اقتصاد هندسي

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للطالبة المبدعة
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إرادة - ثقة - تغيير

Chapter 1

• The purpose in this chapter is to make benefits out of any design exceed it's cost.

• Solutions to engineering problems must:

- Promote the survival of an organization.
- Embody creative ideas.
- Permit identification.
- Translate profitability to the 'bottom line'.

• There are 7 principles of engineering economy:

1- Develop the alternatives
2- Focus on the differences
3- use a consistent viewpoint
4- Use a common measure unit
5- Consider all relevant criteria
6- Make uncertainity explicit
7- Revisit your decisions

• Engineering economic analysis procedure:

Problem definition → Development of alternatives → Development of prospective outcomes → Selection of a decision criterion → Analysis and comparison of alternatives → selection of the preferred alternative → Performance monitoring and postevaluation of results.

تعريف المشكلة ← وضع حلول المشكلة ← النتيجة المستقبلية للحل

وضع معايير لأفضل حل ← مقارنة الحلول بناءً على المعايير ←

اختيار الحل الأفضل ← تقييم النتائج.

* Most engineering economy problems can be solved by using spread sheets.

Chapter 2

Short term alternatives :-

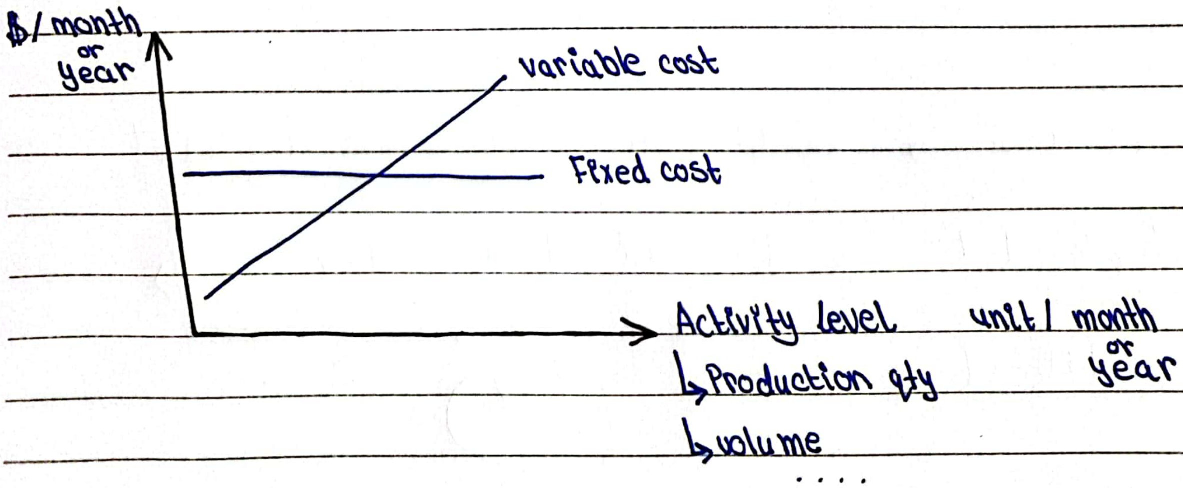
- Study period is short which is less than 1 year (useful life)
- Present economy studies
- Time value of money can be ignored (القدرة الشرائية)

* Examples on short term alternatives :

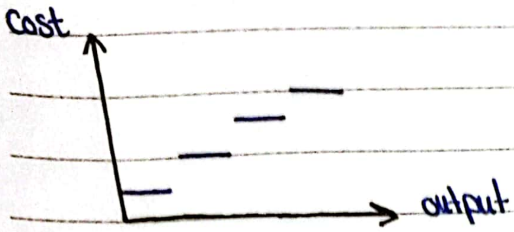
1- site selection for portable project equipment
2- Production of electronic devices
3- Airline operations
4- Best production schedule : materials, machines, operating conditions
5- Buy or Make decisions
   ↳ you either buy a service / thing or you do it yourself.
6- Energy studies

* Check the meanings

→ Variable and Fixed costs :



→ Incremental cost:



• for example,

تكلفة النقل : كل شاحنة بتوسع 100 منتج
و لهذه الشاحنة تكلفة بمقدار 150 $ فاذا
قامت شركة معينة بانتاج 100 منتج او اقل
اذا نحتاج شاحنة واحدة .

* Total cost = Fc + Vc

Fc ↳ Fixed costs

Vc ↳ variable costs : vc = cu × D

cu → cost per unit

D → quantity (demand)

Standard cost = $\frac{\text{Total cost}}{\text{Quantity output}}$

→ Cash and Non-Cash cost:-

inorder to be a cash cost it must follow those rules:

Payment of money — Actual cash flow — Two parties — Transaction

• Non-cash cost (Book cost):

you pay in an indirect way

→ life cycle cost:
→ Initial investment (1) أول المشروع
→ operation and maintenance (2) الصيانة
→ Disposal costs (3) انهاء المشروع

life cycle cost = 1+2+3 → the sum of all of them

* Breck Even Analysis

→ Breck even point :→ profit = zero

* Price and Demand are independent

↳ (when the price is constant and unchangable)

Profit = Total Revenue - Total cost

TR = selling Price (P) x D

Total cost = Fixed costs + variable costs

VC = cost / unit (CU) x D

* At Breck point:

$TR - TC = 0 \rightarrow \therefore TR = TC$

Note:-

when $TR > TC$ ∴ profit

$TR < TC$ ∴ Loss

$TR = TC$ ∴ Breck-even

- At the Breck point

$TR = TC \rightarrow P \times D = FC + (CU \times D)$

$$\rightarrow \boxed{D = \frac{FC}{P - CU}}$$

- Note:- maximum revenue = P x max . quantity → Dmax

Range of profit → from D^* to D max

Breck

Example: Given that Fixed costs in a production facility are $4000 / month
The product has: material cost $3 / unit , labor cost $2 / unit
selling price $11 / unit , Max capacity 1200 unit / month

1- Determine the B.E quantity (D^*):-

$TR = TC \rightarrow P \times D^* = FC + C_v D$

$$D^* = \frac{FC}{P - C_v} = \frac{4000}{11 - 5} = 667 \text{ unit / month}$$

2- Max Revenues at which D :-

$D = D_{max} = 1200$ unit / month

$TR_{max} = P D_{max} = 11 \times 1200 = \$13,200$

3- Max Profit at D :-

$D = D_{max} = 1,200$

Profit = TR - TC

$= PD - (FC + C_v D)$

$= 11 \times 1200 - (4,000 + 5 \times 1200)$

$= \$3,200$

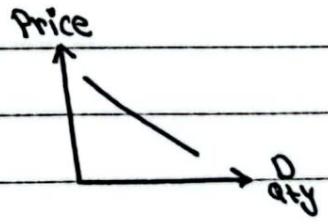
4- Range of Profit:-

$667 < D < 1200$

* Price and Demand are dependent :-

$P = \underbrace{a - bD}_{1}$

$a, b > 0$



* $TR = P \times D \rightarrow P = a - bD$

$TC = FC + C_v D$ (stays the same)

Max Revenues : $D' = \frac{a}{2b}$

Max Profit : $D'' = \frac{a - C_v}{2b}$

Five Apple

Range of Profit : D_1^* to (D_2^* or D_{max})

↳ حسب مين الأقل

$D_{1,2}^* = -bD^2 + (a - C_v)D - f_c = 0$

↳ Breck even quantities

because : (per unit)

Example: Fixed costs \$ 400 k / year (f_c) , variable costs \$ 131.5 / unit (C_v)
Price \$ 600 - 0.05D .

1- Are the price and demand dependenb or independent ?

Price and Demand are dependent

2- Sales volume (quantity) at which profit is a max :-
↳ (optimal sales volume)

$D'' = \frac{a - C_v}{2b} = \frac{600 - 131.5}{2(0.05)} = 4,685$ unit / year

3- Max Profit at D :-
↳ D''

$TR - TC = PD'' - (f_c + C_v D'') = \$147,461$ / year

$P = (a - bD'')$

4- Value of D when revenue is max :-

$D' = \frac{a}{2b} = \frac{600}{2(0.05)} = 6,000$ unit / year

5- Max Revenues :-

$TR_{max} = P \times D'$

$= (600 - 0.05 \times 6,000) \times 6,000$

$= 1,800,000$

Five Apple

6- Breck even quantities:-

$D^*_{1,2} = -bD^2 + (a - C_v)D - Fc = 0$ put on the calculator

$\rightarrow D^*_1 = 2,698 \quad D^*_2 = 6,672$

7- Range of Profit:-

$2,698 < D < 6,000$

↳ because it's smaller

* Note:

$D'' = \frac{D^*_1 + D^*_2}{2}$

* Chapter 4: The Time value of Money

1- Money has earning power (الاستفادة منها و استثمارها)
2- Money has purchasing power (مصاريف + شراء)
3- Money kept uninvested (تحويشهم)
↳ worst alternative

* Return to capital in the form of interest and profit :- (reasons)
- It pays the providers of capital
- payment for the risk the investor takes.
- providing a sufficient return to be financially attractive to the suppliers

* Interest: is money added to the borrowed capital

→ Simple Interest : infrequently used

I (total interest) → linearly proportional to:-

P (loan / amount borrowed) → \$, i (interest rate) ↳ %/year , N (interest periods) ↳ year → years

• F (total amount repaid) = P + I

→ Compound Interest

Time / year	Amount owed BOY	Interest during year	Amount owed EOY
1	\$ 1,000	1,000 x 10% = \$ 100	\$1,100 1,000 (1+10%)
2	\$ 1,100	1,100 x 10% = \$110	\$1,210 1,000(1+10%)(1+10%)
3	\$ 1,210	1,210 x 10% = \$121	\$1,331 $1,000(1+10\%)^3$

$$\therefore F = P(1+i)^n$$

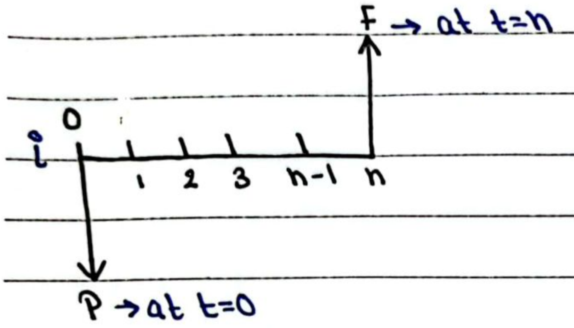
* $F = P(1+i)^n$ → compounded interest rate

at $t=n$ ← F: Total owed amount (future value)

P ↳ at $t=0$
- principal
- original amount
- borrowed capital (present value)

→ Cash flow diagram (important)

1. cash amounts ($)
2. cash timing ($t=0,1,2,\dots,n$)
3. interest rate (i)
4. point of view → lender (+) ⎫
   ↳ borrower (−) ⎭ - opposite



• Example :- An engineer borrows $3,000 at 12% per year compounded annually. What amount should be repaid after 3 years?

→ We have 2 ways to solve:-

1. Compounding tables :

Find F Given P

$F = P(F/P, i, n)$

$F = 3{,}000\,\underbrace{(F/P, i, n)}_{\text{compounding Factor}}$

$= 3{,}000(1.4049) = \$4{,}214$

2. Formula : $F = P(1+i)^n$

$F = 3{,}000(1+0.12)^3 = \$4{,}214$

Example: How much should be deposited now in order to obtain $ 2,500, 6 years from now, when $i = 5\%$?

1. Tables:

Find P Given F

$P = F(P/F, i, n)$

$= 2{,}500\ (0.74622) = \$1{,}865$

2. Formula:

$F = P(1+i)^n \rightarrow P = \dfrac{F}{(1+i)^n}$

$= \dfrac{2{,}500}{(1+0.05)^6}$

$= \$1{,}865$

Example: How much should be deposited now in order to obtain \$ 2,500, 6 years from now, when $i = 5\%$?

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$= \$1,865$

Example: what interest rate doubles your investment in 10 years?

$2\cancel{P} = \cancel{P}(1+i)^{10}$

$i = 2^{\frac{1}{10}} - 1 = 7.177\%$ per year

Example: How long do you have to wait until you double your money if you are paid 10% ?

$2\cancel{P} = \cancel{P}(1+0.10)^n$

$n = \frac{\ln 2}{\ln 1.10} = 7.27 \rightarrow 7$

8 → bcz it's closer to the result (from the tables)

* Second

يودع

Example: Ammar deposits in his account \$3,500 at the end of each year for 5 years. How much will he find in his account at the end of those 5 years? (when $i = 5\%$ per year)

$A = \$3{,}500$

Find F Given A

$F = 3{,}500\,(F/A, 5\%, 5)$

$= 3{,}500\,(5.5256)$

$= \$19{,}340$

or we can solve it by the formula:

$$F = 3500\left[\frac{(1+0.05)^5 - 1}{0.05}\right] = \$19{,}340$$

Example: How much can Derar borrow from a bank at 5% per year, if he can pay \$1,500 at the end of the year for 5 years?

Find P Given A

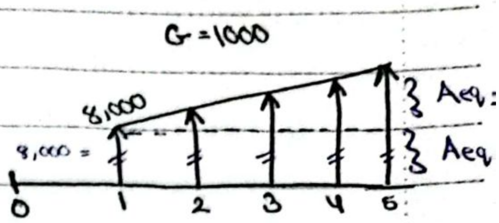
$P = 1{,}500\,(P/A, 5\%, 5)$

$= 1{,}500\,(4.3295)$

$= \$6{,}494$

* Uniform and Geometric series of Payments :

• example: uniform gradient series of payments starts with \$ 8,000 (EOY 1) and increases by \$ 1,000 / year. Find the equivalent annual series of payments to 5 payments of this gradient series of payments, when $i = 12\%$ per year ?



$A_{eq} = A_{eq_1} \oplus A_{eq_2}$

$= 8{,}000 + G(A/G, 12\%, 5)$

$= 8{,}000 + 1{,}000(1.7746)$

$= \$ 9{,}774.6$

→ Find P_{eq} :

$P_{eq} = 9{,}774.6\,(3.6048)$

$= \$ 35{,}235.5$

* increasing G

(+G)

→ Find F_{eq} :

$F_{eq} = 9{,}774.6\,(6.3528)$

$= \$ 62{,}096$

→ ~~wh~~ we can find it by:

Find F Given P

• example : Uniform Gradient Series : $EOY_1 \rightarrow \$ 6{,}000$ $EOY_2 \rightarrow \$ 3{,}000$ when $i = 5\%$, find A_{eq} ?

$G = \dfrac{y_2 - y_1}{x_2 - x_1} = \left| \dfrac{6{,}000 - 3{,}000}{6 - 1} \right| = \$ 600 / yr$

* decreasing G

(−G

$A_{eq} = A_{eq_1} \ominus A_{eq_2}$

$= 6{,}000 - 600\,(2.3579)$

$= \$ 4{,}585$ / year (1~6)

• example : what is the present equivalent of 15 payments having a first year base of \$10,000, increasing by 8% per year, when the interest rate is 12% per year ?

$$P_{eq} = \frac{A_1}{i - f}\left[1 - (1+i)^{-n}(1+f)^n\right] \qquad i \neq f$$

$$= \frac{10{,}000}{12\% - 8\%}\left[1 - (1.12)^{-15}(1.08)^{15}\right]$$

$$= \$105{,}114$$

* if the question was (decreasing by 8% per year) :

$\therefore f = -8\%$

$$P_{eq} = \frac{A_1}{i-(-f)}\left[1 - (1+i)^{-n}(1+(-f))^n\right]$$

$$= \frac{10000}{12\% + 8\%}\left[1 - (1.12)^{-15}(0.92)^{15}\right]$$

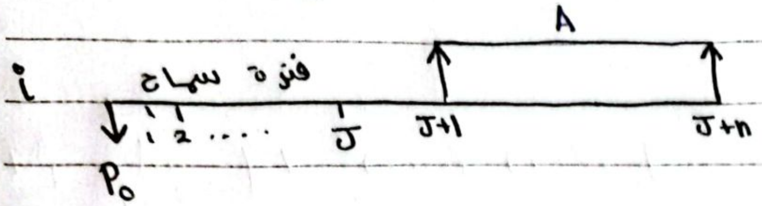
$$= \$47{,}384$$

* if the question was : (increasing by 12% per year)

$$P_{eq} = \frac{A_1 \cdot n}{1+i} = \frac{10{,}000 \times 15}{1 + 12\%} \qquad i = f$$

$$= \$133{,}928$$

*Deferred Payments

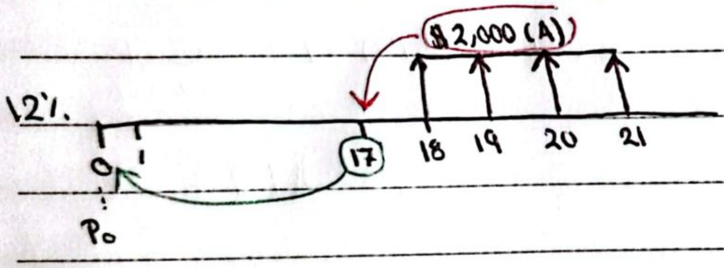


$P_J = A\,(P/A, i, n)$

$F_J = \underset{\uparrow}{\underset{=}{P_J}}$

$P_0 = \underline{F_J}\,(P/F, i, J) \rightsquigarrow P_0 = A\,.(P/A, i, n)\,(P/F, i, J)$

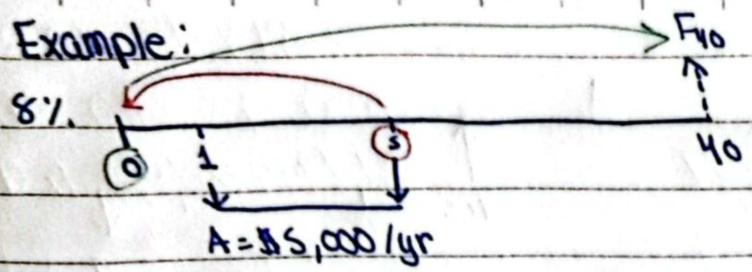
Example: what amount should an engineer deposit in his account today in order to provide for his son's education after 18 yrs from now, when the university costs are \$2,000 / year (1~4) and i = 12% ?



Steps	from	to	factor
1	$\boxed{A_{18-21}}$ =2000	P_{17}	$(P/A, 12, 4)$
2	P_{17}	F_{17}	( ) → في هذه الخطوة بنعكس فقط
3	F_{17}	$\boxed{P_0}$	$(P/F, 12\%, 17)$

$\rightarrow P_0 = 2{,}000\,(P/A, 12\%, 4)(P/F, 12\%, 17) = \$\,885$

Example:



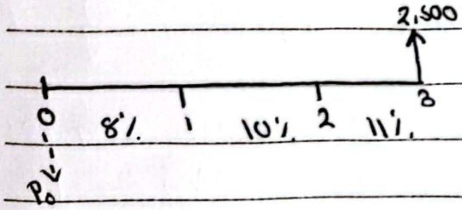
≡1 ≡2

Steps	from	to	factor
1	$\boxed{A_{1-5}}^{=5000}$	P_0	$(P/A, 8\%, 5)$
2	P_0	$\boxed{F_{40}}$	$(F/P, 8\%, 40)$

$\rightarrow F_{40} = 5000(P/A, 8\%, 5)(F/P, 8\%, 40) = \$433,697$

* Variable Interest Rate:

Example: $F_3 = \$2,500$ $i_1 = 8\%$ $i_2 = 10\%$ $i_3 = 11\%$, Find P_o :-



$$P_o = \overset{F_3}{2,500}\ \overset{(3\to2)}{(P/F, 11\%, 1)}\overset{(2\to1)}{(P/F, 10\%, 1)}\overset{(1\to0)}{(P/F, 8\%, 1)}$$
$$= \$1,896$$

→ Find F_1 :-

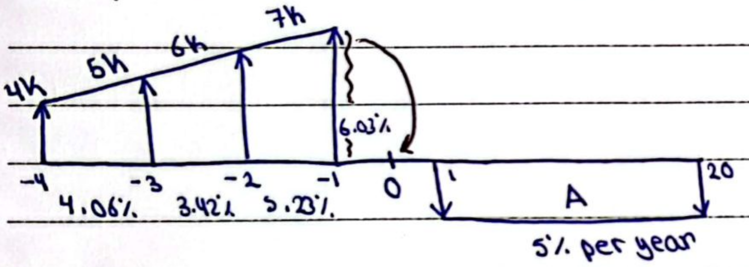
$$F_1 = 2,500\ \overset{(3\to2)}{(P/F, 11\%, 1)}\overset{(2\to1)}{(P/F, 10\%, 1)}$$
$$= \$2,047$$

→ Find F_2 :-

$$F_2 = 2,500\ \overset{(3\to2)}{(P/F, 11\%, 1)} = \$2,252$$

* Note: in this question we moved from larger to smaller, so we used $(P/F, i\%, n)$.

Example:



→ Find $A_{1\sim20}$:

$P_{o_1} = (4,000)(F/P, 4.06\%, 1)(F/P, 3.42\%, 1)(F/P, 5.23\%, 1)(F/P, 6.03\%, 1)$

There are no decimals in the tables so we use this: $(1+i)^n$

$+\ P_{o_2} = (5,000)(F/P, 3.42\%, 1)(F/P, 5.23\%, 1)(F/P, 6.03\%, 1)$

$+\ P_{o_3} = (6,000)(F/P, 5.23\%, 1)(F/P, 6.03\%, 1)$

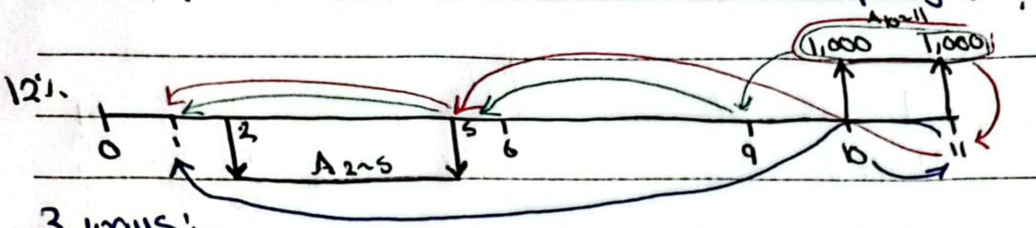
$+\ P_{o_4} = (7,000)(F/P, 6.03\%, 1)$

$\therefore\ P_o = \$24,689$

$A_{1\sim20} = 24,689\ (A/P, 5\%, 20)$

$= \$1,980$ / year

Example: Two receipts of ฿1,000 (EOY 10 & 11) are desired. How much should be deposited (EOY 2~5) ~~A~~ when $i = 12\%$ per year?



3 ways:

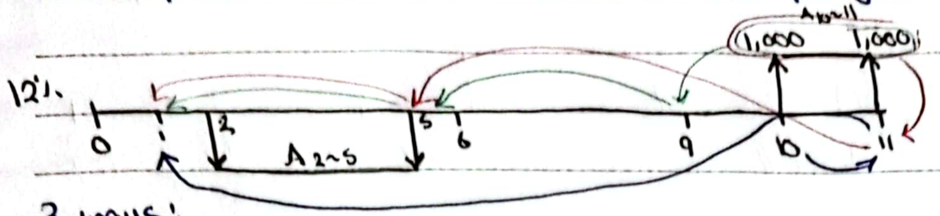
$A_{2\sim5} = 1{,}000\,(F/A, 12\%, 2)\,(P/F, 12\%, 6)\,(A/F, 12\%, 4)$

$A_{2\sim5} = 1{,}000\,(P/A, 12\%, 2)\,(P/F, 12\%, 4)\,(A/F, 12\%, 4)$

$A_{2\sim5} = 1{,}000\,(F/A, 12\%, 2)\,(P/F, 12\%, 10)\,(A/P, 12\%, 4)$

$= ฿225/\text{year} \; (2\sim5)$

Example: Two receipts of ฿1,000 (EOY 10 & 11) are desired. How much should be deposited (EOY 2~5) ~~Aw~~ when $i = 12\%$ per year?



3 ways:

$A_{2\sim 5} = 1{,}000\,(F/A, 12\%, 2)(P/F, 12\%, 6)(A/F, 12\%, 4)$

$A_{2\sim 5} = 1{,}000\,(P/A, 12\%, 2)(P/F, 12\%, 4)(A/F, 12\%, 4)$

$A_{2\sim 5} = 1{,}000\,(F/A, 12\%, 2)(P/F, 12\%, 10)(A/P, 12\%, 4)$

$= ฿\,225$ / year (2~5)

* Frequency of compounding:

- Nominal interest rate $(r) \equiv$ APR (Annual Percentage Rate)
- Effective (Actual) interest rate (i)

depends on: ↳ r (nominal)

↳ M (frequency of compounding)

Example: 12% per year compounded annually

1. Nominal interest rate (r) = 12% per year
2. Frequency of compounding (M) = 1 / year
3. Compounding period $\left(\frac{1}{M}\right)$ = 1 year
4. Effective annual interest rate $(i) = i = \left(1 + \frac{r}{m}\right)^m - 1$ = 12% per year
5. Effective interest rate per compounding period $(i_m) = i_m = \frac{r}{m}$ = 12% / yr

$\therefore$ when $\underline{m = 1} \rightarrow i = i_m = r$

Five Apple

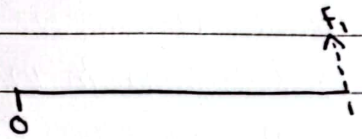
Example: 12% per year compounded monthly

1. Nominal int. rate (r) = 12% per year
2. Freq. of compounding (M) = 12 / year (bcz 1 time monthly)
3. Compounding period ($\frac{1}{M}$) = 1 month
4. Eff. annual int. rate (i) = 12.68% per year
5. Eff. interest rate per comp. period (i_m) = 1% per month

Example: 12% per year compounded monthly

r = 12% per year i = 12.68% i_m = 1% per month

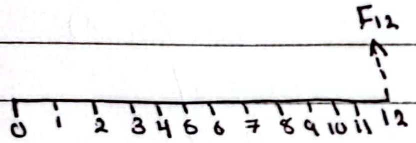
so :-



P = \$1000 year → i used here is i = 12.68%

$F_1 = P(1 + 12.68\%)^1$

or :-



P_0 = \$1000 month → i used here is i_m = 1%

$F_{12} = P(F/P, 1\%, 12)$

$= 1000\,(1 + 1\%)^{12}$

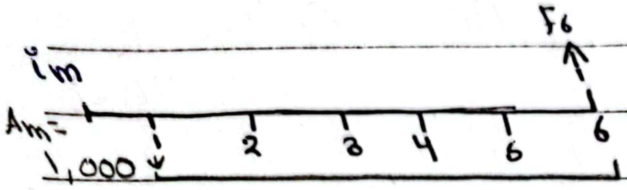
* both of them must have the same answer

* compounded annually M = 1 / year
  - semiannually = 2 / year
  - monthly = 12 / year
  - bimonthly = 6 / year
  - quarterly = 4 / year
  - weekly = 52 / year
  - daily = 365 / year
  - continously = ∞

    ↓ it has its own rule

Example: If you make deposits of \$1,000 every 6 months for 3 years, how much will be accumulated by then if interest is 12% compounded semiannually?

$r = 12\%$ $\quad$ $M = 2/\text{year}$

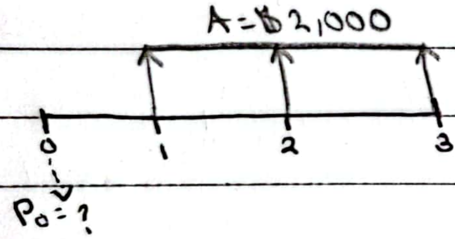


6 months $\rightarrow i_m = \frac{r}{m} = \frac{12\%}{2} = 6\%$

$\therefore F_6 = \underset{\$1,000}{A_m}(F/A, i_m, n_m) = \$6,975$

Example: If you want to earn \$2,000 per year for 3 years, how much should be deposited now if interest is 6% per year compounded quarterly?

$r = 6\%$ $\quad$ $M = 4/\text{year}$



year $\rightarrow i = (1+\frac{r}{m})^m - 1 = 6.1364\%$ per year

$\therefore P_0 = 2000(P/A, i, n) = \$5,333$

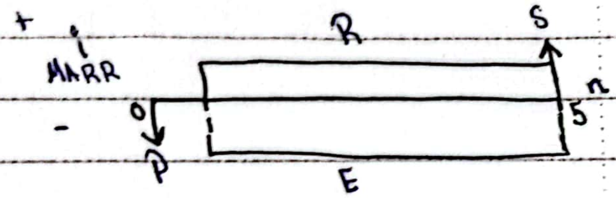
* Final

Ch5: Evaluation of Engineering Alternatives

- Long term alternatives:-

- The study period / useful life > 1 year
- Time value of money is important

* Project cash flow:



t=0 P: Capital investment (-)
(single payment)

t=1~n E: Expenses / costs (-) (multiple payments)
t=1~n R: Revenues sales (+) (multiple payments)
t=n S: Salvage value ($\frac{+}{0}$) (single payment)

- n: study period / useful life

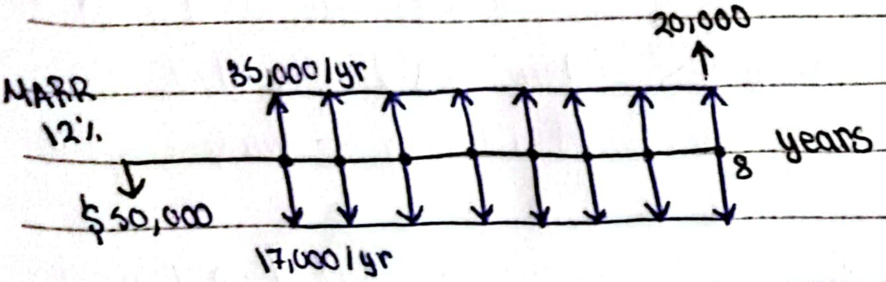
* R+S > P+E → Accepted
(R-E)+S > P ↗
R-E > P-S ↗
↳ Asset's loss in value

Example: A project has a capital investment of \$50,000, annual revenues of \$35,000 per year for 8 years. If the salvage value (market value at the end of the project's life) is estimated to be \$20,000 and MARR is 12%, answer the following questions:-

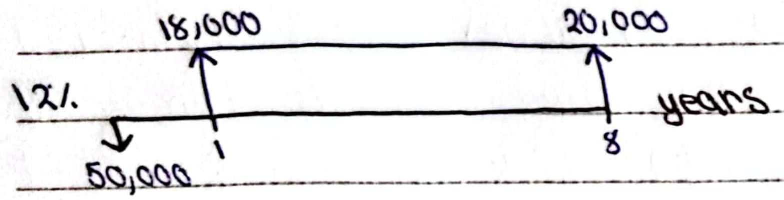
Solution:

P t=0 \$50,000 (-)   i=12%
R t=1~8 \$35,000 / yr (+)   n=8 years
E t=1~8 \$17,000 / yr (-)
S t=8 \$20,000 (+)

1. Construct the cash flow :



2. Determine the project net cash flow :



3. Determine the present worth, future worth, annual worth at the stated MARR

Pw (12%) = -50,000 (1) + 18,000 (P/A, 12%, 8) + 20,000 (P/F, 12%, 8)

= $47,495 > 0 (Accepted project)

Fw (12%) = -50,000 (F/P, 12%, 8) + 18,000 (F/A, 12%, 8) + 20,000 (1)

= $ 117,596 > 0 (Accepted project)

Aw (12%) = -50,000 (A/P, 12%, 8) + 18,000 (1) + 20,000 (A/F, 12%, 8)

= $ 9,560 / yr > 0 (Accepted project)

4. Determine the pw, fw and Aw at 0% :-

Pw (0%) = -50,000 + 18,000 x 8 + 20,000 = $ 114,000

Fw (0%) = -50,000 + 18,000 x 8 + 20,000 = $114,000

Aw (0%) = -50,000 / 8 + 18,000 + 20,000 / 8 = $14,250 / yr

* Payback Period: (PBP)

- الزمن (المدّة اللازمة) لاسترداد رأس المال المدفوع في بداية المشروع
- ليست مقياساً لربحية المشروع (Profitability)
- مقياس للسيولة (liquidity)

• Simple PBP (without interest)

1. Find net cash flow
2. Min θ' :

$$\sum_{t=0}^{\theta'} F_t \geq 0 \ , \ 0 < \theta' \leq n$$

• Discounted PBP (i)

1. Find out net cash flow
2. Min θ'' :

$$\sum_{t=0}^{\theta''} F_t \cdot (P/F, i, t) \geq 0 \ , \quad 0 < \theta'' \leq n$$

• Example: A project has a capital investment of \$50,000, annual revenues of \$35,000 per year for 8 years and annual expenses of \$17,000 per year for 8 years. If the salvage value is estimated to be \$20,000 and MARR is 12%, answer the following questions:-

Solution:-

→ Net cash flow in the previous page

• Determine the simple payback period for this project?

sum of net cash flow $(t=1) = \sum_{t=0}^{1} F(t) = -50{,}000 + 18{,}000 = -32{,}000 < 0$ ✗

sum " " " $(t=2) = \sum_{t=0}^{2} F(t) = -50{,}000 + 18{,}000 + 18{,}000 = -14{,}000 < 0$ ✗

sum " " " $(t=3) = \sum_{t=0}^{3} F(t) = -50{,}000 + 18{,}000 + 18{,}000 + 18{,}000 = 4{,}000 > 0$ ✓

$\therefore \theta' = 3$ years

• Determine the discounted payback period (at 12%) :

$\text{sum at } (t=1) = \sum_{t=0}^{1} F(t) \cdot (P/F, 12\%, t) = -50{,}000 + 18{,}000\,(P/F, 12\%, 1) = -33{,}928 < 0$

$\text{sum at } (t=2) = -33{,}928 + 18{,}000\,(P/F, 12\%, 2) = -19{,}579 < 0$

$\text{sum at } (t=3) = -19{,}579 + 18{,}000\,(P/F, 12\%, 3) = -6{,}767 < 0$

$\text{sum at } (t=4) = -6{,}767 + 18{,}000\,(P/F, 12\%, 4) = 4{,}672 > 0$

$\therefore \theta'' = 4$ years

....

* Rate of Return Methods

• Internal rate of return — العائد الداخلي للمشروع

- For a given project cash flow, MARR :

1. Find the net cash flow
2. Find i^* : $PW(i^*) = 0$
3. if i^* single value :

   $IRR = i^*$

   if $IRR \geq MARR \rightarrow$ project is accepted

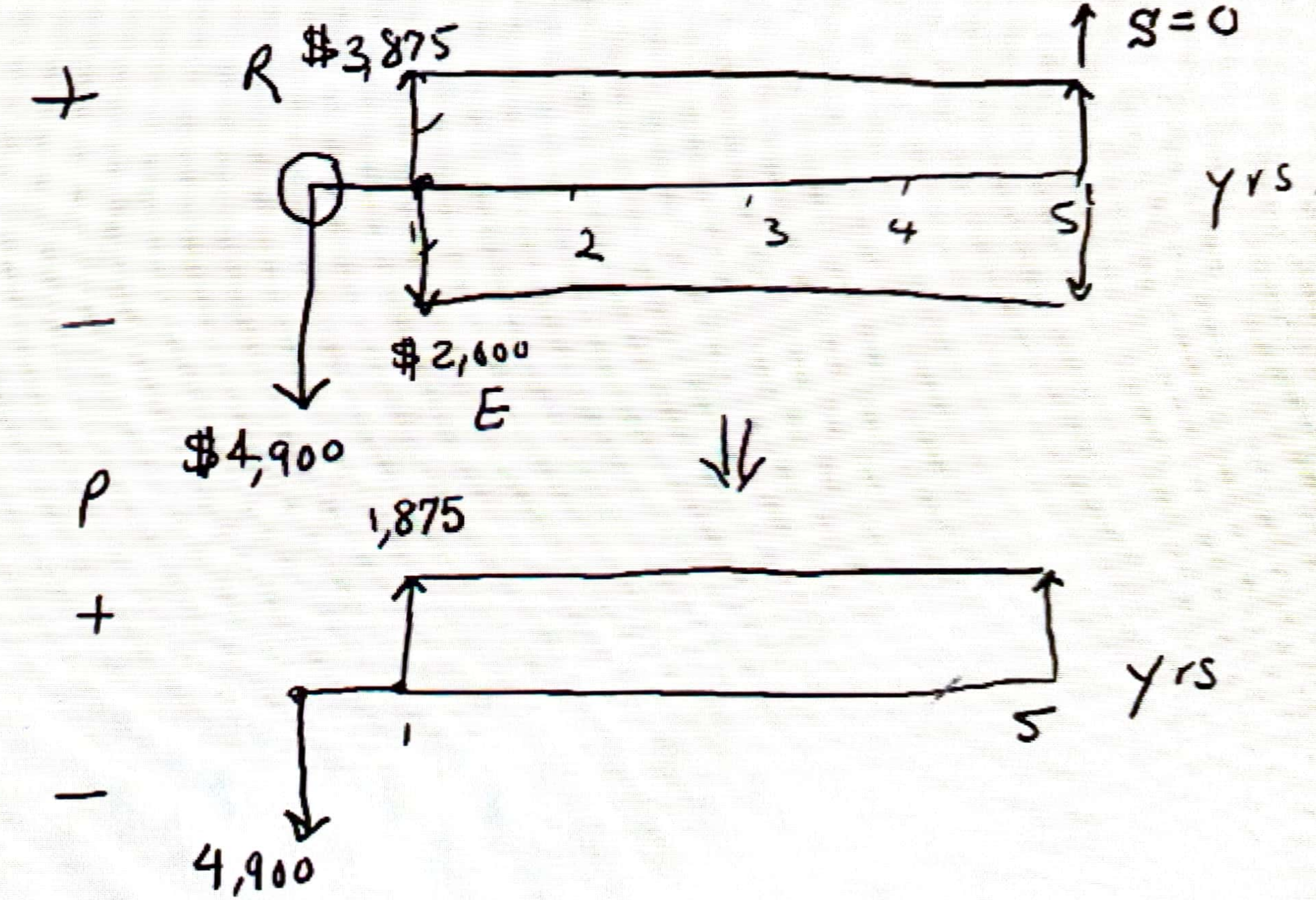
   $< MARR \rightarrow$ project is not accepted

* if i^* has multiple values :
  - Method fails
  - use a different method

Example

Project cash flow

MARR = 25% per year



$$PW(i^*) = 0$$

$$-4{,}900 + 1{,}875\,(P/A,\ i^*,\ 5) = 0 \quad \text{——— (1)}$$

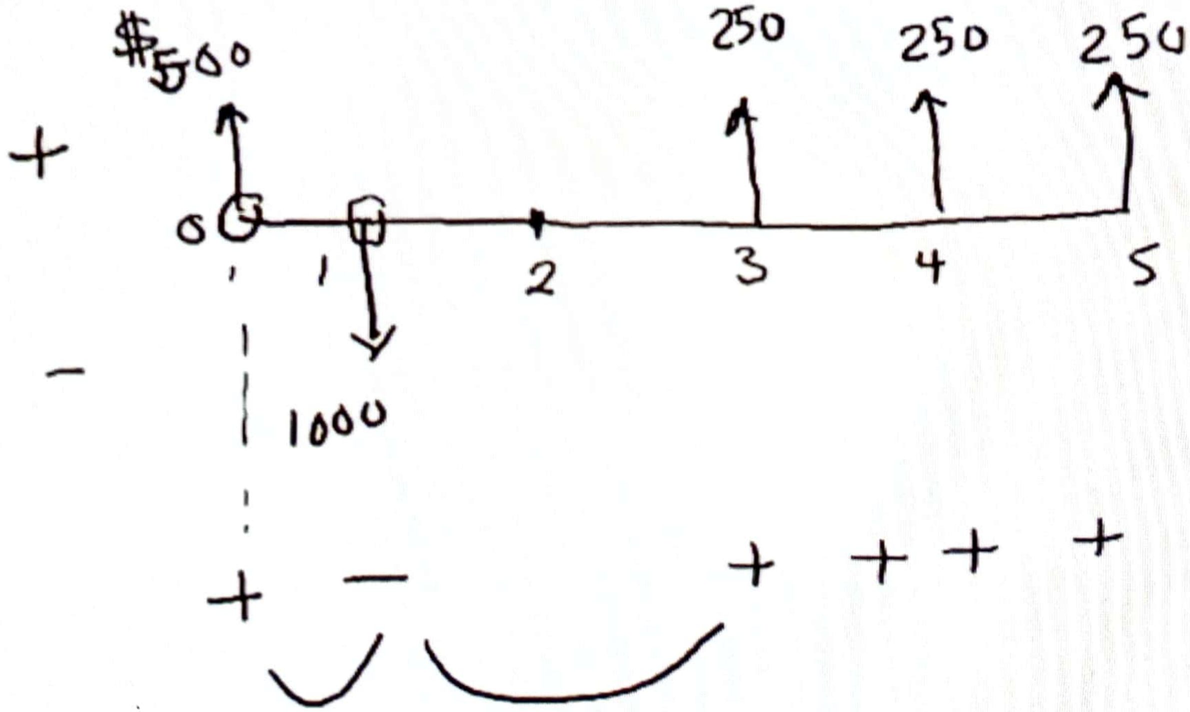
$$-4{,}900 + 1{,}875\,(1+i^*)^{-1} + 1{,}875\,(1+i^*)^{-2} \;----\; 1{,}875\,(1+i^*)^{-5} = 0$$

$$i^* = 26.5\% \longrightarrow IRR = 26.50\%$$

$$MARR = 25\%$$

CamScanner

Example



net cash flow

$PW(i^*) = 0$

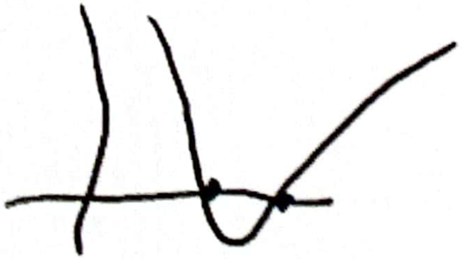
MARR = 35%

$i^*_1 = 30\%$

$i^*_2 = 62\%$

$\text{MARR} = \underline{35\%}$

$\left\{\begin{array}{l} i^*_1 = \underline{30\%} \\ i^*_2 = \underline{62\%} \end{array}\right.$



$i^*_1 < \text{MARR} \longrightarrow$ not accepted

$i^*_2 > \text{MARR} \longrightarrow$ accepted

Method <u>fails</u>.

- External Rate of Return Method (ERR)

For a given project cash flow, ϵ = MARR:

1. Find net cash flow
2. Find: $PW(\epsilon)$, $FW(\epsilon)$
3. Find i^*:

$$F = P(1+i^*)^n$$

4. ERR = i^*

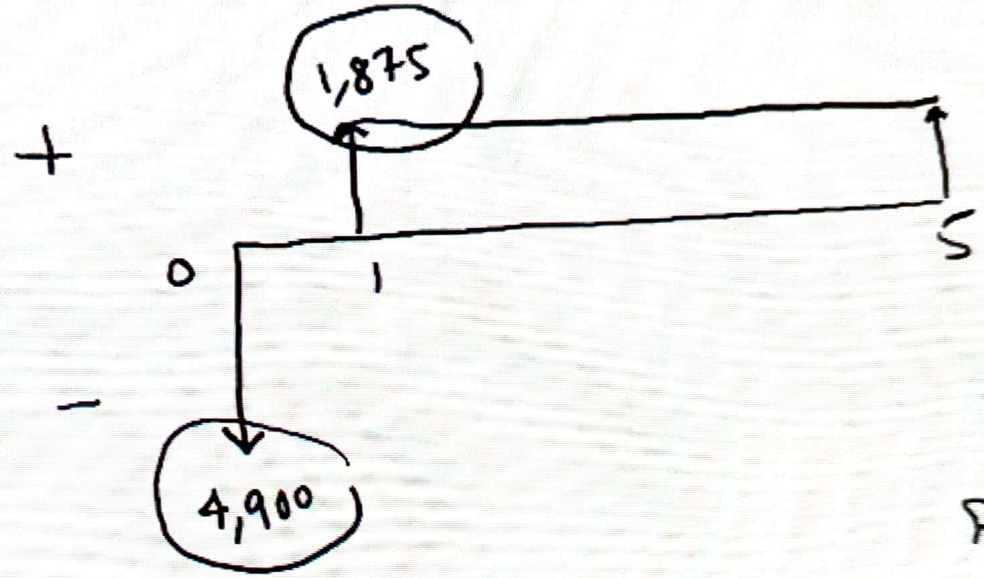
if ERR $\geq$ MARR $\rightarrow$ accepted

$<$ MARR $\rightarrow$ not accepted

CamScanner

Example

MARR = 25%

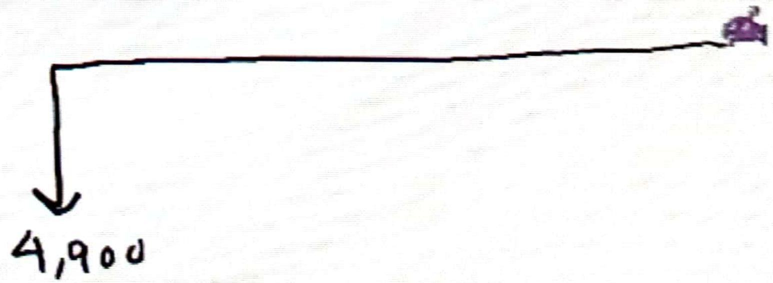


I

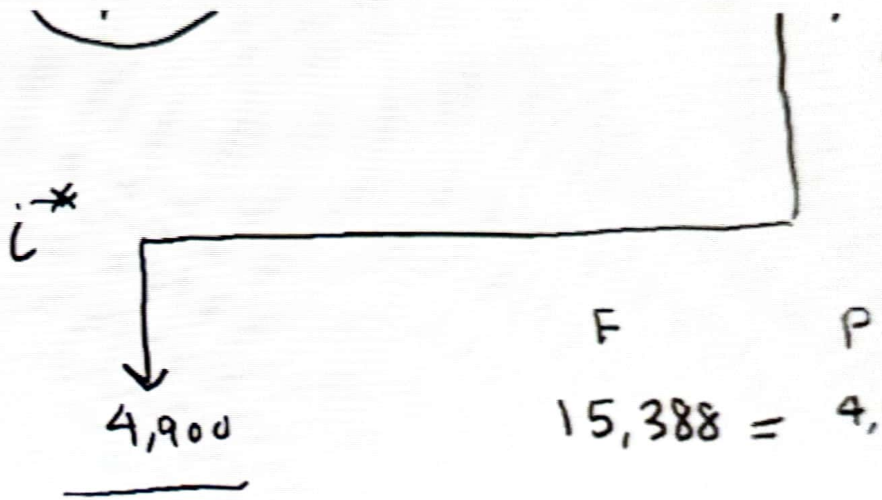
II

$FW_R =$

$PW_E(25\%)$
$= 4{,}900$



$PW_E(25\%)$
$= 4,900$



F P 5

$$15,388 = 4,900(1+i^*)^5$$

$$i^* = 25.7\%$$

$$\text{ERR} = \underline{25.7\%} > \text{MARR}$$

accepted