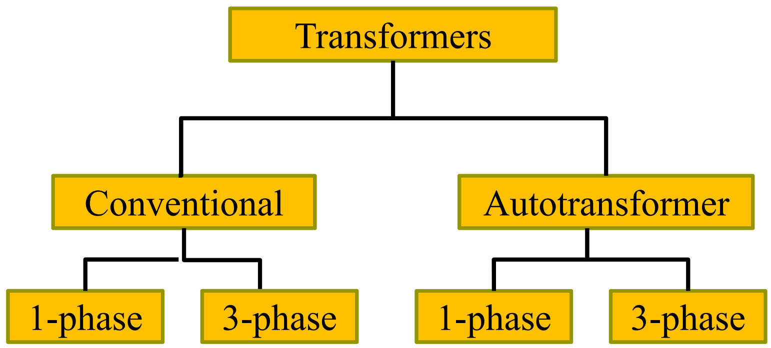
(1) The **magnetization current**, which is the current required to produce the flux in the transformer core.

(2) The **core-loss current**, which is the current required to make up for hysteresis and eddy current losses

Operation of Real Transformers

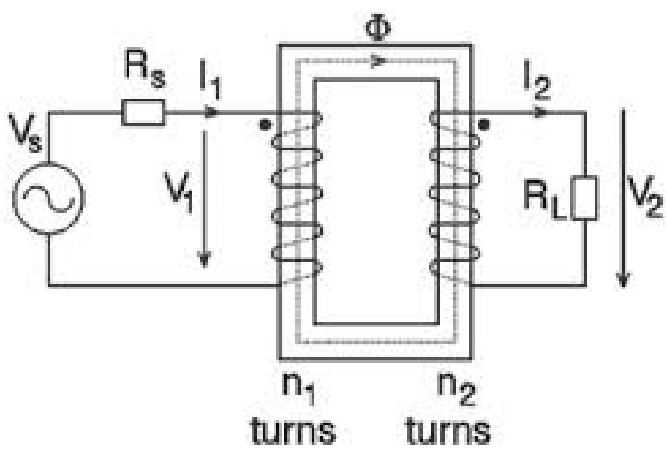
- Notice the total no-load current in the core is called the **excitation current** of the transformer. It is just the sum of the magnetization current and the core-loss current in the core.

Types of Transformers



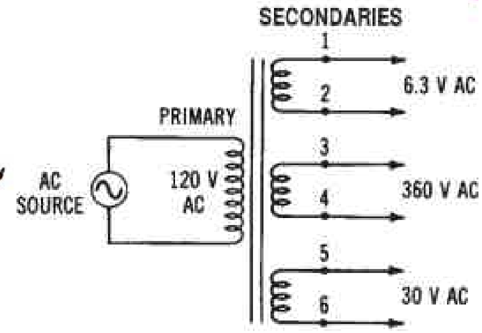
In each type of the transformers there are the tapped type and the variable type

Types of Transformers

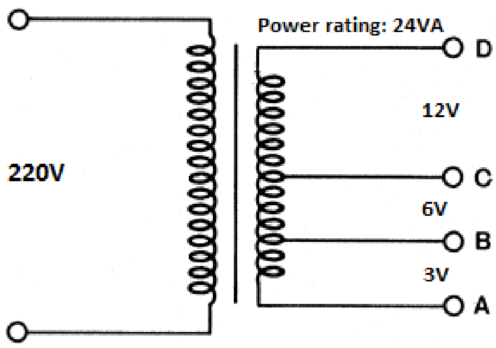


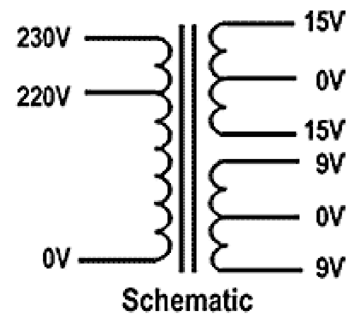
Conventional Single-Phase Transformer

Types of Transformers



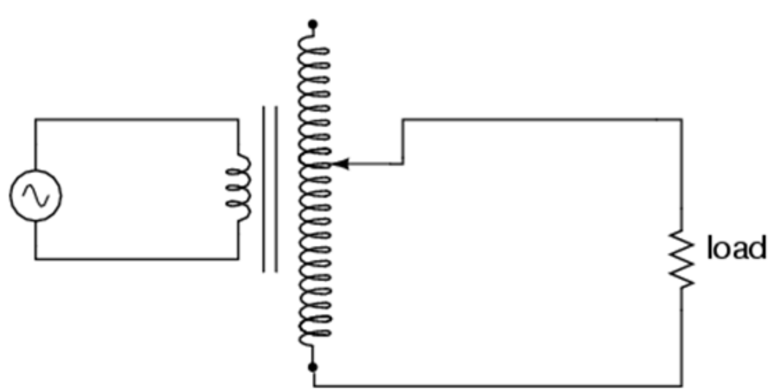
Single-Phase Tapped Transformer



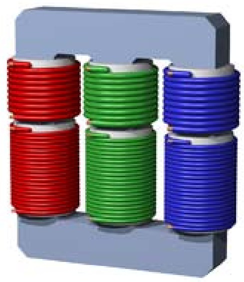


Types of Transformers

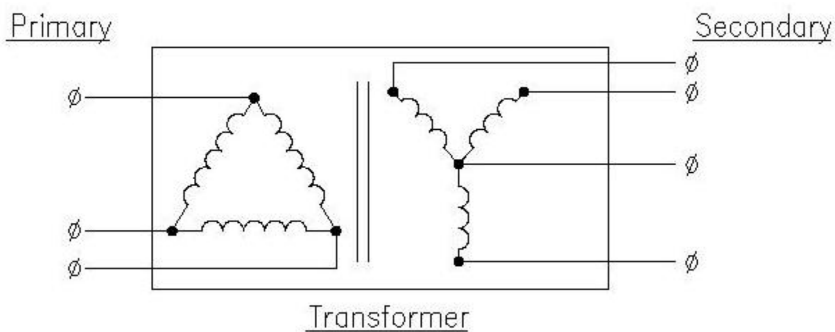
Single-Phase Variable Transformer



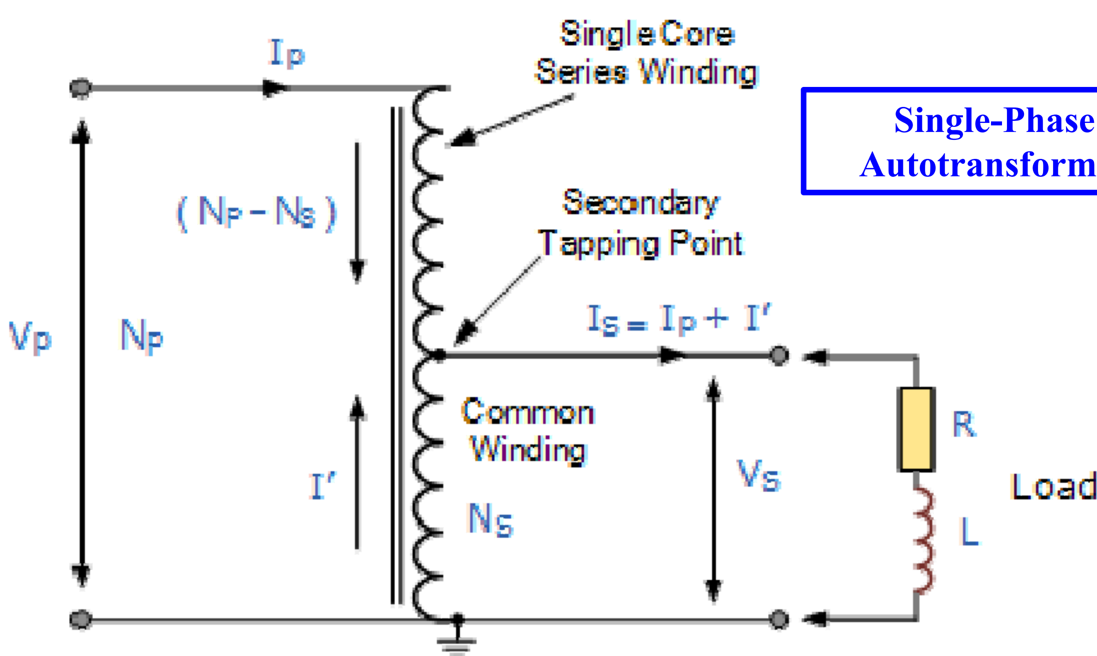
Types of Transformers



Three-Phase Variable Autotransformer



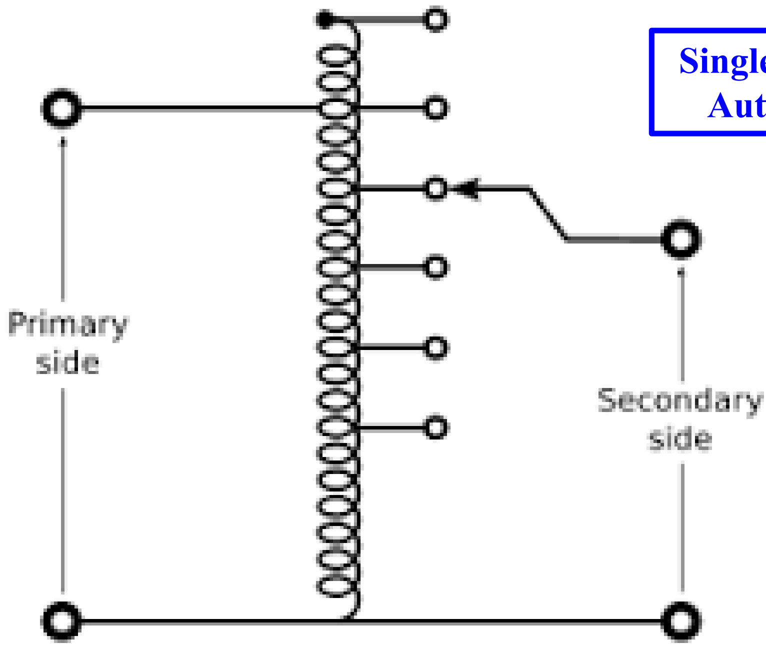
Types of Transformers



Single-Phase Autotransformer

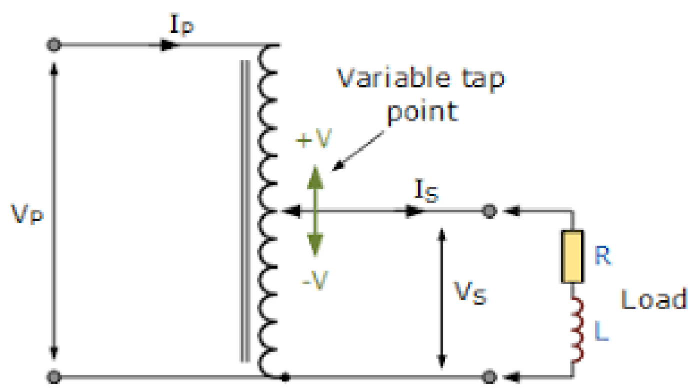
Types of Transformers

Single-Phase Tapped Autotransformer

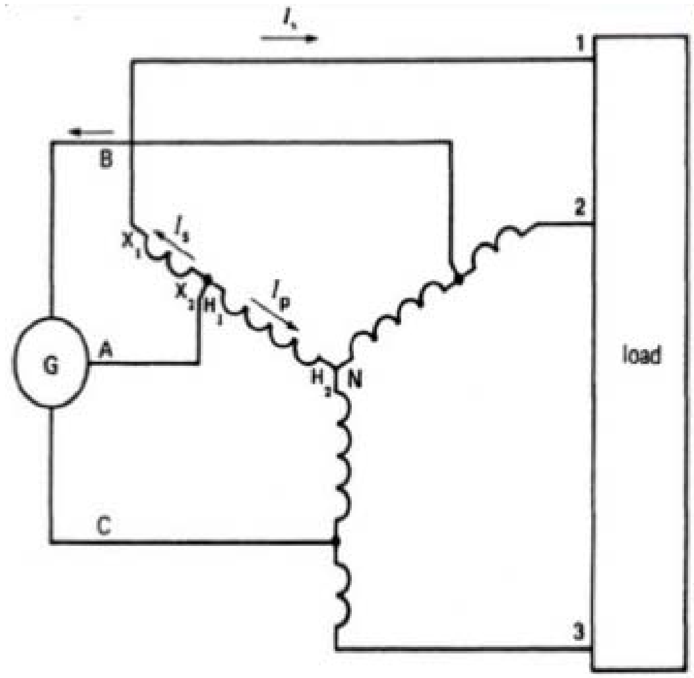


Types of Transformers

Single-Phase Variable Autotransformer



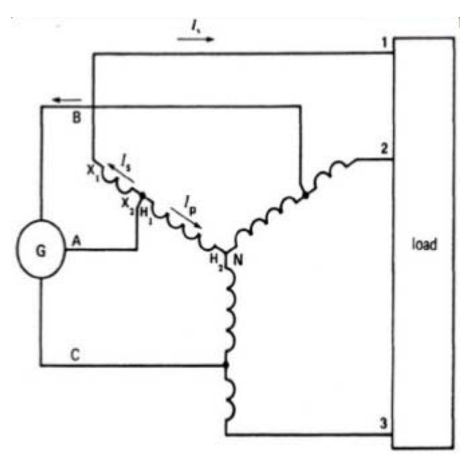
Types of Transformers



Three-Phase Autotransformer

Types of Transformers

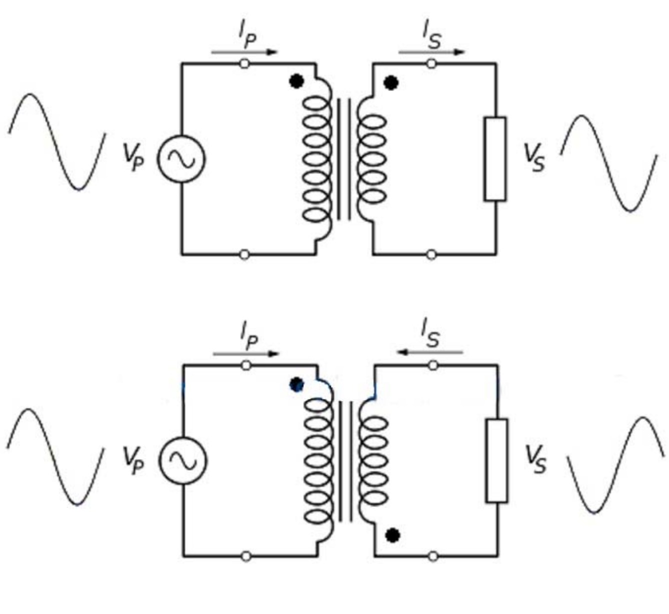
Three-Phase Variable Autotransformer



Types of Transformers

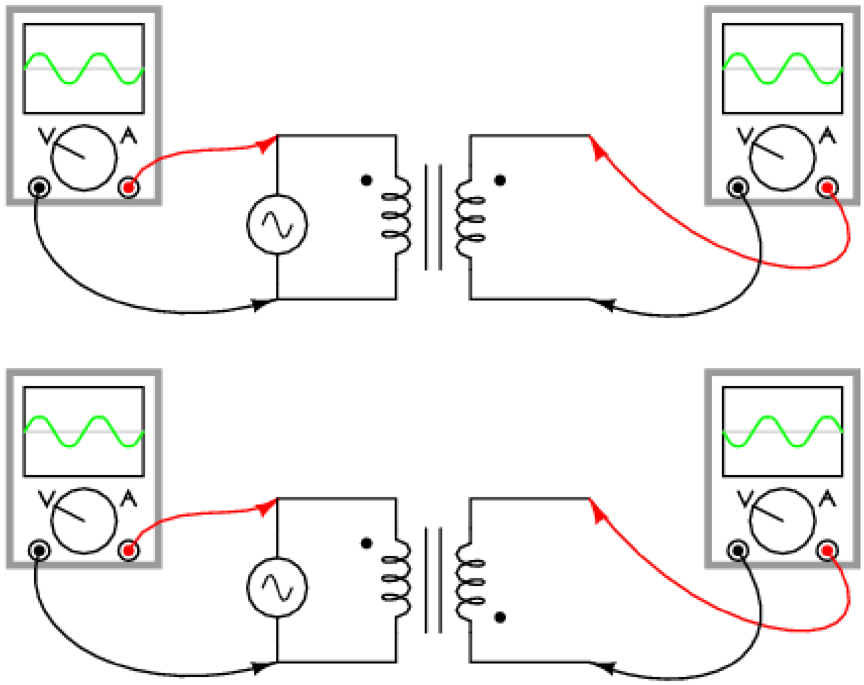
- **Transformer Types**
  https://www.youtube.com/watch?v=-S2s1b2j25o

The Dot Convention

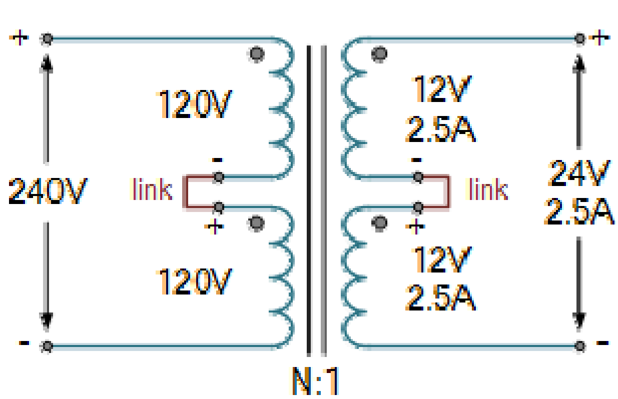


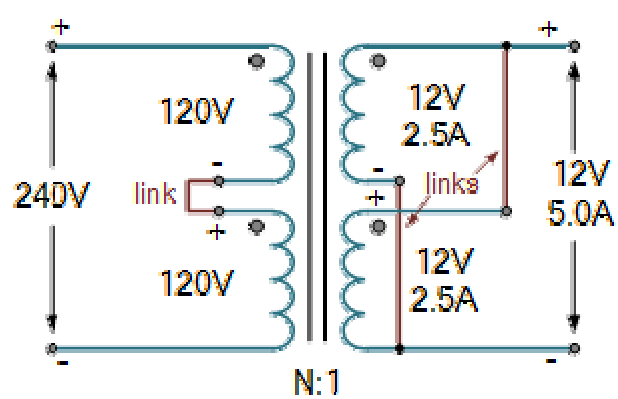
Dot Convention used to show phase relation between primary and secondary voltages and currents in a Transformer.

The Dot Convention

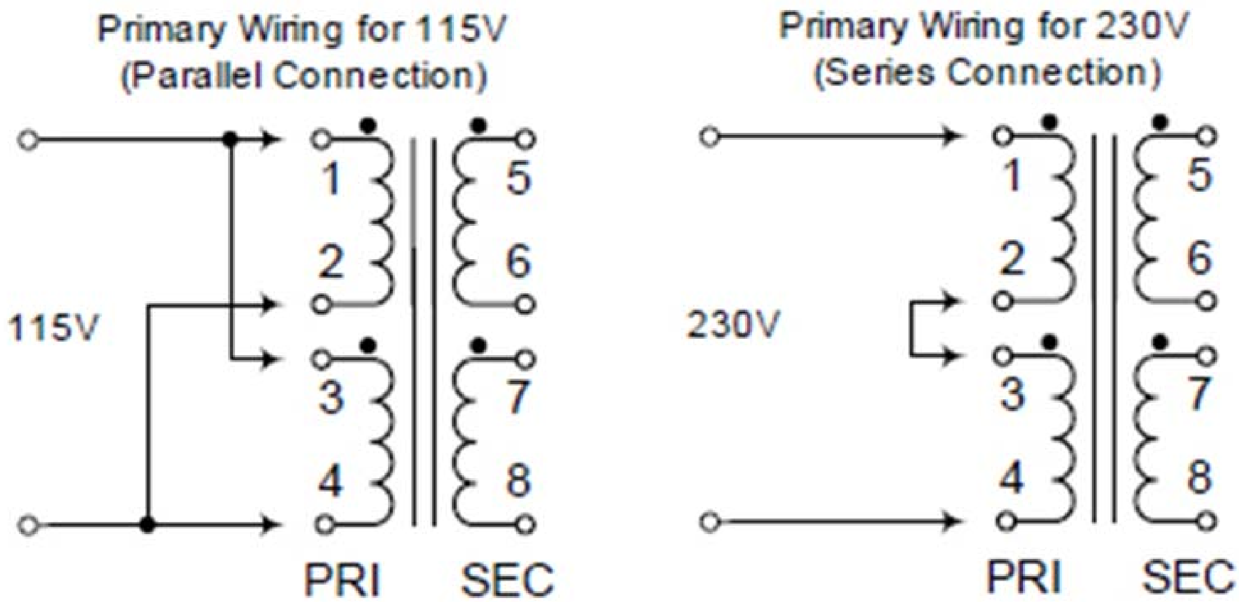


The Dot Convention

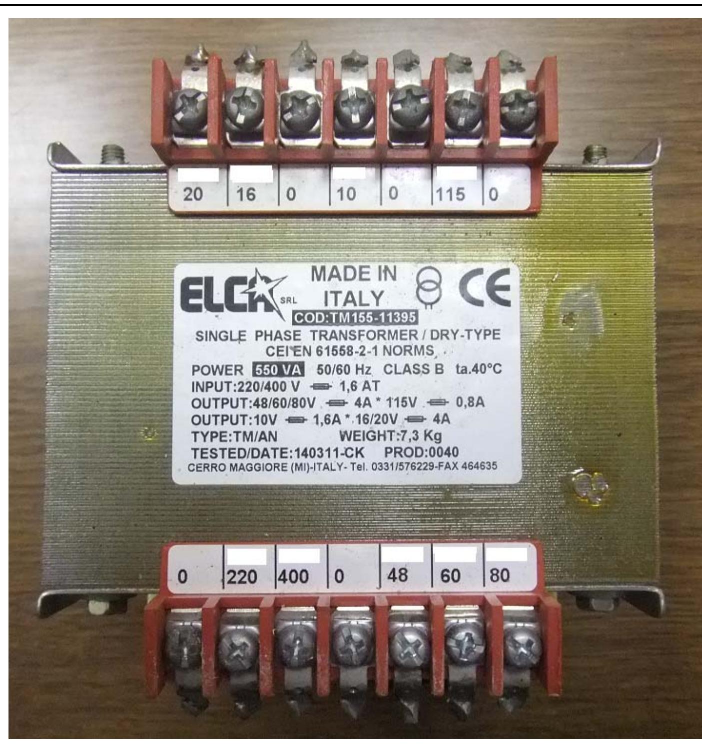




The Dot Convention



The Dot Convention



Equivalent Circuit of a Transformer

- The losses that occur in real transformers have to be accounted for in any accurate model of transformer behavior.
- The major items to be considered in the construction of such a model are

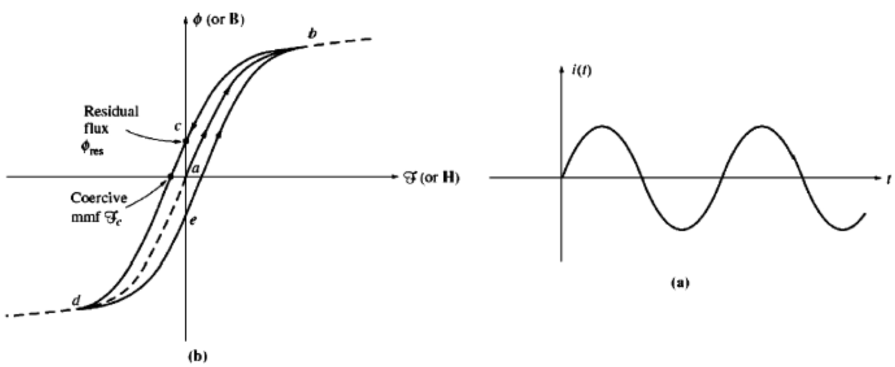
1. **Copper Losses (I^2R).** Copper losses are the resistive heating losses in the primary and secondary windings of the transformer. They are proportional to the square of the current in the windings.

Equivalent Circuit of a Transformer

2. **Eddy Current Losses**. Eddy current losses are resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.
3. **Hysteresis Losses**. Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are a complex, nonlinear function of the voltage applied to the transformer.

Equivalent Circuit of a Transformer

- To understand hysteresis, let's assume an alternating current is applied to a coil wrapped on a core and the flux in the core is initially zero.



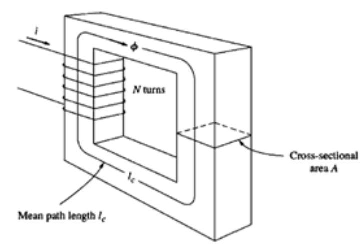


FIGURE 1–3
A simple magnetic core.

FIGURE 1–11
The hysteresis loop traced out by the flux in a core when the current $i(t)$ is applied to it.

Equivalent Circuit of a Transformer

- When the current rises and then falls, ***the flux traces out a different path from the one it followed when the current increased.***
- Notice that the amount of flux present in the core depends not only on the amount of current applied to the windings of the core, but also on the previous history of the flux in the core.
- This dependence on the preceding flux history and the resulting failure to retrace flux paths is called ***hysteresis.***
- Path *bcdeb* traced out in Figure 1-11b as the applied current changes is called a ***hysteresis loop.***

Equivalent Circuit of a Transformer

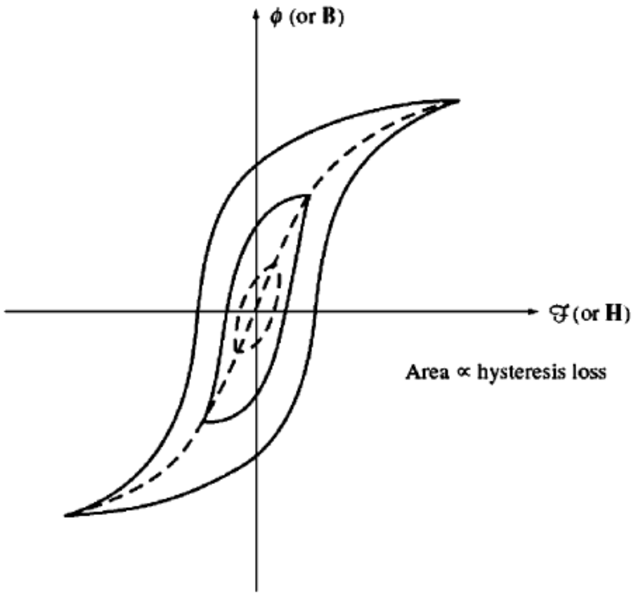


FIGURE 1–13
The effect of the size of magnetomotive force excursions on the magnitude of the hysteresis loss.

The *hysteresis loss* in an iron core is the energy required to accomplish the reorientation of domains during each cycle of the alternating current applied to the core. It can be shown that the area enclosed in the hysteresis loop formed by applying an alternating current to the core is directly proportional to the energy lost in a given ac cycle. The smaller the applied magnetomotive force excursions on the core, the smaller the area of the resulting hysteresis loop and so the smaller the resulting losses.

Equivalent Circuit of a Transformer

- Notice that if a large magnetomotive force is first applied to the core and then removed, the flux path in the core will be *abc*, When the magnetomotive force is removed, the flux in the core ***does not go to zero***.
- Instead, a magnetic field is left in the core. This magnetic field is called the ***residual flux*** in the core.
- It is in precisely this manner that permanent magnets are produced.
- To force the flux to zero, an amount of magnetomotive force known as the ***coercive magnetomotive force*** $\mathcal{F}_c$ must be applied to the core in the opposite direction.

Equivalent Circuit of a Transformer

- Both hysteresis and eddy current losses **cause heating in the core material**, and both losses must be considered in the design of any machine or transformer.
- Since both losses occur within the metal of the core, they are usually lumped together and called **core losses**

Equivalent Circuit of a Transformer

4. **Leakage Flux.** The fluxes ϕ_{LP} and ϕ_{LS} which escape the core and pass through only one of the transformer windings are leakage fluxes. These escaped fluxes produce a self-inductance in the primary and secondary coils, and the effects of this inductance must be accounted for.

- It is possible to construct an equivalent circuit that takes into account all the major imperfections in real transformers. Each major imperfection is considered in turn, and its effect is included in the transformer model.

Equivalent Circuit of a Transformer

- Copper losses are resistive losses in the primary and secondary windings of the transformer core and they are modeled by placing a resistor R_p in the primary circuit of the transformer and a resistor R_s in the secondary circuit.
- The leakage flux is modeled by primary and secondary inductors (where $L = N^2 \wp$ is the self inductance of coils and $\wp = 1/\mathcal{R}$ is the permeance of flux path)

Equivalent Circuit of a Transformer

- The magnetization current i_M can be modeled by a reactance X_M connected across the primary voltage source.
- The core-loss current can be modeled by a resistance R_c connected across the primary voltage source.
- Note that both these currents are really nonlinear, so the inductance X_M and the resistance R_c are, at best, approximations of the real excitation effects.

Equivalent Circuit of a Transformer

Notice that the elements forming the excitation branch are placed inside the primary resistance *Rp* and the primary inductance *Lp*. This is because the voltage actually applied to the core (*i.e.*, elements forming the excitation branch) is really less than the internal voltage drop on the winding elements (resistance and inductance).

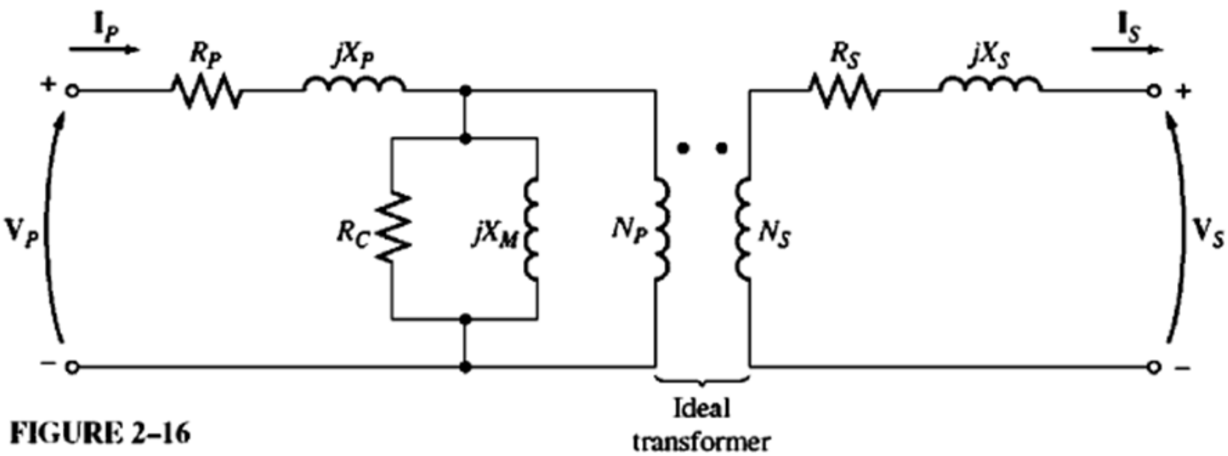


FIGURE 2–16
The model of a real transformer.

Equivalent Circuit of a Transformer

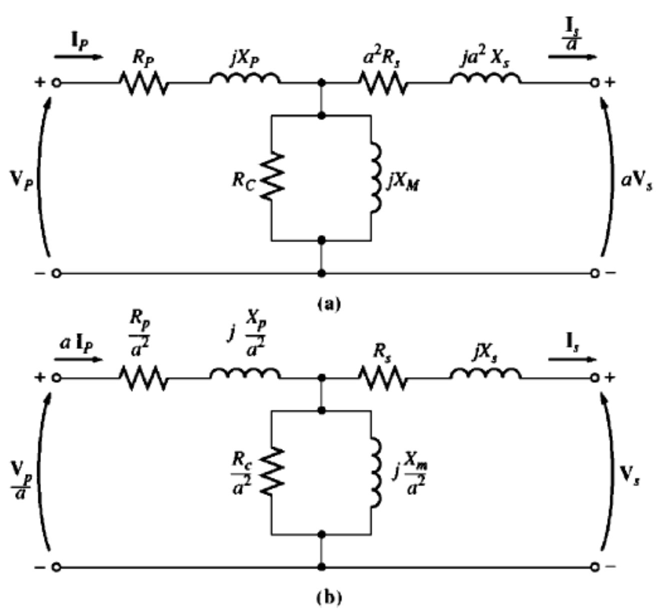


FIGURE 2–17
(a) The transformer model referred to its primary voltage level. (b) The transformer model referred to its secondary voltage level.

Although Figure 2-16 is an accurate model of a transformer, it is not a very useful one. To analyze practical circuits containing transformers, it is normally necessary to convert the entire circuit to an equivalent circuit at a single voltage level. Therefore, the equivalent circuit must be referred either to its primary side or to its secondary side in problem solutions.

Approximate Equivalent Circuits

- The transformer models shown before are often more complex than necessary in order to get good results in practical engineering applications.
- One of the principal complaints about them is that the excitation branch of the model adds another node to the circuit being analyzed, making the circuit solution more complex than necessary.
- The excitation branch has a very small current compared to the load current of the transformers. In fact, it is so small that under normal circumstances it causes a completely negligible voltage drop in ***Rp*** and ***Xp***.

Approximate Equivalent Circuits

- Because this is true, a simplified equivalent circuit can be produced that works almost as well as the original model.
- The excitation branch is simply moved to the front of the transformer, and the primary and secondary impedances are left in series with each other. These impedances are just added, creating the approximate equivalent circuits in Figure 2-18a and b.
- In some applications, the excitation branch may be neglected entirely without causing serious error. In these cases, the equivalent circuit of the transformer reduces to the simple circuits in Figure 2-18c and d.

Approximate Equivalent Circuits

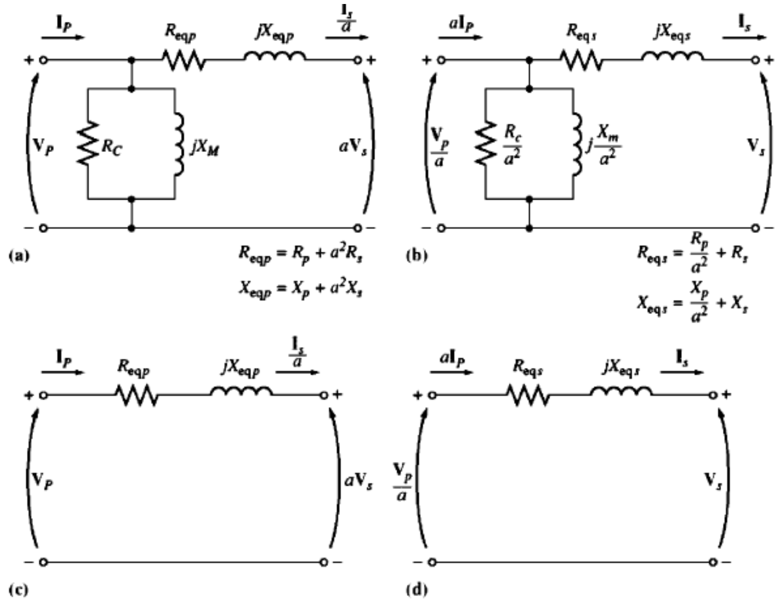


FIGURE 2–18
Approximate transformer models. (a) Referred to the primary side; (b) referred to the secondary side; (c) with no excitation branch, referred to the primary side; (d) with no excitation branch, referred to the secondary side.

Transformer Tests

- It is possible to experimentally determine the values of the inductances and resistances in the transformer model.
- An adequate approximation of these values can be obtained with only two tests:

1. **The open-circuit test**
2. **The short-circuit test**

- Another test can be done to determine the resistances of the primary and secondary windings: **The DC Test.**

Transformer Tests – Open-Circuit

- In the **open-circuit test**, a transformer's secondary winding is open circuited, and its primary winding is connected to a full-rated line voltage.

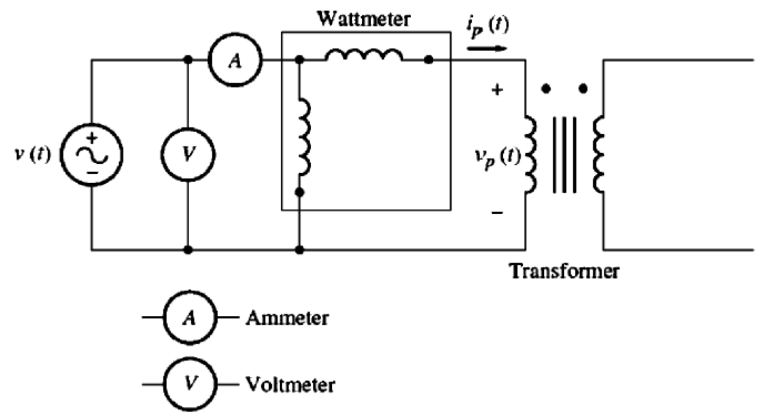


FIGURE 2–19
Connection for transformer open-circuit test.

Transformer Tests – Open-Circuit

- Under the conditions described, all the input current must be flowing through the excitation branch of the transformer.
- The series elements R_p and X_p are too small in comparison to R_c and X_M to cause a significant voltage drop, so essentially all the input voltage is dropped across the excitation branch.
- The easiest way to calculate the values of *Rc* and *XM* is to look first at the **admittance** of the excitation branch, Y_E .

Transformer Tests – Open-Circuit

- The **conductance** of the core- loss resistor is given by

$$G_c = \frac{1}{R_c}$$

- The **susceptance** of the magnetizing inductor is given by

$$B_M = \frac{1}{X_M}$$

- Since these two elements are in parallel, their admittances add, and the total **excitation admittance** is

$$Y_E = G_c - jB_M$$

$$Y_E = \frac{1}{R_c} - j\frac{1}{X_M}$$

Transformer Tests – Open-Circuit

- Hence, the following relations can be obtained

$$|Y_E| = \frac{I_{oc}}{V_{oc}}$$

$$PF = \cos(\theta) = \frac{P_{OC}}{V_{OC} I_{OC}} \quad \theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{P_{OC}}{V_{OC} I_{OC}}\right)$$

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\theta \quad \Rightarrow \quad Y_E = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1}(PF)$$

$$R_c = \frac{1}{G_c} = \frac{1}{Y_E \cos(\theta)}$$

$$X_M = \frac{1}{B_M} = \frac{1}{Y_E \sin(\theta)}$$

Transformer Tests – Short-Circuit

- In the **short-circuit test**, the secondary terminals of the transformer are short circuited, and the primary terminals are connected to a fairly low-voltage

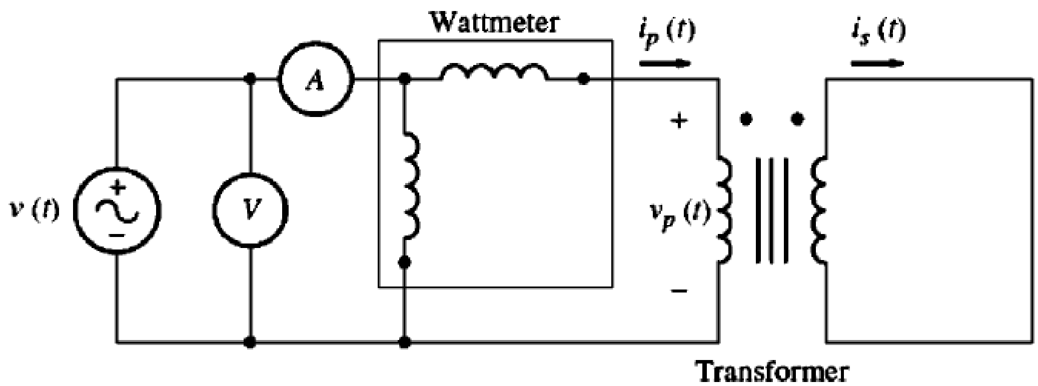


FIGURE 2–20
Connection for transformer short-circuit test.

Transformer Tests – Short-Circuit

- The input voltage is adjusted until the current in the short-circuited windings is equal to its rated value. (Be sure to keep the primary voltage at a safe level. It would not be a good idea to burn out the transformer's windings while trying to test it.)
- Since the input voltage is so low during the short-circuit test, negligible current flows through the excitation branch.
- If the excitation current is ignored, then all the voltage drop in the transformer can be attributed to the series elements in the circuit.

Transformer Tests – Short-Circuit

- The magnitude of the series impedances referred to the primary side of the transformer is

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

- The following relations can be obtained

$$PF = \cos(\theta) = \frac{P_{SC}}{V_{SC} I_{SC}} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{P_{SC}}{V_{SC} I_{SC}}\right)$$

$$Z_{SE} = \frac{V_{SC} \angle 0}{I_{SC} \angle -\theta} = \frac{V_{SC}}{I_{SC}} \angle \theta$$

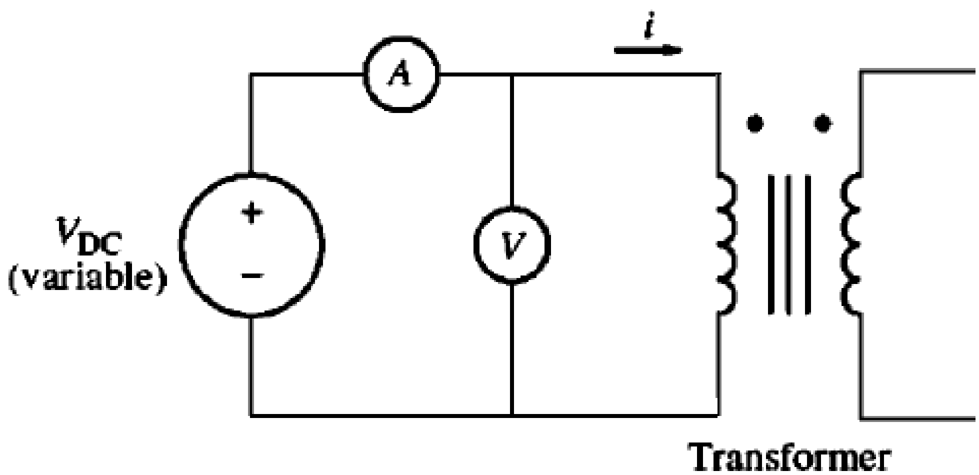
$$Z_{SE} = R_{eq} + jX_{eq} = (R_P + a^2 R_S) + j(X_P + a^2 X_S)$$

Transformer Tests – Short-Circuit

- It is possible to determine the total series impedance referred to the primary side by using this technique, but there is no easy way to split the series impedance into primary and secondary components.
- Fortunately, such separation is not necessary to solve normal problems.
- These same tests may also be performed on the secondary side of the transformer if it is more convenient to do so because of voltage levels or other reasons.

Transformer Tests – DC Test

- In the **DC test**, a transformer's winding is connected to a variable DC power supply. It does not matter what the other winding is connected to.



Transformer Tests – DC Test

- Using Ohm's law and measurements of voltage and current, the internal resistor of the winding can be determined as

$$R = V / I$$

- It should be noted that this test can be done on both sides of the transformer
- Moreover, the DC power supply should be adjusted so that maximum DC current is not exceeded.
- Note that the coil can handle a DC current that is certainly less than the rated AC current.

Transformer Tests

Example 2.2 The equivalent circuit impedances of a 20-kVA, 8000/240-V, 60-Hz transformer are to be determined. The open-circuit test and the short-circuit test were performed on the primary side of the transformer, and the following data were taken:

Open-circuit test (on primary)	Short-circuit test (on primary)
$V_{OC} = 8000$ V	$V_{SC} = 489$ V
$I_{OC} = 0.214$ A	$I_{SC} = 2.5$ A
$V_{OC} = 400$ W	$P_{SC} = 240$ W

Find the impedances of the approximate equivalent circuit referred to the primary side, and sketch that circuit.

Transformer Tests

The power factor during the *open-circuit* test is

$$\text{PF} = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}} \qquad (2\text{–}45)$$

$$= \cos\theta = \frac{400 \text{ W}}{(8000 \text{ V})(0.214 \text{ A})}$$

$$= 0.234 \text{ lagging}$$

The excitation admittance is given by

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1}\text{PF} \qquad (2\text{–}47)$$

$$= \frac{0.214 \text{ A}}{8000 \text{ V}} \angle -\cos^{-1} 0.234$$

$$= 0.0000268 \angle -76.5° \; \Omega$$

$$= 0.0000063 - j0.0000261 = \frac{1}{R_C} - j\frac{1}{X_M}$$

Transformer Tests

Therefore,

$$R_C = \frac{1}{0.0000063} = 159\ \text{k}\Omega$$

$$X_M = \frac{1}{0.0000261} = 38.4\ \text{k}\Omega$$

The power factor during the *short-circuit* test is

$$\text{PF} = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}} \qquad (2\text{–}49)$$

$$= \cos\theta = \frac{240\ \text{W}}{(489\ \text{V})(2.5\ \text{A})} = 0.196\ \text{lagging}$$

The series impedance is given by

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle -\cos^{-1}\text{PF}$$

$$= \frac{489\ \text{V}}{2.5\ \text{A}} \angle 78.7°$$

$$= 195.6 \angle 78.7° = 38.4 + j192\ \Omega$$

Therefore, the equivalent resistance and reactance are

$$R_{eq} = 38.4\ \Omega \qquad X_{eq} = 192\ \Omega$$

The resulting simplified equivalent circuit is shown in Figure 2–21.

Transformer Tests

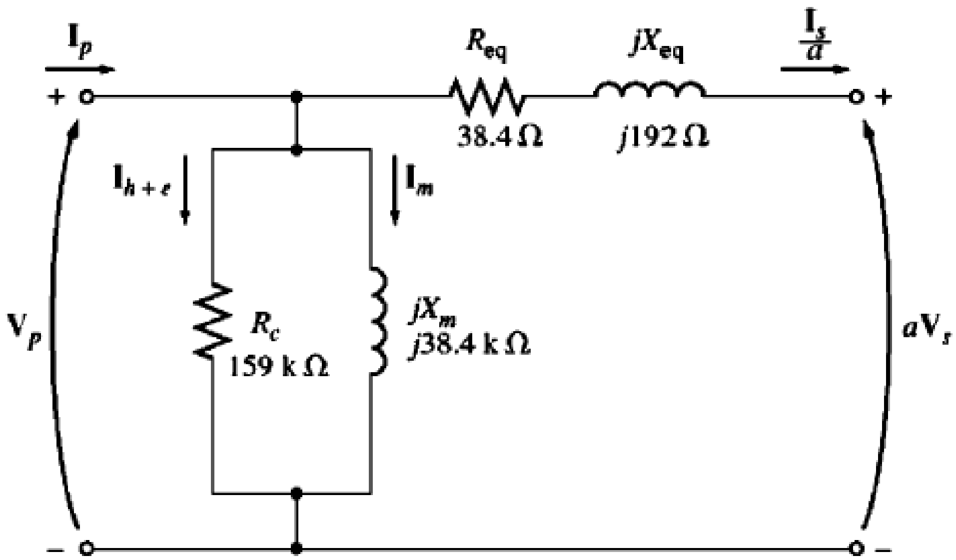


FIGURE 2–21
The equivalent circuit of Example 2–2.

Transformer Tests – Voltage Regulation

- Because a real transformer has series impedances within it, the output voltage of a transformer varies with the load even if the input voltage remains constant.
- To conveniently compare transformers in this respect, it is customary to define a quantity called **Voltage Regulation (VR).**
- Full-load voltage regulation is a quantity that compares the output voltage of the transformer at no load with the output voltage at full load.

Transformer Tests – Voltage Regulation

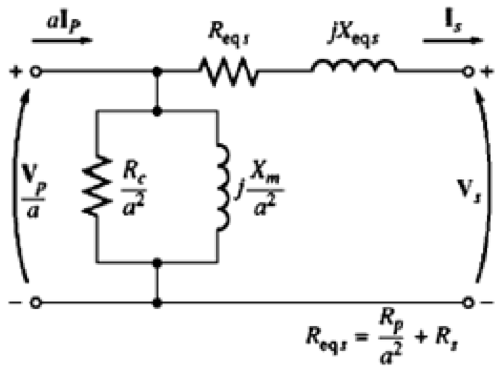
- lt is defined by the equation

$$\mathrm{VR} = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\% \qquad (2\text{–}61)$$

$$\mathrm{VR} = \frac{V_P/a - V_{S,fl}}{V_{S,fl}} \times 100\% \qquad (2\text{–}62)$$

- Usually it is a good practice to have as small a voltage regulation as possible. For an ideal transformer, VR = 0%
- lt is not always a good idea to have a low-voltage regulation, though-sometimes high-impedance and high-voltage regulation transformers are deliberately used to reduce the fault currents in a circuit.

Transformer Tests – Voltage Regulation



$$\frac{\mathbf{V}_P}{a} = \mathbf{V}_S + R_{eq}\mathbf{I}_S + jX_{eq}\mathbf{I}_S \quad (2\text{–}64)$$

Figure 2-26 shows a phasor diagram of a transformer operating at a lagging power factor. It is easy to see that $V_P/a > V_S$ for lagging loads, so the voltage regulation of a transformer with lagging loads must be greater than zero.

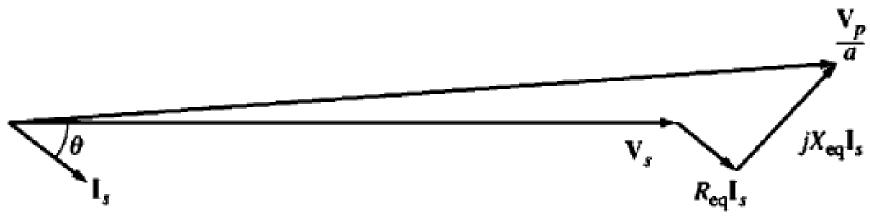


FIGURE 2–26
Phasor diagram of a transformer operating at a lagging power factor.

Transformer Tests – Voltage Regulation

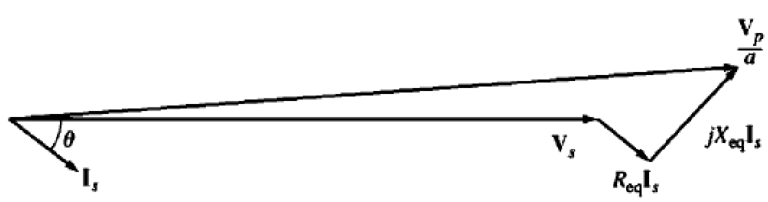
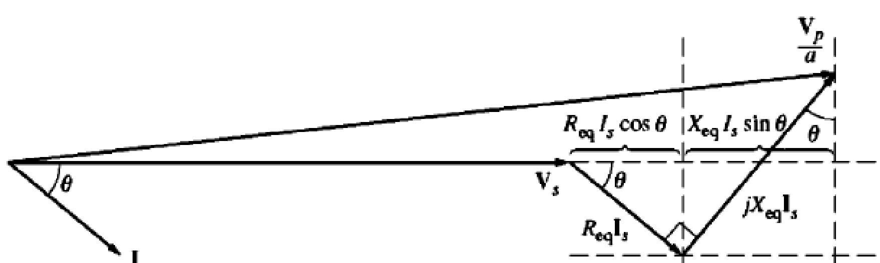


FIGURE 2–26
Phasor diagram of a transformer operating at a lagging power factor.



$$\frac{V_P}{a} \approx V_s + R_{eq}\,I_s\cos\theta + X_{eq}\,I_s\sin\theta$$

FIGURE 2–28
Derivation of the approximate equation for V_P/a.

Transformer Tests – Voltage Regulation

$V_P/a > V_S$ so $VR > 0$

If the secondary current is leading then $V_P/a < V_S$ so $VR < 0$

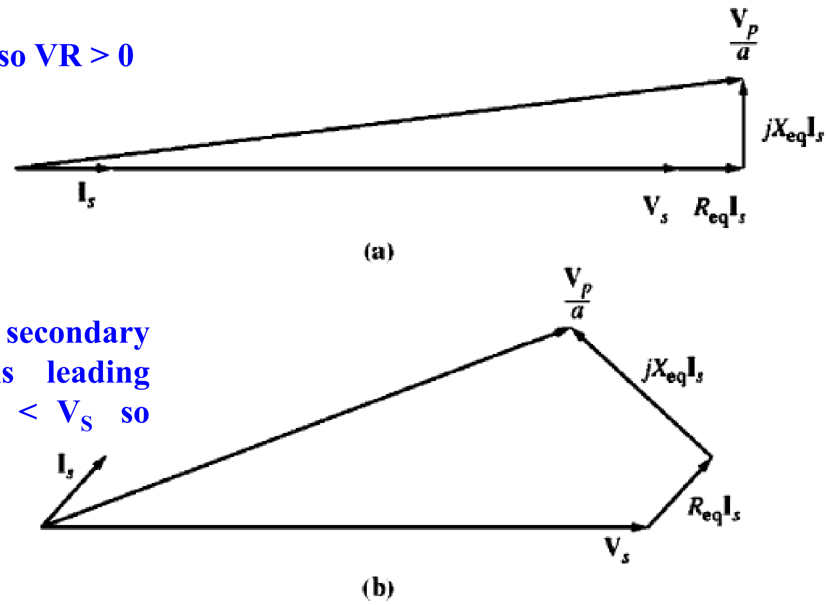


FIGURE 2–27
Phasor diagram of a transformer operating at (a) unity and (b) leading power factor.

Transformer Tests – Efficiency

- Transformers are also compared and judged on their efficiencies. The efficiency of a device is defined by the equation

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \quad (2\text{–}65)$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\% \quad (2\text{–}66)$$

- These equations apply to motors and generators as well as to transformers.
- The transformer equivalent circuits make efficiency calculations easy.

Transformer Tests – Efficiency

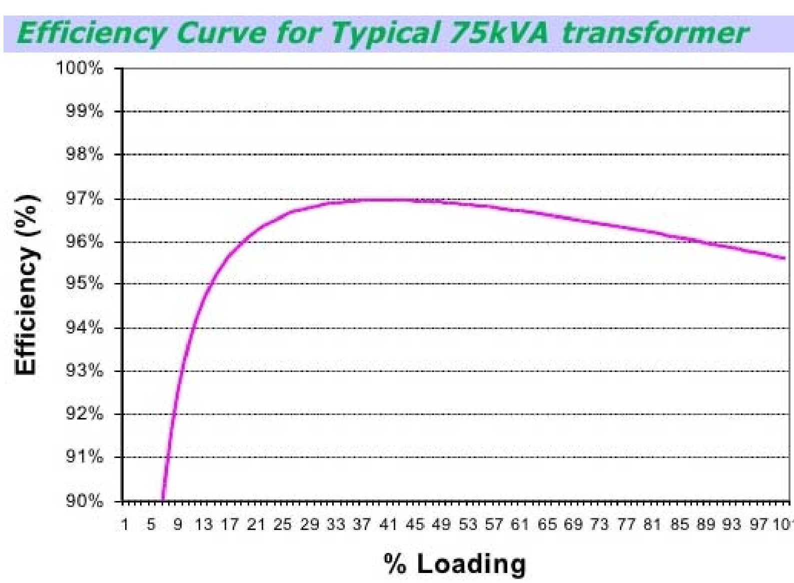
- There are three types of losses present in transformers: **(1)** Copper (I^2R) losses, **(2)** Hysteresis losses, and **(3)** Eddy current losses.
- To calculate the efficiency of a transformer at a given load, just add the losses the equation of efficiency such that

$$P_{out} = V_S I_S \cos \theta_S \qquad (2\text{–}7)$$

$$\eta = \frac{V_S I_S \cos \theta}{P_{Cu} + P_{core} + V_S I_S \cos \theta} \times 100\% \qquad (2\text{–}67)$$

Transformer Tests – Efficiency

The efficiency of a transformer will be **maximum** when the copper losses are equal to core losses (*i.e.*, $P_{cu} = P_{core}$)



Transformer Tests

Example 2.5 A 15-kVA, 2300/230-V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following test data have been taken from the primary side of the transformer:

Open-circuit test	Short-circuit test
$V_{OC} = 2300$ V	$V_{SC} = 47$ V
$I_{OC} = 0.21$ A	$I_{SC} = 6.0$ A
$P_{OC} = 50$ W	$P_{SC} = 160$ W

(a) Find the equivalent circuit of this transformer referred to the high-voltage side. (b) Find the equivalent circuit of this transformer referred to the low-voltage side. (c) Calculate the full-load voltage regulation at 0.8 lagging power factor, 1.0 power factor, and at 0.8 leading power factor. (d) Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading. (e) What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

Transformer Tests

(*a*) The excitation branch values of the transformer equivalent circuit can be calculated from the *open-circuit test* data, and the series elements can be calculated from the *short-circuit test* data. From the open-circuit test data, the open-circuit impedance angle is

$$\theta_{OC} = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}}$$

$$= \cos^{-1}\frac{50\text{W}}{(2300\text{ V})(0.21\text{ A})} = 84^\circ$$

The excitation admittance is thus

$$Y_E = \frac{I_{OC}}{V_{OC}}\angle -84^\circ$$

$$= \frac{0.21\text{ A}}{2300\text{ V}}\angle -84^\circ$$

$$= 9.13 \times 10^{-5}\angle -84^\circ\Omega = 0.0000095 - j0.0000908\ \Omega$$

The elements of the excitation branch referred to the primary are

$$R_C = \frac{1}{0.0000095} = 105\text{ k}\Omega$$

$$X_M = \frac{1}{0.0000908} = 11\text{ k}\Omega$$

Transformer Tests

From the short-circuit test data, the short-circuit impedance angle is

$$\theta_{SC} = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}}$$

$$= \cos^{-1}\frac{160\text{ W}}{(47\text{ V})(6\text{ A})} = 55.4°$$

The equivalent series impedance is thus

$$Z_{SE} = \frac{V_{SC}}{I_{SC}}\angle\theta_{SC}$$

$$= \frac{47\text{ V}}{6\text{ A}}\angle 55.4°\ \Omega$$

$$= 7.833\angle 55.4° = 4.45 + j6.45$$

The series elements referred to the primary are

$$R_{eq} = 4.45\ \Omega \qquad X_{eq} = 6.45\ \Omega$$

This equivalent circuit is shown in Figure 2–29a.

Transformer Tests

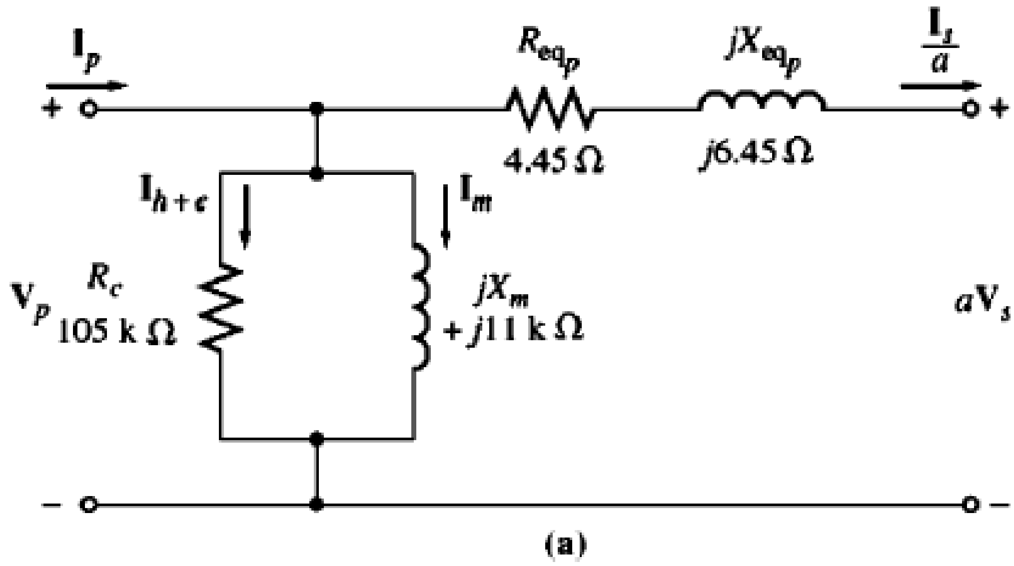


FIGURE 2–29
The transfer equivalent circuit for Example 2–5 referred to (a) its primary side

Transformer Tests

(*b*) To find the equivalent circuit referred to the low-voltage side, it is simply necessary to divide the impedance by a^2. Since $a = N_P/N_S = 10$, the resulting values are

$$R_C = 1050\ \Omega \qquad R_{eq} = 0.0445\ \Omega$$
$$X_M = 110\ \Omega \qquad X_{eq} = 0.0645\ \Omega$$

The resulting equivalent circuit is shown in Figure 2–29b.

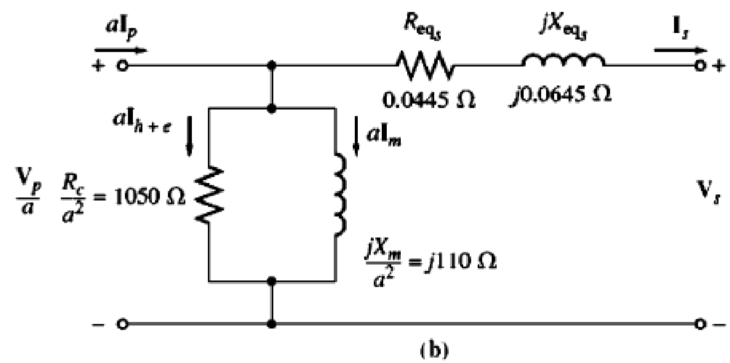


FIGURE 2–29
The transfer equivalent circuit for Example 2–5 referred to (b) its secondary side.

Transformer Tests

(*c*) The full-load current on the secondary side of this transformer is

$$I_{S,\text{rated}} = \frac{S_{\text{rated}}}{V_{S,\text{rated}}} = \frac{15{,}000\ \text{VA}}{230\ \text{V}} = 65.2\ \text{A}$$

To calculate V_P/a, use Equation (2–64):

$$\frac{\mathbf{V}_P}{a} = \mathbf{V}_S + R_{eq}\mathbf{I}_S + jX_{eq}\mathbf{I}_S \qquad (2\text{–}64)$$

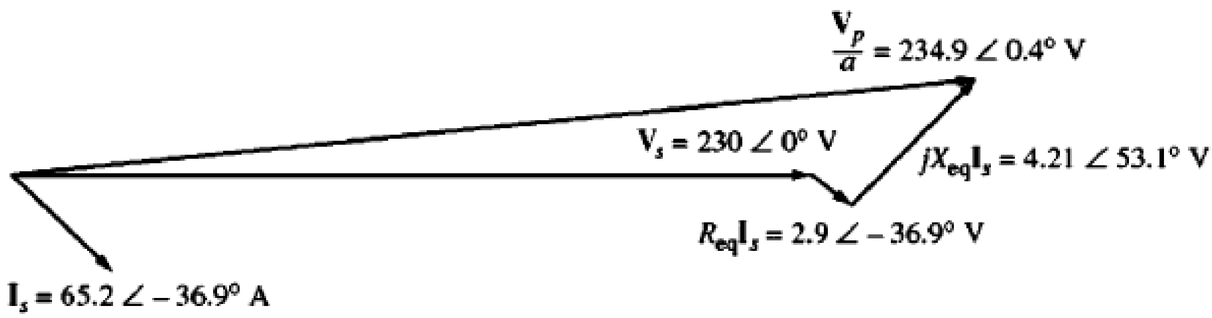
At PF = 0.8 lagging, current $\mathbf{I}_S = 65.2\angle -36.9°$ A. Therefore,

$$\frac{\mathbf{V}_P}{a} = 230\angle 0°\ \text{V} + (0.0445\ \Omega)(65.2\angle -36.9°\ \text{A}) + j(0.0645\ \Omega)(65.2\angle -36.9°\ \text{A})$$
$$= 230\angle 0°\ \text{V} + 2.90\angle -36.9°\ \text{V} + 4.21\angle 53.1°\ \text{V}$$
$$= 230 + 2.32 - j1.74 + 2.52 + j3.36$$
$$= 234.84 + j1.62 = 234.85\angle 0.40°\ \text{V}$$

The resulting voltage regulation is

$$\text{VR} = \frac{V_P/a - V_{S,\text{fl}}}{V_{S,\text{fl}}} \times 100\% \qquad (2\text{–}62)$$
$$= \frac{234.85\ \text{V} - 230\ \text{V}}{230\ \text{V}} \times 100\% = 2.1\%$$

Transformer Tests



Transformer Tests

At PF = 1.0, current $I_S = 65.2 \angle 0°$ A. Therefore,

$$\frac{V_P}{a} = 230 \angle 0° \text{ V} + (0.0445\ \Omega)(65.2 \angle 0° \text{ A}) + j(0.0645\ \Omega)(65.2 \angle 0° \text{ A})$$

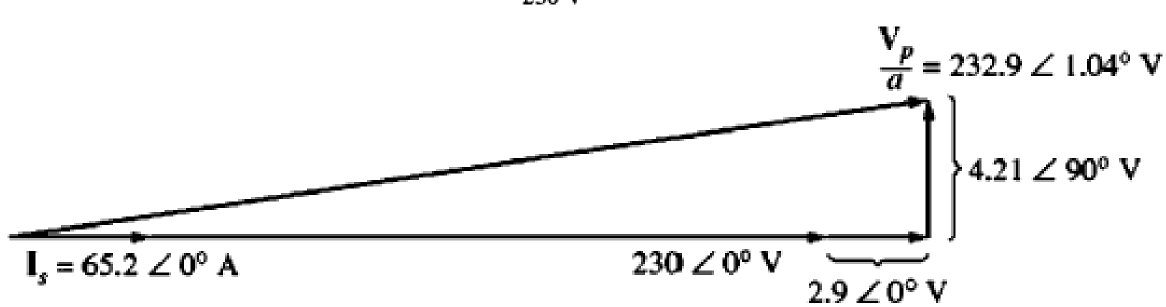
$$= 230 \angle 0° \text{ V} + 2.90 \angle 0° \text{ V} + 4.21 \angle 90° \text{ V}$$

$$= 230 + 2.90 + j4.21$$

$$= 232.9 + j4.21 = 232.94 \angle 1.04° \text{ V}$$

The resulting voltage regulation is

$$\text{VR} = \frac{232.94 \text{ V} - 230 \text{ V}}{230 \text{ V}} \times 100\% = 1.28\%$$



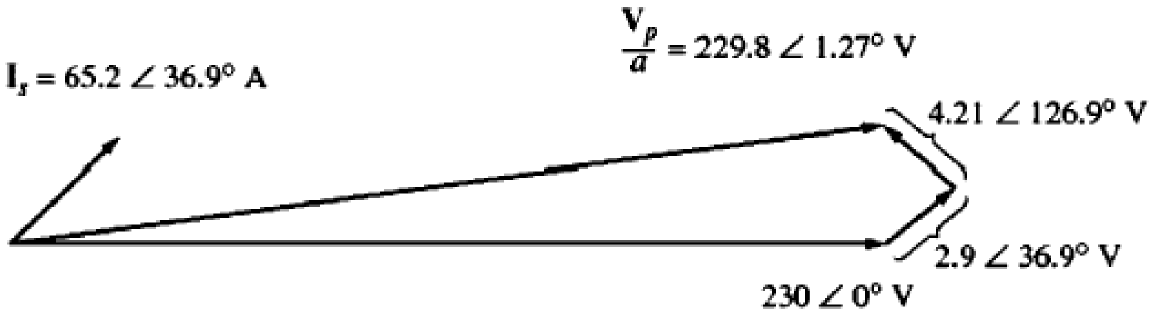
Transformer Tests

At PF = 0.8 leading, current $\mathbf{I}_S = 65.2 \angle 36.9° \text{ A}$. Therefore,

$$\frac{\mathbf{V}_P}{a} = 230 \angle 0° \text{ V} + (0.0445\ \Omega)(65.2 \angle 36.9° \text{ A}) + j(0.0645\ \Omega)(65.2 \angle 36.9° \text{ A})$$
$$= 230 \angle 0° \text{ V} + 2.90 \angle 36.9° \text{ V} + 4.21 \angle 126.9° \text{ V}$$
$$= 230 + 2.32 + j1.74 - 2.52 + j3.36$$
$$= 229.80 + j5.10 = 229.85 \angle 1.27° \text{ V}$$

The resulting voltage regulation is

$$\text{VR} = \frac{229.85 \text{ V} - 230 \text{ V}}{230 \text{ V}} \times 100\% = -0.062\%$$



Transformer Tests

(d) The best way to plot the voltage regulation as a function of load is to repeat the calculations in part *c* for many different loads using MATLAB.

The plot produced by this program is shown in Figure 2–31.

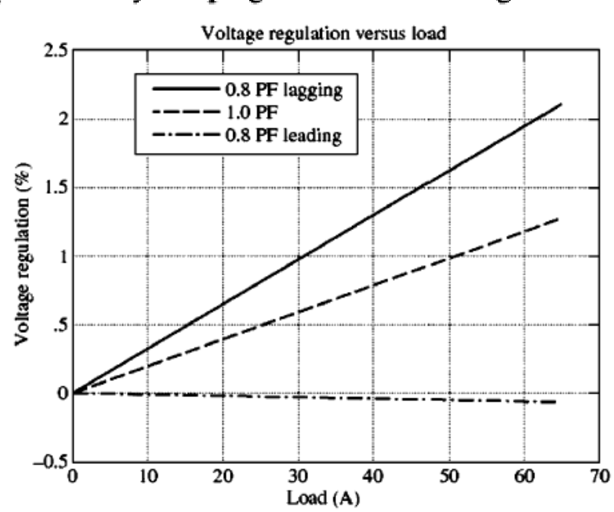


FIGURE 2–31
Plot of voltage regulation versus load for the transformer of Example 2–5.

Transformer Tests

(e) To find the efficiency of the transformer, first calculate its losses. The copper losses are

$$P_{Cu} = (I_S)^2 R_{eq} = (65.2\ A)^2(0.0445\ \Omega) = 189\ W$$

The core losses are given by

$$P_{core} = \frac{(V_P/a)^2}{R_C} = \frac{(234.85\ V)^2}{1050\ \Omega} = 52.5\ W$$

The output power of the transformer at this power factor is

$$P_{out} = V_S I_S \cos\theta$$
$$= (230\ V)(65.2\ A)\cos 36.9° = 12{,}000\ W$$

Therefore, the efficiency of the transformer at this condition is

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} \times 100\% \qquad (2\text{–}68)$$

$$= \frac{12{,}000\ W}{189\ W + 52.5\ W + 12{,}000\ W} \times 100\%$$

$$= 98.03\%$$

Transformer Taps

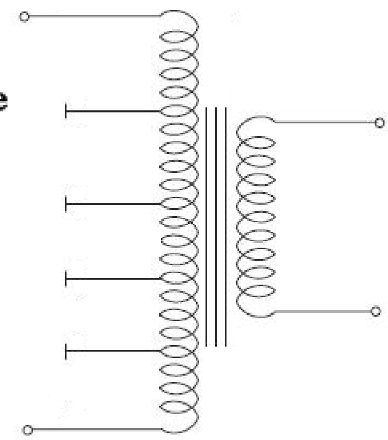
- Distribution transformers have a series of taps in the windings to permit small changes in the turns ratio of the transformer after it has left the factory.
- A typical installation might have four taps in addition to the nominal setting with spacings of 2.5 percent of full-load voltage between them.
- Such an arrangement provides for adjustments up to 5 percent above or below the nominal voltage rating of the transformer.

Transformer Taps

Example 2.6 A 500-kVA, 13,200/480-V distribution transformer has four 2.5 percent taps on its primary winding. What are the voltage ratios of this transformer at each tap setting?

The five possible voltage ratings of this transformer are

+5.0% tap	13,860/480 V
+2.5% tap	13,530/480 V
Nominal rating	13,200/480 V
−2.5% tap	12,870/480 V
−5.0% tap	12,540/480 V



Transformer Taps

- The taps on a transformer permit the transformer to be adjusted in the field to accommodate variations in local voltages.
- However, these taps normally cannot be changed while power is being applied to the transformer. They must be set once and left alone.
- Sometimes a transformer is used on a power line whose voltage varies widely with the load. Such voltage variations might be due to a high line impedance between the generators on the power system and that particular load (perhaps it is located far out in the country).

Transformer Taps

- A constant voltage need to be supplied to loads which are constantly changing.
- Hence, a special transformer called **a tap changing under load (TCUL) transformer** or **voltage regulator** is used.
- Basically, a **TCUL transformer** is a transformer with the ability to change taps while power is connected to it.
- A voltage regulator is a **TCUL transformer** with built-in voltage sensing circuitry that automatically changes taps to keep the system voltage constant. Such special transformers are very common in modem power systems.

The Autotransformer

- On some occasions it is desirable to change voltage levels by only a small amount. For example, it may be necessary to increase a voltage from 110 to 120 V or from 13.2 to 13.8 kV.
- These small rises may be made necessary by voltage drops that occur in power systems a long way from the generators.
- In such circumstances, it is wasteful and excessively expensive to wind a transformer with two full windings, each rated at about the same voltage. A special-purpose transformer, called an **autotransformer** is used instead.

The Autotransformer

- The principal disadvantage of autotransformers is that, unlike ordinary transformers, there is a direct physical connection between the primary and the secondary circuits, so the electrical isolation of the two sides is lost.
- If a particular application does not require electrical isolation, then the autotransformer is a convenient and inexpensive way to tie nearly equal voltages together.

The Autotransformer

$$\mathbf{V}_L = \mathbf{V}_C \quad (2\text{–}70)$$

$$\mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{SE} \quad (2\text{–}71)$$

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} \quad (2\text{–}72)$$

$$\mathbf{I}_H = \mathbf{I}_{SE} \quad (2\text{–}73)$$

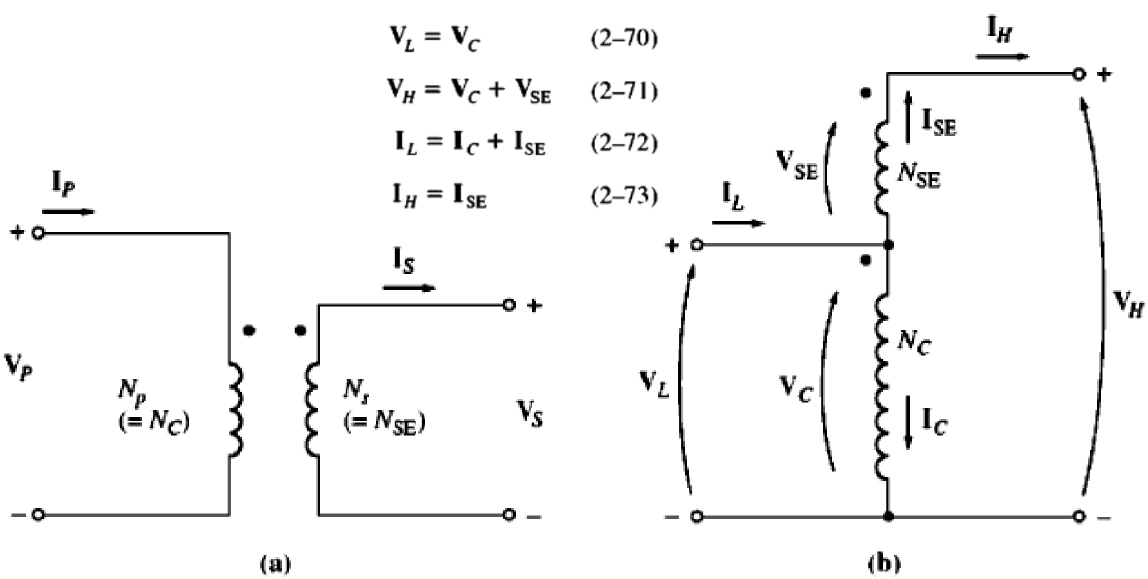
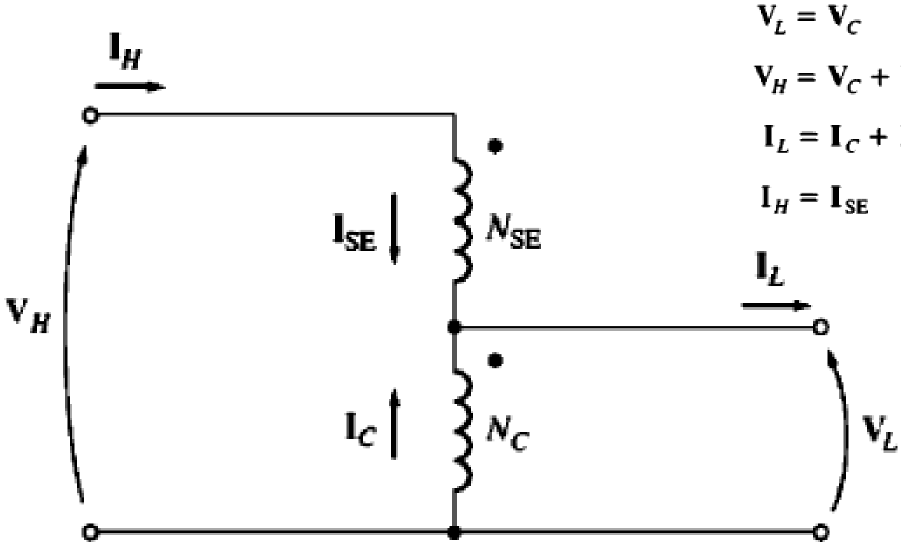


FIGURE 2–32
A transformer with its windings (a) connected in the conventional manner and (b) reconnected as an autotransformer.

The Autotransformer



$$V_L = V_C \quad (2\text{–}70)$$

$$V_H = V_C + V_{SE} \quad (2\text{–}71)$$

$$I_L = I_C + I_{SE} \quad (2\text{–}72)$$

$$I_H = I_{SE} \quad (2\text{–}73)$$

FIGURE 2–33
A step-down autotransformer connection.

The Autotransformer

- Because the transformer coils are physically connected, a different terminology is used for the autotransformer than for other types of transformers.
- The voltage on the common coil is called the **common voltage** V_C, and the current in that coil is called the **common current** I_C.
- The voltage on the series coil is called the **series voltage** V_{SE}, and the current in that coil is called the **series current** I_{SE}.

The Autotransformer

- The voltage and current on the low-voltage side of the transformer are called $\mathbf{V_L}$ and $\mathbf{I_L}$, respectively, while the corresponding quantities on the high-voltage side of the transformer are called $\mathbf{V_H}$ and $\mathbf{I_H}$.
- The primary side of the autotransformer (the side with power into it) can be either the high-voltage side or the low-voltage side, depending on whether the autotransformer is acting as a step-down or a step-up transformer.

The Autotransformer – Voltage Relationship

$$\mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{SE} \tag{2–71}$$

But $\mathbf{V}_C / \mathbf{V}_{SE} = N_C / N_{SE}$, so

$$\mathbf{V}_H = \mathbf{V}_C + \frac{N_{SE}}{N_C}\mathbf{V}_C \tag{2–74}$$

Finally, noting that $\mathbf{V}_L = V_C$, we get

$$\mathbf{V}_H = \mathbf{V}_L + \frac{N_{SE}}{N_C}\mathbf{V}_L$$

$$= \frac{N_{SE} + N_C}{N_C}\mathbf{V}_L \tag{2–75}$$

or

$$\boxed{\frac{\mathbf{V}_L}{\mathbf{V}_H} = \frac{N_C}{N_{SE} + N_C}} \tag{2–76}$$

The Autotransformer – Current Relationship

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} \quad (2\text{–}72)$$

From Equation (2–69), $\mathbf{I}_C = (N_{SE}/N_C)\mathbf{I}_{SE}$, so

$$\mathbf{I}_L = \frac{N_{SE}}{N_C}\mathbf{I}_{SE} + \mathbf{I}_{SE} \quad (2\text{–}77)$$

Finally, noting that $\mathbf{I}_H = \mathbf{I}_{SE}$, we find

$$\mathbf{I}_L = \frac{N_{SE}}{N_C}\mathbf{I}_H + \mathbf{I}_H$$

$$= \frac{N_{SE} + N_C}{N_C}\mathbf{I}_H \quad (2\text{–}78)$$

or

$$\boxed{\frac{\mathbf{I}_L}{\mathbf{I}_H} = \frac{N_{SE} + N_C}{N_C}} \quad (2\text{–}79)$$

The Autotransformer – Apparent Power Rating Advantage

- Not all the power traveling from the primary to the secondary in the autotransformer goes through the windings.
- As a result, if a conventional transformer is reconnected as an autotransformer, it can handle much more power than it was originally rated for.

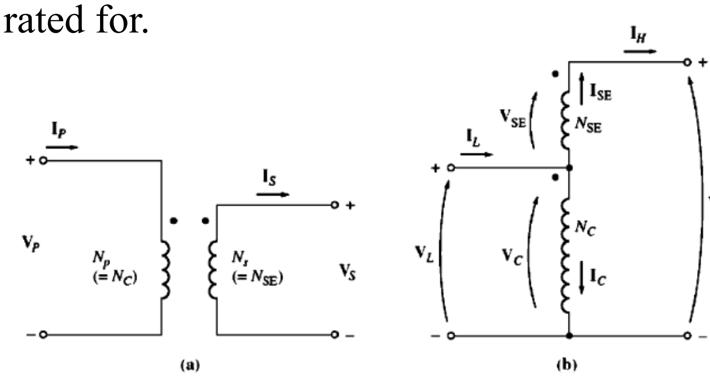


FIGURE 2–32
A transformer with its windings (a) connected in the conventional manner and (b) reconnected as an autotransformer.

The Autotransformer – Apparent Power Rating Advantage

- To understand this idea, let's look at the following relationships

$$S_{in} = V_L I_L \quad \text{and} \quad S_{out} = V_H I_H$$

$$S_{in} = S_{out} = S_{IO}$$

$$S_W = V_C I_C = V_{SE} I_{SE}$$

$$S_W = V_C I_C = V_L (I_L - I_H) = V_L I_L - V_L I_H$$

$$S_W = V_L I_L - V_L I_L \left(\frac{N_C}{N_{SE} + N_C}\right) = V_L I_L \left(1 - \frac{N_C}{N_{SE} + NC}\right)$$

$$S_W = V_L I_L \left(\frac{N_{SE} + NC - N_C}{N_{SE} + NC}\right) = S_{IO} \left(\frac{N_{SE}}{N_{SE} + NC}\right)$$

The Autotransformer – Apparent Power Rating Advantage

- Therefore, the ratio of the apparent power in the primary and secondary of the autotransformer to the apparent power actually traveling through its windings is

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} \qquad (2\text{–}86)$$

- This equation describes the apparent power rating advantage of an autotransformer over a conventional transformer. Here S_{IO} is the apparent power entering the primary and leaving the secondary of the transformer, while S_W is the apparent power actually traveling through the transformer's windings (the rest passes from primary to secondary without being coupled through the transformer's windings). Note that the smaller the series winding, the greater the advantage.

The Autotransformer – Apparent Power Rating Advantage

- For example, a 5000-kVA autotransformer connecting a 110-kV system to a 138-kV system would have an N_C/N_{SE} turns ratio of 110:28. This autotransformer would actually have windings rated at

$$S_W = S_{IO} \frac{N_{SE}}{N_{SE} + N_C} \qquad (2\text{–}85)$$

$$= (5000 \text{ kVA}) \frac{28}{28 + 110} = 1015 \text{ kVA}$$

- The autotransformer would have windings rated at only about 1015kVA, while a conventional transformer doing the same job would need windings rated at 5000kVA.
- The autotransformer could be 5 times smaller than the conventional transformer and also would be much less expensive.
- For this reason, it is very advantageous to build transformers between two nearly equal voltages as autotransformers.

The Autotransformer – Apparent Power Rating Advantage

Example 2.7 A 100-VA 120/12-V transformer is to be connected so as to form a step-up autotransformer. A primary voltage of 120 V is applied to the transformer.

(a) What is the secondary voltage of the transformer?

(b) What is its maximum voltampere rating in this mode of operation?

(e) Calculate the rating advantage of this autotransformer connection over the transformer's rating in conventional 120/12-V operation.

The Autotransformer –
Apparent Power Rating Advantage

To accomplish a step-up transformation with a 120-V primary, the ratio of the turns on the common winding N_C to the turns on the series winding N_{SE} in this transformer must be 120:12 (or 10:1).

(*a*) This transformer is being used as a step-up transformer. The secondary voltage is V_H, and from Equation (2–75),

$$\mathbf{V}_H = \frac{N_{SE} + N_C}{N_C}\mathbf{V}_L \qquad (2\text{–}75)$$

$$= \frac{12 + 120}{120} 120\text{ V} = 132\text{ V}$$

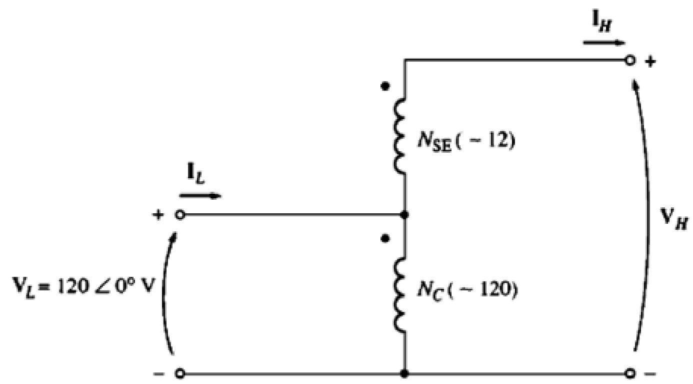


FIGURE 2–34
The autotransformer of Example 2–7.

The Autotransformer –
Apparent Power Rating Advantage

(*b*) The maximum voltampere rating in either winding of this transformer is 100 VA. How much input or output apparent power can this provide? To find out, examine the series winding. The voltage V_{SE} on the winding is 12 V, and the voltampere rating of the winding is 100 VA. Therefore, the *maximum* series winding current is

$$I_{SE,max} = \frac{S_{max}}{V_{SE}} = \frac{100\text{ VA}}{12\text{V}} = 8.33\text{ A}$$

Since I_{SE} is equal to the secondary current I_S (or I_H) and since the secondary voltage $V_S = V_H = 132$ V, the secondary apparent power is

$$S_{out} = V_S I_S = V_H I_H$$

$$= (132\text{ V})(8.33\text{ A}) = 1100\text{ VA} = S_{in}$$

(*c*) The rating advantage can be calculated from part (*b*) or separately from Equation (2–86). From part *b*,

$$\frac{S_{IO}}{S_W} = \frac{1100\text{ VA}}{100\text{ VA}} = 11$$

From Equation (2–86),

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} \qquad (2\text{–}86)$$

$$= \frac{12 + 120}{12} = \frac{132}{12} = 11$$

By either equation, the apparent power rating is increased by a factor of 11.

The Autotransformer – Apparent Power Rating Advantage

- It is not normally possible to just reconnect an ordinary transformer as an autotransformer and use it in the manner of Example 2-7, because the insulation on the low-voltage side of the ordinary transformer may not be strong enough to withstand the full output voltage of the autotransformer connection.
- In transformers built specifically as autotransformers, the insulation on the smaller coil (the series winding) is made just as strong as the insulation on the larger coil.

The Autotransformer – Apparent Power Rating Advantage

- It is common practice in power systems to use autotransformers whenever two voltages fairly close to each other in level need to be transformed, because the closer the two voltages are, the greater the autotransformer power advantage becomes.
- They are also used as variable transformers, where the low-voltage tap moves up and down the winding. This is a very convenient way to get a variable ac voltage.

The Autotransformer – Apparent Power Rating Advantage

Example 2.8 A transformer is rated at 1000kVA, 12/1.2kV, 60 Hz when it is operated as a conventional two-winding transformer. This transformer is to be used as a 13.2/12-kV step-down autotransformer in a power distribution system. In the autotransformer connection, what is the transformer's rating when used in this manner?

The N_C/N_{SE} turns ratio must be 12:1.2 or 10:1. The voltage rating of this transformer will be 13.2/12 kV, and the apparent power (voltampere) rating will be

$$S_{IO} = \frac{N_{SE} + N_C}{N_{SE}} S_W$$

$$= \frac{1 + 10}{1} 1000\ \text{kVA} = 11{,}000\ \text{kVA}$$

Three-Phase Transformers

- Almost all the major power generation and distribution systems in the world today are three-phase AC systems since they play a very important role in modern life.
- Transformers for three-phase circuits can be constructed in one of two ways:

1. One approach is simply to take three single-phase transformers and connect them in a three-phase bank.
2. An alternative approach is to make a three-phase transformer consisting of three sets of windings wrapped on a common core.

Three-Phase Transformers

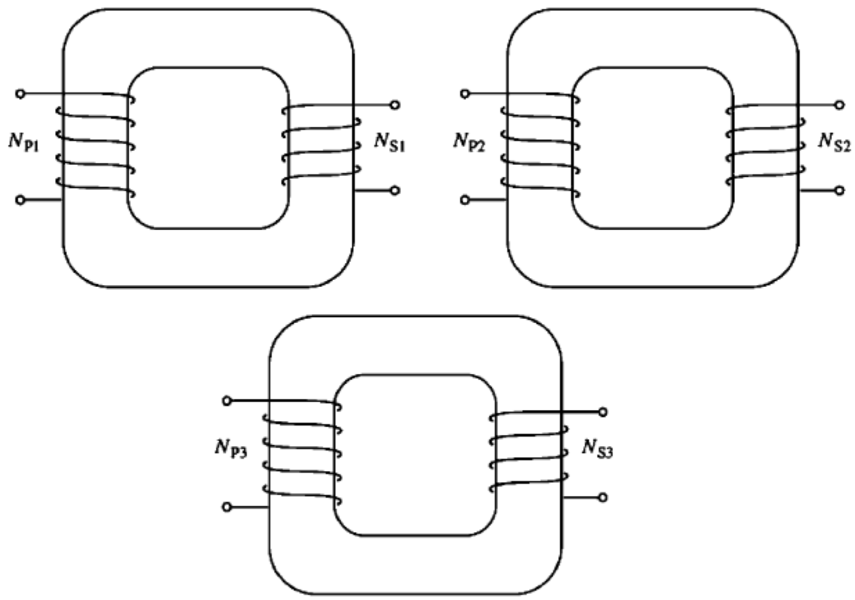


FIGURE 2–36
A three-phase transformer bank composed of independent transformers.

Three-Phase Transformers

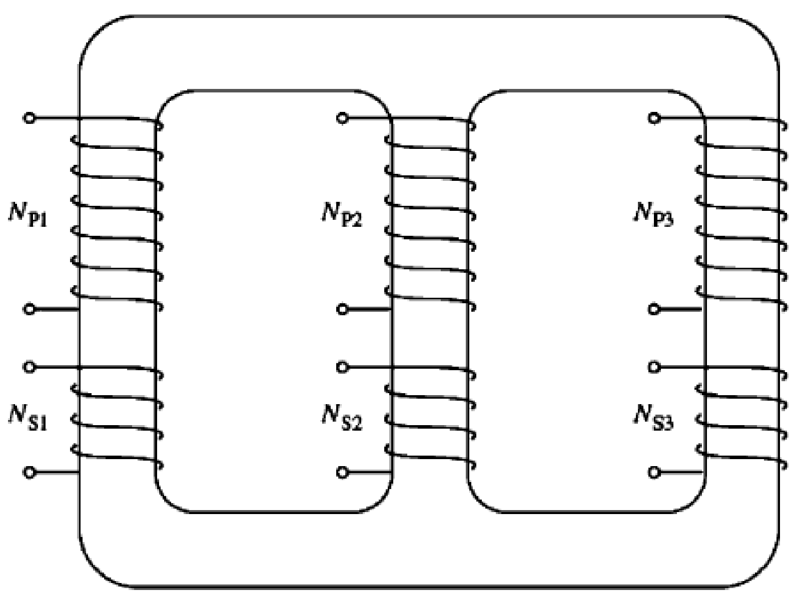


FIGURE 2–37
A three-phase transformer wound on a single three-legged core.

Three-Phase Transformers

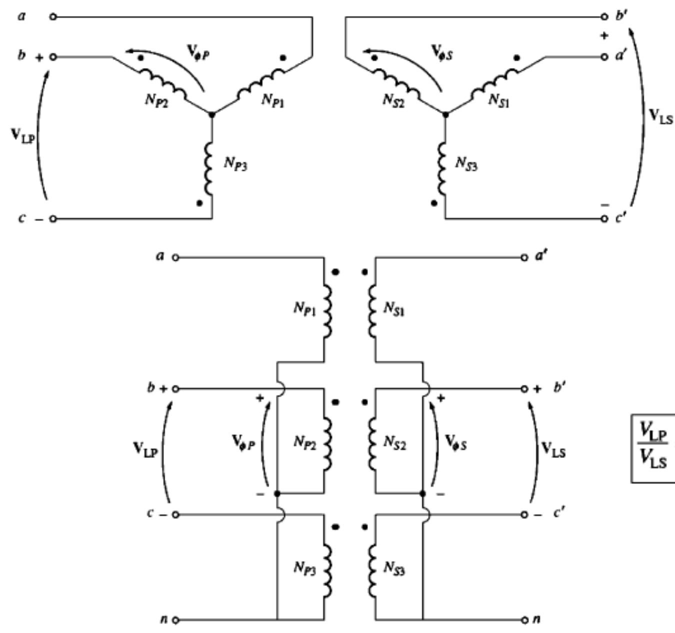
- The construction of a single three-phase transformer is the preferred practice today, since it is lighter, smaller, cheaper, and slightly more efficient.
- The older construction approach was to use three separate transformers. That approach had the advantage that each unit in the bank could be replaced individually in the event of trouble, but that does not outweigh the ad vantages of a combined three-phase unit for most applications.
- However, there are still a great many installations consisting of three single-phase units in service.

Three-Phase Transformers – Connection

- A three-phase transformer consists of three transformers, either separate or combined on one core.
- The primaries and secondaries of any three-phase transformer can be independently connected in either a wye (Y) or a delta (Δ).
- This gives a total of four possible connections for a three-phase transformer bank:

  (1) Wye-Wye (Y-Y)
  (2) Wye-Delta (Y-Δ)
  (3) Delta-Wye (Δ-Y)
  (4) Delta-Delta (Δ-Δ)

Three-Phase Transformers – Connection

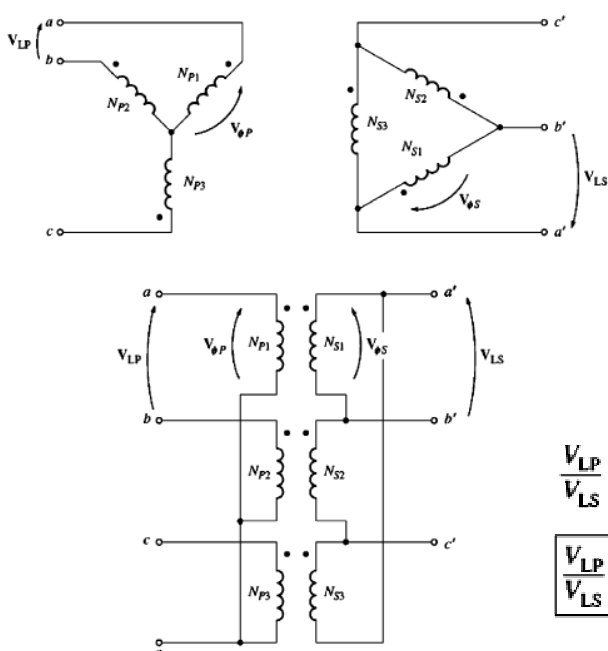


$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a \qquad \text{Y–Y} \qquad (2\text{–}87)$$

(a)

FIGURE 2–38
Three-phase transformer connections and wiring diagrams: (a) Y–Y

Three-Phase Transformers – Connection



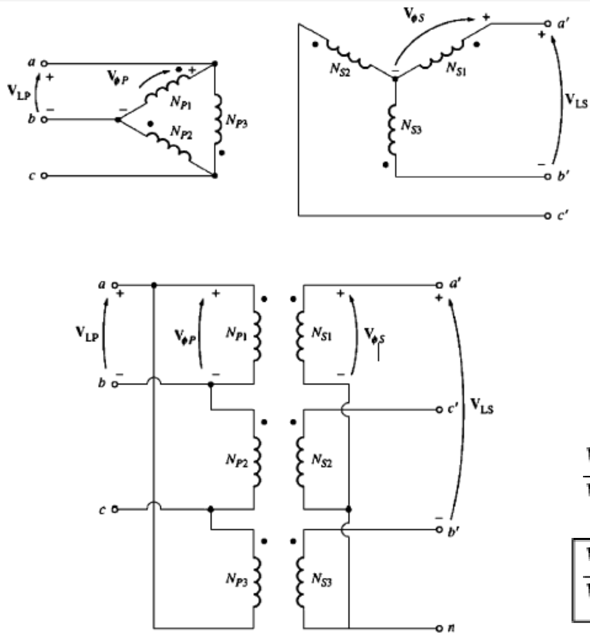
$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}}$$

$$\frac{V_{LP}}{V_{LS}} = \sqrt{3}a \qquad \text{Y–}\Delta \qquad (2\text{–}88)$$

(b)

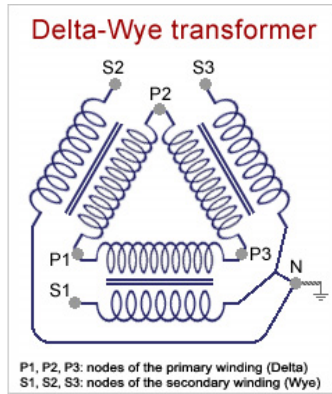
FIGURE 2–38
(b) Y–Δ *(continued)*

Three-Phase Transformers – Connection



(c)

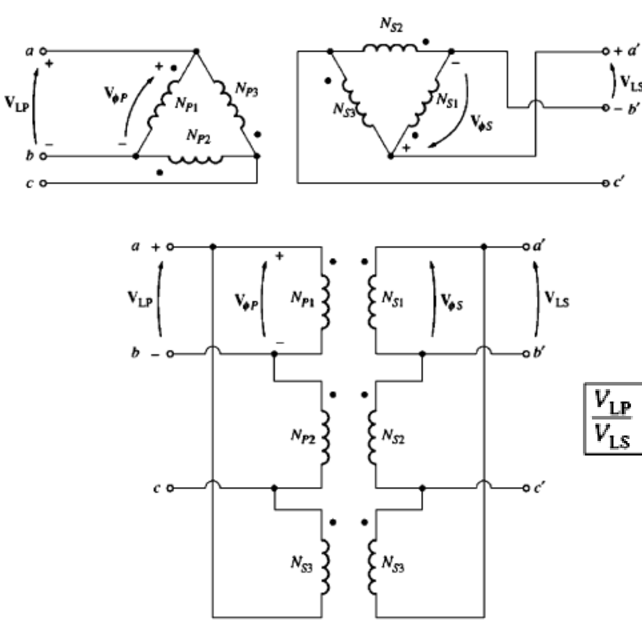
FIGURE 2–38
(c) Δ–Y *(continued)*



$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}}$$

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}}{a} \qquad \Delta-Y} \tag{2–89}$$

Three-Phase Transformers – Connection



(d)

FIGURE 2–38
(d) Δ–Δ *(concluded)*

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \qquad \Delta-\Delta} \tag{2–90}$$

Three-Phase Transformers – Connection

- The key to analyzing any three-phase transformer bank is to look at a single transformer in the bank.
- Any single transformer in the bank behaves exactly like the single-phase transformers already studied.
- The impedance, voltage regulation, efficiency, and similar calculations for three-phase transformers are done on a per-phase basis, using exactly the same techniques already developed for single-phase transformers.

Three-Phase Transformers – Connection

- The Y-Y connection has two very serious problems:

1. If loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.
2. Third-harmonic voltages can be large.

- In practice, very few Y-Y transformers are used, since the same jobs can be done by one of the other types of three-phase transformers.

Three-Phase Transformers – Connection

- The Y-Δ connection has no problem with third-harmonic components in its voltages, since they are consumed in a circulating current on the Δ side.
- This connection is also more stable with respect to unbalanced loads, since the Δ partially redistributes any imbalance that occurs.
- This arrangement does have one problem; the secondary voltage is shifted 30^{o} relative to the primary voltage of the transformer. The fact that a phase shift has occurred can cause problems in paralleling the secondaries of two transformer banks together.

Three-Phase Transformers – Connection

- The phase angles of transformer secondaries must be equal if they are to be paralleled, which means that attention must be paid to the direction of the 30^{o} phase shift occurring in each transformer bank to be paralleled together.
- This Δ-Y connection has the same advantages and the same phase shift as the Y-Δ transformer.
- This Δ-Δ connection has no phase shift associated with it and no problems with unbalanced loads or harmonics.

Instrument Transformers

- Two special -purpose transformers are used with power systems for taking measurements;

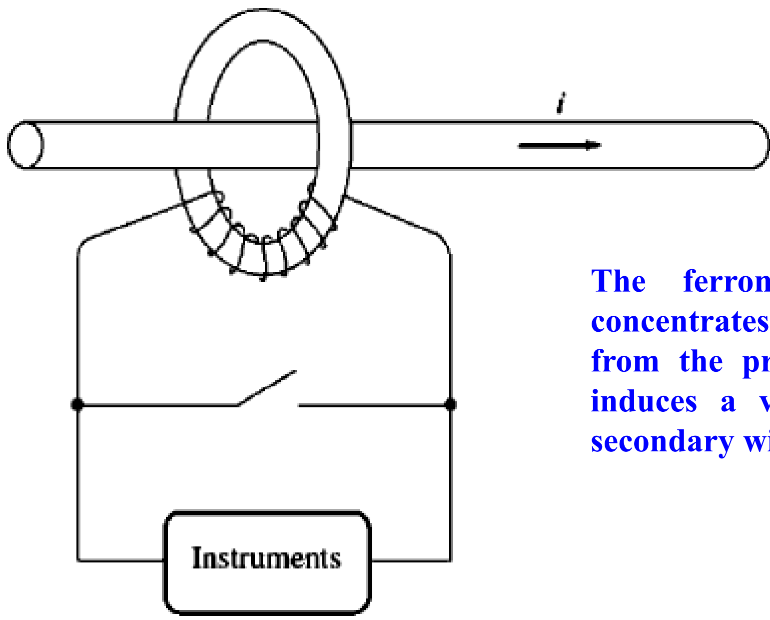
1. Potential transformer (PT)
2. Current transformer (CT)

- A **potential transformer** is a specially wound transformer with a high voltage primary and a low-voltage secondary.
- It has a very low power rating, and its sole purpose is to provide a sample of the power system's voltage to the instruments monitoring it.

Instrument Transformers

- Since the principal purpose of the transformer is voltage sampling, it must be very accurate so as not to distort the true voltage values too badly.
- **Current transformers** sample the current in a line and reduce it to a safe and measurable level.
- The **current transformer** consists of a secondary winding wrapped around a ferromagnetic ring, with the single primary line running through the center of the ring.

Instrument Transformers



The ferromagnetic ring holds and concentrates a small sample of the flux from the primary line. That flux then induces a voltage and current in the secondary winding.

FIGURE 2–50
Sketch of a current transformer.

Instrument Transformers

- A **current transformer** differs from the other transformers in that its windings are loosely coupled. Unlike all the other transformers, the mutual flux ϕ_M in the current transformer is smaller than the leakage flux ϕ_L Because of the loose coupling.
- Nevertheless, the secondary current in a **current transformer** is directly proportional to the much larger primary current, and the device can provide an accurate sample of a line's current for measurement purposes.

Instrument Transformers

- **Current transformer** ratings are given as ratios of primary to secondary current.
- A typical **current transformer** ratio might be 600:5, 800:5, or 1000:5.
- A 5-A rating is standard on the secondary of a **current transformer.**
- It is important to keep a **current transformer** short-circuited at all times, since extremely high voltages can appear across its open secondary terminals.

Instrument Transformers

- In fact, most relays and other devices using the current from a current transformer have a shorting interlock which must be shut before the relay can be removed for inspection or adjustment.
- Without this interlock, very dangerous high voltages will appear at the secondary terminals as the relay is removed from its socket.

Instrument Transformers

- **CT vs PT**
  https://www.youtube.com/watch?v=qynwO9ts-lY&t=28s

Induction Motors

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Introduction

- AC machines are generators that convert mechanical energy to AC electrical energy and motors that convert AC electrical energy to mechanical energy.
- There are two major classes of AC machines (1) synchronous machines and (2) induction machines.
- **Synchronous machines** are motors and generators whose magnetic field current is supplied by a separate DC power source, while **induction machines** are motors and generators whose field current is supplied by magnetic induction (transformer action) into their field windings.

Introduction

- The field circuits of most synchronous and induction machines are located on their rotors.
- Although it is possible to use an induction machine as either a motor or a generator, it has many disadvantages as a generator and so is rarely used in that manner.
- For this reason, **induction machines** are usually referred to as **induction motors**.

Types of Induction Motors

In squirrel cage induction motors:

1. The rotor is simplest and most rugged in construction.
2. Cylindrical laminated core rotor with heavy bars or copper or aluminum or alloys are used for conductors.
3. Rotor conductors or rotor bars are short circuited with end rings.
4. Rotor bars are permanently short circuited and hence it is not possible to connect external resistance in the circuit in series with the rotor conductors.

Types of Induction Motors

5. Cheaper cost.
6. No moving contacts in the rotor.
7. Higher efficiency.
8. Low starting torque. It is 1.5 time full load torque.
9. Speed control by rotor resistance is not possible.
10. Starting current is 5 to 7 times the full load.

Types of Induction Motors

In slip ring (wound rotor) Induction Motor :

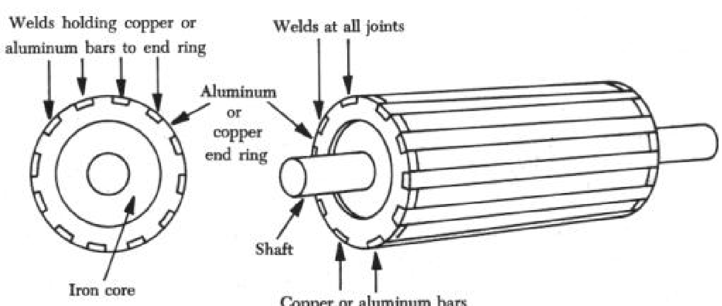
1. The rotor is wound type
2. Carbon brushes and slip rings are provided in the rotor circuit.
3. The rotor construction is not simple.
4. Cylindrical laminated core rotor is wound for as the number of poles of the stator.
5. At starting the 3-phase windings are connected to a star connected rheostat and during running condition, the windings is short circuited at the slip rings.

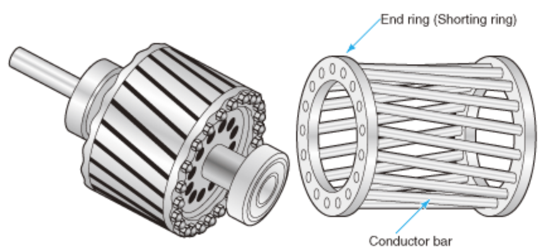
Types of Induction Motors

6. It is possible to insert additional resistance in the rotor circuit. Therefore it is possible to increase the torque (the additional series resistance is used for starting purposes).
7. Cost is slightly higher.
8. comparatively less efficiency.
9. High starting torque. It can be obtained by adding external resistance in the rotor circuit.
10. Speed control by rotor resistance is possible.
11. Less starting current.
12. Much more maintenance is required because of the wear associated with their brushes and slip rings.

Hence, wound-rotor induction motors are rarely used.

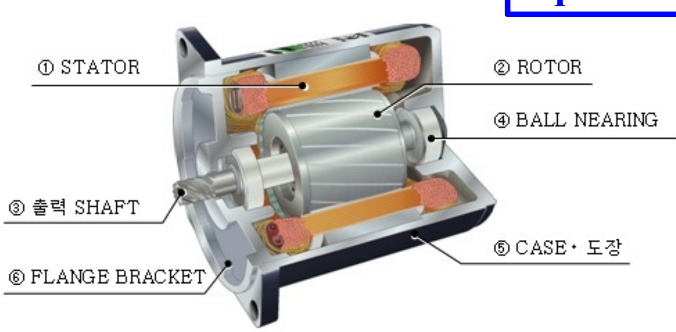
Types of Induction Motors

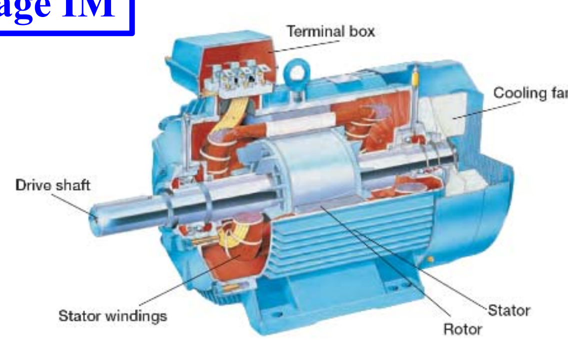




(a) Rotor (b) Squirrel-cage conductor

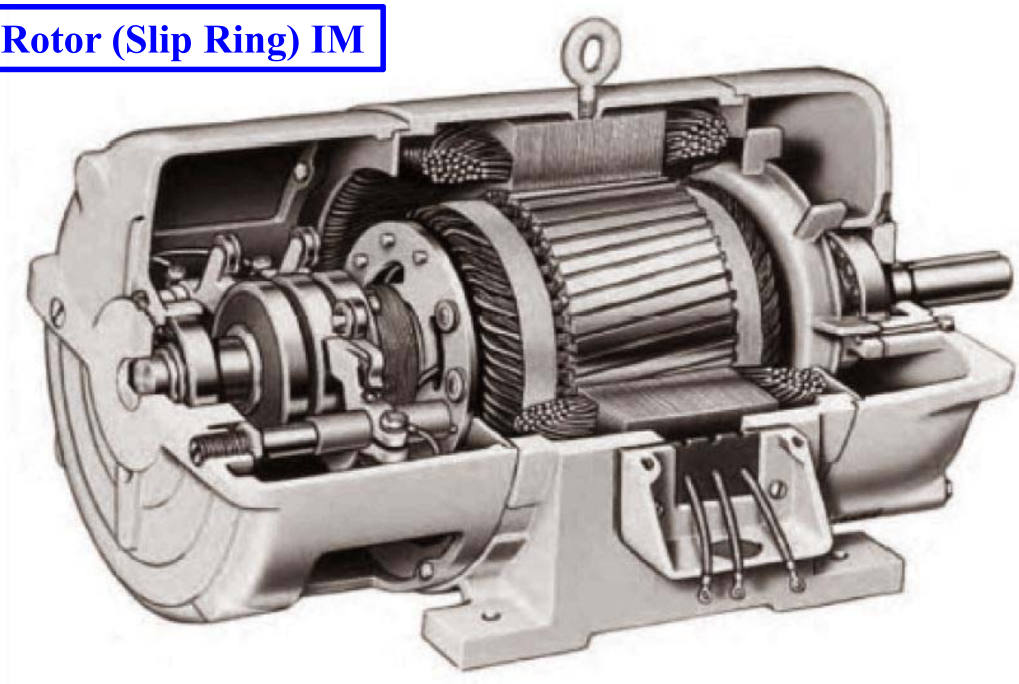
Squirrel Cage IM





Types of Induction Motors

Wound Rotor (Slip Ring) IM

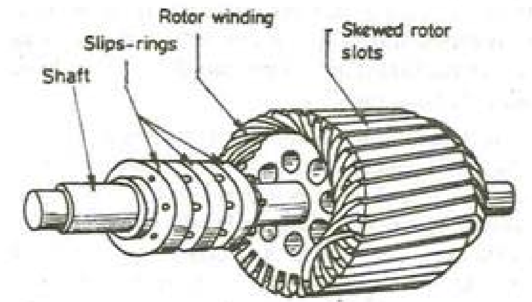


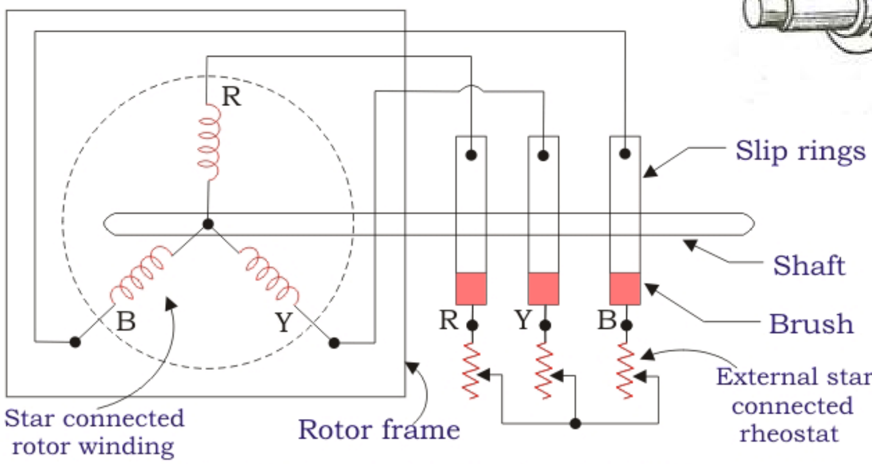
Types of Induction Motors

- Slip rings are metal rings completely encircling the shaft of a machine but insulated from it.
- Each end of the rotor winding is tied to each of the three slip rings on the shaft and a stationary brush rides on each slip ring.
- A "brush" is a block of graphitelike carbon compound that conducts electricity freely but has very low friction so that it doesn't wear down the slip ring.

Types of Induction Motors

Wound Rotor (Slip Ring) IM





Slip Ring Three Phase Induction Motor

Types of Induction Motors

Rotor conductors are skewed because of

(1) Primarily to prevent the cogging phenomenon. It is a phenomenon in which, if the rotor conductors are straight, there are chances of magnetic locking or strong coupling between rotor and stator.

(2) To avoid crawling phenomenon. It is a phenomenon where harmonic components introduces oscillations in torque.

(3) To increased rotor resistance due to comparatively lengthier rotor conductor bars.

(4) To improve the starting torque and starting power factor.

(5) Increasing effective magnetic coupling between Stator and Rotor fluxes

Winding Insulation

- One of the most critical parts of an ac machine design is the insulation of its windings.
- If the insulation of a motor or generator breaks down, the machine shorts out.
- The repair of a machine with shorted insulation is quite expensive, if it is even possible.
- To prevent the winding insulation from breaking down as a result of overheating, it is necessary to limit the temperature of the windings.

Winding Insulation

- This can be partially done by providing a cooling air circulation over them, but ultimately the maximum winding temperature limits the maximum power that can be supplied continuously by the machine.
- Insulation rarely fails from immediate breakdown at some critical temperature.
- Instead, the increase in temperature produces a gradual degradation of the insulation, making it subject to failure from another cause such as shock, vibration, or electrical stress.

Winding Insulation

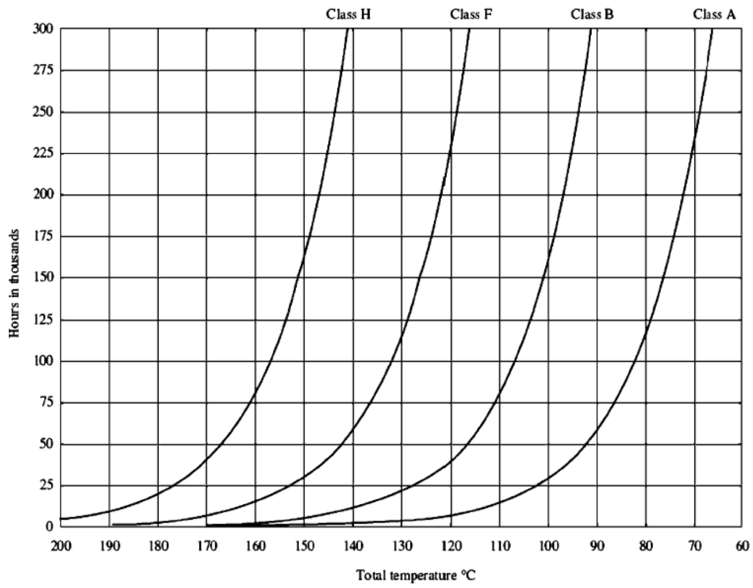
- There was an old rule of thumb that said that the life expectancy of a motor with a given type of insulation is halved for each 10 percent rise in temperature above the rated temperature of the winding. This rule still applies to some extent today.
- To standardize the temperature limits of machine insulation, the **National Electrical Manufacturers Association (NEMA)** in the United States has defined a series of insulation system classes.
- Similar standards have been defined by the **International Electrotechnical Commission (IEC)** and by various national standards organizations in other countries.

Winding Insulation

- Each insulation system class specifies the maximum temperature rise permissible for that class of insulation.
- There are **three common NEMA insulation classes** for integral-horsepower ac motors: **B**, **F**, and **H**.
- Each class represents a higher permissible winding temperature than the one before it. For example, the armature winding temperature rise above ambient temperature in one type of continuously operating ac induction motor must be limited to 80°C for class B, 105°C for class F, and 125°C for class H insulation.

Winding Insulation

- The effect of operating temperature on insulation life for a typical machine can be quite dramatic.



This curve shows the mean life of a machine in thousands of hours versus the temperature of the windings, for several different insulation classes.

FIGURE 4–20
Plot of mean insulation life versus winding temperature for various insulation classes. *(Courtesy of Marathon Electric Company.)*

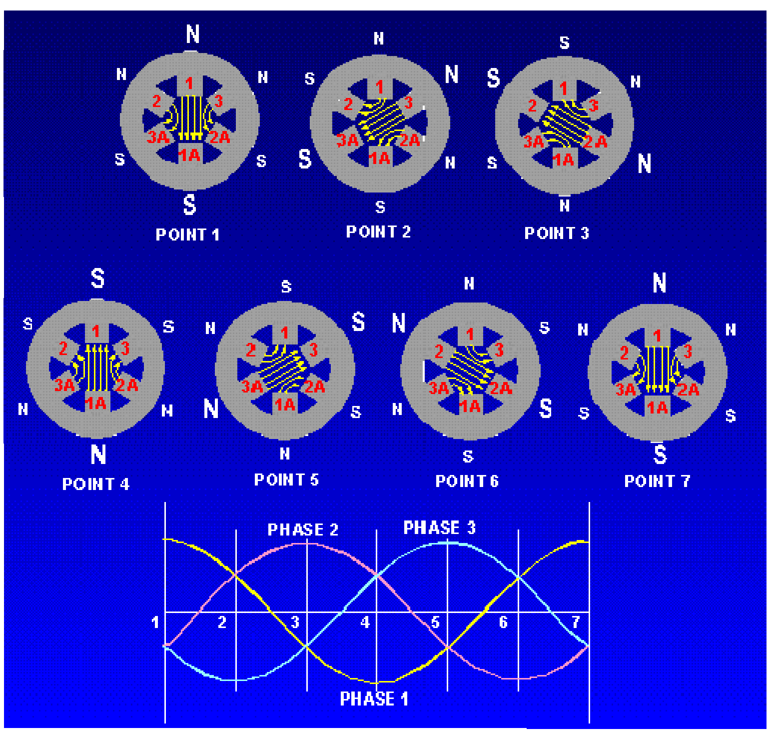
The Rotating Magnetic Field

- It is not possible to induce torque in the induction motor without having a rotating magnetic field.
- In this case, the induced torque in the rotor would cause it to constantly "chase" the stator magnetic field around in a circle.
- The fundamental principle of AC machine operation is that if a three-phase set of currents, each of equal magnitude and differing in phase by 120°, flows in a three-phase winding, then it will produce a rotating magnetic field of constant magnitude.

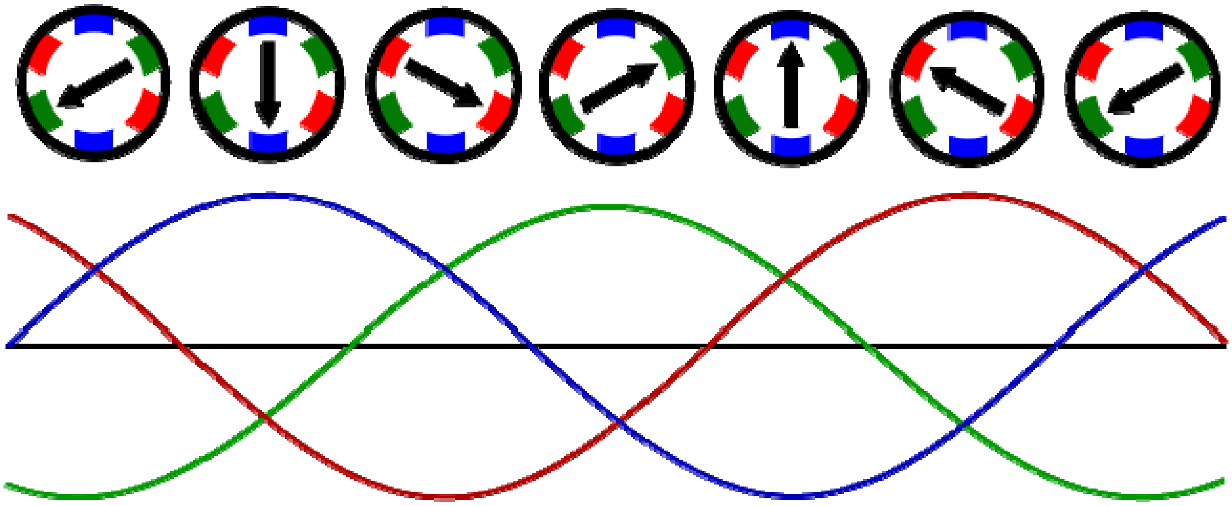
The Rotating Magnetic Field

- The three-phase winding consists of three separate windings spaced 120 electrical degrees apart around the surface of the machine.
- The rotating magnetic field concept is illustrated in the simplest case by an empty stator containing just three coils, each 120^o apart.
- Since such a winding produces only one north and one south magnetic pole, it is a two pole winding.

The Rotating Magnetic Field

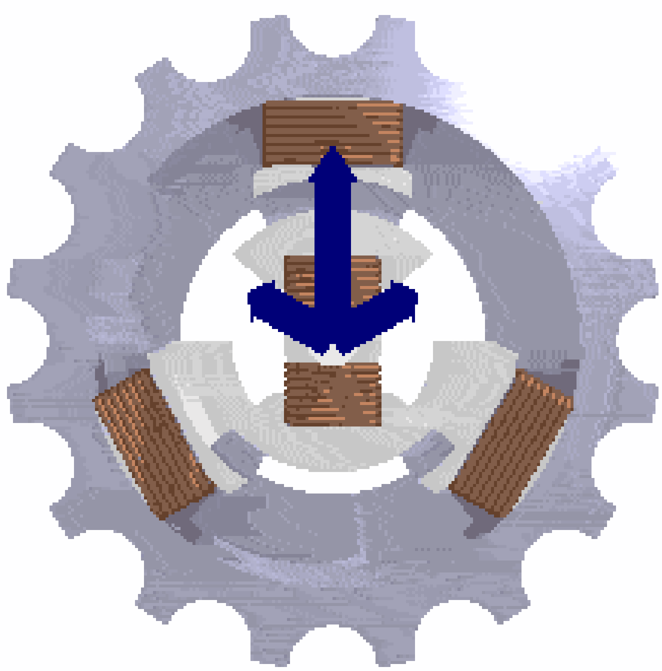


The Rotating Magnetic Field



Notice that although the direction of the magnetic field has changed, the magnitude is constant. The magnetic field is maintaining a constant magnitude while rotating in a counterclockwise direction.

The Rotating Magnetic Field



Basic Concepts of Operation

- When a three-phase set of voltages are applied to the stator, a three-phase set of stator currents flow.
- These currents produce a magnetic field $\mathbf{B_S}$, which is rotating as explained earlier.
- The speed of the magnetic field's rotation is given by

$$n_{sync} = \frac{120 f_e}{P} \qquad (7\text{–}1)$$

where $\mathbf{f_e}$ is the system frequency in hertz and **P** is the number of poles in the machine.

- This rotating magnetic field $\mathbf{B_S}$ passes over the rotor bars and induces a voltage in them.

Basic Concepts of Operation

- The voltage induced in a given rotor bar is given by the equation

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45)$$

where **V** is the velocity of the bar relative to the magnetic field, **B** is the magnetic flux density vector, and **l** is the length of conductor in the magnetic field

- It is the relative motion of the rotor compared to the stator magnetic field that produces induced voltage in a rotor bar.

Basic Concepts of Operation

- This results in a current flow. However, since the rotor assembly is inductive, the peak rotor current lags behind the peak rotor voltage.
- The rotor current flow produces a rotor magnetic field $\mathbf{B_R}$.
- Finally, the induced torque in the machine is given by

$$\tau_{ind} = k\mathbf{B}_R \times \mathbf{B}_S \qquad (4\text{–}58)$$

- Hence, the resulting torque causes the rotor to accelerate in the direction of rotating magnetic field.

Basic Concepts and Equations

- As it is clear that the torque induced in an induction motor depends on four factors:

(1) The strength of the rotor magnetic field

(2) The strength of the external magnetic field

(3) The sine of the angle between them

(4) A constant representing the construction of the machine (*i.e.*, geometry). Note that **k = K/μ** and **K** is a constant dependent on the construction of the machine.

- In fact, the parameter **k** will not be constant since **μ** varies with the amount of magnetic saturation in the machine.

Basic Concepts and Equations

- There is a finite upper limit to the motor's speed.
- If the induction motor's rotor were turning at synchronous speed, then the rotor bars would be stationary relative to the magnetic field and there would be no induced voltage.
- If $\mathbf{e_{ind}}$ was equal to zero, then there would be no rotor current and no rotor magnetic field.
- With no rotor magnetic field, the induced torque would be zero, and the rotor would slow down as a result of friction losses.
- An induction motor can thus speed up to near-synchronous speed, but it can never exactly reach synchronous speed.

Basic Concepts and Equations

- Note that in normal operation both the rotor and stator magnetic fields $\mathbf{B_R}$ and $\mathbf{B_S}$ rotate together at synchronous speed $\mathbf{n_{Sync}}$ while the rotor itself turns at a slower speed.
- Two terms are commonly used to define the relative motion of the rotor and the magnetic fields

(1) One is slip speed, defined as the difference between synchronous speed and rotor speed:

$$n_{\text{slip}} = n_{\text{sync}} - n_m \qquad (7\text{–}2)$$

where $\mathbf{n_{Slip}}$ is the slip speed of the machine, $\mathbf{n_{Sync}}$ is the speed of the rotating magnetic fields, and $\mathbf{n_m}$ is the mechanical shaft speed of motor

Basic Concepts and Equations

(2) The other is term used to describe the relative motion is **slip**, which is the relative speed expressed on a percentage basis, and it is defined as

$$s = \frac{n_{slip}}{n_{sync}}(\times 100\%) \quad (7\text{–}3)$$

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}}(\times 100\%)} \quad (7\text{–}4)$$

$$\boxed{s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}}(\times 100\%)} \quad (7\text{–}5)$$

- Notice that if the rotor turns at synchronous speed, s = 0, while if the rotor is stationary, s = 1.
- All normal motor speeds fall somewhere between those two limits.

Basic Concepts and Equations

- It is possible to express the mechanical speed of the rotor shaft in terms of synchronous speed and slip.
- Solving Equations (7-4) and (7-5) for mechanical speed yields

$$\boxed{n_m = (1 - s)n_{sync}} \quad (7\text{–}6)$$

$$\boxed{\omega_m = (1 - s)\omega_{sync}} \quad (7\text{–}7)$$

- An induction motor works by inducing voltages and currents in the rotor of the machine, and for that reason it has sometimes been called a **rotating transformer**.

Basic Concepts and Equations

- Like a transformer, the primary (stator) induces a voltage in the secondary (rotor), but unlike a transformer, the secondary frequency is not necessarily the same as the primary frequency.
- If the rotor of a motor is locked so that it cannot move, then the rotor will have the same frequency as the stator.
- On the other hand, if the rotor turns at synchronous speed, the frequency on the rotor will be zero.

Basic Concepts and Equations

- At $n_m = 0$ r/min, the rotor frequency $f_r = f_e$, and the slip $s = 1$. At $n_m = n_{Sync}$ the rotor frequency $f_r = 0$ Hz, and the slip $s = 0$.
- For any speed in between, the rotor frequency is directly proportional to the difference between the speed of the magnetic field n_{Sync} and the speed of the rotor n_m.
- Based on the definition of the slip, the rotor frequency can be expressed as

$$\boxed{f_r = sf_e} \qquad (7\text{–}8)$$

Basic Concepts and Equations

- Several alternative forms of this expression exist that are sometimes useful. One of the more common expressions is derived by substituting Equation (7-4) for the slip into Equation (7-8) and then substituting for n_{Sync} in the denominator of the expression:

$$f_r = \frac{n_{sync} - n_m}{n_{sync}} f_e$$

- But $n_{Sync} = 120.f_e / P$ [from Equation (7- 1)], so

$$f_r = (n_{sync} - n_m)\frac{P}{120 f_e} f_e$$

$$\boxed{f_r = \frac{P}{120}(n_{sync} - n_m)} \qquad (7\text{–}9)$$

Basic Concepts and Equations

YouTube

- **How Does an Induction Motor Work ?**
  https://www.youtube.com/watch?v=AQqyGNOP_3o&t=28s

Basic Concepts and Equations

- **Slip ring Induction Motor, How it works ?**
  https://www.youtube.com/watch?v=JPn5Ou-N0b0&t=308s

Basic Concepts and Equations

Example 7.1 208-V, 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent.

(a) What is the synchronous speed of this motor?

(b) What is the rotor speed of this motor at the rated load?

(c) What is the rotor frequency of this motor at the rated load?

(d) What is the shaft torque of this motor at the rated load?

Basic Concepts and Equations

(*a*) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P} \qquad (7\text{–}1)$$

$$= \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

(*b*) The rotor speed of the motor is given by

$$n_m = (1 - s)n_{\text{sync}} \qquad (7\text{–}6)$$

$$= (1 - 0.05)(1800\text{ r/min}) = 1710\text{ r/min}$$

(*c*) The rotor frequency of this motor is given by

$$f_r = sf_e = (0.05)(60\text{ Hz}) = 3\text{ Hz} \qquad (7\text{–}8)$$

Alternatively, the frequency can be found from Equation (7–9):

$$f_r = \frac{P}{120}(n_{\text{sync}} - n_{\text{m}}) \qquad (7\text{–}9)$$

$$= \frac{4}{120}(1800\text{ r/min} - 1710\text{ r/min}) = 3\text{ Hz}$$

Basic Concepts and Equations

(*d*) The shaft load torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{(10\text{ hp})(746\text{ W/hp})}{(1710\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})} = 41.7\text{ N}\bullet\text{m}$$

The shaft load torque in English units is given by Equation (1–17):

$$\tau_{\text{load}} = \frac{5252P}{n}$$

where τ is in pound-feet, P is in horsepower, and n_m is in revolutions per minute. Therefore,

$$\tau_{\text{load}} = \frac{5252(10\text{ hp})}{1710\text{ r/min}} = 30.7\text{ lb}\bullet\text{ft}$$

Equivalent Circuit

- Because the induction of voltages and currents in the rotor circuit of an induction motor is essentially a transformer operation, the equivalent circuit of an induction motor will turn out to be very similar to the equivalent circuit of a transformer.
- An induction motor is called a **singly excited machine** (as opposed to a doubly excited synchronous machine), since power is supplied to only the stator circuit.
- It is possible to derive the equivalent circuit of an induction motor from a knowledge of transformers and from the variation of rotor frequency with speed in induction motors.

Equivalent Circuit

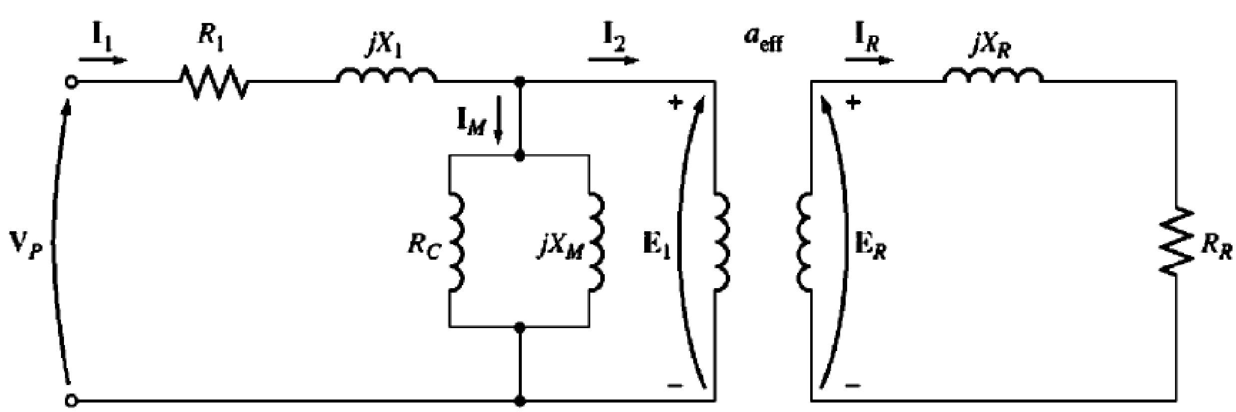


FIGURE 7–7
The transformer model or an induction motor, with rotor and stator connected by an ideal transformer of turns ratio a_{eff}.

Equivalent Circuit

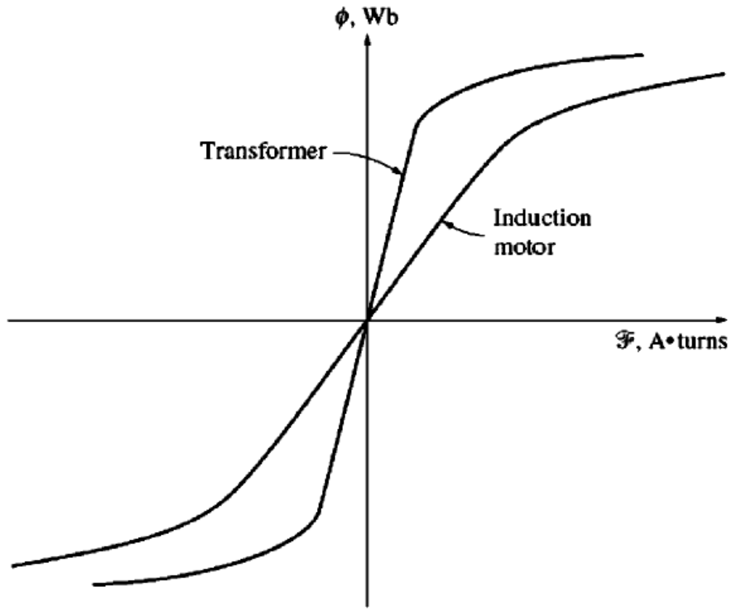


FIGURE 7–8
The magnetization curve of an induction motor compared to that of a transformer.

Equivalent Circuit

- Notice that the slope of the induction motor's magnetomotive force-flux curve is much shallower than the curve of a good transformer.
- This is because there must be an air gap in an induction motor, which greatly increases the reluctance of the flux path and therefore reduces the coupling between primary and secondary windings.
- The higher reluctance caused by the air gap means that a higher magnetizing current is required to obtain a given flux level.

Equivalent Circuit

- Therefore, the magnetizing reactance X_M in the equivalent circuit will have a much smaller value (or the susceptance B_M will have a much larger value) than it would in an ordinary transformer.
- The primary internal stator voltage E_1 is coupled to the secondary E_R by an ideal transformer with an effective turns ratio a_{eff}
- The effective turns ratio a_{eff} is fairly easy to determine for a wound-rotor motor. It is basically the ratio of the conductors per phase on the stator to the conductors per phase on the rotor, modified by any pitch and distribution factor differences.

Equivalent Circuit

- It is rather difficult to see a_{eff} clearly in the cage of a case rotor motor because there are no distinct windings on the cage rotor. In either case, there is an effective turns ratio for the motor.
- The voltage E_R produced in the rotor, in turn, produces a current flow in the shorted rotor (or secondary) circuit of the machine.
- The primary impedances and the magnetization current of the induction motor are very similar to the corresponding components in a transformer equivalent circuit.

Equivalent Circuit

- An induction motor equivalent circuit differs from a transformer equivalent circuit primarily in the effects of varying rotor frequency on the rotor voltage $\mathbf{E_R}$ and the rotor impedances $\mathbf{R_R}$ and $\mathbf{jX_R}$
- In general, the greater the relative motion between the rotor and the stator magnetic fields, the greater the resulting rotor voltage and rotor frequency.
- The largest relative motion occurs when the rotor is stationary, called the **locked-rotor** or **blocked-rotor** condition, so the largest voltage and rotor frequency arc induced in the rotor at that condition.

Equivalent Circuit

- The smallest voltage (0 V) and frequency (0 Hz) occur when the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion.
- The magnitude and frequency of the voltage induced in the rotor at any speed between these extremes is directly proportional to the slip of the rotor.
- Therefore, if the magnitude of the induced rotor voltage at locked-rotor conditions is called $\mathbf{E_{RO}}$, the
- magnitude of the induced voltage at any slip will be given by the equation

$$E_R = sE_{R0} \tag{7–10}$$

Equivalent Circuit

- The frequency of the induced voltage at any slip will be given by the equation

$$f_r = sf_e \qquad (7\text{–}8)$$

- This voltage is induced in a rotor containing both resistance and reactance.
- The rotor resistance R_R is a constant (except for the skin effect), independent of slip, while the rotor reactance is affected in a more complicated way by slip.

Equivalent Circuit

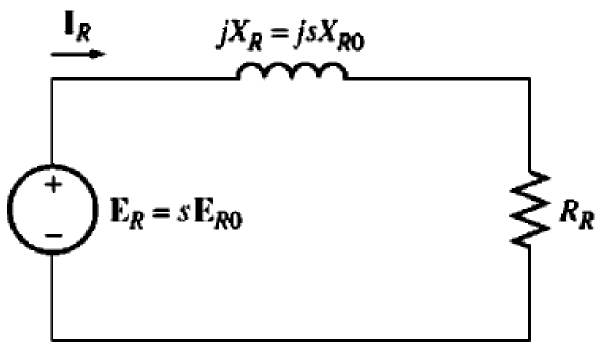
- The reactance of an induction motor rotor depends on the inductance of the rotor and the frequency of the voltage and current in the rotor. With a rotor inductance of L_R, the rotor reactance is given by

$$X_R = \omega_r L_R = 2\pi f_r L_R$$

$$\begin{aligned} X_R &= 2\pi s f_e L_R \\ &= s(2\pi f_e L_R) \\ &= sX_{R0} \qquad (7\text{–}11) \end{aligned}$$

where X_{RO} is the blocked-rotor rotor reactance.

Equivalent Circuit



$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jX_R}$$

$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jsX_{R0}} \qquad (7\text{–}12)$$

$$\mathbf{I}_R = \frac{\mathbf{E}_{R0}}{R_R/s + jX_{R0}} \qquad (7\text{–}13)$$

FIGURE 7–9
The rotor circuit model of an induction motor.

Notice from Equation (7-1 3) that it is possible to treat all of the rotor effects due to varying rotor speed as being caused by a varying impedance supplied with power from a constant-voltage source ERO

Equivalent Circuit

$$Z_{R,\text{eq}} = R_R/s + jX_{R0}$$

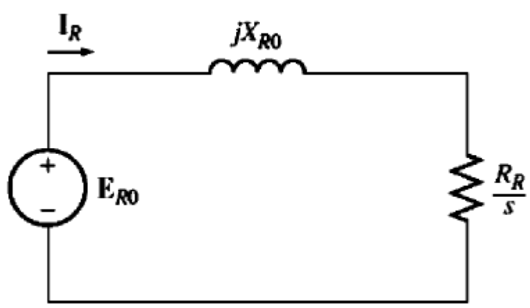


FIGURE 7–10
The rotor circuit model with all the frequency (slip) effects concentrated in resistor R_R.

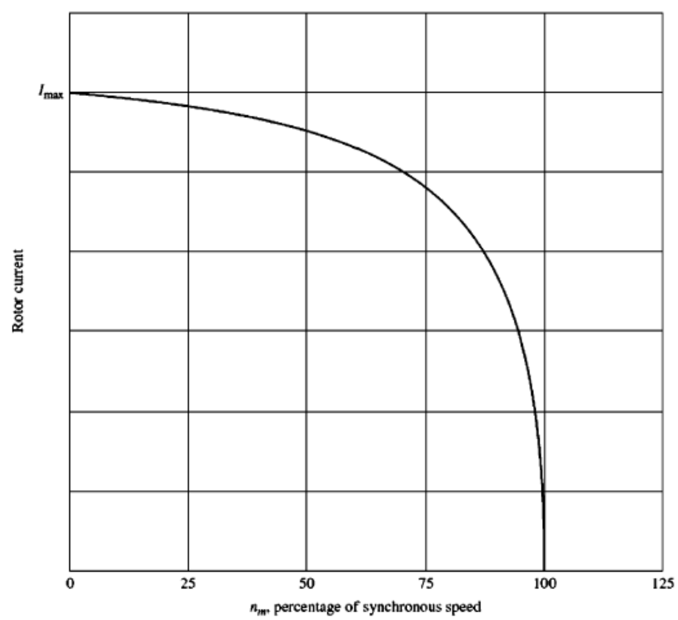


FIGURE 7–11
Rotor current as a function of rotor speed.

Equivalent Circuit

- Note that the rotor voltage is constant $\mathbf{E_{RO}}$ and the rotor impedance $\mathbf{Z_{R,eq}}$ contains all the effects of varying rotor slip.
- Notice that at very low slips the resistive term $\mathbf{R_R/s >> X_{RO}}$, so the rotor resistance predominates and the rotor current varies linearly with slip.
- At high slips, $\mathbf{X_{RO}}$ is much larger than $\mathbf{R_R/s}$, and the rotor current approaches a steady-state value as the slip becomes very large.

Equivalent Circuit

- In an ordinary transformer, the voltages, currents, and impedances on the secondary side of the device can be referred to the primary side by means of the turns ratio of the transformer:

$$\mathbf{V}_P = \mathbf{V}'_S = a\mathbf{V}_S \tag{7–15}$$

$$\mathbf{I}_P = \mathbf{I}'_S = \frac{\mathbf{I}_S}{a} \tag{7–16}$$

$$Z'_S = a^2 Z_S \tag{7–17}$$

Equivalent Circuit

- Exactly the same sort of transformation can be done for the induction motor's rotor circuit. If the effective turns ratio of an induction motor is **a_{eff}** , then the transformed rotor voltage becomes

$$\mathbf{E}_1 = \mathbf{E}'_R = a_{\text{eff}}\mathbf{E}_{R0} \quad (7\text{–}18)$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_R}{a_{\text{eff}}} \quad (7\text{–}19)$$

$$Z_2 = a_{\text{eff}}^2\left(\frac{R_R}{s} + jX_{R0}\right) \quad (7\text{–}20)$$

$$R_2 = a_{\text{eff}}^2 R_R \quad (7\text{–}21)$$

$$X_2 = a_{\text{eff}}^2 X_{R0} \quad (7\text{–}22)$$

Equivalent Circuit

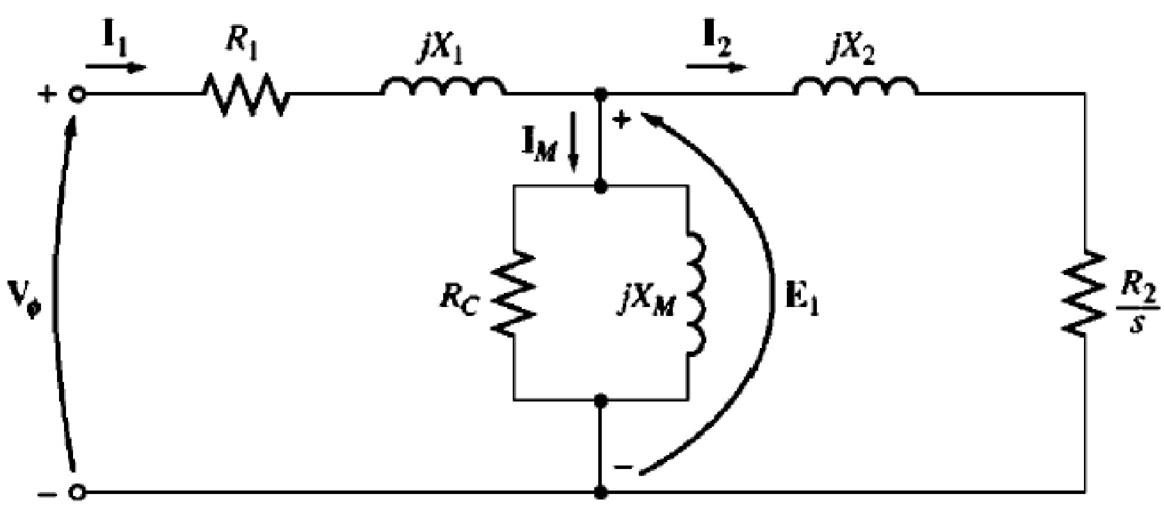


FIGURE 7–12
The per-phase equivalent circuit of an induction motor.

Equivalent Circuit

- The rotor resistance R_R and the locked-rotor rotor reactance X_{RO} are very difficult or impossible to determine directly on cage rotors, and the effective turns ratio a_{eff} is also difficult to obtain for cage rotors.
- Fortunately, though, it is possible to make measurements that will directly give the referred resistance and reactance R_2 and X_2, even though R_R, X_{RO} and a_{eff} are not known separately.

Power Flow and Losses

- The efficiency of an AC machine is defined by the equation

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \qquad (4\text{–}62)$$

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\% \qquad (4\text{–}63)$$

- The Losses that occur in AC machines can be divided into four basic categories:

1. Electrical or copper losses (I^2R losses)
2. Core losses
3. Mechanical losses
4. Stray load losses

Power Flow and Losses

- **Electrical or Copper Losses** are the resistive heating losses that occur in the stator (armature) and rotor (field) windings of the machine.
- **Core Losses** are the hysteresis losses and eddy current losses occurring in the metal of the motor. These losses vary as the square of the flux density **(B^2)** and, for the stator, as the 1.5th power of the speed of rotation of the magnetic fields **($n^{1.5}$)** .
- **Mechanical Losses** are the losses associated with mechanical effects (friction and windage). These losses vary as the cube of the speed of rotation of the machine.

Power Flow and Losses

- The mechanical and core losses of a machine are often lumped together and called the **no-load rotational loss** of the machine. At no load, all the input power must be used to overcome these losses. Therefore, measuring the input power to the stat or of an ac machine acting as a motor at no load will give an approximate value for these losses.
- **Stray (Miscellaneous) Losses** are losses that cannot be placed in one of the previous categories. No matter how carefully losses are accounted for, some always escape inclusion in one of the above categories. All such losses are lumped into stray losses. For most machines, stray losses are taken by convention to be I percent of full load.

Power Flow and Losses

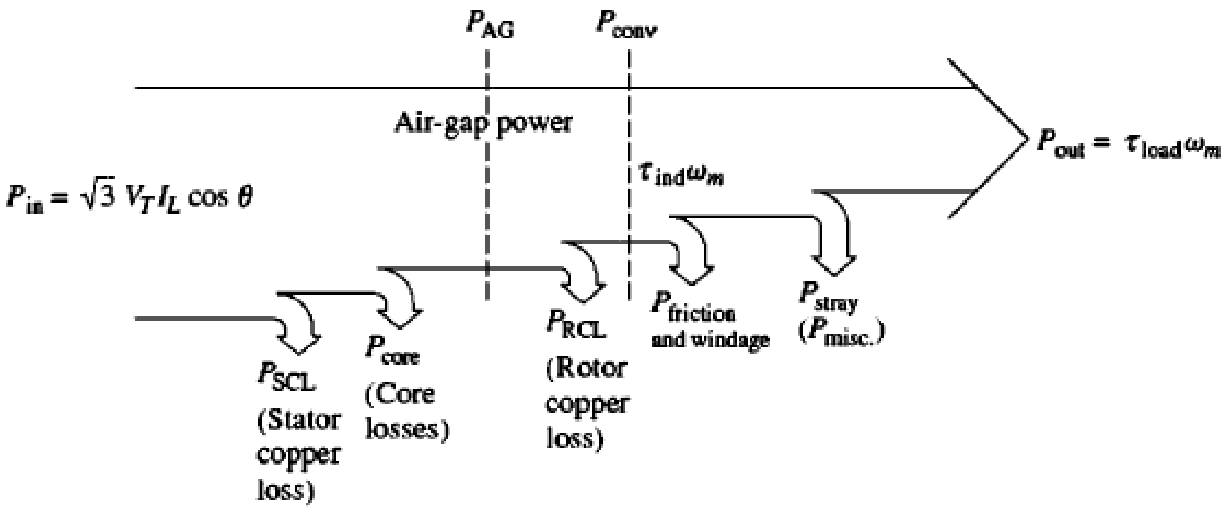


FIGURE 7–13
The power-flow diagram of an induction motor.

Power Flow and Losses

- The core losses of an induction motor come partially from the stator circuit and partially from the rotor circuit.
- Since an induction motor normally operates at a speed near synchronous speed, the relative motion of the magnetic fields over the rotor surface is quite slow, and the rotor core losses are very tiny compared to the stator core losses.
- Since the largest fraction of the core losses comes from the stator circuit, all the core losses are lumped together at that point on the diagram.
- These losses are represented in the induction motor equivalent circuit by the resistor **R_C**

Power Flow and Losses

- The higher the speed of an induction motor, the higher its friction, windage, and stray losses.
- On the other hand, the higher the speed of the motor (up to n_{Sync}), the lower its core losses.
- The total rotational losses of a motor are often considered to be constant with changing speed, since the component losses change in opposite directions with a change in speed.

Power Flow and Losses

Example 7.2 A 480-V, 60-Hz, 50-hp, three-phase induction motor is drawing 60A at 0.85 PF lagging. The stator copper losses are 2kW, and the rotor copper losses are 700W. The friction and windage losses are 600W, the core losses are 1800W, and the stray losses are negligible. Find the following quantities:

(a) The air-gap power P_{AG}

(b) The power converted P_{conv}

(c) The output power P_{out}

(d) The efficiency of the motor

Power Flow and Losses

To answer these questions, refer to the power-flow diagram for an induction motor (Figure 7–13).

(*a*) The air-gap power is just the input power minus the stator I^2R losses. The input power is given by

$$P_{\text{in}} = \sqrt{3} V_T I_L \cos\theta$$
$$= \sqrt{3}(480\text{ V})(60\text{ A})(0.85) = 42.4\text{ kW}$$

From the power-flow diagram, the air-gap power is given by

$$P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} - P_{\text{core}}$$
$$= 42.4\text{ kW} - 2\text{ kW} - 1.8\text{ kW} = 38.6\text{ kW}$$

(*b*) From the power-flow diagram, the power converted from electrical to mechanical form is

$$P_{\text{conv}} = P_{\text{AG}} - P_{\text{RCL}}$$
$$= 38.6\text{ kW} - 700\text{ W} = 37.9\text{ kW}$$

Power Flow and Losses

(*c*) From the power-flow diagram, the output power is given by

$$P_{\text{out}} = P_{\text{conv}} - P_{\text{F\&W}} - P_{\text{misc}}$$
$$= 37.9\text{ kW} - 600\text{ W} - 0\text{ W} = 37.3\text{ kW}$$

or, in horsepower,

$$P_{\text{out}} = (37.3\text{ kW})\frac{1\text{ hp}}{0.746\text{ kW}} = 50\text{ hp}$$

(*d*) Therefore, the induction motor's efficiency is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$
$$= \frac{37.3\text{ kW}}{42.4\text{ kW}} \times 100\% = 88\%$$

Power Flow and Losses

- From the equivalent circuit of an induction motor and power flow diagram, the following relations can be obtained

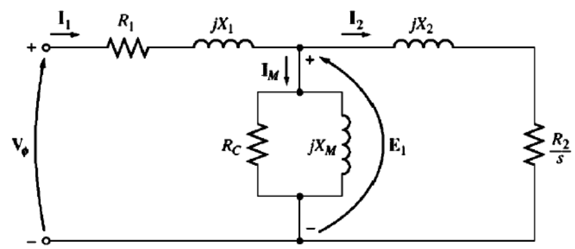


FIGURE 7–12
The per-phase equivalent circuit of an induction motor.

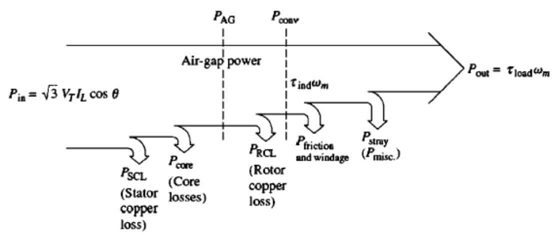


FIGURE 7–13
The power-flow diagram of an induction motor.

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{eq}} \qquad (7\text{–}23)$$

where

$$Z_{eq} = R_1 + jX_1 + \frac{1}{G_C - jB_M + \dfrac{1}{V_2/s + jX_2}} \qquad (7\text{–}24)$$

Power Flow and Losses

$$P_{SCL} = 3I_1^2R_1 \qquad (7\text{–}25)$$

$$P_{core} = 3E_1^2G_C \qquad (7\text{–}26)$$

$$P_{AG} = P_{in} - P_{SCL} - P_{core} \qquad (7\text{–}27)$$

$$P_{AG} = 3I_2^2\frac{R_2}{s} \qquad (7\text{–}28)$$

$$P_{RCL} = 3I_R^2R_R \qquad (7\text{–}29)$$

$$P_{RCL} = 3I_2^2R_2 \qquad (7\text{–}30)$$

Power Flow and Losses

Developed Mechanical Power ⇒

$$P_{\text{conv}} = P_{\text{AG}} - P_{\text{RCL}}$$

$$= 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2$$

$$= 3I_2^2 R_2\left(\frac{1}{s} - 1\right)$$

$$\boxed{P_{\text{conv}} = 3I_2^2 R_2\left(\frac{1-s}{s}\right)} \qquad (7\text{–}31)$$

Note that the lower the slip of the motor, the lower the rotor losses in the machine. Note also that if the rotor is not turning, the slip S = 1 and the air-gap power is entirely consumed in the rotor.

$$P_{\text{RCL}} = sP_{\text{AG}} \qquad (7\text{–}32)$$

$$P_{\text{conv}} = P_{\text{AG}} - P_{\text{RCL}}$$

$$= P_{\text{AG}} - sP_{\text{AG}}$$

$$\boxed{P_{\text{conv}} = (1-s)P_{\text{AG}}} \qquad (7\text{–}33)$$

Power Flow and Losses

$$\boxed{P_{\text{out}} = P_{\text{conv}} - P_{\text{F\&W}} - P_{\text{misc}}} \qquad (7\text{–}34)$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} \qquad (7\text{–}35)$$

$$\tau_{\text{ind}} = \frac{(1-s)P_{\text{AG}}}{(1-s)\omega_{\text{sync}}}$$

Developed Torque

$$\boxed{\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}} \qquad (7\text{–}36)$$

Power Flow and Losses

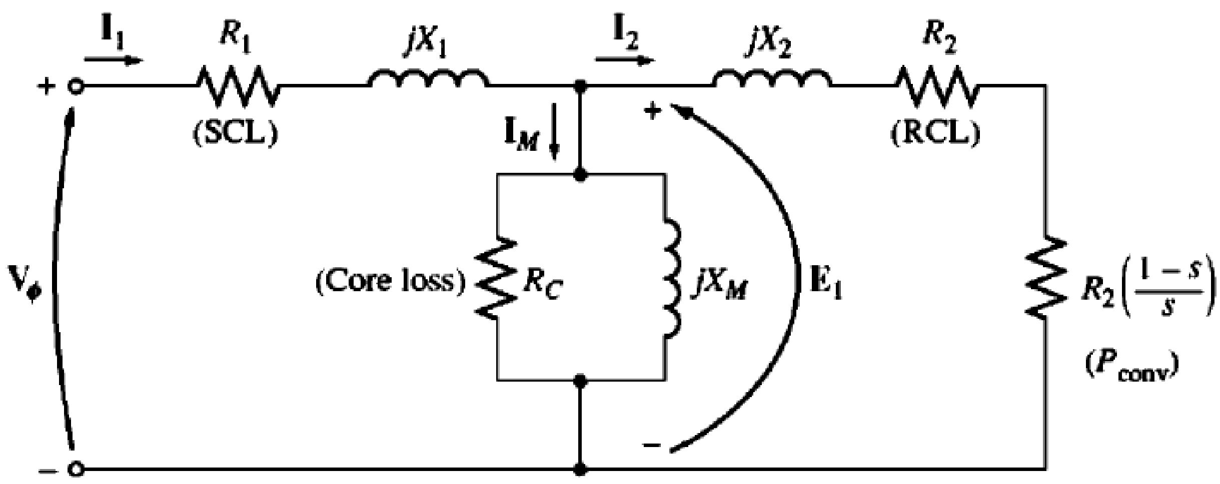


FIGURE 7–14
The per-phase equivalent circuit with rotor losses and P_{core} separated.

Power Flow and Losses

Example 7.3 A 460-V, 25-hp. 60-'Hz. four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$R_1 = 0.641\ \Omega$, $X_1 = 1.106\ \Omega$, $R_2 = 0.332\ \Omega$, $X_2 = 0.464\ \Omega$, $X_M = 26.3\ \Omega$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency. find the motor's

(a) Speed
(b) Stator current
(c) Power factor
(d) P_{conv} and P_{out}
(e) T_{ind} and T_{load}
(f) Efficiency

Power Flow and Losses

(a) The synchronous speed is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60 \text{ Hz})}{4 \text{ poles}} = 1800 \text{ r/min}$$

or $$\omega_{sync} = (1800 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 188.5 \text{ rad/s}$$

The rotor's mechanical shaft speed is

$$\begin{aligned} n_m &= (1 - s)n_{sync} \\ &= (1 - 0.022)(1800 \text{ r/min}) = 1760 \text{ r/min} \end{aligned}$$

or $$\begin{aligned} \omega_m &= (1 - s)\omega_{sync} \\ &= (1 - 0.022)(188.5 \text{ rad/s}) = 184.4 \text{ rad/s} \end{aligned}$$

Power Flow and Losses

(b) To find the stator current, get the equivalent impedance of the circuit. The first step is to combine the referred rotor impedance in parallel with the magnetization branch, and then to add the stator impedance to that combination in series. The referred rotor impedance is

$$\begin{aligned} Z_2 &= \frac{R_2}{s} + jX_2 \\ &= \frac{0.332}{0.022} + j0.464 \\ &= 15.09 + j0.464 \ \Omega = 15.10\angle 1.76° \ \Omega \end{aligned}$$

The combined magnetization plus rotor impedance is given by

$$\begin{aligned} Z_f &= \frac{1}{1/jX_M + 1/Z_2} \\ &= \frac{1}{-j0.038 + 0.0662\angle -1.76°} \\ &= \frac{1}{0.0773\angle -31.1°} = 12.94\angle 31.1° \ \Omega \end{aligned}$$

Power Flow and Losses

Therefore, the total impedance is

$$Z_{tot} = Z_{stat} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1^\circ\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6^\circ\ \Omega$$

The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{tot}}$$
$$= \frac{266\angle 0^\circ\ \text{V}}{14.07\angle 33.6^\circ\ \Omega} = 18.88\angle -33.6^\circ\ \text{A}$$

(c) The power motor power factor is

$$\text{PF} = \cos 33.6^\circ = 0.833 \qquad \text{lagging}$$

Power Flow and Losses

(d) The input power to this motor is

$$P_{in} = \sqrt{3} V_T I_L \cos\theta$$
$$= \sqrt{3}(460\ \text{V})(18.88\ \text{A})(0.833) = 12{,}530\ \text{W}$$

The stator copper losses in this machine are

$$P_{SCL} = 3 I_1^2 R_1$$
$$= 3(18.88\ \text{A})^2(0.641\ \Omega) = 685\ \text{W}$$

The air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} = 12{,}530\ \text{W} - 685\ \text{W} = 11{,}845\ \text{W}$$

Therefore, the power converted is

$$P_{conv} = (1 - s)P_{AG} = (1 - 0.022)(11{,}845\ \text{W}) = 11{,}585\ \text{W}$$

The power P_{out} is given by

$$P_{out} = P_{conv} - P_{rot} = 11{,}585\ \text{W} - 1100\ \text{W} = 10{,}485\ \text{W}$$
$$= 10{,}485\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = 14.1\ \text{hp}$$

Power Flow and Losses

(e) The induced torque is given by

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

$$= \frac{11{,}845\ \text{W}}{188.5\ \text{rad/s}} = 62.8\ \text{N}\bullet\text{m}$$

and the output torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{10{,}485\ \text{W}}{184.4\ \text{rad/s}} = 56.9\ \text{N}\bullet\text{m}$$

(In English units, these torques are 46.3 and 41.9 lb-ft, respectively.)

(f) The motor's efficiency at this operating condition is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{10{,}485\ \text{W}}{12{,}530\ \text{W}} \times 100\% = 83.7\%$$

Torque–Speed Characteristics

From utilizing the induction motor equivalent circuit and applying Thevenin theorem, the torque-speed characteristic curve can be obtained

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2/s}{\omega_{\text{sync}}[(R_{\text{TH}} + R_2/s)^2 + (X_{\text{TH}} + X_2)^2]}$$

$$V_{\text{TH}} \approx V_\phi \frac{X_M}{X_1 + X_M}$$

$$R_{\text{TH}} \approx R_1 \left(\frac{X_M}{X_1 + X_M}\right)^2$$

$$X_{\text{TH}} \approx X_1$$

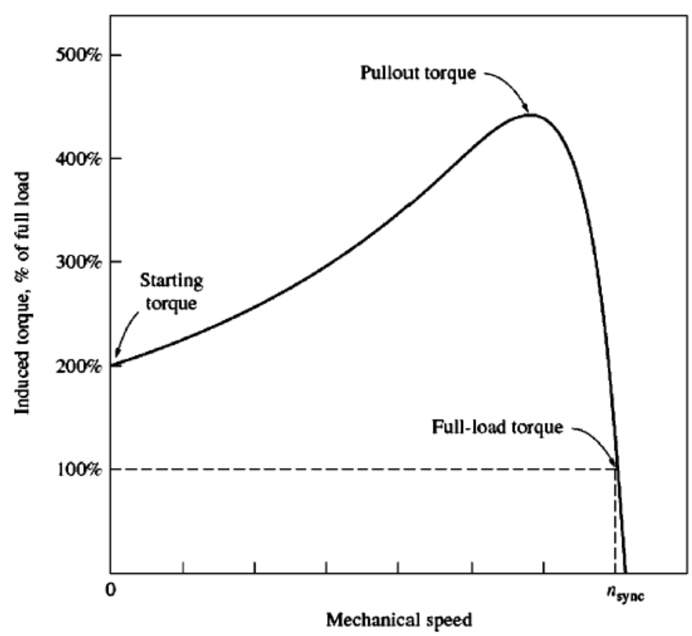


FIGURE 7–19
A typical induction motor torque–speed characteristic curve.

Torque–Speed Characteristics

- This characteristic curve can be divided roughly into three regions.
- The first region is the **low-slip region** of the curve. In the **low-slip region**, the motor slip increases approximately linearly with increased load, and the rotor mechanical speed decreases approximately linearly with load. In this region of operation, the rotor reactance is negligible, so the rotor power factor is approximately unity, while the rotor current increases linearly with slip. The entire normal steady-state operating range of an induction motor is included in this linear low-slip region. Thus in normal operation, an induction motor has a linear speed droop.

Torque–Speed Characteristics

- The second region on the induction motor's curve can be called the **moderate-slip region**. In the **moderate-slip region**, the rotor frequency is higher than before, and the rotor reactance is on the same order of magnitude as the rotor resistance. In this region, the rotor current no longer increases as rapidly as before, and the power factor starts to drop. The **peak torque (the pullout torque)** of the motor occurs at the point where, for an incremental increase in load, the increase in the rotor current is exactly balanced by the decrease in the rotor power factor.

Torque–Speed Characteristics

- The third region on the induction motor's curve is called the **high-slip region**. In the **high-slip region**, the induced torque actually decreases with increased load, since the increase in rotor current is completely overshadowed by the decrease in rotor power factor.

Torque–Speed Characteristics

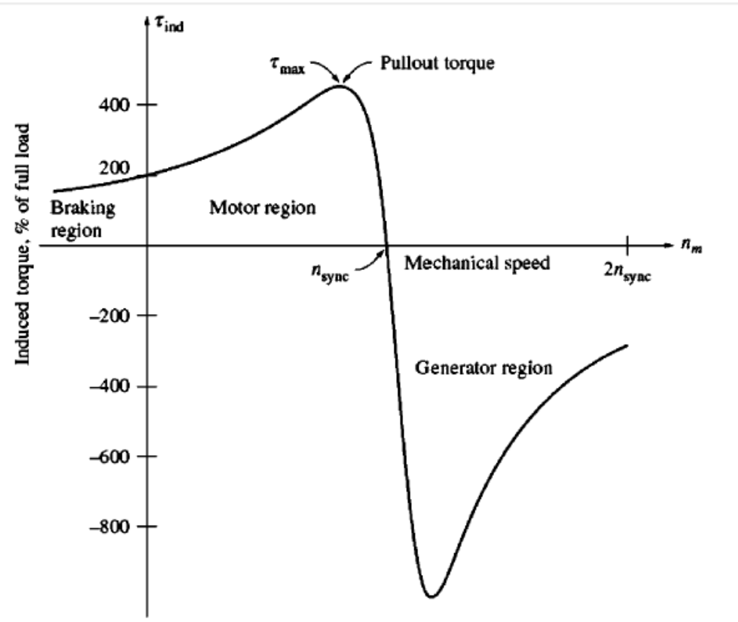


FIGURE 7–20
Induction motor torque–speed characteristic curve, showing the extended operating ranges (braking region and generator region).

Torque–Speed Characteristics

- From the torque-speed characteristic curve, the following can be observed

1. The induced torque of the motor is zero at synchronous speed.
2. The torque- speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.

Torque–Speed Characteristics

3. There is a maximum possible torque that cannot be exceeded. This torque is called the **pullout torque** or **breakdown torque**, is 2 to 3 times the rated full load torque of the motor.
4. The starting torque on the motor is slightly larger than its full-load torque, so this motor will start carrying any load that it can supply at full power.
5. Notice that the torque on the motor for a given slip varies as the square of the applied voltage. This fact is useful in one form of induction motor speed control.

Torque–Speed Characteristics

6. If the rotor of the induction motor is driven faster than synchronous speed, then the direction of the induced torque in the machine reverses and the machine becomes a generator, converting mechanical power to electric power.
7. If the motor is turning backward relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor. The act of switching two phases in order to stop the motor very rapidly is called **plugging**.

Torque–Speed Characteristics

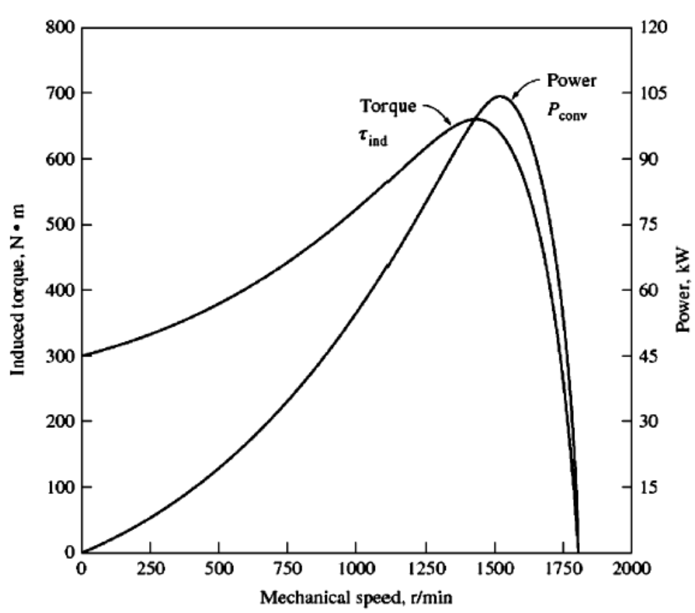


FIGURE 7–21
Induced torque and power converted versus motor speed in revolutions per minute for an example four-pole induction motor.

Since the air-gap power is equal to the power consumed in the resistor R_2/S, the maximum induced torque will occur when the power consumed by that resistor is maximum.

Torque–Speed Characteristics

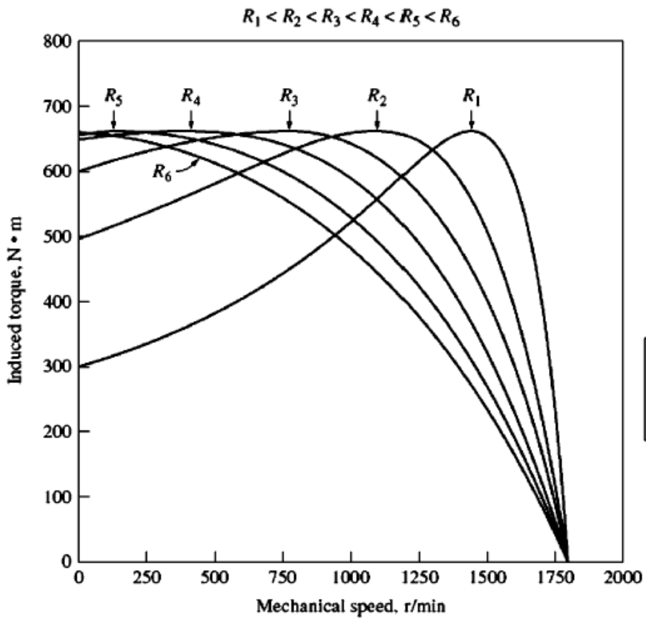


FIGURE 7–22
The effect of varying rotor resistance on the torque–speed characteristic of a wound-rotor induction motor.

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}$$

Torque–Speed Characteristics

Example 7.3 A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(a) What is the motor's slip?
(b) What is the induced torque in the motor in [N.m] under these conditions?
(c) What will the operating speed of the motor be if its torque is doubled?
(d) How much power will be supplied by the motor when the torque is doubled?

(a) The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(50 \text{ Hz})}{2 \text{ poles}} = 3000 \text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{sync} - n_m}{n_{sync}} (\times 100\%) \qquad (7\text{–}4)$$

$$= \frac{3000 \text{ r/min} - 2950 \text{ r/min}}{3000 \text{ r/min}} (\times 100\%)$$

$$= 0.0167 \text{ or } 1.67\%$$

Torque–Speed Characteristics

(*b*) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$
$$= \frac{15\ kW}{(2950\ r/min)(2\pi\ rad/r)(1\ min/60\ s)}$$
$$= 48.6\ N \cdot m$$

(*c*) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

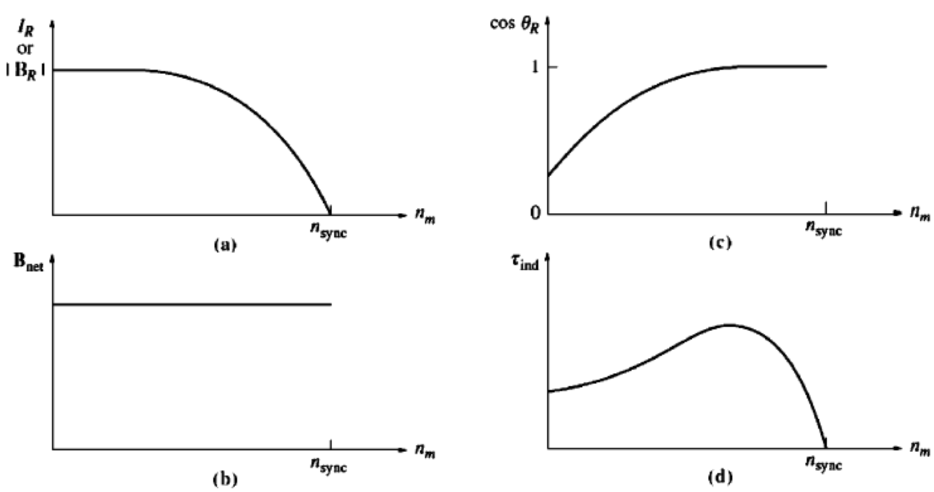
$$n_m = (1 - s)n_{sync} = (1 - 0.0333)(3000\ r/min) = 2900\ r/min$$

(*d*) The power supplied by the motor is given by

$$P_{conv} = \tau_{ind}\omega_m$$
$$= (97.2\ N \cdot m)(2900\ r/min)(2\pi\ rad/r)(1\ min/60\ s)$$
$$= 29.5\ kW$$

Torque–Speed Characteristics

Curves that represent the characteristics of induction motors

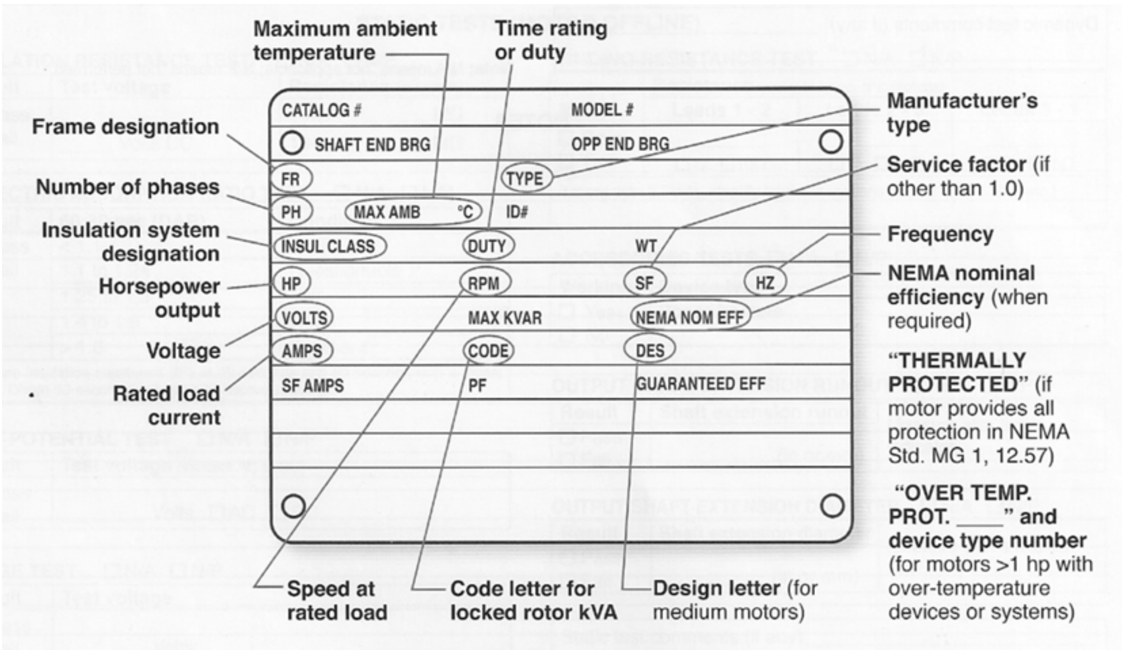


$$PF_R = \cos\theta_R$$

$$PF_R = \cos\left(\tan^{-1}\frac{sX_{R0}}{R_R}\right)$$

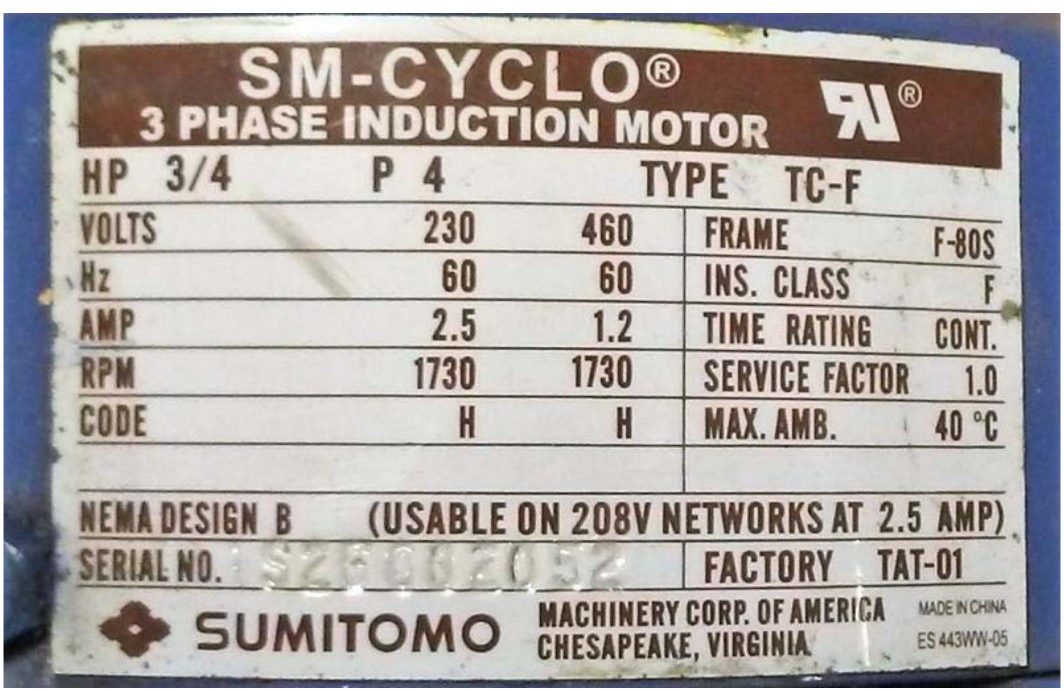
FIGURE 7–16
Graphical development of an induction motor torque–speed characteristic. (a) Plot of rotor current (and thus $|B_R|$) versus speed for an induction motor; (b) plot of net magnetic field versus speed for the motor; (c) plot of rotor power factor versus speed for the motor; (d) the resulting torque–speed characteristic.

Motor Tag / Nameplate



Note: Nameplates may also include the manufacturer's name and usually its principal location.

Motor Tag / Nameplate



Motor Tag / Nameplate