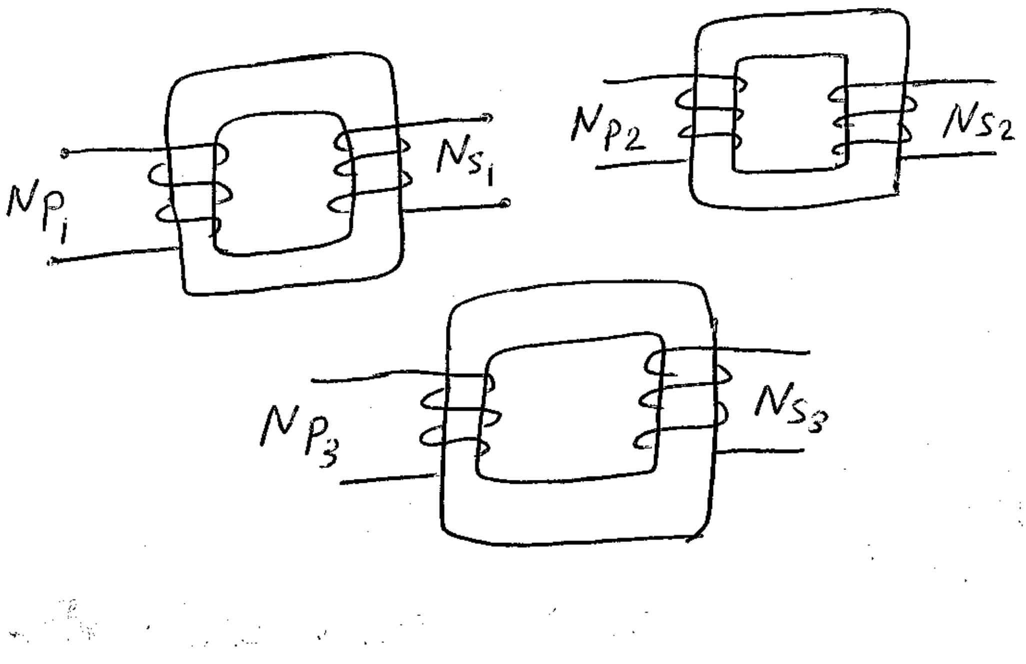
Three-Phase Transformers

* Almost all the major power generation and distribution systems in the world today are three-phase ac systems.

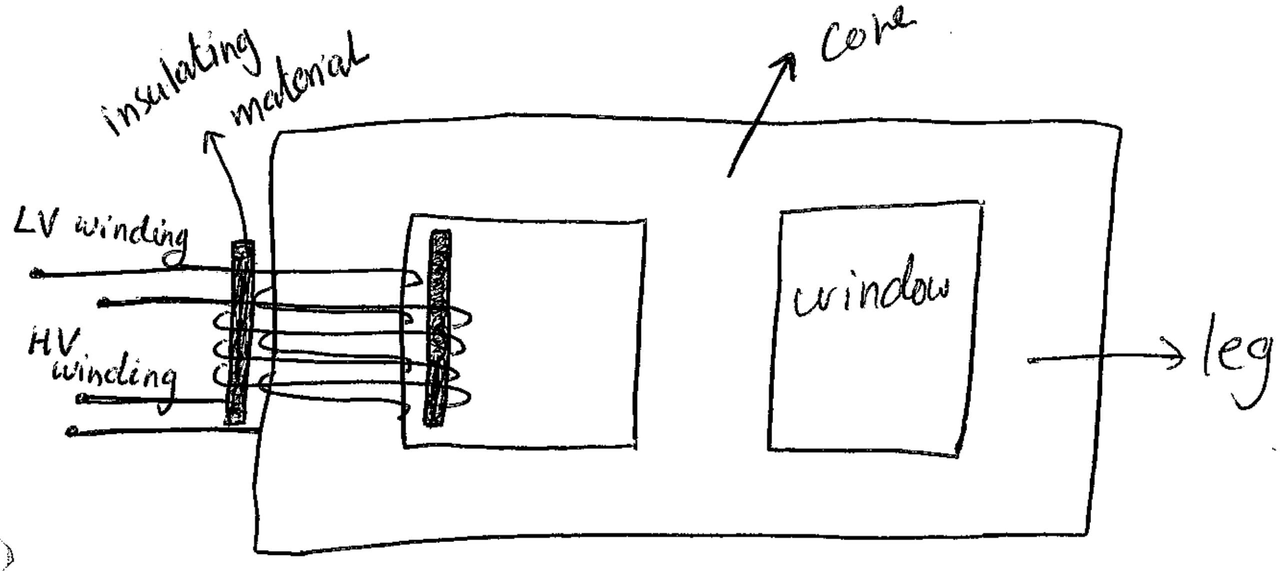
* Three-phase transformers can be constructed in one or two ways:

1. three single-phase transformers



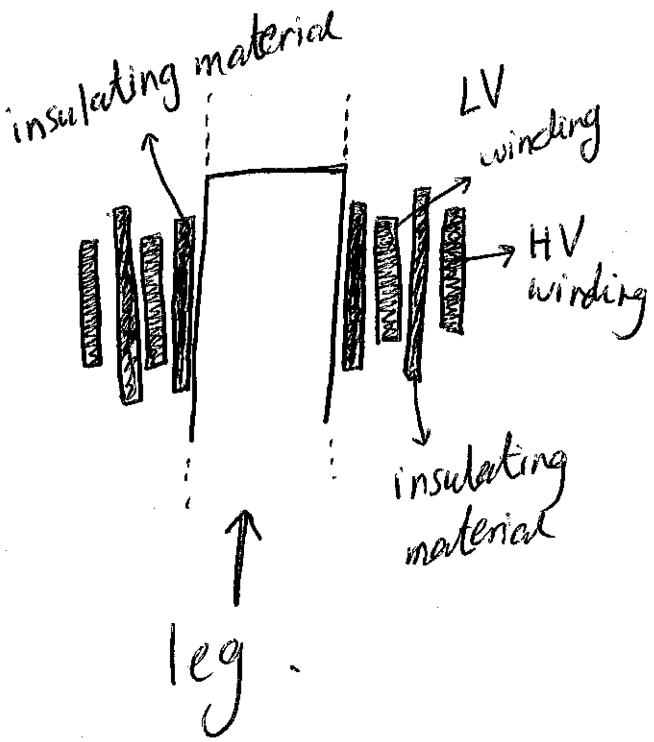
This arrangement:
each unit could be replaced individually in case of trouble.

2 single three-phase transformer

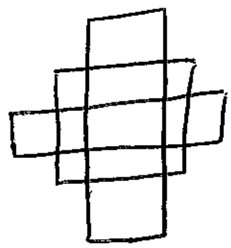


This arrangement is:

lighter, smaller, cheaper and more efficient.



leg side view



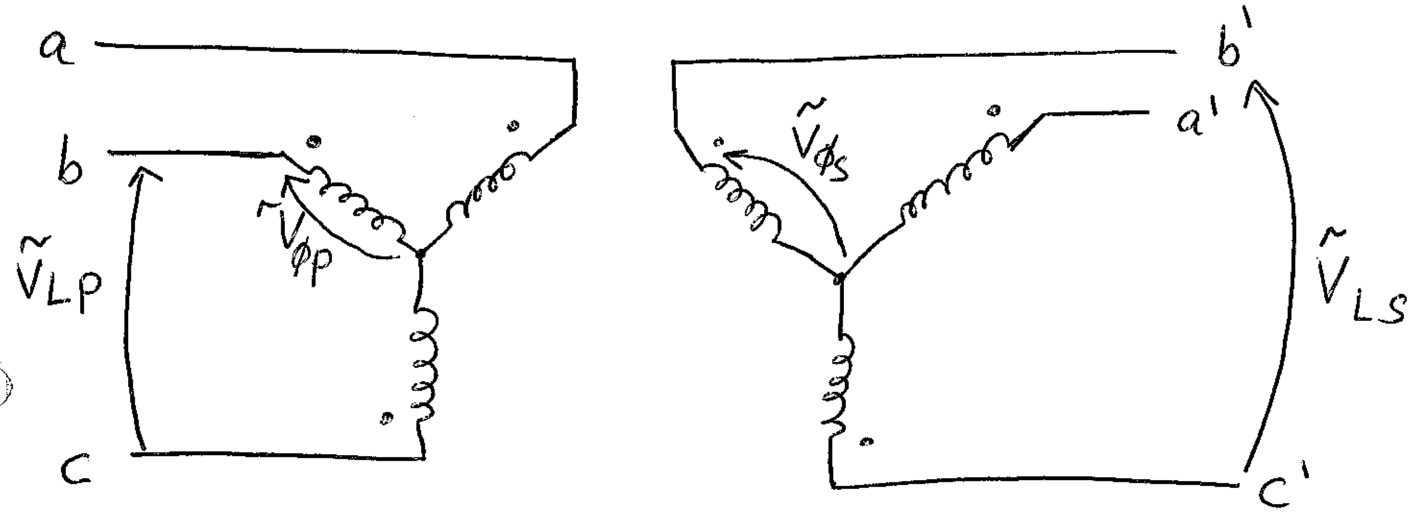
leg top view

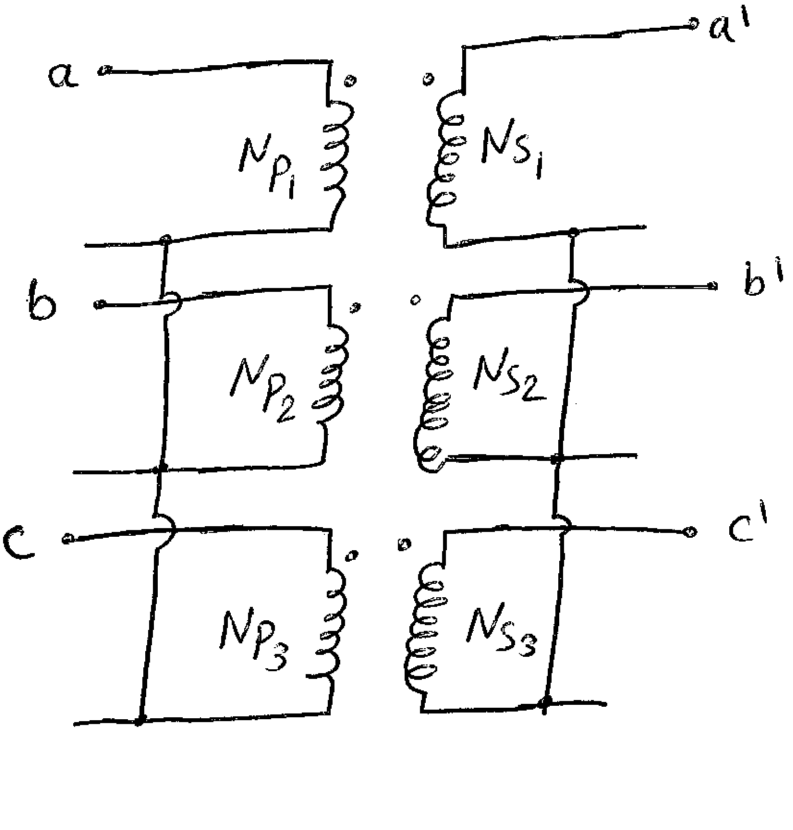
advantages: For the same cross-sectional area, the circle has the lowest circumference ⇒ lower winding resistance, higher efficiency, ~~higher~~ lower VR

and lower cooling power

Three-phase Transformer Connections

① Y–Y Connection





$$V_{LP} = \sqrt{3}\, V_{\phi P} \quad , \quad V_{LS} = \sqrt{3}\, V_{\phi S}$$

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}\, V_{\phi P}}{\sqrt{3}\, V_{\phi S}} = a$$

* Thy Y-Y connection has two very serious problems:

① If the loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.

② There is a serious problem with the third harmonic voltages. Due to the nonlinearity of the core, there are always some third-harmonic components. These third harmonic components are in phase.

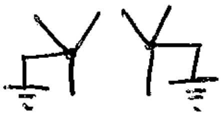
$$V_{3h_a} = \hat{V}_{3h} \sin(3(\omega t))$$

$$V_{3h_b} = \hat{V}_{3h} \sin(3(\omega t - 120^\circ))$$

$$V_{3h_c} = \hat{V}_{3h} \sin(3(\omega t + 120^\circ))$$

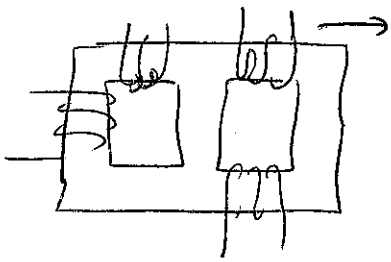
$$V_{3ha} + V_{3h_b} + V_{3h_c} = \hat{V}_{3h} \sin 3\omega t + \hat{V}_{3h} \sin(3\omega t - 360^\circ) + \hat{V}_{3h} \sin(3\omega t + 360^\circ) = 3\hat{V}_{3h}.$$

* Both the unbalance problem and the third harmonic problem can be solved by:



① Solidly ground the neutrals of the transformers. This permits the additive third harmonic components to cause a current flow in the neutral instead of building up large voltages.

② Add third (tertiary) winding connected in Δ to the transformer. If a third Δ-connected winding is added to the transformer, then the third-harmonic components of the voltages in the Δ will add up, causing a circulating current flow within the winding. This suppresses the third harmonic components of voltage.



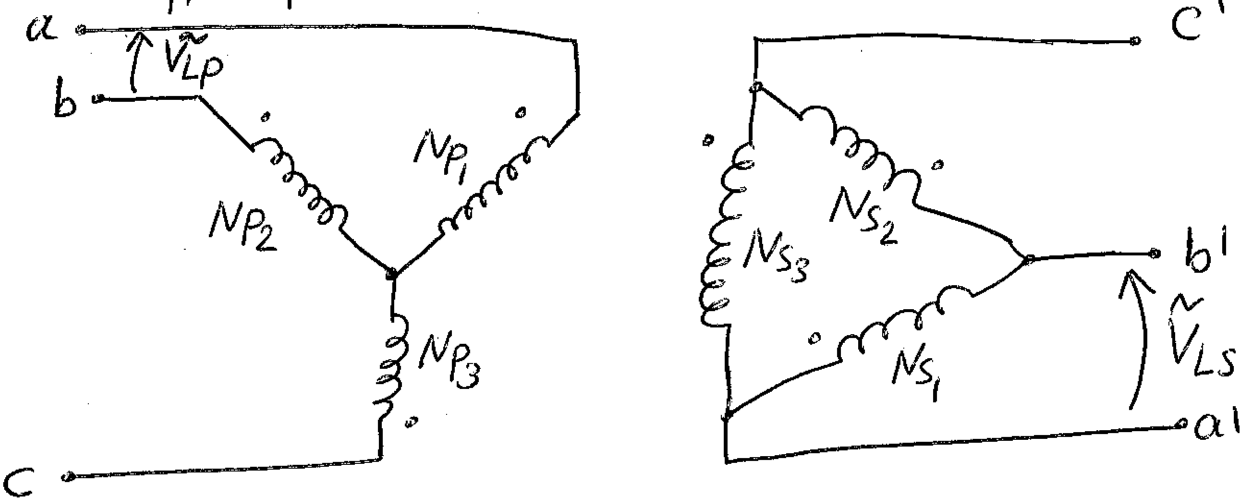
Note:

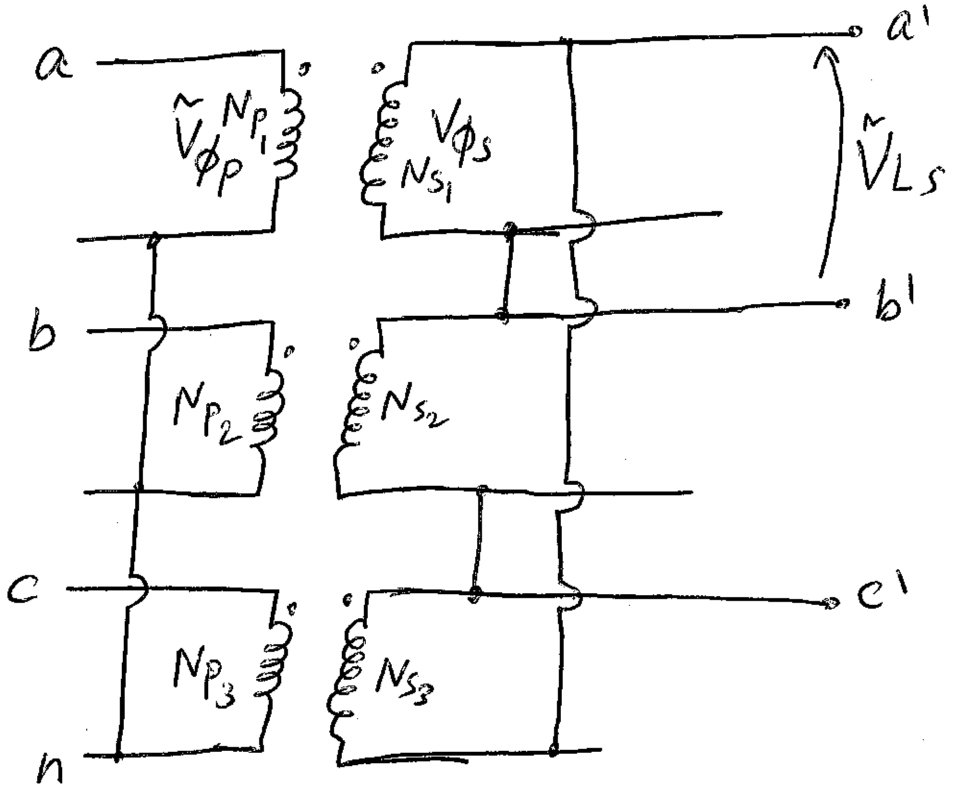
* Tertiary winding is often used to supply lights and auxiliary power to the substation where it is located. They have to be large enough to handle the large circulating currents.

* Due to all mensioned above reasons, Y-Y connection is rarly used. (should be avoided).

2 Y_Δ Connection

The Y-Δ connection is :





$$\frac{V_{Lp}}{V_{Ls}} = \frac{\sqrt{3}V_{\phi p}}{V_{\phi s}} = \sqrt{3}a$$

* The Y-Δ connection has no problem with third-harmonic components in its voltages since they are consumed in a circulating current on the Δ side. This connection is also more stable with respect to unbalanced loads sine the Δ connection partially redistributes any imbalance that occurs.

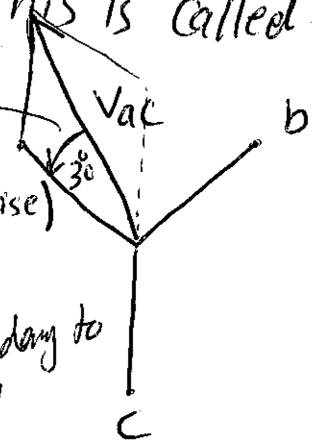
* In Y-Δ connection, the secondary is either lagging or leading the primary by 30°. The secondary voltage is lagging the primary by 30° if the system phase sequence is abc (+ve sequence). On the other hand, the secondary voltage leads the primary by 30° if the system phase sequence is acb. (-ve sequence).

This is called "Group Connection" as shown below:

from line to phase (counterclockwise)

⇓

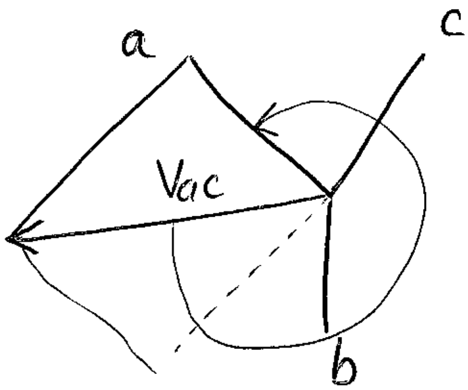
From the secondary to the primary and counterclockwise.



$V_{ac} = V_a - V_c$

YD_1

$1 \times 30 = 30°$



$V_{ac} = V_a - V_c$

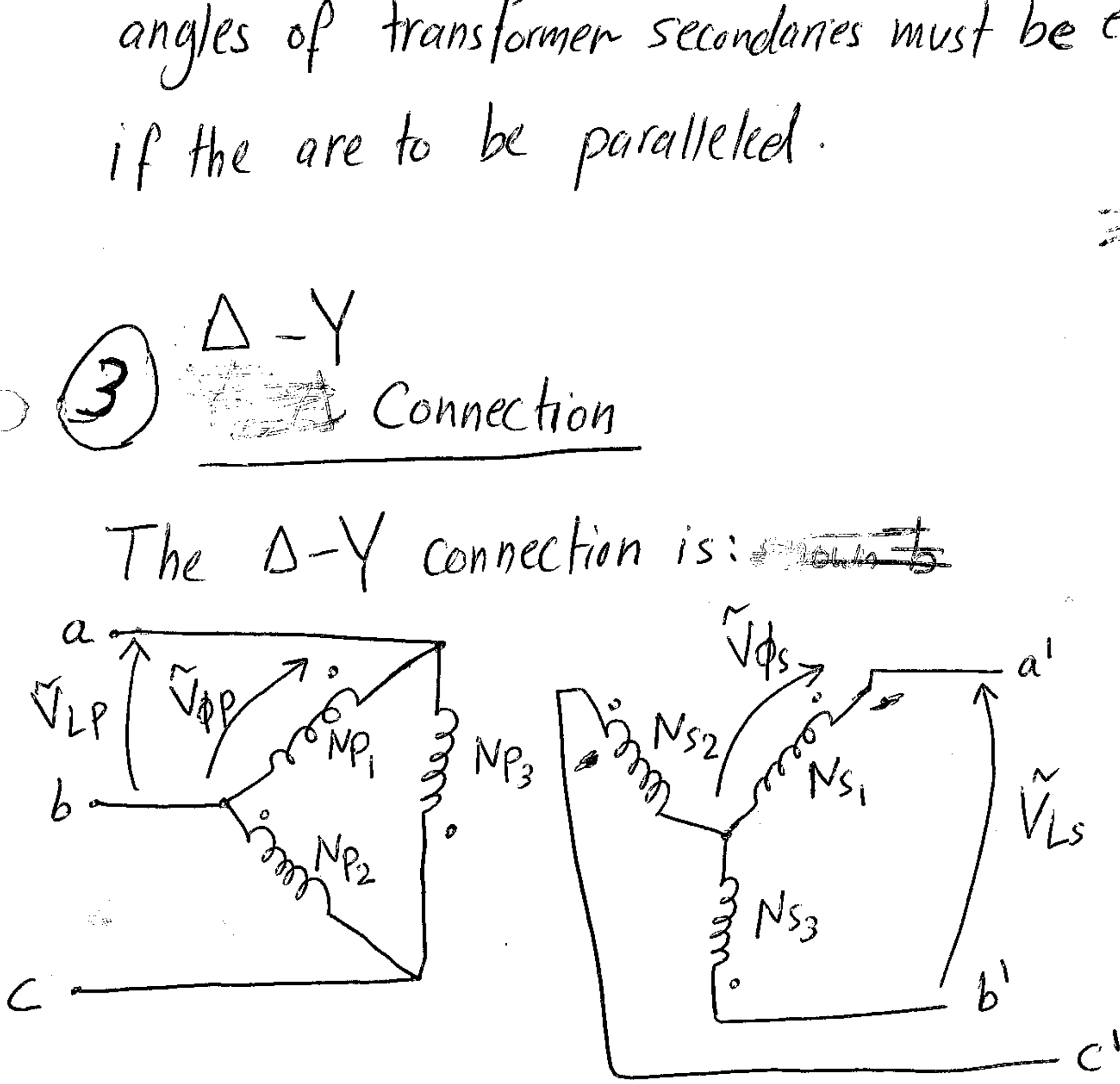
YD_{11}

$11 * 30 = 330°$

* The Group Connection is very important if the transformers are to be paralled. The phase angles of transformer secondaries must be equal if the are to be paralleled.

3 Δ-Y Connection

The Δ-Y connection is:



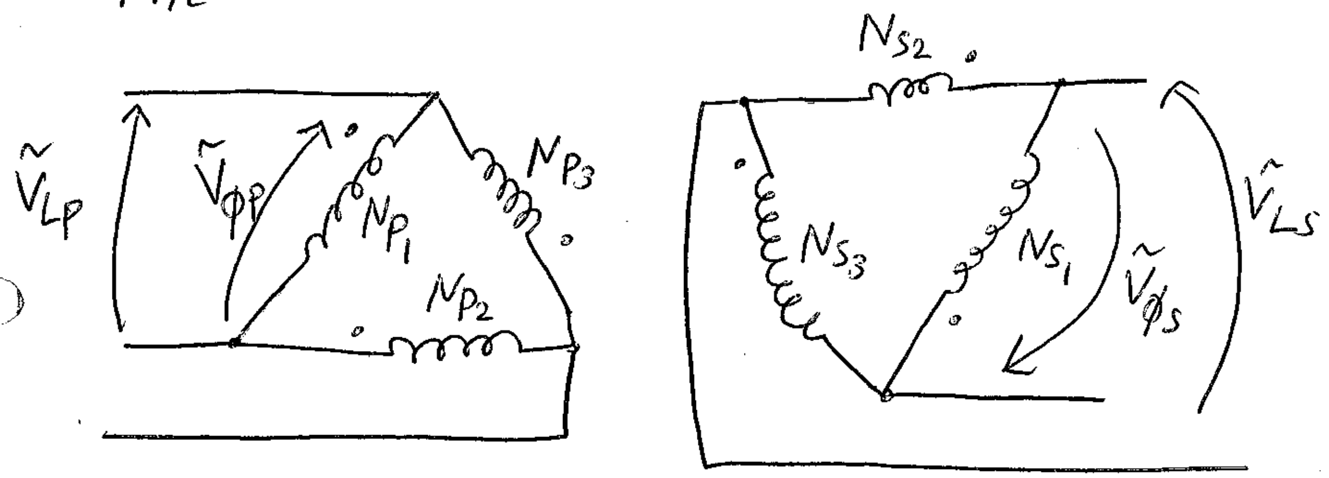
$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}\, V_{\phi S}} = \frac{1}{\sqrt{3}} a$$

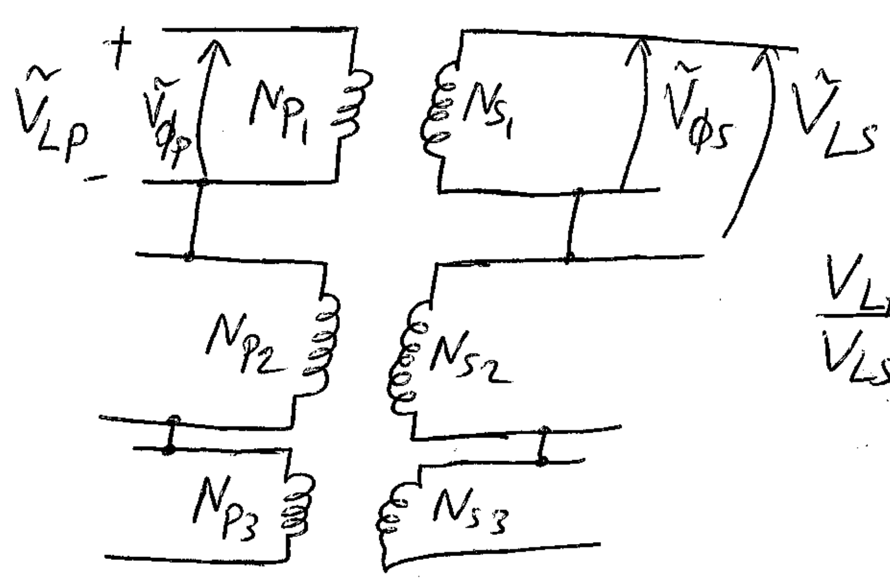
* This connection has the same advantages of the Y-Δ connection. As for the group connection, it is similar to the previous one as well.

4 Δ-Δ Connection

see the back of the page for group connection

The Δ-Δ connection is:





$$\frac{V_{LP}}{V_{Ls}} = \frac{V_{\phi P}}{V_{\phi s}} = a$$

* Similar to Y-Y, the Δ-Δ has no phase shift associated with it. unlike Y-Y, the Δ-Δ has no problems with the unbalanced loads or harmonics.

The per-unit system for three-phase Transformers

Let:

S_{base} : total base voltampere value of the transformer bank

$$S_{1\phi,base} = \frac{S_{base}}{3}$$

$$I_{\phi,base} = \frac{S_{1\phi,base}}{V_{\phi,base}}$$

$$\boxed{I_{\phi,base} = \frac{S_{base}}{3V_{\phi,base}}}$$

$$Z_{base} = \frac{(V_{\phi,base})^2}{S_{1\phi,base}}$$

$$\boxed{Z_{base} = \frac{3(V_{\phi,base})^2}{S_{base}}}$$

$V_{L,base} = V_{\phi,base}$ if the windings are connected in Δ.

$V_{L,base} = \sqrt{3}\, V_{\phi,base}$ if the windings are connected in Y.

Ex A 50 kVA 13800/208 V Δ-Y distribution transformer has a resistance of 1% and a reactance of 7% per unit.

(a) What is the transformer's phase impedance referred to the high voltage side?

(b) Calculate the voltage regulation at full load and 0.8 PF lagging using the calculated high-side impedance

(c) " " " " " " " " " using the per-unit system

(a) referred to the high voltage side means referred to the primary side.

$$Z_{base} = \frac{3(V_{\phi,base})^2}{S_{base}}$$

For the primary side, (high voltage side),

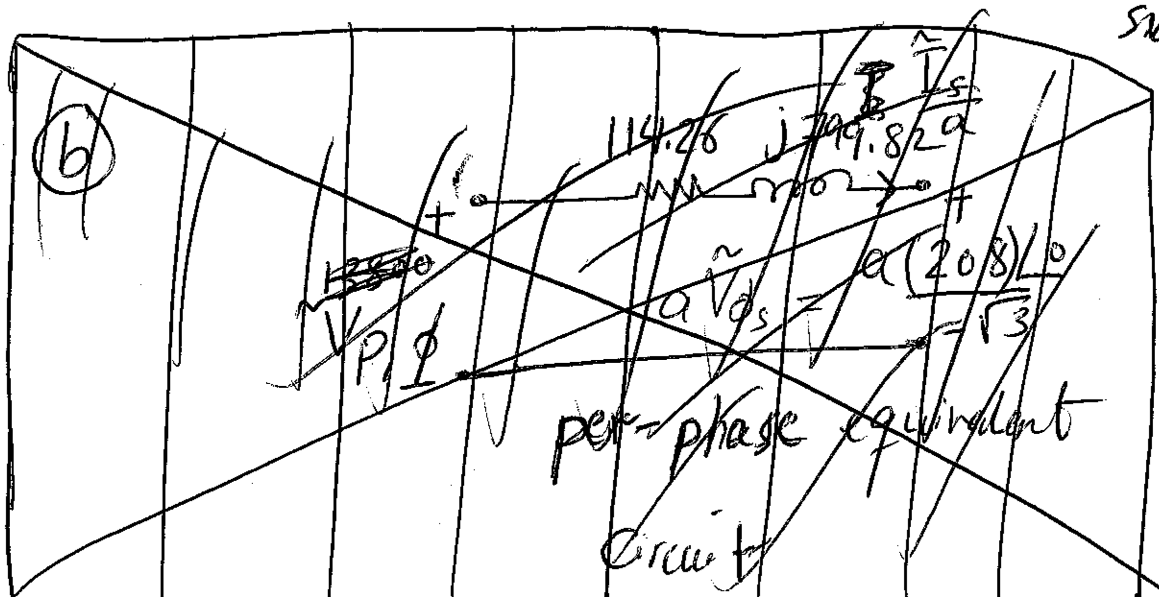
$$V_\phi = V_L = V_{\phi,base} = V_{L,base} = 13{,}800\,V.$$

$$Z_{base} = \frac{(3)(13800)^2}{50\times10^3} = 11426\ \Omega.$$

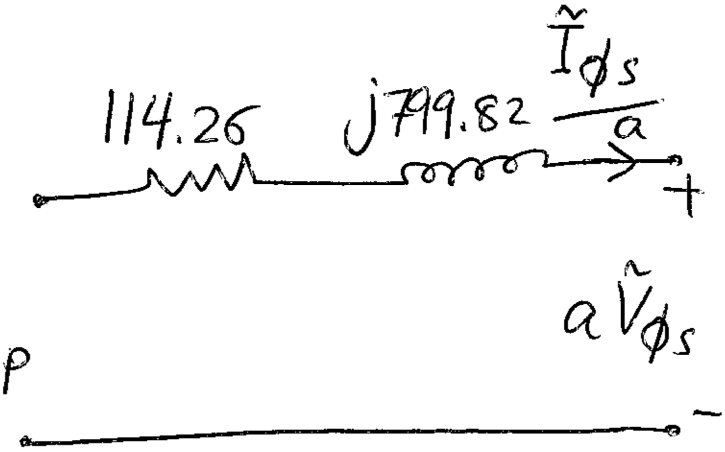
$$\tilde{Z}_{eq} = 0.01 + j0.07\ pu$$

$$\tilde{Z}_{eq} = (0.01 + j0.07)(11426)$$

$$= 114.26 + j\,799.82\ \Omega \longrightarrow$$ referred to the primary side.



(b)



(13)

$$V_{\phi s} = 208/\sqrt{3} = 120.23\,V$$

$$50\times10^3 = \sqrt{3}\,V_L I_L$$
$$= \sqrt{3}\,V_{L_s} I_{L_s}$$

$$I_{L_s} = I_{\phi s} = \frac{50\times10^3}{(\sqrt{3})(208)} = 138.95\,A.$$

$$a = \frac{V_{\phi p}}{V_{\phi s}} = \frac{13800}{208/\sqrt{3}} = 114.78$$

$$\frac{\tilde{I}_{\phi s}}{a} = \frac{138.95}{114.78}\angle -36.87^\circ = 1.21\angle -36.87^\circ$$

$$\tilde{V}_{\phi p} = a\tilde{V}_{\phi s} + \left(\frac{\tilde{I}_{\phi s}}{a}\right)(114.26 + j799.82)$$

$$= (114.78)(120.23\angle 0) + (1.21\angle -36.87^\circ)$$
$$(114.26 + j799.82)$$

$$\tilde{V}_{\phi p} = 14506\angle 2.73^\circ$$

$$VR = \frac{|\tilde{V}_{\phi p}| - |a\tilde{V}_{\phi s}|}{|a\tilde{V}_{\phi s}|} \times 100$$

$$= \frac{14506 - 13800}{13800} \times 100 = 5.1\,\% .$$

ⓒ $\tilde{V}_p = 1\angle 0 + (1\angle -36.87^\circ)(0.01 + j0.07)$

$$\tilde{V}_p = 1.051\angle 2.73^\circ$$

$$VR = \frac{1.051 - 1.0}{1.0} \times 100 = 5.1\,\%$$

→ one of the questions of the chapter

<u>Ex</u> A 100,000 KVA 230/115 KV Δ-Δ 3-ϕ power transformer has a perunit resistance of 0.02 pu and a per-unit reactance of 0.055 pu. The excitation branch elements are $R_C = 120$ pu and $X_M = 18$ pu.

ⓐ If the transformer supplies a load of 80MW at 0.85 PF lagging, draw the phasor diagram of one phase of the transformer.

(b) Calculate the voltage regulation of the transformer under these operating conditions.?

(c) " " " efficiency " "

" " " " "

(d) Calculate the impedances of the transformer referred to the primary side.

$$S_{base} = 100 \times 10^6 \ VA \Rightarrow S_{1\phi, base} = 33.33 \times 10^6 \ VA$$

$$V_{\phi, base} = 230 \times 10^3$$

$$Z_{base} = \frac{(V_{\phi, base})^2}{S_{1\phi, base}} = \frac{(230 \times 10^3)^2}{33.33 \times 10^6} = 1587.2 \ \Omega$$

$=$

(a) $\dfrac{80 \times 10^6}{100 \times 10^6} = 0.8 \ pu$

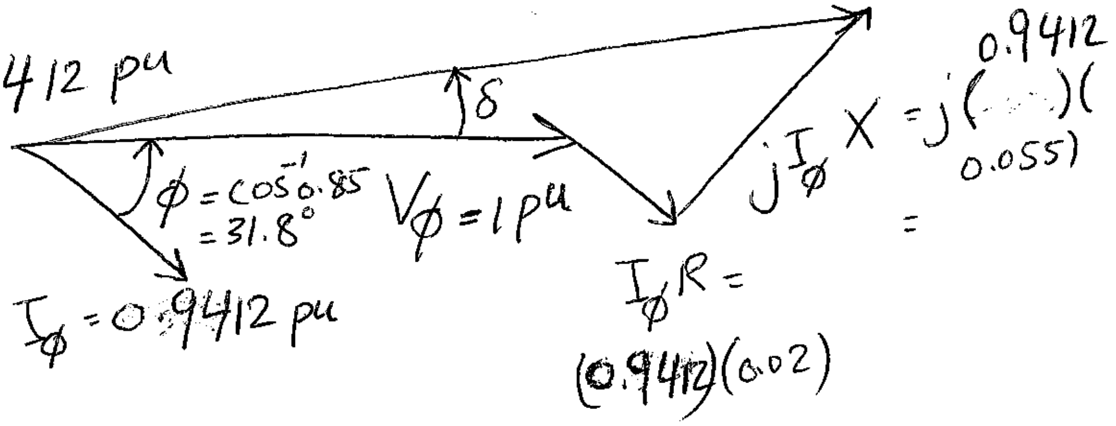
$$P = \sqrt{3} \ V_L \ I_L \cos\phi = 3 \ V_\phi \ I_\phi \cos\phi$$

$$\frac{P}{S_{base}} = \frac{3 V_\phi I_\phi \cos\phi}{S_{base}} = \frac{3 V_\phi \ I_\phi \cos\phi}{3 \ S_{1\phi, base}} = \frac{3 V_\phi I_\phi \cos\phi}{3 V_{\phi, base} \ I_{\phi, base}}$$

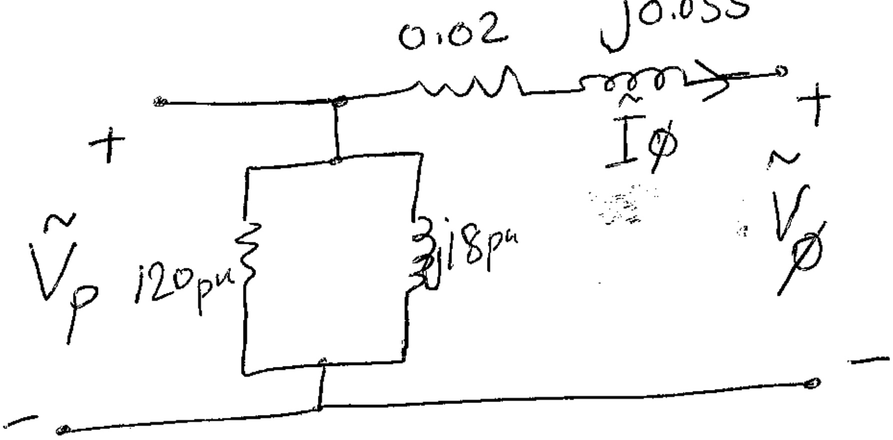
$$P_{pu} = V_{\phi \, pu} \ I_{\phi \, pu} \cos\phi$$

$$\frac{80\times10^6}{100\times10^6} = V_{\phi pu}\, I_{\phi pu} \cos\phi \Rightarrow 0.8 = (1)(I_{\phi pu})(0.85)$$ (1b)

$I_{\phi pu} = 0.9412$ pu



(b)



$$a = \frac{230}{115} = 2$$

$$\tilde{I}_\phi = 0.9412\angle{-31.8^\circ} \text{ pu}$$

$$\tilde{V}_\phi = 1\text{pu}$$

$$\tilde{V}_p = (0.02 + j0.055)(0.9412\angle{-31.8^\circ}) + 1\angle 0$$

$$= (0.02 + j0.055)(0.8 - j0.565) + 1$$

$$= 1.0471 + j0.0327 = 1.0476\angle 1.788^\circ$$

$$VR = \frac{1.0476 - 1}{1} = 4.76\%$$

(17)

(c) $P_{in} = P_{out} + |I_\phi|^2(0.02) + \frac{|\tilde{V}_P|^2}{120}$

$= 0.8 + (0.9412)^2(0.02) + \frac{(1.0476)^2}{120}$

$= 0.8269$ pu

$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{0.8}{0.8269} = 96.75\%$

(d) $Req_P = (0.02)(Z_{base}) = (0.02)(529.12) = 10.6\,\Omega$

$Xeq_P = (0.055)(529.12) = 29.1\,\Omega$

$R_C = (120)(529.12) = 6.35 \times 10^4\,\Omega$

$X_M = (18)(529.12) = 9.52 \times 10^3\,\Omega$

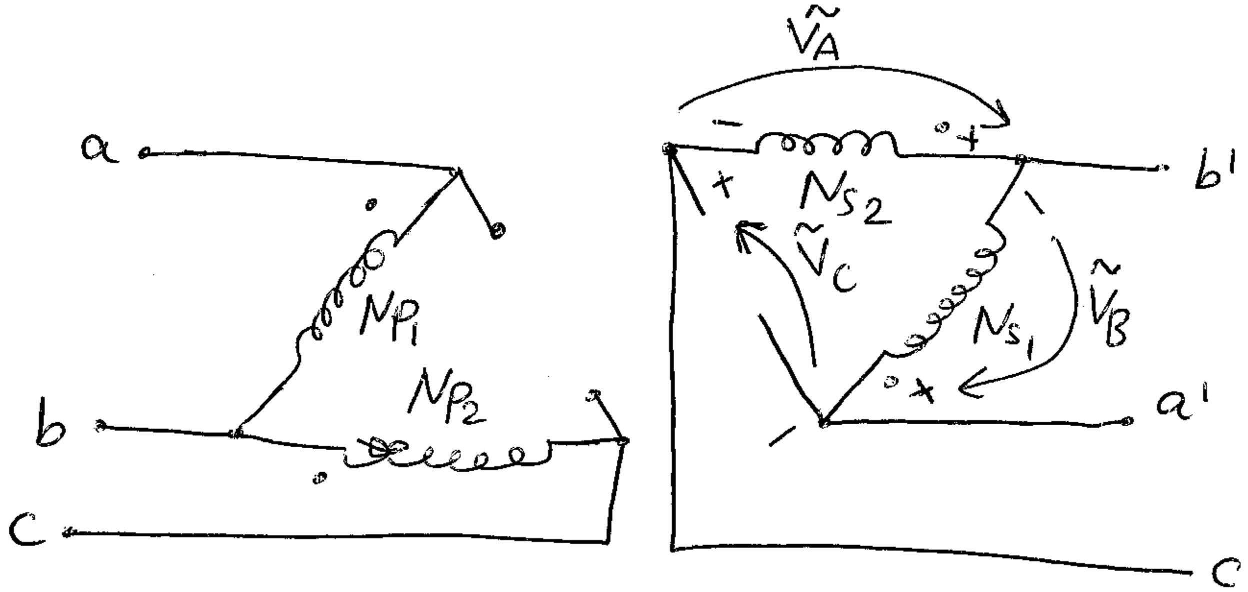
Three-phase Transformermation using Two Transformers

* There are ways to perform three-phase transformation with only two transformers. All techniques that do so involve a reduction in the power-handling capability of the transformers, but they may be justified by certain economic situations. Some of the more important two-transformer connections are:

1. The Open-Δ (or V–V) connection.
2. The open-Y-open-Δ connection.
3. The Scott-T connection.
4. The three-phase T connection

* The open-Δ (or V–V) connection

* Suppose that a Δ-Δ transformer bank composed of separate transformers has a damaged phase that must be removed for repair. The resulting situation is :

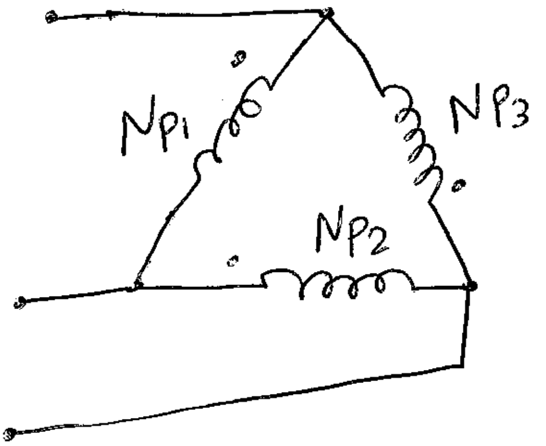


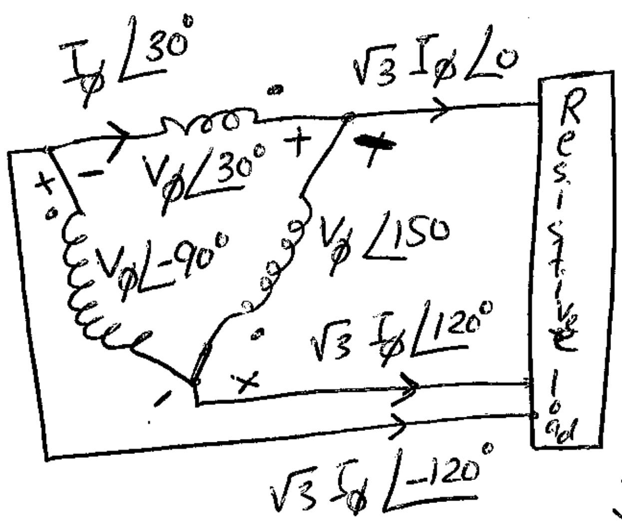
$$-\tilde{V}_C \cancel{+} \tilde{V}_A - \tilde{V}_B = 0$$

$$\tilde{V}_C = -\tilde{V}_A - \tilde{V}_B = -V\angle 0 - V\angle -120^\circ$$

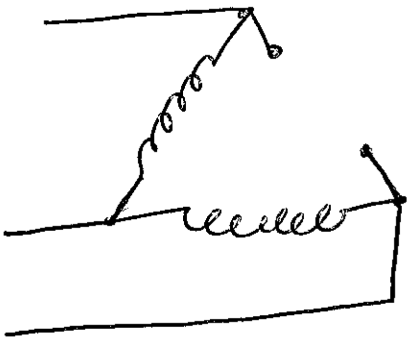
$$= V\angle 120 \ V$$

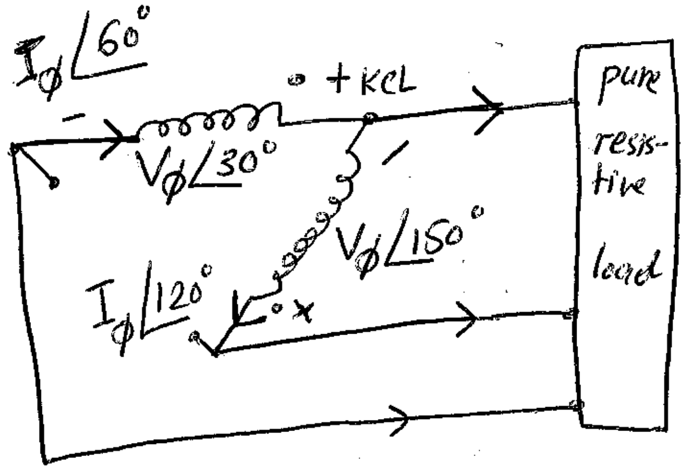
∴ the three-phase is still balanced. What about the ~~outpu~~ output power?! For power calculations, lets check the following circuit:





⇓





solving this circuit with one phase removed

$$P_1 = V_\phi I_\phi \cos(150-120) = V_\phi I_\phi \cos 30$$

$$= \frac{\sqrt{3}}{2} V_\phi I_\phi \Rightarrow$$ power developed by one winding.

$$P_2 = V_\phi I_\phi \cos(60-30) = \frac{\sqrt{3}}{2} V_\phi I_\phi \Rightarrow$$ power developed by the other winding

$$P_1 + P_2 = \frac{\sqrt{3}}{2} V_\phi I_\phi + \frac{\sqrt{3}}{2} V_\phi I_\phi = \sqrt{3} V_\phi I_\phi \Rightarrow$$ total power developed by the two windings

∴ The total power developed in this case is $\frac{\sqrt{3} V_\phi I_\phi}{3 V_\phi I_\phi} = \frac{1}{\sqrt{3}} = 0.577 =$ 57.7% of the original power developed by the healty system.

* What about the reactive power?

$$Q_1 = V_\phi I_\phi \sin(150° - 120°) = V_\phi I_\phi \sin 30° = 0.5 V_\phi I_\phi$$

$$Q_2 = V_\phi I_\phi \sin(30 - 60°) = -0.5 V_\phi I_\phi$$

$$Q_{total} = 0.5 V_\phi I_\phi - 0.5 V_\phi I_\phi = 0.$$

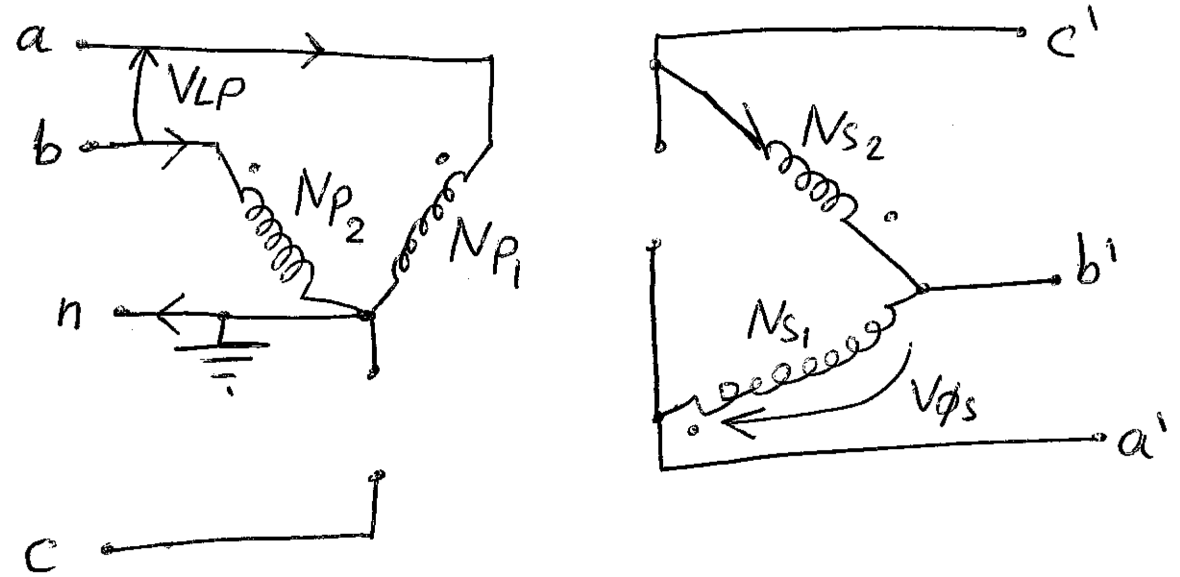
pure resistive load

* So the total reactive power developed is zero. Thus one transformer is producing reactive power which the other one is consuming.

~~exchange of energy between the two transformers that limits the power output to 57.7% of~~ the original power.

The Open-Y-Open-Δ Connection

* This type of connection is shown below.



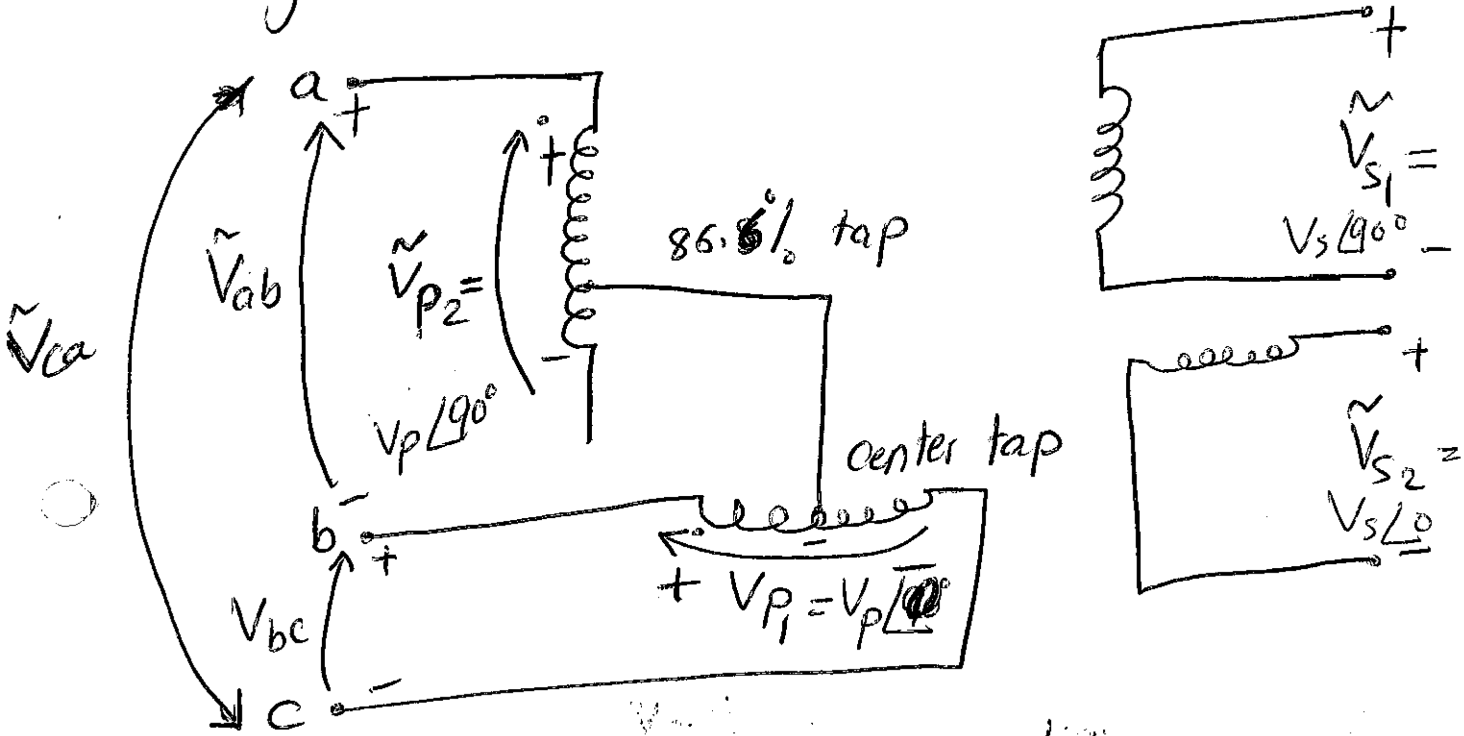
* This connection is very similar to the open-delta connection except that the primary voltages are derived from two phases and the neutral.
A major disadvantage of this connection is that a very large return current must flow in the neutral of the primary circuit.

The Scott-T Connection

forget it

* The Scott-T connection is a way to derive two phases 90° apart from a three-phase power supply. It consists of two single-phase transformers with identical ratings. One has a tap on its primary winding at 86.6% of full-load voltage.

They are connected as shown below:



$$-\tilde{V}_{ab} + \frac{86.6}{100} V_p\angle 90^\circ + 0.5\, V_p\angle 0^\circ = 0$$

$$\tilde{V}_{ab} = V_p\angle 120^\circ$$

Similarly,

$$\tilde{V}_{bc} = V_p\angle 0^\circ$$

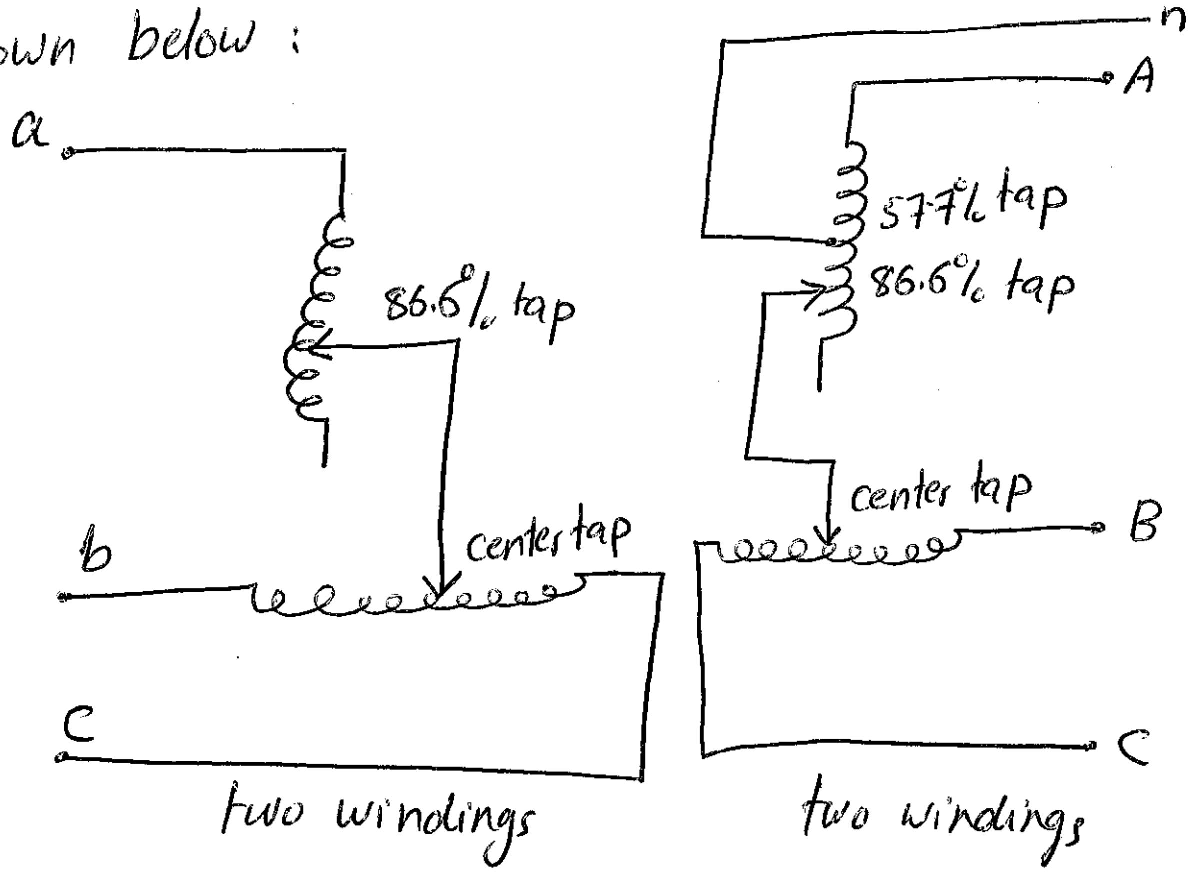
and $\tilde{V}_{ca} = V\angle -120^\circ$

* The Scott-T connection is an old arrangement as the ~~conversion to~~ two phase power is limited to only certain control applications.

The Three-Phase T Connection

forget it

The connection of the three-phase T is shown below:



~~major advantage~~

* It has two windings for both the primary & secondary which is similar to the open-Δ (delta) and open-wye-open-delta connections.

* One major advantage of the three-phase T connection over the other three-phase two transformer connections (the open-Δ and open-Y-open-Δ) is that a neutral can be connected to both the primary and the secondary sides of the transformer bank.

Transformer Ratings and Related Problems

Transformers have four major ratings: S, V, I and f.

The Voltage and Frequency Ratings of a Transformer

The induced voltage inside the winding of a transformer is:

$$E_{rms} = 4.44\,\phi f N$$

$$\phi = \phi_M \sin wt$$

$$E = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(\phi_M \sin wt)$$

$$= -N\phi w \cos wt$$

$$E_{rms} = \frac{N\phi w}{\sqrt{2}} = \frac{N\phi 2\pi f}{\sqrt{2}}$$

$$E_{rms} = 4.44\,\phi f N$$

* suppose that the transformer will run at a frequency lower than the nominal frequency, then

$$E_{rms_1} = 4.44\,\phi_1 f_1 N \longrightarrow \text{nominal}$$

$$E_{rms_2} = 4.44\,\phi_2 f_2 N \longrightarrow f_2 < f_1$$

$$\frac{E_{rms_1}}{E_{rms_2}} = \frac{\phi_1 f_1}{\phi_2 f_2}$$

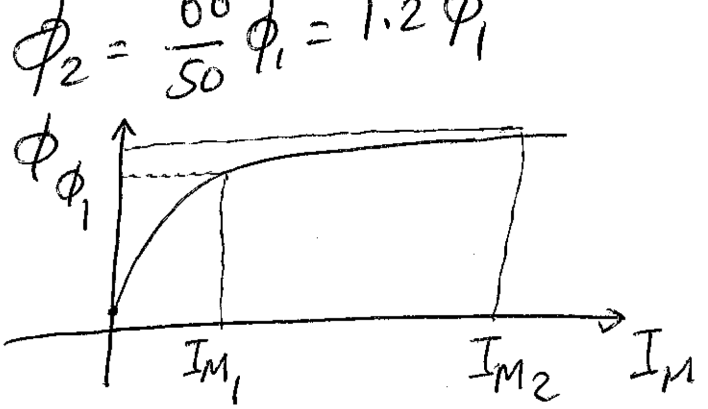
If the same voltage is applied, then

$$1 = \frac{\phi_1 f_1}{\phi_2 f_2} \Rightarrow \phi_1 f_1 = \phi_2 f_2$$

$$\phi_2 = \frac{f_1}{f_2}\phi_1$$

assume $f_1 = 60\,Hz$, $f_2 = 50\,Hz$

$$\phi_2 = \frac{60}{50}\phi_1 = 1.2\,\phi_1$$



$I_{M2} \gg I_{M1}$

* Due to saturation, the magnetization current in case of f_2 (lower frequency) is much higher than with the nominal frequency. This causes excessive over heat.

* To overcome this problem, derate the applied Voltage with the same ratio of the two frequencies as:

$$E_{rms_2} = \frac{f_2}{f_1} E_{rms_1}$$

This approach is called "derating" and widly applied at the transformers which are designed to run at USA (60 Hz) when they are brought to countries which apply the Europe system (50 Hz).

The Apparent Power Ratings of a Transformer

The actual voltampere rating of a transformer may be more than a single value. In real transformers, there may be a voltampere rating for the transformer by itself, and another (higher) rating for the transformer with forced cooling. or if the transformer will operate in an outside freezing temperature. The idea is that the hot-spot temperature in the transformer windings must be limited to protect the life of the transformer as overheating the coils of a transformer drastically shortens the life of its insulation.

The problem of Current Inrush

Current Inrush ≡ the current withdrawn by transformer at starting. suppose that the voltage :

$$V(t) = V_M \sin(\omega t + \theta)$$

is applied at the moment the transformer is first connected to the power line. The maximum flux reached on the first half-cycle of the the applied voltage depends on the phase of the voltage at the time the voltage is applied. If the initial voltage is :

$$V(t) = V_M \sin(\omega t + 90°) = V_M \cos\omega t$$

and if the initial flux in the core is zero, then the maximum flux during the first half-cycle will be:

$$\phi_{max} = \frac{1}{N_P}\int_0^{\pi/\omega} V_M \cos\omega t \, dt = \frac{V_M}{N_P \omega} \sin\omega \Big]_0^{\pi/\omega}$$

$$\phi_{max} = \frac{V_M}{\omega N_P} \sin\left(\frac{\omega\pi}{\omega} - 0\right) = 0 .$$

This causes no special problems. But if the applied voltage happens to be:

$$V(t) = V_M \sin\omega t \ \ V$$

the maximum flux during the first half-cycle is:

$$\phi_{max} = \frac{1}{N_P}\int_0^{\pi/\omega} V_M \sin\omega t \, dt = \frac{2V_M}{\omega N_P}$$

* This maximum flux is twice the normal steady-state flux. Coming back to the magnetization curve, it is easy to see that doubling the maximum flux in the core results in an enormous magnetization current. In fact, for part of the cycle the transformer looks like a short circuit and a very large current flows.

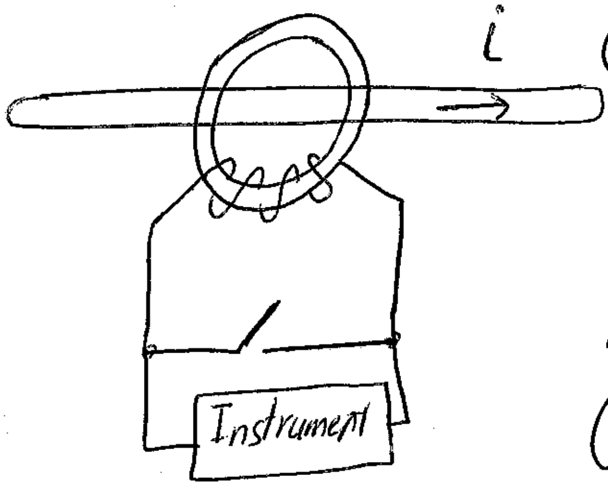
* For any values of the angle between 0° and 90°, there some excess current flow. The applied phase angle of the voltage is not normally controlled on starting, so there can be huge inrush currents during the first several cycles after the transformer is connect to the line.

Instrument Transformers

potential (voltage) transformer (VT)

1. step-down
2. very lower power rating
3. accurate
4. used to provide a sample of the power system voltage for measuring purposes

current transformer (CT)



1. step-down
2. used to provide a sample of the power system current for measuring purposes (current measurement)

* It is important to keep a current transformer short-circuited at all times since extremely high voltages can appear across its open secondary terminals.

Notes on AC Machines

The Effect of Coil Pitch on AC Machines

Stators

* The stator of AC machines exists in synchronous machine, induction machine and AC permanent machines and therefore it is important to know the details of the stator and its winding distribution.

* Sinusoidal air gap flux density distribution produces sinusoidal induced voltage. In general air gap flux density distribution is not sinusoidal and therefore harmonics do exist in the induced voltage of AC generators. To reduce these unwanted harmonics several techniques are used. Of these techniques what is called "fractional-pitch windings".

The Pitch of a Coil $\rightarrow \rho_p$

* The pole pitch is the angular distance between two adjacent poles on a machine. In mechanical degrees

$$\rho_p = \frac{360^\circ}{P}$$

where ρ_p is the pole pitch in mechanical degrees and P is the number of poles. Regardless of the number of poles on the machine, a pole pitch is always 180 electrical degrees.

* If the stator coil spans across the same angle (extends) as the pole pitch, it is called a full-pitch coil. If the stator coil spans across an angle smaller than a pole pitch, it is called a fractional-pitch coil. The pitch of a fractional-pitch coil is often expressed as a fraction indicating the portion of the pole pitch it spans. For example, a $\frac{5}{6}$ pitch coil spans five-sixths of the

the distance between two adjacent poles.

Or it can be expressed as :

$$\rho = \frac{\theta_M}{\rho_P} \times 180^\circ$$

coil pitch

θ_m: mechanical angle covered by the coil in degrees.

ρ_P: machine pole pitch in mechanical degrees.

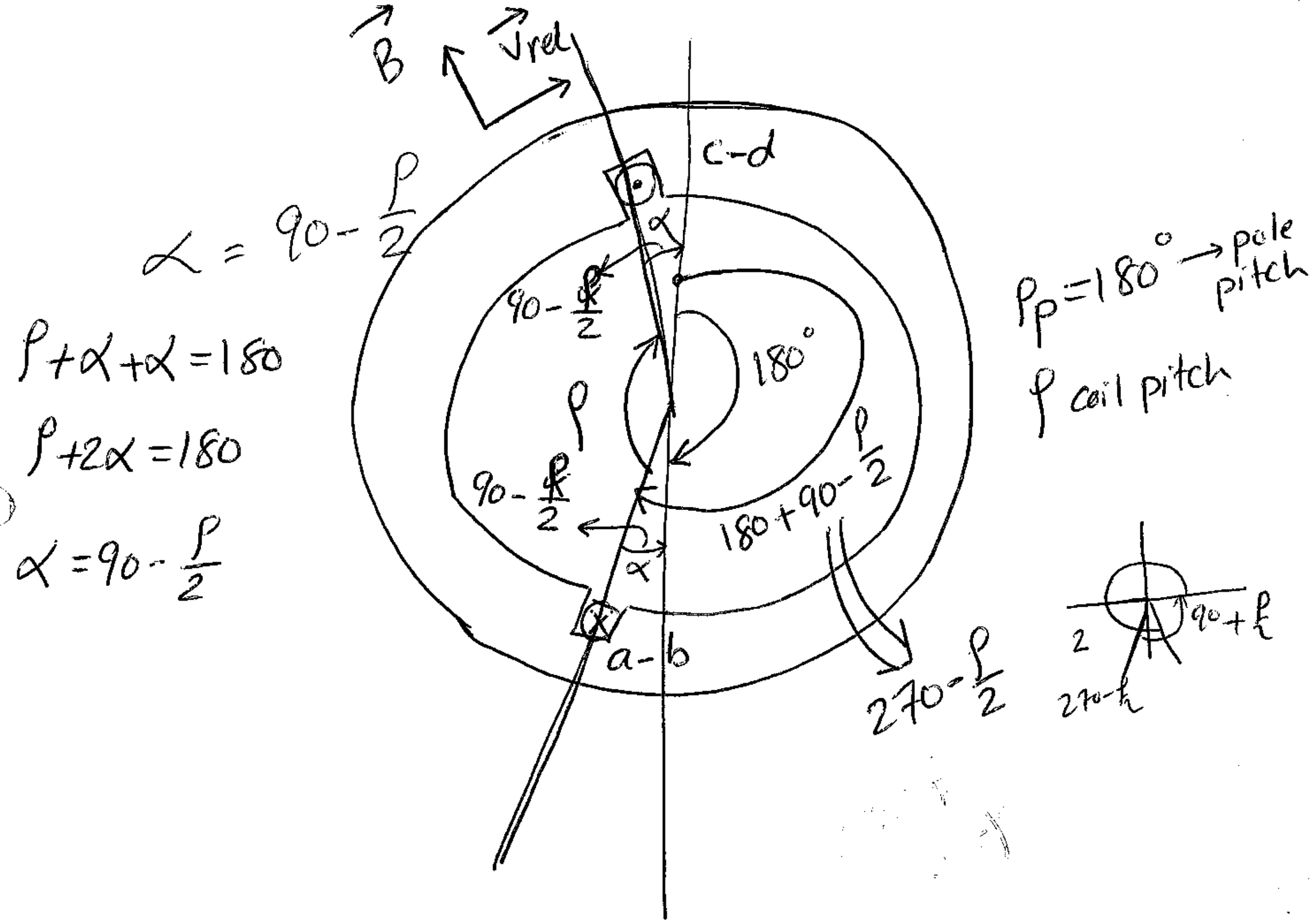
$$\cancel{\rho = \frac{\theta_m P}{2} \times 180^\circ}$$

~~P : number of poles~~

~~* Windings employing fractional-pitch coils are known as chorded windings.~~

* Windings employing fractional-pitch coils are known as chorded windings.

The Induced Voltage of a Fractional-Pitch Coil



* The magnitude of the flux density vector $\vec{B}$ in the air gap between the rotor and the stator varies sinusoidally with mechanical angle while the directon of $\vec{B}$ is always ~~~~ radially outward. If α is the angle measured from the direction of the peak rotor flux density, then the magnitude of the flux density vector $\vec{B}$ at a point around the rotor is:

$$B = B_M \cos \alpha$$

* Since the rotor is rotating at an angular velocity ω_m, the magnitude of the flux density $\vec{B}$ at any angle α around the stator is:

$$B = B_M \cos(\omega_m t - \alpha)$$

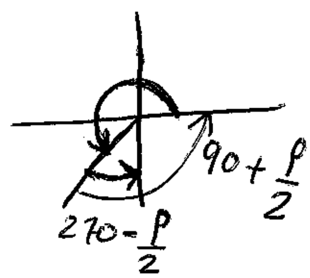
* The equation of the induced voltage e_{ind} is

$$e_{ind} = (\vec{V} \times \vec{B}) \cdot \vec{I}$$

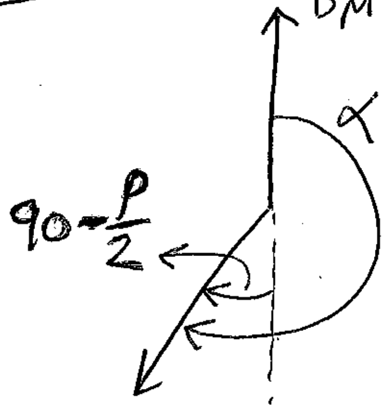
$\vec{V}$: velocity of the wire relative to the magnetic field.

$\vec{B}$: magnetic flux density of the field

$\vec{I}$: length of wire.



① <u>segment ab</u>



$$180 + 90 - \frac{\rho}{2} = 270 - \frac{\rho}{2}$$
$$= 90 + \frac{\rho}{2}$$

$$\alpha = 90 + \frac{\rho}{2}$$

$$e_{ba} = (\vec{V} \times \vec{B}) \cdot \vec{I} = VB\ell$$
$$= -VB_M \cos\alpha \, \ell$$
$$= -VB_M \cos\left[\omega_m t - \left(90^\circ \pm \frac{\rho}{2}\right)\right] \ell$$
$$= -VB_M \ell \cos\left(\omega_m t - 90^\circ + \frac{\rho}{2}\right)$$

where the negative sign comes from the fact that the voltage is built up with a polarity opposite to the assumed polarity.

② segment bc. The voltage on segment bc is zero since the vector quantity $\vec{V} \times \vec{B}$ is $\perp$ to $\vec{I}$.

$$e_{cb} = (\vec{V} \times \vec{B}) \cdot \vec{I} = 0$$

③ segment cd. For segment cd, the angle $\alpha = 90 - \frac{\rho}{2}$

$$e_{dc} = (\vec{V} \times \vec{B}) \cdot \vec{I} = VB\ell = VB_M \cos\left[\omega_m t - \left(90^\circ - \frac{\rho}{2}\right)\right] \ell$$

$$e_{dc} = VB_M \ell \cos\left(\omega_m t - 90^\circ + \frac{\rho}{2}\right)$$

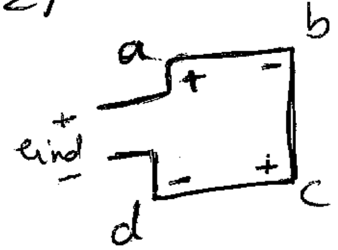
④ segement da.

$$e_{ind} = (\vec{V} \times \vec{B}) \cdot \vec{I} = 0$$

* The total voltage for the whole fractional-pitch single-turn coil is :

$$e_{ind} = e_{ba} + e_{dc}$$
$$= -VB_M l \cos(\omega_m t - 90° - \tfrac{\rho}{2}) +$$
$$VB_M l \cos(\omega_m t - 90° + \tfrac{\rho}{2})$$

By trigenometric functions



$$-e_{ind} + e_{ab} + e_{cd} = 0$$

$$\boxed{e_{ind} = 2VB_M l \cos\omega_m t \sin\frac{\rho}{2}}$$

let $K_p = \sin\frac{\rho}{2}$ → pitch factor

$\omega = 2\pi f$

$V = r\omega$

$$e_{ind} = 2 V B_M l \cos\omega_m t \, K_p$$

$r\omega$

$\phi = Brl$

$$e_{ind} = K_p \phi \omega \cos\omega t \Rightarrow \text{For } N_c \text{ turns}$$
$$e_{ind} = N_c \phi \omega K_p \cos\omega t$$

and its peak voltage is

$$E_{max} = N_c \phi \omega K_p = 2\pi N_c \phi f K_p$$

Therefore, the rms voltage of any phase ~~s~~ of this three-phase stator is :

$$E_{rms} = E_A = \sqrt{2}\,\pi N_C k_p \phi f .$$

* For machines with more than two poles

$$\boxed{K_p = \sin \frac{\theta_m P}{2}}$$

→ pitch factor

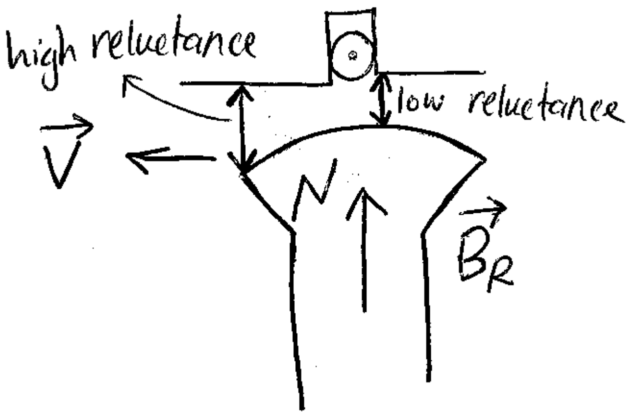
θ_m → mech. angle covered by the coil in degrees

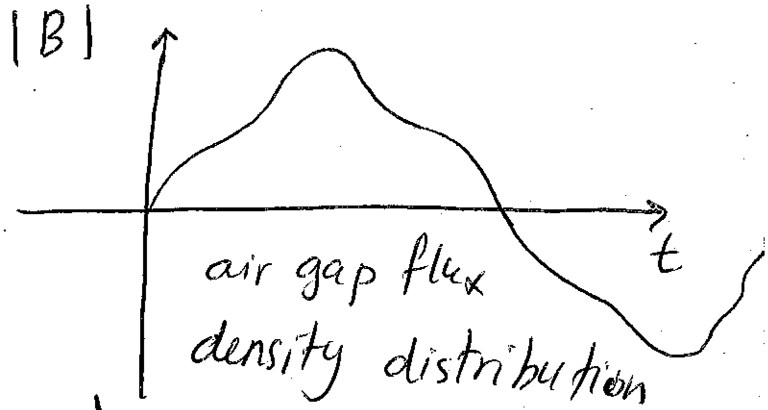
P → coil pitch in mechanical degrees

$$K_p = \sin\left(\frac{\rho}{2}\right)$$

Harmonic Problems and Fractional-Pitch Windings

*In salient pole machines, the air gap flux density is not ~~sinus~~ pure sinusoidal. It looks like:





and ~~as~~ so is the induced voltage.

* Only odd harmonics exist in the waveform of the induced Voltage i.e. third, fifth , seventh, ninth and so on and so forth. The third harmonic and its multiples are removed in both cases of Y and Δ connected stator. In Y-connected stator, the third harmonics and their multiples are in phase and go the neutral which is usually grounded. In Δ-connected stator, the third harmonics and their multiples circulates inside the Δ loop and do not appear at the output of the generator. As the number of the harmonic increases, it magnitude decreases and therefore only the fifth and seventh components are of prime concern which should be reduced.

* If ρ represents the electrical angle spanned by the coil at its fundamental frequency and ν is the number of the harmonic being examined, then the coil will span $\nu\rho$ electrical degrees at that harmonic frequency.

Therefore, the pitch factor of the coil at the harmonic frequency is:

$$K_p = \sin\frac{\overset{\text{nu}}{\nu}\rho}{2}$$ ← pitch factor

<u>Ex</u> A three-phase, two pole stator has coils with a $\frac{5}{6}$ pitch. What are the pitch factors for the harmonics presented in ~~the~~ this machine's coils? Does this pitch help suppress the harmonic content of the generated voltage?

$$\rho_p = \frac{360}{P} = 180^\circ \text{ pole-pitch.}$$

$$\rho = \frac{\theta_m}{\rho_p} \times 180^\circ \qquad \theta_m = \frac{5}{6} \times 180 = 150^\circ$$

$$\rho = \frac{150^\circ}{180^\circ} \times 180^\circ = 150^\circ \qquad \rho = \frac{5}{6} \rho_p$$

Fundamental $K_p = \sin \frac{\rho}{2} = \sin \frac{150}{2} = 0.966$.

3rd harmonic $K_p = \sin \frac{3\rho}{2} = \sin \underbrace{\frac{(3)(150)}{2} = -0.707}$

does not appear on the three-phase output.

5th harmonic $K_p = \sin \frac{(5)(150)}{2} = 0.259$

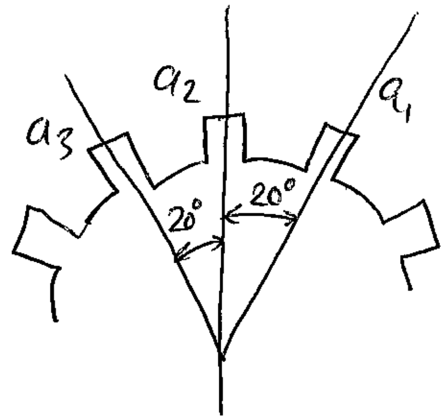
7th harmonic $K_p = \sin \frac{(7)(150)}{2} = 0.259$

9th harmonic $K_p = \underbrace{\frac{(9)(150)}{2} = -0.707}$

multiple of the third harmonic $\equiv$ does not appear at the output.

Therefore, employing fractional-pitch windings will drastically reduce the harmonic content of the machines output voltage while causing only a small decrease in its fundamental voltage.

Distribution Windings and Distribution Factor



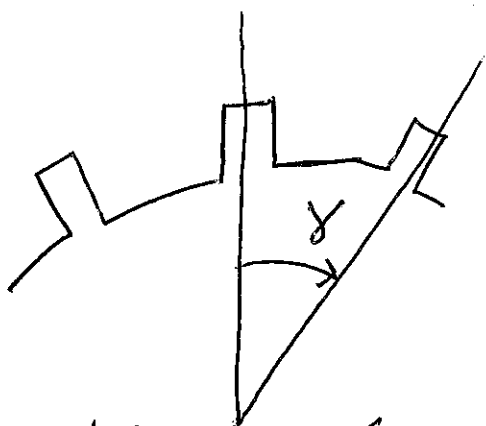
$\tilde{E}_{a_2} = E\angle 0$

$\tilde{E}_{a_1} = E\angle -20^\circ$

$\tilde{E}_{a_3} = E\angle 20^\circ$

$\tilde{E}_a = \tilde{E}_{a_1} + \tilde{E}_{a_2} + \tilde{E}_{a_3} = 2.879\,E\angle 0$

$|\tilde{E}_a| = 2.879\,E$. $\Rightarrow$ normally $\frac{2.879}{3} = K_d$



$$K_d = \frac{\sin(n\gamma/2)}{n\sin(\gamma/2)}$$

γ: angle between two successive slots (slot pitch)

n: no. of slots / phase / pole

* If $n = 3$, $\gamma = 20^\circ$ (18 slots)

$(3)(2)(3) = 18$ slots

slots/pole/phase

$$K_d = \frac{\sin((3)(20)/2)}{3\sin(20/2)} = 0.96$$

* The pitch factor & the distribution factor of a winding are sometimes combined for ease of use into a single <u>winding factor</u> K_W

$$K_W = K_p K_d$$

$$E_A = \sqrt{2}\pi N_p \phi f \underbrace{K_W}_{K_p K_d}$$

Ex. A simple two pole three phase Y-connected synchronous machine stator is used to make a generator. It has a double-layer coil construction with four stator coils per phase. Each coil consists of 10 turns. The windings have an electrical pitch of 150°. The rotor (and the magnetic field) is rotating at 3000 r/min and the flux per pole in this machine is 0.019 Wb.

(a) What is the slot pitch of this stator in mechanical degrees? In electrical degrees?

(b) How many slots do the coils of this stator span?

(c) What is the magnitude of the phase voltage of one phase of this machine's stator?

(d) What is the machine's terminal voltage?

(e) How much suppression does the fractional-pitch winding give for the 5^{th} harmonic component of the voltage relative to the decrease in its fundamental component?

Sol.

(a) $\gamma = \frac{360}{12} = 30^\circ$. mechanical and electrical degrees.

(b) $\frac{150}{180} = \frac{5}{6} \Rightarrow$ every coil spans 5 slots.

(c) $E_A = 4.44 \phi f N K_W$

$$f = \frac{nP}{120} = \frac{(3000)(2)}{120} = 50HZ.$$

$\phi = 0.019\,Wb$ (flux per pole)

$$N = 4 \frac{\text{coils}}{\text{phase}} * \frac{10 \text{ turns}}{\text{coil}} = 40 \text{ turns/phace}$$

$$K_W = K_p K_d$$

$$K_p = \sin\left(\frac{\gamma P}{2}\right) = \sin\left(\frac{(1)(150)}{2}\right) = 0.966$$

no. of stator slots per pole per phase

$$K_d = \frac{\sin(n\gamma/2)}{n \sin(\gamma/2)} = \frac{\sin((2)(\frac{30}{2}))}{2\sin(\frac{30}{2})} = 0.966$$

$$E_A = E_{rms} = (4.44)(0.019)(40)(50)(0.966)(0.966)$$
$$= 157V.$$

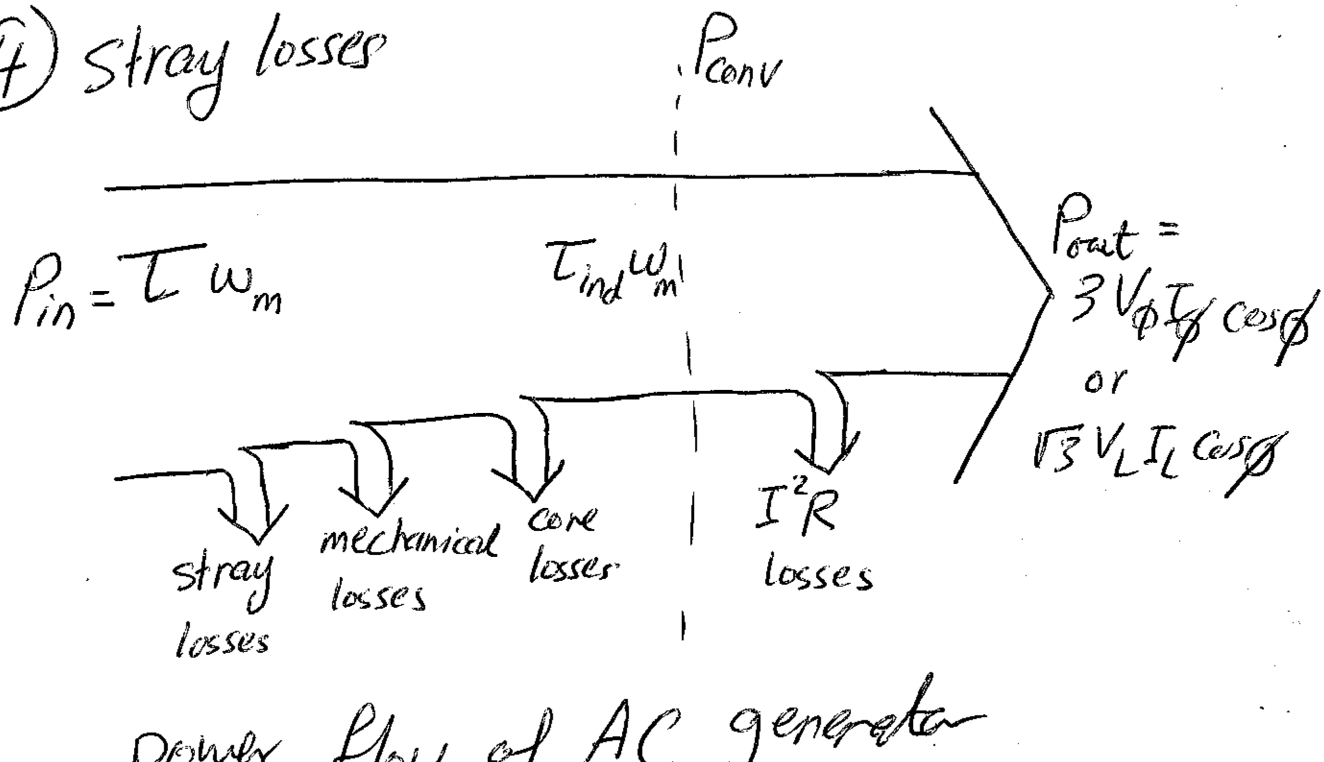
(d) $V_T = (\sqrt{3})(157) = 272V$

(e) $K_p = \sin\frac{(5)(150)}{2} = 0.259$.

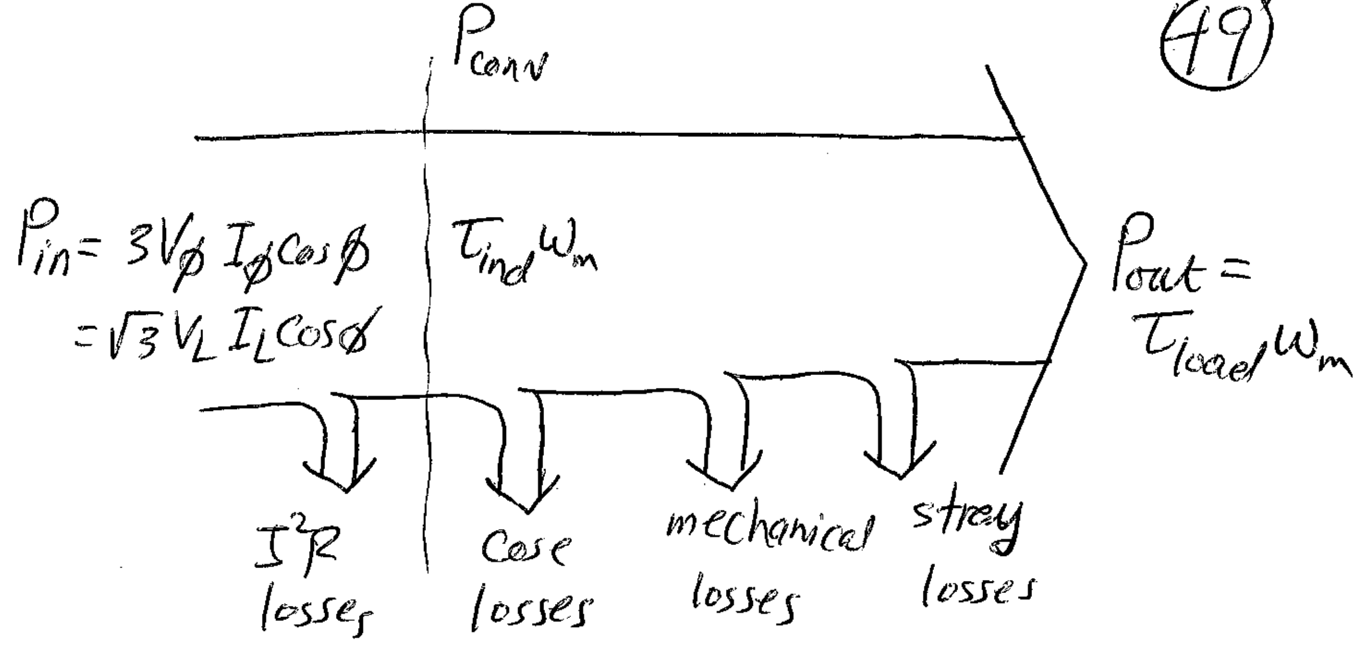
AC Machines Power Flows and Losses

The losses in ac machines fall into the same categories as the losses in dc machines:

1. Rotor and stator copper losses (I^2R)
2. Core losses
3. Mechanical losses
4. stray losses



power flow of AC generator



power flow of AC motor

Synchronous Generators

Construction

* synchronous machines are classified according to the type of the rotor into:
  1. cylindrical rotor type.
  2. Salient pole type.

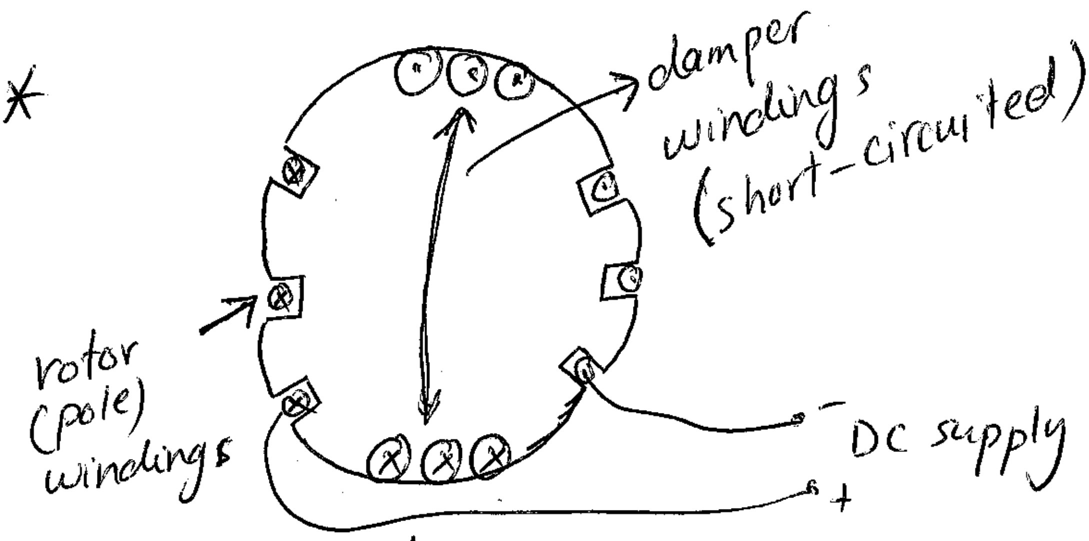
* In both types, the stator has the same construction.

* For rotor and stator constructions, see the figures in the book.

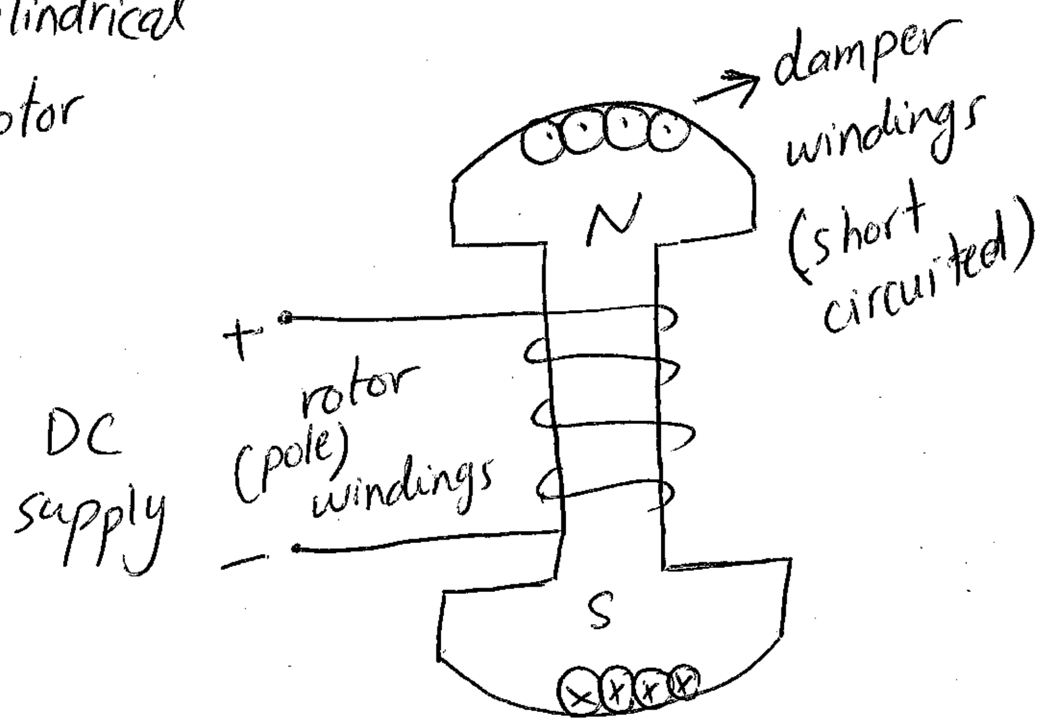
* In synchronous machine, the rotor is fed by dc power supply called the excitation and the output is a three-phase ac power in case of generator.

* Damper windings are windings placed in slots curved in the rotor (poles) short circuited from both the beginning and the end. They are used to damp oscillations in case of transients. In steady-state, the induced voltage and current inside them are zero.

*



cylindrical rotor



salient pole rotor

* There two common approches to supply dc power supply to the rotor of synchronous generator :

① supply the dc power from an external dc source to the rotor by means of slip rings and brushes.

② supply the dc power from a special dc power source mounted directly on the shaft of the generator.

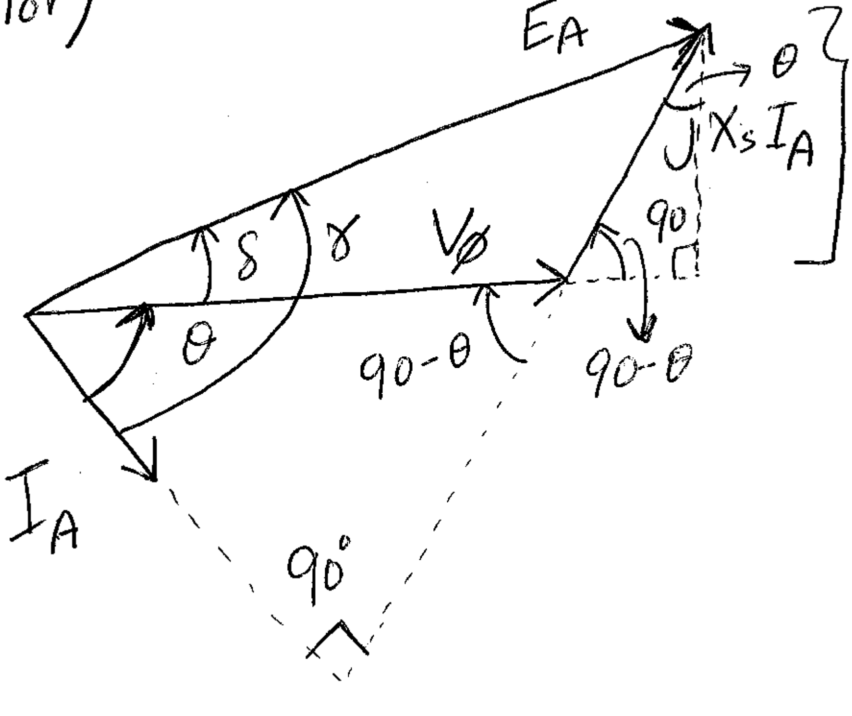
* For different excitation systems used to supply dc power supply to the rotor of synch. machine, see the book... pilot exciter!

power and Torque in synchronous Generators

* The source of the power in synchronous generator is called "prime mover" (mechanical engine). It must have the basic property that its speed is almost constant regardless of the power demand.

R_A is ignored ($X_s \gg R_A$)

* Lets examine the <u>simplified</u> phasor diagram of synchronous generator load by inductive load (lagging power factor)



$E_A \sin\delta = X_s I_A \cos\theta$

$$\therefore E_A \sin\delta = X_s \underbrace{I_A \cos\theta}$$

$$P_{out} = 3V_\phi \underbrace{I_A \cos\theta}$$

$$= 3V_\phi \frac{E_A \sin\delta}{X_s}$$

$$= \frac{3V_\phi E_A}{X_s} \sin\delta$$

* Since $R_A = 0$ (ignored) then $P_{conv} = P_{out}$.

* δ is called the "Torque angle". If $\delta = 90°$,
"power angle"

then $P_{out} = P_{max}$

$$\underbrace{P_{max} = \frac{3V_\phi E_A}{X_s}}_{\text{static stability limit}}, \quad \delta = 90°.$$

static stability limit. (very important in power system stability)

* Normally P_{out} is very far from P_{max} as at full load δ is normally typically between $15°$ to $20°$.

* As for the induced torque τ_{ind}:

$$\tau_{ind} = \frac{3V_\phi E_A \sin\delta}{\omega_m X_s}$$

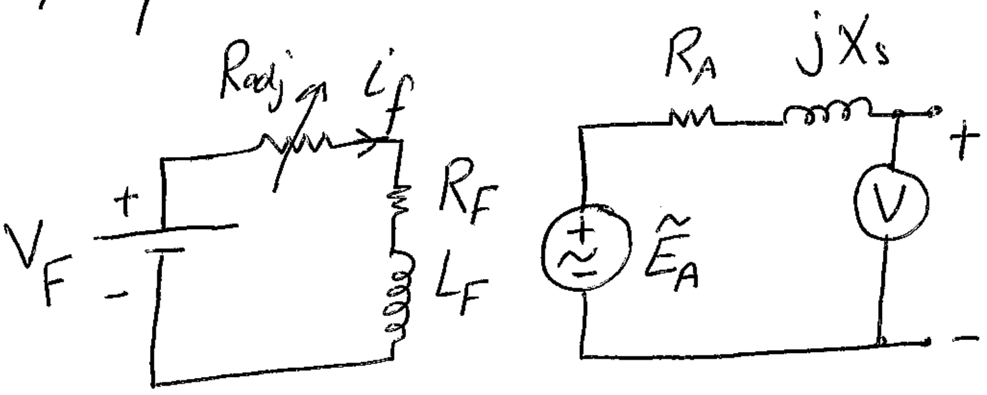
Measuring Synchronous Generator Model Parameters

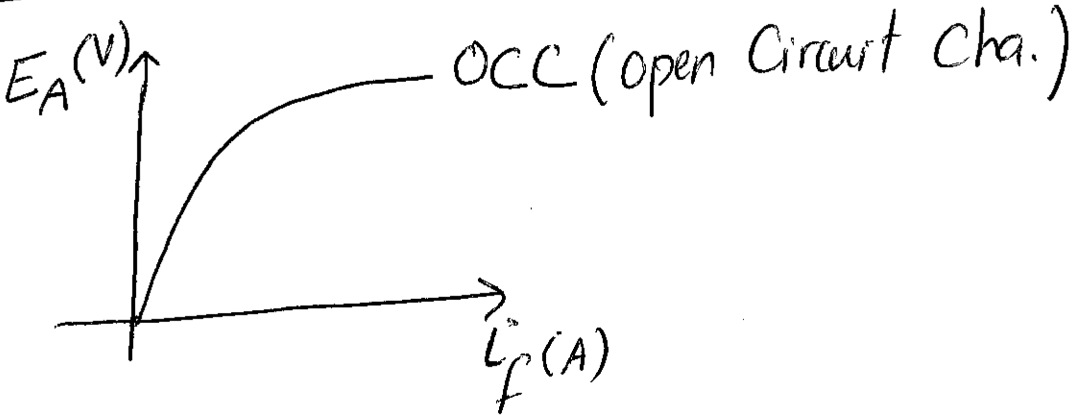
* The equivalent circuit of a synchronous generator contains three quantities that must be determined in order to completely describe the behavior of a real synchronous generator:

  1. The relationship between field current and flux and therefore between i_f & E_A.
  2. The synchronous reactance.
  3. The armature resistance.

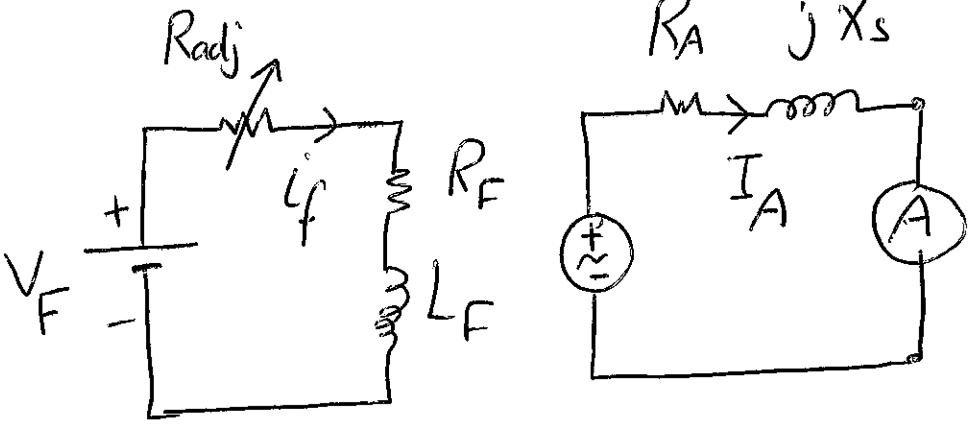
* This can be done by what is called "Open-Circuit Test" & "Short Circuit Test".

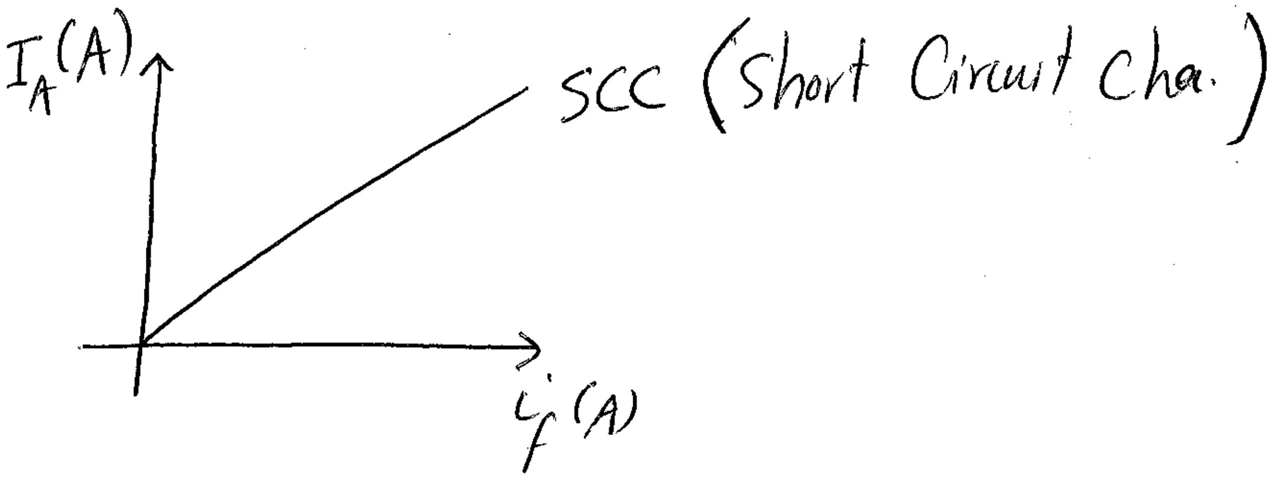
* Open Circuit Test





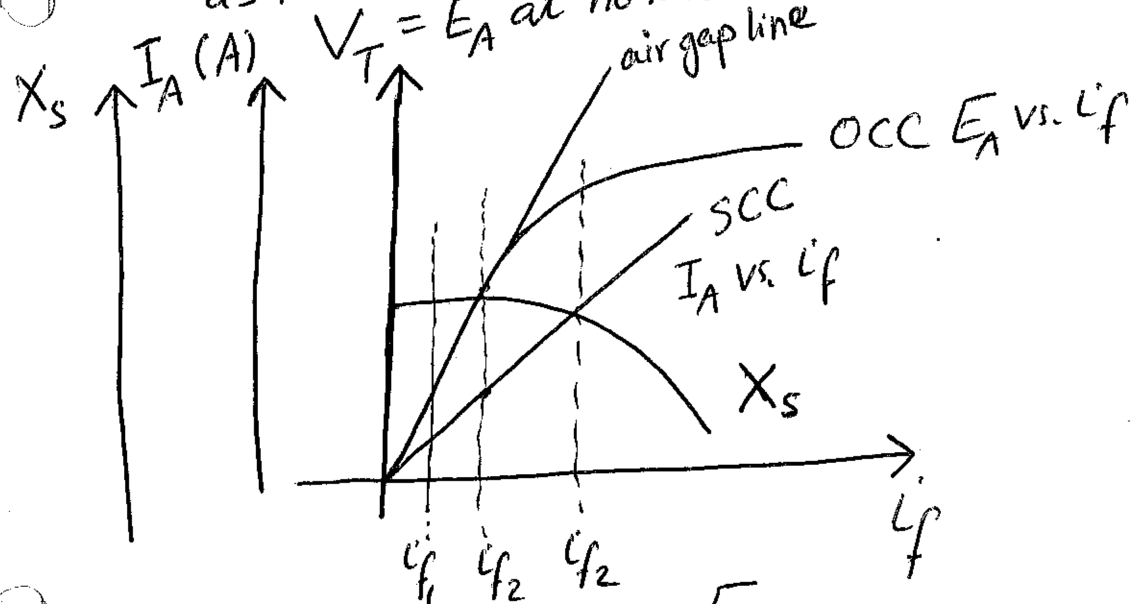
* Short Circuit Test





* The armature resistance R_A can be measured by ~~$~~ applying dc power supply to the stator and measuring the ~~resis~~ voltage & the current.

* The values of X_s and Z_s are determined as:



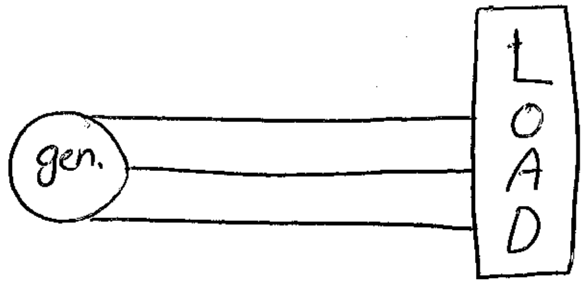
$$Z_s = \sqrt{R_A^2 + X_s^2} = \frac{E_A}{I_A}$$

$$X_s \approx \frac{E_A}{I_A}$$

* Note the effect of saturation on the value of X_s (at high values of field current)

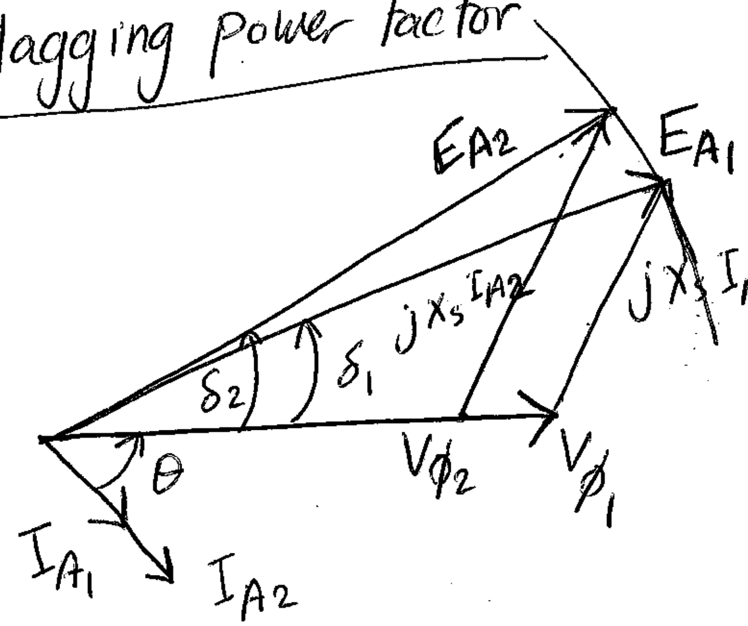
* There is a straight forward example in the book. See it.

The Synchronous Generator Operating Alone



* Lets study the cha. of synchronous generator operating alone using phasor diagram as follows:

① with lagging power factor



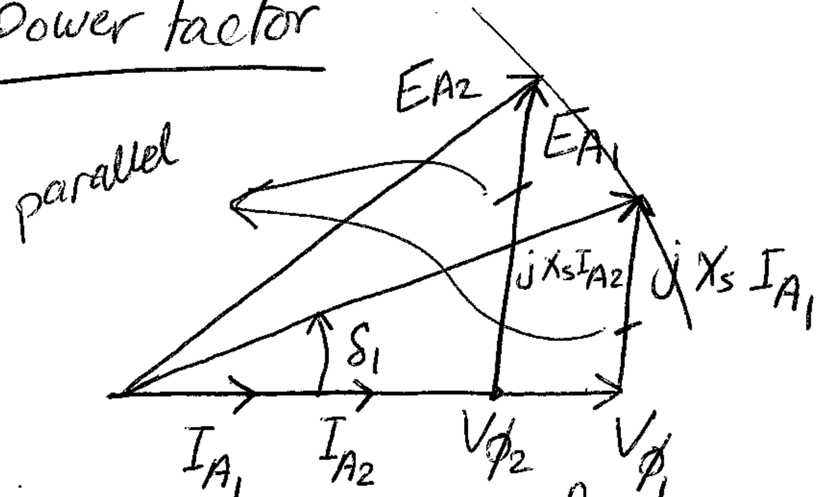
$V_{\phi 2} < V_{\phi 1}$

* assumptions: no changes on $|\tilde{E}_A|$ (fixed E_A)
  no changes on power factor i.e. θ
  no changes on ω.

* As the load increases (demand for P_{out} and/or Q_{out} increases), the output current I_A increases.

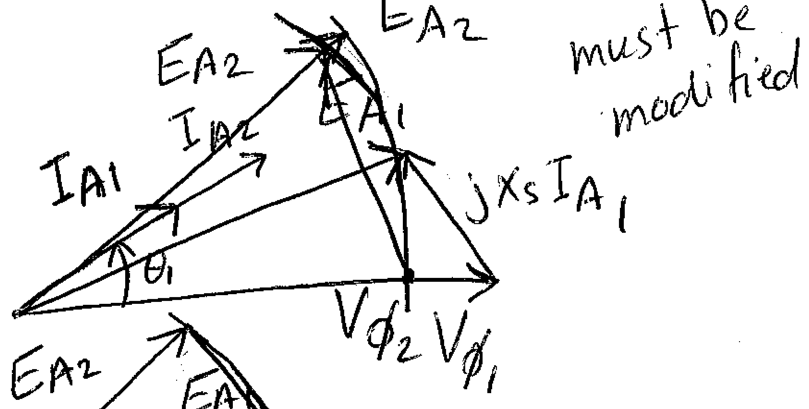
*Conclusion : as the load increases, the terminal voltage decreases for the same E_A.

② Unity power factor



* The same assumptions as before.
* As the load increases, the terminal Voltage decreases for the same E_A.

③ Leading Power Factor



* As the load increases, the terminal Voltage increases.

* $VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$

$VR > 0$ in case of R & RL loads.

$VR < 0$ in case of RC loads.

* How to keep constant V_ϕ (terminal voltage)?

This is done by what is called "Automatic Voltage Regulator" (AVR). When the load increases, V_ϕ decreases (in case of R and RL loads). To return V_ϕ to its original value, E_A must increase. To accomplish this, the field current must increase by decreasing the field resistance R_F.

Ex. A 480V 60 Hz Y-connected six pole synchronous generator has a per-phase synchronous reactance of 1Ω. Its full-load armature current is 60A at 0.8 PF lagging. This generator has friction and windage losses of 1.5KW and core losses of 1KW at 60 Hz at full load. Since the armature resistance is being ignored, assume that the I^2R losses are negligible. The field current has been adjusted so that the terminal voltage is 480V at no load.

ⓐ What is the speed of rotation of this generator?

ⓑ What is the terminal voltage of this generator if the following are true:

1. It is loaded with the rated current at 0.8 PF lagging.
2. It is loaded with the rated current at 1.0 PF.
3. It is loaded with the rated current at 0.8 PF leading

ⓒ What is the efficiency of this generator when it is operating at the rated current and 0.8 PF lagging.

(d) How much shaft torque must be applied by the prime mover at full load? How large is the induced countertorque?

(e) What is the voltage regulation of this generator at 0.8 PF lagging? At 1.0 PF? At 0.8 PF leading?

Sol.

(a) $f = \frac{nP}{120} \Rightarrow n = \frac{120f}{P} = \frac{(120)(60)}{6} = 1200 \text{ rpm}.$

$$\omega = \frac{2\pi n}{60} = \frac{(2\pi)(1200)}{60} = 125.7 \text{ rad/s}.$$

(b) (1) $\tilde{I}_A = 60\angle -36.87^\circ$

$$\tilde{E}_A = \tilde{V}_T + \tilde{I}_A(R_A + jX_s)$$

$$\frac{480}{\sqrt{3}}\angle\delta = V_T\angle 0 + (60\angle -36.87^\circ)(j1)$$

$$277.5\angle\delta = V_T\angle 0 + 60\angle -36.87^\circ \, 1\angle 90$$

$$277.5\cos\delta + j277.5\sin\delta = V_T + 60\angle 53.13^\circ$$

$$277.5\cos\delta + j277.5\sin\delta = V_T + 60\cos 53.13^\circ + j60\sin 53.13^\circ$$

$$277.5\cos\delta = V_T + 60\cos 53.13^\circ$$

$$277.5\sin\delta = 60\sin 53.13^\circ$$

$$\sin\delta = \frac{60\sin 53.13^\circ}{277.5} \Rightarrow \delta =$$

$V_T = 236.8\ V$ phase-neutral (phase voltage)

$V_T = \sqrt{3}(236.8) = 410V$ line-line. (line voltage)

(2) $\tilde{E}_A = \tilde{V}_T + \tilde{I}_A (jX_s)$

$$\tilde{I}_A = 60\angle 0$$

$$E_A\angle\delta = V_T\angle 0 + (60\angle 0)(j1)$$

$$E_A\cos\delta + jE_A\sin\delta = V_T + j60$$

$$277.5\cos\delta + j277.5\sin\delta = V_T + j60$$

$$\Downarrow$$

$V_T = 270.4V$ phase voltage

$V_T = \sqrt{3}(270.4) = 468.4V$ line voltage

(3) $\hat{I}_A = 60\angle 36.87^\circ$

$$277.5\angle\delta = V_T\angle 0 + (60\angle 36.87^\circ)(j1)$$

$V_T = 308.8\,V$ phase voltage.

$V_T = (\sqrt{3})(308.8) = 535\,V$ Line voltage.

(c) $\eta = \frac{P_{out}}{P_{in}} \times 100\%$

$P_{out} = \sqrt{3}\, V_L\, I_L \cos\phi = (\sqrt{3})(410)(60)(0.8)$

$= 34046.4\,W$

$P_{in} = 34046.4 + 1500 + 1000 = 36546.4\,W$

$\eta = \frac{34046.4}{36546.4} \times 100\% = 93.2\%$.

(d) $P_{in} = T\omega \Rightarrow T = \frac{36546.4}{\frac{(2\pi)(1200)}{60}} = 290.8\,Nm$.

$T_{counter} = \frac{P_{out}}{\omega} = \frac{34046.4}{\frac{(2\pi)(1200)}{60}} = 271.3\,Nm$.

(e) **At 0.8 PF lagging**

$$VR = \frac{277.5 - 236.8}{236.8} \times 100\%$$

$$= 17.2\%$$

At 1 PF

$$VR = \frac{277.5 - 270.4}{270.4} \times 100\%$$

$$= 2.6\%$$

At 0.8 PF leading

$$VR = \frac{277.5 - 308.8}{308.8} \times 100 = -10.3\%$$

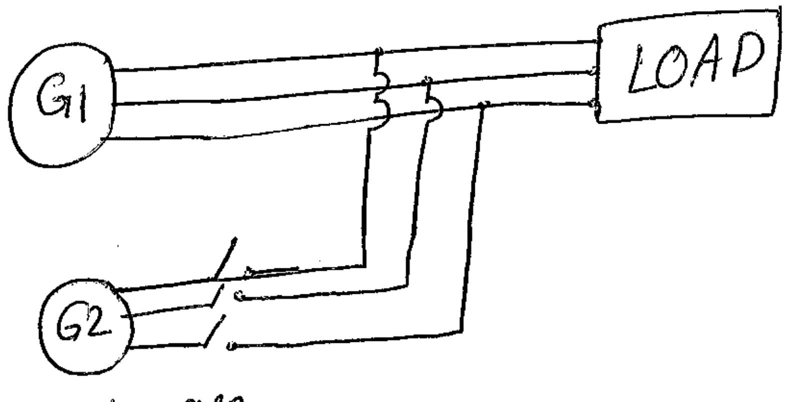
First First → **Parallel operation of AC Generators** First

Whay are synchronous generators operated in parallel?

1. Several generators can supply a bigger load than one machine.
2. increases the reliability as the failure of any one doesn't cause a total power loss to the load.
3. Having many generators operating in parallel allows one or more to be removed for maintenance.

4 If only one generator is used and it is not operating at near full load, then it will be relatively inefficient. But with several smaller machines operating near full load, it is more efficient.

The conditions Required for Paralleling



oncoming gen.

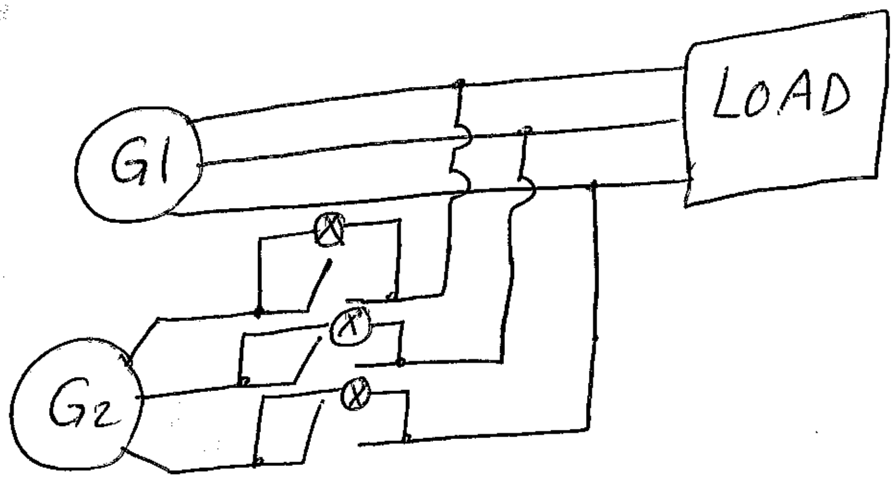
The following paralleling conditions must be met:

1. The rms line voltages of the two generators must be equal.
2. They must have the same phase sequence.
3. The phase angles must be equal
4. The frequency of the oncoming generator must be slightly higher than the frequency of the running generator.

The General Procedure for Paralleling Generators

First Using Voltmeters, the field current of the oncoming generator should be adjusted until its terminal voltage is equal to the line voltage of the running system.

Second The phase sequence of the oncoming generator must be checked to be like that of the running system. One possible way is by the three-light-bulb method. This method is shown below in this figure:



If the three bulbs get bright and dark together, then the systems have the same phase sequence.

Third The frequency of the oncoming generator can be measured by frequency meter or speed meter. It must be slightly higher than that of the running system.

* Another way of paralleling is by using what is called "synchroscope".

* In large power stations and huge power systems, the paralleling process is fully automated and computerized.

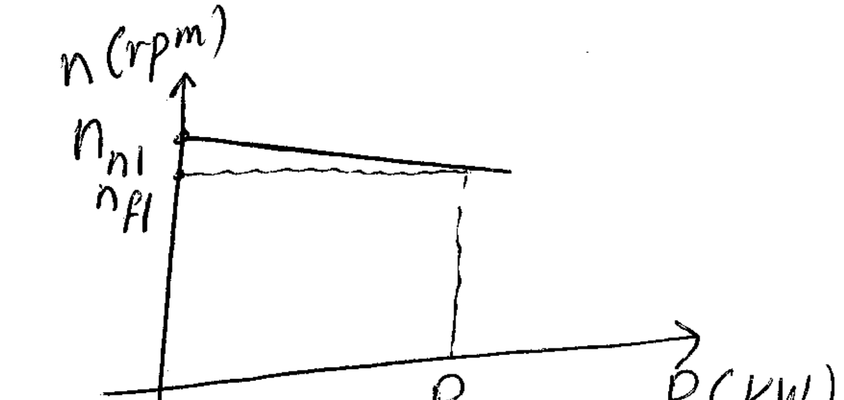
Frequency-Power and Voltage- Reactive Power Characteristics of a Synchronous Generator

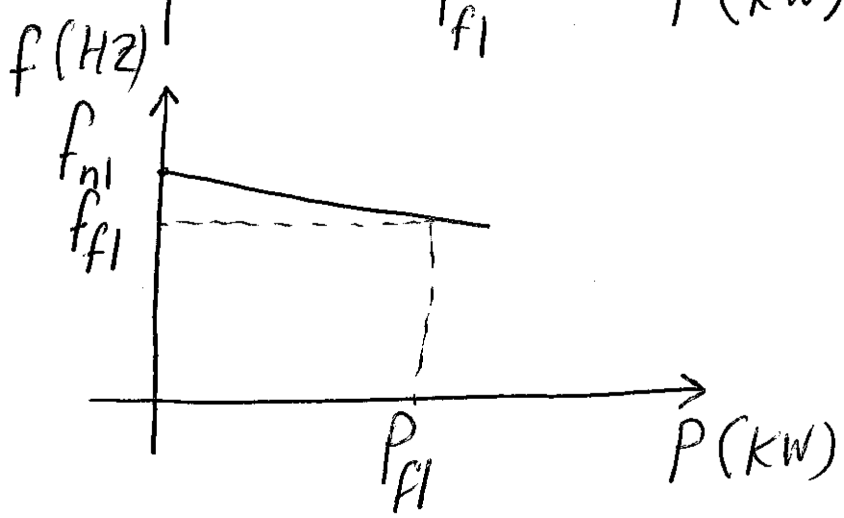
* Synchronous Generators are driven by "prime mover" which could be steam turbine, diesel engine, gas turbine, water turbine and wind turbines.

* All prime movers tend to behave in a similar fashion — as the power drawn from them increases, the speed at which they turn decreases. This decrease is in general nonlinear. Some mechanism is usually included to make the decrease in speed linear with an increase in power demand. The Speed Drop (SD) or SR is:

$$SD = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

SD is typically about 2 to 4%.





* Normally, the output power of synch. generator as function of ~~the~~ its frequency is given by:

$$P = S_p \left(f_{nl} - f_{sys} \right) \Rightarrow \text{Frequency - Power Cha.}$$

P: output power of generator

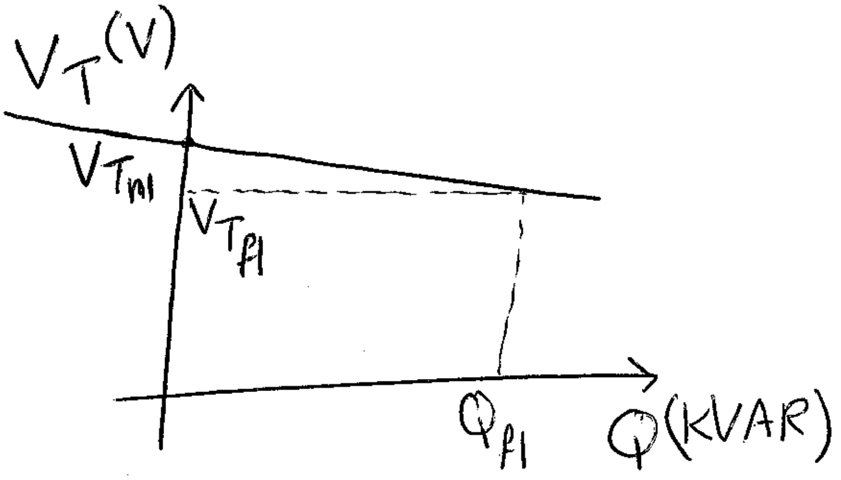
S_p: Slope of curve KW/Hz or MW/Hz

f_{nl}: no-load frequency of generator.

f_{sys}: operating frequency of the system 50 Hz or 60 Hz

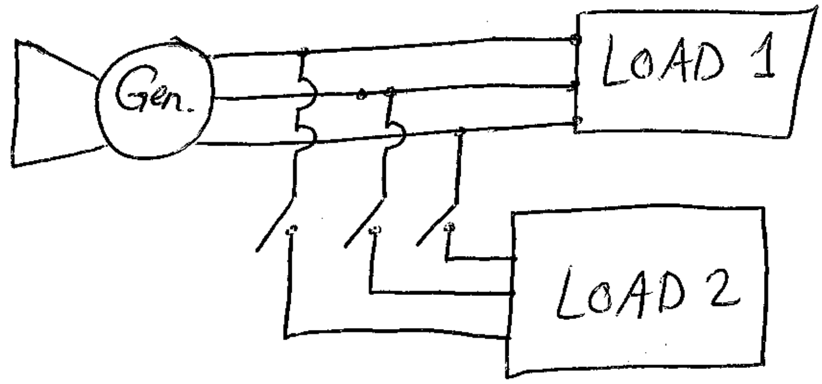
~~of the synchronous Generator as function of the~~

* A similar relation for the terminal voltage of synchronous generator V_T as function of the output reactive power Q can be derived. Generally it is a nonlinear relation but the voltage regulator makes it linear.



* It is important to realize that when a single generator is operating alone, the P and Q supplied by the generator is the amount demanded by the load connected to the generator. Therefore, for any given real power, the governer set points controls the generator's operating frequency f_e and for any given reactive power, the field current controls the generator's terminal voltage V_T.

Ex. The following figure shows a generator supplying a load. A second load is to be connected in parallel with the first one. The generator has a no-load frequency of 61.0 Hz and a slope S_p of 1 MW/Hz. Load 1 consumes 1000 KW at 0.8 PF lagging. Load 2 consumes 800 KW at 0.707 PF lagging.



ⓐ Before the switch is closed, what is the operating frequency of the system?

ⓑ After load 2 is connected, what is the operating frequency of the system.

ⓒ After load 2 is connected, what action can an operator take to restore the system frequency to 60 Hz.

a)

$$P = S_P(f_{nl} - f_{sys})$$

$$1000\,000 = 1000\,000\,(61 - f_{sys})$$

$$f_{sys} = 60\ Hz.$$

b)

$$1000\,000 + 800\,000 = 1000\,000\,(61 - f_{sys})$$

$$f_{sys} = 59.2\ Hz.$$

c) The operator should increase the governor set point by 0.8 such that the no-load frequency beomes $61 + 0.8 = 61.8\ Hz.$

To summarize what is going on in synchronous gen:

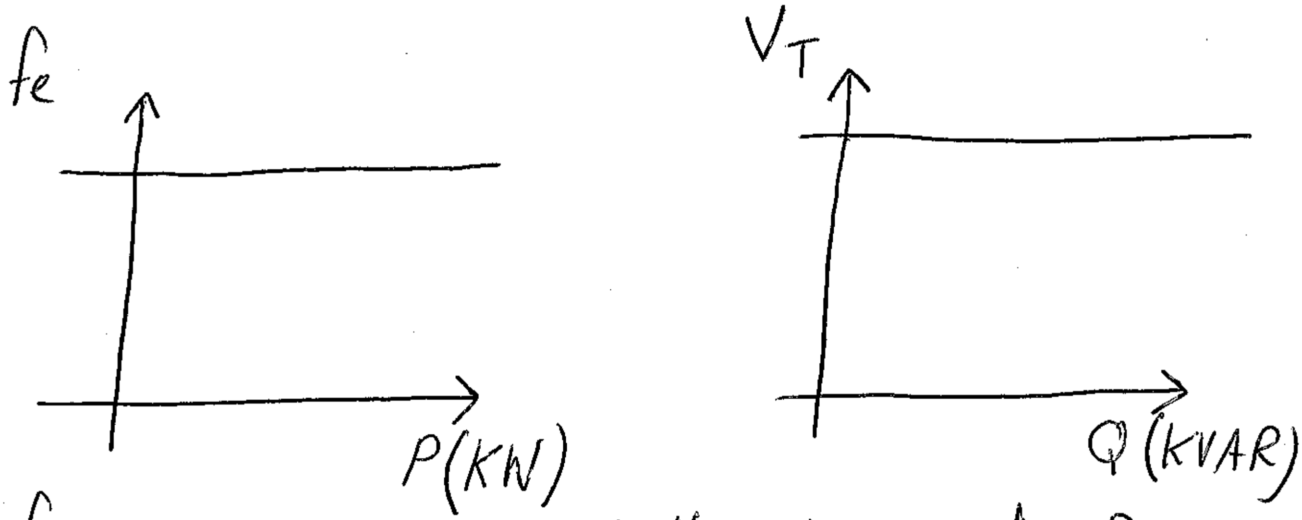
1) The real and reactive power supplied by the generator is the amount demanded by the attached load.

2) The governor set points of the generator controls the operating frequency of the power system.

3. The field current (or the field regulator set points) controls the terminal voltage of the power system.

Operation of Generators in Parallel with Large Power Systems

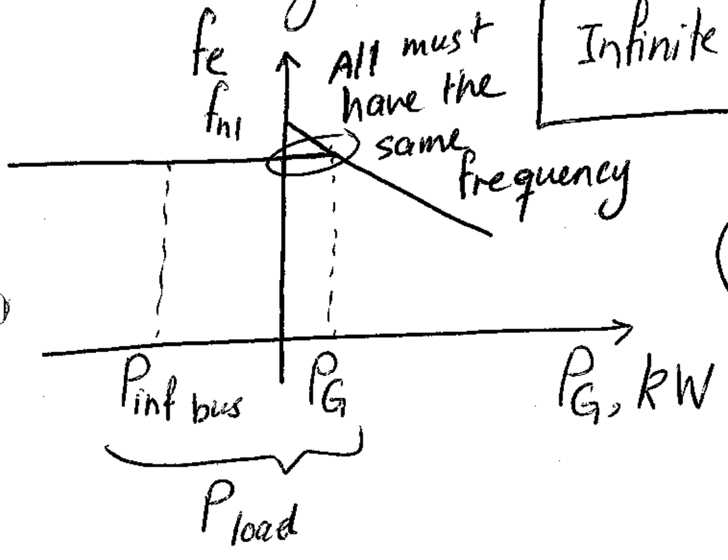
Infinite Bus: A so large power system where its voltage and frequency do not vary regardless of how much real and reactive power is drawn from or supplied to it. Its power-frequency and reactive power-voltage characteristics ar shown below:

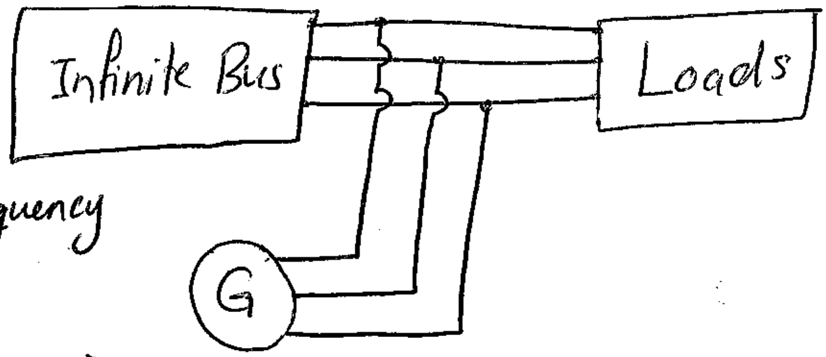


frequency vs. Power & Voltage vs. reactive power of inifinite bus

* Infinite Bus: Constant voltage constant frequency bus.

* When a generator is connected in parallel with another generator or a large system, the frequency and terminal voltage of all the machines must be the same since their output conductors are tied together. Therefore, their real power-frequency and reactive power-voltage characteristics can be plotted back to back with a common vertical axis. Such a sketch is sometimes called informally "house diagram".

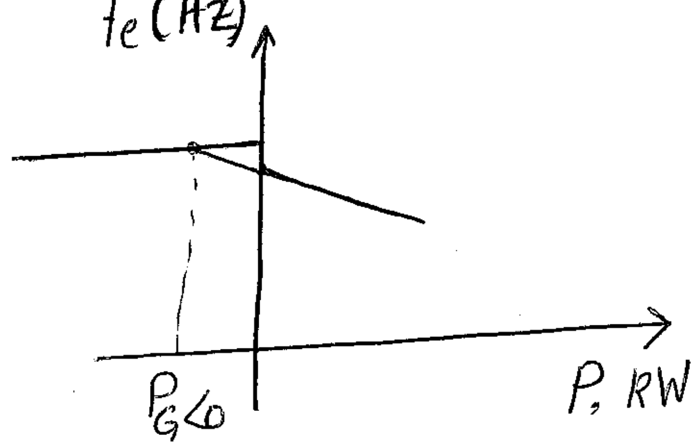




"house diagram"

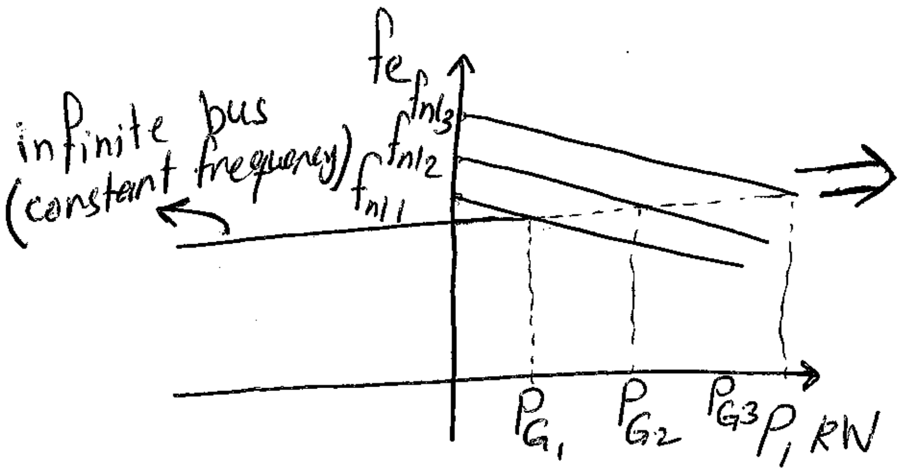
frequency vs. power for a synch. generator in parallel with an infinite bus

* Assume that the generator has just been paralleled with a no-load frequency lower than the system frequency as shown below in the following figure.



Clearly the output power of the generator is negative which means that the power is being consumed by the machine i.e. it is the motor mode of operation. Many real generators have a reverse-power trip connected to them as a protection system against reversing the direction of power flow i.e if such a situation takes place, the generator is directly disconnected from the grid.

* Once the generator has been connected, what happens when its governor set points are increased? The effect of this increase is to shift the no-load frequency of the generator upward and to increase the output power of the generator as shown below in the house diagram:



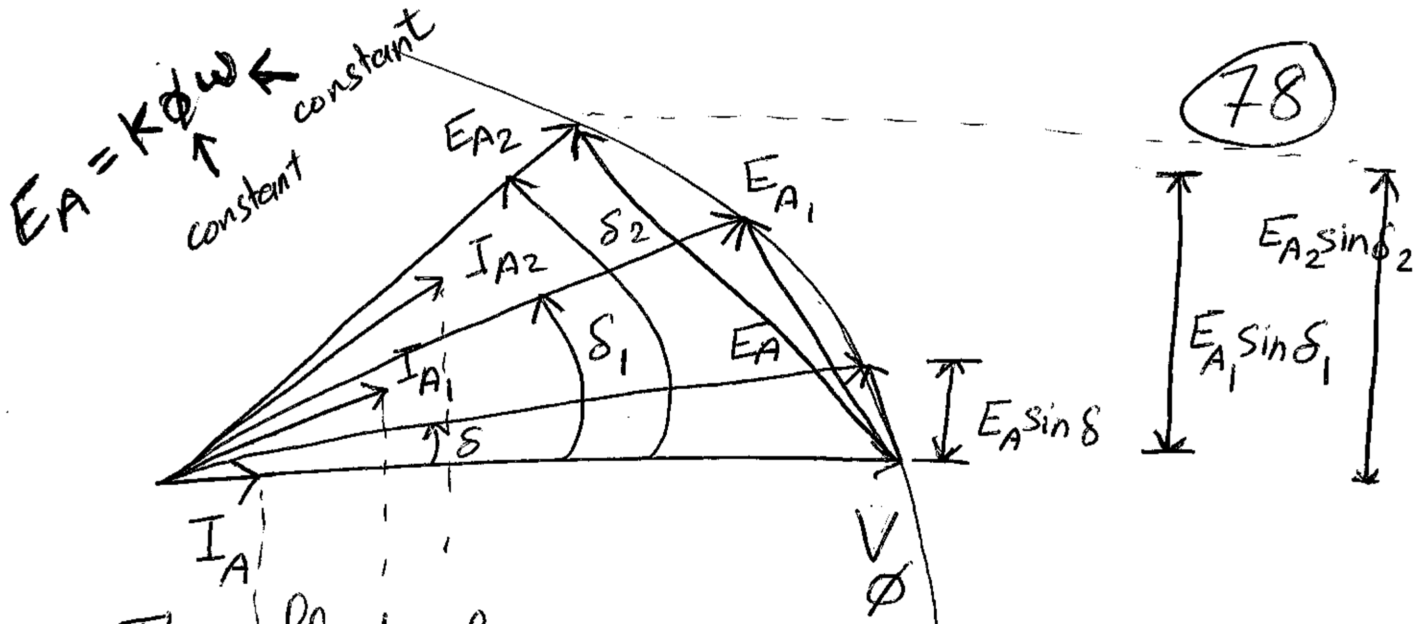
The effect of increasing the governor set point

* The phasor diagram of this case is shown below. Notice that $\underline{E_A \sin \delta}$ (which is proportional to the power supplied as long as $\underline{\underline{V_\phi}}$ is constant)

because it is infinite bus.

$$P_{out} = \frac{3 V_\phi E_A \sin \delta}{X_s}$$

has increased while the magnitude of E_A ($= K\phi\omega$) remains constant since both the field current I_F and the speed of rotation ω are constant.



The effect of increasing the governor set points (Constant field current I_F, Constant rotational speed ω) (Constant terminal Voltage)

* The previous investigation shows the effect of changing the governor set point while keeping the field current unchanged. Now what happens if the field current increases and governor set point is kept unchanged. In this case the input power to the generator is constant and therefore

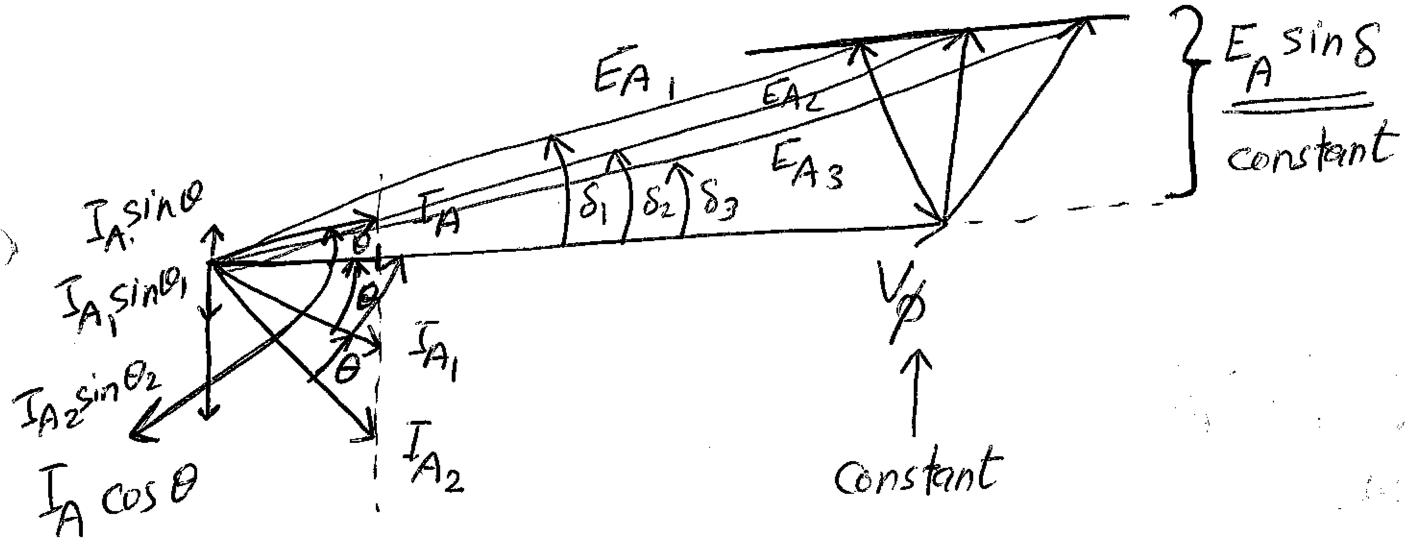
$I_A \cos\theta$ & $E_A \sin\delta$ cannot change

$$P_{out} = \frac{3 V_\phi E_A \sin\delta}{X_s}$$
V_ϕ, X_s: constant

$$P_{out} = 3 V_\phi I_\phi \cos\theta$$
V_ϕ: Constant

When the field current is increased, the flux ϕ increases and therefore $E_A = K\phi\uparrow\omega$ increases.

If E_A inreases but $E_A \sin\delta$ is constant then E_A must slide along the line of constant power as shown below:



$I_A \cos\theta$ is constant, but $I_A \sin\theta$ increases with higher I_A. The output reactive power Q increases.

<u>Conclusion</u> Increasing the field current in a synchronous operating in parallel with an infinite bus increases the reactive power output of the generator.

summary

When a generator is operating in parallel with an infinite bus:

① The frequency and terminal voltage of the generator are controlled by the system to which it is connected.

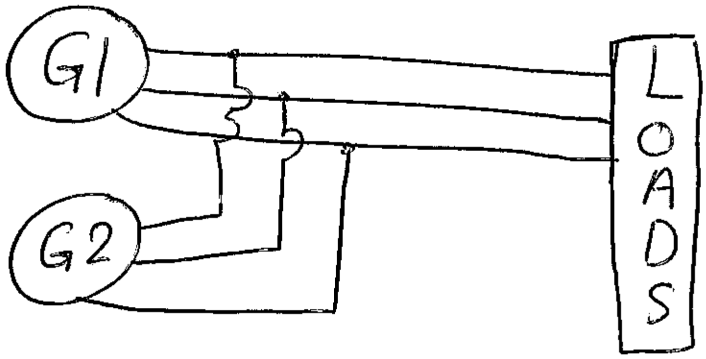
② The governer set points of the generator control the real power supplied by the generator to the system.

③ The field current in the generator controls the reactive power supplied by the generator to the system.

→ 1st
1st

Operation of Generators in Parallel with Other Generators of the same size

If a generator is connected in parallel with another one of the same size, then we have the following system:



stand alone power system

Two generators connected in parallel feeding electrical loads.

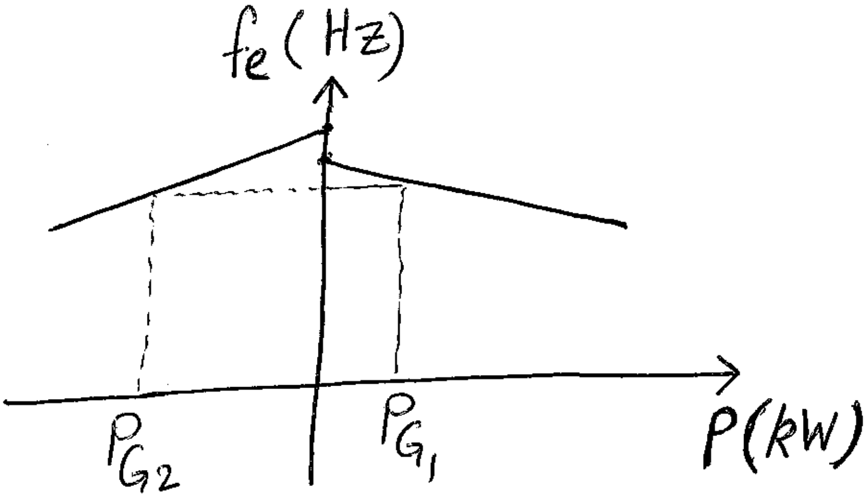
* In this system, the sum of the real and reactive powers supplied by the two generators must equal the P and Q demanded by the loads. The power and frequency are not constant. Here the total power P_{tot} (which equal to P_{load}) is given by:

$$P_{tot} = P_{load} = P_{G_1} + P_{G_2}$$

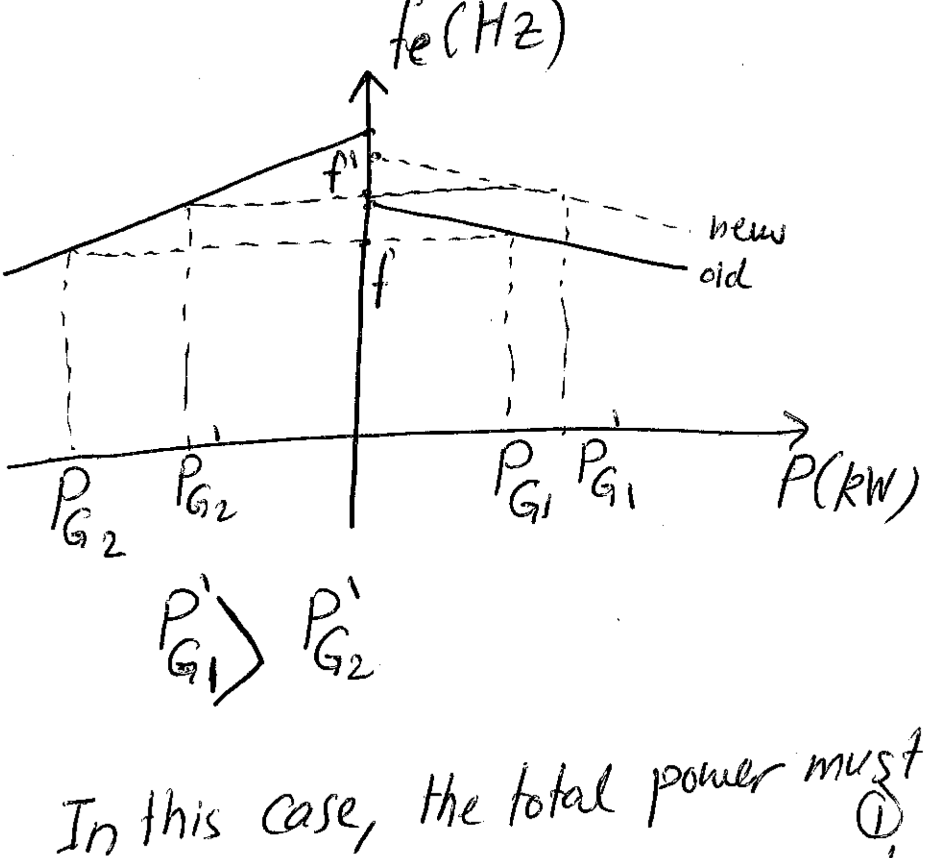
and Q_{tot}:

$$Q_{tot} = Q_{G_1} + Q_{G2}$$

The house diagram in this case is:



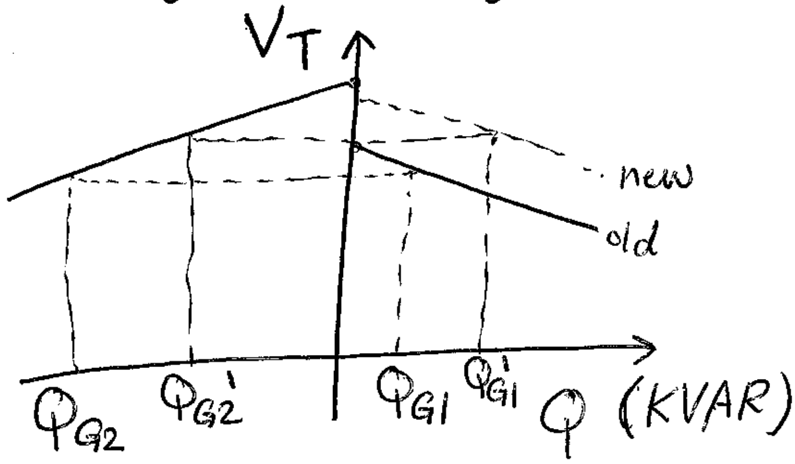
What happens if the governor set point of any generator ~~is~~ is increased? This is explained in the following House diagram:



$P'_{G_1} > P'_{G_2}$

In this case, the total power must remain constant ① and therefore P_{G_1} increases and P_{G_2} decreases and ② the frequency of the system increases.

What happens if the field current of G_2 is increased? The result is shown below in the Voltage vs. Q diagram:



$Q_{G1}' > Q_{G2}'$

* Q_{total} must remain fixed equal to the total reactive power consumed by the load.

① The system terminal voltage increases

② The reactive power Q supplied by that generator increases while the reactive power supplied the other generator decreases.

Ex. Two generators are supplying a load.

Generator 1 has a no-load frequency of 61.5 Hz and a slope S_{P_1} of 1MW/Hz. Generator 2 has a no-load frequency of 61.0 Hz and a slope S_{P_2} of 1MW/HZ. The two generators are supplying a real load totaling of 2.5MW at 0.8 PF lagging.

ⓐ Calculate the frequency of the system.

$$P_{tot} = P_1 + P_2$$

$$2.5\times10^6 = S_{P_1}(f_{nl_1} - f_{sys}) + S_{P_2}(f_{nl_2} - f_{sys})$$

$$2.5\times10^6 = 1\times10^6\,\frac{W}{Hz}(61.5 - f_{sys}) + 1\times10^6\,\frac{W}{Hz}(61 - f_{sys})$$

$$f_{sys} = 60\,Hz.$$

$$P_1 = 1\times10^6\,\frac{W}{Hz}(61.5 - 60) = 1.5\,MW.$$

$$P_2 = 1\times10^6\,\frac{W}{Hz}(61 - 60) = 1\,MW.$$

85

(b) Suppose an additional 1MW load were attached to this power system. What would the new system frequency be? How much power would G_1 and G_2 supply now?

$$P_{tot} = S_{P_1}(f_{nl_1} - f_{sys}) + S_{P_2}(f_{nl_2} - f_{sys})$$

$$3.5 \times 10^6 = 1 \times 10^6 (61.5 - f_{sys}) + 1 \times 10^6 (61 - f_{sys})$$

$$f_{sys} = 59.5 \text{ Hz}.$$

$$\left.\begin{aligned} P_1 &= (1 \times 10^6)(61.5 - 59.5) = 2\text{MW} \\ P_2 &= (1 \times 10^6)(61 - 59.5) = 1.5\text{MW} \end{aligned}\right\}$$

(c) With the operating of part (b), what will the system frequency and generator powers be if the governer set points on G_2 are increased by 0.5Hz.

$$P_{tot} = S_{P_1}(f_{nl_1} - f_{sys}) + S_{P_2}(f_{nl_2} - f_{sys})$$

$$3.5\times10^6 = (1\times10^6)(61.5 - f_{sys}) + (1\times10^6)(61.5 - f_{sys})$$

$$f_{sys} = 59.75 \text{ Hz.}$$

$$P_1 = (1\times10^6)(61.5 - 59.75) = 1.75\,MW.$$

$$P_2 = (1\times10^6)(61.5 - 59.75) = 1.75\,MW.$$

Note that the system frequency rose, the power of G_2 rose and the power of G_1 fell.

* To summarize in the case of two generators operating together:

① The system is constrained in that the total power supplied by the two generators together must equal the amount consumed by the load. Neither f_{sys} nor V_T is constrained to be constant.

(2) To adjust the real power sharing between the generators without changing f_{sys} simultaneously increase the governer set points on one generator while decreasing the governer set points on the other generator.

The machine whose governer set point was increased will provide ~~...~~ more power of the load.

(3) To adjust f_{sys} without changing the real power sharing, simultaneously increase or decrease both generators' governer set points.

(4) To adjust the reactive power sharing between generators without charging V_T, simultaneously increase the field current on one generator while decreasing the field current on the other. The machine whose field current was increased will provide more reactive power to the load.

(5) To adjust V_T without changing the reactive power sharing, simultaneously increase or decrease both generators' field current.

Synchronous Generator Transients

* Transients of synchronous generators come as a result of :
  1. shaft torque is applied.
  2. output load suddenly changes.
  3. the generator is paralleled with a running power system.
  4. short circuit across the terminals of the generator.

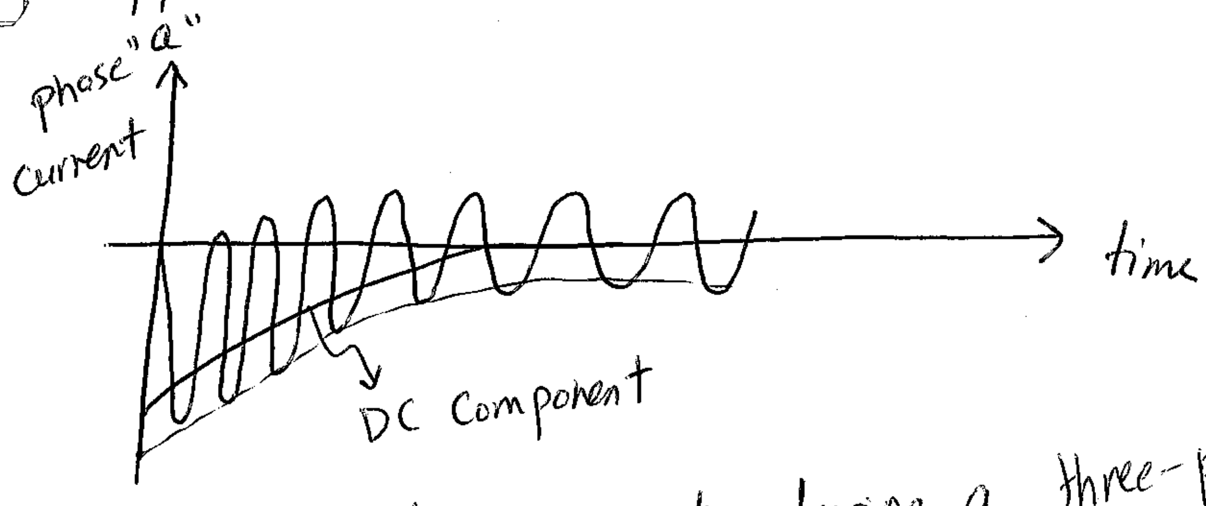
* If the generator was turning at a speed higher than the synchronous speed when it was paralleled with the power system. At that instant the induced current of the stator produces torque opposite to the direction of motion. This torque slows down the generator until it finally turns at synchronous speed with the rest of the power system.

* Similarly, if the generator were turning at a speed lower than synchronous speed when it was paralleled with the power system, then the induced

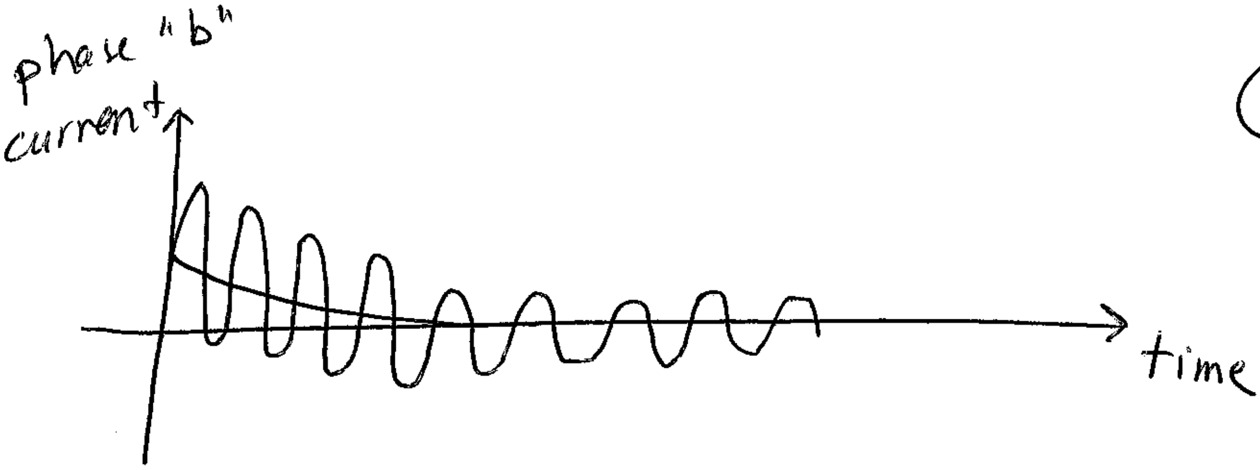
as a result of the stator current which is in the direction of motion would speed up the rotor until it again began turning at synchronous speed

Short-Circuit Transients in Synchronous Generator

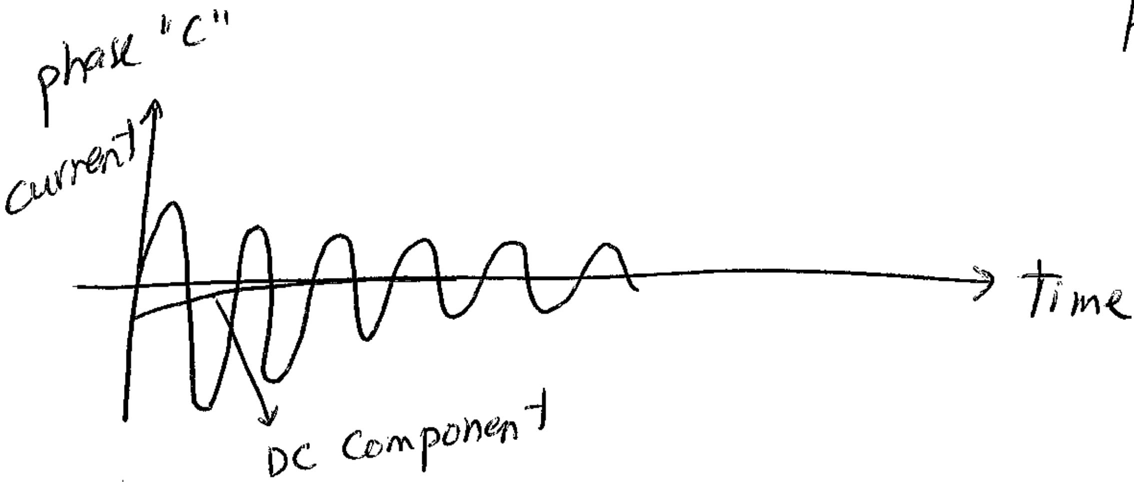
* The severest transient condition that can occur in a synchronous generator is the three-phase short circuit. When this fault occurs the resulting current flow in the phases of the generator can appear as shown below:



phase "a" current during a three-phase fault (symmetrical fault).

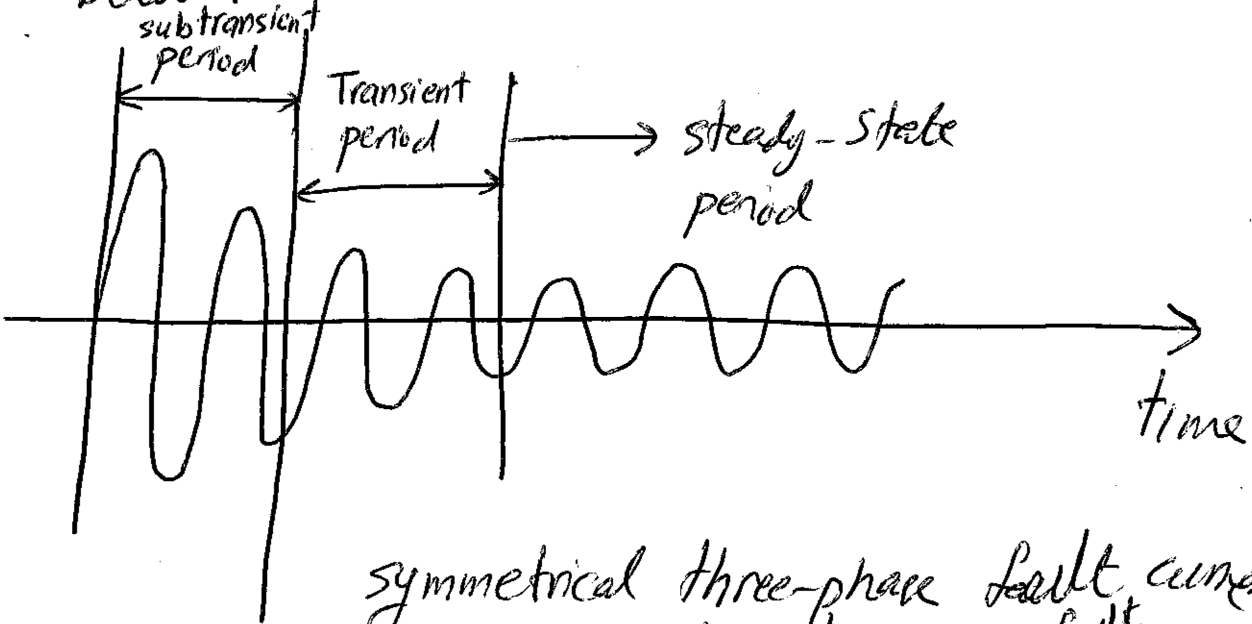


phase "b" current during three-phase fault



phase "c" current during three-phase fault.

* The symmetrical three-phase current is shown below:



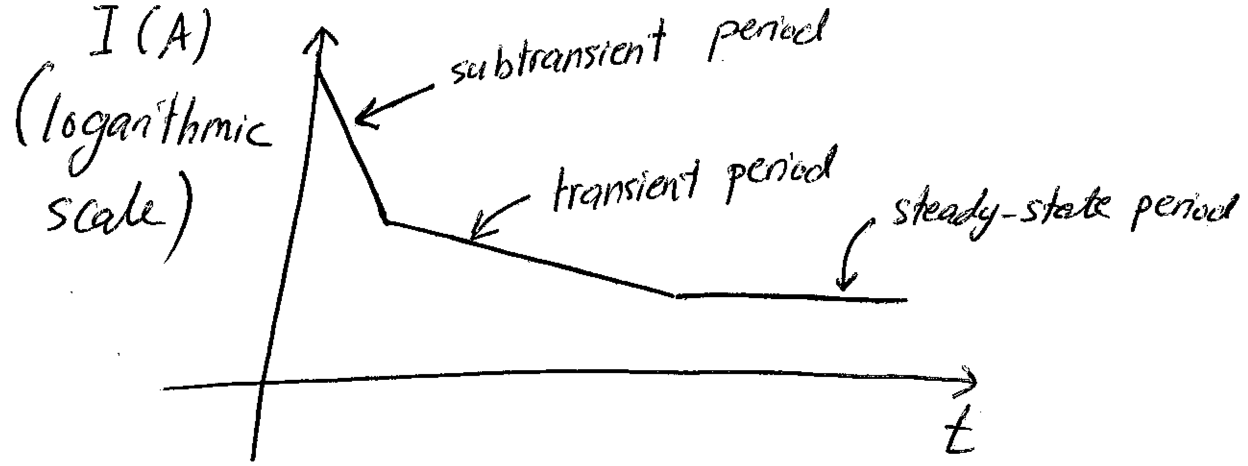
symmetrical three-phase fault current
"summation of all three phase fault currents"

* Where did the dc currents come from?

The synchronous generator is basically inductive. Recall that a current cannot change instantaneously in an inductor. When the fault occurs, the ac component current jumps to a very large value, but the total current cannot change at that instant. The dc component appears such that the sum of the ac and dc components just after the fault equals the ac current flowing just before the fault.

* The ac symmetrical component of current is divided roughly into three periods. <u>Subtransient period</u> where the ac current is very large and falls very rapidly during the first cycle or so. In <u>transient period</u>, the current continues to fall at a slower rate until it reaches a steady-state.

* The rms value of the ac component of
the current is plotted as function of time as:



subtransient period

I'', T'', $X'' = \frac{E_A}{I''}$ ⇒ subtransient reactance

I'': subtransient current

T'': Time constant of subtransient current

I'' is about 10 time the size of the steady-state fault current.

* I'' is caused by the damper windings of the generator. and T'' is the time constant of the damper windings.

transient period

I' , T'

transient current — transient time constant

$$X' = \frac{E_A}{I'} \Rightarrow \text{Transient reactance}$$

* I' is caused by the dc field circuit and T' is the time constant of the dc field winding.

$$T' > T'' \Rightarrow$$ Transient period is larger than the subtransient period

* I' is often 5 times the steady-state fault current.

Steady-State Period Current

$$I_{ss} = \frac{E_A}{X_s} \quad \text{steady state}$$

* The rms magnitude of the ac fault current in a synchronous generator is:

$$I(t) = (I'' - I')e^{-t/T''} + (I' - I)e^{-t/T'} + I_{ss}$$

* There is a straight forward Example ....
see it !

Synchronous Generator Ratings

* ratings : certain basic limits to the speed and power that may be obtained from the machine. The purpose of the ratings is to protect the generator from damage due to improper operation.

* Typical ratings on a synchronous machine are:
1 voltage 2 frequency 3 speed 4 apparent power (kVA)
5 power factor 6 field current 7 service factor

The Voltage, Speed and Frequency Ratings

* The rated frequency depends on the power system that the generator will be connected to it. Typical frequencies are 50Hz (Europe & Asia), 60 Hz (USA) and 400Hz in special control applications.

$$f = \frac{nP}{120} \quad P \rightarrow \text{no. of poles}$$

* The voltage depends on the flux, speed and the number of turns (mechanical construction).

For a given mechanical construction and speed, the higher the flux the high the voltage. The flux is limited by the field current. The voltage is limited by the breakdown value of the winding inselation.

* As for the frequency, if a 60Hz generator is to be run at 50 Hz, then derating has to be done with a scale of $\frac{50}{60} = 83.3\%$ i.e. the voltage has to be reduced by 83.3%. Opposite effect happens when a 50 Hz generator is to be run at 60 Hz.

Apparent Power and Power-Factor Ratings

* The power limit is determined by the mechanical torque on the shaft of the machine and the heating of the machine's windings. Practically, the shaft is strong enough to mechanically handle much larger steady-state power than the machine is rated for. So the practical parameter ~~is the~~ for the power limit is the heating in the machine windings.

* There are two windings → armature winding
  → field winding

* $I_{A,max}$ sets the apparent power rating S_{rated}.

$$S_{rated} = 3 V_{\phi,rated} I_{A,max}$$

$$S_{rated} = \sqrt{3} V_{T,rated} I_{L,max}$$

* The heating of the armature windings due to the armature current doesn't depend on the power factor of the current.

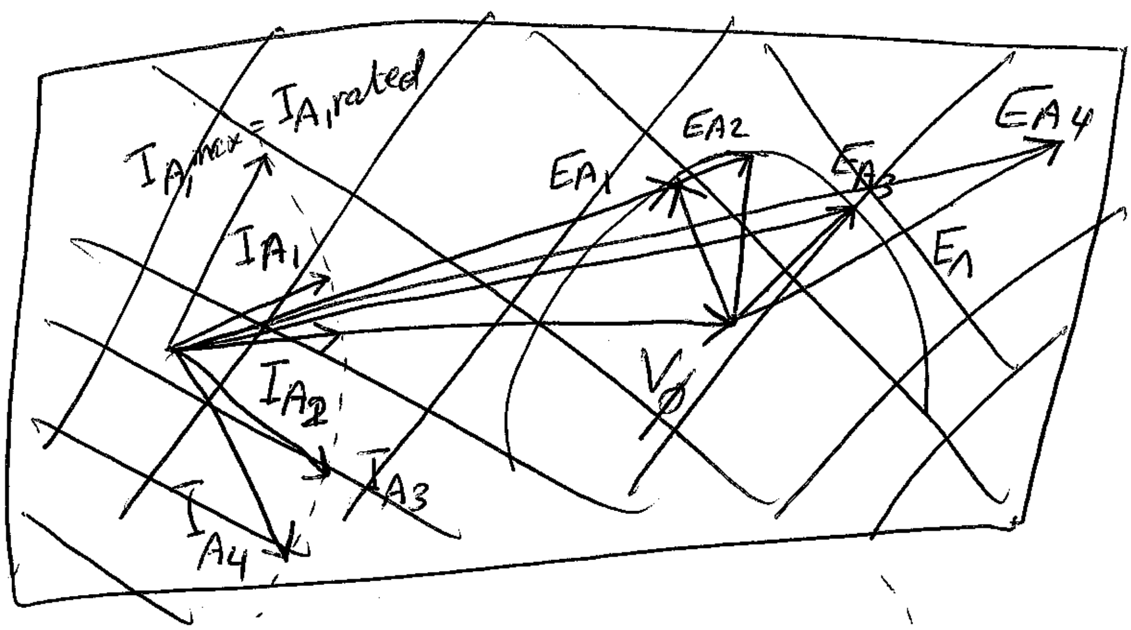
$$\underbrace{P_{SCL}}_{\text{stator copper losses}} = 3 I_A^2 R_A$$

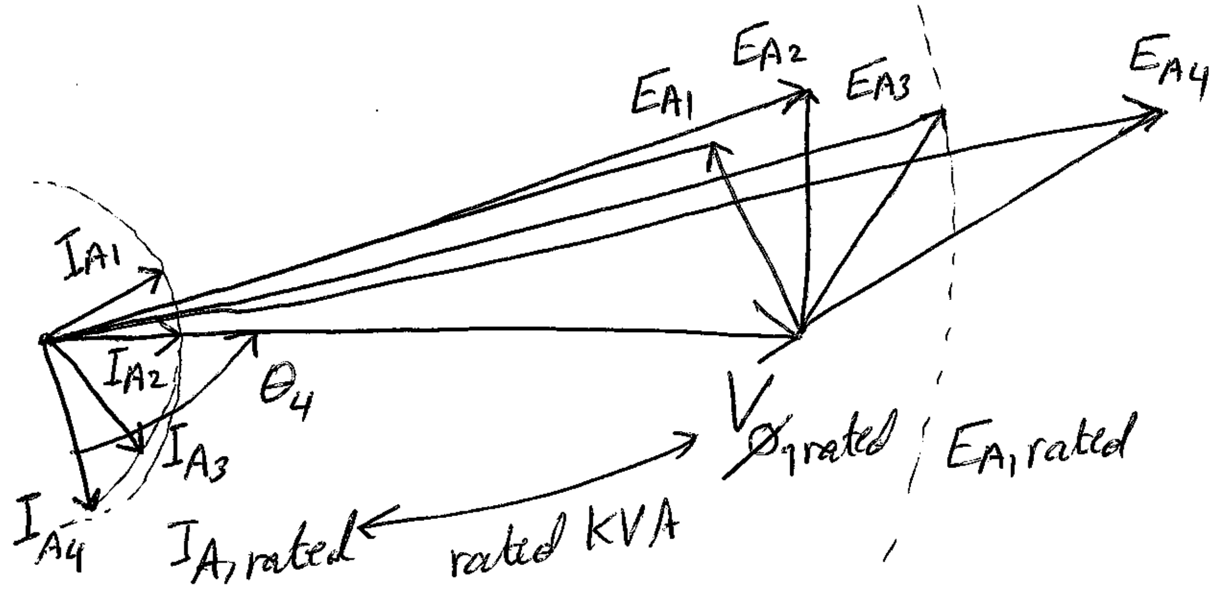
Similarly for the rotor circuit:

$P_{RCL} = I_F^2 R_F \Rightarrow$ The maximum allowable heating sets a maximum field current for the machine. This also sets the maximum E_A as $E_A = K\phi\omega$. ~~B~~ at a certain rotational speed.

* The effect of having a maximum I_F (98)

and a maximum E_A translates directly into a restriction on the lowest acceptable power factor of the generator when it is operating at the rated KVA ~~K~~ as shown below:





at I_{A4} corresponding to θ_4, E_{A4} exceeds the rated value.

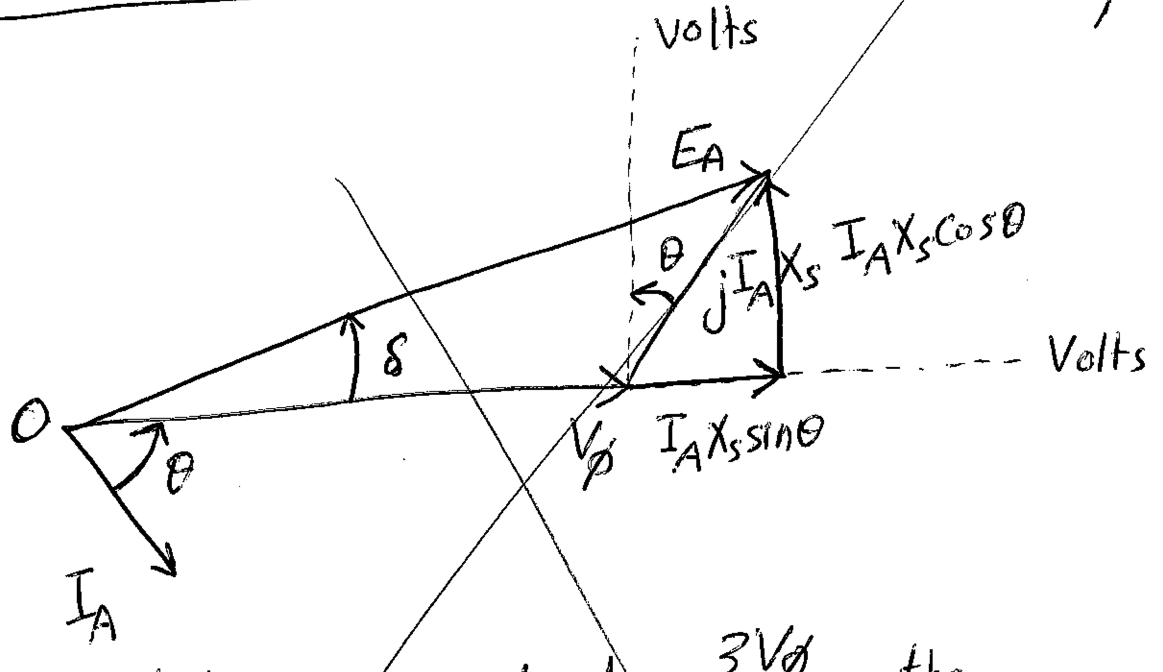
constant $3V_{\phi\,rated} I_{\phi\,rated}$

* Therefore, at the rated KVA, for large power factor (greater than certain value) the generated voltage exceeds its limit.

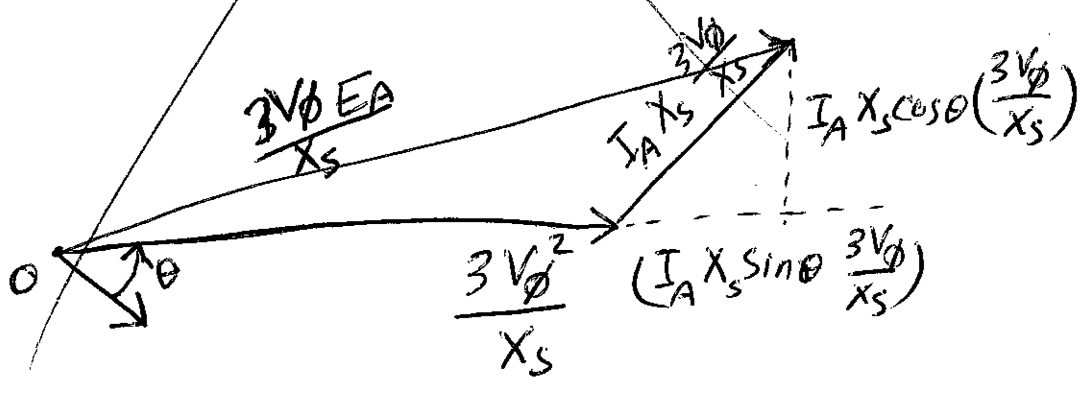
rated $KVA = 3\, V_{\phi,rated}\, I_{\phi,rated}$

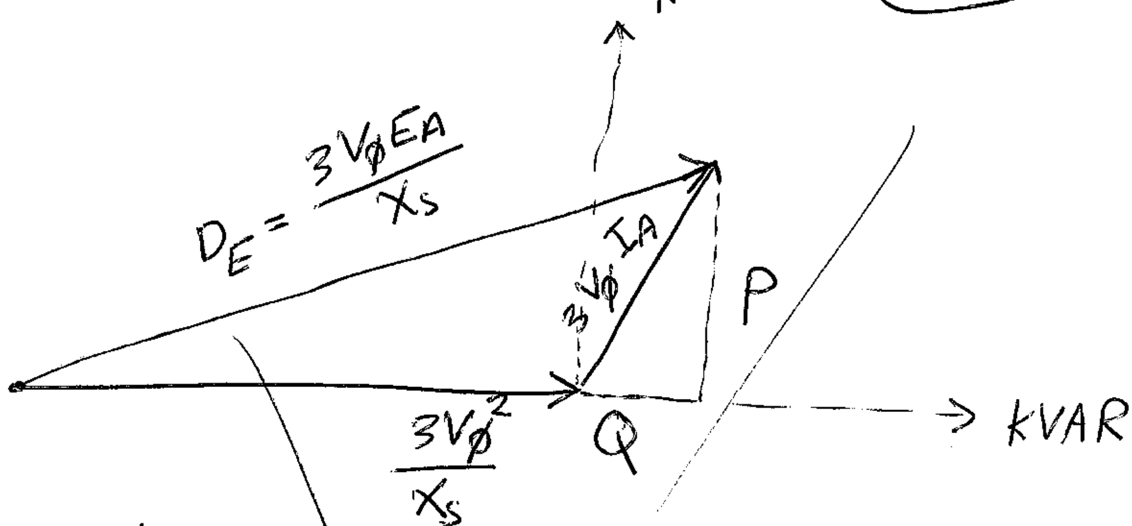
Forget

Synchronous Generator Capability Curves

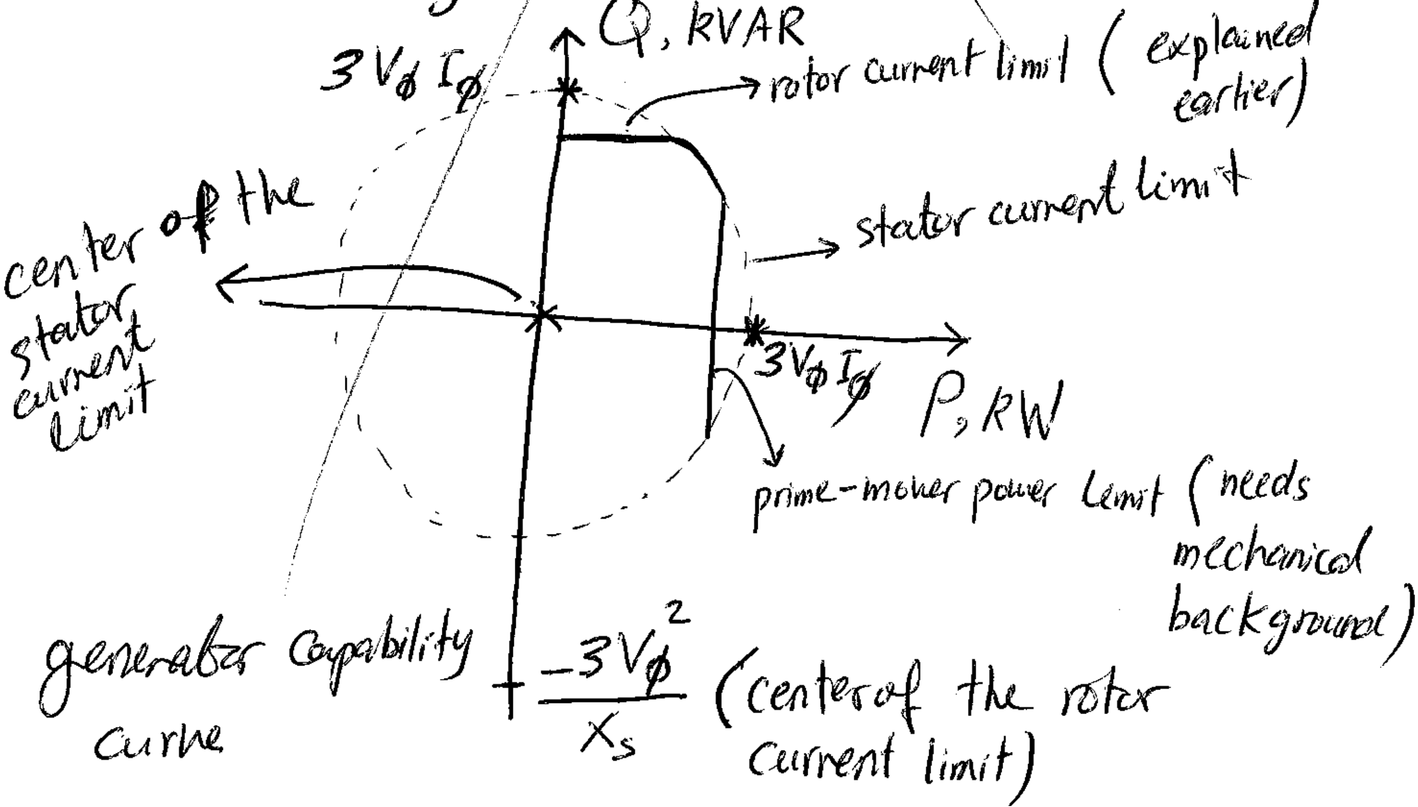


If we multiply all parameters by $\frac{3V_\phi}{X_s}$, then





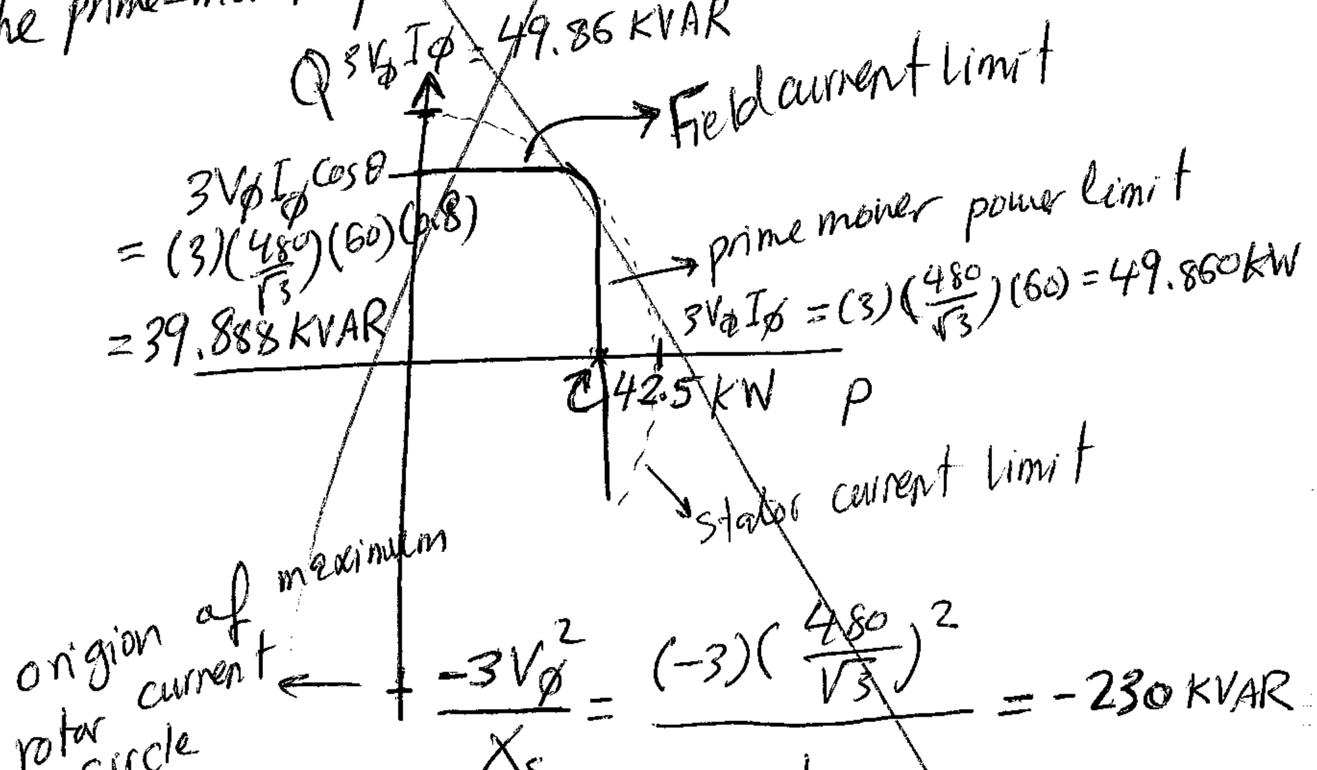
Capability ~~curve~~ diagram: a plot of complex power $S = P + jQ$, $\underbrace{|S| = \sqrt{P^2 + Q^2}}_{\text{Circle}}$. It is derived from the phasor diagram of the generator assuming that V_ϕ is constant at the machine's rated voltage.



Forget 101

Ex. A 480V 50Hz Y-connected six-pole synchronous generator is rated at 50KVA at 0.8 PF. It has a synchronous reactance of 1.0 Ω per phase. Assume that this generator is connected to a steam turbine capable of supplying up to 45KW. The friction and windage losses are 1.5KW and the core losses are 1.0KW.

ⓐ Sketch the capability curve for this generator including the prime-mover power limit.



$$P_{max, out} = 45KW - 1.5KW - 1.0KW = 42.5KW$$

(b) A current of 56 A at 0.7 PF lagging (102)

produces a real power of :

$$P = (3)\left(\frac{480}{\sqrt{3}}\right)(56)(0.7) = 32.6\ KW$$

$$Q = (3)\left(\frac{480}{\sqrt{3}}\right)(56)\sin(\cos^{-1}(0.7))$$

$$= 33.2\ KVAR$$

plotting this point on the capability diagram shows that it is safely within the maximum I_A curve but outside the maximum I_F curve. Therefore, this point is not a safe operating condition.

(c) What is the maximum amount of reactive power this generator can produce?

$$Q = D_E - \cancel{230KVA}\ Q = \frac{3E_A V_\phi}{X_S} - \frac{3V_\phi^2}{X_S}$$

$$= 263\ KVAR - 230KVAR = 33KVAR$$

(d) If the generator is 30KW of real power, what is the maximum amount of reactive power that can be simultaneously supplied?

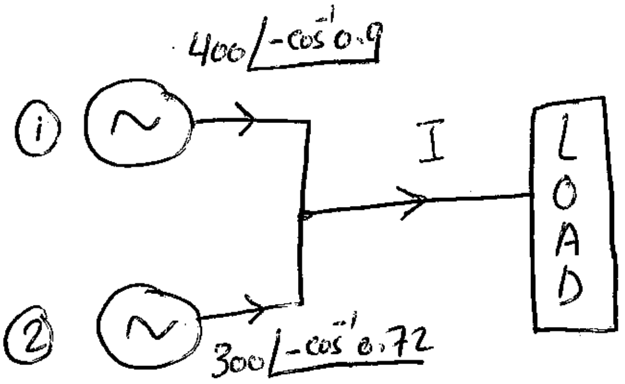
~~Coming back to the capability curve, one can find that this value is about 31.5KVAR.~~

Short-Time Operation and Service Factor

* Typical synchronous generator is often able to supply up to 300% of its rated power for a while. This ability to supply power above the rated amount is used to supply momentary power surges during motor starting and similar load transients. The higher the power over the rated value, the shorter the time a machine can tolerate it.

* The service factor is defined as the ratio of the actual maximum power of the machine to its nameplate rating. A generator with a service factor of 1.15 can actually be operated at 115% of the rated load without harm.

Q Two identical 600KVA 480V synchronous generators are connected in parallel to supply a load. The prime movers of the two generators happen to have different speed droop characteristics. When the field currents of the two generators are equal, one delivers 400A at 0.9 PF lagging while the other delivers 300A at 0.72 PF lagging.



ⓐ What are the real power and the reactive power supplied by each generator to the load?

$$P_1 = \sqrt{3} V_L I_L \cos\theta_1 = (\sqrt{3})(480)(400)(0.9)$$
$$= 299298.4 \, W$$

$$P_2 = \sqrt{3} V_L I_L \cos\theta_2 = (\sqrt{3})(480)(300)(0.72)$$
$$= 179579 \, W$$

$$Q_1 = \sqrt{3} V_L I_L \sin\theta_1 = (\sqrt{3})(480)(400)\sin(\cos^{-1}(0.9))$$
$$= 144960 \text{ VAR}$$

$$Q_2 = \sqrt{3} V_L I_L \sin\theta_2 = \sqrt{3}(480)(300)\sin(\cos^{-1}(0.72))$$
$$= 173090 \text{ VAR}$$

(b) $\tilde{I} = 400\angle -\cos^{-1}0.9 + 300\angle -\cos^{-1}0.72$

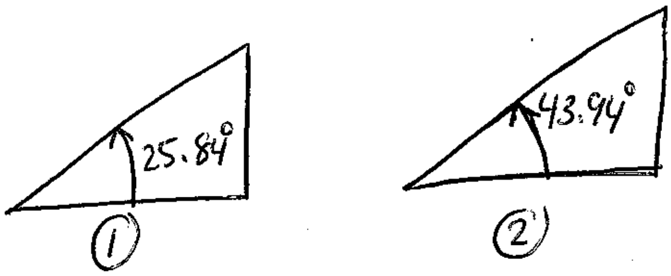
$= 360 - j174.36 + 216 - j208.19$

$= 691.46\angle -33.59$

$PF = \cos(-33.59) = 0.833$

What is the overall power factor of the load?

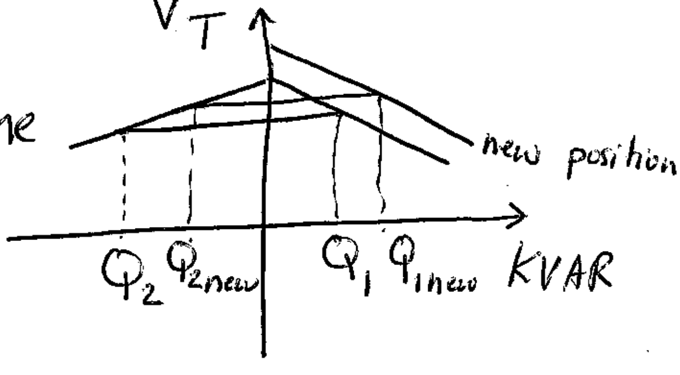
(c)



as θ increases Q increases

In what direction must the field current on each generator be adjusted in order for them to operate at the same power factor?

The



∴ The field current of the first generator has to be increased. such that its Q increases and the field current of the second generator has to decrease such that its Q decreases.

Q// A 20 MVA 13.8 KV 0.8 PF lagging Y-connected synchronous generator has a negligible armature resistance and a synchronous reactance of 0.7 pu. The generator is connected in parallel with a 60 HZ 13.8 KV infinite bus which capable of supplying or consuming any amount of real or reactive power with no change in frequency or terminal voltage.

ⓐ What is the synchronous reactance of the generator in ohms?

$$S_{base} = \sqrt{3}\, V_{L,base}\, I_{L,base} \Rightarrow I_{L,base} = \frac{S_{base}}{\sqrt{3}\, V_{L,base}}$$

$$Z_{base} = \frac{V_{\phi,base}}{I_{\phi,base}} =$$

$$= \frac{20 \times 10^6}{(\sqrt{3})(13.8 \times 10^3)}$$

$$= 836.74\ A$$

$$Z_{base} = \frac{13.8 \times 10^3 / \sqrt{3}}{836.74}$$

$$= 9.5\ \Omega$$

$$X_s = (0.7)(9.5) = 6.67\ \Omega$$

(b) $$\tilde{E}_A = \tilde{V}_T + \tilde{I}_A (jX_s)$$

$$= 1\angle 0 + (1\angle -\cos^{-1} 0.8)(j0.7)$$

$$= 1 + (0.8 - j0.6)(j0.7)$$

$$= 1.42 + j0.56 = 1.5264\angle 21.52^\circ \text{ pu}$$

calculate $\tilde{E}_A$? (pu)

(c) $$|\tilde{E}_A| = (1.5264)(13.8 \times 10^3) = 21064\ V$$

calculate E_A in Volts.

(d) if the internal generated voltage is reduced by a factor of 0.05 i.e the new value is 0.95 of the previous one, what will be the new armature current?

$$E_{A_1} \sin\delta_1 = E_{A_2} \sin\delta_2$$

$$E_{A_1} \sin 21.52^\circ = 0.95\, E_{A_1} \sin\delta_2$$

$$\sin 21.52^\circ = 0.95 \sin\delta_2$$

$$\delta_2 =$$

$$\hat{E}_A = \tilde{V}_T + \hat{I}_A j0.7$$

$$(0.95)(1.5264)\angle\delta_2 = 1\angle 0 + \tilde{I}_A j0.7$$

$$\tilde{I}_A =$$

(e) Repeat the previous part for 10%, 15%, 20% and 25% reduction in $|\tilde{E}_A|$.

Do it ...!

(f) plot the magnitude of the armature current $|\tilde{I}_A|$ as a function of $|\tilde{E}_A|$

Do it ...!

You may need MATLAB for this task.

Synchronous Motor

* Synchronous Motor has the same construction of synchronous generator i.e.:

  (1) cylindrical rotor

  (2) salient pole rotor

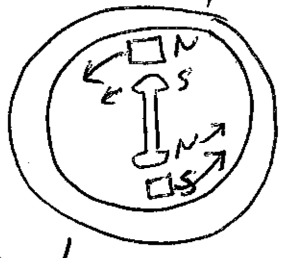
and a three-phase distributed slotted stator.

* DC power supply must be supplied to the rotor.

principle of Operation

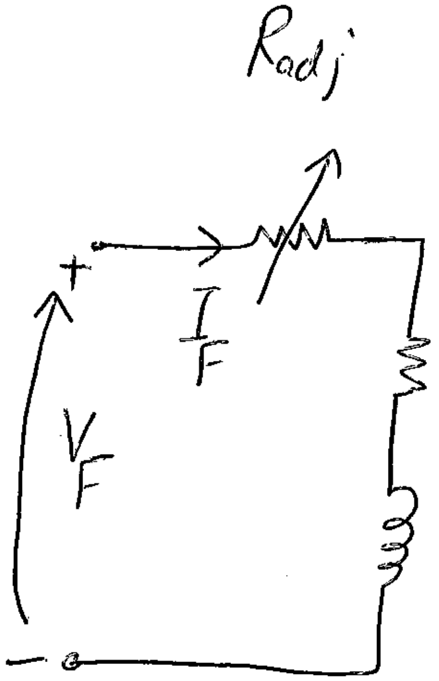
* A three-phase currents in three-phase stator windings produce a rotating magnetic field in the air gap of synchronous motor rotating at synchronous speed n_s. $f_s = \frac{n_s P}{120}$

The rotor field due to the rotor DC current tends to line up with the stator field just as two bar magnets will tend to line up if placed near each other. Since the stator magnetic field is rotating, the rotor magnetic field (and the rotor itself) will constantly try to catch up. The larger the angle between the two magnetic fields (up to a certain maximum), the greater the torque on the rotor of the machine.

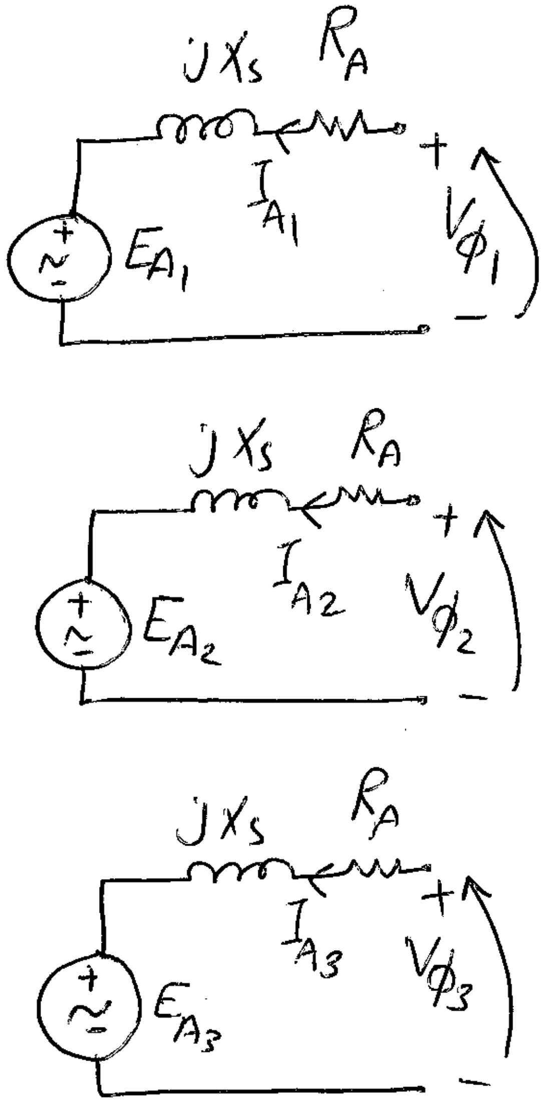


The Equivalent Circuit of Synchronous Motor

A synchronous motor is the same in all aspects as a synchronous generator, except that the direction of power flow is reversed and therefore the direction of current flow in the stator is reversed. The Equivalent circuit is:

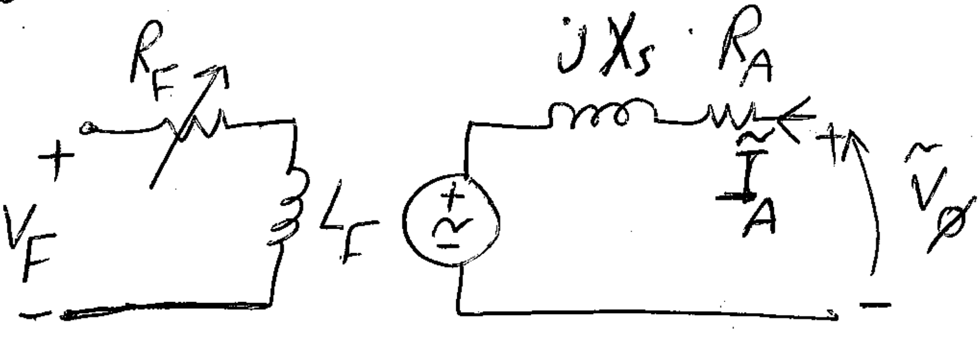


field circuit



armature circuit

Full equivalent circuit of three-phase synchronous motor



per-phase equivalent circuit of synchronous motor

* The stator may be Y- or Δ- connected.

$$\tilde{V}_\phi = \tilde{E}_A + j X_s \tilde{I}_A + R_A \tilde{I}_A$$

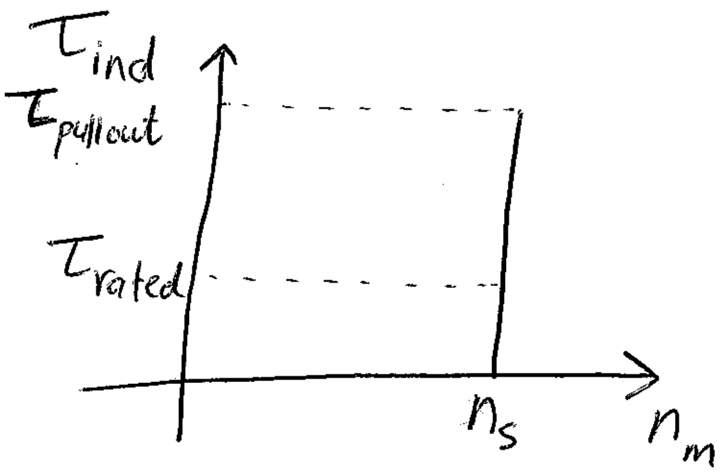
or $$\tilde{E}_A = V_\phi - j X_s \tilde{I}_A - R_A \tilde{I}_A$$

Note: In a motor the induced torque is in the direction of motion and in a generator the induced torque is a counter torque opposing the direction of motion.

Steady-State Synchronous Motor Operation

The synchronous motor Torque-Speed Characteristic Curve

The speed of rotation of synchronous motor depends on the applied electrical frequency so the speed of the motor will be constant regardless of the load. The resulting torque-speed characteristic is:



$$\tau_{ind} = \frac{3V_\phi E_A \sin\delta}{\omega_s X_s}$$ $\Rightarrow$ already derived from the phasor diagram

The maximum (pullout torque) occurs when $\delta = 90°$. Normall full load torques are much less than that. Pullout torque may typically be three times the full load torque of the machine. τ_{max} is:

$$\tau_{max} = \frac{3V_\phi E_A}{\omega_s X_s}$$ (pullout torque)

* When the load torque applied on the shaft of a synchronous motor exceeds the pullout torque, the rotor can no longer remain locked to the stator rotating magnetic field. The resulting huge torque surges, first one way and then the other way causing the whole motor to vibrate severly. This might cause what is called "loss of synchronism".

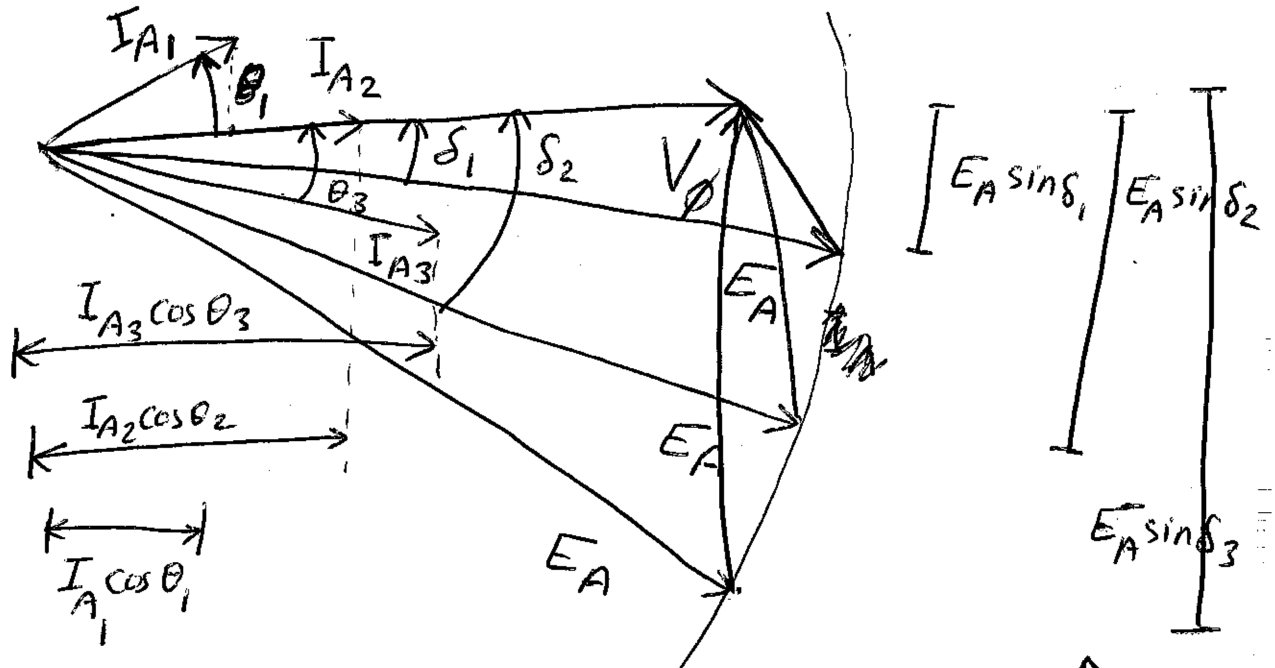
The Effect of Load changes on a Synchronous Motor

In this part, the induced voltage $|\tilde{E}_A|$ is constant as the flux has not changed (Constant I_F). The terminal voltage and frequency of the motor are also constants. As the load increases the developed torque of the motor must increase to match the load torque.

* As the load torque increases the induced torque $\tau_{ind} = \dfrac{3 V_\phi E_A \sin\delta}{\omega_s X_s}$ increases to match the torque needed. $|\tilde{E}_A|$, $|\tilde{V}_\phi|$ and ω_s are constants. Therefore $\underline{E_A \sin\delta}$ must increase.

$P_{in} = 3 V_\phi \underbrace{I_A \cos\theta}$ and therefore $\underline{I_A \cos\theta}$ must also increase. On the phasor diagram:

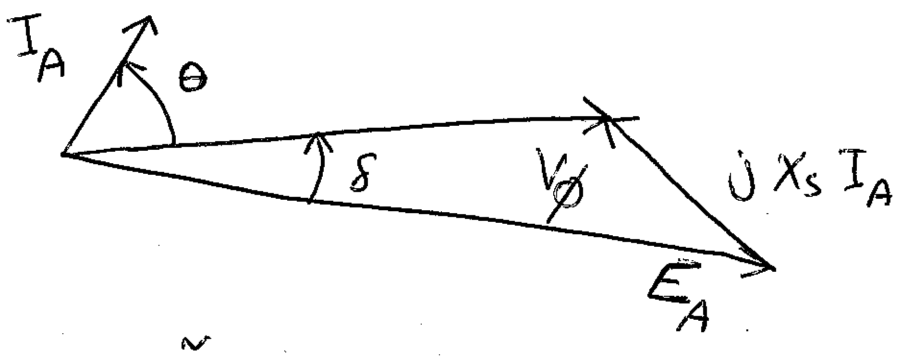
$\tilde{V}_\phi = \tilde{E}_A + j\tilde{I}_A X_s$



$\therefore$ As the load increases $I_A \cos\theta$ increases $\Rightarrow I_A \uparrow$ and $\sin\delta \uparrow \Rightarrow \sin\delta \uparrow \Rightarrow \delta \uparrow$ and the motor becomes more and more lagging.

Ex. A 208V 45 KVA 0.8 PF leading Δ-connected 60 Hz synchronous motor has a synchronous reactance of $2.5\,\Omega$ and a negligible armature resistance ($R_A = 0$). Its friction and windage losses are 1.5 KW and its core losses are 1.0 KW. Initially the shaft is supplying a 15 hp load and the motor's power factor is 0.80 leading.

ⓐ Sketch the phasor diagram of this motor and find the values of I_A, I_L and E_A. Now $\cos\theta = 0.8$ leading



$$\tilde{V}_\phi = \tilde{E}_A + jX_s \tilde{I}_A$$

$$\tilde{E}_A = \tilde{V}_\phi - jX_s \tilde{I}_A$$

$P_{out} = 15\text{ hp} = 15 \times 746 = 11.19\text{ kW}$

$P_{in} = P_{out} + P_{friction} + P_{core}$

$= 11.19 \times 10^3 + 1.5 \times 10^3 + 1 \times 10^3 =$

$= 13.69\text{ kW}$

$P_{in} = \sqrt{3}\, V_L\, I_L \cos\theta$

$13.69 \times 10^3 = \sqrt{3}\,(208)(I_L)(0.8)$

$I_L = 47.5\text{A}$

$$\tilde{I}_\phi = \frac{47.5}{\sqrt{3}} \angle \cos^{-1} 0.8$$

$$= 27.4 \angle 36.87^\circ \text{ A}$$

$$\tilde{E}_A = 208 \angle 0 - (j2.5)(27.4)\angle 36.87^\circ$$

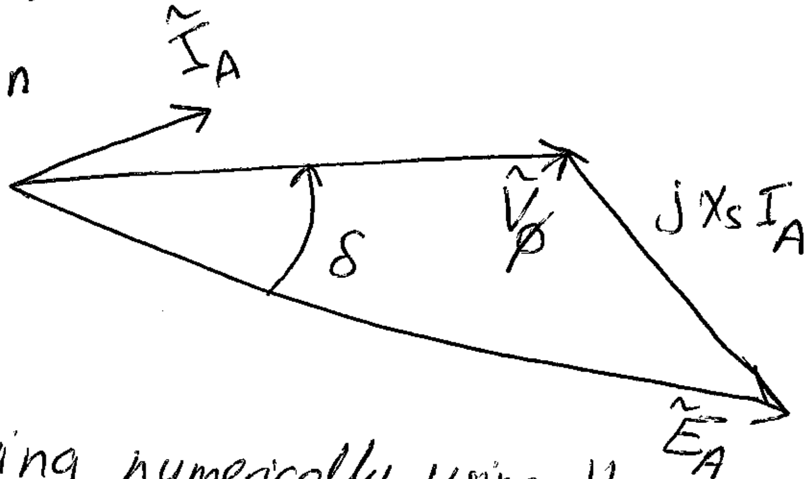
$$= 249.1 - j54.8\text{V} = 255 \angle -12.4^\circ$$

$$\left.\begin{aligned} \delta &= -12.4^\circ \\ \theta &= 36.87^\circ \end{aligned}\right\}$$

$|\tilde{E}_A| = 255\text{ V}$

$|\tilde{V}_\phi| = 208\text{V}$

(b) assume that the shaft load is now increased to 30hp. Sketch the behavior of the phasor diagram in response to this change.



Solving numerically using the same approach of the previous part gives:

$$|\tilde{E}_A| = 255$$

$$\delta = -23^\circ$$

$$|\tilde{V}_\phi| = 208^\circ$$

$$\theta = 15^\circ$$

(c) $\tilde{V}_\phi = \tilde{E}_A + jX_s \tilde{I}_A$

$$\tilde{E}_A = \tilde{V}_\phi - jX_s \tilde{I}_A$$

$$255\angle\delta = 208 - (j2.5)\, I_A\angle\theta$$

$$P_{in} = P_{out} + P_{mech} + P_{core}$$

$$= (30)(746) + 1.5\times10^3 + 1\times10^3$$

$$= 24.88\,KW$$

$$P_{in} = \sqrt{3}\, V_L\, I_L \cos\theta$$

$$24.88\times10^3 = (\sqrt{3})(208)\, \underline{I_L \cos\theta}$$

two unknows

$$P_{in} = \frac{3 V_\phi E_A}{X_s}\sin\delta$$

$$24.88\times10^3 = \frac{(3)(208)(255)}{2.5}\sin\delta$$

$\rightarrow$ same as before

$$\delta = 23^\circ$$

$$255\angle 23^\circ = 208 - j2.5\,\tilde{I}_A$$

$$\tilde{I}_A = \frac{255\angle 23^\circ - 208}{-j2.5} = 41.2\angle 15^\circ\ A$$

$$\tilde{I}_L = \sqrt{3}\ 41.2 = 71.4\,A$$

$$PF = \cos 15^\circ = 0.966 \text{ leading}$$

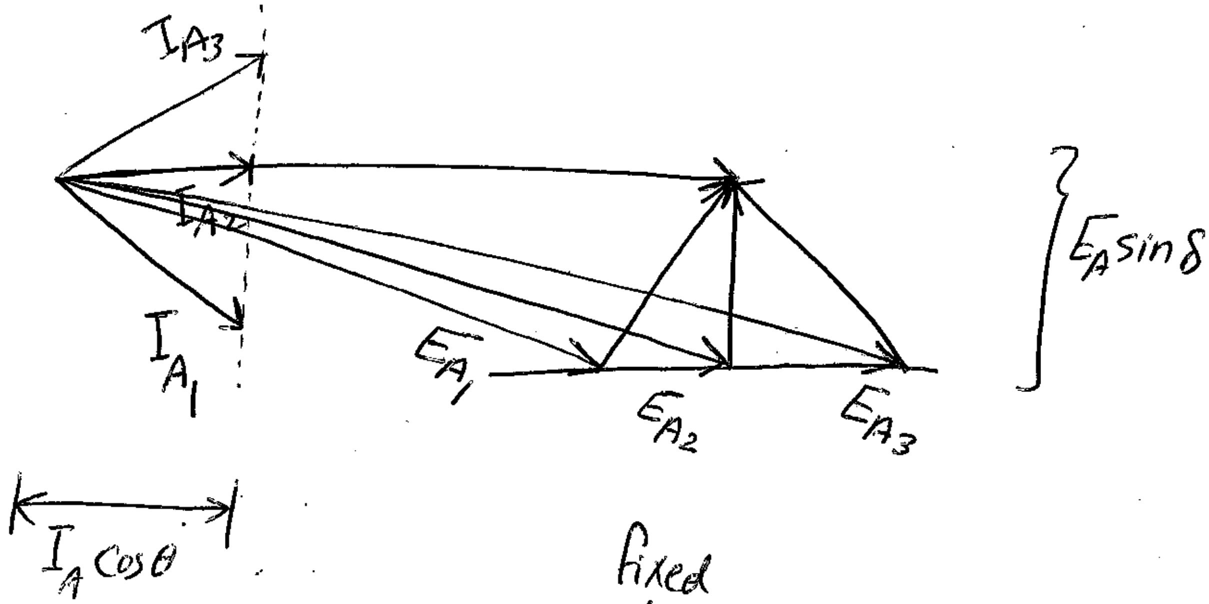
The Effect of Field Current Changes on a Synchronous Motor

$$P_{in} = 3V_\phi I_A \cos\theta = \frac{3V_\phi E_A \sin\delta}{X_s}$$

In this part, $\underbrace{V_\phi,\ P_{out},\ P_{in},\ \omega_m}_{\text{constants}}$,

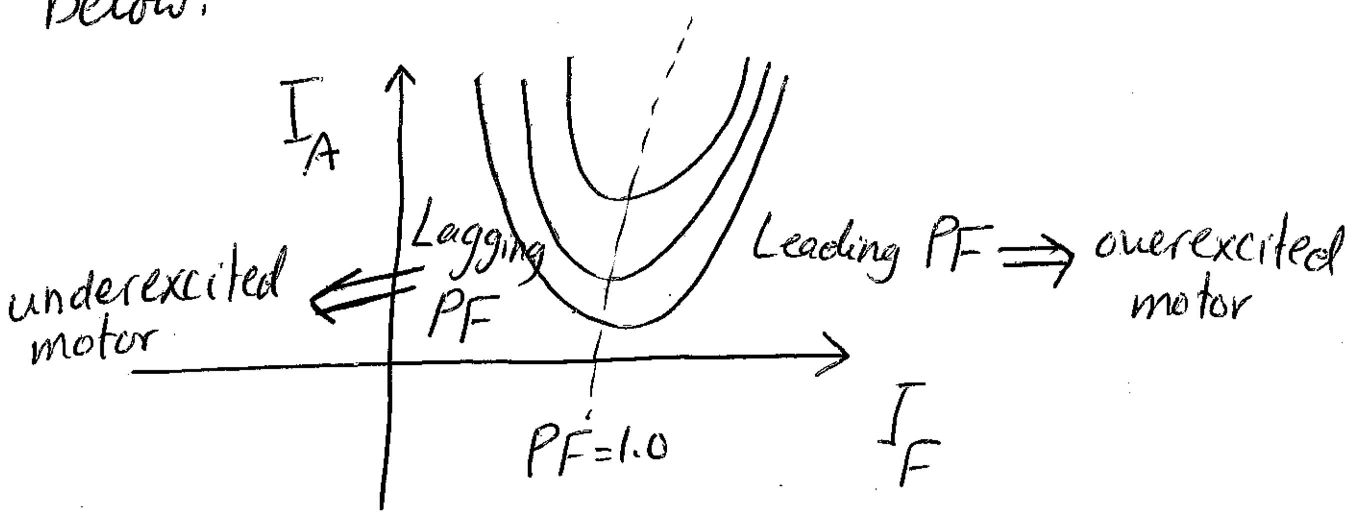
$\left.\begin{array}{l} E_A \sin\delta \ \& \\ I_A \cos\theta \end{array}\right\}$ constants

$I_F \uparrow \Rightarrow E_A \uparrow$ but $E_A \sin\delta$ is fixed



$$P_{in} = 3V_\phi \underbrace{I_A \cos\theta}_{\text{fixed}} = \frac{3V_\phi \overbrace{E_A \sin\delta}^{\text{fixed}}}{X_s}$$

* As the field current increases for a fixed output load, the armature current decreases and the power factor becomes less and less lagging until unity. On further increase of the field current, $|\tilde{E}_A|$ continoues increasing, I_A increases and the power factor becomes more and more leading. A plot of I_A as function of I_F is called the V-curves of synchronous motor as shown below:



Synchronous motor V-Curves

* Therefore, by controlling the field current of synchronous motor, the reactive supplied to or consumed by the motor can be controlled.

* Lagging Power Factor motor consumes reactive power
* Leading " " " supplies reactive power.

Ex. The 208 V 45KVA 0.8PF leading Δ-connected 60Hz synchronous motor of the previous example is supplying a 15hp load with an initial PF of 0.85 lagging.

@ Sketch the initial phasor diagram of this motor and find the values of $\tilde{I}_A$ and $\tilde{E}_A$

$$P_{in} = 3 V_\phi I_A \cos\theta$$

$$(15)(746) + 1.5\times10^3 + 1\times10^3 = (3)(208)(I_A)(0.85)$$

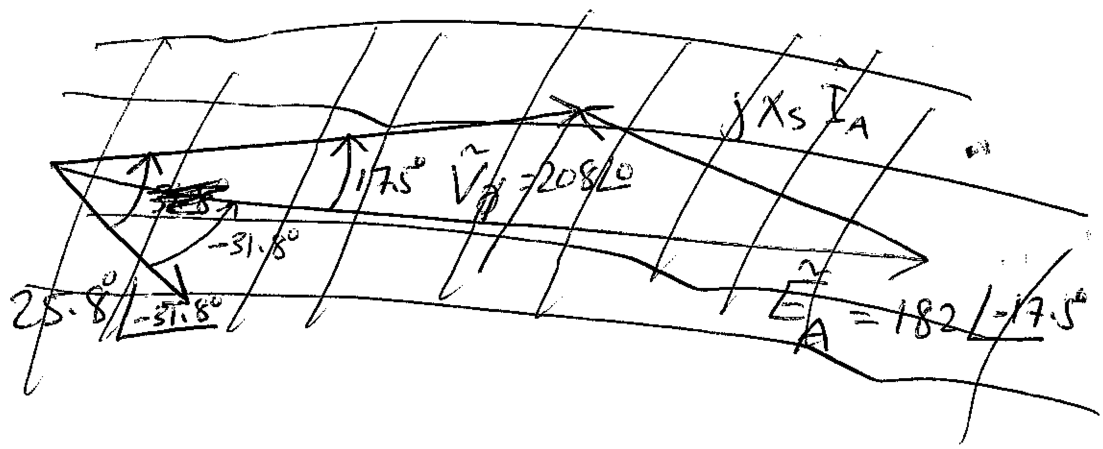
$$I_A = 25.8A$$

$$\tilde{I}_A = 25.8\angle{-\cos^{-1}0.85} = 25.8\angle{-31.8^\circ}$$

$$\tilde{E}_A = \tilde{V}_\phi - jX_s\tilde{I}_A$$

$$= 208 - (j2.5)(25.8)\angle -31.8^\circ$$

$$= 182\angle -17.5^\circ$$



(b) If the motor's flux is increased by 25%, sketch the new phasor diagram of the motor. What are $\tilde{E}_A$, $\tilde{I}_A$ and the PF of the motor now?

$$|\tilde{E}_A| = (1.25)(182) = 227.5\,V$$

$$E_{A_1}\sin\delta_1 = E_{A_2}\sin\delta_2$$

$$\delta_2 = \sin^{-1}\left[\frac{E_{A_1}\sin\delta_1}{E_{A_2}}\right] = \sin^{-1}\left[\frac{182\sin(-17.5^\circ)}{227.5}\right]$$

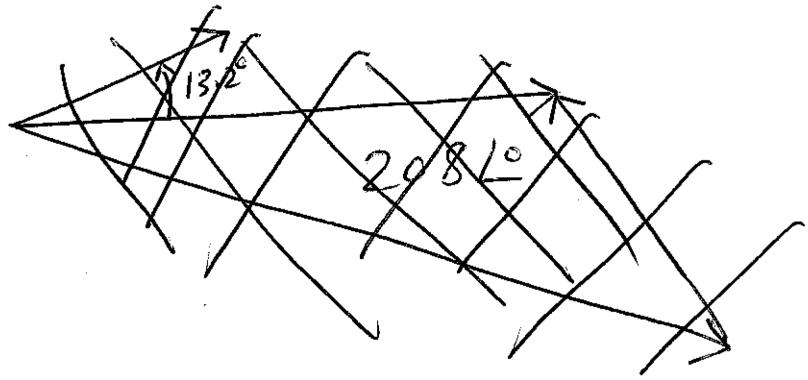
$$= -13.9^\circ$$

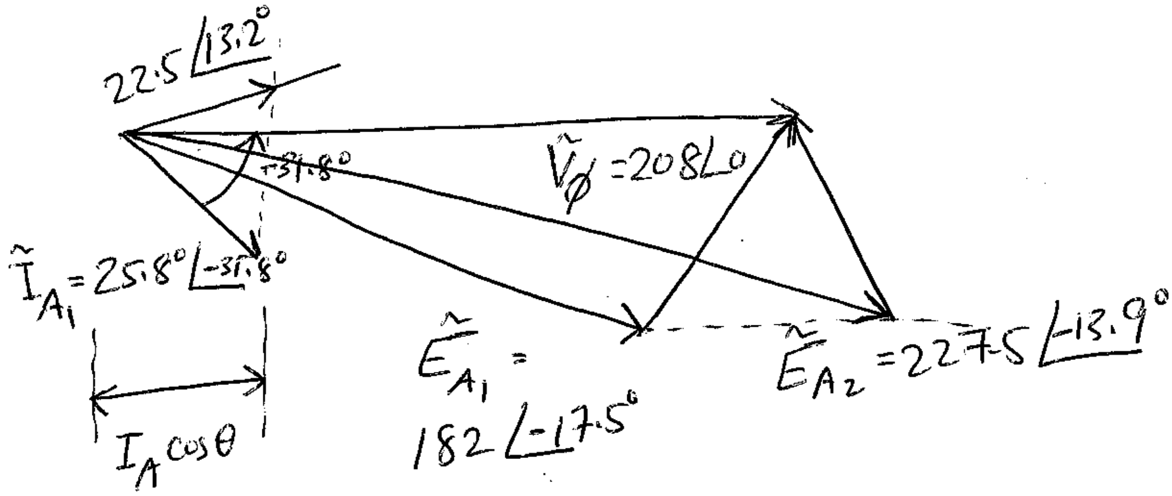
$$\tilde{V}_\phi = \tilde{E}_A + jX_S \tilde{I}_A$$

$$\tilde{I}_A = \frac{\tilde{V}_\phi - \tilde{E}_A}{jX_s} = \frac{208\angle 0 - 227.5\angle{-13.9^\circ}}{j2.5}$$

$$= 22.5\angle 13.2^\circ$$

$$PF = \cos(13.2) = 0.974 \text{ leading}$$

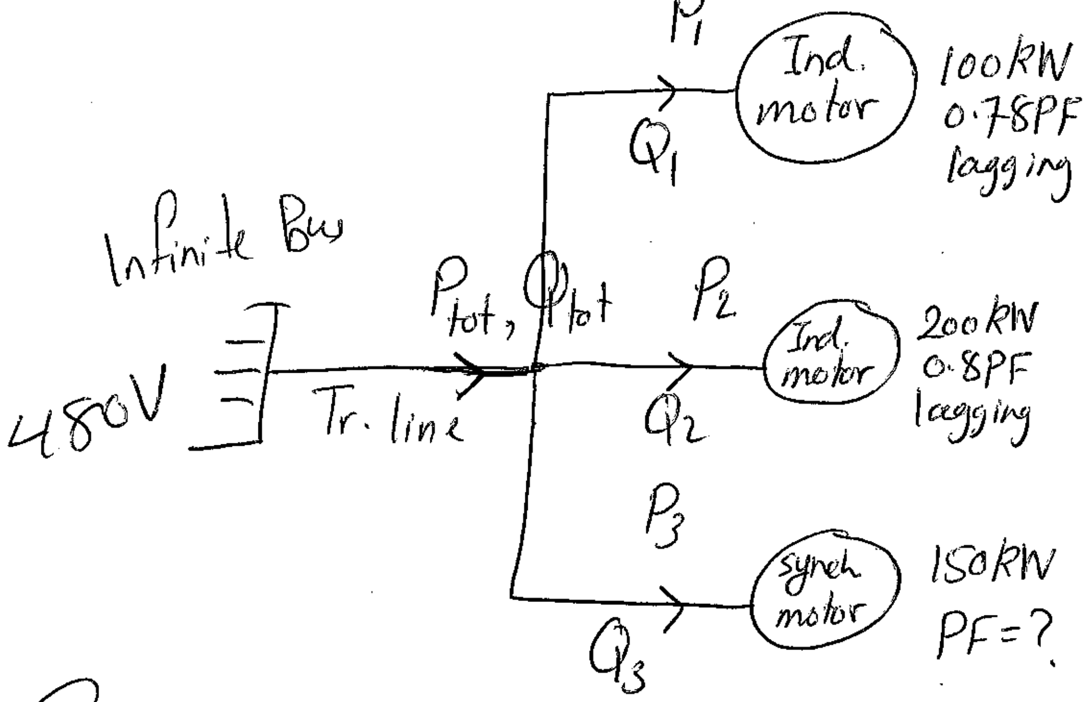




The synchronous Motor & Power Factor Correction

* Overexcited synchronous motor operates ~~at~~ with leading power factor. This actually can be used as power factor corrector in power systems. The good thing is that the power factor of synchronous motor is controllable using the field current.

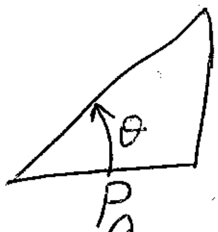
<u>Ex</u> The infinite bus of the following figure Operates at 480V. Load #1 is an induction motor consuming 100kW at 0.78 PF lagging. Load 2 is an induction motor consuming 200 kW at 0.8PF lagging. Load 3 is a synchronous motor whose real power consumption is 150 kW.



(a) If the synchronous motor is adjusted to operate at 0.85 PF lagging, what is the transmission line current in this system.

$$P_1 = 100\,kW, \quad P_2 = 200\,kW, \quad P_3 = 150\,kW$$

$$Q_1 = \tan\theta_1 P_1$$
$$= \tan(\cos^{-1} 0.78)\, 100 \times 10^3$$
$$= 80.2\ kVAR$$



$$\tan\theta = \frac{Q}{P}$$

$$Q_2 = \tan\theta_2 P_2$$
$$= \tan(\cos^{-1} 0.8)\, 200 \times 10^3$$
$$= 150\ kVAR$$

$$Q_3 = \tan\theta_3 \, P_3$$
$$= \tan(\cos^{-1} 0.85)\, 150 \times 10^3$$
$$= 93 \text{ KVAR}$$

$$P_{tot} = P_1 + P_2 + P_3$$
$$= 100 + 200 + 150$$
$$= 450 \text{ KW}$$

$$Q_{tot} = 80.2 + 150 + 93 = 323.2 \text{ KVAR}$$

$$\theta_{tot} = \tan^{-1} \frac{Q_{tot}}{P_{tot}}$$

$$\cos\theta_{tot} = \cos\left(\tan^{-1} \frac{Q_{tot}}{P_{tot}}\right)$$
$$= \cos\left(\tan^{-1} \frac{323.2}{450}\right)$$
$$= 0.812 \text{ lagging}$$

$$P_{tot} = \sqrt{3}\, V_L \, I_L \cos\theta_{tot}$$

$$450 \times 10^3 = \sqrt{3}(480)(I_L)(0.812)$$

$$I_L = 667 \text{ A}$$

(b) If the synchronous motor is adjusted to operate at 0.85 PF leading, what is the transmission line current in this system.

$$Q_3 = +P_3 \tan(-\cos^{-1} 0.85)$$
$$= (150 \times 10^3) \tan(-\cos^{-1} 0.85)$$
$$= -93 \text{ kVAR}$$

$$P_{tot} = P_1 + P_2 + P_3$$
$$= 450 \text{ kW} \quad (\text{same as before})$$

$$Q_{tot} = Q_1 + Q_2 + Q_3$$
$$= 80.2 \text{ kVAR} + 150 \text{ kVAR} - 93 \text{ kVAR}$$
$$= 137.2 \text{ kVAR}$$

$$\theta_{tot} = \tan^{-1}\left(\frac{Q_{tot}}{P_{tot}}\right)$$

$$PF_{tot} = \cos\theta_{tot} = \cos\left(\tan^{-1}\frac{Q_{tot}}{P_{tot}}\right)$$
$$= \cos\left(\tan^{-1}\frac{137.2}{450}\right) = 0.957 \text{ lagging}$$

$$P_{tot} = \sqrt{3}\, V_L I_L \cos\theta_{tot}$$

$$I_L = \frac{P_{tot}}{\sqrt{3}\, V_L \cos\theta_{tot}} = \frac{450\times10^3}{(\sqrt{3})(480)(0.957)}$$

$$= 566\,A.$$

(c) Assume that the transmission line losses are given by $\underbrace{P_{LL}}_{\text{line losses}} = 3 I_L^2 R_L$

How do the transmission line losses compare in the two cases?

$$P_{LL_a} = (3)(667)^2 R_L$$

$$P_{LL_b} = (3)(566)^2 R_L$$

$$\frac{566^2}{667^2} = \frac{320356}{444889} = 0.72$$

So the losses in the second case has been reduced by a factor of 0.28 despite the fact that the real power consumed has not changed.

Starting Synchronous Motor

Synchronous motor is <u>not a self starting motor.</u> It is almost impossible for the rotor to line up with the stator rotating mmf by itself. Three basic approaches are used to safely start a synchronous motor:

(1) Reduce the speed of the stator rotating magnetic field to a low enough value that the rotor can accelerate and catch up with the stator mmf. This can be done by reducing the frequency of the applied electric power.

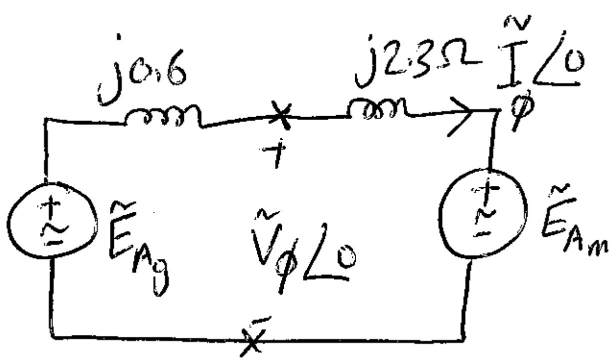
(2) Use an external prime mover to accelerate the synchronous motor up to synchronous speed. Turn off or disconnect the prime mover after turning on the rotor field circuit.

(3) Use damper windings or amortisseur windings. In this approach the motor is started an induction motor as the damper windings are short circuits until the rotor reaches almost the synchronous speed. The power circuit of the rotor is then turned. on and the rotor will line up with stator rotating magnetic field

Note: Damper windings are winding placed on the rotor of synchronous machine short circuited from the two sides. They are used to damp oscillations under transient conditions. In steady-state the current through them are zero. They are very important in keeping the synchronism of synchronous generators and motors and therefore in keeping the stability of power systems.

Q A 480V 375KVA 0.8 PF lagging Y-connected synchronous generator has a synchronous reactance of 0.6Ω and a negligible armature resistance. This generator is supplying power to a 480V 100KVA 0.8PF leading Y-connected synchronous motor with a synchronous reactance of 2.3Ω and $R_A = 0\,\Omega$. The generator is adjusted to have a terminal voltage of 480V when the motor is drawing the rated power at unity power factor.

(a) Calculate the magnitudes and angles of $\tilde{E}_A$ for both machines.



$$100\times10^3 = \sqrt{3}\, V_L I_L \cos\theta$$

$$100\times10^3 = \sqrt{3}\,(480)\, I_\phi \Rightarrow I_\phi = 120.28\text{ A}$$

$$\tilde{I}_\phi = 120.28\angle 0$$

$$\tilde{V}_\phi = \frac{480}{\sqrt{3}}\angle 0$$

$$\tilde{E}_{Ag} = (j0.6)(120.28\angle 0) + \frac{480}{\sqrt{3}}$$

$$= 286.37\angle 14.59^\circ$$

$$\tilde{V}_\phi = (120.28\angle 0)(j2.3) + \tilde{E}_{Am}$$

$$\frac{480}{\sqrt{3}} = (120.28)(j2.3) + \tilde{E}_{Am}$$

$$\tilde{E}_{Am} = 391.6\angle -44.9^\circ$$

(b) $|\tilde{E}_{Am}| = (1.1)(391.6) = 430.72\ V$ - if the flux is increased by 10%.

as far as the load has not changed, then the power will not change ~~and so the~~

$$E_{Am_1}\sin\delta_1 = E_{Am_2}\sin\delta_2$$

$$-391.6\sin(44.9) = (430.72)\sin\delta_2$$

$$\delta_2 = -39.92^\circ$$

$$P_1 = P_2$$

$$3V_\phi I_\phi \cos\theta = \frac{3V_{\phi_2} E_{A_{m2}}}{X_s} \sin\delta_2$$

$$100\times10^3 = \frac{(3)(V_{\phi 2})(430.72)}{2.3} \sin(39.92)$$

$V_{\phi_2} = 277.36 \Rightarrow$ The terminal voltage has not changed.

(c) $$\tilde{I}_\phi = \frac{277.36 - 430.72\angle{-39.92^\circ}}{j2.3}$$

$$= 122.38\angle{-169.1} = 122.38\angle{10.89^\circ}$$

$PF = \cos(10.89^\circ) = 0.98$, leading.

9. A 440V three-phase Y-connected synchronous motor has a synchronous reactance of $1.5\,\Omega$ per phase. The field current has been adjusted so that the torque angle δ is $30°$ when the power supplied by the generator is 90KW.

(a) What is the magnitude of the internal generated voltage $|\tilde{E}_A|$ in this machine.

$$P = \frac{3 V_\phi E_A \sin\delta}{X_s}$$

$$90 \times 10^3 = \frac{(3)\left(\frac{440}{\sqrt{3}}\right)(E_A)}{1.5} \sin 30$$

$$E_A = 354.28\,V.$$

(b)
$$\tilde{I}_\phi = \frac{\tilde{V}_\phi - \tilde{E}_A}{jX_s} = \frac{\frac{440}{\sqrt{3}} - 354.28\angle -30°}{j1.5}$$

$$= \frac{254.33 - 306.82 + j177.14}{j1.5}$$

$$= 123.17\angle 16.5°$$

$PF = \cos(16.5°) = 0.9588$ leading

(c) $P_{max} = \frac{3V_\phi E_A}{X_S} =$

$$= \frac{(3)(254.33)(354.28)}{1.5}$$

$$= 180208\ W$$

~~static stability Limit~~ (max. power)

static stability power limit (max. power)

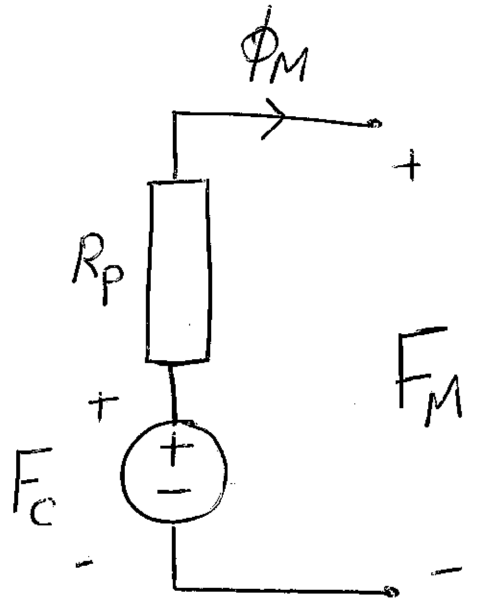
Permanent Magnet (PM) Materials & Permanent Magnet (PM) Machines

* The flux in permanent magnet machines is established by the magnets. Both the internal generated voltage and the induced torque is proportional to the flux. A permanent magnet can be regerded as a flux source and the magnetic field can be calculated by means of the magnetic circuit which is analogous to a simple electric circuit as:

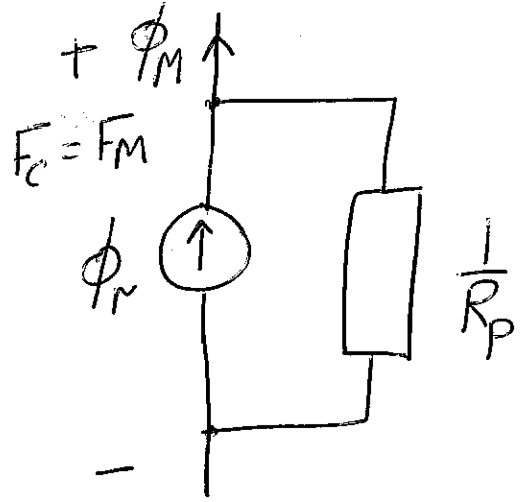
Magnetic Circuit parameter	Electric Circuit parameter
Flux (Wb or Vs)	Current (A)
MMF (A or A.turns)	Voltage (V)
Reluctance (A/Wb)	Resistance (Ω)

* A permanent magnet can be represented by a Thevenin equivalent circuit which comprises an MMF source in series with an internal reluctance R_p or by

a Norton equivalent circuit comprising a flux source in parallel with an internal permeance $\left(\frac{1}{R_P}\right)$.

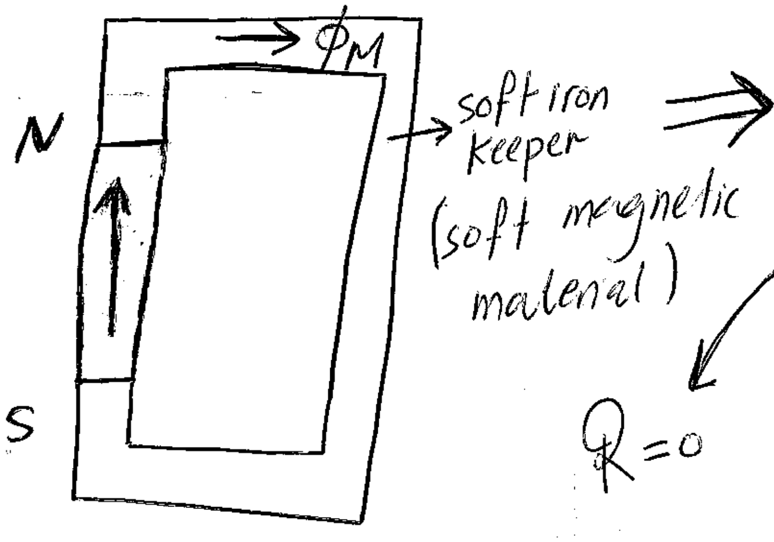


Thevenin



Norton

* The magnet can be short circuited by connecting a soft iron keeper across its poles. This ensures that the mmf across its terminals is zero and the magnet is operating at the short circuit point.



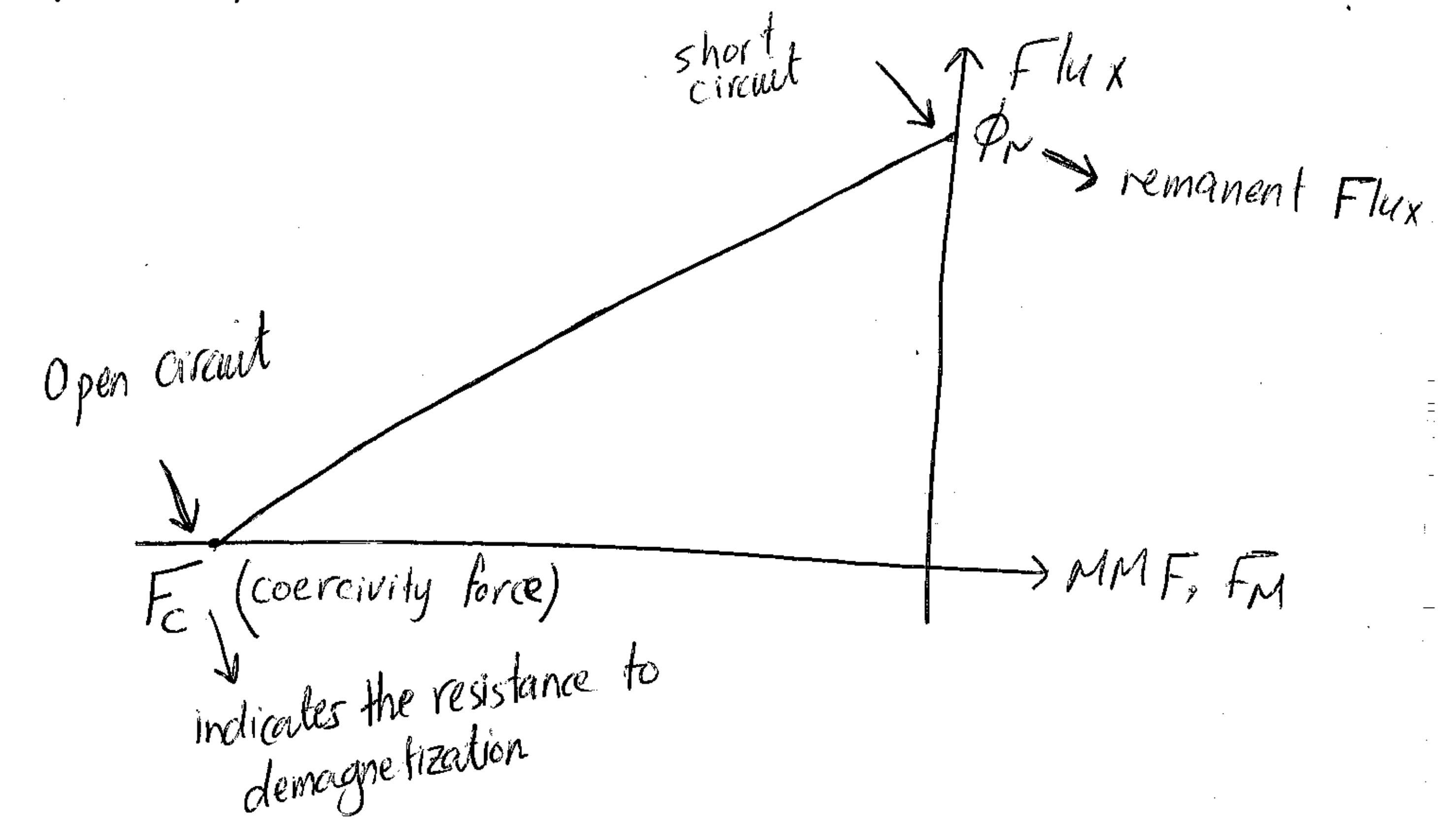
$\Rightarrow$ if the soft iron keeper is infinitely permeable, then $\phi_M = \phi_r$

$R = 0$

* The magnet is open circuited if the flux leaving the magnet poles is zero.

S [→] N $\Rightarrow \phi_M = 0$

* F_c is called the coercive MMF because it is the MMF required to coerce the magnet to produce zero flux. It directly expresses the resistance of the magnet to demagnetization. The characteristics of a permanent magnet can be expressed graphically in terms of the Flux/MMF relationship at the terminals of the pole faces as shown below:



* The remanent flux ϕ_r and coercive MMF depend not only on the material properties but also on the dimensions of the magnet:

$$\phi_r = B_r A_M \rightarrow \text{(magnet pole area)}$$

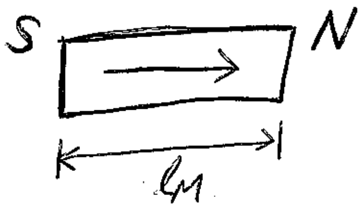
B_r: material property (remanent flux density)

A_M: magnet dimensions

$$F_c = H_c l_M$$

H_c: coercivity

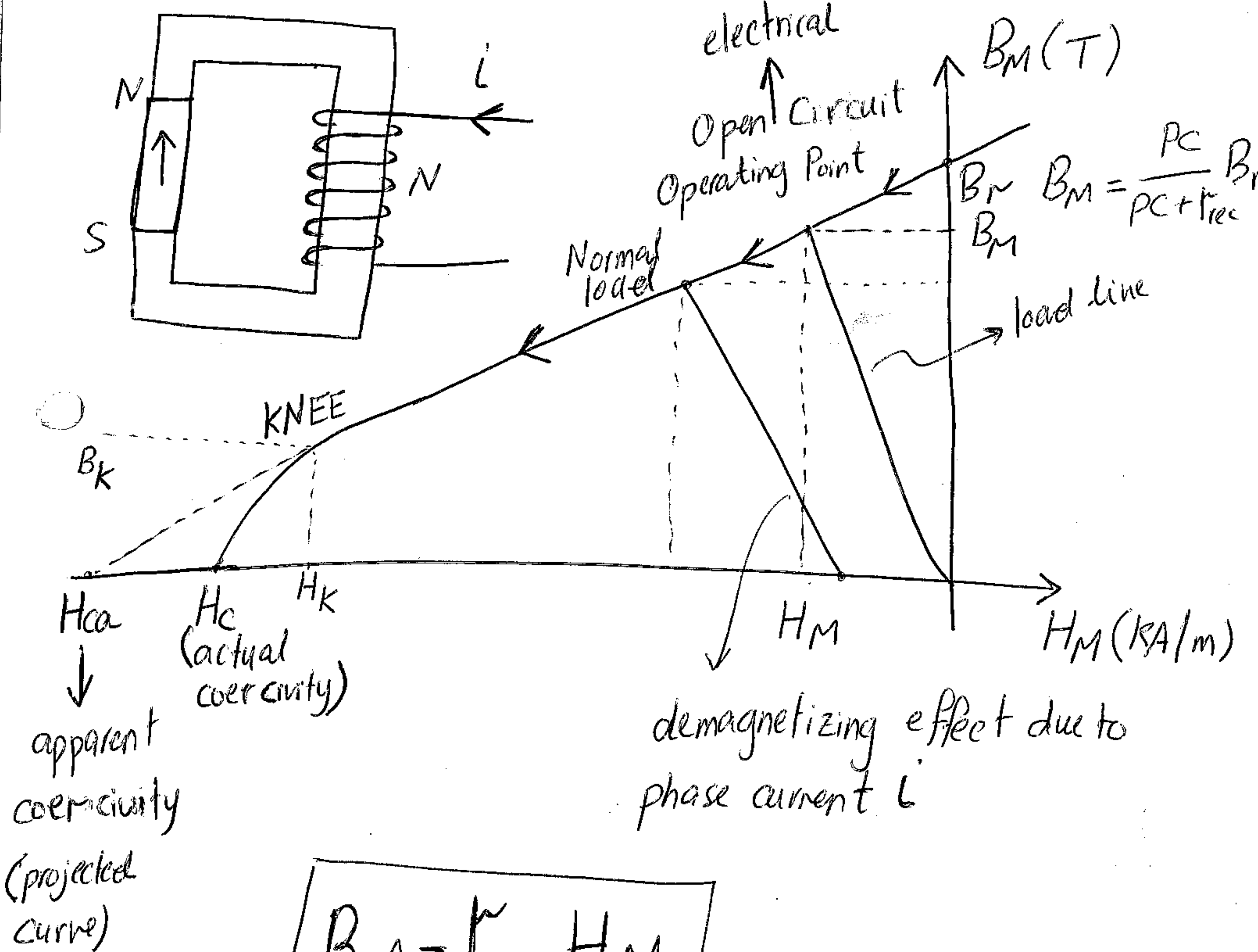
l_M: length of the material in the direction of magnetization (dimensions)



* If the vertical axis is scaled by $1/A_M$ and the horizontal axis is scaled by $1/l_M$, then the result will be a relationship between B_M and H_M.

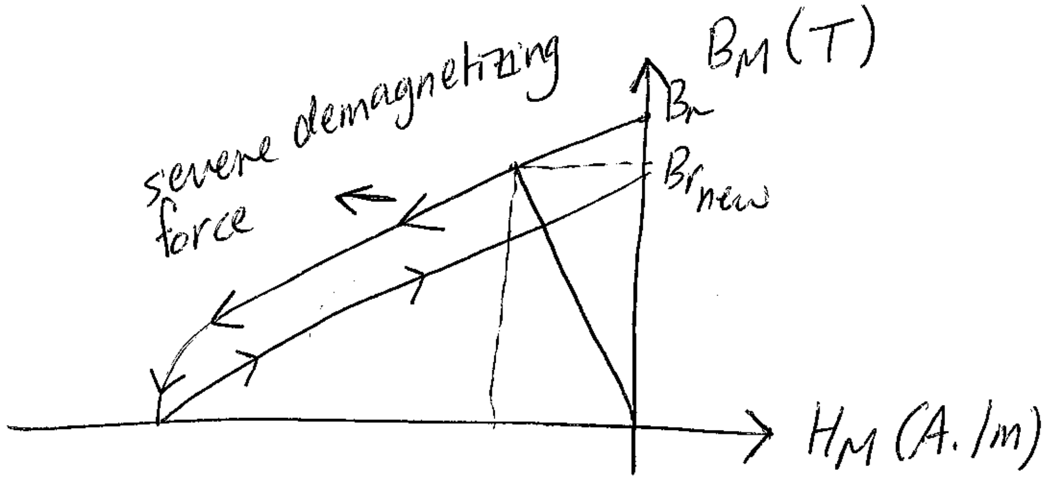
$$\phi_M = B_M A_M \qquad F_M = H_M l_M$$

- The graph of B_M vs. H_M is shown below:



$$\boxed{B_M = \mu_{rec} H_M}$$

* For permanent magnets, if the operating point is forced below the knee by severe demagnetizing force, then the magnet recovers along a lower recoil line when it is removed as shown below in the figure:

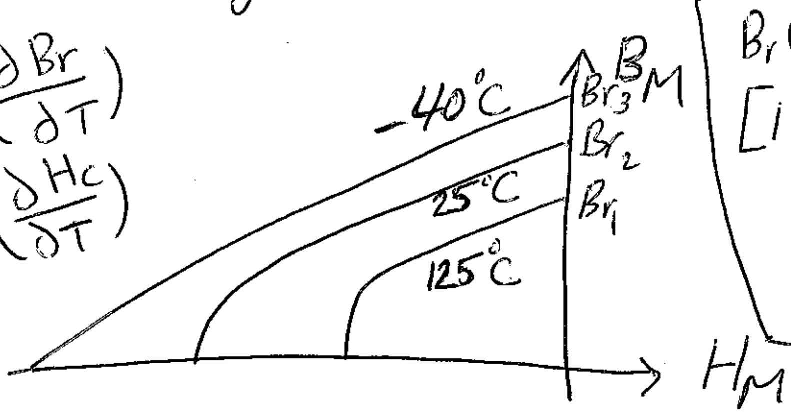


* The effect of temperature on the B/H characteristics of the permanent magnet material is:

reversible temperature coefficient

$$\alpha_{Br} = \left(\frac{1}{Br}\right)\left(\frac{\partial Br}{\partial T}\right)$$

$$\alpha_{H_c} = \left(\frac{1}{Hc}\right)\left(\frac{\partial Hc}{\partial T}\right)$$



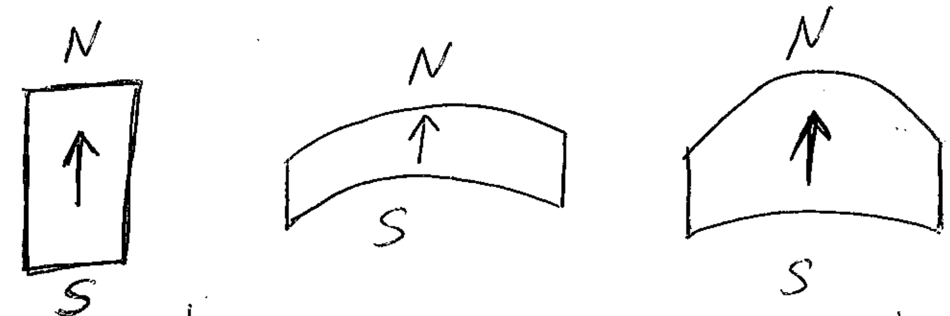
$$B_r(T) = B_r(20) \times \left[1 + \alpha_{Br}(T-20)/100\right]$$

↑ reversible temperature coefficient

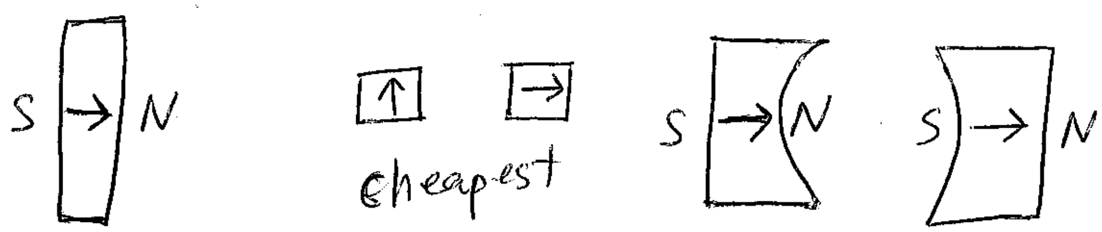
<u>or</u> the effect of temperature is normally specified in terms of the <u>reversible temperature coefficient</u> α_{Br}

* Shapes of permanent magnets:

(1)



cheapest — most expensive



cheapest — cheapest — most expensive

* Types of permanent magnets:

commonly used because there are low $(BH)_{max}$ (maximum energy product)

(1) Ferrites → inexpensive

(2) AlNiCo (Aluminum-Nickel-Cobalt)

(3) SmCo (Samarium-Cobalt) → higher $(BH)_{max}$

rare-earth

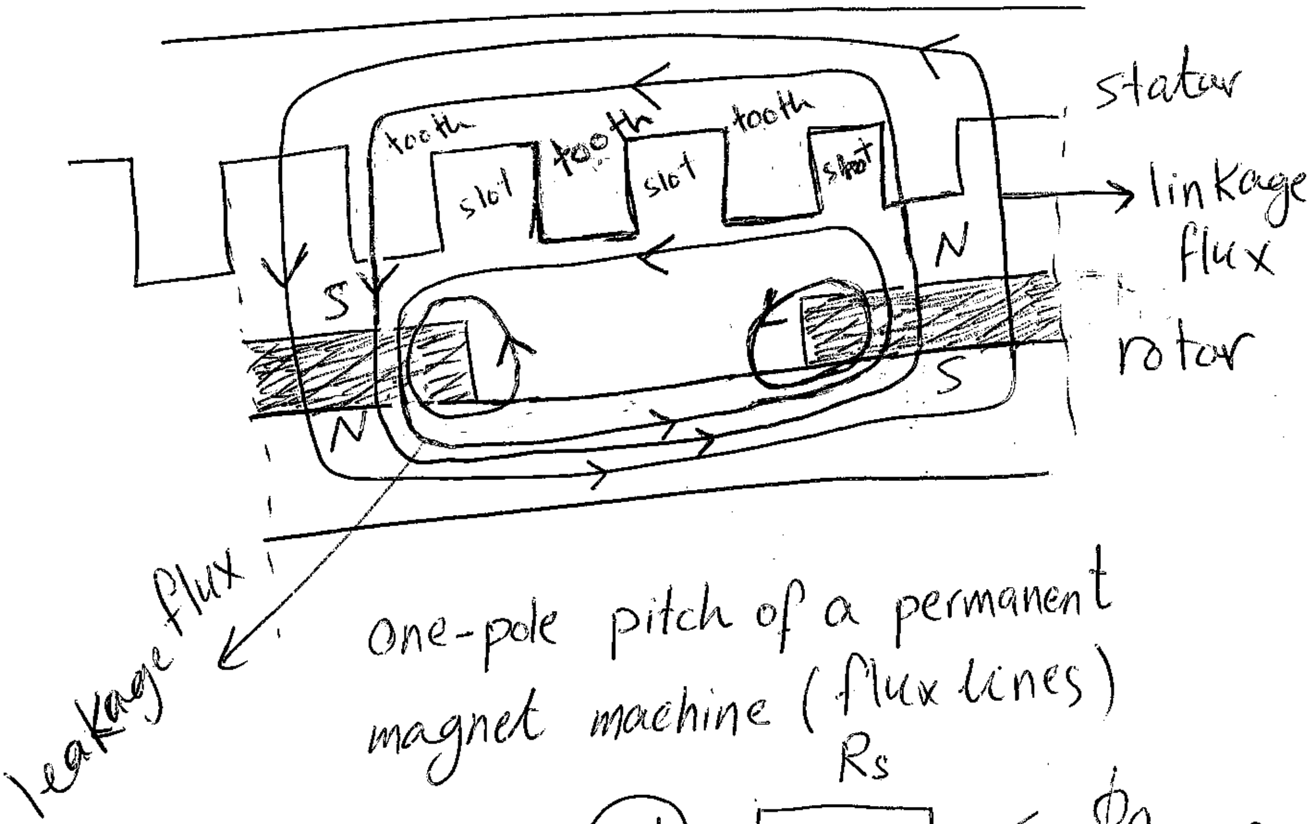
⇓ military applications ⟷ → more stable from temperature point of view

(4) NdFeB (Neodymium-Iron-Boron) → highest $(BH)_{max}$

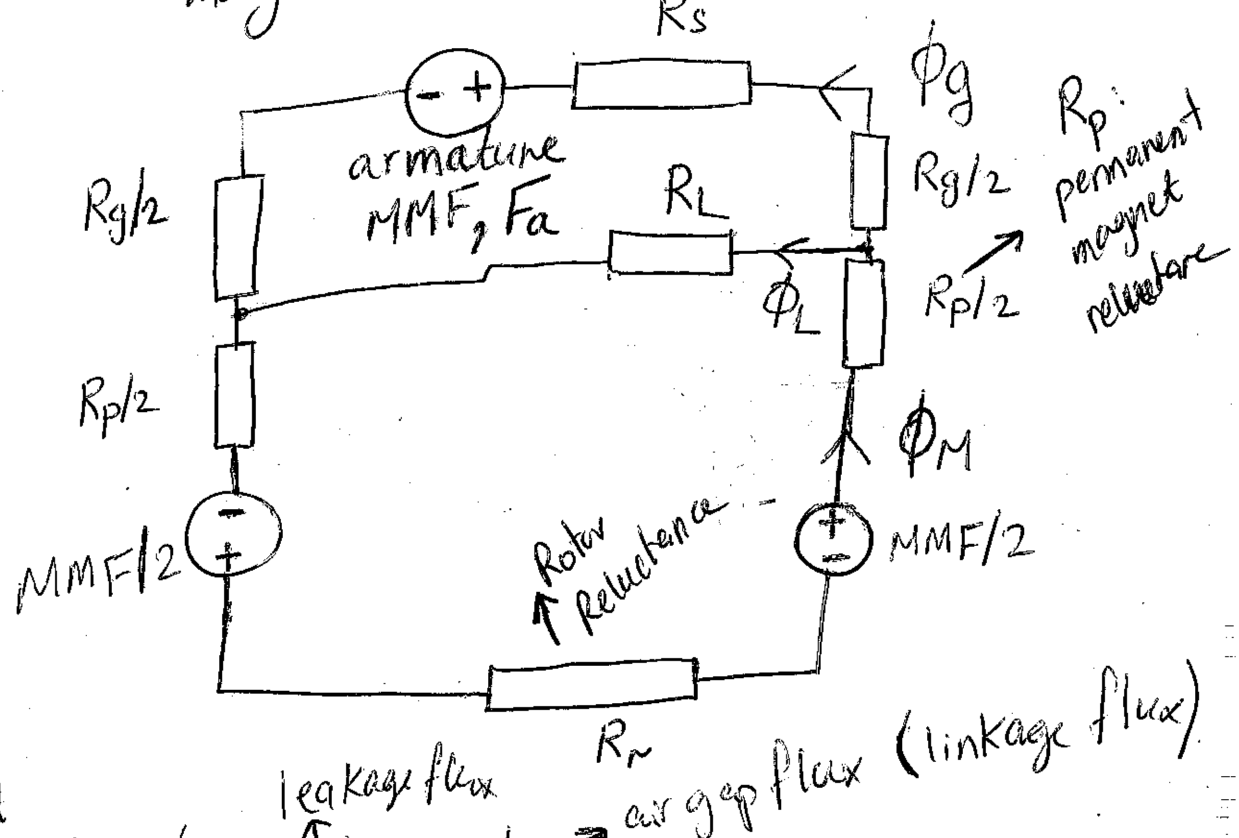
rare-earth

→ loses some char. as a result of high temp.

Approximate Calculations of Flux



one-pole pitch of a permanent magnet machine (flux lines)



$$\underset{\text{magnet flux}}{\phi_M} = \underset{\text{leakage flux}}{\phi_L} + \underset{\text{air gap flux (linkage flux)}}{\phi_g}$$

Magnetic Equivalent Circuit of one pole pitch

* The leakage flux coefficient f_{LKG} is defined as the ratio of air gap flux to magnet flux

$$f_{LKG} = \frac{\phi_g}{\phi_M} = \frac{\phi_g}{\phi_g + \phi_L}$$

$f_{LKG} < 1 \Rightarrow$ "Typical rule of thumb" value is 0.9.

$\Downarrow$

It depends on the configuration of the motor

* Fa is normally initially assumed zero (open electrical-circuit conditions).

* $R_p = \dfrac{l_M}{\mu_o \mu_{rec} A_M}$

l_M: length of the permanent magnet in the direction of magnetization

A_M: pole magnet area.

$$R_g = \frac{g'}{\mu_0 A_g} = \frac{K_c g}{\mu_0 \beta_{ir} r_g l_{stk}}$$

g': effective air gap length determined by the use of the Carter's coefficient

K_c: Carter's coefficient

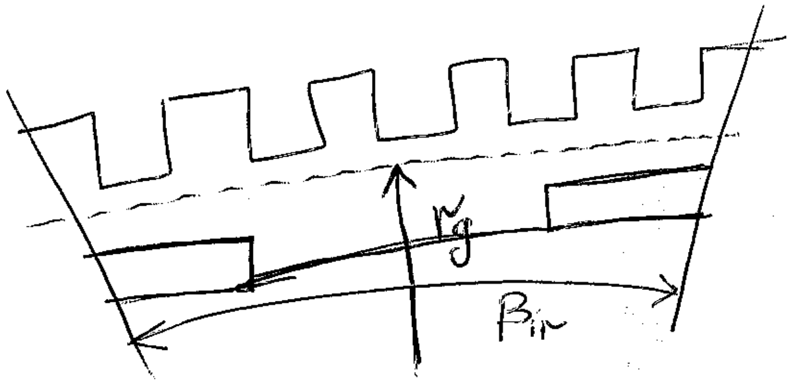
g: physical air gap length

r_g: midway radius through the physical air gap

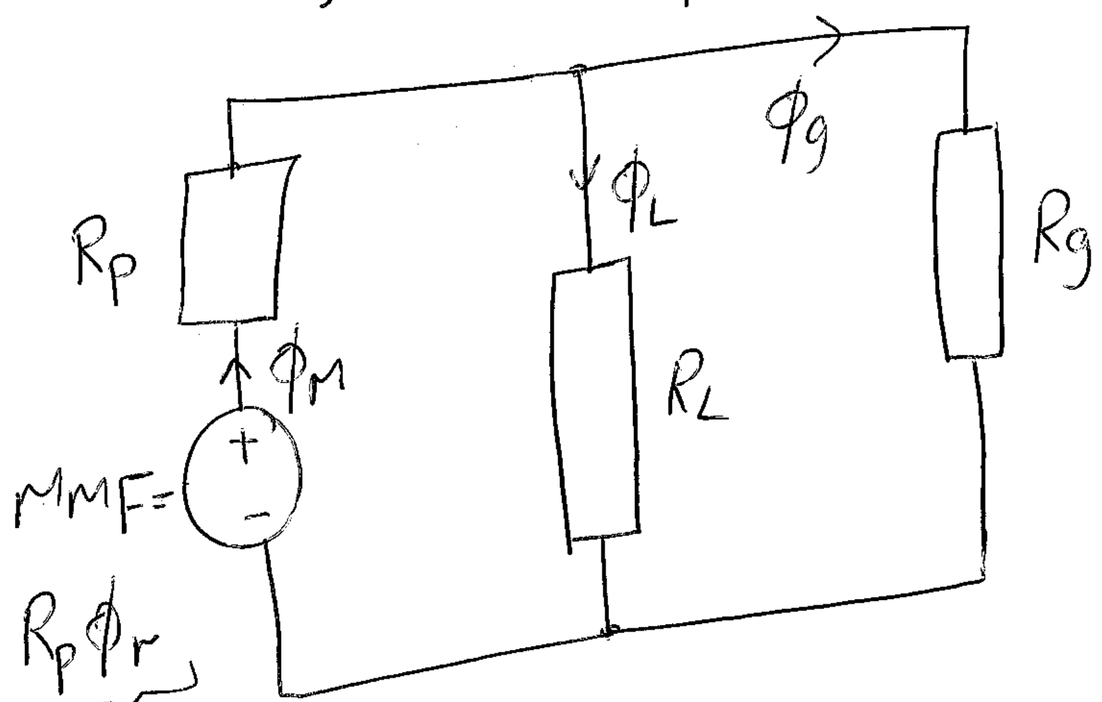
β_{ir}: arc angle of the rotor iron facing the stator (mechanical radius)

A_g: rotor iron area facing the stator equals $\underline{r_g \beta_{ir} l_{stk}}$

l_{stk}: stack (active) length of the machine



* Normally the stator and rotor are infinitely permeable ($\mu_r \rightarrow \infty$) and therefore $R_s = R_r = 0$. The armature mmf F_a is also neglected (treated as open electrical circuit conditions). The equivalent circuit will be:



From the Norton equivalent

$$\phi_g = \phi_M \frac{R_L}{R_L + R_g}$$

$$\phi_M = \frac{MMF}{R_p + (R_L \| R_g)}$$

$$\phi_M = \frac{\phi_r R_p}{R_p + \frac{R_L R_g}{R_L + R_g}}$$

but

$$\phi_M = \phi_g \frac{R_L + R_g}{R_L}$$

$$\phi_g = \phi_r \frac{R_p}{R_p + \frac{R_L R_g}{R_L + R_g}} \frac{R_L}{R_L + R_g}$$

$$f_{LKG} = \frac{\phi_g}{\phi_M} = \frac{R_L}{R_L + R_g}$$

$$\therefore \quad \phi_g = \frac{f_{LKG}}{1 + f_{LKG}(R_g \| R_p)} \phi_r$$

but $\phi_r = B_r A_M$, $\phi_g = B_g A_g$ and therefore

$$B_g = \frac{f_{LKG} \frac{A_M}{A_g}}{1 + \mu_{rec} f_{LKG} \frac{A_M}{A_g} \frac{g'}{l_M}} B_r$$

* Having $f_{LKG} < 1$ means that the air-gap flux density is reduced compared to the value it would have if there were no leakage. The corresponding flux density B_M can be determined as:

$$\boxed{\begin{aligned} \phi_g &= f_{LKG} \phi_M \\ B_g A_g &= f_{LKG} B_M A_M \end{aligned}}$$

$$B_M = B_g \frac{1}{f_{LKG}} \frac{A_g}{A_M}$$

* a convenient formula for the permeance coefficient PC is:

$$PC = \frac{1}{f_{LKG}} \frac{l_M}{g'} \frac{A_g}{A_M}$$

* Another useful relationship for the permeance coefficient is:

$$B_M = \frac{PC}{PC + \mu_{rec}} B_r$$

* The operating point of the magnet can now be determined either graphically from the B/H characteristics of the magnet or by calculating H_M from the equation that describes the demagnetization characteristics:

$$B_M = \mu_{rec} \mu_0 H_M + B_r \qquad \underline{\underline{B_M > B_K}} \Rightarrow$$

[linear region]

* μ_{rec} is normally <u>one</u> for hard magnets. A PC value of 5 would be typical giving

$$\boxed{B_M = 0.83 B_r}$$