

Design of Mechatronics Systems (DoMS)

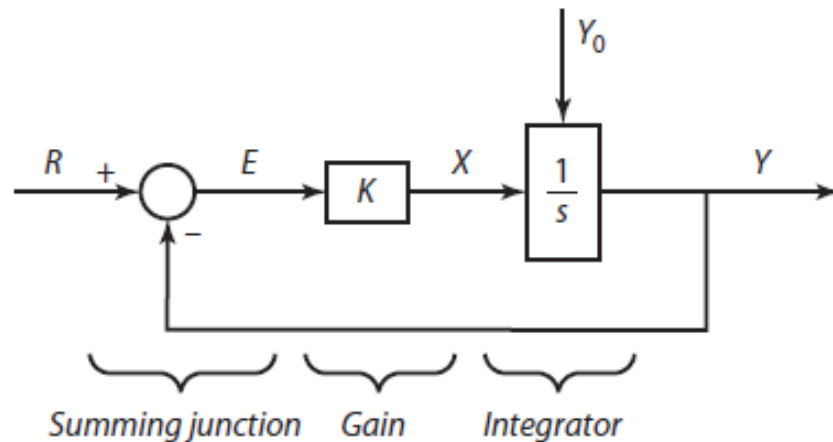
# Modeling and simulation of physical systems

Block diagram model

Dr. Mohammad R. Hayajneh

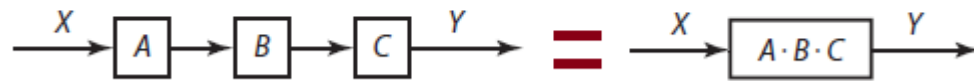
# Introduction

- *Simulation* is the process of *solving* any model on a computer.
- Since block diagram models are so widely used, we will use block diagrams for all modeling tasks.
- Block diagram models consist of two fundamental objects; *signal wires* and *blocks*.

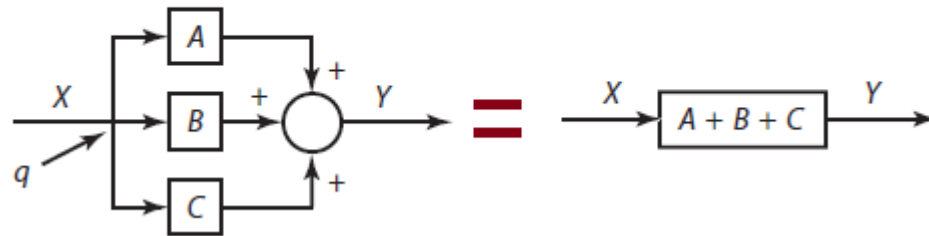


# Block Diagrams and Manipulations (Review)

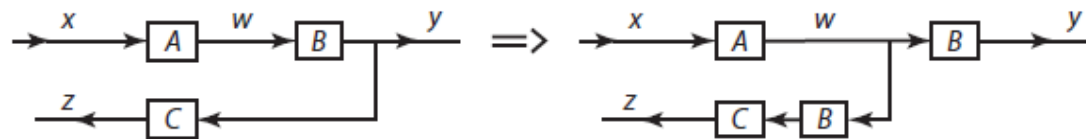
## SERIES MANIPULATION



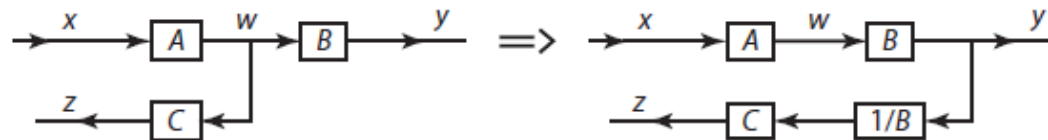
## PARALLEL MANIPULATION



## PICK-OFF POINT SHIFTED UPSTREAM

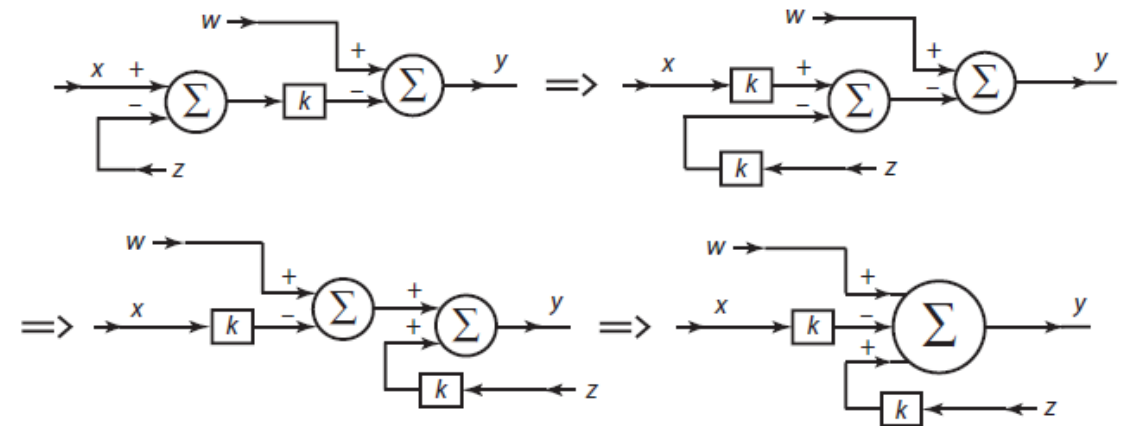


## PICK-OFF POINT SHIFTED DOWNSTREAM

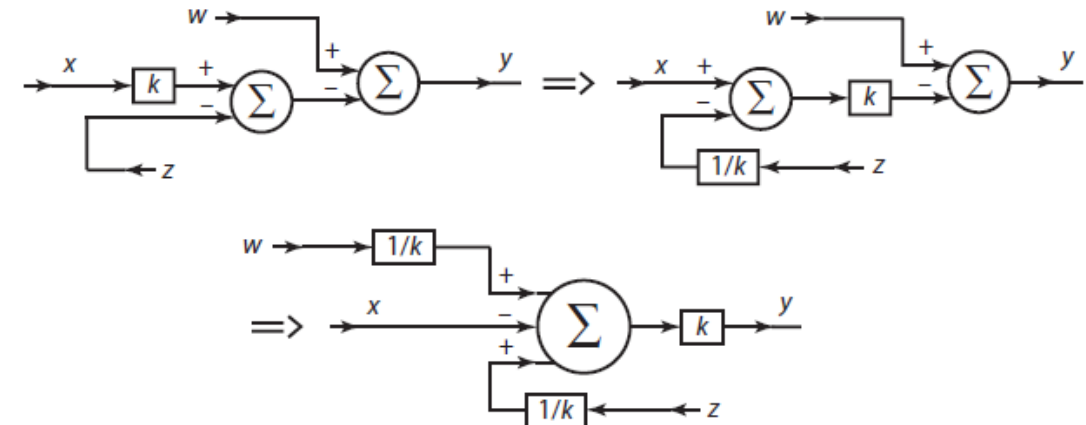


## MOVING BLOCKS UPSTREAM THROUGH A SUMMING JUNCTION

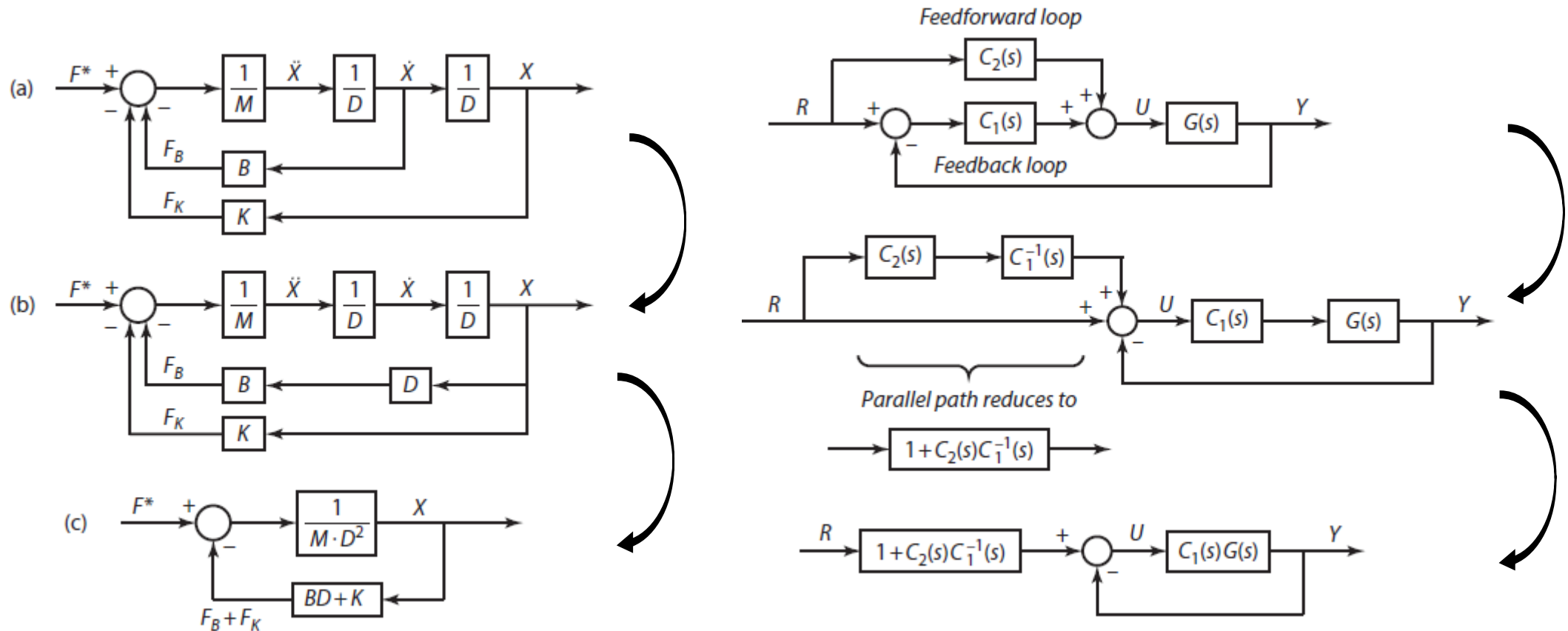
$$y = k(A + B) = kA + kB$$



## MOVING BLOCKS DOWNSTREAM THROUGH A SUMMING JUNCTION



# Simple Feedback Diagram Reduction (Review)



# Simulation

- **Simulation is the process through which the model equations are numerically solved.**
- Most visual simulation environments perform three basic functions.
  - ***Graphical Editing***: Used for the creation, editing, storage, and retrieval of models. Also used to create model inputs, orchestrate the simulation, and to present the model results.
  - ***Analysis***: Used to obtain transfer functions, compute frequency response, and evaluate sensitivity to disturbances.
  - ***Simulation***: Numerical solution of the block diagram model.
- All visual modeling environments include the simulation function. Some of the most commonly used environments are MATLAB/Simulink (Mathworks) and LabVIEW (National Instruments).

# Simulation Process

- The simulation process consists of three steps.
- **Step 1. Initialization:** In the initialization step, the equations for each block in the system model are *sorted* according to the pattern in which the blocks are connected.
- **Step 2. Iteration:** In the iteration step, differential equations present in the model are solved using numerical integration and/or differentiation, and the simulation time is advanced.
- **Step 3. Termination:** Results are presented in the termination step along with any other post-processed calculations

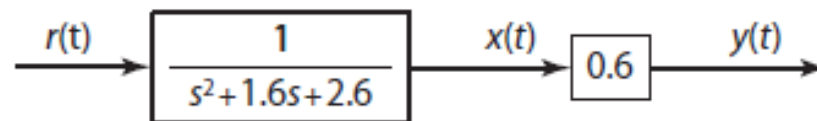
# Transfer Function Conversion to Block Diagram Model

$$T(s) = \frac{Y(s)}{R(s)} = \frac{3}{5s^2 + 8s + 13} = \frac{\text{Num}(s)}{\text{Den}(s)}; \quad y(0) = 2, \dot{y}(0) = -2$$

- **Step 1.** Make highest  $s$ -power in Den coefficient equal to 1.

$$T(s) = \frac{Y(s)}{R(s)} = \frac{3}{5} \frac{1}{s^2 + 8/5s + 13/5} = 0.6 \frac{1}{s^2 + 1.6s + 2.6}$$

Create the state variable,  $x(t)$ , by “sliding” the numerator part of the transfer function into a new block located to the right of the denominator part of the transfer function



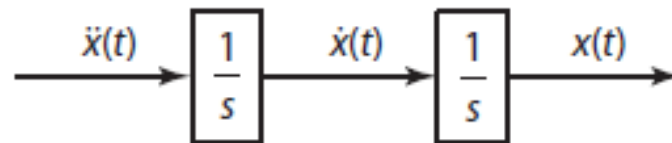
- **Step 2.** From step 1, write the state equation (SE) as the differential equation relating the input,  $r(t)$ , to the state,  $x(t)$ , as

$$\text{SE: } \frac{x(t)}{r(t)} = \frac{1}{s^2 + 1.6s + 2.6}$$

or

$$\frac{d^2x(t)}{dt^2} + 1.6 \frac{dx(t)}{dt} + 2.6x(t) = r(t)$$

- **Step 3.** Begin constructing the block diagram by placing  $n$ -integrator blocks in series and connect them from left to right. The input to the leftmost integrator block will be the highest derivative of the state equation, in this case  $\frac{d^2x}{dt^2}$ , and the output of the rightmost integrator block will be  $x(t)$ . Using our example system, there are two integrators, and they are written as

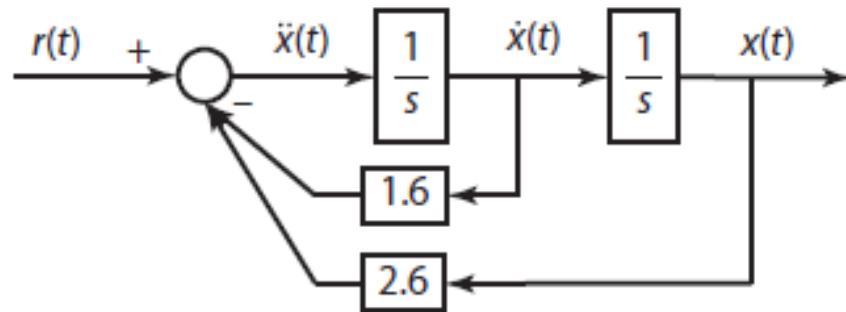




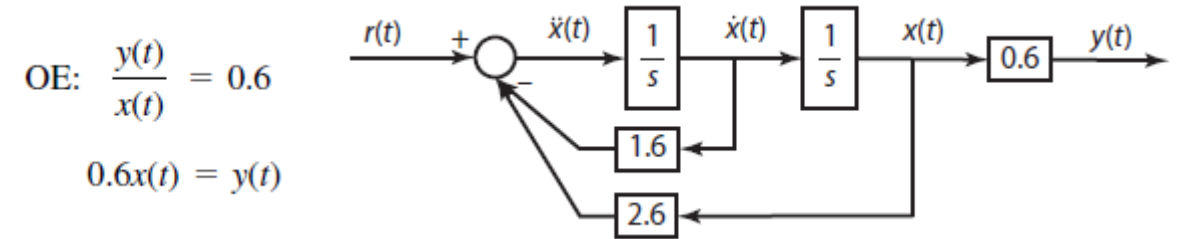
- **Step 4.** Solve the state equation from step 2 for the highest derivative of the state variable.

$$\frac{d^2x(t)}{dt^2} = -1.6 \frac{dx(t)}{dt} - 2.6x(t) + r(t)$$

- we implement the above form of the state equation onto the block diagram started in step 3.



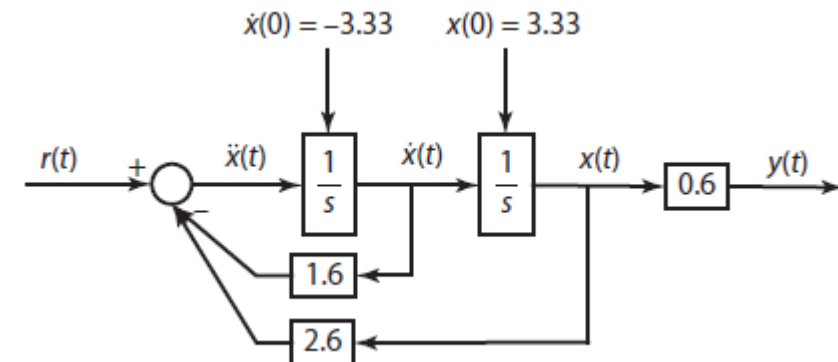
- **Step 5.** write the output equation (OE) as the differential equation relating the output,  $y(t)$ , to the state,  $x(t)$ , and its derivatives



- **Step 6.** Add the initial conditions to the block diagram in step 5

$$0.6x(0) = y(0) = 2 \rightarrow x(0) = 3.33$$

$$0.6\dot{x}(0) = \dot{y}(0) = -2 \rightarrow \dot{x}(0) = -3.3333$$

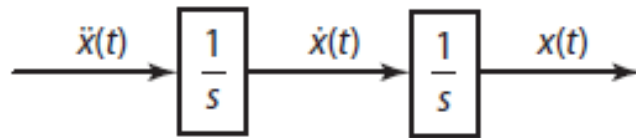


# ODE to Block Diagram

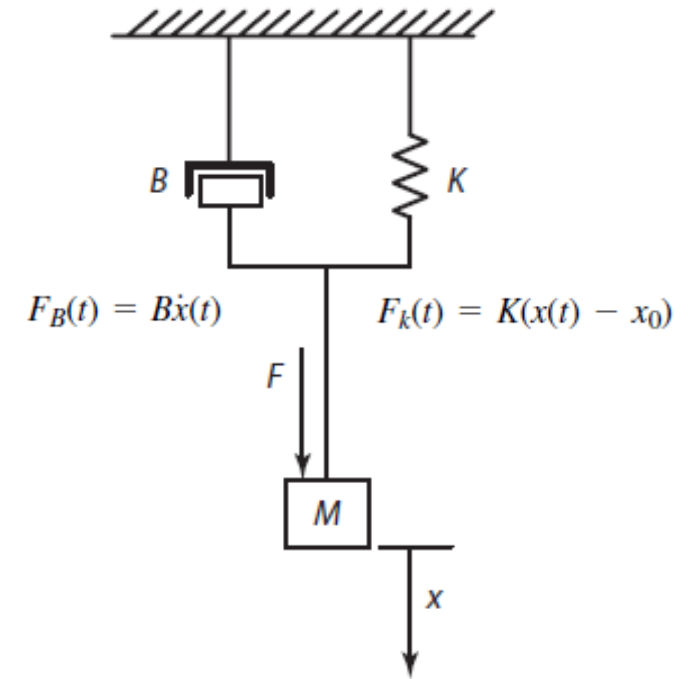
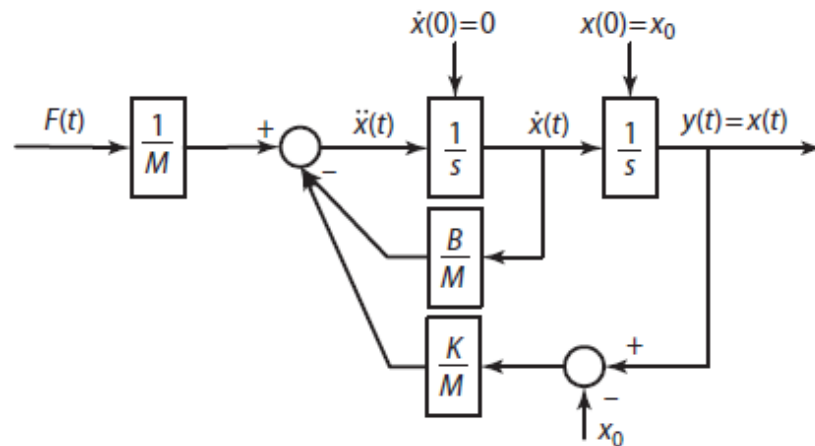
- The ODE of the system

$$\ddot{x}(t) = -\frac{B}{M}\dot{x}(t) - \frac{K}{M}(x(t) - x_0) + \frac{1}{M}F(t)$$

- Second order system needs two integrators



- Complete the block diagram



$$\sum F(t) = M\ddot{x}(t) = F(t) - F_k(t) - F_B(t)$$

$$F(t) - B\dot{x}(t) - K(x(t) - x_0) = M\ddot{x}(t)$$

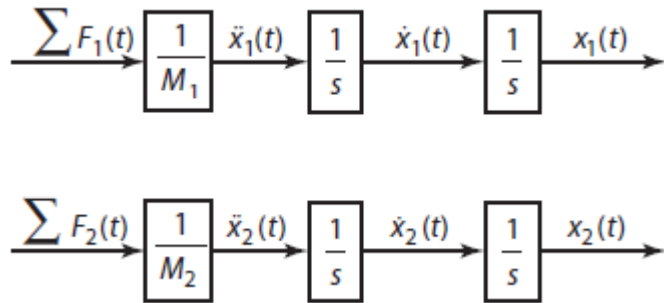
# Two-Mass Mechanical System (Illustration)

- The forces on each mass

$$\text{Mass 1: } \ddot{x}_1(t) = \frac{1}{M_1} \sum F_1(t)$$

$$\text{Mass 2: } \ddot{x}_2(t) = \frac{1}{M_2} \sum F_2(t)$$

Represent equations by the block diagram

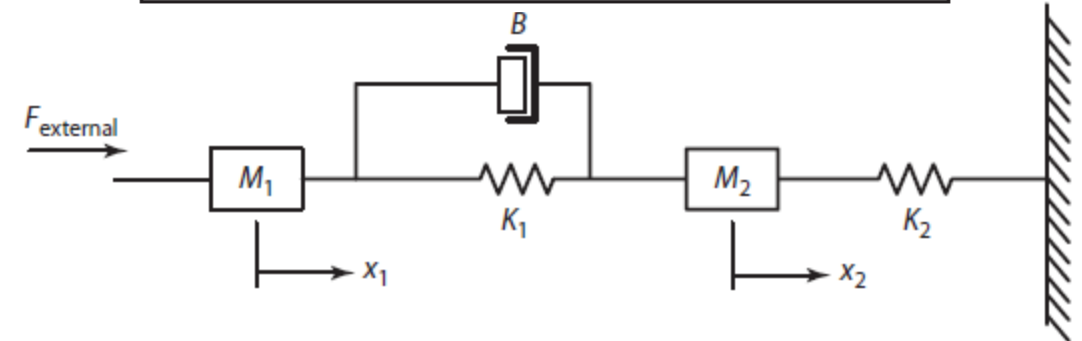
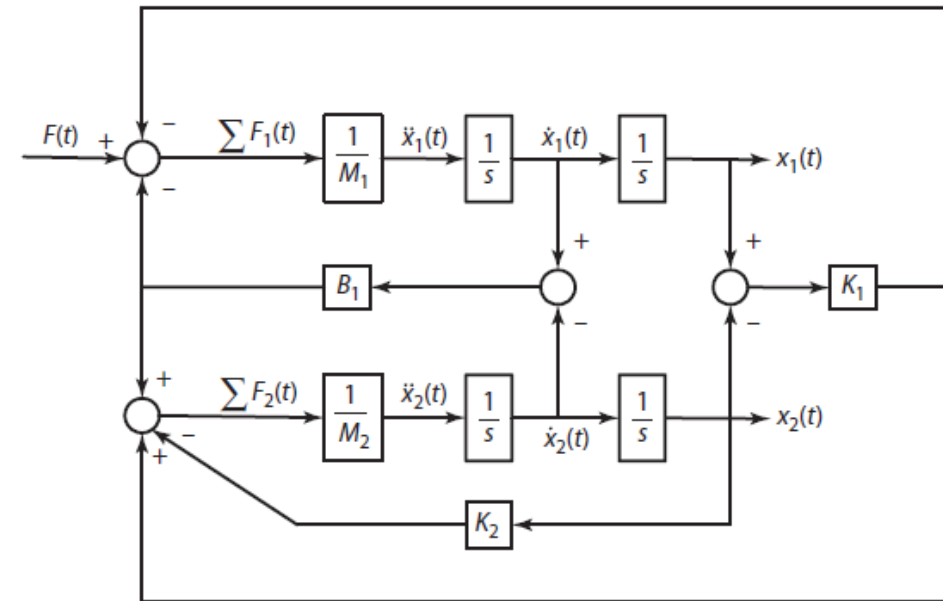


- The total force in terms of components

$$\sum F_1(t) = F_1(t) - K_1(x_1(t) - x_2(t)) - B(\dot{x}_1(t) - \dot{x}_2(t))$$

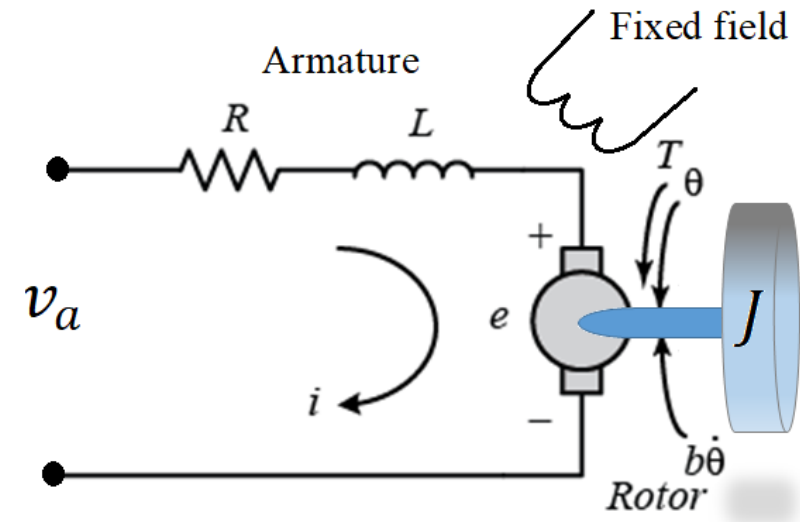
$$\sum F_2(t) = K_1(x_1(t) - x_2(t)) + B(\dot{x}_1(t) - \dot{x}_2(t)) - K_2x_2(t)$$

- Implement the total block diagram



# DC Motor Model

- In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field.
- Assumptions: The magnetic field is constant
- The motor torque is proportional to only the armature current  $i$  by a constant factor  $k_t$  as  $T_m = k_t i$
- We can derive the following governing equations based on Newton's 2nd law and Kirchhoff's voltage law.



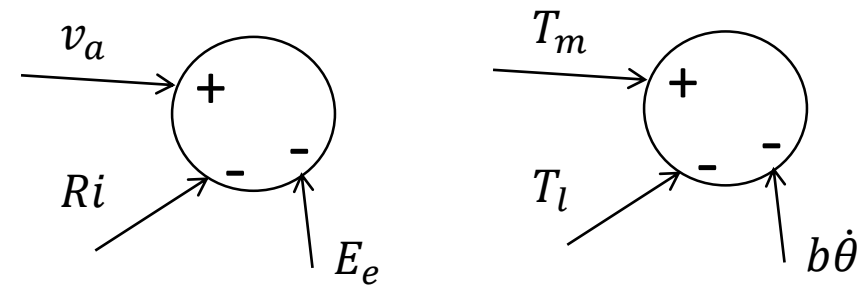
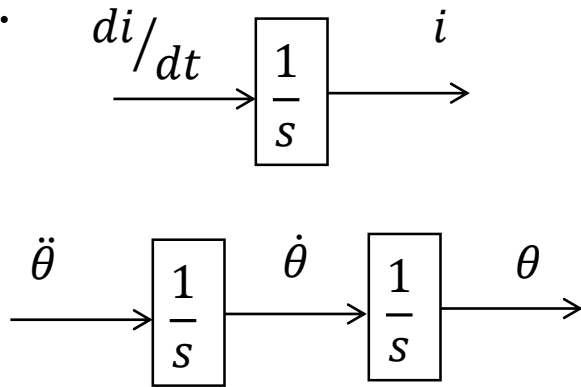
$$T_m - T_l - b\dot{\theta} = J\ddot{\theta}, \quad \text{Mechanical Equation}$$

$$L \frac{di}{dt} = -Ri + v_a - E_e \quad \text{Electrical Equation}$$

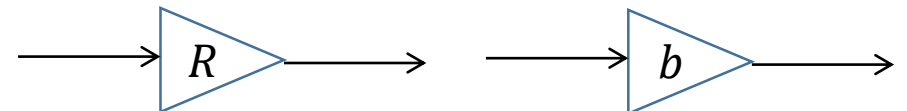
- The back emf,  $E_e$ , is proportional to the angular velocity of the shaft by a constant factor  $K_e$ , where  $E_e = k_e \dot{\theta}$ .

# DC Motor Simulink model ( Procedure)

- Placing integrator blocks in series number of integrators equals to highest derivative in each ODE. Using our example system, there are two integrators for mechanical equation, and one for the electrical equation.
- For each subsystem above, add terms to solve for the highest derivative:

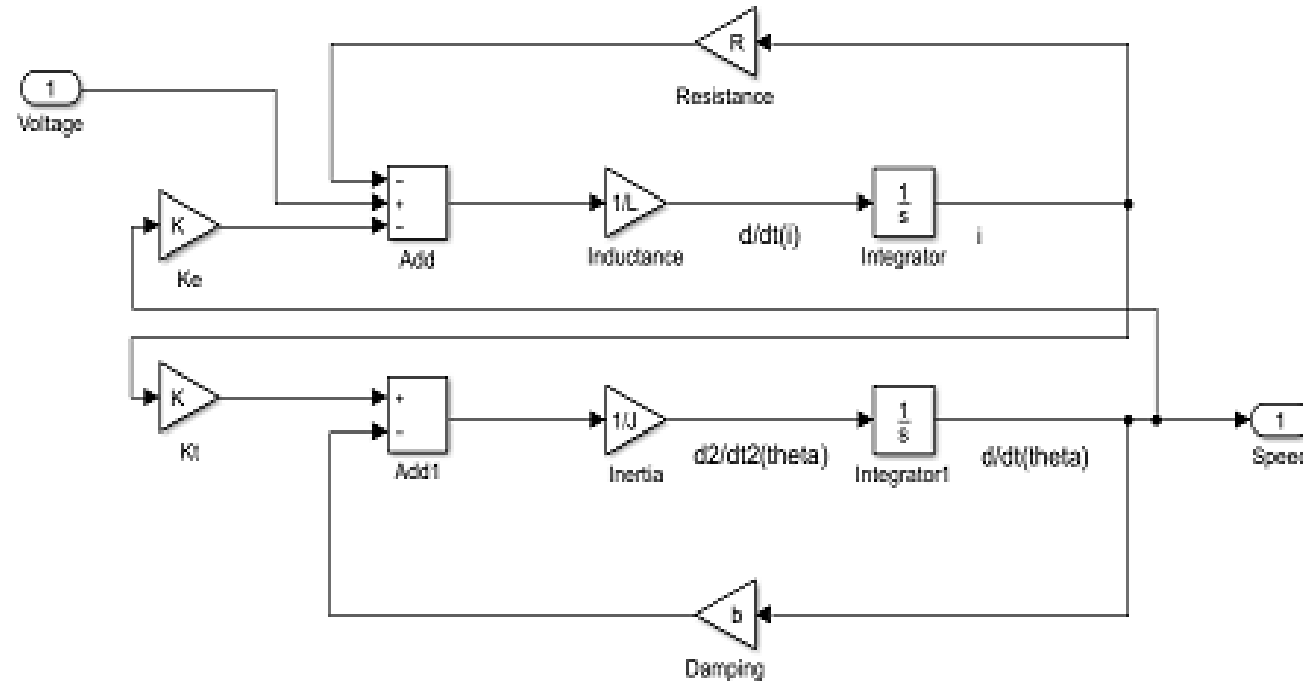


- Solve for variables such as  $T_m$  and  $E_e$  to be functions of  $i$  and  $\theta$ .
- Replace each constant such as  $R, b, k_t, k_v$  by the gain symbol:



# DC Motor Simulink model

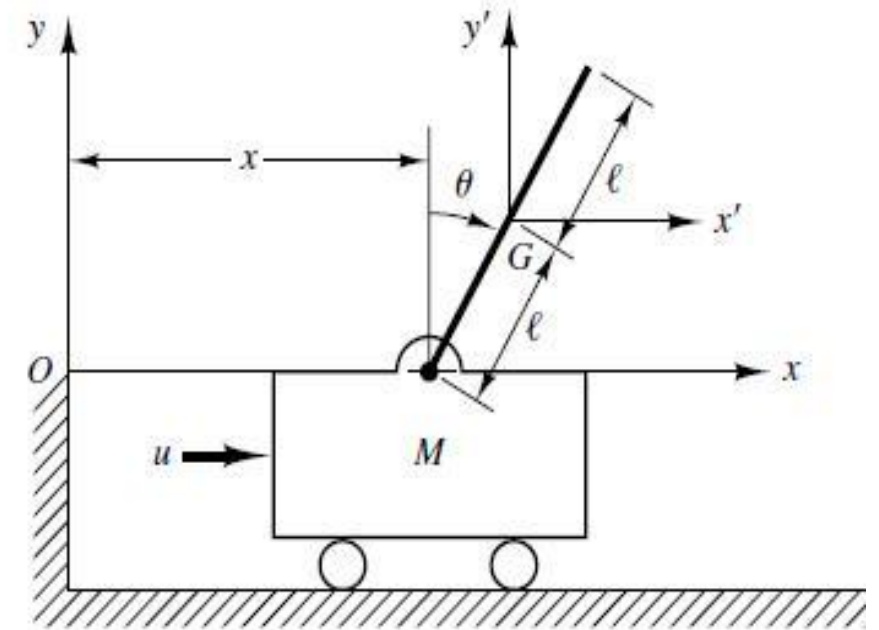
- The full model of the DC motor can be represented in Simulink as follow: Try it in MATLAB



Note: One integrator is used to solve for the mechanical equation.

# Inverted Pendulum (IP)

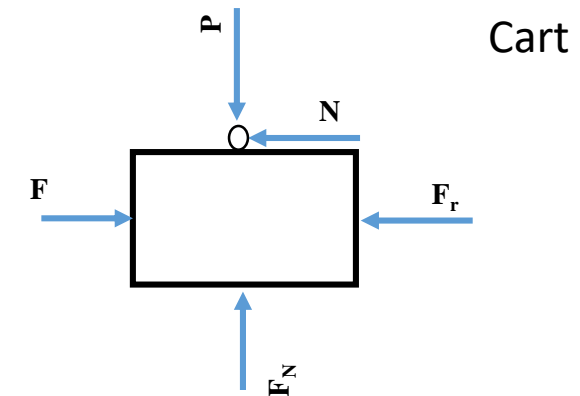
- An inverted pendulum is a pendulum that has its centre of mass above its pivot point.
- It is naturally unstable and must be actively balanced in order to remain upright. It is necessary to keep the pendulum vertically aligned with the centre of mass to avoid its falling over.
- The inverted pendulum is a typical problem in dynamics and control theory and is used as a benchmark for testing control methods.
- A feedback control system can be used to stabilize the pendulum in this inverted position to monitor the pendulum's angle and to move the position of the pivot point sideways when the pendulum starts to fall over, to keep it balanced.



# Mathematical model of IP

## • Forces on the cart

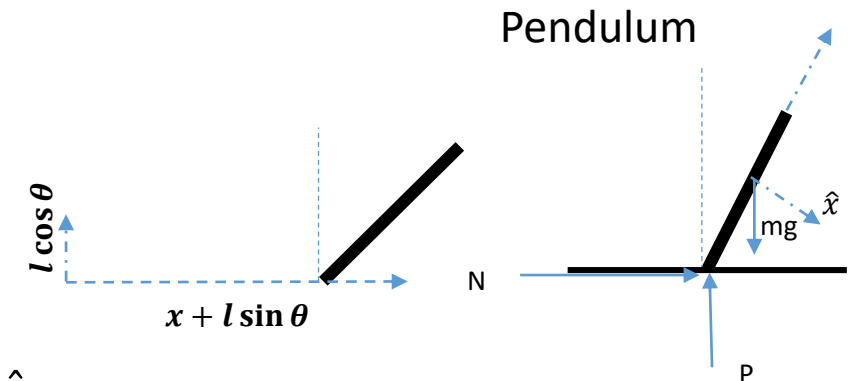
- $\sum F_x = M\ddot{x} = F - F_r - N \dots\dots\dots (1)$
- $\sum F_y = M\ddot{y} = 0 = F_N - P \dots\dots\dots (2)$ 
  - $N$  and  $P$  are reaction forces at the joint
  - $F$  is the normal force applied to the cart
  - Note: no useful information can be given by equation (2) in  $y$  direction



## • Forces on the Pendulum

The position of the point mass (G) can be given in inertial coordinates as:

- $r = (x + l \sin \theta)\hat{i} + (l \cos \theta) \hat{j}$
  - $\dot{r} = (\dot{x} + l \dot{\theta} \cos \theta)\hat{i} - (l \dot{\theta} \sin \theta) \hat{j}$
  - $\ddot{r} = (\ddot{x} - l \dot{\theta}^2 \sin \theta + l \ddot{\theta} \cos \theta)\hat{i} - (l \dot{\theta}^2 \cos \theta + l \ddot{\theta} \sin \theta) \hat{j}$
- $\underbrace{\hspace{15em}}_{\ddot{r}_x} \qquad \underbrace{\hspace{15em}}_{\ddot{r}_y}$





# IP modeling

- Two equations can be written in the x-direction and the y-direction. The reaction forces are positive as applied to the pendulum. The linear motion of the pendulum at point (G) in x and y direction can be given by the following equations:

$$\bullet \sum F_p x = m \ddot{r}_x = N = m(\ddot{x} - l \dot{\theta}^2 \sin \theta + l \ddot{\theta} \cos \theta) \quad \dots\dots\dots (3)$$

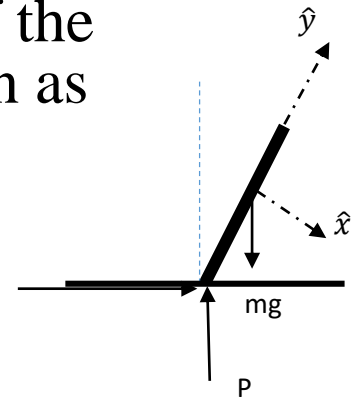
$$\bullet \sum F_p y = m \ddot{r}_y = P = m(-l \dot{\theta}^2 \cos \theta - l \ddot{\theta} \sin \theta) + mg \quad \dots\dots\dots (4)$$

Substitute (3) in (1) yields:

$$\bullet (M + m)\ddot{x} = F + ml \dot{\theta}^2 \sin \theta - ml \ddot{\theta} \cos \theta - b\dot{x})$$

- Now we solve for the pendulum equation of motion in the body frame of the pendulum  $\hat{x}$  and  $\hat{y}$ . The acceleration of the pendulum in its frame is given as

$$\begin{aligned} \bullet \ddot{\hat{x}} &= \ddot{r}_x \cos \theta + \ddot{r}_y \sin \theta \\ &= (\ddot{x} - l \dot{\theta}^2 \sin \theta + l \ddot{\theta} \cos \theta) \cos \theta - (l \dot{\theta}^2 \cos \theta + l \ddot{\theta} \sin \theta) \sin \theta \\ &= (\ddot{x} \cos \theta - l \dot{\theta}^2 \sin \theta \cos \theta + l \ddot{\theta} \cos^2 \theta) + (l \dot{\theta}^2 \sin \theta \cos \theta + l \ddot{\theta} \sin^2 \theta) \\ &= \ddot{x} \cos \theta + l \ddot{\theta} \end{aligned}$$



# PI Modeling

- The forces on the pendulum

- $\sum F_{\hat{x}} = m\ddot{\hat{x}} = N \cos \theta - P \sin \theta + mg \sin \theta$

- $m(\ddot{x} \cos \theta + l \ddot{\theta}) = N \cos \theta - P \sin \theta + mg \sin \theta$  ..... (5)

- The generated torque around the point G is given by

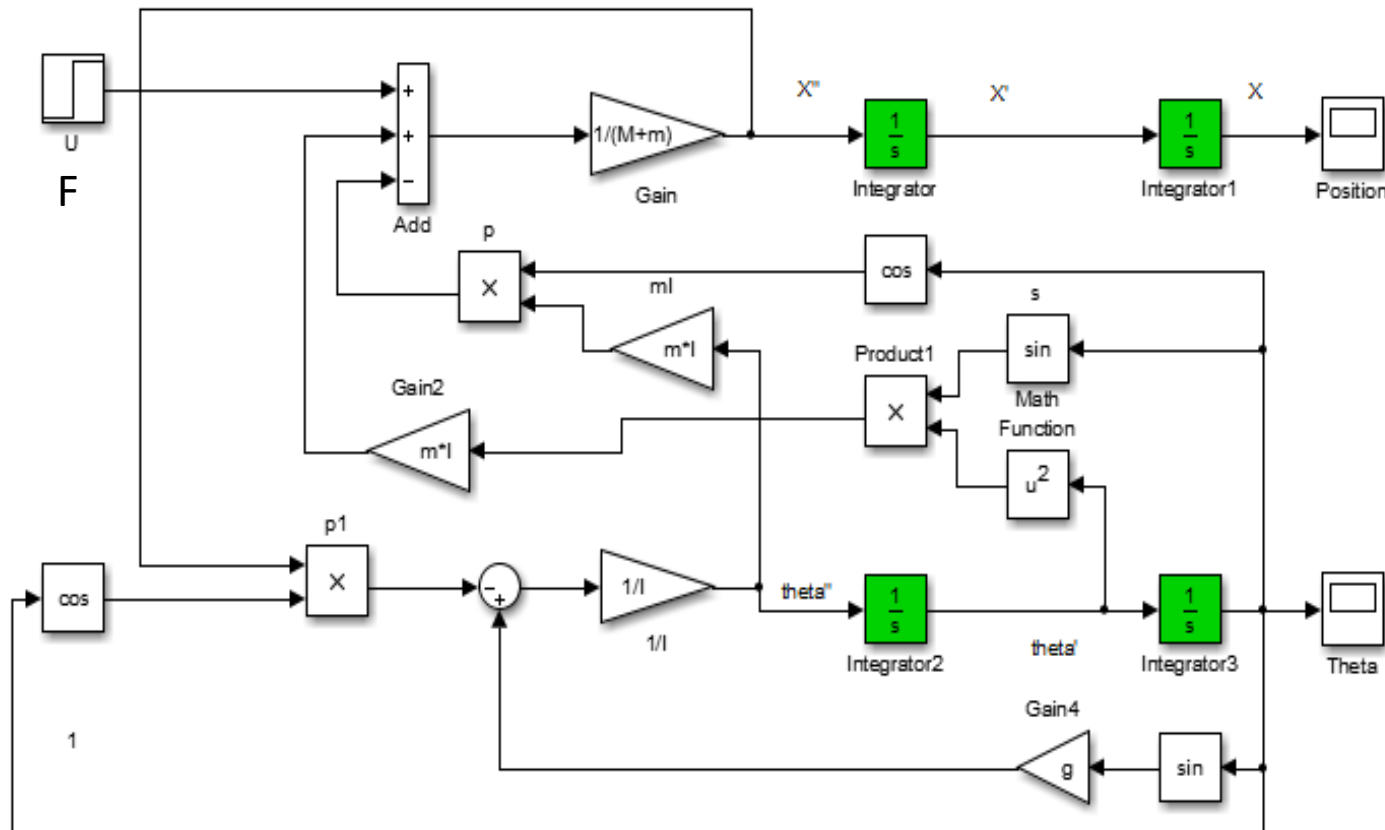
- $\sum T_G = I\ddot{\theta} = -Pl \sin \theta + Nl \cos \theta \rightarrow (N \cos \theta - P \sin \theta) = \frac{I\ddot{\theta}}{l}$  ..... (6)

- By substituting (6) in (5) yields

- $(ml^2 + I) \ddot{\theta} = ml(g \sin \theta - \ddot{x} \cos \theta)$

# PI Simulink Model

- The full Simulink block diagram is illustrated as shown in the figure below

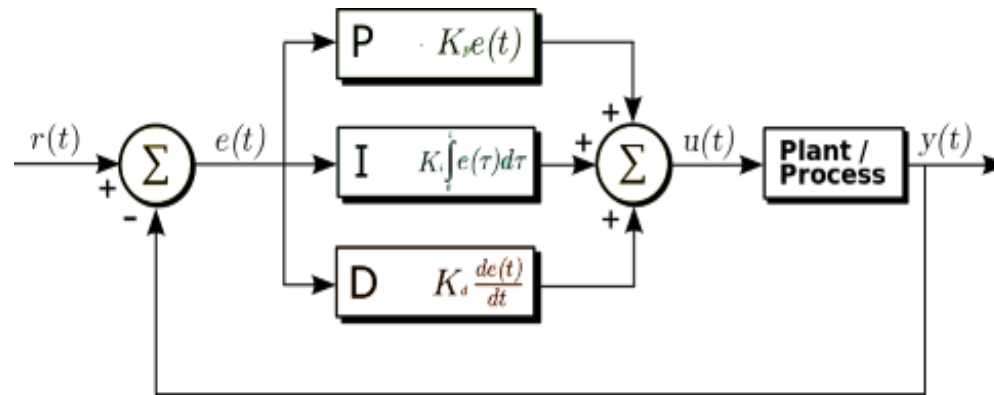


$$(M + m)\ddot{x} = F + ml\dot{\theta}^2 \sin \theta - ml\ddot{\theta} \cos \theta - b\dot{x}$$

$$(ml^2 + I)\ddot{\theta} = ml(g \sin \theta - \ddot{x} \cos \theta)$$

# PID Controller model

- The Proportional-Integral-Derivative (PID) controller is used in several robotics and industrial applications. The structure of the PID controller is illustrated in the figure below.

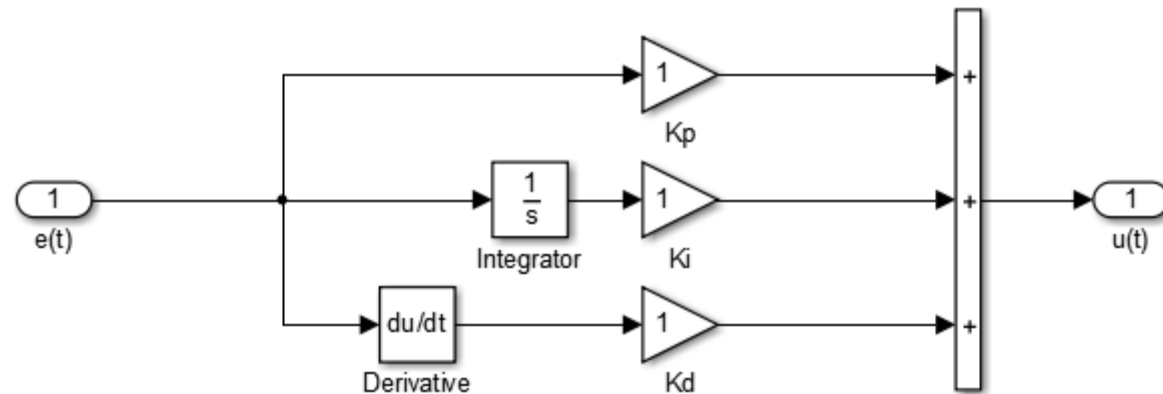


- The controller calculates the error value  $e(t)$  as the difference between the desired response  $r(t)$  and the measured variable  $y(t)$  (the output) and applies a correction on the **actuating signal** or the control variable  $u(t)$  based on proportional, integral, and derivative terms.

# PID Simulink model

$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{d e(t)}{dt}$$

- $e(t) = r(t) - y(t)$
- Where
- $K_p, K_i$ , and  $K_d$  are positive gains. The stability in the process is achieved by selecting the suitable values of these gains.
- The PID controller can be implemented by a Simulink model in MATLAB as shown in the figure below:



# Controlling IP by PID

- In the inverted pendulum problem, we need to achieve the stability of the system by controlling the position of the cart and the angle of the pendulum from the vertical axis. The whole system is actuated by one force applied on the cart. The pendulum angle  $\theta$  should maintain around zero while the cart is moving in horizontal direction  $x$ .
- We will use one PID to control the pendulum angle and another PID for the cart position to generate the sufficient force  $u(t)$  for the total system.

