تقدم لجنة ElCoM الاكاديمية

تلخيص لمادة:

الات كهربائية

جزيل الشكر للطالب:

تولين صبرة

$* \, In\ rotary\ machines, speed\ is\ measured\ in\ (radian\ per\ second[r/s])$
$or\ (revolution\ per\ minute\ [rpm])$

$$1[rpm] = \frac{\pi}{30}[r/s]$$

$Where\ [r/s] < [rpm]$

$* \, In\ all\ motors\ and\ generators, electric\ power\ is\ measured\ in\ Watts[W]$
$On\ the\ other\ hand, mechanical\ power\ is\ measured\ either\ in\ watts[W]$
$or\ in\ Horsepower[hp]$

$$1[hp] = 746[W]$$

$Where\ [hp] < [W]$

$* \, It\ should\ be\ noted\ that\ the\ mechanical\ power\ can\ be\ computed$
$from\ the\ torque\ and\ speed\ as:$

$$Power\ [W] = Torque\ [N.m] \times speed\ [r/s]$$

$* \, Electrical\ power\ can\ be\ computed\ as:$

$-DC\ Machines \rightarrow Power[W] = current[A] \times voltage[V]$

$-AC\ Machines - 1\theta^{(one\ phase)} \rightarrow$

$$Active\ power[W] = current[A] \times voltage[V] \times cos\theta$$

$-AC\ Machines - 3\theta \rightarrow$

$$Active\ power[W] = \sqrt{3} \times line\ current[A] \times line\ voltage[V] \times cos\theta$$

$$Active\ power[W] = 3 \times phase\ current[A] \times phase\ voltage[V] \times cos\theta$$

Magnetic field

$*$ The total flux in the core due to the current i in the winding is

$\emptyset = \text{BA} = \mu \frac{NiA}{L_c}, N \rightarrow Num\ of\ turns, i \rightarrow the\ current\ in\ the\ coil, A \rightarrow the\ cross -$ $sectional\ area\ of\ the\ magnatic\ field, L_c \rightarrow the\ average\ path\ in\ which\ the\ mag\ field\ flows, \mu \rightarrow$ $permeability$

Where $\emptyset$(magnatic field)in Webers[Web], B(magnatic field dencity)in Webers per square meter[Web/m^2]

$known\ as\ Tesla[T]$

**right hand rule:-four fingers in the direction of the current*

-the thumb in the direction of the magnatic field

-palm of the hand in the direction of the force

In the magnetic circuit:

$*$ The reluctance of a magnetic circuit is the counterpart of electrical resistance, and its units are ampere − turns per weber

$\emptyset = BA = \mu HA$ $\mathcal{F} \rightarrow magnetomotive\ force\ of\ circuit$

$\emptyset = \mu \left(\frac{Ni}{L_c}\right) A = Ni \left(\frac{\mu A}{L_c}\right)$ $\emptyset \rightarrow fluc\ of\ circuit, \Re \rightarrow reluctance$

$\emptyset = \mathcal{F} \left(\frac{\mu A}{L_c}\right) = \frac{\mathcal{F}}{\Re}$ $H \rightarrow magnetic\ field\ intensity$

$B = \mu H$

Linear DC Machine

$*\ the\ equation\ for\ the\ force\ on\ a\ wire\ in\ the\ presence\ of\ a\ magnetic\ field$:

$F = i(L \times B) \rightarrow F = iLBsin\theta$

$where\ F \rightarrow force\ on\ wire, i \rightarrow magnitude\ of\ current\ in\ wire, B \rightarrow magnetic\ flux\ density\ vector$

$L \rightarrow length\ of\ wire. with\ direction\ of\ L\ defined\ to\ be\ in\ the\ direction\ of\ current\ flow$

$*\ the\ equation\ for\ the\ voltage\ induced\ on\ a\ wire\ moving\ in\ a\ magnetic\ field:$

$$e_{ind} = (v \times B).L \rightarrow e_{ind} = (VBsin\theta_1)L\ cos\theta_2$$

$where\ e_{ind} \rightarrow voltage\ induced\ in\ wire, v \rightarrow velocity\ of\ the\ wire, B \rightarrow magnetic\ flux\ dencity\ vector, L \rightarrow length\ of\ conductor\ in\ the\ magnetic\ field$

$*\ Kirchhoff's\ voltage\ law\ for\ this\ machine$

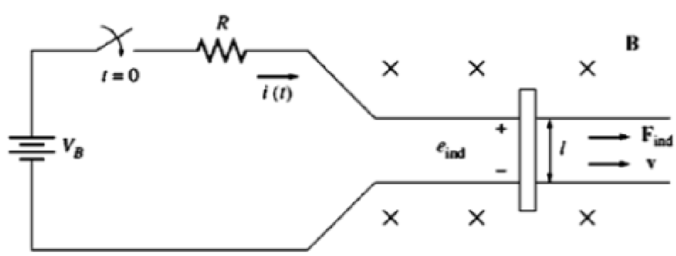
$$V_B - iR - e_{ind} = 0 \rightarrow V_B = e_{ind} + iR = 0$$

**Newton's law for the bar across the tracks:*

$$F_{net} = ma$$

Linear DC Machine cases:

First case: (Ideal motor)



1. $closing\ the\ switch\ produces\ a\ current\ flow\ i = \frac{V_B}{R}\ (starting\ current)$

2. $the\ current\ flow\ produces\ an\ induced\ force\ on\ the\ bar\ given\ be$

$$F_{ind} = iLB$$

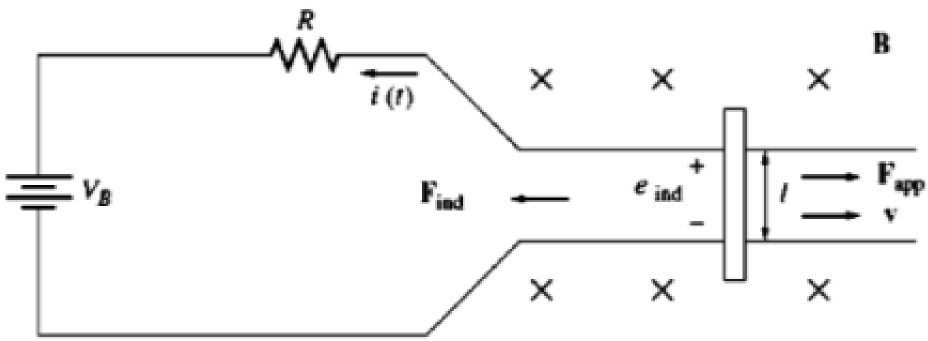
3. The bar accelerates due to the induced force as $a = \frac{F_{ind}}{m} = \frac{\Delta v}{\Delta t}$

4. The bar moves to the right, producing an induced voltage $e_{ind} = vBL$ *as it speeds up*

5. This induced voltage reduces the current flow $i = (v_B - e_{ind} \uparrow)/R$

6. The induced force is thus decreased $F_{ind} = \text{i} \downarrow \text{LB}$ until eventually $F_{ind} = 0$ At that $point.\, e_{ind} = V_B$, i = 0, and the bar moves at a constant no − load speed $v_{ss} = V_B/BL$ where a = 0 (note that $F_{ind} = 0$)

Second case: (real case of DC motor)



$* if\ a\ force\ F_{load}\ is\ apllied\ to\ the\ bar\ in\ OPPOSITE\ direction\ of\ motion$

$hence\ the\ net\ force\ is\ F_{net} = -F_{load}\ and\ the\ following\ events\ occure:$

1. The bar decelerates due to the load force as $\text{a} = F_{net}/m = \Delta v/\Delta t\ since\ F_{net} < 0$ $note\ that\ F_{load} < F_{ind}$

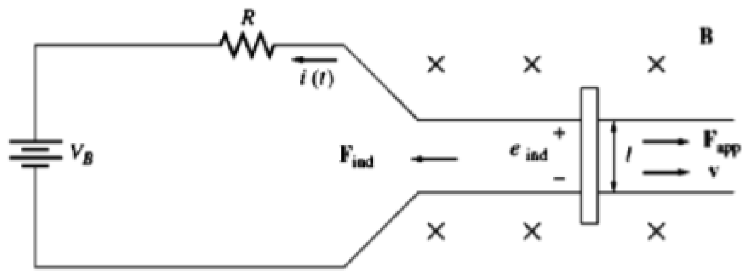
2. The effect of this net force will reduce $e_{ind} = v \downarrow BL\ as\ the\ bar\ slows\ down$

3. This induced voltage increases the current flow $\text{i} = (V_B - e_{ind} \downarrow)/R$

4. The induced force is thus increased $\text{F} = \text{i} \uparrow \text{LB}$ until $|F_{ind}| = |F_{load}|\ at\ a\ lower\ speed\ v$

5. An amount of electric power equal to $e_{ind}i$ is now being converted to mechanical power equal to $F_{ind}v$, and the machine is acting as a motor.

Third case:



$*$ If a force F_{app} is applied to the bar in the SAME direction of motion, hence the net force is $F_{net} = F_{app}$ and the following events occur:

1. The bar acelerates due to the load force as a $= F_{net}/m = \Delta v/\Delta t$ $since$ $F_{net} > 0$

2. The effect of this net force will increase $e_{ind} = v \uparrow BL$ as the bar $speeds$ up

3. This induced voltage reverses the direction of the current flow i $= (V_B - e_{ind} \uparrow)/R$ $since$ $(V_B < e_{ind})$

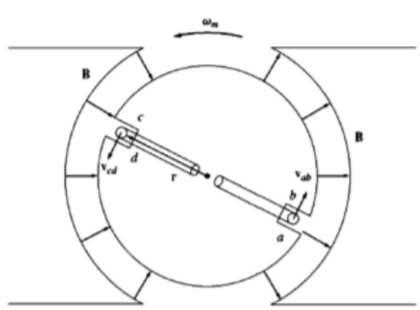
4. The induced force is thus reverses its direction due to the reversed current $-F_{ind} = -iLB$ $until$ $|F_{ind}| = |F_{app}|$. $Note$ $that$ $F_{net} = F_{app} - F_{ind}$ in $this$ $case$

5. An amount of mechanical power equal to $F_{app}v$ is now being converted To electric power equal to $e_{ind}i$, and the machine is acting as a generator.

DC Machines-simple rotating loop

$*$ If the rotor of this machine is rotated, a voltage will be induced in the wire loop.

$*$ To determine the total voltage e_{tot} on the loop, we need to examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by



$$e_{ba} = (v \times B).L = \begin{cases} vBL & \text{positive into page under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases} \quad \text{....(1)}$$

$e_{cb} = 0,$ *because of thetas in sin and cos* .....(2)

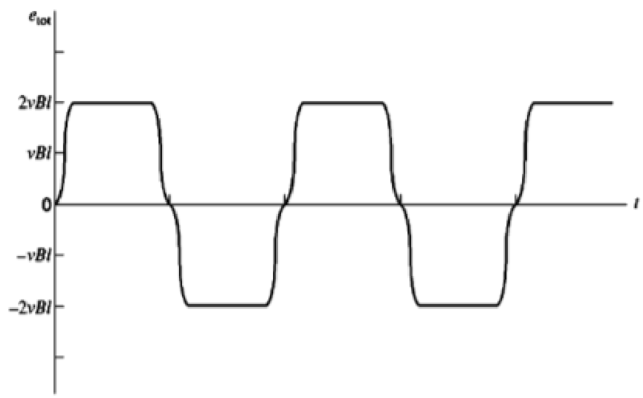
$$e_{dc} = (v \times B).L = \begin{cases} vBL & \text{positive out of page under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases} \quad \text{....(3)}$$

$e_{ad} = 0,$ *because of thetas in sin and cos* .....(4)

The total induced voltage on the loop e_{ind} *is given by*:

$$e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$$e_{ind} = \begin{cases} 2vBL & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \quad \text{......(5)}$$



Given $v = r\omega, \emptyset = A_pB,$ *and pole surface area of* $A_p = \pi rL,$ *an alternative expression can be obtained.*

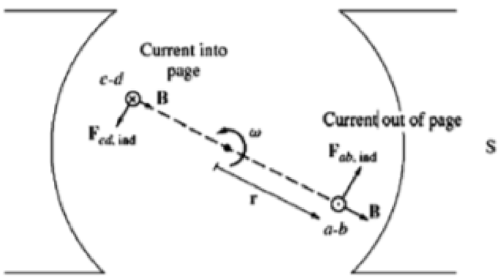
$$e_{ind} = \begin{cases} 2r\omega BL & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$e_{ind} = \begin{cases} \frac{2}{\pi} A_pB\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$e_{ind} = \begin{cases} \frac{2}{\pi} \emptyset\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \quad \text{.......(6)}$$

-This is for single loop (*speed of rotor* $\omega = \frac{V_B}{2rLB}$ *when* $V_B = e_{ind}$)

$*$ ***Suppose a battery is connected to the machine how the torque is developed inside it.*****(P11 in slides)**



$F_{ab} = i(L \times B) = iLB\ (tangent\ to\ direction\ of\ motion)$...**(7)**

$The\ torque\ on\ the\ rotor\ caused\ by\ this\ force\ is$

$\tau_{ab} = rFsin\theta = r(iLB)sin90° = riLB\ CCW$....**(8)**

$F_{bc} = i(L \times B) = 0\ since\ L\ is\ parallel\ to\ B$....**(9)**

$Therfore\ \tau_{bc} = 0\ \ldots\ldots(10)$

$F_{cd} = i(L \times B) = iLB\ (tangent\ to\ direction\ of\ motion)$ ....**(11)**

$The\ torque\ on\ the\ rotor\ caused\ by\ this\ force\ is$

$\tau_{cd} = rFsin\theta = r(iLB)sin90° = riLB\ CCW$....**(12)**

$F_{da} = i(L \times B) = 0\ since\ L\ is\ parallel\ to\ B$....**(13)**

$Therfore\ \tau_{da} = 0\ \ldots\ldots(14)$

$*\ the\ resulting\ total\ induced\ torque\ on\ the\ loop\ is\ given\ by$

$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$

$$\tau_{ind} = \begin{cases} 2riLB & under\ the\ pole\ faces \\ 0 & beyond\ the\ pole\ edges \end{cases} \text{....(15)}$$

$By\ using\ A_p \approx \pi rL\ and\ \emptyset = A_pB$

$$\tau_{ind} = \begin{cases} \frac{2}{\pi}\emptyset i & under\ the\ pole\ faces \\ 0 & beyond\ the\ pole\ edges \end{cases} \text{....(16)},\ P = \tau\omega$$

this is in single loop if it's more than this $\frac{2}{\pi}$ *will be represented as another constant* (K)

Power flow and losses

$*$ ***the copper losses for the armature and field windings are given by:***

Armature loss: $P_A = I^2{}_A R_A$

Where $P_A \rightarrow$ *armature loss,* $I_A \rightarrow$ *armature current,* $R_A \rightarrow$ *armature resistance*

Field loss: $P_F = I^2{}_F R_F$

Where $P_F \rightarrow$ *field circuit loss,* $I_F \rightarrow$ *field current,* $R_F \rightarrow$ *field resistance*

$*$ ***the brush losses it is given by the equation:***

$$P_{BD} = V_{BD} I_A$$

where $P_{BD} \rightarrow$ *brush drop loss,* $V_{BD} \rightarrow$ *brush voltage drop,* $I_A \rightarrow$ *armature current*

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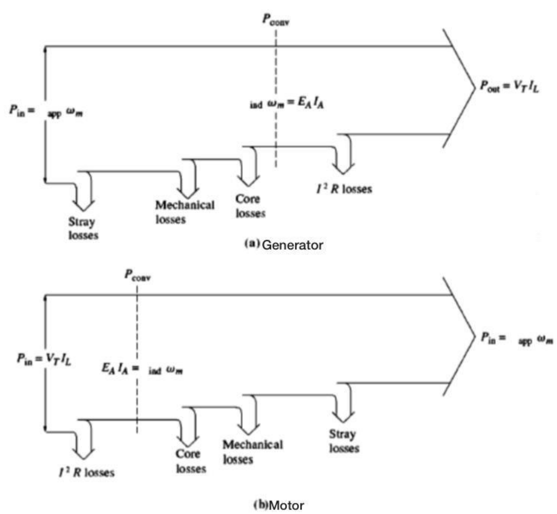


$$P_{conv} = \tau_{ind} \omega_m$$

$$P_{conv} = E_A I_A$$

$*$ ***The efficiency of a DC machine is defined by the equation***

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%$$

DC Machines-Motors

$* \, the \; speed \; regulation \; (SR) of \; a \; motor \; is \; defined \; by$

$$SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \times 100\% \;\; (nl \rightarrow no \; load, fl \rightarrow full \; load)$$

$$SR = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

Separately Excited and Shunt DC Motors

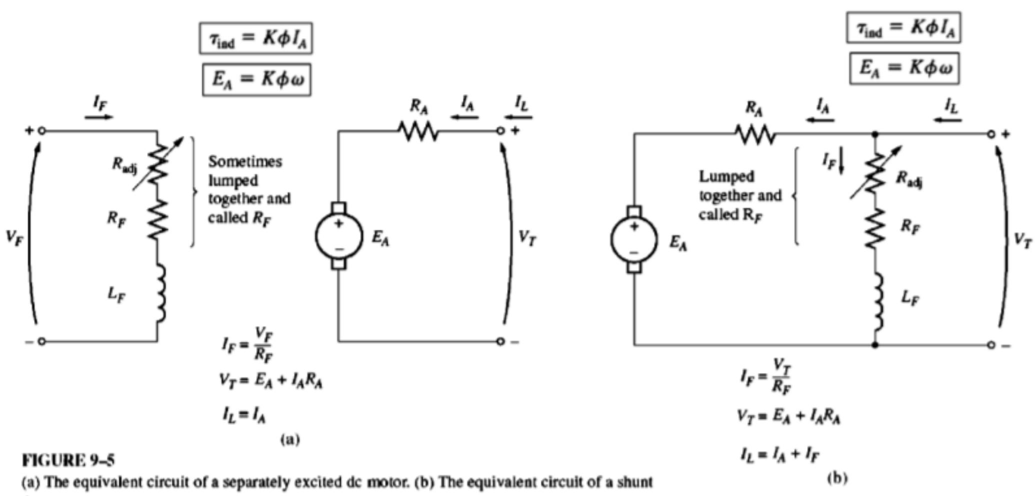


FIGURE 9–5
(a) The equivalent circuit of a separately excited dc motor. (b) The equivalent circuit of a shunt dc motor.

$$* \, Speed \; of \; the \; motor \rightarrow \frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

#Changing the Field Resistor Control Method:

1. $Increaseing \; R_F \; causes \; I_F = (V_F / R_F \uparrow) \; to \; decrease.$

2. $Decreasing \; I_F \; decreases \; \emptyset$

3. $Decreasing \; \emptyset \; lowers \; E_A = (K\emptyset \downarrow \omega).$

4. $Decreasing \; E_A \; increases \; I_A = ((V_T - E_A \downarrow)/R_A)$

5. $Increasing \; I_A \; increase \; \tau_{ind} = (K\emptyset \downarrow I_A \uparrow), with \; the \; change$
$in \; I_A \; dominant \; over \; the \; change \; in \; flux.$

6. $Increasing \; \tau_{ind} \; makes \; \tau_{ind} > \tau_{load}, and \; the \; speed \; \omega \; increases$

7. $Increasing \; \omega \; increases \; E_A = (K\emptyset\omega \uparrow) \; again.$

$8. \boldsymbol{Increasing\ E_A\ decreases\ I_A}$

$9. \boldsymbol{Decreasing\ I_A\ decreases\ \tau_{ind}\ untill\ \tau_{ind} = \tau_{load}\ at\ a\ higher\ speed\ \omega}$

#Changing the Armature Voltage Control Method

$1. \boldsymbol{Increasing\ V_T\ increases\ I_A = ((V_T \uparrow - E_A)/R_A)}$

$2. \boldsymbol{Increasing\ I_A\ increases\ \tau_{ind} = (K\emptyset I_A \uparrow)}$

$3. \boldsymbol{Increasing\ \tau_{ind}\ , makes\ \tau_{ind} > \tau_{load}\ and\ the\ speed\ \omega\ increases}$

$4. \boldsymbol{Increasing\ \omega\ increases\ E_A = (K\emptyset\omega \uparrow)}$

$5. \boldsymbol{Increasing\ E_A\ decreases\ I_A = ((V_T - E_A \uparrow)/R_A)}$

$6. \boldsymbol{Decreasing\ I_A\ decreases\ \tau_{ind}\ until\ \tau_{ind} = \tau_{load}\ at\ a\ higher\ speed\ \omega}$

#Inserting a Resistor in Series with the Armature Circuit

$1. \boldsymbol{Increasing\ R_A\ dicreases\ I_A = ((V_T - E_A)/R_A \uparrow)}$

$2. \boldsymbol{Decreasing\ I_A\ decreases\ \tau_{ind} = (K\emptyset I_A \downarrow)}$

$3. \boldsymbol{decreasing\ \tau_{ind}\ , makes\ \tau_{ind} < \tau_{load}\ and\ the\ speed\ \omega\ decreases}$

$4. \boldsymbol{decreasing\ \omega\ decreases\ E_A = (K\emptyset\omega \downarrow)}$

$5. \boldsymbol{Decreasing\ E_A\ increases\ I_A = ((V_T - E_A \downarrow)/R_A)}$

$6. \boldsymbol{Increasing\ I_A\ increases\ \tau_{ind}\ until\ \tau_{ind} = \tau_{load}\ at\ a\ lower\ speed\ \omega}$

$\boldsymbol{the\ internal\ generated\ voltage\ of\ a\ DC\ machine\ with\ its\ speed\ expressed\ in\ revolution\ per\ minute\ is\ given\ by}$

$$\boldsymbol{e_A = k'\emptyset n}$$

$\boldsymbol{the\ relationship\ between\ the\ speeds\ and\ internal\ generated\ voltages\ of\ the\ motor\ at\ two\ different\ conditions\ is\ thus}$

$$\boldsymbol{\frac{E_{A2}}{E_{A1}} = \frac{K'\emptyset n_2}{K'\emptyset n_1} \rightarrow n_2 = \frac{E_{A2}}{E_{A1}} n_1}$$

$\boldsymbol{The\ induced\ torque\ for\ any\ other\ load\ can\ be\ found\ from}$

$\boldsymbol{the\ fact\ P_{conv} = E_A I_A = \tau_{ind}\omega \rightarrow}$

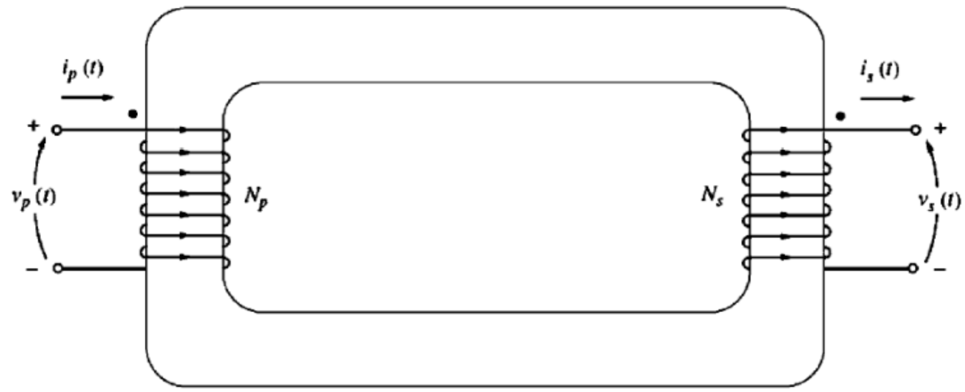
$$\boldsymbol{\tau_{ind} = \frac{E_A I_A}{\omega}}$$

$\boldsymbol{the\ effective\ shunt\ field\ current\ of\ the\ motor\ is}$

$$\boldsymbol{I^*{}_F = I_F - \frac{\mathcal{F}_{AR}}{N_F}\ ,\mathcal{F}_{AR}[A.turns]}$$

#Transformers:

*Ideal transformer



(a)

FIGURE 2–4
(a) Sketch of an ideal transformer.

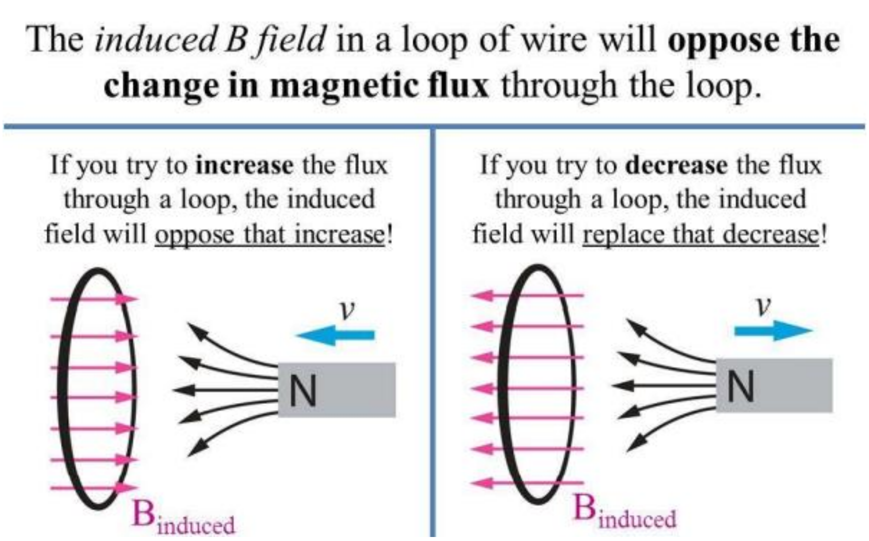
$* \ Faraday's \ law \ of \ induction$

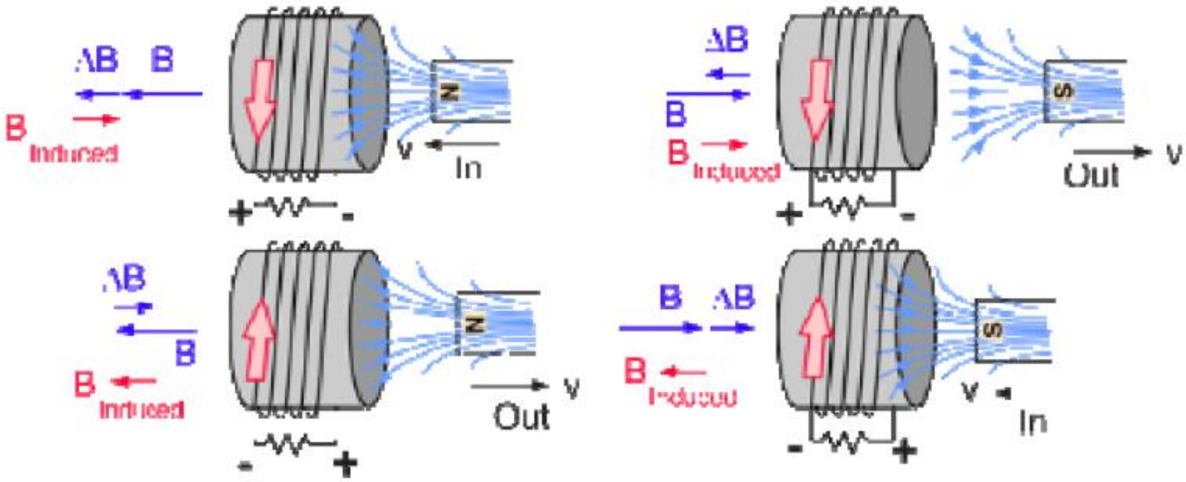
$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

ε: $induced \ emf$, N: $numper \ of \ loops$, $\Delta\Phi/\Delta t$: $rate \ of \ change \ of \ flux \ with \ time$

$minus \ sign \ from \ lenz's \ law$

$* \ Lenz's \ Law$





$* \; Since\ the\ magetomotive\ forces\ are\ equal\ in\ primary\ and\ secondary\ windings$

$$\mathcal{F}_P = \Phi\mathcal{R} = \mathcal{F}_S \rightarrow N_P i_P = N_S i_S \rightarrow \frac{N_P}{N_S} = \frac{i_S}{i_P} = a$$

$where\ \boldsymbol{a}\ is\ the\ \boldsymbol{transformer\ turns\ ratio}$

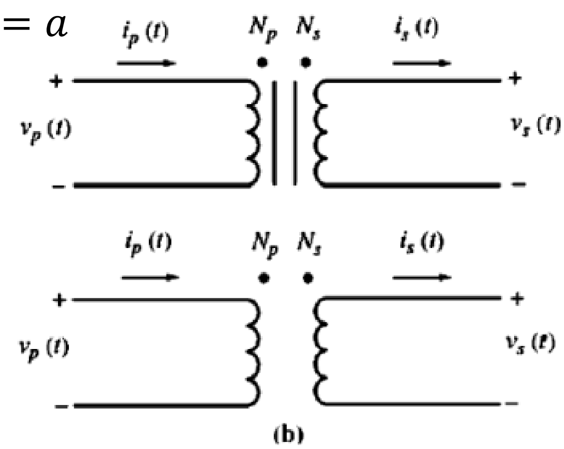


FIGURE 2–4
(b) Schematic symbols of a transformer.

$* \; From\ faraday's\ laws$

$V\ is\ proportional\ to\ N\ (V \propto N)$

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = a$$

$$* \; V_P i_P = V_S i_S \rightarrow S_{in} = S_{out}$$

$$* \; P_{in} = V_P i_P(\theta_P), * \; P_{out} = V_S i_S(\theta_S)$$

$* \; since\ V_p i_p = V_s i_s\ and\ since\ voltage\ and\ current\ angles\ are\ unaffected\ by\ an\ ideal\ transformer\ then\ \theta_p = \theta_s$

$*$ The primary and secondary windings of an ideal transformer have the same power factor then $P_{in} = P_{out}$ *and then* $Q_{in} = Q_{out}$

$\#If\ N_P < N_S \rightarrow V_P < V_S \rightarrow i_P > i_S\ (step - up\ transformer)$

$\#If\ N_P > N_S \rightarrow V_P > V_S \rightarrow i_P < i_S\ (step - down\ transformer)$

$*$ The impedance of a device or an element is defined as the ratio of the phasor Voltage across it to the phasor current flowing through it

$$Z_L = \frac{V_L}{I_L}$$

$$* Z'_L = \frac{V_P}{I_P} = \frac{aV_S}{I_S/a} = a^2 \frac{V_S}{I_S}$$

$$\rightarrow Z'_L = a^2 Z_L$$

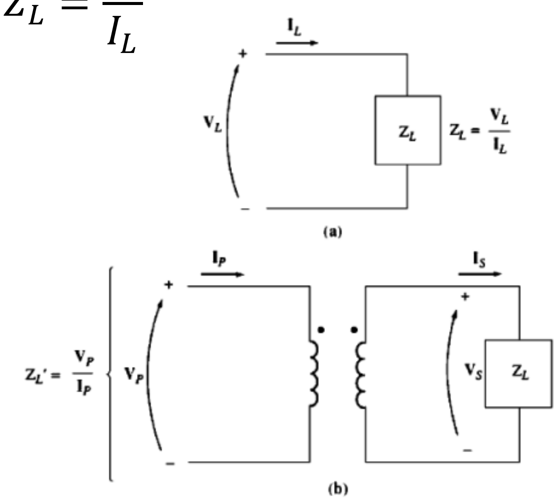


FIGURE 2–5
(a) Definition of impedance. (b) Impedance scaling through a transformer.

$$* I_{line} = \frac{V}{Z_{line} + Z_{load}}, V_{load} = I_{line} Z_{load}, P_{loss} = (I_{line})^2 R_{line}, Z_{eq} = Z_{line} + Z'_{load}$$

#Operation of real transformer

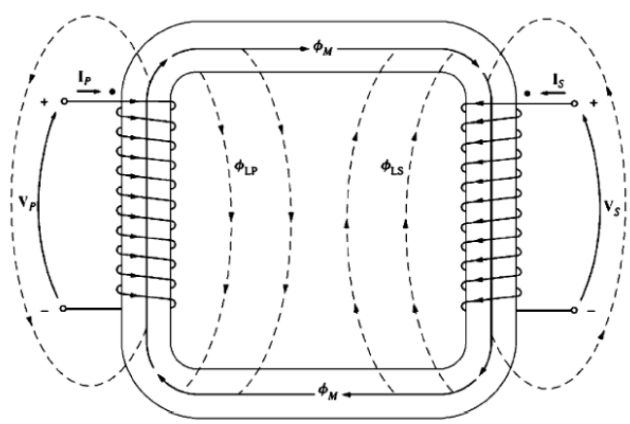


FIGURE 2–10
Mutual and leakage fluxes in a transformer core.

#The questions about this case will be concept questions

∗ The flux in the primary coil of the transformer can thus be divided into two components: a mutual flux, which remains in the core and links both windings, and a small leakage flux, which passes through the primary winding but returns through the air, bypassing the secondary winding:

$$\bar{\Phi}_P = \Phi_M + \Phi_{LP}$$

$\bar{\Phi}_P: total\ average\ primary\ flux, \Phi_M: flux\ component\ linking\ both\ primary\ and\ secondary\ coils,$
$\Phi_{LP}: primary\ leakage\ flux$

∗ There is a similar division of flux in the secondary winding between mutual flux and leakage flux which passes through the secondary winding but returns through the air, by passing the primary winding:

$$\bar{\Phi}_S = \Phi_M + \Phi_{LS}$$

$\bar{\Phi}_S: total\ average\ secondary\ flux, \Phi_M: flux\ component\ linking\ both\ primary\ and\ secondary\ coils,$
$\Phi_{LS}: secondary\ leakage\ flux$

∗ Since in a well designed transformer $\Phi_M \gg \Phi_{LP}$ and $\Phi_M \gg \Phi_{LS}$ the ratio of the total voltage on the primary of a transformer to the total voltage on the secondary of a transformer is approximately

∗ The smaller the leakage fluxes of the transformer are, the closer the total transformer voltage ratio approximates that of the ideal transformer

$$\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a$$

∗ $Open - circuit\ test$:

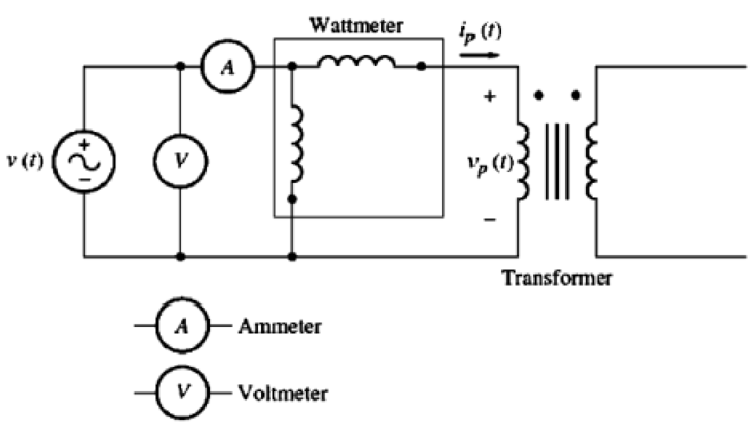


FIGURE 2–19
Connection for transformer open-circuit test.

- The conductance of the core − loss resistor is given by

$$G_c = \frac{1}{R_c}$$

- The susceptance of the magnetizing inductor is given by

$$B_M = \frac{1}{X_M}$$

- Since these two elements are in parallel, their admittances add, and the total excitation admittance is

$$Y_E = G_c - jB_M$$

$$Y_E = \frac{1}{R_c} - j\frac{1}{X_M}$$

- the following relations can be obtained

$$|Y_E| = \frac{I_{oc}}{V_{oc}}$$

$$PF = \cos(\theta) = \frac{P_{OC}}{V_{OC}I_{OC}} \Rightarrow \theta = cos^{-1}\left(\frac{P_{OC}}{V_{OC}I_{OC}}\right)$$

$$Y_E = \frac{I_{oc}}{V_{oc}}\angle - \theta \Rightarrow Y_E = \frac{I_{oc}}{V_{oc}}\angle - cos^{-1}(PF)$$

$$R_c = \frac{1}{G_c} = \frac{1}{Y_E cos(\theta)}$$

$$X_M = \frac{1}{B_M} = \frac{1}{Y_E sin(\theta)}$$

$* Short - circuit\ test$:

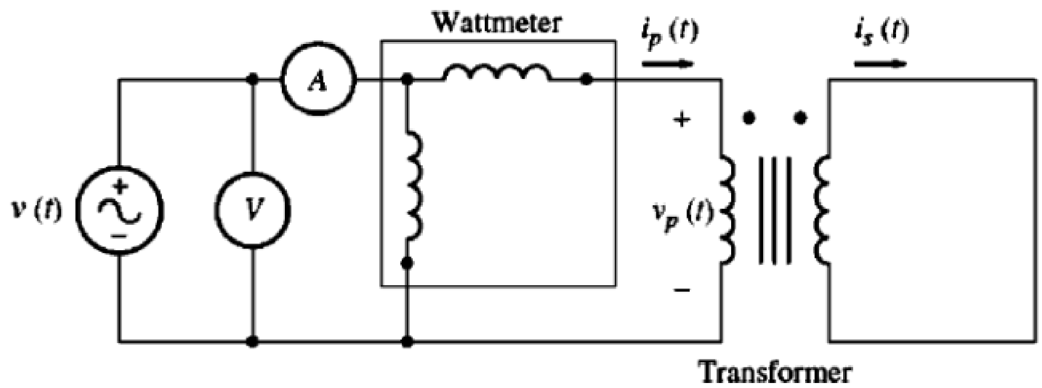


FIGURE 2–20
Connection for transformer short-circuit test.

∘ The magnitude of the series impedances referred to the primary side of the transformer is

$$|Z_{SE}| = \frac{V_{sc}}{I_{sc}}$$

∘ $The\ following\ relations\ can\ be\ obtaind$

$$PF = \cos(\theta) = \frac{P_{SC}}{V_{SC}I_{SC}} \Rightarrow \theta = cos^{-1}\left(\frac{P_{SC}}{V_{SC}I_{SC}}\right)$$

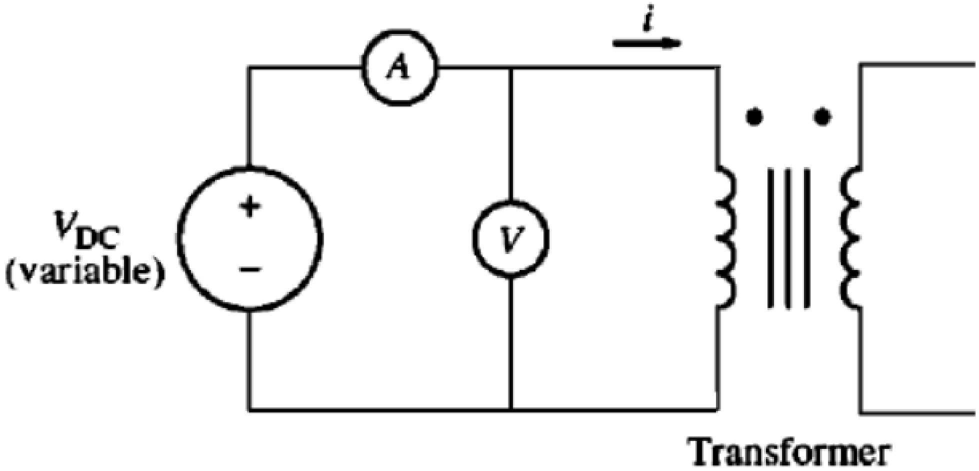
$$Z_{SE} = \frac{V_{SC}\angle 0}{I_{SC}\angle - \theta} = \frac{V_{SC}}{I_{SC}}\angle\theta$$

$$R_{eq} = Z_{SE}cos(\theta)$$

$$X_{eq} = Z_{SE}sin(\theta)$$

$$\underline{Z_{SE} = R_{eq} + jX_{eq}} = (R_P + a^2R_S) + j(X_P + a^2X_S)$$

$* \, DC \; test$:



- Using Ohm's law and measurements of voltage and current, the internal resistor of the winding can be determined as

R = V / I

- It should be noted that this test can be done on both sides of the transformer
- Moreover, the DC power supply should be adjusted so that maximum DC current is not exceeded.
- **Note that the coil can handle a DC current that is certainly less than the rated AC current.**

$* \, \boldsymbol{Voltage\ regulation\ (VR)}$

$$\boldsymbol{VR = \frac{V_{S.nl} - V_{S.fl}}{V_{S.fl}} \times 100\%}$$

$$\boldsymbol{VR = \frac{V_P/a - V_{S.fl}}{V_{S.fl}} \times 100\%}$$

$$\circ \boldsymbol{\frac{V_P}{a} = V_S + R_{eq}I_S + jX_{eq}I_S}$$

$* \boldsymbol{Efficiency}$

Transformers are also compared and judged on their efficiencies. The efficiency of a device is defined by the equation

$$\boldsymbol{\eta = \frac{P_{out}}{P_{in}} \times 100\%}$$

$$\boldsymbol{\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%}$$

$$\boldsymbol{\eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%}$$

To calculate the efficiency of a transformer at a given load, just add the losses the equation of efficiency such that

$$\boldsymbol{P_{out} = V_S I_S cos\theta_S}$$

$$\boldsymbol{\eta = \frac{V_S I_S cos\theta_S}{P_{Cu} + P_{core} + V_S I_S cos\theta_S} \times 100\%}$$

$$\boldsymbol{P_{Cu} = (I_S)^2 R_{eq}}$$

$$\boldsymbol{P_{core} = \frac{(V_P/a)^2}{R_C}}$$

$* \ In\ rotary\ machines, speed\ is\ measured\ in\ (radian\ per\ second[r/s])$
$or\ (revolution\ per\ minute\ [rpm])$

$$1[rpm] = \frac{\pi}{30}[r/s]$$

$Where\ [r/s] < [rpm]$

$* \ In\ all\ motors\ and\ generators, electric\ power\ is\ measured\ in\ Watts[W]$
$On\ the\ other\ hand, mechanical\ power\ is\ measured\ either\ in\ watts[W]$
$or\ in\ Horsepower[hp]$

$$1[hp] = 746[W]$$

$Where\ [hp] < [W]$

$* \ It\ should\ be\ noted\ that\ the\ mechanical\ power\ can\ be\ computed$
$from\ the\ torque\ and\ speed\ as:$

$Power\ [W] = Torque\ [N.m] \times speed\ [r/s]$

$* \ Electrical\ power\ can\ be\ computed\ as:$

$-DC\ Machines \rightarrow Power[W] = current[A] \times voltage[V]$

$-AC\ Machines - 1\theta^{(one\ phase)} \rightarrow$

$Active\ power[W] = current[A] \times voltage[V] \times cos\theta$

$-AC\ Machines - 3\theta \rightarrow$

$Active\ power[W] = \sqrt{3} \times line\ current[A] \times line\ voltage[V] \times cos\theta$

$Active\ power[W] = 3 \times phase\ current[A] \times phase\ voltage[V] \times cos\theta$

Magnetic field

$*$ The total flux in the core due to the current i in the winding is

$\emptyset = \mathrm{BA} = \mu \frac{NiA}{L_c}$, $N \rightarrow Num\ of\ turns, i \rightarrow the\ current\ in\ the\ coil, A \rightarrow the\ cross -$ $sectional\ area\ of\ the\ magnatic\ field, L_c \rightarrow the\ average\ path\ in\ which\ the\ mag\ field\ flows, \mu \rightarrow$ $permeability$

Where $\emptyset$(magnatic field)in Webers[Web], B(magnatic field dencity)in Webers per square meter[Web/m^2]

$known\ as\ Tesla[T]$

**right hand rule:-four fingers in the direction of the current*

-the thumb in the direction of the magnatic field

-palm of the hand in the direction of the force

In the magnetic circuit:

$*$ The reluctance of a magnetic circuit is the counterpart of electrical resistance, and its units are ampere − turns per weber

$\emptyset = BA = \mu HA$

$\emptyset = \mu \left(\frac{Ni}{L_c}\right) A = Ni \left(\frac{\mu A}{L_c}\right)$

$\emptyset = \mathcal{F} \left(\frac{\mu A}{L_c}\right) = \frac{\mathcal{F}}{\Re}$

$B = \mu H$

$\mathcal{F} \rightarrow magnetomotive\ force\ of\ circuit$

$\emptyset \rightarrow fluc\ of\ circuit, \Re \rightarrow reluctance$

$H \rightarrow magnetic\ field\ intensity$

Linear DC Machine

$*\ the\ equation\ for\ the\ force\ on\ a\ wire\ in\ the\ presence\ of\ a\ magnetic\ field$:

$F = i(L \times B) \rightarrow F = iLBsin\theta$

$where\ F \rightarrow force\ on\ wire, i \rightarrow magnitude\ of\ current\ in\ wire, B \rightarrow magnetic\ flux\ density\ vector$

$L \rightarrow length\ of\ wire. with\ direction\ of\ L\ defined\ to\ be\ in\ the\ direction\ of\ current\ flow$

$*\ the\ equation\ for\ the\ voltage\ induced\ on\ a\ wire\ moving\ in\ a\ magnetic\ field$:

$$e_{ind} = (v \times B). L \rightarrow e_{ind} = (VBsin\theta_1)L\ cos\theta_2$$

$where\ e_{ind} \rightarrow voltage\ induced\ in\ wire, v \rightarrow velocity\ of\ the\ wire, B \rightarrow magnetic\ flux\ dencity\ vector, L \rightarrow length\ of\ conductor\ in\ the\ magnetic\ field$

$*\ Kirchhoff's\ voltage\ law\ for\ this\ machine$

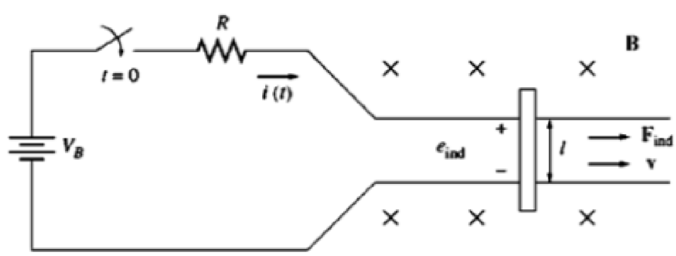
$$V_B - iR - e_{ind} = 0 \rightarrow V_B = e_{ind} + iR = 0$$

**Newton's law for the bar across the tracks:*

$$F_{net} = ma$$

Linear DC Machine cases:

First case: (Ideal motor)



1. $closing\ the\ switch\ produces\ a\ current\ flow\ i = \frac{V_B}{R}$ $(starting\ current)$

2. $the\ current\ flow\ produces\ an\ induced\ force\ on\ the\ bar\ given\ be$

$$F_{ind} = iLB$$

3. The bar accelerates due to the induced force as a $= \frac{F_{ind}}{m} = \frac{\Delta v}{\Delta t}$

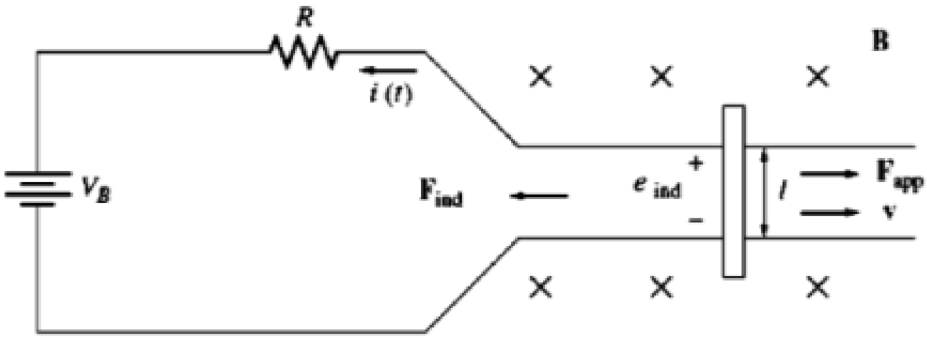
4. The bar moves to the right, producing an induced voltage $e_{ind} = vBL\ as\ it\ speeds\ up$

5. This induced voltage reduces the current flow $i = (v_B - e_{ind} \uparrow)/R$

6. The induced force is thus decreased F_{ind} = i ↓ LB until eventually $F_{ind} = 0$ At that $point.\, e_{ind} = V_B$, i = 0, and the bar moves at a constant no − load speed

$v_{ss} = V_B/BL$ where a = 0 (note that $F_{ind} = 0$)

Second case: (real case of DC motor)



$* \, if\ a\ force\ F_{load}\ is\ apllied\ to\ the\ bar\ in\ OPPOSITE\ direction\ of\ motion$

$hence\ the\ net\ force\ is\ F_{net} = -F_{load}\ \ and\ the\ following\ events\ occure$:

1. The bar decelerates due to the load force as a = $F_{net}/m = \Delta v/\Delta t\ since\ F_{net} < 0$ $note\ that\ F_{load} < F_{ind}$

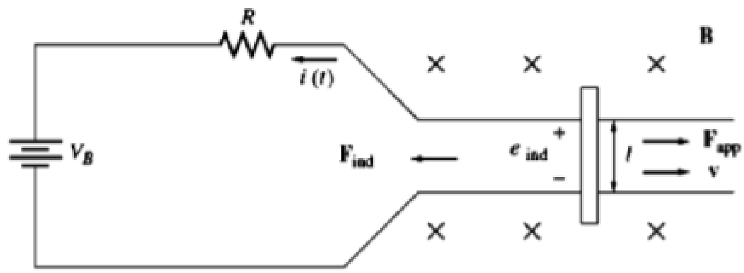
2. The effect of this net force will reduce $e_{ind} = v \downarrow BL\ as\ the\ bar\ slows\ down$

3. This induced voltage increases the current flow i = $(V_B - e_{ind} \downarrow)/R$

4. The induced force is thus increased F = i ↑ LB until $|F_{ind}| = |F_{load}|\ at\ a\ lower\ speed\ v$

5. An amount of electric power equal to $e_{ind} i$ is now being converted to mechanical power equal to $F_{ind} v$, and the machine is acting as a motor.

Third case:



$*$ If a force F_{app} is applied to the bar in the SAME direction of motion, hence the net force is $F_{net} = F_{app}$ and the following events occur:

1. The bar acelerates due to the load force as a $= F_{net}/m = \Delta v/\Delta t \; since \; F_{net} > 0$

2. The effect of this net force will increase $e_{ind} = v \uparrow BL \; as \; the \; bar \; speeds \; up$

3. This induced voltage reverses the direction of the current flow i $= (V_B - e_{ind} \uparrow)/R$ $since \; (V_B < e_{ind})$

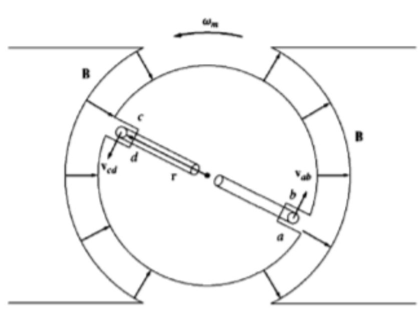
4. The induced force is thus reverses its direction due to the reversed current $-F_{ind} = -iLB \; until \; |F_{ind}| = |F_{app}|. Note \; that \; F_{net} = F_{app} - F_{ind} \; in \; this \; case$

5. An amount of mechanical power equal to $F_{app}v$ is now being converted To electric power equal to $e_{ind}i$, and the machine is acting as a generator.

DC Machines-simple rotating loop

$*$ If the rotor of this machine is rotated, a voltage will be induced in the wire loop.

$*$ To determine the total voltage e_{tot} on the loop, we need to examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by



$$e_{ba} = (v \times B).L = \begin{cases} vBL \text{ positive into page under the pole face} \\ 0 \qquad \text{beyond the pole edges} \end{cases} \text{....(1)}$$

$e_{cb} = 0$, *because of thetas in sin and cos* .....(2)

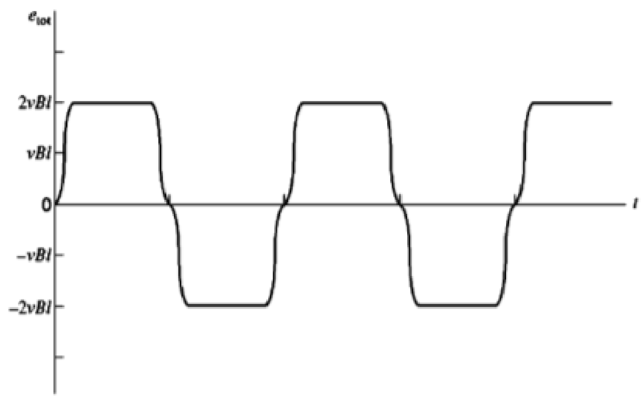
$$e_{dc} = (v \times B).L = \begin{cases} vBL \text{ positive out of page under the pole face} \\ 0 \qquad \text{beyond the pole edges} \end{cases} \text{....(3)}$$

$e_{ad} = 0$, *because of thetas in sin and cos* .....(4)

The total induced voltage on the loop e_{ind} *is given by*:

$$e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$$e_{ind} = \begin{cases} 2vBL \qquad \text{under the pole faces} \\ 0 \qquad \text{beyond the pole edges} \end{cases} \text{......(5)}$$



Given $v = r\omega$, $\emptyset = A_pB$, *and pole surface area of* $A_p = \pi rL$, *an alternative expression can be obtained.*

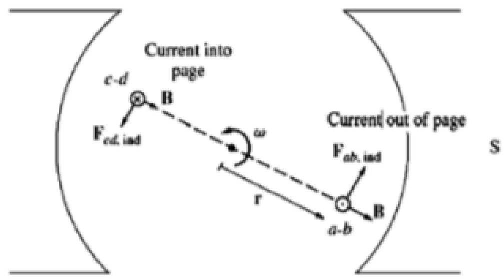
$$e_{ind} = \begin{cases} 2r\omega BL \qquad \text{under the pole faces} \\ 0 \qquad \text{beyond the pole edges} \end{cases}$$

$$e_{ind} = \begin{cases} \frac{2}{\pi} A_p B\omega \qquad \text{under the pole faces} \\ 0 \qquad \text{beyond the pole edges} \end{cases}$$

$$e_{ind} = \begin{cases} \frac{2}{\pi} \emptyset\omega \qquad \text{under the pole faces} \\ 0 \qquad \text{beyond the pole edges} \end{cases} \text{.......(6)}$$

-This is for single loop ($speed\ of\ rotor\ \omega = \frac{V_B}{2rLB}\ when\ V_B = e_{ind}$)

$* \, Suppose\ a\ battery\ is\ connected\ to\ the\ machine\ how\ the\ torque\ is\ developed\ inside\ it.$ **(P11 in slides)**



$F_{ab} = i(L \times B) = iLB\ (tangent\ to\ direction\ of\ motion)$…**(7)**

$The\ torque\ on\ the\ rotor\ caused\ by\ this\ force\ is$

$\tau_{ab} = rFsin\theta = r(iLB)sin90° = riLB\ CCW$….**(8)**

$F_{bc} = i(L \times B) = 0\ since\ L\ is\ parallel\ to\ B$….**(9)**

$Therfore\ \tau_{bc} = 0 \ldots\ldots(10)$

$F_{cd} = i(L \times B) = iLB\ (tangent\ to\ direction\ of\ motion)$ ….**(11)**

$The\ torque\ on\ the\ rotor\ caused\ by\ this\ force\ is$

$\tau_{cd} = rFsin\theta = r(iLB)sin90° = riLB\ CCW$….**(12)**

$F_{da} = i(L \times B) = 0\ since\ L\ is\ parallel\ to\ B$….**(13)**

$Therfore\ \tau_{da} = 0 \ldots\ldots(14)$

$* \, the\ resulting\ total\ induced\ torque\ on\ the\ loop\ is\ given\ by$

$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$

$\tau_{ind} = \begin{cases} 2riLB & under\ the\ pole\ faces \\ 0 & beyond\ the\ pole\ edges \end{cases}$ ***….(15)***

$By\ using\ A_p \approx \pi rL\ and\ \emptyset = A_pB$

$\tau_{ind} = \begin{cases} \frac{2}{\pi}\emptyset i & under\ the\ pole\ faces \\ 0 & beyond\ the\ pole\ edges \end{cases}$ ***….(16),*** $P = \tau\omega$

this is in single loop if it's more than this $\frac{2}{\pi}$ *will be represented as another constant* (K)

Power flow and losses

$*$ ***the copper losses for the armature and field windings are given by:***

Armature loss: $P_A = I^2{}_A R_A$

Where $P_A \rightarrow$ ***armature loss,*** $I_A \rightarrow$ ***armature current,*** $R_A \rightarrow$ ***armature resistance***

Field loss: $P_F = I^2{}_F R_F$

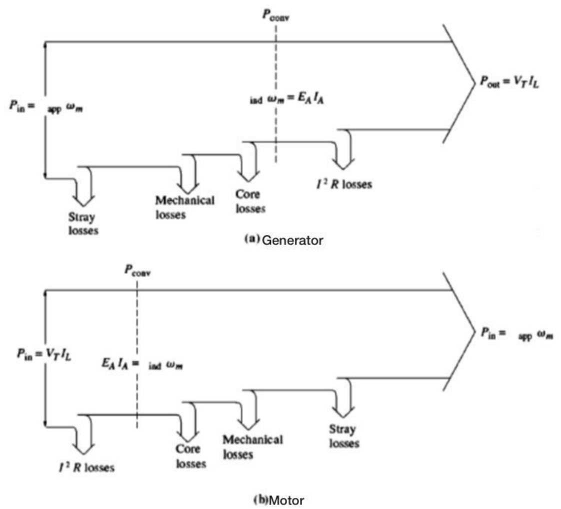
Where $P_F \rightarrow$ ***field circuit loss,*** $I_F \rightarrow$ ***field current,*** $R_F \rightarrow$ ***field resistance***

$*$ ***the brush losses it is given by the equation:***

$$P_{BD} = V_{BD} I_A$$

where $P_{BD} \rightarrow$ ***brush drop loss,*** $V_{BD} \rightarrow$ ***brush voltage drop,*** $I_A \rightarrow$ ***armature current***

Page(42 in slides)



(a) Generator

(b) Motor

$$P_{conv} = \tau_{ind} \omega_m$$

$$P_{conv} = E_A I_A$$

$*$ ***The efficiency of a DC machine is defined by the equation***

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \qquad \eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%$$

DC Machines-Motors

$*$ *the speed regulation (SR) of a motor is defined by*

$$SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \times 100\% \;\; (nl \rightarrow no\ load, fl \rightarrow full\ load)$$

$$SR = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

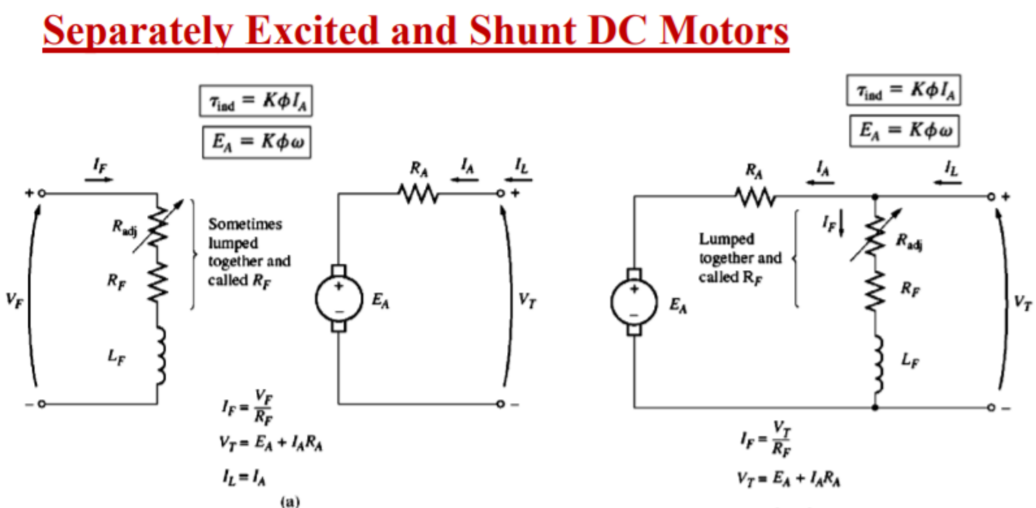


FIGURE 9–5
(a) The equivalent circuit of a separately excited dc motor. (b) The equivalent circuit of a shunt dc motor.

$$* \ Speed\ of\ the\ motor \rightarrow \frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

#Changing the Field Resistor Control Method:

1. *Increaseing* R_F *causes* $I_F = (V_F/R_F \uparrow)$ *to decrease.*

2. *Decreasing* I_F *decreases* $\emptyset$

3. *Decreasing* $\emptyset$ *lowers* $E_A = (K\emptyset \downarrow \omega)$.

4. *Decreasing* E_A *increases* $I_A = ((V_T - E_A \downarrow)/R_A)$

5. *Increasing* I_A *increase* $\tau_{ind} = (K\emptyset \downarrow I_A \Uparrow)$, *with the change in* I_A *dominant over the change in flux.*

6. *Increasing* τ_{ind} *makes* $\tau_{ind} > \tau_{load}$, *and the speed* ω *increases*

7. *Increasing* ω *increases* $E_A = (K\emptyset\omega \uparrow)$ *again.*

$8. \textit{Increasing } E_A \textit{ decreases } I_A$

$9. \textit{Decreasing } I_A \textit{ decreases } \tau_{ind} \textit{ untill } \tau_{ind} = \tau_{load} \textit{ at a higher}$

$\textit{speed } \omega$

#Changing the Armature Voltage Control Method

$1. \textit{Increasing } V_T \textit{ increases } I_A = ((V_T \uparrow - E_A)/R_A)$

$2. \textit{Increasing } I_A \textit{ increases } \tau_{ind} = (K\emptyset I_A \uparrow)$

$3. \textit{Increasing } \tau_{ind}\textit{, makes } \tau_{ind} > \tau_{load} \textit{ and the speed } \omega \textit{ increases}$

$4. \textit{Increasing } \omega \textit{ increases } E_A = (K\emptyset\omega \uparrow)$

$5. \textit{Increasing } E_A \textit{ decreases } I_A = ((V_T - E_A \uparrow)/R_A)$

$6. \textit{Decreasing } I_A \textit{ decreases } \tau_{ind} \textit{ until } \tau_{ind} = \tau_{load} \textit{ at a higher speed } \omega$

#Inserting a Resistor in Series with the Armature Circuit

$1. \textit{Increasing } R_A \textit{ dicreases } I_A = ((V_T - E_A)/R_A \uparrow)$

$2. \textit{Decreasing } I_A \textit{ decreases } \tau_{ind} = (K\emptyset I_A \downarrow)$

$3. \textit{decreasing } \tau_{ind}\textit{, makes } \tau_{ind} < \tau_{load} \textit{ and the speed } \omega \textit{ decreases}$

$4. \textit{decreasing } \omega \textit{ decreases } E_A = (K\emptyset\omega \downarrow)$

$5. \textit{Decreasing } E_A \textit{ increases } I_A = ((V_T - E_A \downarrow)/R_A)$

$6. \textit{Increasing } I_A \textit{ increases } \tau_{ind} \textit{ until } \tau_{ind} = \tau_{load} \textit{ at a lower speed } \omega$

---

the internal generated voltage of a DC machine with its speed expressed in revolution per minute is given by

$$e_A = k'\emptyset n$$

---

the relationship between the speeds and internal generated voltages of the motor at two different conditions is thus

$$\frac{E_{A2}}{E_{A1}} = \frac{K'\emptyset n_2}{K'\emptyset n_1} \rightarrow n_2 = \frac{E_{A2}}{E_{A1}} n_1$$

$\boldsymbol{The\ induced\ torque\ for\ any\ other\ load\ can\ be\ found\ from}$

$\boldsymbol{the\ fact\ P_{conv} = E_A I_A = \tau_{ind}\omega \rightarrow}$

$$\boldsymbol{\tau_{ind} = \frac{E_A I_A}{\omega}}$$

$\boldsymbol{the\ effective\ shunt\ field\ current\ of\ the\ motor\ is}$

$$\boldsymbol{I^*_F = I_F - \frac{\mathcal{F}_{AR}}{N_F}\ , \mathcal{F}_{AR}[A.turns]}$$

$*\ \boldsymbol{If\ E_{A1} = E_{A2}\ the\ equation\ of\ motor}\ speed\ reduces\ as$

$$\boldsymbol{1 = \frac{\emptyset_2 n_2}{\emptyset_1 n_1} \rightarrow n_1 = \frac{\emptyset_1}{\emptyset_2} n_2}$$

$\boldsymbol{Series\ DC\ Motors}$

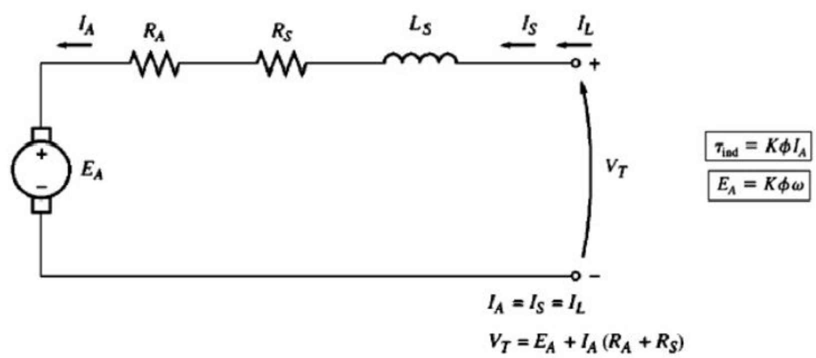


FIGURE 9–20
The equivalent circuit of a series dc motor.

$*$ The output characteristic of series DC motors can be derived from the induced voltage and torque equations of the motor plus Kirchhoff′s voltage law such as

$\boldsymbol{\phi = cI_S = cI_A\ since\ I_A = I_S\ then\ \phi = cI_A}$

$\boldsymbol{Recall\ \tau_{ind} = I_A K\phi = cKI_A^2 \Longrightarrow\ \tau_{ind} \propto I_A^2}$

$\boldsymbol{Note\ that\ I_A = \sqrt{\frac{\tau_{ind}}{cK}}\ and\ \phi = c\sqrt{\frac{\tau_{ind}}{cK}} = \sqrt{\frac{c\tau_{ind}}{K}}}$

$$V_T = E_A + I_A(R_A + R_S) = K\phi\omega + I_A(R_A + R_S)$$

$*$ **The output characteristic of series DC motors can be derived from the induced voltage and torque equations of the motor plus Kirchhoff′s voltage law such as**

$$V_T = K\sqrt{\frac{c\tau_{ind}}{K}}\,\omega + \sqrt{\frac{\tau_{ind}}{cK}}\,(R_A + R_S) \Longrightarrow \omega = \frac{V_T}{\sqrt{cK}}\frac{1}{\sqrt{\tau_{ind}}} - \frac{(R_A + R_S)}{cK}$$

#Transformers:

*Ideal transformer

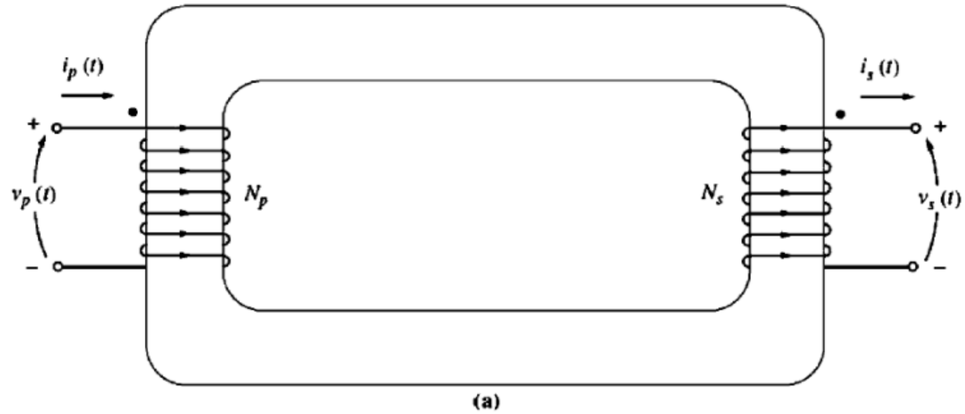


FIGURE 2–4
(a) Sketch of an ideal transformer.

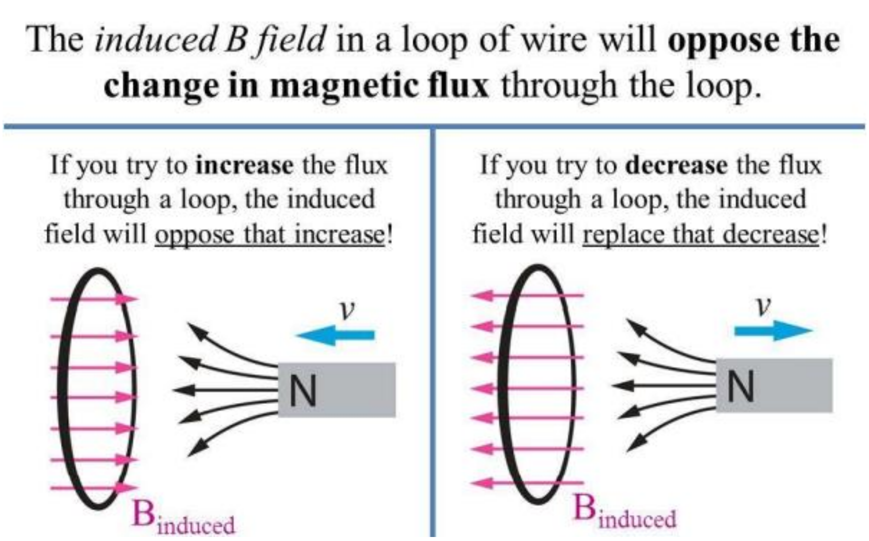
$* \; Faraday's \; law \; of \; induction$

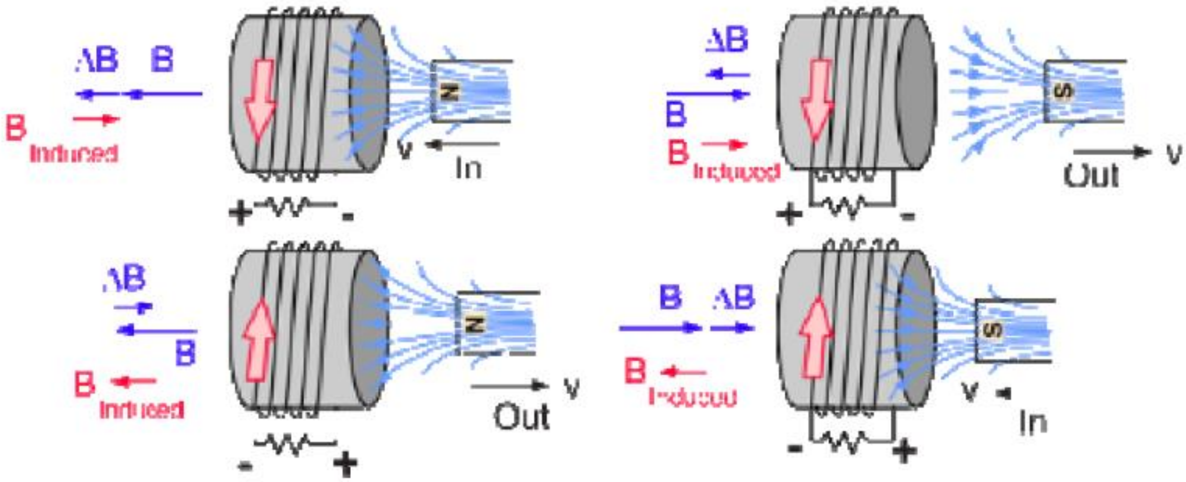
$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

ε: $induced \; emf$, N: $numper \; of \; loops$, $\Delta\Phi/\Delta t$: $rate \; of \; change \; of \; flux \; with \; time$

$minus \; sign \; from \; lenz's \; law$

$* \; Lenz's \; Law$





$* \, Since \, the \, magetomotive \, forces \, are \, equal \, in \, primary \, and \, secondary \, windings$

$$\mathcal{F}_P = \Phi\mathcal{R} = \mathcal{F}_S \rightarrow N_P i_P = N_S i_S \rightarrow \frac{N_P}{N_S} = \frac{i_S}{i_P} = a$$

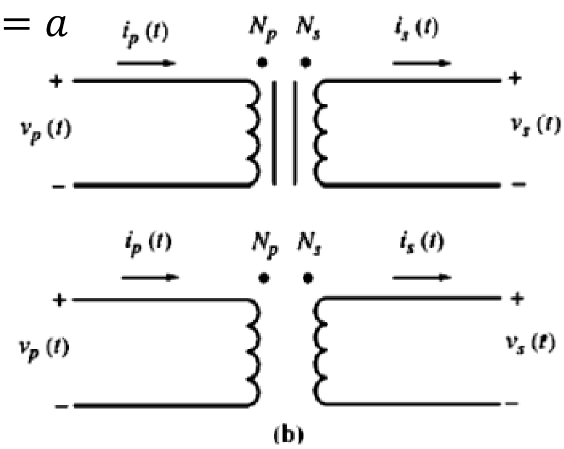


FIGURE 2–4
(b) Schematic symbols of a transformer.

$where \, \boldsymbol{a} \, is \, the \, \boldsymbol{transformer \, turns \, ratio}$

$* \, From \, faraday's \, laws$

$V \, is \, proportional \, to \, N \, (V \propto N)$

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = a$$

$* \, V_P i_P = V_S i_S \rightarrow S_{in} = S_{out}$

$* \, P_{in} = V_P i_P(\theta_P), * \, P_{out} = V_S i_S(\theta_S)$

$* \, since \, V_p i_p = V_s i_s \, and \, since \, voltage \, and \, current \, angles \, are \, unaffected \, by \, an \, ideal \, transformer \, then \, \theta_p = \theta_s$

$*$ The primary and secondary windings of an ideal transformer have the same power factor then $P_{in} = P_{out}$ *and then* $Q_{in} = Q_{out}$

$\#If\ N_P < N_S \rightarrow V_P < V_S \rightarrow i_P > i_S\ (step-up\ transformer)$

$\#If\ N_P > N_S \rightarrow V_P > V_S \rightarrow i_P < i_S\ (step-down\ transformer)$

∗ The impedance of a device or an element is defined as the ratio of the phasor Voltage across it to the phasor current flowing through it

$$Z_L = \frac{V_L}{I_L}$$

$$* Z'_L = \frac{V_P}{I_P} = \frac{aV_S}{I_S/a} = a^2 \frac{V_S}{I_S}$$

$$\rightarrow Z'_L = a^2 Z_L$$

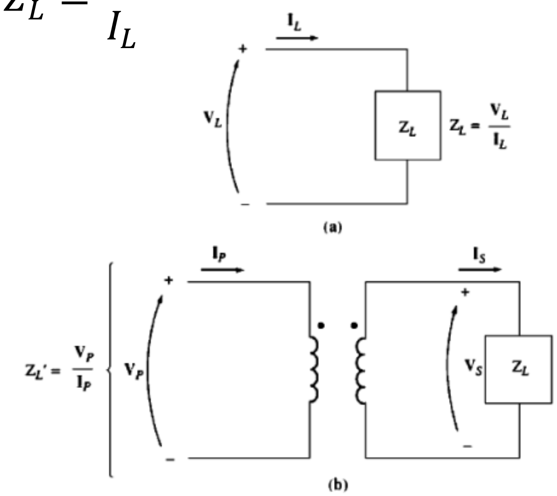


FIGURE 2–5
(a) Definition of impedance. (b) Impedance scaling through a transformer.

$$* I_{line} = \frac{V}{Z_{line} + Z_{load}}, V_{load} = I_{line} Z_{load}, P_{loss} = (I_{line})^2 R_{line}, Z_{eq} = Z_{line} + Z'_{load}$$

#Operation of real transformer

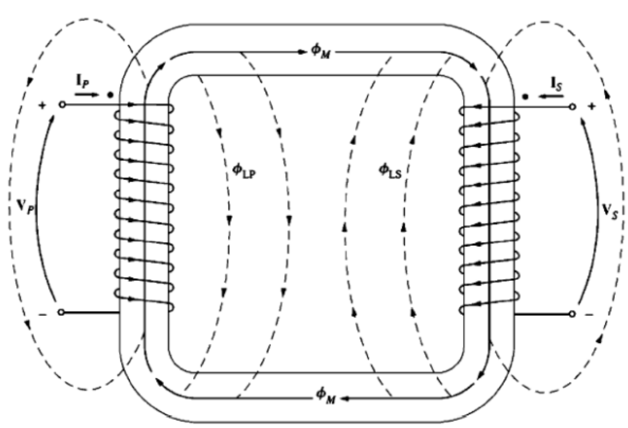


FIGURE 2–10
Mutual and leakage fluxes in a transformer core.

#The questions about this case will be concept questions

* The flux in the primary coil of the transformer can thus be divided into two components: a mutual flux, which remains in the core and links both windings, and a small leakage flux, which passes through the primary winding but returns through the air, bypassing the secondary winding:

$$\bar{\Phi}_P = \Phi_M + \Phi_{LP}$$

$\bar{\Phi}_P$: *total average primary flux*, Φ_M: *flux component linking both primary and secondary coils*, Φ_{LP}: *primary leakage flux*

* There is a similar division of flux in the secondary winding between mutual flux and leakage flux which passes through the secondary winding but returns through the air, by passing the primary winding:

$$\bar{\Phi}_S = \Phi_M + \Phi_{LS}$$

$\bar{\Phi}_S$: *total average secondary flux*, Φ_M: *flux component linking both primary and secondary coils*, Φ_{LS}: *secondary leakage flux*

* Since in a well designed transformer $\Phi_M \gg \Phi_{LP}$ and $\Phi_M \gg \Phi_{LS}$ the ratio of the total voltage on the primary of a transformer to the total voltage on the secondary of a transformer is approximately

* The smaller the leakage fluxes of the transformer are, the closer the total transformer voltage ratio approximates that of the ideal transformer

$$\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a$$

* *Open − circuit test*:

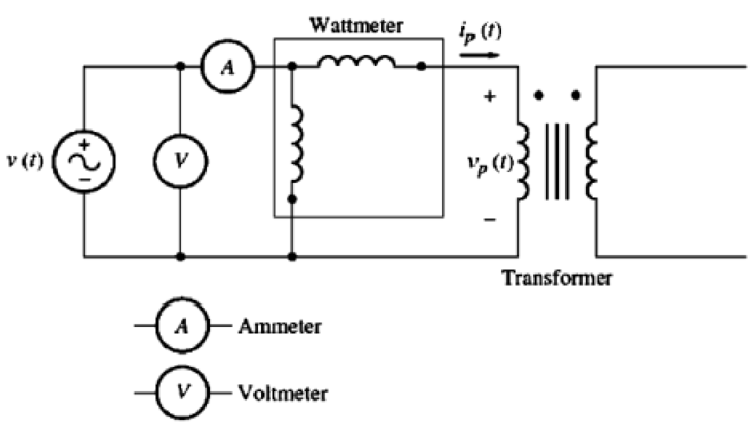


FIGURE 2–19
Connection for transformer open-circuit test.

- The conductance of the core − loss resistor is given by

$$G_c = \frac{1}{R_c}$$

- The susceptance of the magnetizing inductor is given by

$$B_M = \frac{1}{X_M}$$

- Since these two elements are in parallel, their admittances add, and the total excitation admittance is

$$Y_E = G_c - jB_M$$

$$Y_E = \frac{1}{R_c} - j\frac{1}{X_M}$$

- the following relations can be obtained

$$|Y_E| = \frac{I_{oc}}{V_{oc}}$$

$$PF = \cos(\theta) = \frac{P_{OC}}{V_{OC}I_{OC}} \Rightarrow \theta = cos^{-1}\left(\frac{P_{OC}}{V_{OC}I_{OC}}\right)$$

$$Y_E = \frac{I_{oc}}{V_{oc}}\angle - \theta \Rightarrow Y_E = \frac{I_{oc}}{V_{oc}}\angle - cos^{-1}(PF)$$

$$R_c = \frac{1}{G_c} = \frac{1}{Y_E cos(\theta)}$$

$$X_M = \frac{1}{B_M} = \frac{1}{Y_E sin(\theta)}$$

$* \, Short - circuit \; test$:

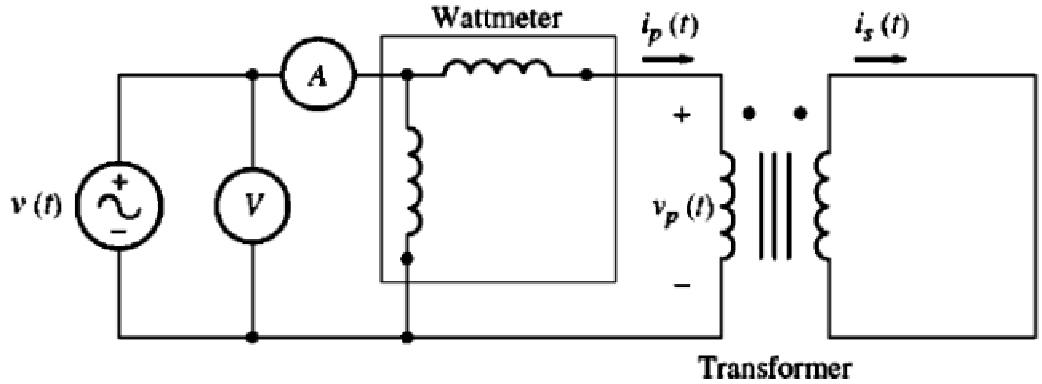


FIGURE 2–20
Connection for transformer short-circuit test.

∘ The magnitude of the series impedances referred to the primary side of the transformer is

$$|Z_{SE}| = \frac{V_{sc}}{I_{sc}}$$

∘ *The following relations can be obtaind*

$$PF = \cos(\theta) = \frac{P_{SC}}{V_{SC}I_{SC}} \Rightarrow \theta = cos^{-1}\left(\frac{P_{SC}}{V_{SC}I_{SC}}\right)$$

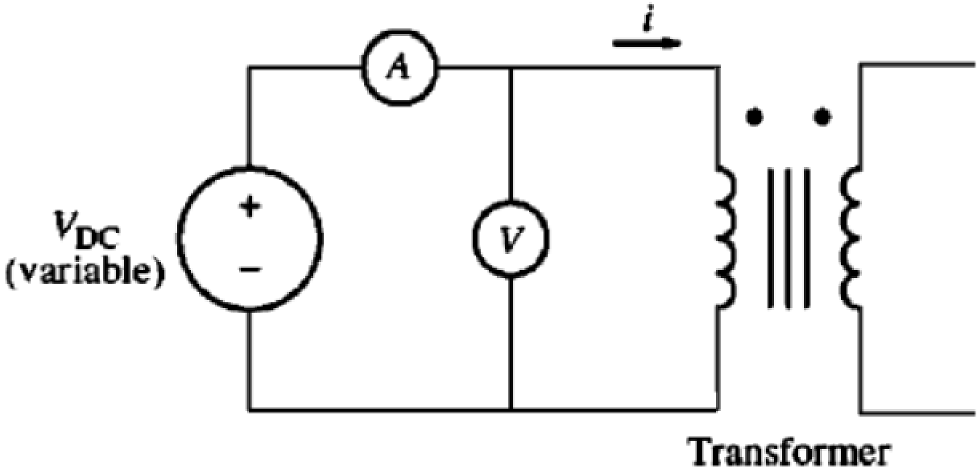
$$Z_{SE} = \frac{V_{SC}\angle 0}{I_{SC}\angle -\theta} = \frac{V_{SC}}{I_{SC}}\angle\theta$$

$$R_{eq} = Z_{SE}cos(\theta)$$

$$X_{eq} = Z_{SE}sin(\theta)$$

$$Z_{SE} = R_{eq} + jX_{eq} = (R_P + a^2R_S) + j(X_P + a^2X_S)$$

$* \ DC \ test$:



- Using Ohm's law and measurements of voltage and current, the internal resistor of the winding can be determined as

R = V / I

- It should be noted that this test can be done on both sides of the transformer
- Moreover, the DC power supply should be adjusted so that maximum DC current is not exceeded.
- **Note that the coil can handle a DC current that is certainly less than the rated AC current.**

$* \ \boldsymbol{Voltage\ regulation\ (VR)}$

$$\boldsymbol{VR = \frac{V_{S.nl} - V_{S.fl}}{V_{S.fl}} \times 100\%}$$

$$\boldsymbol{VR = \frac{V_P/a - V_{S.fl}}{V_{S.fl}} \times 100\%}$$

$$\circ \ \boldsymbol{\frac{V_P}{a} = V_S + R_{eq}I_S + jX_{eq}I_S}$$

$*\ Efficiency$

Transformers are also compared and judged on their efficiencies. The efficiency of a device is defined by the equation

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%$$

To calculate the efficiency of a transformer at a given load, just add the losses the equation of efficiency such that

$$P_{out} = V_S I_S cos\theta_S$$

$$\eta = \frac{V_S I_S cos\theta_S}{P_{Cu} + P_{core} + V_S I_S cos\theta_S} \times 100\%$$

$$P_{Cu} = (I_S)^2 R_{eq}$$

$$P_{core} = \frac{(V_P/a)^2}{R_C}$$

#Autotransformer:

$$\mathbf{V}_L = \mathbf{V}_C \qquad (2\text{–}70)$$

$$\mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{SE} \qquad (2\text{–}71)$$

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} \qquad (2\text{–}72)$$

$$\mathbf{I}_H = \mathbf{I}_{SE} \qquad (2\text{–}73)$$

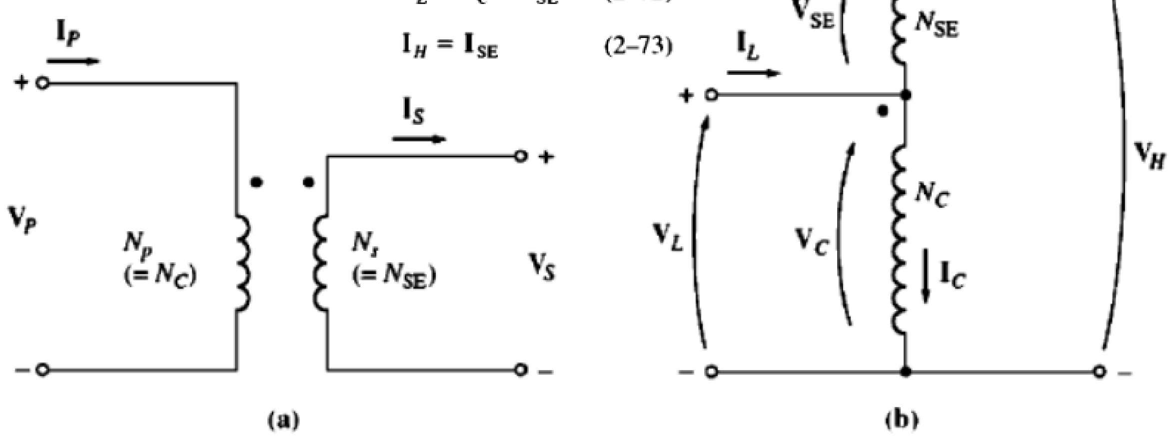


FIGURE 2–32
A transformer with its windings (a) connected in the conventional manner and (b) reconnected as an autotransformer.

$$\mathbf{V}_L = \mathbf{V}_C \qquad (2\text{–}70)$$

$$\mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{SE} \qquad (2\text{–}71)$$

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} \qquad (2\text{–}72)$$

$$\mathbf{I}_H = \mathbf{I}_{SE} \qquad (2\text{–}73)$$

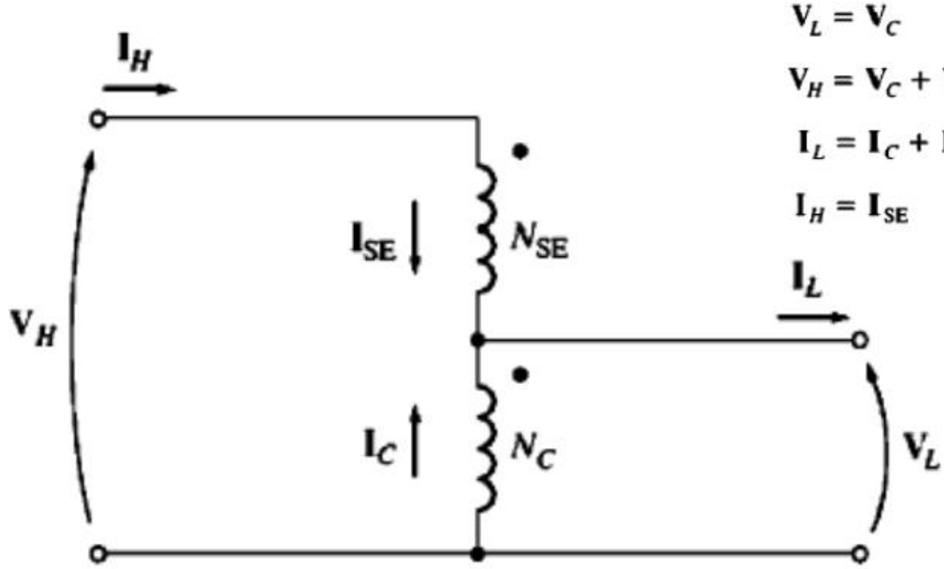


FIGURE 2–33
A step-down autotransformer connection.

*Voltage relationship:

$V_H = V_C + V_{SE}$ But $V_C/V_{SE} = N_C/N_{SE}$, $so \rightarrow$

$$V_H = V_C + \frac{N_{SE}}{N_C} V_C, noting\ that\ V_L = V_C\ then\ we\ get \rightarrow$$

$$V_H = V_L + \frac{N_{SE}}{N_C} V_L = \frac{N_{SE} + N_C}{N_C} V_L \rightarrow$$

$$\frac{V_L}{V_H} = \frac{N_C}{N_{SE} + N_C}$$

---

*Current relationship:

$I_L = I_C + I_{SE}$, $also$ $I_C = (N_{SE}/N_C)I_{SE}$, $so \rightarrow$

$$I_L = \frac{N_{SE}}{N_C} I_{SE} + I_{SE}, Noting\ that\ I_H = I_{SE}, then\ we\ get \rightarrow$$

$$I_L = \frac{N_{SE}}{N_C} I_H + I_H = \frac{N_{SE} + N_C}{N_C} I_H \rightarrow$$

$$\frac{I_L}{I_H} = \frac{N_{SE} + N_C}{N_C}$$

---

*Apparent Power Rating Advantage

$S_{in} = V_L I_L$ and $S_{out} = V_H I_H$

$$S_{in} = S_{out} = S_{IO}$$

$$S_W = V_C I_C = V_{SE} I_{SE}$$

$$S_W = V_C I_C = V_L(I_L - I_H) = V_L I_L - V_L I_H$$

$$S_W = V_L I_L - V_L I_L \left(\frac{N_C}{N_{SE} + N_C}\right) = V_L I_L \left(1 - \frac{N_C}{N_{SE} + NC}\right)$$

$$S_W = V_I L_I \left(\frac{N_{SE} + NC - N_C}{N_{SE} + NC}\right) = S_{IO} \left(\frac{N_{SE}}{N_{SE} + NC}\right)$$

$*$ Therefore, the ratio of the apparent power in the primary and secondary of the autotransformer to the apparent power actually traveling through its windings is

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} > 1$$

#Three-Phase Transformers (Connection)

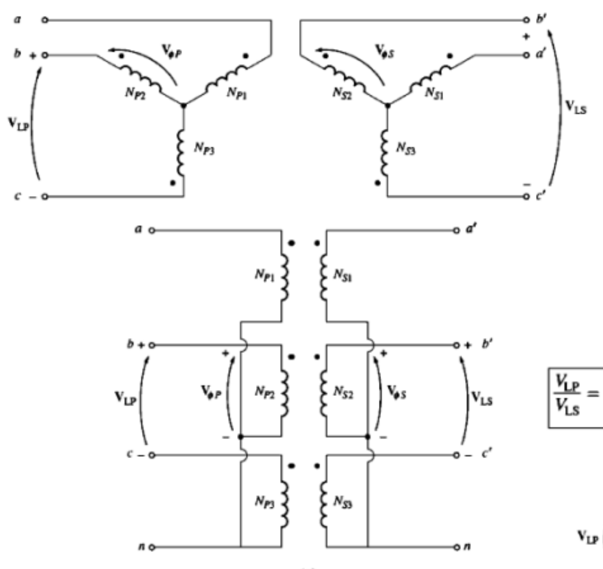
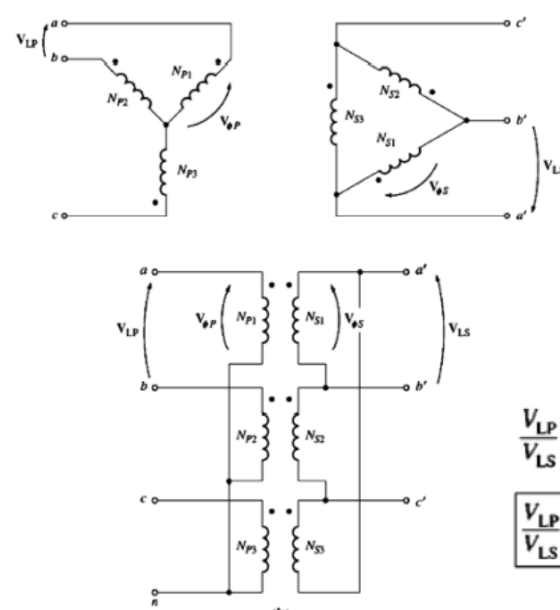


FIGURE 2-38
Three-phase transformer connections and wiring diagrams: (a) Y–Y

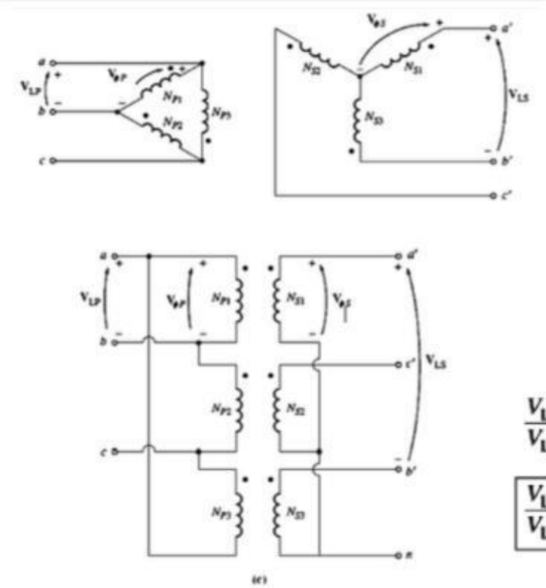
$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a \qquad \text{Y–Y}$$

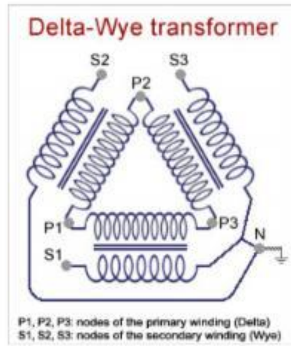


$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}}$$

$$\frac{V_{LP}}{V_{LS}} = \sqrt{3}a \qquad \text{Y–}\Delta$$

FIGURE 2-38
(b) Y–Δ (*continued*)

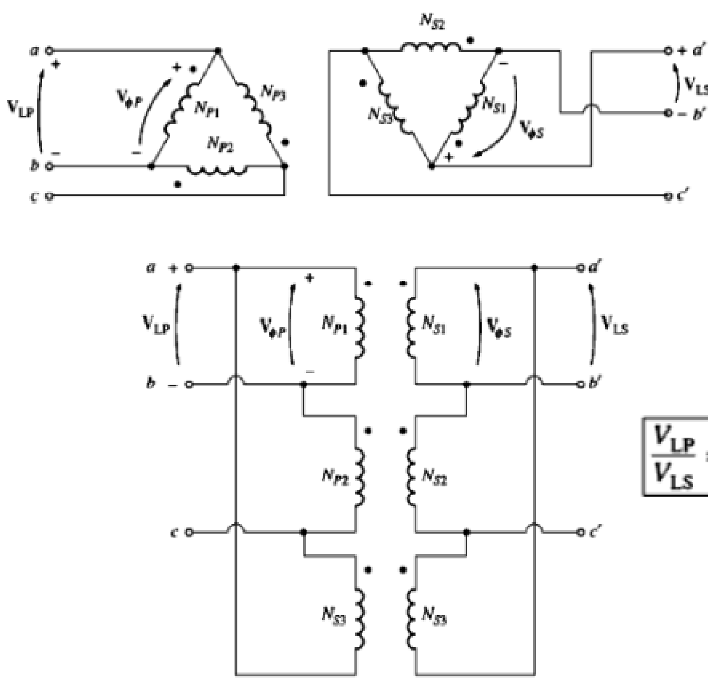




$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}}$$

$$\frac{V_{LP}}{V_{LS}} = \frac{a}{\sqrt{3}} \qquad \Delta-Y$$

FIGURE 2-38
(c) Δ–Y (continued)



(d)

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \qquad \Delta-\Delta$$

FIGURE 2-38
(d) Δ–Δ *(concluded)*

∗ The impedance, voltage regulation, efficiency, and similar calculations for three − phase transformers are done on a per − phase basis, using exactly the same techniques already developed for single − phase transformers.

*Basic Concepts of Operation

$The\ speed\ of\ the\ magnetic\ field's\ rotation\ is\ given\ by$

$$n_{sync} = \frac{120f_e}{P}, f_e: system\ frequency\ in\ hertz, P: num\ of\ poles$$

The voltage induced in a given rotor bar is given by the equation

$$e_{ind} = (v \times B) \cdot l$$

$v: the\ velocity\ of\ the\ bar\ relative\ to\ the\ magnetic\ field,$

$B: the\ magnetic\ flux\ density\ vector,$

$l: the\ length\ of\ conductor\ in\ the\ magnetic\ field$

The induced torque in the machine is given by

$$\tau_{ind} = kB_R \times B_S$$

$Note\ that\ k = K/\mu\ and\ K\ is\ a\ constant\ dependent\ on\ the\ construction\ of\ the\ machine$

In fact, the parameter k will not be constant since μ varies with the amount

of magnetic saturation in the machine.

Two terms are commonly used to define the relative motion of the rotor

and the magnetic fields

(1)One is slip speed, defined as the difference between synchronous

speed and rotor speed:

$$n_{slip} = n_{sync} - n_m$$

n_{slip}: the slip speed of the machine, n_{sync}: the speed of the rotating magnetic fields,

n_m: the mechanical shaft speed of motor

(2)The other is term used to describe the relative motion is slip, which is the

relative speed expressed on a percentage basis, and it is defined as

$$s = \frac{n_{slip}}{n_{sync}} \times 100\% \rightarrow s = \frac{n_{sync} - n_m}{n_{sync}} \times 100\% \rightarrow s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} \times 100\%$$

*If the rotor turns at synchronous speed s = 0, while if the rotor is stationary s = 1.

$*$ It is possible to express the mechanical speed of the rotor shaft in terms of synchronous speed and slip.

By Solving the previous Equations for mechanical speed yields

$n_m = (1 - s)n_{sync}$

$\omega_m = (1 - s)\omega_{sync}$

$* At\ n_m = 0\ r/min\ the\ rotor\ frequency\ f_r = f_e and\ the\ slip\ s = 1.$

$* At\ n_m = n_{sync}\ the\ rotor\ frequency\ f_r = 0Hz\ and\ the\ slip\ s = 0$

$*$ Based on the definition of the slip, the rotor frequency can be expressed as

$f_r = sf_e$

*From the previous equations:

$$f_r = \frac{n_{sync} - n_m}{n_{sync}} f_e, but\ n_{sync} = 120f_e/P\ \ so \rightarrow$$

$$f_r = (n_{sync} - n_m)\frac{P}{120f_e} f_e \rightarrow f_r = \frac{P}{120}(n_{sync} - n_m)$$

---

*Equivalent Circuit

—magnitude of the induced voltage at any slip will be given by the equation

$E_R = sE_{RO}$

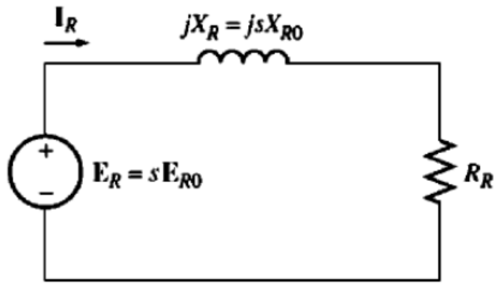
—The frequency of the induced voltage at any slip will be given by the equation

$f_r = sf_e$

—The reactance of an induction motor rotor depends on the inductance of the rotor and the frequency of the voltage and current in the rotor. With a rotor inductance of L_R, the rotor reactance is given by

$X_R = \omega_r L_R = 2\pi f_r L_R \rightarrow X_R = 2\pi s f_e L_R = s(2\pi f_e L_R) = sX_{RO}$

$Where\ X_{RO}$ is the blocked − rotor rotor reactance.



$$I_R = \frac{E_R}{R_R + jX_R}$$

$$\boxed{I_R = \frac{E_R}{R_R + jsX_{R0}}} \qquad (7\text{–}12)$$

$$\boxed{I_R = \frac{E_{R0}}{R_R/s + jX_{R0}}} \qquad (7\text{–}13)$$

FIGURE 7–9
The rotor circuit model of an induction motor.

$$Z_{R,eq} = R_R/s + jX_{R0}$$

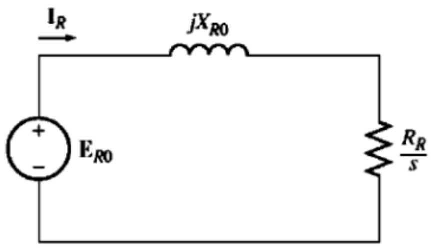


FIGURE 7–10
The rotor circuit model with all the frequency (slip) effects concentrated in resistor R_R.

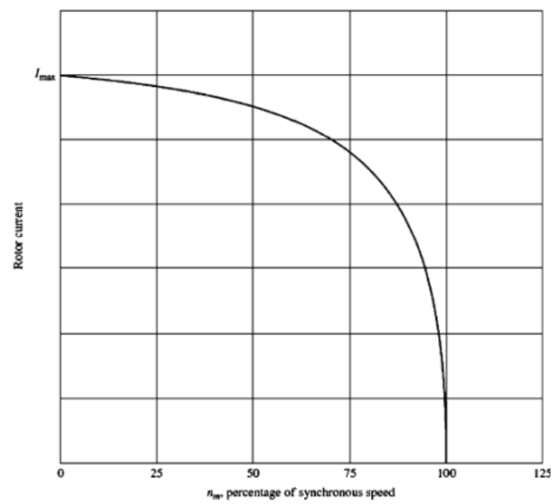


FIGURE 7–11
Rotor current as a function of rotor speed.

—In an ordinary transformer, the voltages, currents, and impedances on the secondary side of the device can be referred to the primary side by means of the turns ratio of the transformer:

$V_P = V'_S = aV_S$

$I_P = I'_S = \frac{I_S}{a}$

$Z'_S = a^2 Z_S$

—If the effective turns ratio of an induction motor is a_{eff} , then the transformed rotor voltage becomes

$E_1 = E'_R = a_{eff} E_{RO}$

$I_2 = \frac{I_R}{a_{eff}}$

$$Z_2 = a^2{}_{eff}\left(\frac{R_R}{s} + jX_{RO}\right)$$

$$R_2 = a^2{}_{eff}R_R$$

$$X_2 = a^2{}_{eff}X_{RO}$$

—The efficiency of an AC machine is defined by the equation

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%$$

$$P_{in} = \sqrt{3}V_T I_L cos\theta$$

$$P_{out} = \tau_{load}\omega_m$$

- From the equivalent circuit of an induction motor and power flow diagram, the following relations can be obtained

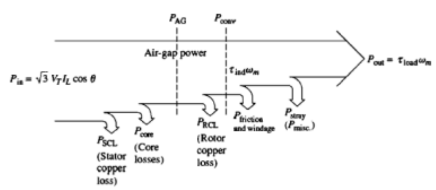


FIGURE 7–13
The power-flow diagram of an induction motor.

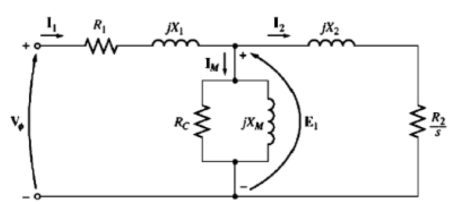


FIGURE 7–12
The per-phase equivalent circuit of an induction motor.

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{eq}} \qquad (7\text{–}23)$$

where

$$Z_{eq} = R_1 + jX_1 + \frac{1}{G_C - jB_M + \frac{1}{V_2/s + jX_2}} \qquad (7\text{–}24)$$

$$P_{SCL} = 3I_1^2 R_1$$

$$P_{core} = 3E_1^2 G_C$$

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

$$P_{RCL} = 3I_R^2 R_R$$

$$P_{RCL} = 3I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2$$

$$= 3I_2^2 R_2\left(\frac{1}{s} - 1\right)$$

$$P_{conv} = 3I_2^2 R_2\left(\frac{1 - s}{s}\right)$$

$$P_{RCL} = sP_{AG}$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= P_{AG} - sP_{AG}$$

$$P_{conv} = (1 - s)P_{AG}$$

$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

$$\tau_{ind} = \frac{(1 - s)P_{AG}}{(1 - s)\omega_{sync}}$$

Developed Torque

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

---

*Torque-Speed Characteristic

From utilizing the induction motor equivalent circuit and applying Thevenin theorem, the torque-speed characteristic curve can be obtained

$$\tau_{ind} = \frac{3V_{TH}^2 R_2/s}{\omega_{sync}[(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2]}$$

$$V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}$$

$$R_{TH} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2$$

$$X_{TH} \approx X_1$$

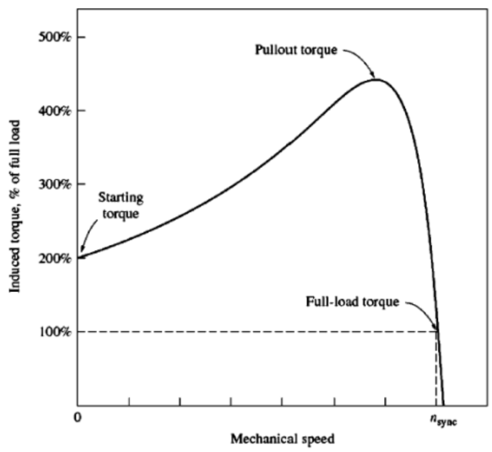


FIGURE 7–19
A typical induction motor torque–speed characteristic curve.

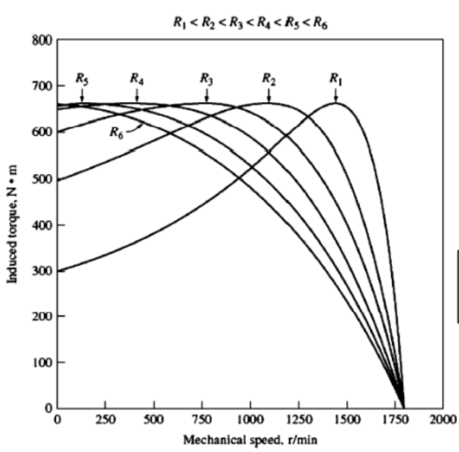
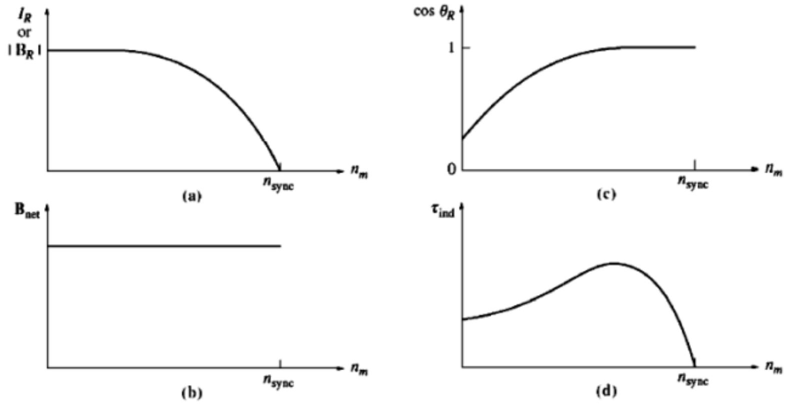


FIGURE 7–22
The effect of varying rotor resistance on the torque–speed characteristic of a wound-rotor induction motor.

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}$$

Curves that represent the characteristics of induction motors



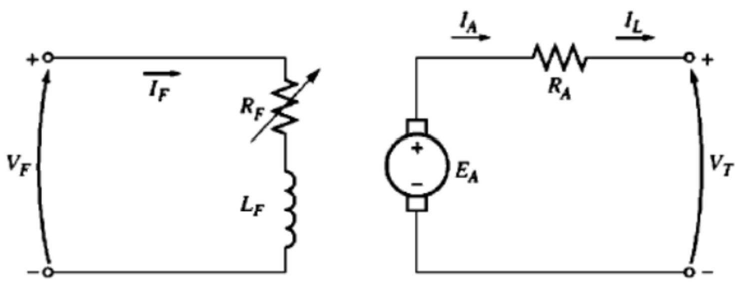
$PF_R = \cos\theta_R$

$$PF_R = \cos\left(\tan^{-1}\frac{sX_{R0}}{R_R}\right)$$

FIGURE 7–16
Graphical development of an induction motor torque–speed characteristic. (a) Plot of rotor current (and thus $|B_R|$) versus speed for an induction motor; (b) plot of net magnetic field versus speed for the motor; (c) plot of rotor power factor versus speed for the motor; (d) the resulting torque–speed characteristic.

*Generators

Separately Excited DC Generators



$I_L = I_A$
$V_T = E_A - I_A R_A$
$I_F = \dfrac{V_F}{R_F}$

FIGURE 9–44
A separately excited dc generator.

For a constant speed $\quad V_T = E_A - I_A R_A \quad$ (9–41)

- Since the internal generated voltage E_A is given by the equation $E_A = K\phi{\downarrow}\omega$, there are two possible ways to control the voltage of this generator:

1. **Change the speed:** If ω increases, then $E_A = K\phi\omega\uparrow$ increases, so $V_T = E_A\uparrow - I_A R_A$ increases too.
2. **Change the filed current:** If R_F is decreased, then the field current increases ($I_F = V_F/R_F\downarrow$). Therefore, the flux ϕ in the machine increases. As the flux rises, $E_A = K\phi\uparrow\omega$ must rise too, so $V_T = E_A\uparrow - I_A R_A$ increases.

- In many applications, the speed ran ge of the prime mover is quite limited, so the terminal voltage is most commonly controlled by changing the field current.

—Voltage regulation (VR) is defined by the equation

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

—The total magnetomotive force in a separately excited generator is the field circuit magnetomotive force less the magnetomotive force due to armature reaction (AR):

$$\mathcal{F}_{net} = N_F I_F - \mathcal{F}_{AR}$$

—The equivalent field current of a separately excited DC generator is given by

$$I^*{}_F = I_F - \frac{\mathcal{F}_{AR}}{N_F}$$

—the difference between the speed of the magnetization curve and the real speed of the generator must be taken into account:

$$\frac{E_A}{E_{AO}} = \frac{n}{n_O}$$