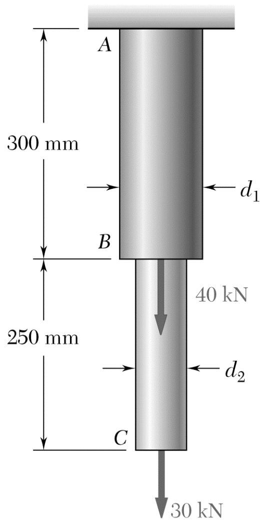
CHAPTER 1

PROBLEM 1.1



Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod AB and 150 MPa in rod BC, determine the smallest allowable values of d_1 and d_2.

SOLUTION

(*a*) <u>Rod *AB*</u>

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4} d_1^2} = \frac{4P}{\pi d_1^2}$$

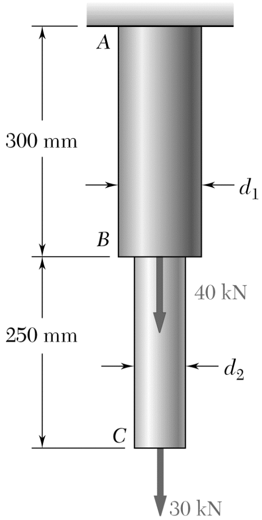
$$d_1 = \sqrt{\frac{4P}{\pi \sigma_{AB}}} = \sqrt{\frac{(4)(70 \times 10^3)}{\pi (175 \times 10^6)}} = 22.6 \times 10^{-3} \text{m} \qquad d_1 = 22.6 \text{ mm} \blacktriangleleft$$

(*b*) <u>Rod *BC*</u>

$$P = 30 \text{ kN} = 30 \times 10^3 \text{N}$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4} d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi \sigma_{BC}}} = \sqrt{\frac{(4)(30 \times 10^3)}{\pi (150 \times 10^6)}} = 15.96 \times 10^{-3} \text{m} \qquad d_2 = 15.96 \text{ mm} \blacktriangleleft$$



PROBLEM 1.2

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{N}$$

$$A = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{mm}^2 = 1.9635 \times 10^{-3} \text{m}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{70 \times 10^3}{1.9635 \times 10^{-3}} = 35.7 \times 10^6 \text{Pa}$$

$\sigma_{AB} = 35.7$ MPa ◀

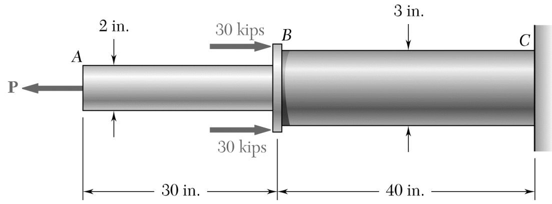
(*b*) Rod *BC*

$$P = 30 \text{ kN} = 30 \times 10^3 \text{N}$$

$$A = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{m}^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{30 \times 10^3}{706.86 \times 10^{-6}} = 42.4 \times 10^6 \text{Pa}$$

$\sigma_{BC} = 42.4$ MPa ◀



PROBLEM 1.3

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{3.1416}$$

$$= 0.31831P$$

$$A_{BC} = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$$

$$\sigma_{BC} = \frac{(2)(30) - P}{A_{AB}}$$

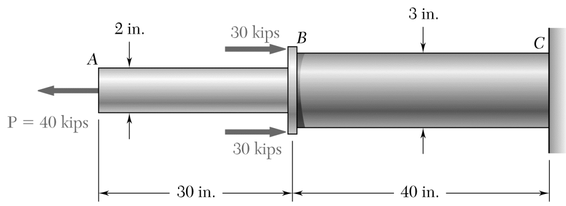
$$= \frac{60 - P}{7.0686} = 8.4883 - 0.14147P$$

Equating σ_{AB} to $2\sigma_{BC}$

$$0.31831P = 2(8.4883 - 0.14147P)$$

$P = 28.2$ kips ◀

PROBLEM 1.4



In Prob. 1.3, knowing that $P = 40$ kips, determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

PROBLEM 1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

SOLUTION

(*a*) Rod *AB*

$P = 40$ kips (tension)

$$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi(2)^2}{4} = 3.1416 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{40}{3.1416} \qquad \sigma_{AB} = 12.73 \text{ ksi} \blacktriangleleft$$

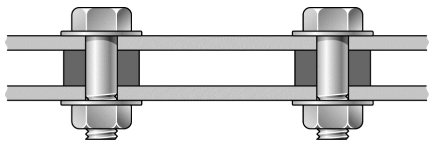
(*b*) Rod *BC*

$F = 40 - (2)(30) = -20$ kips, i.e., 20 kips compression.

$$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi(3)^2}{4} = 7.0686 \text{ in}^2$$

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-20}{7.0686} \qquad \sigma_{BC} = -2.83 \text{ ksi} \blacktriangleleft$$

PROBLEM 1.5



Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time, the spacer pushes that plate upward with a compressive force P_s in order to maintain equilibrium.

$$P_b = P_s$$

For the bolt, $\sigma_b = \dfrac{F_b}{A_b} = \dfrac{4P_b}{\pi d_b^2}$ or $P_b = \dfrac{\pi}{4}\sigma_b d_b^2$

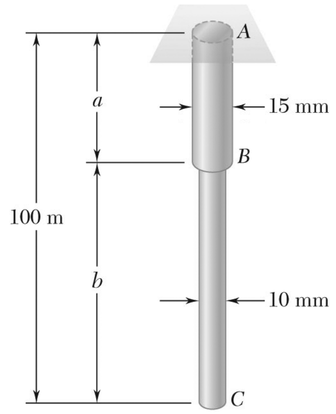
For the spacer, $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \dfrac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)} d_b = \sqrt{\left(1 + \frac{200}{130}\right)}(16) \qquad d_s = 25.2 \text{ mm} ◀$$

PROBLEM 1.6



Two brass rods AB and BC, each of uniform diameter, will be brazed together at B to form a nonuniform rod of total length 100 m, which will be suspended from a support at A as shown. Knowing that the density of brass is 8470 kg/m^3, determine (a) the length of rod AB for which the maximum normal stress in ABC is minimum, (b) the corresponding value of the maximum normal stress.

SOLUTION

Areas: $A_{AB} = \frac{\pi}{4}(15\text{ mm})^2 = 176.71\text{ mm}^2 = 176.71\times 10^{-6}\text{m}^2$

$$A_{BC} = \frac{\pi}{4}(10\text{ mm})^2 = 78.54\text{ mm}^2 = 78.54\times 10^{-6}\text{m}^2$$

From geometry, $b = 100 - a$

Weights: $W_{AB} = \rho g\, A_{AB}\ell_{AB} = (8470)(9.81)(176.71\times 10^{-6})a = 14.683a$

$$W_{BC} = \rho g\, A_{BC}\ell_{BC} = (8470)(9.81)(78.54\times 10^{-6})(100-a) = 652.59 - 6.526a$$

Normal stresses:

At A, $P_A = W_{AB} + W_{BC} = 652.59 + 8.157a$ (1)

$$\sigma_A = \frac{P_A}{A_{AB}} = 3.6930\times 10^6 + 46.160\times 10^3 a$$

At B, $P_B = W_{BC} = 652.59 - 6.526a$ (2)

$$\sigma_B = \frac{P_B}{A_{BC}} = 8.3090\times 10^6 - 83.090\times 10^3 a$$

(a) Length of rod AB. The maximum stress in ABC is minimum when $\sigma_A = \sigma_B$ or

$$4.6160\times 10^6 - 129.25\times 10^3 a = 0$$

$a = 35.71\text{ m}$ $\qquad \ell_{AB} = a = 35.7\text{ m}$ ◀

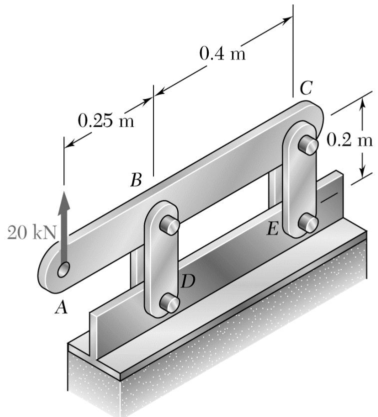
(b) Maximum normal stress.

$$\sigma_A = 3.6930\times 10^6 + (46.160\times 10^3)(35.71)$$

$$\sigma_B = 8.3090\times 10^6 - (83.090\times 10^3)(35.71)$$

$\sigma_A = \sigma_B = 5.34\times 10^6\text{ Pa}$ $\qquad \sigma = 5.34\text{ MPa}$ ◀

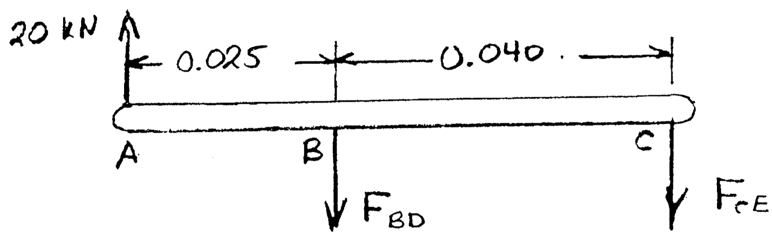
PROBLEM 1.7



Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$\Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{N} \qquad \text{Link } BD \text{ is in tension.}$$

$$\Sigma M_B = 0: \quad -(0.040)F_{CE} - (0.025)(20 \times 10^3) = 0$$

$$F_{CE} = -12.5 \times 10^3 \text{N} \qquad \text{Link } CE \text{ is in compression.}$$

Net area of one link for tension $= (0.008)(0.036 - 0.016) = 160 \times 10^{-6}\text{m}^2$.

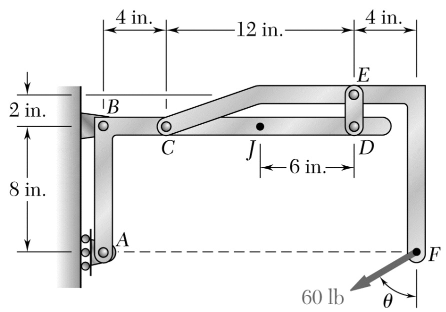
For two parallel links, $A_{\text{net}} = 320 \times 10^{-6}\text{m}^2$

(*a*) $\sigma_{BD} = \dfrac{F_{BD}}{A_{\text{net}}} = \dfrac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.56 \times 10^6$ $\qquad \sigma_{BD} = 101.6 \text{ MPa}$ ◀

Area for one link in compression $= (0.008)(0.036) = 288 \times 10^{-6}\text{m}^2$.

For two parallel links, $A = 576 \times 10^{-6}\text{m}^2$

(*b*) $\sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.70 \times 10^{-6}$ $\qquad \sigma_{CE} = -21.7 \text{ MPa}$ ◀

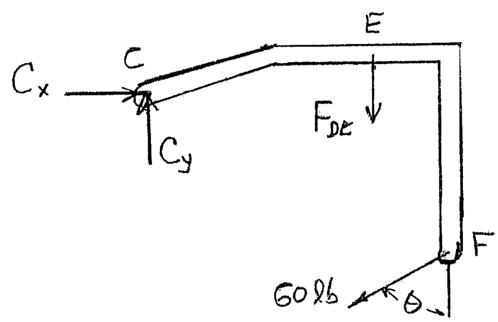


PROBLEM 1.8

Knowing that the link DE is $\frac{1}{8}$ in. thick and 1 in. wide, determine the normal stress in the central portion of that link when (*a*) $\theta = 0°$, (*b*) $\theta = 90°$.

SOLUTION

Use member CEF as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad -12\, F_{DE} - (8)(60 \sin\theta) - (16)(60 \cos\theta) = 0$$

$$F_{DE} = -40 \sin\theta - 80 \cos\theta \text{ lb.}$$

$$A_{DE} = (1)\left(\frac{1}{8}\right) = 0.125 \text{ in.}^2$$

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

(*a*) $\underline{\theta = 0}$: $F_{DE} = -80$ lb.

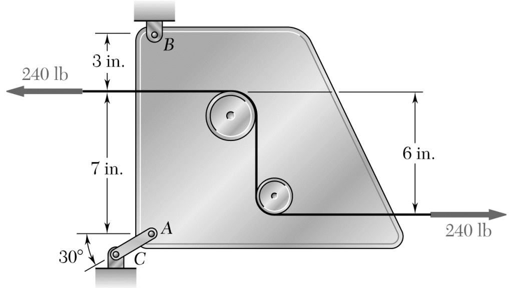
$$\sigma_{DE} = \frac{-80}{0.125}$$

$\sigma_{DE} = -640$ psi ◀

(*b*) $\underline{\theta = 90°}$: $F_{DE} = -40$ lb.

$$\sigma_{DE} = \frac{-40}{0.125}$$

$\sigma_{DE} = -320$ psi ◀

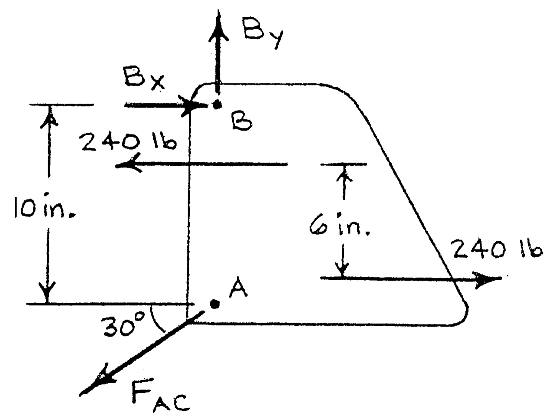


PROBLEM 1.9

Link AC has a uniform rectangular cross section $\frac{1}{16}$ in. thick and $\frac{1}{4}$ in. wide. Determine the normal stress in the central portion of the link.

SOLUTION

Free Body Diagram of Plate



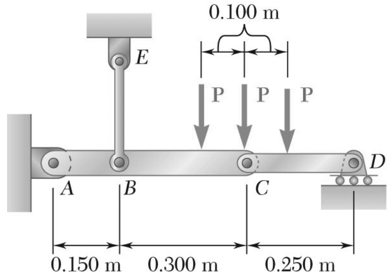
Note that the two 240-lb forces form a couple of moment

(240 lb)(6 in.) = 1440 lb · in.

$$+\circlearrowleft \Sigma M_B = 0: \quad 1440 \text{ lb} \cdot \text{in} - (F_{AC} \cos 30°)(10 \text{ in.}) = 0$$
$$F_{AC} = 166.277 \text{ lb.}$$

Area of link: $$A_{AC} = \left(\frac{1}{16} \text{ in.}\right)\left(\frac{1}{4} \text{ in.}\right) = 0.015625 \text{ in.}^2$$

Stress: $$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{166.277}{0.015625} = 10640 \text{ psi}$$ $\sigma_{AC} = 10.64$ ksi ◀

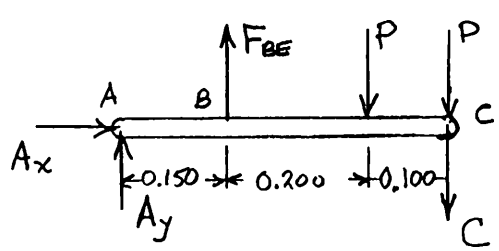


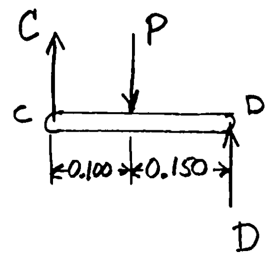
PROBLEM 1.10

Three forces, each of magnitude $P = 4$ kN, are applied to the mechanism shown. Determine the cross-sectional area of the uniform portion of rod *BE* for which the normal stress in that portion is +100 MPa.

SOLUTION

Draw free body diagrams of *AC* and *CD*.





<u>Free Body *CD*</u>: $\overset{\curvearrowleft}{+}\Sigma M_D = 0: \quad 0.150P - 0.250C = 0$

$$C = 0.6P$$

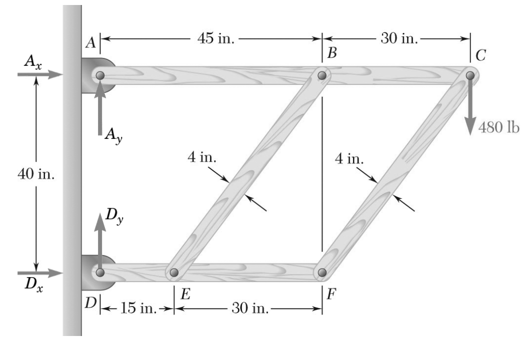
<u>Free Body *AC*</u>: $\overset{\curvearrowleft}{+} M_A = 0: \quad 0.150F_{BE} - 0.350P - 0.450P - 0.450C = 0$

$$F_{BE} = \frac{1.07}{0.150}P = 7.1333\,P = (7.133)(4\text{ kN}) = 28.533\text{ kN}$$

<u>Required area of *BE*</u>: $\sigma_{BE} = \dfrac{F_{BE}}{A_{BE}}$

$$A_{BE} = \frac{F_{BE}}{\sigma_{BE}} = \frac{28.533\times 10^3}{100\times 10^6} = 285.33\times 10^{-6}\text{m}^2$$

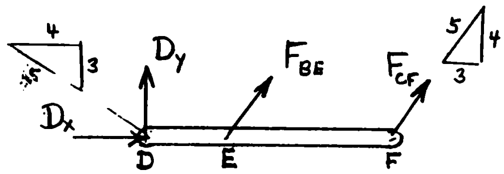
$$A_{BE} = 285\text{ mm}^2 \blacktriangleleft$$



PROBLEM 1.11

The frame shown consists of *four* wooden members, *ABC, DEF, BE,* and *CF*. Knowing that each member has a 2 × 4-in. rectangular cross section and that each pin has a 1/2-in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.

SOLUTION



Add support reactions to figure as shown.

Using entire frame as free body,

$$\Sigma M_A = 0: \quad 40\,D_x - (45 + 30)(480) = 0$$

$$D_x = 900 \text{ lb.}$$

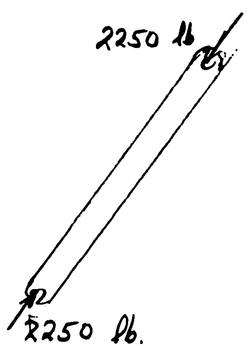
Use member DEF as free body.

Reaction at D must be parallel to F_{BE} and F_{CF}.

$$D_y = \frac{4}{3} D_x = 1200 \text{ lb.}$$

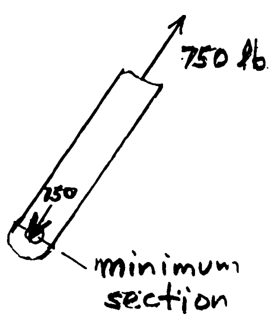
$$\Sigma M_F = 0: \quad -(30)\left(\frac{4}{5} F_{BE}\right) - (30 + 15)D_Y = 0$$

$$F_{BE} = -2250 \text{ lb.}$$



$$\Sigma M_E = 0: \quad (30)\left(\frac{4}{5} F_{CE}\right) - (15)D_Y = 0$$

$$F_{CE} = 750 \text{ lb.}$$



Stress in compression member *BE*

Area: $A = 2 \text{ in} \times 4 \text{ in} = 8 \text{ in}^2$

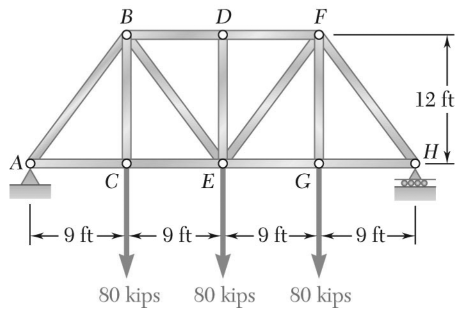
(*a*) $\sigma_{BE} = \dfrac{F_{BE}}{A} = \dfrac{-2250}{8}$ $\qquad \sigma_{BE} = -281 \text{ psi}$ ◀

Minimum section area occurs at pin.

$$A_{\min} = (2)(4.0 - 0.5) = 7.0 \text{ in}^2$$

Stress in tension member *CF*

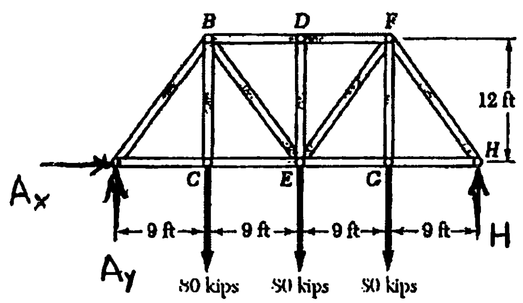
(*b*) $\sigma_{CF} = \dfrac{F_{CF}}{A_{\min}} = \dfrac{750}{7.0}$ $\qquad \sigma_{CF} = 107.1 \text{ psi}$ ◀



PROBLEM 1.12

For the Pratt bridge truss and loading shown, determine the average normal stress in member *BE*, knowing that the cross-sectional area of that member is 5.87 in^2.

SOLUTION



Use entire truss as free body.

$$+\circlearrowleft \Sigma M_H = 0: \quad (9)(80) + (18)(80) + (27)(80) - 36A_y = 0$$

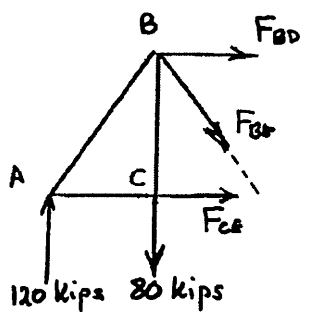
$$A_y = 120 \text{ kips}$$

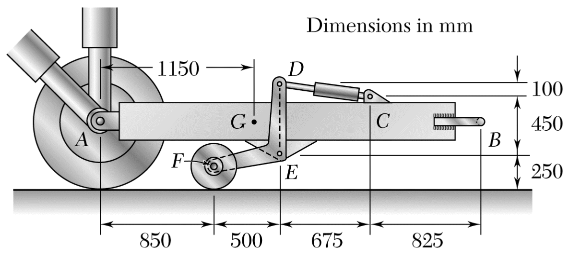
Use portion of truss to the left of a section cutting members *BD*, *BE*, and *CE*.

$$+\uparrow \Sigma F_y = 0: \quad 120 - 80 - \frac{12}{15}F_{BE} = 0 \qquad \therefore\ F_{BE} = 50 \text{ kips}$$

$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{50 \text{ kips}}{5.87 \text{ in}^2}$$

$$\sigma_{BE} = 8.52 \text{ ksi} \blacktriangleleft$$

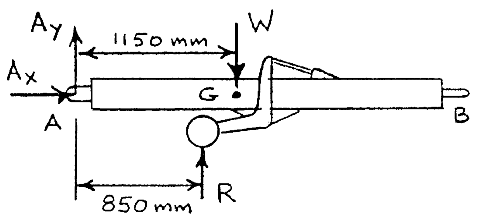




PROBLEM 1.13

An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

SOLUTION

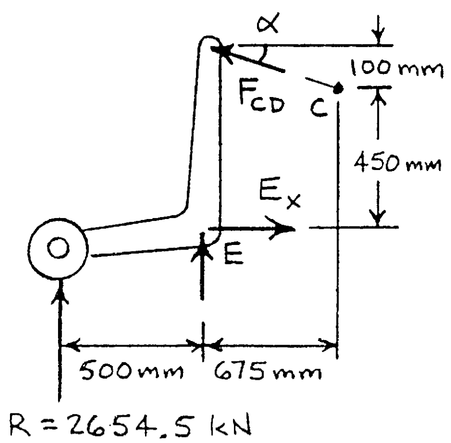


FREE BODY – ENTIRE TOW BAR:

$$W = (200\text{ kg})(9.81\text{ m/s}^2) = 1962.00\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 850R - 1150(1962.00\text{ N}) = 0$$

$$R = 2654.5\text{ N}$$



FREE BODY – BOTH ARM & WHEEL UNITS:

$$\tan\alpha = \frac{100}{675} \qquad \alpha = 8.4270°$$

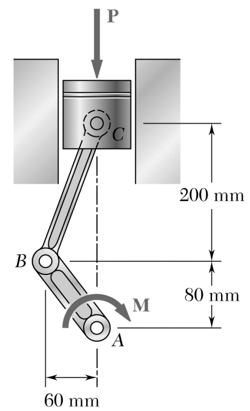
$$+\circlearrowleft \Sigma M_E = 0: \quad (F_{CD}\cos\alpha)(550) - R(500) = 0$$

$$F_{CD} = \frac{500}{550\cos 8.4270°}(2654.5\text{ N})$$

$$= 2439.5\text{ N} \quad \text{(comp.)}$$

$$\sigma_{CD} = -\frac{F_{CD}}{A_{CD}} = -\frac{2439.5\text{ N}}{\pi(0.0125\text{ m})^2}$$

$$= -4.9697\times 10^6\text{ Pa} \qquad \sigma_{CD} = -4.97\text{ MPa} \blacktriangleleft$$



PROBLEM 1.14

A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.

SOLUTION

Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

$$+\circlearrowleft \Sigma M_A = 0: \quad (0.280 \text{ m})H - 1500 \text{ N} \cdot \text{m} = 0$$

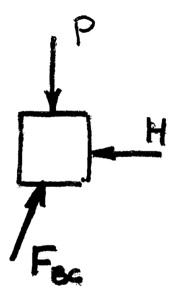
$$H = 5.3571 \times 10^3 \text{N}$$

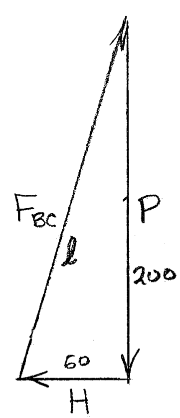
Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BE} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3 \text{N}$$

(*a*) $P = 17.86$ kN ◀





$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643 \times 10^3 \text{ N}$$

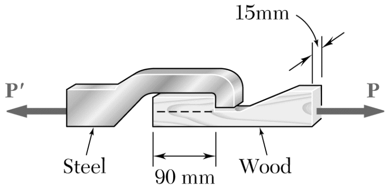
Rod *BC* is a compression member. Its area is

$$450 \text{ mm}^2 = 450 \times 10^{-6} \text{m}^2$$

Stress,

$$\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6 \text{Pa}$$

(*b*) $\sigma_{BC} = -41.4$ MPa ◀



PROBLEM 1.15

When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

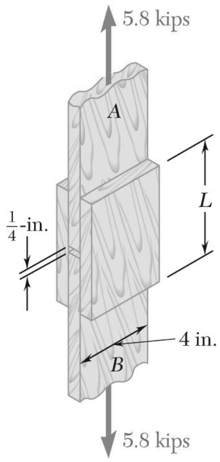
SOLUTION

Area being sheared: $A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{m}^2$

Force: $P = 8 \times 10^3 \text{ N}$

Shearing stress: $\tau = \dfrac{P}{A} - \dfrac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^6 \text{ Pa}$

$\tau = 5.93 \text{ MPa}$ ◀



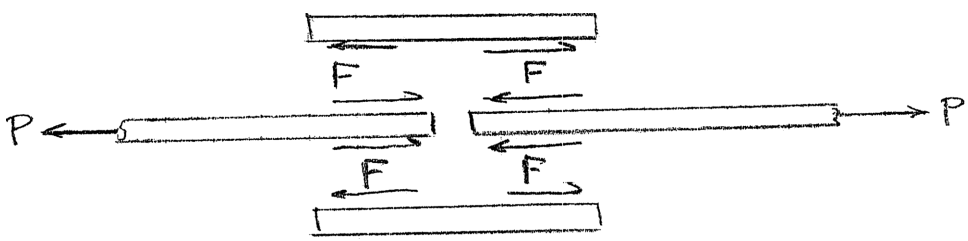
PROBLEM 1.16

The wooden members A and B are to be joined by plywood splice plates, that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be $\frac{1}{4}$ in., determine the smallest allowable length L if the average shearing stress in the glue is not to exceed 120 psi.

SOLUTION

There are four separate areas that are glued. Each of these areas transmits one half the 5.8 kip force. Thus

$$F = \frac{1}{2}P = \frac{1}{2}(5.8) = 2.9 \text{ kips} = 2900 \text{ lb.}$$



Let l = length of one glued area and w = 4 in. be its width.

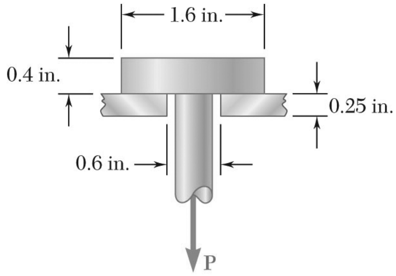
For each glued area, $A = lw$

Average shearing stress: $\tau = \dfrac{F}{A} = \dfrac{F}{lw}$

The allowable shearing stress is $\tau = 120$ psi

Solving for l, $l = \dfrac{F}{\tau w} = \dfrac{2900}{(120)(4)} = 6.0417$ in.

Total length L: $L = l + (\text{gap}) + l = 6.0417 + \dfrac{1}{4} + 6.0417$ $\quad L = 12.33$ in. ◀

	**PROBLEM 1.17** A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load **P** that can be applied to the rod.

SOLUTION

For steel:

$$A_1 = \pi dt = \pi(0.6)(0.4)$$
$$= 0.7540 \text{ in}^2$$

$$\tau_1 = \frac{P}{A} \therefore P = A_1\tau_1 = (0.7540)(18)$$
$$= 13.57 \text{ kips}$$

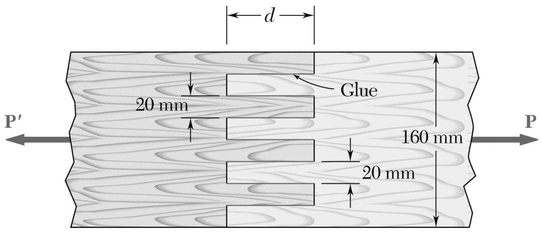
For aluminum:

$$A_2 = \pi dt = \pi(1.6)(0.25) = 1.2566 \text{ in}^2$$

$$\tau_2 = \frac{P}{A_2} \therefore P = A_2\tau_2 = (1.2566)(10) = 12.57 \text{ kips}$$

Limiting value of P is the smaller value, so $P = 12.57$ kips ◀

PROBLEM 1.18



Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.

SOLUTION

Seven surfaces carry the total load $P = 7.6 \text{ kN} = 7.6 \times 10^3$.

Let $t = 22$ mm.

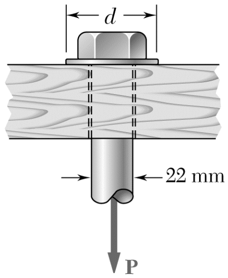
Each glue area is $A = dt$

$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3}\,\text{m}^2$$

$$= 1.32404 \times 10^3\,\text{mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2 \qquad d = 60.2 \text{ mm} \blacktriangleleft$$

PROBLEM 1.19



The load **P** applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter *d* of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

SOLUTION

Steel rod: $A = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6}\text{m}^2$

$$\sigma = 35 \times 10^6 \text{Pa}$$

$$P = \sigma A = (35 \times 10^6)(380.13 \times 10^{-6})$$

$$= 13.305 \times 10^3 \text{N}$$

Washer: $\sigma_b = 5 \times 10^6 \text{Pa}$

Required bearing area:

$$A_b = \frac{P}{\sigma_b} = \frac{13.305 \times 10^3}{5 \times 10^6} = 2.6609 \times 10^{-3}\text{m}^2$$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$

$$d^2 = d_i^2 + \frac{4A_b}{\pi}$$

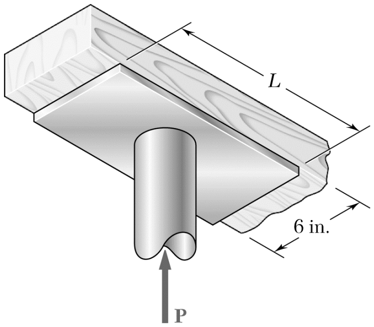
$$= (0.025)^2 + \frac{(4)(2.6609 \times 10^{-3})}{\pi}$$

$$= 4.013 \times 10^{-3}\text{m}^2$$

$$d = 63.3 \times 10^{-3}\text{m}$$

$d = 63.3$ mm ◀

PROBLEM 1.20



The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

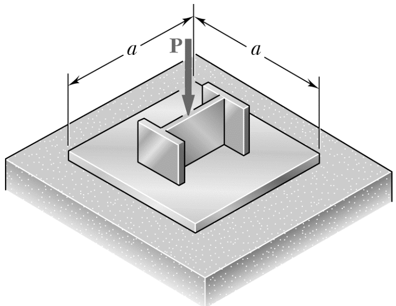
SOLUTION

Bearing area: $A_b = Lw$

$$\sigma_b = \frac{P}{A_b} = \frac{P}{Lw}$$

$$L = \frac{P}{\sigma_b w} = \frac{20 \times 10^3}{(400)(6)} = 8.33 \text{ in.}$$

$L = 8.33$ in. ◀



PROBLEM 1.21

An axial load **P** is supported by a short W8 × 40 column of cross-sectional area $A = 11.7\,\text{in.}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side *a* of the plate that will provide the most economical and safe design.

SOLUTION

For the column $\sigma = \dfrac{P}{A}$ or

$$P = \sigma A = (30)(11.7) = 351 \text{ kips}$$

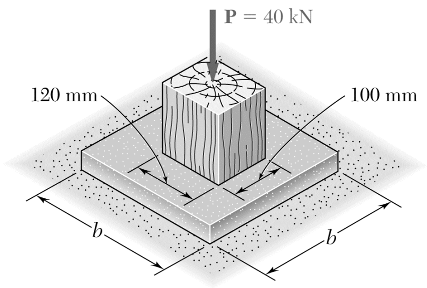
For the $a \times a$ plate, $\sigma = 3.0$ ksi

$$A = \frac{P}{\sigma} = \frac{351}{3.0} = 117 \text{ in}^2$$

Since the plate is square, $A = a^2$

$$a = \sqrt{A} = \sqrt{117}$$

$a = 10.82$ in. ◀



PROBLEM 1.22

A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

SOLUTION

(*a*) Bearing stress on concrete footing.

$$P = 40\,\text{kN} = 40 \times 10^3\,\text{N}$$

$$A = (100)(120) = 12 \times 10^3\,\text{mm}^2 = 12 \times 10^{-3}\,\text{m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.333 \times 10^6\,\text{Pa}$$

3.33 MPa ◀

(*b*) Footing area. $P = 40 \times 10^3\,\text{N}$ $\sigma = 145\text{ kPa} = 45 \times 10^3\,\text{Pa}$

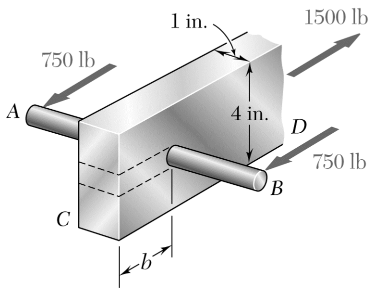
$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586\text{ m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525\text{ m}$$

$b = 525$ mm ◀

PROBLEM 1.23



A $\frac{5}{8}$-in.-diameter steel rod *AB* is fitted to a round hole near end *C* of the wooden member *CD*. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance *b* for which the average shearing stress is 100 psi on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

SOLUTION

(*a*) Maximum normal stress in the wood

$$A_{\text{net}} = (1)\left(4 - \frac{5}{8}\right) = 3.375 \text{ in.}^2$$

$$\sigma = \frac{P}{A_{\text{net}}} = \frac{1500}{3.375} = 444 \text{ psi} \qquad \sigma = 444 \text{ psi} \blacktriangleleft$$

(*b*) Distance *b* for $\tau = 100$ psi

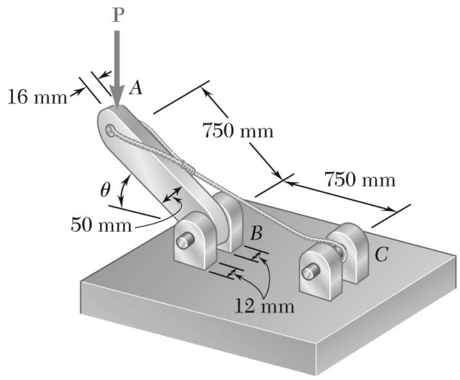
For sheared area see dotted lines.

$$\tau = \frac{P}{A} = \frac{P}{2bt}$$

$$b = \frac{P}{2t\tau} = \frac{1500}{(2)(1)(100)} = 7.50 \text{ in.} \qquad b = 7.50 \text{ in.} \blacktriangleleft$$

(*c*) Average bearing stress on the wood

$$\sigma_b = \frac{P}{A_b} = \frac{P}{dt} = \frac{1500}{\left(\frac{5}{8}\right)(1)} = 2400 \text{ psi} \qquad \sigma_b = 2400 \text{ psi} \blacktriangleleft$$

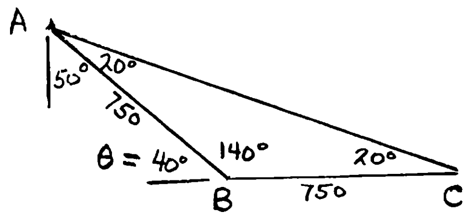


PROBLEM 1.24

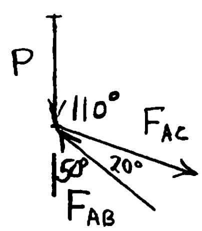
Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at *B* if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member *AB* at *B*, (*c*) the corresponding average bearing stress in each of the support brackets at *B*.

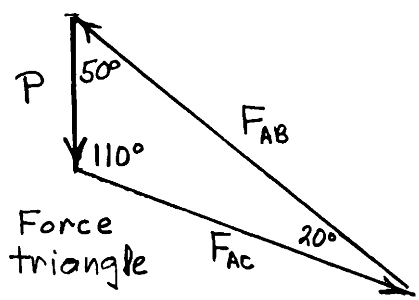
SOLUTION

Geometry: Triangle *ABC* is an isoseles triangle with angles shown here.



Use joint A as a free body.





Law of sines applied to force triangle

$$\frac{P}{\sin 20°} = \frac{F_{AB}}{\sin 110°} = \frac{F_{AC}}{\sin 50°}$$

$$F_{AB} = \frac{P \sin 110°}{\sin 20°}$$

$$= \frac{(9)\sin 110°}{\sin 20°} = 24.73 \text{ kN}$$

PROBLEM 1.24 *(Continued)*

(*a*) Allowable pin diameter.

$$\tau = \frac{F_{AB}}{2A_P} = \frac{F_{AB}}{2\frac{\pi}{4}d^2} = \frac{2F_{AB}}{\pi d^2} \quad \text{where } F_{AB} = 24.73 \times 10^3\,\text{N}$$

$$d^2 = \frac{2F_{AB}}{\pi\tau} = \frac{(2)(24.73 \times 10^3)}{\pi(120 \times 10^6)} = 131.18 \times 10^{-6}\,\text{m}^2$$

$$d = 11.45 \times 10^{-3}\,\text{m}$$

11.45 mm ◀

(*b*) Bearing stress in *AB* at *A*.

$$A_b = td = (0.016)(11.45 \times 10^{-3}) = 183.26 \times 10^{-6}\,\text{m}^2$$

$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{24.73 \times 10^3}{183.26 \times 10^{-6}} = 134.9 \times 10^6$$

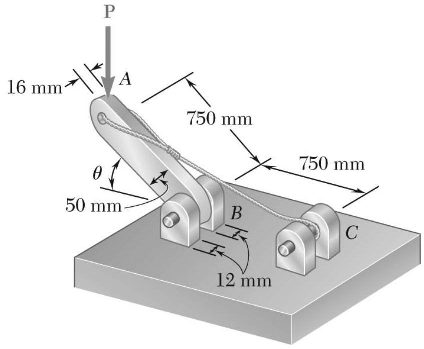
134.9 MPa ◀

(*c*) Bearing stress in support brackets at *B*.

$$A = td = (0.012)(11.45 \times 10^{-3}) = 137.4 \times 10^{-6}\,\text{m}^2$$

$$\sigma_b = \frac{\frac{1}{2}F_{AB}}{A} = \frac{(0.5)(24.73 \times 10^3)}{137.4 \times 10^{-6}} = 90.0 \times 10^6$$

90.0 MPa ◀

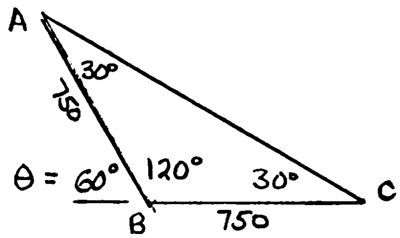


PROBLEM 1.25

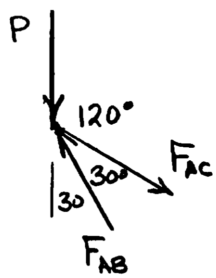
Determine the largest load **P** which may be applied at A when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at B must not exceed 120 MPa and that the average bearing stress in member AB and in the bracket at B must not exceed 90 MPa.

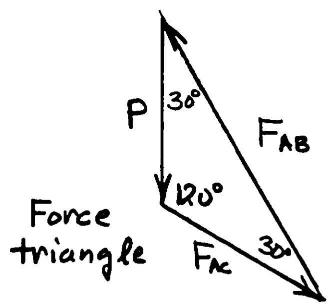
SOLUTION

Geometry: Triangle ABC is an isosceles triangle with angles shown here.



Use joint A as a free body.





Law of sines applied to force triangle

$$\frac{P}{\sin 30°} = \frac{F_{AB}}{\sin 120°} = \frac{F_{AC}}{\sin 30°}$$

$$P = \frac{F_{AB} \sin 30°}{\sin 120°} = 0.57735\, F_{AB}$$

$$P = \frac{F_{AC} \sin 30°}{\sin 30°} = F_{AC}$$

PROBLEM 1.25 *(Continued)*

If shearing stress in pin at B is critical,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\,\text{m}^2$$

$$F_{AB} = 2A\tau = (2)(78.54 \times 10^{-6})(120 \times 10^6) = 18.850 \times 10^3\,\text{N}$$

If bearing stress in member AB at bracket at A is critical,

$$A_b = td = (0.016)(0.010) = 160 \times 10^{-6}\,\text{m}^2$$

$$F_{AB} = A_b\sigma_b = (160 \times 10^{-6})(90 \times 10^6) = 14.40 \times 10^3\,\text{N}$$

If bearing stress in the bracket at B is critical,

$$A_b = 2td = (2)(0.012)(0.010) = 240 \times 10^{-6}\,\text{m}^2$$

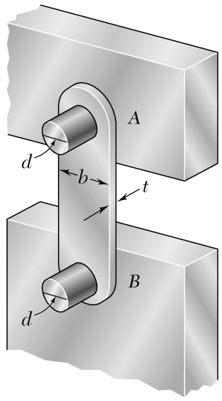
$$F_{AB} = A_b\sigma_b = (240 \times 10^{-6})(90 \times 10^6) = 21.6 \times 10^3\,\text{N}$$

Allowable F_{AB} is the smallest, i.e., $14.40 \times 10^3\text{N}$

Then from Statics $\quad P_{\text{allow}} = (0.57735)(14.40 \times 10^3)$

$$= 8.31 \times 10^3\,\text{N}$$

8.31 kN ◀



PROBLEM 1.26

Link AB, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (*a*) the diameter d of the pins, (*b*) the average bearing stress in the link.

SOLUTION

Rod AB is in compression.

$$A = bt \quad \text{where} \quad b = 50 \text{ mm} \quad \text{and} \quad t = 6 \text{ mm}$$

$$A = (0.050)(0.006) = 300 \times 10^{-6} \text{m}^2$$

$$P = -\sigma A = -(-140 \times 10^6)(300 \times 10^{-6})$$

$$= 42 \times 10^3 \text{N}$$

For the pin,

$$A_p = \frac{\pi}{4} d^2 \quad \text{and} \quad \tau = \frac{P}{A_p}$$

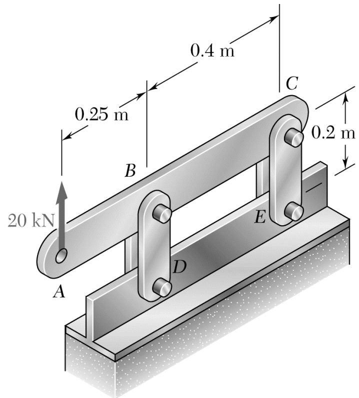
$$A_p = \frac{P}{\tau} = \frac{42 \times 10^3}{80 \times 10^6} = 525 \times 10^{-6} \text{m}^2$$

(*a*) Diameter d

$$d = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{(4)(525 \times 10^{-6})}{\pi}} = 2.585 \times 10^{-3} \text{m} \qquad d = 25.9 \text{ mm} \blacktriangleleft$$

(*b*) Bearing stress

$$\sigma_b = \frac{P}{dt} = \frac{42 \times 10^3}{(25.85 \times 10^{-3})(0.006)} = 271 \times 10^6 \text{ Pa} \qquad \sigma_b = 271 \text{ MPa} \blacktriangleleft$$



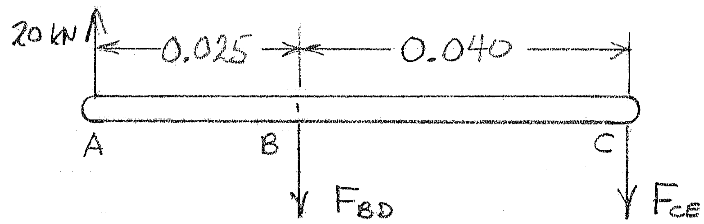
PROBLEM 1.27

For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10×50-mm uniform rectangular cross section.

PROBLEM 1.7 Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{N}$$

(*a*) Shear pin at *B* $\qquad \tau = \dfrac{F_{BD}}{2A}$ for double shear,

where $\qquad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{m}^2$

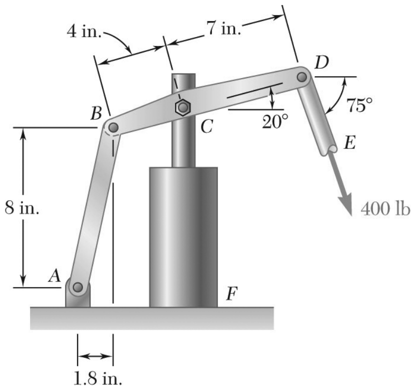
$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad \tau = 80.8 \text{ MPa} \blacktriangleleft$$

(*b*) Bearing: link *BD* $\qquad A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad \sigma_b = 127.0 \text{ MPa} \blacktriangleleft$$

(*c*) Bearing in *ABC* at *B* $\qquad A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{m}^2$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad \sigma_b = 203 \text{ MPa} \blacktriangleleft$$



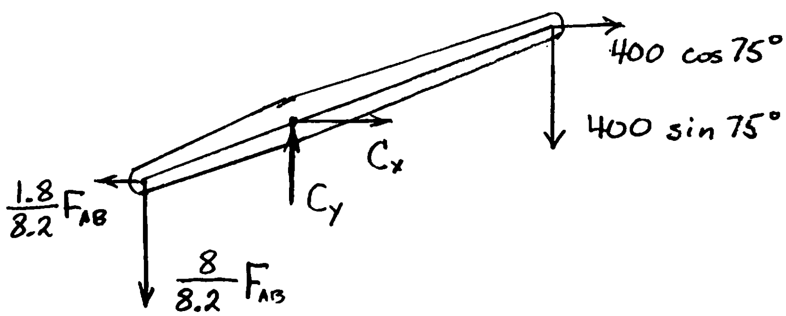
PROBLEM 1.28

The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is $\frac{5}{8}$ in. thick and is connected to the vertical rod by a $\frac{3}{8}$-in.-diameter bolt. Determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.

SOLUTION

Use member *BCD* as a free body, and note that *AB* is a two force member.

$$l_{AB} = \sqrt{8^2 + 1.8^2} = 8.2 \text{ in.}$$



$$+\circlearrowleft \Sigma M_C = 0: \quad (4\cos 20°)\left(\frac{8}{8.2}F_{AB}\right) - (4\sin 20°)\left(\frac{1.8}{8.2}F_{AB}\right)$$

$$-(7\cos 20°)(400\sin 75°) - (7\sin 20°)(400\cos 75°) = 0$$

$$3.36678F_{AB} - 2789.35 = 0 \qquad \therefore\ F_{AB} = 828.49 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{1.8}{8.2}F_{AB} + C_x + 400\cos 75° = 0$$

$$C_x = \frac{(1.8)(828.49)}{8.2} - 400\cos 75° = 78.34 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{8}{8.2}F_{AB} + C_y - 400\sin 75° = 0$$

$$C_y = \frac{(8)(828.49)}{8.2} + 400\sin 75° = 1194.65 \text{ lb}$$

$$C = \sqrt{C_x^2 + C_y^2} = 1197.2 \text{ lb}$$

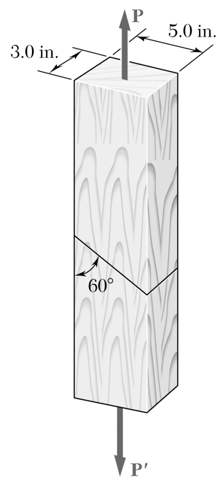
PROBLEM 1.28 *(Continued)*

(*a*) Shearing stress in the bolt: $P = 1197.2$ lb $\quad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}\left(\dfrac{3}{8}\right)^2 = 0.11045 \text{ in}^2$

$$\tau = \frac{P}{A} = \frac{1197.2}{0.11045} = 10.84 \times 10^3 \text{psi} = \qquad 10.84 \text{ ksi} \blacktriangleleft$$

(*b*) Bearing stress at *C* in member *BCD*: $P = 1197.2$ lb $\quad A_b = dt = \left(\dfrac{3}{8}\right)\left(\dfrac{5}{8}\right) = 0.234375 \text{ in}^2$

$$\sigma_b = \frac{P}{A_b} = \frac{1197.2}{0.234375} = 5.11 \times 10^3 \text{psi} = \quad 5.11 \text{ ksi} \blacktriangleleft$$



PROBLEM 1.29

The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

SOLUTION

$$P = 1400 \text{ lb} \qquad \theta = 90° - 60° = 30°$$

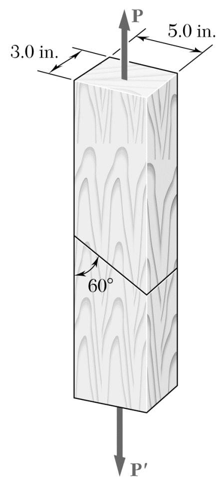
$$A_0 = (5.0)(3.0) = 15 \text{ in}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(1400)(\cos 30°)^2}{15}$$

$\sigma = 70.0$ psi ◀

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1400)\sin 60°}{(2)(15)}$$

$\tau = 40.4$ psi ◀



PROBLEM 1.30

Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding shearing stress in the splice.

SOLUTION

$$A_0 = (5.0)(3.0) = 15 \text{ in}^2$$

$$\theta = 90° - 60° = 30°$$

$$\sigma = \frac{P\cos^2\theta}{A_0}$$

(*a*)

$$P = \frac{\sigma A_0}{\cos^2\theta} = \frac{(75)(15)}{\cos^2 30°} = 1500 \text{ lb}$$

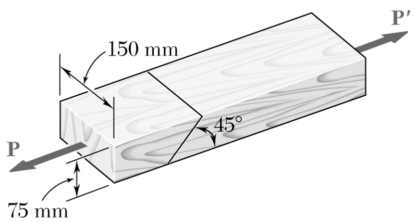
$P = 1.500$ kips ◀

(*b*)

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1500)\sin 60°}{(2)(15)}$$

$\tau = 43.3$ psi ◀

PROBLEM 1.31



Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

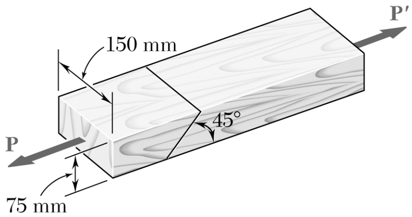
$$P = 11 \text{ kN} = 11 \times 10^3 \text{N}$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(11\times 10^3)\cos^2 45°}{11.25\times 10^{-3}} = 489\times 10^3 \text{Pa} \qquad \sigma = 489 \text{ kPa} \blacktriangleleft$$

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(11\times 10^3)(\sin 90°)}{(2)(11.25\times 10^{-3})} = 489\times 10^3 \text{Pa} \qquad \tau = 489 \text{ kPa} \blacktriangleleft$$

PROBLEM 1.32



Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding tensile stress in the splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$\tau = 620 \text{ kPa} = 620 \times 10^3 \text{Pa}$$

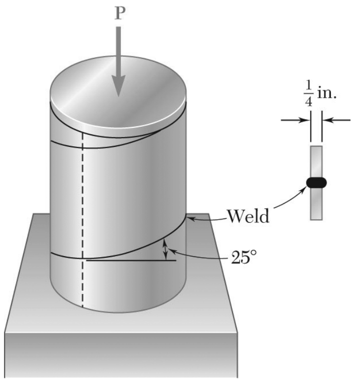
$$\tau = \frac{P \sin 2\theta}{2A_0}$$

(*a*) $$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(11.25 \times 10^{-3})(620 \times 10^3)}{\sin 90°}$$

$$= 13.95 \times 10^3 \text{N} \qquad P = 13.95 \text{ kN} \blacktriangleleft$$

(*b*) $$\sigma = \frac{P \cos^2 \theta}{A_0} = \frac{(13.95 \times 10^3)(\cos 45°)^2}{11.25 \times 10^{-3}}$$

$$= 620 \times 10^3 \text{Pa} \qquad \sigma = 620 \text{ kPa} \blacktriangleleft$$



PROBLEM 1.33

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 12\,\text{ksi}$ and $\tau = 7.2\,\text{ksi}$, determine the magnitude P of the largest axial force that can be applied to the pipe.

SOLUTION

$$d_o = 12\text{ in.} \quad r_o = \frac{1}{2}d_o = 6\text{ in.}$$

$$r_i = r_o - t = 6 - 0.25 = 5.75\text{ in.}$$

$$A_0 = \pi(r_o^2 - r_i^2) = \pi(6^2 - 5.75^2) = 9.228\text{ in}^2$$

$$\theta = 25°$$

Based on $|\sigma| = 12$ ksi: $\sigma = \dfrac{P}{A_0}\cos^2\theta$

$$P = \frac{A_0\sigma}{\cos^2\theta} = \frac{(9.228)(12\times10^3)}{\cos^2 25°} = 134.8\times10^3\text{ lb}$$

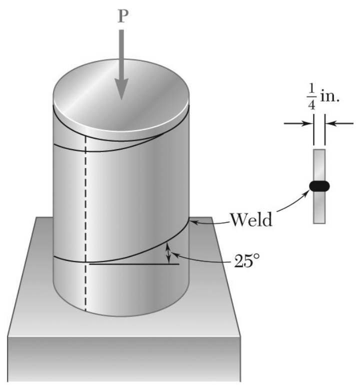
Based on $|\tau| = 7.2$ ksi: $\tau = \dfrac{P}{2A_0}\sin 2\theta$

$$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(9.288)(7.2\times10^3)}{\sin 50°} = 174.5\times10^3\text{ lb}$$

The smaller calculated value of P is the allowable value.

$$P = 134.8\times10^3\text{ lb}$$

$P = 134.8$ kips ◀



PROBLEM 1.34

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that a 66 kip axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

SOLUTION

$$d_o = 12 \text{ in.} \quad r_o = \frac{1}{2} d_o = 6 \text{ in.}$$

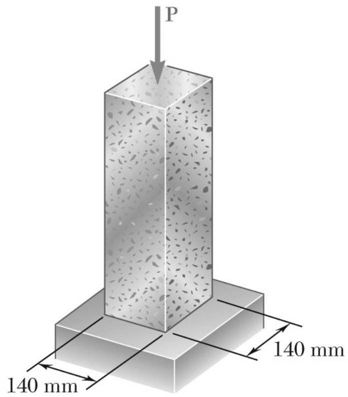
$$r_i = r_o - t = 6 - 0.25 = 5.75 \text{ in.}$$

$$A_0 = \pi(r_o^2 - r_i^2) = \pi(6^2 - 5.75^2) = 9.228 \text{ in}^2$$

$$\theta = 25°$$

Normal stress: $$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(66 \times 10^3)\cos^2 25°}{9.228} = 5875 \text{ psi}$$ $\sigma = 5.87$ ksi ◀

Shearing stress: $$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(66 \times 10^3)\sin 50°}{(2)(9.228)} = 2739 \text{ psi}$$ $\tau = 2.74$ ksi ◀



PROBLEM 1.35

A 1060-kN load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

SOLUTION

$$A_0 = (140 \text{ mm})(140 \text{ mm}) = 19.6 \times 10^3 \text{mm}^2 = 19.6 \times 10^{-3} \text{m}^2$$

$$P = 1060 \times 10^3 \text{N}$$

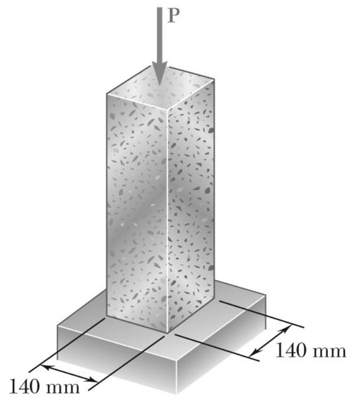
$$\sigma = \frac{P}{A_0} \cos^2 \theta = \frac{1060 \times 10^3}{19.6 \times 10^{-3}} \cos^2 \theta = 54.082 \times 10^6 \cos^2 \theta$$

(*a*) Maximum tensile stress = 0 at $\theta = 90°$.

Maximum compressive stress = 54.1×10^6 at $\theta = 0°$. $|\sigma|_{\text{max}} = 54.1 \text{ MPa}$ ◀

(*b*) Maximum shearing stress:

$$\tau_{\text{max}} = \frac{P}{2A_0} = \frac{1060 \times 10^3}{(2)(19.6 \times 10^{-3})} = 27.0 \times 10^6 \text{ Pa at } \theta = 45°. \qquad \tau_{\text{max}} = 27.0 \text{ MPa} ◀$$



PROBLEM 1.36

A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 18 MPa, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on that surface, (*d*) the maximum value of the normal stress in the block.

SOLUTION

$$A_0 = (140\text{ mm})(140\text{ mm}) = 19.6\times 10^3\text{ mm}^2 = 19.6\times 10^{-3}\text{ m}^2$$

$$\tau_{\max} = 18\text{ MPa} = 18\times 10^6\text{ Pa}$$

$$\theta = 45°\text{ for plane of }\tau_{\max}$$

(*a*) <u>Magnitude of *P*</u>. $\tau_{\max} = \dfrac{|P|}{2A_0}$ so $P = 2A_0\,\tau_{\max}$

$$P = (2)(19.6\times 10^{-3})(18\times 10^6) = 705.6\times 10^3\text{ N}$$

$P = 706\text{ kN}$ ◀

(*b*) <u>Orientation</u>. $\sin 2\theta$ is maximum when $2\theta = 90°$ $\qquad \theta = 45°$ ◀

(*c*) <u>Normal stress at $\theta = 45°$</u>.

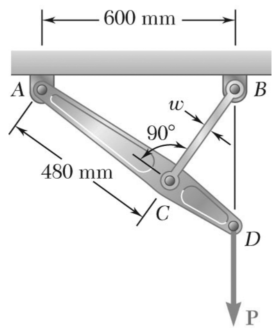
$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(705.8\times 10^3)\cos^2 45°}{19.6\times 10^{-3}} = 18.00\times 10^6\text{Pa}$$

$\sigma = 18.00\text{ MPa}$ ◀

(*d*) Maximum normal stress: $\sigma_{\max} = \dfrac{P}{A_0}$

$$\sigma_{\max} = \frac{705.8\times 10^3}{19.6\times 10^{-3}} = 36.0\times 10^6\text{Pa}$$

$\sigma_{\max} = 36.0\text{ MPa}$ (compression) ◀

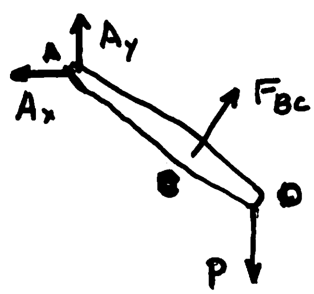


PROBLEM 1.37

Link BC is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load **P**?

SOLUTION

Use bar ACD as a free body and note that member BC is a two-force member.



$$\Sigma M_A = 0:$$

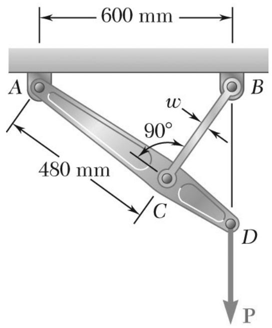
$$(480)F_{BC} - (600)P = 0$$

$$F_{BC} = \frac{600}{480}P = \frac{(600)(16 \times 10^3)}{480} = 20 \times 10^3 \text{N}$$

Ultimate load for member BC: $F_U = \sigma_U A$

$$F_U = (480 \times 10^6)(0.006)(0.025) = 72 \times 10^3 \text{N}$$

Factor of safety: $\text{F.S.} = \dfrac{F_U}{F_{BC}} = \dfrac{72 \times 10^3}{20 \times 10^3}$ $\qquad$ $\text{F.S.} = 3.60$ ◀

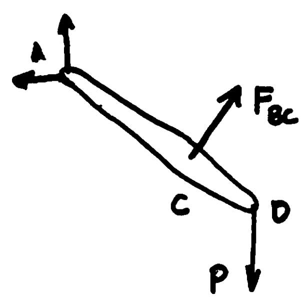


PROBLEM 1.38

Link *BC* is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width *w* if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3?

SOLUTION

Use bar *ACD* as a free body and note that member *BC* is a two-force member.



$$\Sigma M_A = 0:$$

$$(480)F_{BC} - 600P = 0$$

$$F_{BC} = \frac{600P}{480} = \frac{(600)(20\times 10^3)}{480} = 25\times 10^3\,\text{N}$$

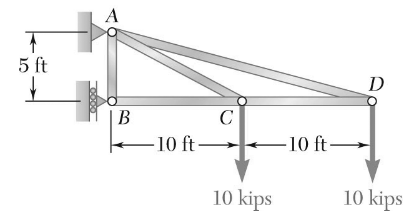
For a factor of safety F.S. = 3, the ultimate load of member *BC* is

$$F_U = (\text{F.S.})(F_{BC}) = (3)(25\times 10^3) = 75\times 10^3\,\text{N}$$

But $F_U = \sigma_U A \quad \therefore A = \dfrac{F_U}{\sigma_U} = \dfrac{75\times 10^3}{450\times 10^6} = 166.67\times 10^{-6}\text{m}^2$

For a rectangular section $\quad A = wt$ or $w = \dfrac{A}{t} = \dfrac{166.67\times 10^{-6}}{0.006}$

$w = 27.8\times 10^{-3}\text{m or } 27.8\text{ mm}$ ◀



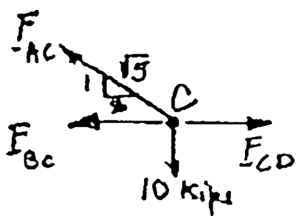
PROBLEM 1.39

A $\frac{3}{4}$-in.-diameter rod made of the same material as rods AC and AD in the truss shown was tested to failure and an ultimate load of 29 kips was recorded. Using a factor of safety of 3.0, determine the required diameter (*a*) of rod AC, (*b*) of rod AD.

SOLUTION

<u>Forces in AC and AD.</u>

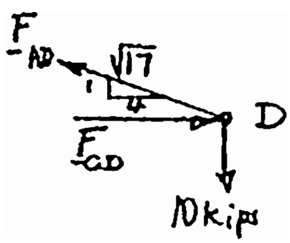
<u>Joint C</u>:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{AC} - 10 \text{ kips} = 0$$

$$F_{AC} = 22.36 \text{ kips } T$$

<u>Joint D</u>:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{17}} F_{AD} - 10 \text{ kips} = 0$$

$$F_{AD} = 41.23 \text{ kips } T$$

<u>Ultimate stress</u>. From test on $\frac{3}{4}$-in. rod:

$$\sigma_U = \frac{P_U}{A} = \frac{29 \text{ kips}}{\frac{1}{4}\pi(\frac{3}{4})^2} = 65.64 \text{ ksi}$$

<u>Allowable stress</u>:

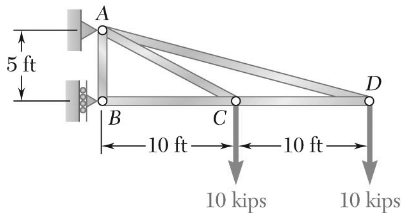
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{65.64 \text{ ksi}}{3.0} = 21.88 \text{ ksi}$$

(*a*) <u>Diameter of rod AC.</u>

$$\sigma_{\text{all}} = \frac{F_{AC}}{\frac{1}{4}\pi d^2} \qquad d^2 = \frac{4F_{AC}}{\pi\sigma_{\text{all}}} = \frac{4(22.36)}{\pi(21.88)} = 1.301 \qquad d = 1.141 \text{ in.} \blacktriangleleft$$

(*b*) <u>Diameter of rod AD.</u>

$$d^2 = \frac{4F_{AD}}{\pi\sigma_{\text{all}}} = \frac{4(41.23)}{\pi(21.88)} = 2.399 \qquad d = 1.549 \text{ in.} \blacktriangleleft$$



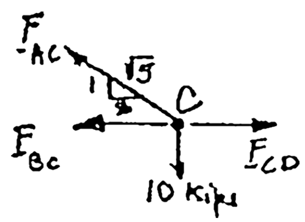
PROBLEM 1.40

In the truss shown, members *AC* and *AD* consist of rods made of the same metal alloy. Knowing that *AC* is of 1-in. diameter and that the ultimate load for that rod is 75 kips, determine (*a*) the factor of safety for *AC*, (*b*) the required diameter of *AD* if it is desired that both rods have the same factor of safety.

SOLUTION

Forces in AC and *AD*.

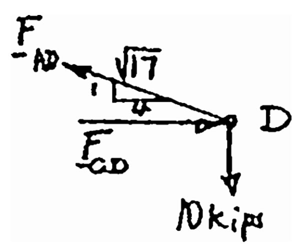
Joint C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{AC} - 10 \text{ kips} = 0$$

$$F_{AC} = 22.36 \text{ kips } T$$

Joint *D*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{17}} F_{AD} - 10 \text{ kips} = 0$$

$$F_{AD} = 41.23 \text{ kips } T$$

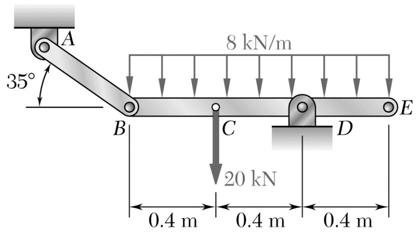
(*a*) Factor of safety for *AC*. $\quad \text{F.S.} = \dfrac{P_U}{F_{AC}} \quad \text{F.S.} = \dfrac{75 \text{ kips}}{22.36 \text{ kips}}$ $\quad \text{F.S.} = 3.35$ ◀

(*b*) For the same factor of safety in *AC* and *AD*, $\sigma_{AD} = \sigma_{AC}$.

$$\frac{F_{AD}}{A_{AD}} = \frac{F_{AC}}{A_{AC}}$$

$$A_{AD} = \frac{F_{AD}}{F_{AC}} A_{AC} = \frac{41.23}{22.36} \frac{\pi}{4} (1)^2 = 1.4482 \text{ in}^2$$

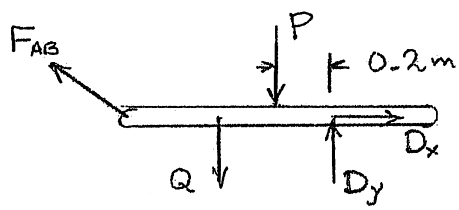
Required diameter: $\quad d_{AD} = \sqrt{\dfrac{4A_{AD}}{\pi}} = \sqrt{\dfrac{(4)(1.4482)}{\pi}}$ $\quad d_{AD} = 1.358 \text{ in.}$ ◀



PROBLEM 1.41

Link AB is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for AB for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at A and B.

SOLUTION



$$P = (1.2)(8) = 9.6 \text{ kN}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(0.8)(F_{AB}\sin 35°) + (0.2)(9.6) + (0.4)(20) = 0$$

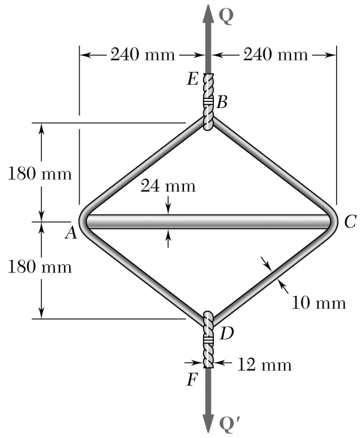
$$F_{AB} = 21.619 \text{ kN} = 21.619 \times 10^3 \text{N}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{\text{ult}}}{F.S.}$$

$$A_{AB} = \frac{(F.S.)F_{AB}}{\sigma_{\text{ult}}} = \frac{(3.50)(21.619 \times 10^3)}{450 \times 10^6}$$

$$= 168.1 \times 10^{-6} \text{m}^2$$

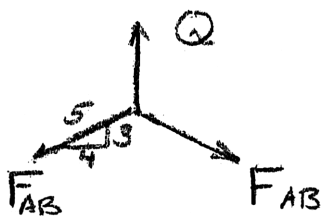
$$A_{AB} = 168.1 \text{ mm}^2 \blacktriangleleft$$



PROBLEM 1.42

A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa and that the ultimate strength of the aluminum used for the rod is 260 MPa, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

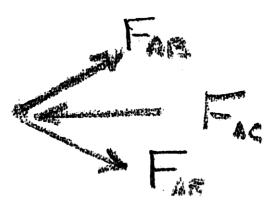
SOLUTION



Using joint *B* as a free body and considering symmetry,

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0 \quad Q = \frac{6}{5} F_{AB}$$

Using joint *A* as a free body and considering symmetry,



$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$

Based on strength of cable *BF*:

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{N}$$

Based on strength of steel loop:

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

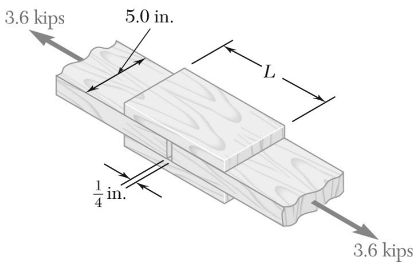
$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{N}$$

Based on strength of rod *AC*:

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2 = \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{N}$$

Actual ultimate load Q_U is the smallest, $\therefore \quad Q_U = 45.24 \times 10^3$ N

Allowable load: $$Q = \frac{Q_U}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3 \text{N}$$ $Q = 15.08$ kN ◀



PROBLEM 1.43

Two wooden members shown, which support a 3.6 kip load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 360 psi and the clearance between the members is $\frac{1}{4}$ in. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.

SOLUTION

There are 4 separate areas of glue. Let l be the length of each area and $w = 5\text{ in.}$ its width. Then the area is $A = lw$.

Each glue area transmits one half of the total load.

$$F = \left(\frac{1}{2}\right)(3.6\text{ kips}) = 1.8\text{ kips}$$

Required ultimate load for each glue area:

$$F_U = (F.S.)\,F = (2.75)(1.8) = 4.95\text{ kips}$$

Required length of each glue area:

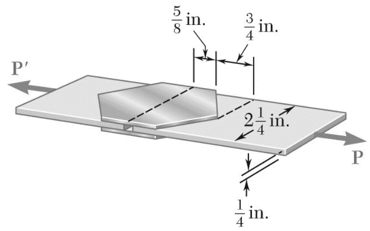
$$F_U = \tau_U A = \tau_U l w$$

$$l = \frac{F_U}{\tau_U w} = \frac{4.95 \times 10^3}{(360)(5)} = 2.75\text{ in.}$$

Total length of splice: $L = l + \frac{1}{4}\text{ in.} + l$

$$L = 2.75 + 0.25 + 2.75$$

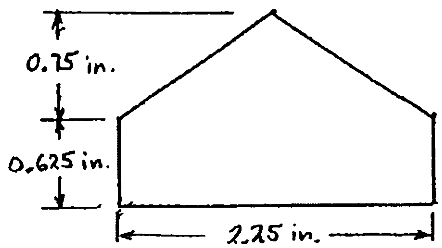
$L = 5.75\text{ in.}$ ◀



PROBLEM 1.44

Two plates, each $\frac{1}{8}$ in. thick, are used to splice a plastic strip as shown. Knowing that the ultimate shearing stress of the bonding between the surface is 130 psi, determine the factor of safety with respect to shear when $P = 325$ lb.

SOLUTION

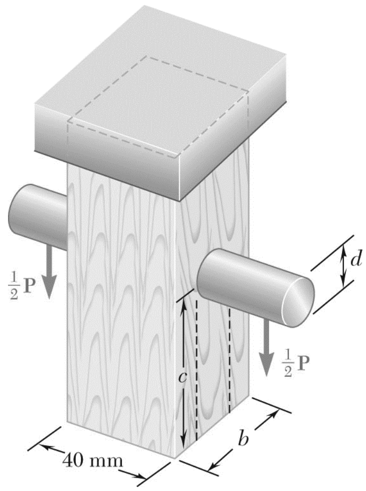


Bond area: (See figure)

$$A = \frac{1}{2}(2.25)(0.75) + (2.25)(0.625) = 2.25 \text{ in}^2$$

$$P_U = 2A\tau_U = (2)(2.25)(130) = 585 \text{ lb.}$$

$$F.S. = \frac{P_U}{P} = \frac{585}{325} = 1.800 \blacktriangleleft$$



PROBLEM 1.45

A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40\text{ mm}$, $c = 55\text{ mm}$, and $d = 12\text{ mm}$, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin:

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3\text{N}$$

Based on tension in wood:

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$= (0.040)(0.040-0.012)(60\times10^6)$$

$$= 67.2\times10^3\text{N}$$

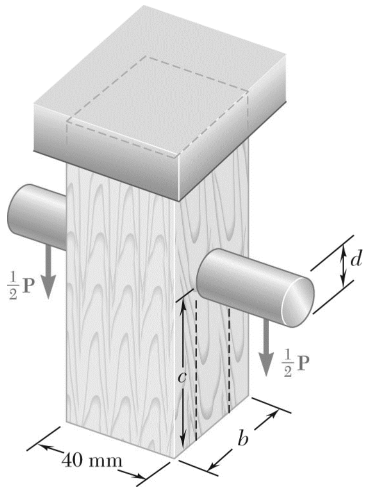
Based on double shear in the wood:

$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$

$$= 33.0\times10^3\text{N}$$

Use smallest $\quad P_U = 32.8\times10^3\text{N}$

Allowable: $\quad P = \dfrac{P_U}{F.S.} = \dfrac{32.8\times10^3}{3.2} = 10.25\times10^3\text{N}$ $\quad$ 10.25 kN ◀



PROBLEM 1.46

For the support of Prob. 1.45, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20$ kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of *b* and *c* if the factor of safety for the wooden members is the same as that found in part *a* for the pin.

PROBLEM 1.45 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

$$P = 20 \text{ kN} = 20 \times 10^3 \text{N}$$

(*a*) Pin: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 2.01.06 \times 10^{-6}\text{m}^2$

Double shear: $\tau = \frac{P}{2A} \quad \tau_U = \frac{P_U}{2A}$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(145 \times 10^6) = 58.336 \times 10^3 \text{N}$$

$$F.S. = \frac{P_U}{P} = \frac{58.336 \times 10^3}{20 \times 10^3}$$

$F.S. = 2.92$ ◀

(*b*) Tension in wood: $P_U = 58.336 \times 10^3 \text{N}$ for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where} \quad w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^{-3}\text{m}$$

$b = 40.3$ mm ◀

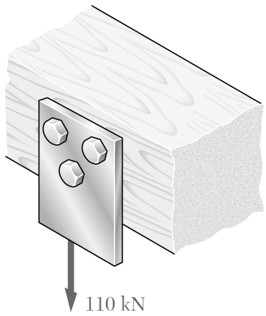
Shear in wood: $P_U = 58.336 \times 10^3 \text{N}$ for same F.S.

Double shear; each area is $A = wc$ $\quad \tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$

$$c = \frac{P_U}{2w\tau_U} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3}\text{m}$$

$c = 97.2$ mm ◀

PROBLEM 1.47



Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.

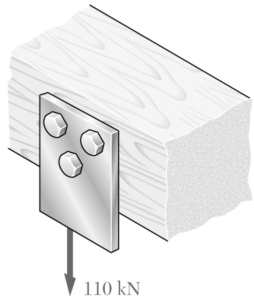
SOLUTION

For each bolt, $P = \dfrac{110}{3} = 36.667 \text{ kN}$

Required: $P_U = (F.S.)P = (3.35)(36.667) = 122.83 \text{ kN}$

$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi\tau_U}} = \sqrt{\frac{(4)(122.83\times10^3)}{\pi(360\times10^6)}} = 20.8\times10^{-3}\text{m} \qquad d = 20.8 \text{ mm} \blacktriangleleft$$



PROBLEM 1.48

Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.

SOLUTION

For each bolt, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2 = 254.47 \times 10^{-6} \text{m}^2$

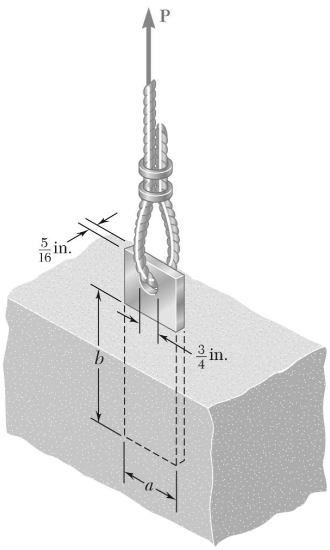
$$P_U = A\tau_U = (254.47 \times 10^{-6})(360 \times 10^6)$$
$$= 91.609 \times 10^3 \text{N}$$

For the three bolts, $P_U = (3)(91.609 \times 10^3) = 274.83 \times 10^3 \text{N}$

Factor of safety:

$$F.S. = \frac{P_U}{P} = \frac{274.83 \times 10^3}{110 \times 10^3}$$

$F.S. = 2.50$ ◀

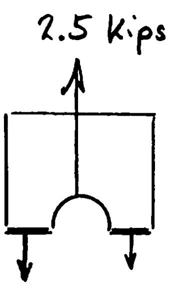


PROBLEM 1.49

A steel plate $\frac{5}{16}$ in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)

SOLUTION

Based on tension in plate:



$$A = (a - d)t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a - d)t}{P}$$

Solving for *a*,

$$a = d + \frac{(F.S.)P}{\sigma_U t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)\left(\frac{5}{16}\right)}$$

(*a*) $a = 1.550$ in. ◀

Based on shear between plate and concrete slab,

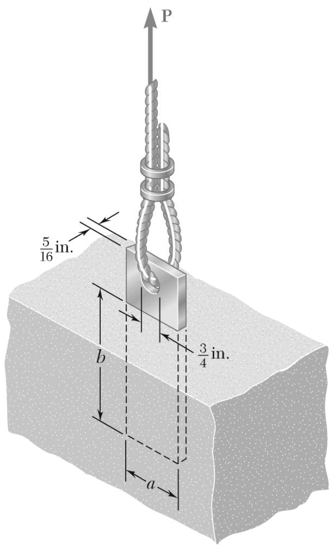
$$A = \text{perimeter} \times \text{depth} = 2(a + t)b \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = 2\tau_U (a + t)b \qquad F.S. = \frac{P_U}{P}$$

Solving for *b*,

$$b = \frac{(F.S.)P}{2(a + t)\tau_U} = \frac{(3.6)(2.5)}{(2)\left(1.550 + \frac{5}{16}\right)(0.300)}$$

(*b*) $b = 8.05$ in. ◀



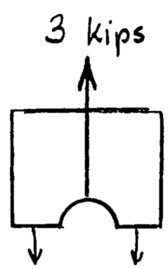
PROBLEM 1.50

Determine the factor of safety for the cable anchor in Prob. 1.49 when $P = 3$ kips, knowing that $a = 2$ in. and $b = 7.5$ in.

PROBLEM 1.49 A steel plate $\frac{5}{16}$ in, thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)

SOLUTION

Based on tension in plate:



$$A = (a - d)t$$

$$= \left(2 - \frac{3}{4}\right)\left(\frac{5}{16}\right) = 0.3906 \text{ in}^2$$

$$P_U = \sigma_U A$$

$$= (36)(0.3906) = 14.06 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{14.06}{3} = 4.69$$

Based on shear between plate and concrete slab:

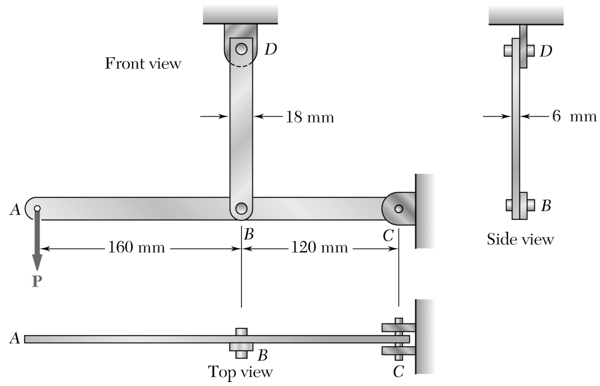
$$A = \text{perimeter} \times \text{depth} = 2(a + t)b = 2\left(2 + \frac{5}{16}\right)(7.5)$$

$$A = 34.69 \text{ in}^2 \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = (0.300)(34.69) = 10.41 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{10.41}{3} = 3.47$$

Actual factor of safety is the smaller value. $F.S. = 3.47$ ◀

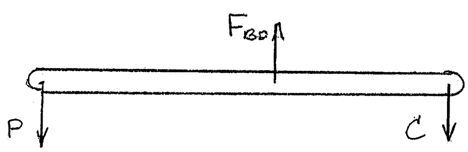


PROBLEM 1.51

In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at A. Note that link BD is not reinforced around the pin holes.

SOLUTION

Use free body ABC.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.280P - 0.120\ F_{BD} = 0$$

$$P = \frac{3}{7}F_{BD} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.160P - 0.120C = 0$$

$$P = \frac{3}{4}C \qquad (2)$$

Tension on net section of link BD.

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{400 \times 10^6}{3}\right)(6 \times 10^{-3})(18 - 10)(10^{-3}) = 6.40 \times 10^3\,\text{N}$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(10 \times 10^{-3})^2 = 3.9270 \times 10^3\,\text{N}$$

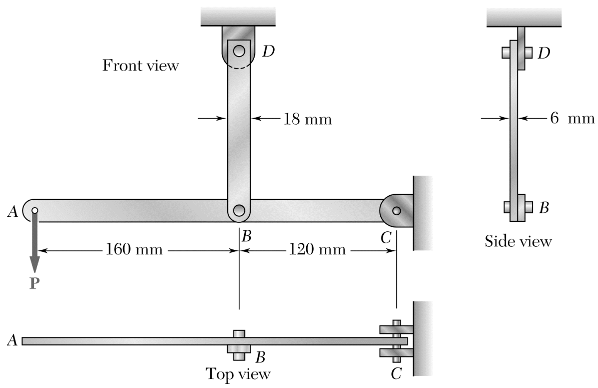
Smaller value of F_{BD} is $3.9270 \times 10^3\,\text{N}$.

From (1) $\quad P = \left(\frac{3}{7}\right)(3.9270 \times 10^3) = 1.683 \times 10^3\,\text{N}$

Shear in pin at C. $\quad C = 2\tau A_{\text{pin}} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(6 \times 10^{-3})^2 = 2.8274 \times 10^3\,\text{N}$

From (2) $\quad P = \left(\frac{3}{4}\right)(2.8274 \times 10^3) = 2.12 \times 10^3\,\text{N}$

Smaller value of P is allowable value. $\quad P = 1.683 \times 10^3\,\text{N}$ $\qquad P = 1.683\ \text{kN}$ ◀



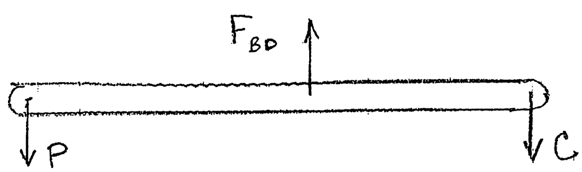
PROBLEM 1.52

Solve Prob. 1.51, assuming that the structure has been redesigned to use 12-mm-diameter pins at B and D and no other change has been made.

PROBLEM 1.51 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at A. Note that link BD is not reinforced around the pin holes.

SOLUTION

Use free body ABC.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.280P - 0.120\,F_{BD} = 0$$

$$P = \frac{3}{7}F_{BD} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.160P - 0.120C = 0$$

$$P = \frac{3}{4}C \qquad (2)$$

Tension on net section of link BD.

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-12)(10^{-3}) = 4.80\times10^3\,\text{N}$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(12\times10^{-3})^2 = 5.6549\times10^3\,\text{N}$$

Smaller value of F_{BD} is 4.80×10^3 N.

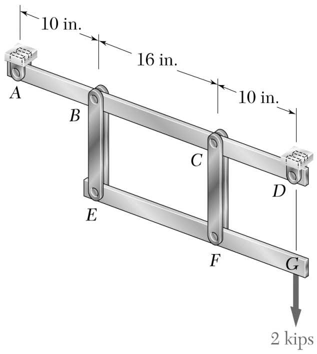
From (1), $\quad P = \left(\frac{3}{7}\right)(4.80\times10^3) = 2.06\times10^3\,\text{N}$

Shear in pin at C. $\quad C = 2\tau A_{\text{pin}} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3\,\text{N}$

From (2), $\quad P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3\,\text{N}$

Smaller value of P is the allowable value. $\quad P = 2.06\times10^3\,\text{N}$ $\qquad P = 2.06\text{ kN}$ ◀

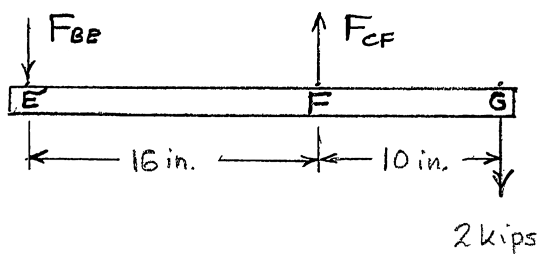
PROBLEM 1.53



Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a uniform rectangular cross section $\frac{1}{4}$ in. thick and 1 in. wide, and is made of a steel with an ultimate strength in tension of 60 ksi. The pins at *C* and *F* each have a $\frac{1}{2}$-in. diameter and are made of a steel with an ultimate strength in shear of 25 ksi. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

SOLUTION

Use member EFG as free body.



$$+\circlearrowleft \Sigma M_E = 0: \quad 16F_{CF} - (26)(2) = 0$$

$$F_{CF} = 3.25 \text{ kips}$$

Failure by tension in links *CF*. (2 parallel links)

Net section area for 1 link: $A = (b - d)t = (1 - \frac{1}{2})(\frac{1}{4}) = 0.125 \text{ in}^2$

$$F_U = 2A\sigma_U = (2)(0.125)(60) = 15 \text{ kips}$$

Failure by double shear in pins.

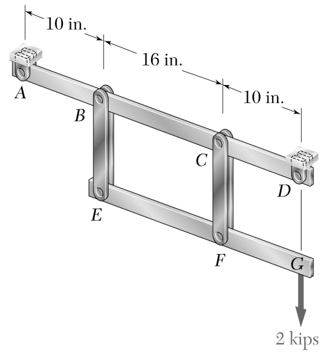
$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.196350 \text{ in}^2$$

$$F_U = 2A\tau_U = (2)(0.196350)(25) = 9.8175 \text{ kips}$$

Actual ultimate load is the smaller value. $F_U = 9.8175$ kips

Factor of safety: $F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{9.8175}{3.25}$ $\quad F.S. = 3.02$ ◀

PROBLEM 1.54

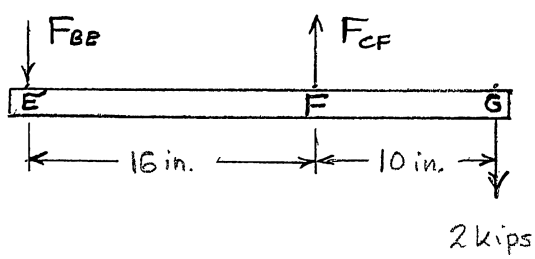


Solve Prob. 1.53, assuming that the pins at C and F have been replaced by pins with a $\frac{3}{4}$-in diameter.

PROBLEM 1.53 Each of the two vertical links CF connecting the two horizontal members AD and EG has a uniform rectangular cross section $\frac{1}{4}$ in. thick and 1 in. wide, and is made of a steel with an ultimate strength in tension of 60 ksi. The pins at C and F each have a $\frac{1}{2}$-in. diameter and are made of a steel with an ultimate strength in shear of 25 ksi. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.

SOLUTION

Use member EFG as free body.



$$+\circlearrowright \Sigma M_E = 0: \quad 16F_{CF} - (26)(2) = 0$$

$$F_{CF} = 3.25 \text{ kips}$$

Failure by tension in links CF. (2 parallel links)

Net section area for 1 link: $\quad A = (b - d)t = (1 - \frac{3}{4})(\frac{1}{4}) = 0.0625 \text{ in}^2$

$$F_U = 2A\sigma_U = (2)(0.0625)(60) = 7.5 \text{ kips}$$

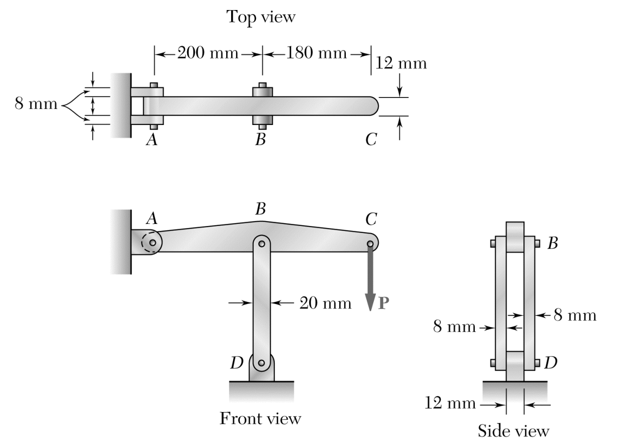
Failure by double shear in pins.

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

$$F_U = 2A\tau_U = (2)(0.44179)(25) = 22.09 \text{ kips}$$

Actual ultimate load is the smaller value. $F_U = 7.5$ kips

Factor of safety: $\quad F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{7.5}{3.25} \qquad F.S. = 2.31$ ◀

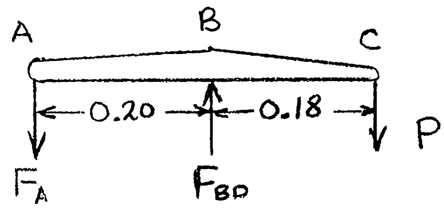


PROBLEM 1.55

In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\,F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin *A*: $A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.008)^2 = 50.266\times 10^{-6}\,\mathrm{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times 10^6)(50.266\times 10^{-6})}{3.0} = 3.351\times 10^3\,\mathrm{N}$$

$$P = \frac{10}{9}F_A = 3.72\times 10^3\,\mathrm{N}$$

Based on double shear in pins at *B* and *D*: $A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.012)^2 = 113.10\times 10^{-6}\,\mathrm{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times 10^6)(113.10\times 10^{-6})}{3.0} = 7.54\times 10^3\,\mathrm{N}$$

$$P = \frac{10}{19}F_{BD} = 3.97\times 10^3\,\mathrm{N}$$

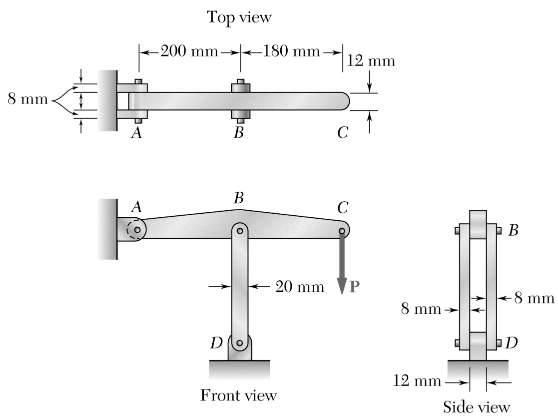
Based on compression in links *BD*: For one link, $A = (0.020)(0.008) = 160\times 10^{-6}\,\mathrm{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times 10^6)(160\times 10^{-6})}{3.0} = 26.7\times 10^3\,\mathrm{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04\times 10^3\,\mathrm{N}$$

Allowable value of *P* is smallest, $\therefore P = 3.72\times 10^3\,\mathrm{N}$ $\qquad P = 3.72\ \mathrm{kN}$ ◀

PROBLEM 1.56

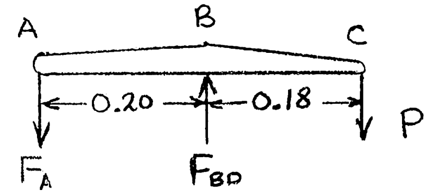


In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

PROBLEM 1.55 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use ABC as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\, F_A - 0.18P = 0 \qquad P = \frac{10}{9} F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20 F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19} F_{BD}$$

Based on double shear in pin A: $\quad A = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\,\text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times 10^6)(78.54\times 10^{-6})}{3.0} = 5.236\times 10^3\,\text{N}$$

$$P = \frac{10}{9} F_A = 5.82 \times 10^3\,\text{N}$$

Based on double shear in pins at B and D: $\quad A = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\,\text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times 10^6)(113.10\times 10^{-6})}{3.0} = 7.54\times 10^3\,\text{N}$$

$$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\,\text{N}$$

Based on compression in links BD: For one link, $A = (0.020)(0.008) = 160 \times 10^{-6}\,\text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times 10^6)(160\times 10^{-6})}{3.0} = 26.7\times 10^3\,\text{N}$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\,\text{N}$$

Allowable value of P is smallest, $\therefore P = 3.97 \times 10^3\,\text{N}$ $\qquad P = 3.97\ \text{kN}$ ◀

PROBLEM 1.57

The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 160 lb and each of the window washers is assumed to weigh 195 lb with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

(*a*) $$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\phi}$$

$$= \frac{(1.2)\left(\frac{1}{2}\times 160\right) + (1.5)\left(\frac{3}{4}\times 2\times 195\right)}{0.85}$$

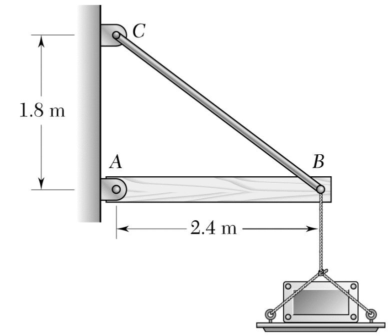
$P_U = 629$ lb ◀

Conventional factor of safety.

$$P = P_D + P_L = \frac{1}{2}\times 160 + 0.75\times 2\times 195 = 372.5 \text{ lb}$$

(*b*) $$F.S. = \frac{P_U}{P} = \frac{629}{372.5}$$

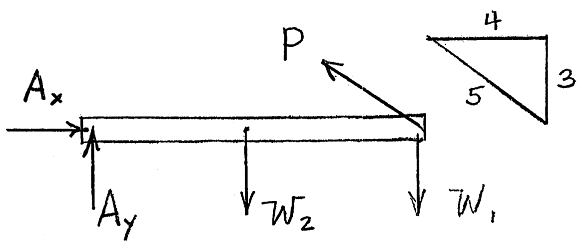
$F.S. = 1.689$ ◀



PROBLEM 1.58

A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (a) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (b) What is the corresponding conventional factor of safety for rod BC?

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0: \quad (2.4)\frac{3}{5}P - 2.4W_1 - 1.2W_2$$

$$\therefore \quad P = \frac{5}{3}W_1 + \frac{5}{6}W_2$$

For dead loading, $\quad W_1 = (40)(9.81) = 392.4 \text{ N}, \quad W_2 = (50)(9.81) = 490.5 \text{ N}$

$$P_D = \left(\frac{5}{3}\right)(392.4) + \left(\frac{5}{6}\right)(490.5) = 1.0628 \times 10^3 \text{N}$$

For live loading, $\quad W_1 = mg \quad W_2 = 0 \quad P_L = \frac{5}{3}mg$

From which $\quad m = \frac{3}{5}\frac{P_L}{g}$

Design criterion. $\quad \gamma_D P_D + \gamma_L P_L = \phi P_U$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^{-3})}{1.6}$$

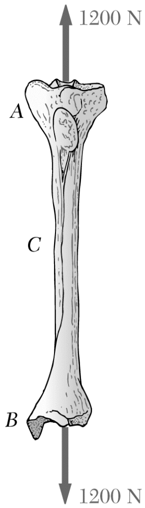
$$= 5.920 \times 10^3 \text{N}$$

(a) Allowable load. $\quad m = \frac{3}{5}\frac{5.92 \times 10^3}{9.81}$ $\qquad m = 362 \text{ kg}$ ◀

Conventional factor of safety.

$$P = P_D + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3 \text{N}$$

(b) $\quad F.S. = \frac{P_U}{P} = \frac{12 \times 10^3}{6.983 \times 10^3}$ $\qquad F.S. = 1.718$ ◀



PROBLEM 1.59

A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $\quad A = \frac{\pi}{4}(d_1^2 - d_2^2)$

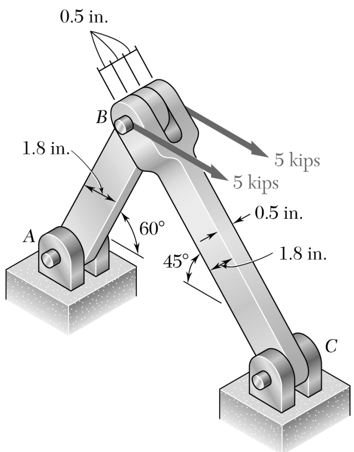
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

$$= 222.9 \times 10^{-6}\,\text{m}^2$$

$$d_2 = 14.93 \times 10^{-3}\,\text{m}$$

$d_2 = 14.93$ mm ◀

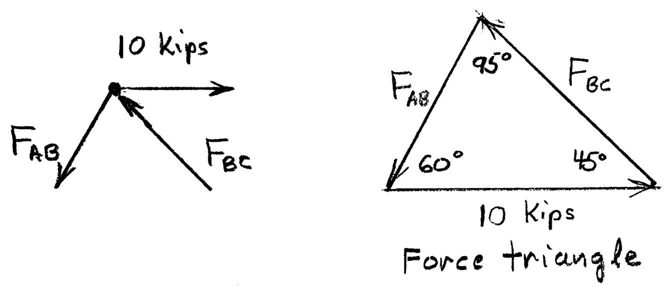


PROBLEM 1.60

Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (a) in link AB, (b) in link BC.

SOLUTION

Use joint B as free body.



Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ}$$

$$F_{AB} = 7.3205 \text{ kips}$$

$$F_{BC} = 8.9658 \text{ kips}$$

Link AB is a tension member.

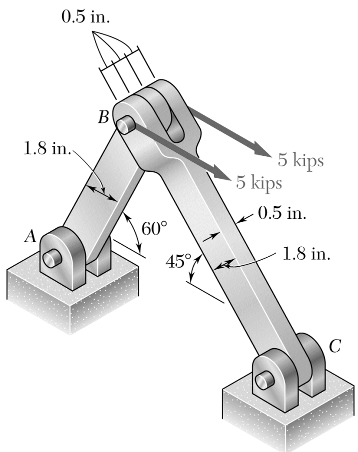
Minimum section at pin. $A_{\text{net}} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$

(a) Stress in AB: $\sigma_{AB} = \dfrac{F_{AB}}{A_{\text{net}}} = \dfrac{7.3205}{0.5}$ $\qquad \sigma_{AB} = 14.64 \text{ ksi}$ ◀

Link BC is a compression member.

Cross sectional area is $A = (1.8)(0.5) = 0.9 \text{ in}^2$

(b) Stress in BC: $\sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-8.9658}{0.9}$ $\qquad \sigma_{BC} = -9.96 \text{ ksi}$ ◀



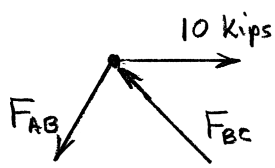
PROBLEM 1.61

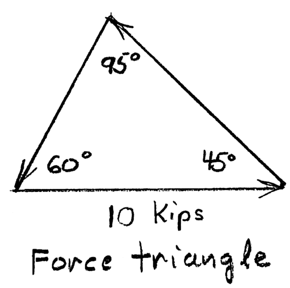
For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

PROBLEM 1.60 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

SOLUTION

Use joint *B* as free body.





Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \qquad F_{BC} = 8.9658 \text{ kips}$$

(*a*) Shearing stress in pin at *C*. $\tau = \dfrac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \qquad \tau = 8.92 \text{ ksi} \blacktriangleleft$$

PROBLEM 1.61 *(Continued)*

(*b*) Bearing stress at C in member BC. $\sigma_b = \frac{F_{BC}}{A}$

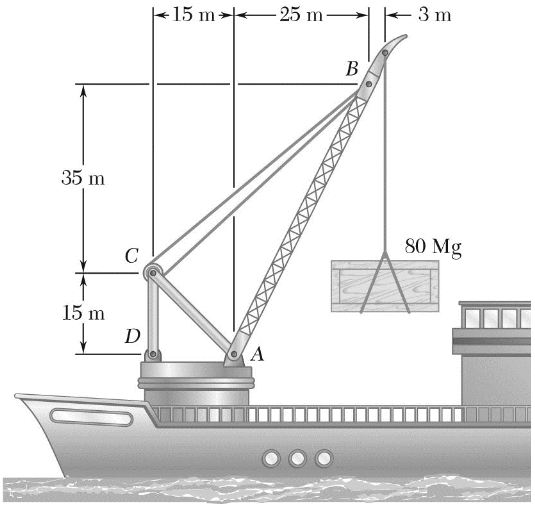
$$A = td = (0.5)(0.8) = 0.4 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \qquad \sigma_b = 22.4 \text{ ksi} \blacktriangleleft$$

(*c*) Bearing stress at B in member BC. $\sigma_b = \frac{F_{BC}}{A}$

$$A = 2td = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \qquad \sigma_b = 11.21 \text{ ksi} \blacktriangleleft$$



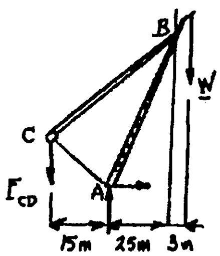
PROBLEM 1.62

In the marine crane shown, link *CD* is known to have a uniform cross section of 50 × 150 mm. For the loading shown, determine the normal stress in the central portion of that link.

SOLUTION

Weight of loading: $W = (80\text{ Mg})(9.81\text{ m/s}^2) = 784.8\text{ kN}$

Free Body: Portion *ABC*

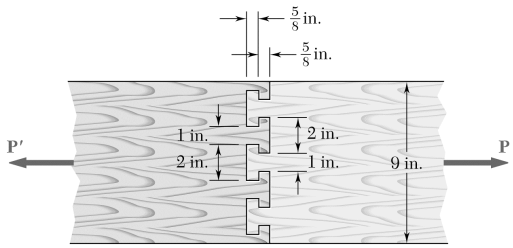


$$+\circlearrowleft \sum M_A = 0: \quad F_{CD}(15\text{ m}) - W(28\text{ m}) = 0$$

$$F_{CD} = \frac{28}{15}W = \frac{28}{15}(784.8\text{ kN})$$

$$F_{CD} = +1465\text{ kN}$$

$$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{+1465 \times 10^3\text{ N}}{(0.050\text{ m})(0.150\text{ m})} = +195.3 \times 10^6\text{ Pa} \qquad \sigma_{CD} = +195.3\text{ MPa} \blacktriangleleft$$



PROBLEM 1.63

Two wooden planks, each $\frac{1}{2}$ in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 1.20 ksi, determine the magnitude P of the axial load that will cause the joint to fail.

SOLUTION

Six areas must be sheared off when the joint fails. Each of these areas has dimensions $\frac{5}{8}$ in. $\times \frac{1}{2}$ in., its area being

$$A = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16} \text{ in}^2 = 0.3125 \text{ in}^2$$

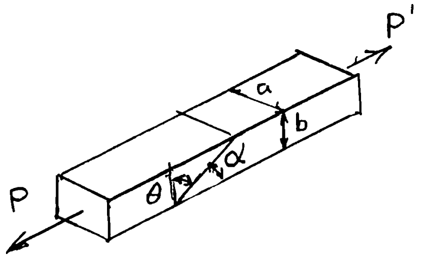
At failure, the force carried by each area is

$$F = \tau A = (1.20 \text{ ksi})(0.3125 \text{ in}^2) = 0.375 \text{ kips}$$

Since there are six failure areas,

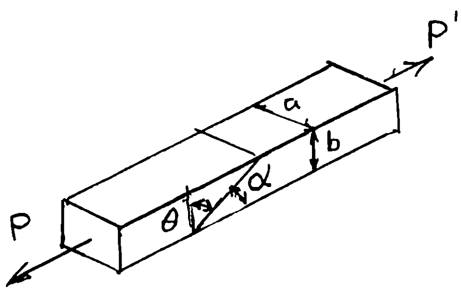
$$P = 6F = (6)(0.375) \qquad P = 2.25 \text{ kips} \blacktriangleleft$$

PROBLEM 1.64



Two wooden members of uniform rectangular cross section of sides $a = 100$ mm and $b = 60$ mm are joined by a simple glued joint as shown. Knowing that the ultimate stresses for the joint are $\sigma_U = 1.26$ MPa in tension and $\tau_U = 1.50$ MPa in shear, and that $P = 6$ kN, determine the factor of safety for the joint when (*a*) $\alpha = 20°$, (*b*) $\alpha = 35°$, (*c*) $\alpha = 45°$. For each of these values of α, also determine whether the joint will fail in tension or in shear if P is increased until rupture occurs.

SOLUTION



Let $\theta = 90° - \alpha$ as shown.

From the text book:

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

or $$\sigma = \frac{P}{A_0}\sin^2\alpha \tag{1}$$

$$\tau = \frac{P}{A_0}\sin\alpha\cos\alpha \tag{2}$$

$$A_0 = ab = (100 \text{ mm})(60 \text{ mm}) = 6000 \text{ mm}^2 = 6\times 10^{-3}\text{m}^2$$

$$\sigma_U = 1.26\times 10^6 \text{ Pa} \qquad \tau_U = 1.50\times 10^6\text{Pa}$$

Ultimate load based on tension across the joint:

$$(P_U)_\sigma = \frac{\sigma_U A_0}{\sin^2\alpha} = \frac{(1.26\times 10^6)(6\times 10^{-3})}{\sin^2\alpha} = \frac{7560}{\sin^2\alpha} = \frac{7.56}{\sin^2\alpha}\text{kN}$$

Ultimate load based on shear across the joint:

$$(P_U)_\tau = \frac{\tau_U A_0}{\sin\alpha\cos\alpha} = \frac{(1.50\times 10^6)(6\times 10^{-3})}{\sin\alpha\cos\alpha} = \frac{9000}{\sin\alpha\cos\alpha} = \frac{9.00}{\sin\alpha\cos\alpha}\text{kN}$$

(*a*) $\alpha = 20°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 20°} = 64.63$ kN

$$= (P_U)_\tau = \frac{9.00}{\sin 20° \cos 20°} = 28.00 \text{ kN}$$

The smaller value governs. The joint will <u>fail in shear</u> and $P_U = 28.00$ kN.

$$F.S. = \frac{P_U}{P} = \frac{28.00}{6}$$

$F.S. = 4.67$ ◀

(*b*) $\alpha = 35°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 35°} = 22.98$ kN

$$(P_U)_\tau = \frac{9.00}{\sin 35° \cos 35°} = 19.155 \text{ kN}$$

The joint will <u>fail in shear</u> and $P_U = 19.155$ kN.

$$F.S. = \frac{P_U}{P} = \frac{19.155}{6}$$

$F.S. = 3.19$ ◀

(*c*) $\alpha = 45°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 45°} = 15.12$ kN

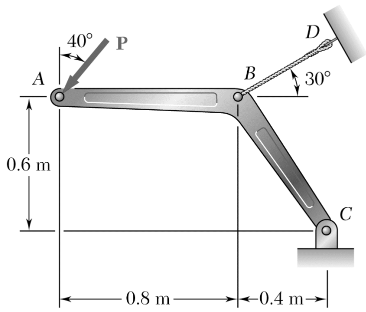
$$(P_U)_\tau = \frac{9.00}{\sin 45° \cos 45°} = 18.00 \text{ kN}$$

The joint will <u>fail in tension</u> and $P_U = 15.12$ kN.

$$F.S. = \frac{P_U}{P} = \frac{15.12}{6}$$

$F.S. = 2.52$ ◀

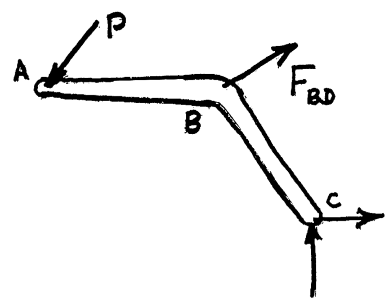
PROBLEM 1.65



Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable *BD* is 100 kN, determine the factor of safety with respect to cable failure.

SOLUTION

Use member *ABC* as a free body, and note that member *BD* is a two-force member.



$$+\circlearrowleft\ \Sigma M_c = 0: \quad (P\cos 40°)(1.2) + (P\sin 40°)(0.6) - (F_{BD}\cos 30°)(0.6) - (F_{BD}\sin 30°)(0.4) = 0$$

$$1.30493P - 0.71962F_{BD} = 0$$

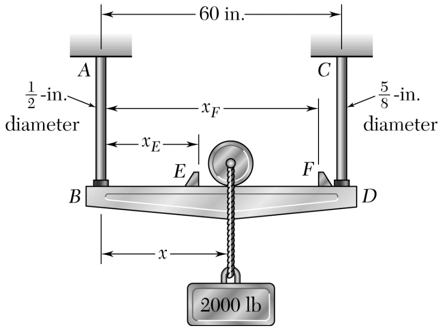
$$F_{BD} = 1.81335\ P = (1.81335)(16\times 10^3) = 29.014\times 10^3\,\text{N}$$

$$F_U = 100\times 10^3\,\text{N}$$

$$F.S. = \frac{F_U}{F_{BD}} = \frac{100\times 10^3}{29.014\times 10^3}$$

$F.S. = 3.45$ ◀

PROBLEM 1.66



The 2000-lb load can be moved along the beam *BD* to any position between stops at *E* and *F*. Knowing that $\sigma_{all} = 6$ ksi for the steel used in rods *AB* and *CD*, determine where the stops should be placed if the permitted motion of the load is to be as large as possible.

SOLUTION

Permitted member forces:

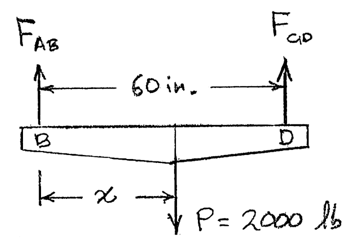
$$AB: \quad (F_{AB})_{max} = \sigma_{all} A_{AB} = (6)\left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)^2$$

$$= 1.17810 \text{ kips}$$

$$CD: \quad (F_{CD})_{max} = \sigma_{all} A_{CD} = (6)\left(\frac{\pi}{4}\right)\left(\frac{5}{8}\right)^2$$

$$= 1.84078 \text{ kips}$$

Use member *BEFD* as a free body.



$$P = 2000 \text{ lb} = 2.000 \text{ kips}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(60)F_{AB} + (60 - x_E)P = 0$$

$$60 - x_E = \frac{60 F_{AB}}{P} = \frac{(60)(1.17810)}{2.000}$$

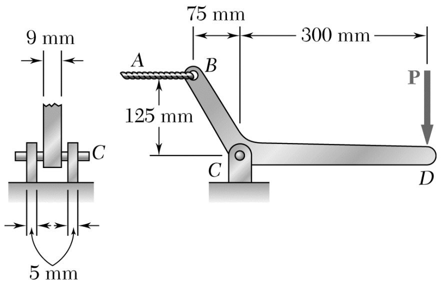
$$= 35.343$$

$$x_E = 24.7 \text{ in.} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 60F_{CD} - x_F P = 0$$

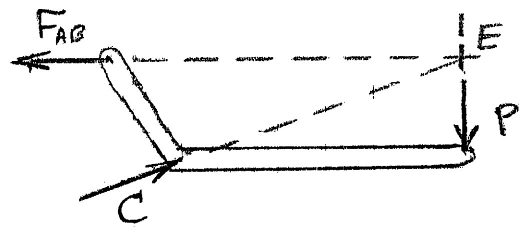
$$x_F = \frac{60F_{CD}}{P} = \frac{(60)(1.84078)}{2.000}$$

$$x_F = 55.2 \text{ in.} \blacktriangleleft$$

	**PROBLEM 1.67** Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at *C* for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at *C*, (*c*) the corresponding bearing stress in each support bracket at *C*.

SOLUTION

Draw free body diagram of *BCD*. Since *BCD* is a 3-force member, the reaction at *C* is directed toward Point *E*, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325 \text{ mm}$

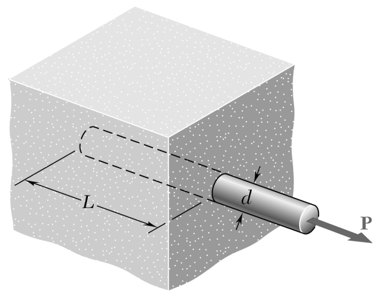
$$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325}C - P = 0 \quad C = 2.6P = (2.6)(750) = 1950 \text{ N}$$

(*a*) $\tau_{\text{pin}} = \dfrac{\frac{1}{2}C}{A_{\text{pin}}} = \dfrac{\frac{1}{2}C}{\frac{\pi}{4}d^2} \quad d = \sqrt{\dfrac{2C}{\pi\tau_{\text{pin}}}} = \sqrt{\dfrac{(2)(1950)}{\pi(40\times10^6)}} = 5.57\times10^{-3}\text{m}$ $\quad d = 5.57 \text{ mm}$ ◀

(*b*) $\sigma_b = \dfrac{C}{A_b} = \dfrac{C}{dt} = \dfrac{1950}{(5.57\times10^{-3})(9\times10^{-3})} = 38.9\times10^6\text{Pa}$ $\quad \sigma_b = 38.9 \text{ MPa}$ ◀

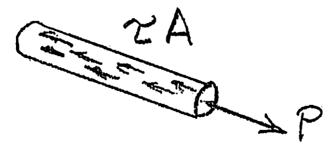
(*c*) $\sigma_b = \dfrac{\frac{1}{2}C}{A_b} = \dfrac{C}{2dt} = \dfrac{1950}{(2)(5.57\times10^{-3})(5\times10^{-3})} = 35.0\times10^6\text{Pa}$ $\quad \sigma_b = 35.0 \text{ MPa}$ ◀

PROBLEM 1.68



A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

SOLUTION



For shear, $A = \pi dL$

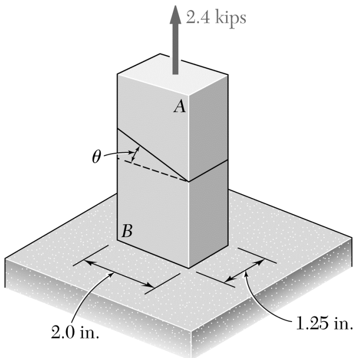
$$P = \tau_{\text{all}} A = \tau_{\text{all}} \pi dL$$

For tension, $A = \frac{\pi}{4} d^2$

$$P = \sigma_{\text{all}} A = \sigma_{\text{all}} \left(\frac{\pi}{4} d^2 \right)$$

Equating, $\tau_{\text{all}} \pi dL = \sigma_{\text{all}} \frac{\pi}{4} d^2$

Solving for L, $L_{\min} = \sigma_{\text{all}} d / 4\tau_{\text{all}}$ ◀



PROBLEM 1.69

The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

SOLUTION

$$A_0 = (2.0)(1.25) = 2.50 \text{ in.}^2$$

$$P = 2.4 \text{ kips}$$

$$P_U = (F.S.)P = 7.2 \text{ kips}$$

Based on tensile stress:

$$\sigma_U = \frac{P_U}{A_0}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_U A_0}{P_U} = \frac{(2.5)(2.50)}{7.2} = 0.86806$$

$$\cos\theta = 0.93169 \qquad \theta = 21.3° \qquad \theta > 21.3°$$

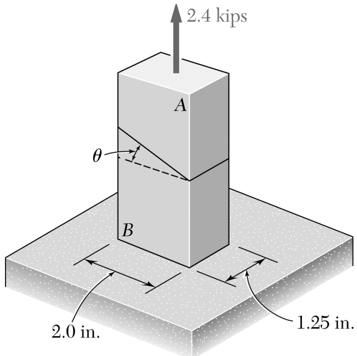
Based on shearing stress: $\tau_U = \dfrac{P_U}{A_0}\sin\theta\cos\theta = \dfrac{P_U}{2A_0}\sin 2\theta$

$$\sin 2\theta = \frac{2A_0\tau_U}{P_U} = \frac{(2)(2.50)(1.3)}{7.2} = 0.90278$$

$$2\theta = 64.52° \qquad \theta = 32.3° \qquad \theta < 32.3°$$

Hence, $21.3° < \theta < 32.3°$ ◀

PROBLEM 1.70



The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

SOLUTION

$$A_0 = (2.0)(1.25) = 2.50 \text{ in}^2$$

At the optimum angle, $\quad (F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\quad \sigma = \dfrac{P}{A_0}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \dfrac{\sigma_U A_0}{\cos^2\theta}$

$$(F.S.)_\sigma = \frac{P_{U,\sigma}}{P} = \frac{\sigma_U A_0}{P\cos^2\theta}$$

Shearing stress: $\tau = \dfrac{P}{A_0}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \dfrac{\tau_U A_0}{\sin\theta\cos\theta}$

$$(F.S.)_\tau = \frac{P_{U,\tau}}{P} = \frac{\tau_U A_0}{P\sin\theta\cos\theta}$$

Equating: $\dfrac{\sigma_U A_0}{P\cos^2\theta} = \dfrac{\tau_U A_0}{P\sin\theta\cos\theta}$

Solving: $\quad \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{\tau_U}{\sigma_U} = \dfrac{1.3}{2.5} = 0.520 \qquad$ (*a*) $\quad \theta_{\text{opt}} = 27.5° \blacktriangleleft$

(*b*) $\quad P_U = \dfrac{\sigma_U A_0}{\cos^2\theta} = \dfrac{(12.5)(2.50)}{\cos^2 27.5°} = 7.94 \text{ kips}$

$$F.S. = \frac{P_U}{P} = \frac{7.94}{2.4} \qquad F.S. = 3.31 \blacktriangleleft$$

CHAPTER 2

PROBLEM 2.1

An 80-m-long wire of 5-mm diameter is made of a steel with $E = 200$ GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (*a*) the largest allowable tension in the wire, (*b*) the corresponding elongation of the wire.

SOLUTION

(*a*) $\sigma_U = 400\times10^6\,\text{Pa}\qquad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(5)^2 = 19.635\ \text{mm}^2 = 19.635\times10^{-6}\ \text{m}^2$

$$P_U = \sigma_U A = (400\times10^6)(19.635\times10^{-6}) = 7854\ \text{N}$$

$$P_{\text{all}} = \frac{P_U}{F.S} = \frac{7854}{3.2} = 2454\ \text{N}$$

$P_{\text{all}} = 2.45\ \text{kN}$ ◀

(*b*) $\delta = \dfrac{PL}{AE} = \dfrac{(2454)(80)}{(19.635\times10^{-6})(200\times10^9)} = 50.0\times10^{-3}\,\text{m}$

$\delta = 50.0\ \text{mm}$ ◀

PROBLEM 2.2

A steel control rod is 5.5 ft long and must not stretch more than 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29\times 10^6$ psi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding normal stress caused by the load.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}: \quad 0.04 \text{ in.} = \dfrac{(2000 \text{ lb})(5.5\times 12 \text{ in.})}{A(29\times 10^6 \text{ psi})}$

$$A = \frac{1}{4}\pi d^2 = 0.11379 \text{ in}^2$$

$$d = 0.38063 \text{ in.}$$

$d = 0.381$ in. ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{2000 \text{ lb}}{0.11379 \text{ in}^2} = 17580 \text{ psi}$

$\sigma = 17.58$ ksi ◀

PROBLEM 2.3

Two gage marks are placed exactly 10 in. apart on a $\frac{1}{2}$-in.-diameter aluminum rod with $E = 10.1\times10^6$ psi and an ultimate strength of 16 ksi. Knowing that the distance between the gage marks is 10.009 in. after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

SOLUTION

(*a*) $\delta = 10.009 - 10.000 = 0.009$ in.

$$\varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} \qquad \sigma = \frac{E\delta}{L} = \frac{(10.1\times10^6)(0.009)}{10} = 9.09\times10^3 \text{ psi} \qquad \sigma = 9.09 \text{ ksi} \blacktriangleleft$$

(*b*) $F.S. = \dfrac{\sigma_U}{\sigma} = \dfrac{16}{9.09}$ $\qquad F.S. = 1.760 \blacktriangleleft$

PROBLEM 2.4

An 18-m-long steel wire of 5-mm diameter is to be used in the manufacture of a prestressed concrete beam. It is observed that the wire stretches 45 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$, or $P = \dfrac{\delta AE}{L}$

with $A = \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi(0.005)^2 = 19.6350 \times 10^{-6}\,\text{m}^2$

$$P = \frac{(0.045\text{ m})(19.6350\times10^{-6}\,\text{m}^2)(200\times10^9\,\text{N/m}^2)}{18\text{ m}} = 9817.5\text{ N}$$

$P = 9.82$ kN ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{9817.5\text{ N}}{19.6350\times10^{-6}\text{ m}^2} = 500\times10^6\,\text{Pa}$ $\qquad \sigma = 500$ MPa ◀

PROBLEM 2.5

A polystyrene rod of length 12 in. and diameter 0.5 in. is subjected to an 800-lb tensile load. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the elongation of the rod, (*b*) the normal stress in the rod.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.5)^2 = 0.19635 \text{ in}^2$$

(*a*) $\delta = \dfrac{PL}{AE} = \dfrac{(800)(12)}{(0.19635)(0.45 \times 10^6)} = 0.1086$ $\qquad \delta = 0.1086 \text{ in.} \blacktriangleleft$

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{800}{0.19635} = 4074 \text{ psi}$ $\qquad \sigma = 4.07 \text{ ksi} \blacktriangleleft$

PROBLEM 2.6

A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3$ GPa and that the length of the thread increases by 1.1%, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

SOLUTION

(*a*) Strain: $\varepsilon = \dfrac{\delta}{L} = \dfrac{1.1}{100} = 0.011$

Stress: $\sigma = E\varepsilon = (3.3 \times 10^9)(0.011) = 36.3 \times 10^6\,\text{Pa}$

$$\sigma = \frac{P}{A}$$

Area: $A = \dfrac{P}{\sigma} = \dfrac{8.5}{36.3 \times 10^6} = 234.16 \times 10^{-9}\,\text{m}^2$

Diameter: $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(234.16 \times 10^{-9})}{\pi}} = 546 \times 10^{-6}\,\text{m}$ $\qquad d = 0.546$ mm ◀

(*b*) Stress: $\qquad \sigma = 36.3$ MPa ◀

PROBLEM 2.7

Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod. Knowing that, with an axial load of 6000 N acting on the rod, the distance between the gage marks is 250.18 mm, determine the modulus of elasticity of the aluminum used in the rod.

SOLUTION

$$\delta = \Delta L = L - L_0 = 250.18 - 250.00 = 0.18 \text{ mm}$$

$$\varepsilon = \frac{\delta}{L_0} = \frac{0.18 \text{ mm}}{250 \text{ mm}} = 0.00072$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{6000}{113.097 \times 10^{-6}} = 53.052 \times 10^6 \text{ Pa}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{53.052 \times 10^6}{0.00072} = 73.683 \times 10^9 \text{ Pa}$$

$E = 73.7$ GPa ◀

PROBLEM 2.8

An aluminum pipe must not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the maximum allowable normal stress is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE};$

Thus, $$L = \frac{EA\delta}{P} = \frac{E\delta}{\sigma} = \frac{(10.1 \times 10^6)(0.05)}{14 \times 10^3}$$

$L = 36.1$ in. ◀

(*b*) $\sigma = \dfrac{P}{A};$

Thus, $$A = \frac{P}{\sigma} = \frac{127.5 \times 10^3}{14 \times 10^3}$$ $A = 9.11\ \text{in}^2$ ◀

PROBLEM 2.9

An aluminum control rod must stretch 0.08 in. when a 500-lb tensile load is applied to it. Knowing that $\sigma_{\text{all}} = 22$ ksi and $E = 10.1\times 10^6$ psi, determine the smallest diameter and shortest length that can be selected for the rod.

SOLUTION

$$P = 500 \text{ lb}, \quad \delta = 0.08 \text{ in.} \quad \sigma_{\text{all}} = 22\times 10^3 \text{ psi}$$

$$\sigma = \frac{P}{A} < \sigma_{\text{all}} \quad A > \frac{P}{\sigma_{\text{all}}} = \frac{500}{22\times 10^3} = 0.022727 \text{ in}^2$$

$$A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.022727)}{\pi}} \qquad d_{\min} = 0.1701 \text{ in.} \blacktriangleleft$$

$$\sigma = E\varepsilon = \frac{E\delta}{L} < \sigma_{\text{all}}$$

$$L > \frac{E\delta}{\sigma_{\text{all}}} = \frac{(10.1\times 10^6)(0.08)}{22\times 10^3} = 36.7 \text{ in.} \qquad L_{\min} = 36.7 \text{ in.} \blacktriangleleft$$

PROBLEM 2.10

A square yellow-brass bar must not stretch more than 2.5 mm when it is subjected to a tensile load. Knowing that $E = 105$ GPa and that the allowable tensile strength is 180 MPa, determine (*a*) the maximum allowable length of the bar, (*b*) the required dimensions of the cross section if the tensile load is 40 kN.

SOLUTION

$\sigma = 180 \times 10^6 \text{ Pa} \quad P = 40 \times 10^3 \text{N}$

$E = 105 \times 10^9 \text{ Pa} \quad \delta = 2.5 \times 10^{-3} \text{m}$

(*a*) $$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$L = \frac{E\delta}{\sigma} = \frac{(105\times 10^9)(2.5\times 10^{-3})}{180\times 10^6} = 1.45833 \text{ m}$$

$L = 1.458 \text{ m}$ ◀

(*b*) $$\sigma = \frac{P}{A}$$

$$A = \frac{P}{\sigma} = \frac{40 \times 10^3}{180 \times 10^6} = 222.22 \times 10^{-6} \text{m}^2 = 222.22 \text{ mm}^2$$

$A = a^2 \quad a = \sqrt{A} = \sqrt{222.22}$ $\qquad a = 14.91 \text{ mm}$ ◀

PROBLEM 2.11

A 4-m-long steel rod must not stretch more than 3 mm and the normal stress must not exceed 150 MPa when the rod is subjected to a 10-kN axial load. Knowing that $E = 200$ GPa, determine the required diameter of the rod.

SOLUTION

$L = 4 \text{ m}$

$\delta = 3 \times 10^{-3} \text{ m}, \qquad \sigma = 150 \times 10^6 \text{ Pa}$

$E = 200 \times 10^9 \text{ Pa}, \quad P = 10 \times 10^3 \text{ N}$

Stress: $\qquad \sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{10 \times 10^3}{150 \times 10^6} = 66.667 \times 10^{-6} \text{m}^2 = 66.667 \text{ mm}^2$$

Deformation: $\qquad \delta = \dfrac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(10 \times 10^3)(4)}{(200 \times 10^9)(3 \times 10^{-3})} = 66.667 \times 10^{-6} \text{m}^2 = 66.667 \text{ mm}^2$$

The larger value of A governs: $\qquad A = 66.667 \text{ mm}^2$

$$A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(66.667)}{\pi}}$$

$d = 9.21 \text{ mm}$ ◀

PROBLEM 2.12

A nylon thread is to be subjected to a 10-N tension. Knowing that $E = 3.2$ GPa, that the maximum allowable normal stress is 40 MPa, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

SOLUTION

Stress criterion:

$$\sigma = 40 \text{ MPa} = 40 \times 10^6 \text{ Pa} \quad P = 10 \text{ N}$$

$$\sigma = \frac{P}{A}: \quad A = \frac{P}{\sigma} = \frac{10 \text{ N}}{40 \times 10^6 \text{ Pa}} = 250 \times 10^{-9} \text{m}^2$$

$$A = \frac{\pi}{4} d^2: \quad d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{250 \times 10^{-9}}{\pi}} = 564.19 \times 10^{-6} \text{m}$$

$$d = 0.564 \text{ mm}$$

Elongation criterion:

$$\frac{\delta}{L} = 1\% = 0.01$$

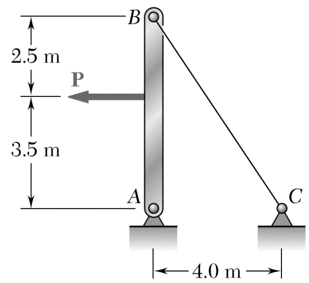
$$\delta = \frac{PL}{AE}:$$

$$A = \frac{P/E}{\delta/L} = \frac{10 \text{ N}/3.2 \times 10^9 \text{ Pa}}{0.01} = 312.5 \times 10^{-9} \text{ m}^2$$

$$d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{312.5 \times 10^{-9}}{\pi}} = 630.78 \times 10^{-6} \text{m}^2$$

$$d = 0.631 \text{ mm}$$

The required diameter is the larger value: $d = 0.631$ mm ◀



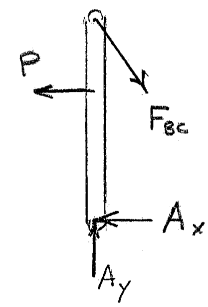
PROBLEM 2.13

The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar AB as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$$

$$P = 0.9509 F_{BC}$$

Considering allowable stress: $\sigma = 190 \times 10^6 \text{ Pa}$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

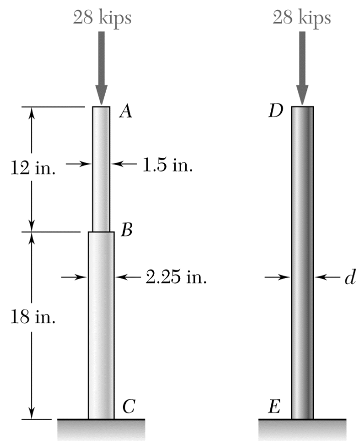
Considering allowable elongation: $\delta = 6 \times 10^{-3}$ m

$$\delta = \frac{F_{BC} L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$$P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N}$$

$P = 1.988$ kN ◀



PROBLEM 2.14

The aluminum rod ABC ($E = 10.1\times 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 29\times 10^6$ psi) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

SOLUTION

Deformation of aluminum rod.

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E}$$

$$= \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{28\times 10^3}{10.1\times 10^6}\left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right)$$

$$= 0.031376 \text{ in.}$$

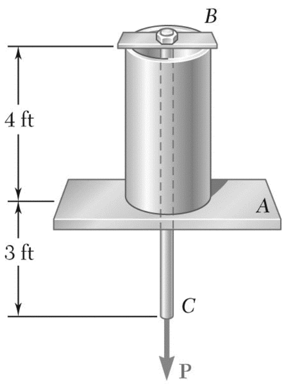
Steel rod. $\delta = 0.031376$ in.

$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(28\times 10^3)(30)}{(29\times 10^6)(0.031376)} = 0.92317 \text{ in}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{28\times 10^3}{24\times 10^3} = 1.1667 \text{ in}^2$$

Required area is the larger value. $A = 1.1667 \text{ in}^2$

Diameter: $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(1.6667)}{\pi}}$ $d = 1.219$ in. ◀



PROBLEM 2.15

A 4-ft section of aluminum pipe of cross-sectional area 1.75 in^2 rests on a fixed support at A. The $\frac{5}{8}$-in.-diameter steel rod BC hangs from a rigid bar that rests on the top of the pipe at B. Knowing that the modulus of elasticity is 29×10^6 psi for steel, and 10.4×10^6 psi for aluminum, determine the deflection of point C when a 15-kip force is applied at C.

SOLUTION

Rod BC:

$$L_{BC} = 7 \text{ ft} = 84 \text{ in.} \quad E_{BC} = 29 \times 10^6 \text{ psi}$$

$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.625)^2 = 0.30680 \text{ in}^2$$

$$\delta_{C/B} = \frac{PL_{BC}}{E_{BC} A_{BC}} = \frac{(15 \times 10^3)(84)}{(29 \times 10^6)(0.30680)} = 0.141618 \text{ in.}$$

Pipe AB:

$$L_{AB} = 4 \text{ ft} = 48 \text{ in.} \quad E_{AB} = 10.4 \times 10^6 \text{ psi}$$

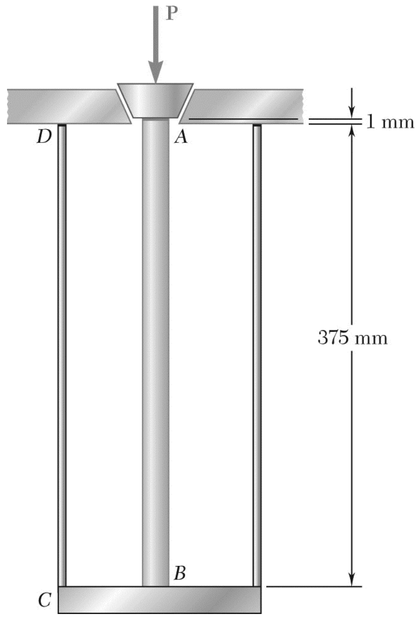
$$A_{AB} = 1.75 \text{ in}^2$$

$$\delta_{B/A} = \frac{PL_{AB}}{E_{AB} A_{AB}} = \frac{(15 \times 10^3)(48)}{(10.4 \times 10^6)(1.75)} = 39.560 \times 10^{-3} \text{ in.}$$

Total:

$$\delta_C = \delta_{B/A} + \delta_{C/B} = 39.560 \times 10^{-3} + 0.141618 = 0.181178 \text{ in.}$$

$$\delta_C = 0.1812 \text{ in.} \blacktriangleleft$$



PROBLEM 2.16

The brass tube AB ($E = 105$ GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder ($E = 72$ GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.

SOLUTION

Shortening of brass tube AB:

$$L_{AB} = 375 + 1 = 376 \text{ mm} = 0.376 \text{ m} \quad A_{AB} = 140 \text{ mm}^2 = 140 \times 10^{-6} \text{ m}^2$$

$$E_{AB} = 105 \times 10^9 \text{ Pa}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(0.376)}{(105 \times 10^9)(140 \times 10^{-6})} = 25.578 \times 10^{-9} P$$

Lengthening of aluminum cylinder CD:

$$L_{CD} = 0.375 \text{ m} \quad A_{CD} = 250 \text{ mm}^2 = 250 \times 10^{-6} \text{ m}^2 \quad E_{CD} = 72 \times 10^9 \text{ Pa}$$

$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(0.375)}{(72 \times 10^9)(250 \times 10^{-6})} = 20.833 \times 10^{-9} P$$

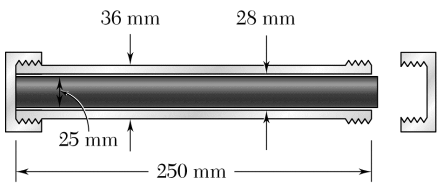
Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = 0.001$ m

$$0.001 = (25.578 \times 10^{-9} + 20.833 \times 10^{-9}) P$$

$$P = 21.547 \times 10^3 \text{ N}$$

$P = 21.5$ kN ◀

PROBLEM 2.17



A 250-mm-long aluminum tube $(E = 70\text{ GPa})$ of 36-mm outer diameter and 28-mm inner diameter can be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod $(E = 105\text{ GPa})$ of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

SOLUTION

$$A_{\text{tube}} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12\text{ mm}^2 = 402.12\times10^{-6}\,\text{m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87\text{ mm}^2 = 490.87\times10^{-6}\,\text{m}^2$$

$$\delta_{\text{tube}} = \frac{PL}{E_{\text{tube}}A_{\text{tube}}} = \frac{P(0.250)}{(70\times10^9)(402.12\times10^{-6})} = 8.8815\times10^{-9}\,P$$

$$\delta_{\text{rod}} = -\frac{PL}{E_{\text{rod}}A_{\text{rod}}} = \frac{P(0.250)}{(105\times10^6)(490.87\times10^{-6})} = -4.8505\times10^{-9}\,P$$

$$\delta^* = \left(\frac{1}{4}\text{turn}\right)\times1.5\text{ mm} = 0.375\text{ mm} = 375\times10^{-6}\,\text{m}$$

$$\delta_{\text{tube}} = \delta^* + \delta_{\text{rod}} \quad \text{or} \quad \delta_{\text{tube}} - \delta_{\text{rod}} = \delta^*$$

$$8.8815\times10^{-9}\,P + 4.8505\times10^{-9}\,P = 375\times10^{-6}$$

$$P = \frac{0.375\times10^{-3}}{(8.8815+4.8505)(10^{-9})} = 27.308\times10^3\,\text{N}$$

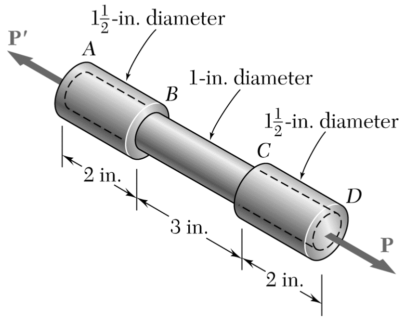
(*a*) $$\sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{27.308\times10^3}{402.12\times10^{-6}} = 67.9\times10^6\,\text{Pa}$$ $\sigma_{\text{tube}} = 67.9\text{ MPa}$ ◀

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{27.308\times10^3}{490.87\times10^{-6}} = -55.6\times10^6\,\text{Pa}$$ $\sigma_{\text{rod}} = -55.6\text{ MPa}$ ◀

(*b*) $$\delta_{\text{tube}} = (8.8815\times10^{-9})(27.308\times10^3) = 242.5\times10^{-6}\,\text{m}$$ $\delta_{\text{tube}} = 0.2425\text{ mm}$ ◀

$$\delta_{\text{rod}} = -(4.8505\times10^{-9})(27.308\times10^3) = -132.5\times10^{-6}\,\text{m}$$ $\delta_{\text{rod}} = -0.1325\text{ mm}$ ◀

PROBLEM 2.18



The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

SOLUTION

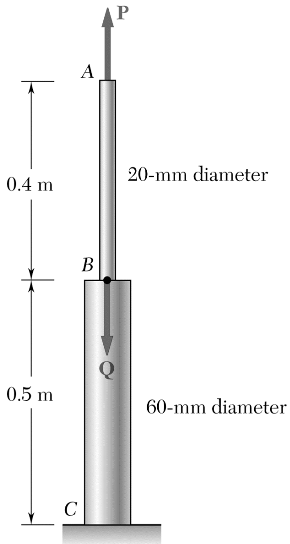
(*a*) $\delta = \Sigma \dfrac{P_i L_i}{A_i E_i} = \dfrac{P}{E} \Sigma \dfrac{L_i}{A_i}$

$$P = E\delta \left(\Sigma \frac{L_i}{A_i} \right)^{-1} \quad A_i = \frac{\pi}{4} d_i^2$$

	L, in.	d, in.	A, in^2	L/A, in^{-1}
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

$P = (29 \times 10^6)(0.002)(6.083)^{-1} = 9.353 \times 10^3$ lb $\quad$ $P = 9.53$ kips ◀

(*b*) $\delta_{BC} = \dfrac{PL_{BC}}{A_{BC} E} = \dfrac{P}{E} \dfrac{L_{BC}}{A_{BC}} = \dfrac{9.535 \times 10^3}{29 \times 10^6}(3.8197)$ $\quad$ $\delta = 1.254 \times 10^{-3}$ in. ◀



PROBLEM 2.19

Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

SOLUTION

(*a*) $A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\,\text{m}^2$

$A_{BC} = \frac{\pi}{4}d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}\,\text{m}^2$

Force in member *AB* is *P* tension.

<u>Elongation</u>:

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})} = 72.756\times10^{-6}\,\text{m}$$

Force in member *BC* is $Q - P$ compression.

<u>Shortening</u>:

$$\delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})} = 2.5263\times10^{-9}(Q-P)$$

For zero deflection at *A*, $\delta_{BC} = \delta_{AB}$

$$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q-P = 28.8\times10^3\,\text{N}$$

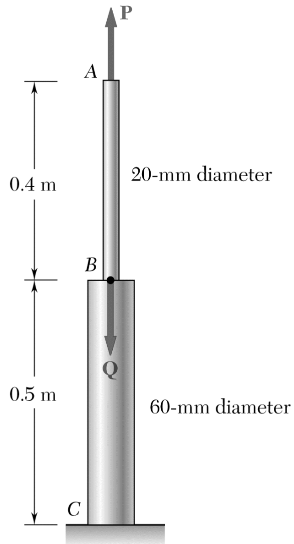
$$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3\,\text{N}$$

$Q = 32.8$ kN ◀

(*b*) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}\,\text{m}$

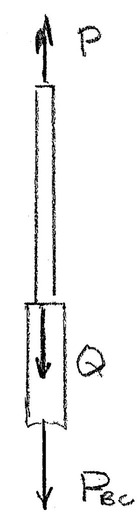
$\delta_{AB} = 0.0728\text{ mm}\downarrow$ ◀

PROBLEM 2.20



The rod ABC is made of an aluminum for which $E = 70$ GPa. Knowing that $\mathbf{P} = 6$ kN and $\mathbf{Q} = 42$ kN, determine the deflection of (*a*) point A, (*b*) point B.

SOLUTION



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$$

$$P_{AB} = P = 6 \times 10^3\,\text{N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3\,\text{N}$$

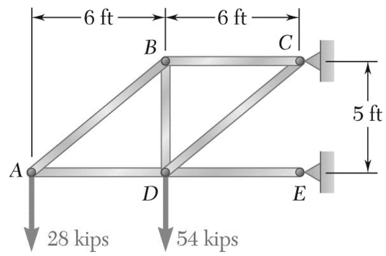
$$L_{AB} = 0.4\text{ m} \quad L_{BC} = 0.5\text{ m}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)} = 109.135 \times 10^{-6}\,\text{m}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)} = -90.947 \times 10^{-6}\,\text{m}$$

(*a*) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6}\,\text{m} = 18.19 \times 10^{-6}\,\text{m}$ $\qquad \delta_A = 0.01819\text{ mm} \uparrow$ ◀

(*b*) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6}\,\text{m} = -0.0909\text{ mm}$ $\qquad$ or $\qquad \delta_B = 0.0919\text{ mm} \downarrow$ ◀



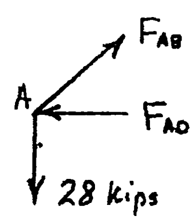
PROBLEM 2.21

Members AB and BC are made of steel $(E = 29 \times 10^6 \text{psi})$ with cross-sectional areas of 0.80 in^2 and 0.64 in^2, respectively. For the loading shown, determine the elongation of (*a*) member AB, (*b*) member BC.

SOLUTION

(*a*) $L_{AB} = \sqrt{6^2 + 5^2} = 7.810 \text{ ft} = 93.72 \text{ in.}$

Use joint A as a free body.



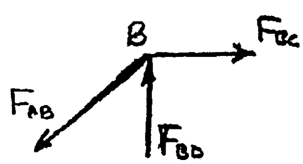
$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.810} F_{AB} - 28 = 0$$

$$F_{AB} = 43.74 \text{ kip} = 43.74 \times 10^3 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{EA_{AB}} = \frac{(43.74 \times 10^3)(93.72)}{(29 \times 10^6)(0.80)}$$

$\delta_{AB} = 0.1767 \text{ in.}$ ◀

(*b*) Use joint B as a free body.

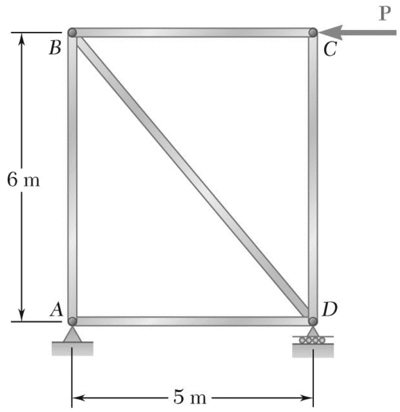


$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} - \frac{6}{7.810} F_{AB} = 0$$

$$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60 \text{ kip} = 33.60 \times 10^3 \text{lb.}$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{EA_{BC}} = \frac{(33.60 \times 10^3)(72)}{(29 \times 10^6)(0.64)}$$

$\delta_{BC} = 0.1304 \text{ in.}$ ◀



PROBLEM 2.22

The steel frame (E = 200 GPa) shown has a diagonal brace BD with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member BD is not to exceed 1.6 mm.

SOLUTION

$$\delta_{BD} = 1.6 \times 10^{-3}\,\text{m}, \quad A_{BD} = 1920\ \text{mm}^2 = 1920 \times 10^{-6}\,\text{m}^2$$

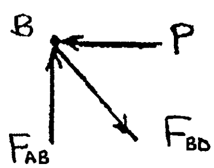
$$L_{BD} = \sqrt{5^2 + 6^2} = 7.810\ \text{m}, \quad E_{BD} = 200 \times 10^9\,\text{Pa}$$

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}}$$

$$F_{BD} = \frac{E_{BD} A_{BD} \delta_{BD}}{L_{BD}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81}$$

$$= 78.67 \times 10^3\,\text{N}$$

Use joint B as a free body. $\xrightarrow{+} \Sigma F_x = 0$:



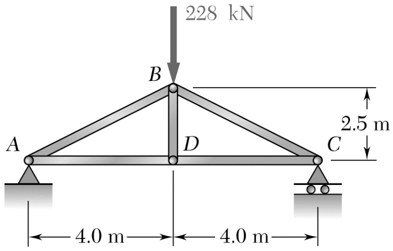
$$\frac{5}{7.810} F_{BD} - P = 0$$

$$P = \frac{5}{7.810} F_{BD} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

$$= 50.4 \times 10^3\,\text{N}$$

$P = 50.4$ kN ◀

PROBLEM 2.23



For the steel truss $(E = 200\text{ GPa})$ and loading shown, determine the deformations of the members *AB* and *AD*, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

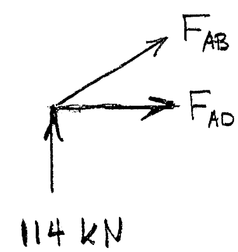
SOLUTION

Statics: Reactions are 114 kN upward at *A* and *C*.

<u>Member *BD*</u> is a zero force member.

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717\text{ m}$$

Use joint *A* as a free body.



$$+\uparrow \Sigma F_y = 0:\ 114 + \frac{2.5}{4.717}F_{AB} = 0$$

$$F_{AB} = -215.10\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0:\ F_{AD} + \frac{4}{4.717}F_{AB} = 0$$

$$F_{AD} = -\frac{(4)(-215.10)}{4.717} = 182.4\text{ kN}$$

<u>Member *AB*</u>:

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA_{AB}} = \frac{(-215.10\times 10^3)(4.717)}{(200\times 10^9)(2400\times 10^{-6})}$$

$$= -2.11\times 10^{-3}\text{m}$$

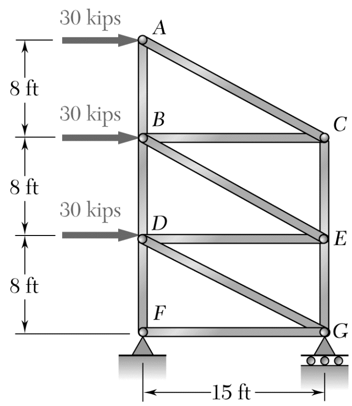
$\delta_{AB} - 2.11\text{ mm}$ ◀

<u>Member *AD*</u>:

$$\delta_{AD} = \frac{F_{AD}L_{AD}}{EA_{AD}} = \frac{(182.4\times 10^3)(4.0)}{(200\times 10^9)(1800\times 10^{-6})}$$

$$= 2.03\times 10^{-3}\text{m}$$

$\delta_{AD} = 2.03\text{ mm}$ ◀

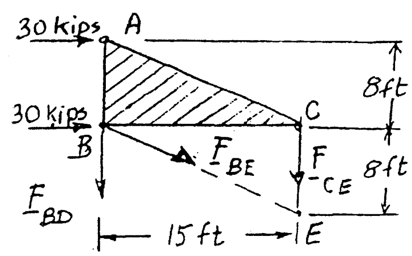


PROBLEM 2.24

For the steel truss $(E = 29 \times 10^6 \text{ psi})$ and loading shown, determine the deformations of the members *BD* and *DE*, knowing that their cross-sectional areas are 2 in^2 and 3 in^2, respectively.

SOLUTION

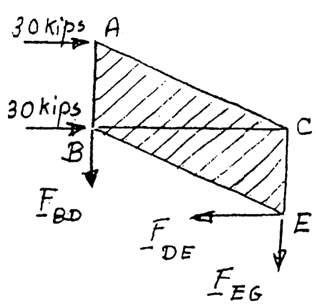
Free body: Portion *ABC* of truss



$$+\circlearrowleft \Sigma M_E = 0: \ F_{BD}(15 \text{ ft}) - (30 \text{ kips})(8 \text{ ft}) - (30 \text{ kips})(16 \text{ ft}) = 0$$

$$F_{BD} = +48.0 \text{ kips}$$

Free body: Portion *ABEC* of truss

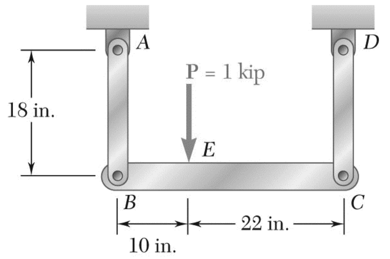


$$\xrightarrow{+} \Sigma F_x = 0: \ 30 \text{ kips} + 30 \text{ kips} - F_{DE} = 0$$

$$F_{DE} = +60.0 \text{ kips}$$

$$\delta_{BD} = \frac{PL}{AE} = \frac{(+48.0 \times 10^3 \text{ lb})(8 \times 12 \text{ in.})}{(2 \text{ in}^2)(29 \times 10^6 \text{ psi})} \qquad \delta_{BD} = +79.4 \times 10^{-3} \text{ in.} \blacktriangleleft$$

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+60.0 \times 10^3 \text{ lb})(15 \times 12 \text{ in.})}{(3 \text{ in}^2)(29 \times 10^6 \text{ psi})} \qquad \delta_{DE} \ +124.1 \times 10^{-3} \text{ in.} \blacktriangleleft$$

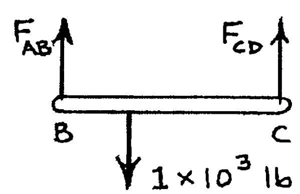


PROBLEM 2.25

Each of the links AB and CD is made of aluminum $(E = 10.9 \times 10^6 \text{psi})$ and has a cross-sectional area of 0.2 in.^{2}. Knowing that they support the rigid member BC, determine the deflection of point E.

SOLUTION

Free body BC:



$$+\circlearrowleft \Sigma M_C = 0: \; -(32)F_{AB} + (22)(1\times 10^3) = 0$$

$$F_{AB} = 687.5 \text{ lb}$$

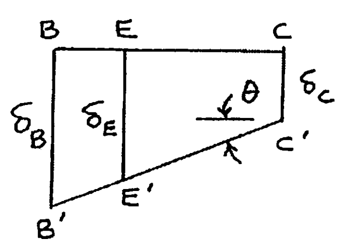
$$+\uparrow \Sigma F_y = 0: \; 687.5 - 1\times 10^3 + F_{CD} = 0$$

$$F_{CD} = 312.5 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB}\,L_{AB}}{EA} = \frac{(687.5)(18)}{(10.9\times 10^6)(0.2)} = 5.6766\times 10^{-3} \text{ in} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}\,L_{CD}}{EA} = \frac{(312.5)(18)}{(10.9\times 10^6)(0.2)} = 2.5803\times 10^{-3} \text{ in} = \delta_C$$

Deformation diagram:



$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{L_{BC}} = \frac{3.0963\times 10^{-3}}{32}$$

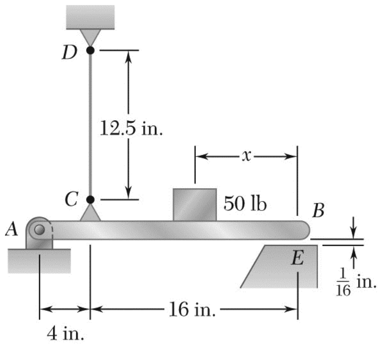
$$= 96.759\times 10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + L_{EC}\theta$$

$$= 2.5803\times 10^{-3} + (22)(96.759\times 10^{-6})$$

$$= 4.7090\times 10^{-3} \text{ in}$$

$$\delta_E = 4.71\times 10^{-3} \text{ in} \downarrow \blacktriangleleft$$



PROBLEM 2.26

The length of the $\frac{3}{32}$-in.-diameter steel wire CD has been adjusted so that with no load applied, a gap of $\frac{1}{16}$ in. exists between the end B of the rigid beam ACB and a contact point E. Knowing that $E = 29 \times 10^6$ psi, determine where a 50-lb block should be placed on the beam in order to cause contact between B and E.

SOLUTION

Rigid beam ACB rotates through angle θ to close gap.

$$\theta = \frac{1/16}{20} = 3.125 \times 10^{-3} \text{ rad}$$

Point C moves downward.

$$\delta_C = 4\theta = 4(3.125 \times 10^{-3}) = 12.5 \times 10^{-3} \text{ in.}$$

$$\delta_{CD} = \delta_C = 12.5 \times 10^{-3} \text{ in.}$$

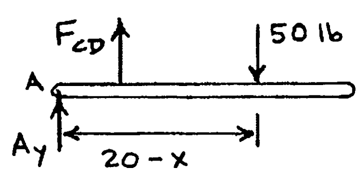
$$A_{CD} = \frac{\pi}{d} d^2 = \frac{\pi}{4}\left(\frac{3}{32}\right)^2 = 6.9029 \times 10^{-3} \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$$

$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(29 \times 10^6)(6.9029 \times 10^{-3})(12.5 \times 10^{-3})}{12.5}$$

$$= 200.18 \text{ lb}$$

Free body ACB:

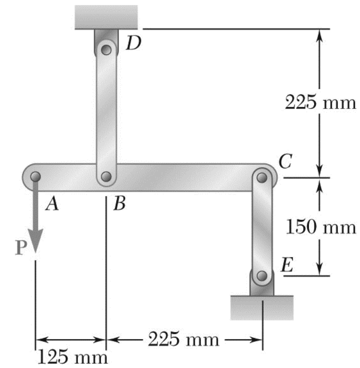


$$+\circlearrowleft \Sigma M_A = 0: \quad 4F_{CD} - (50)(20 - x) = 0$$

$$20 - x = \frac{(4)(200.18)}{50} = 16.0144$$

$$x = 3.9856 \text{ in.}$$

For contact, $x < 3.99$ in. ◀

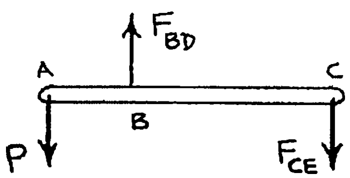


PROBLEM 2.27

Link BD is made of brass $(E = 105\text{ GPa})$ and has a cross-sectional area of 240 mm^2. Link CE is made of aluminum $(E = 72\text{ GPa})$ and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member ABC determine the maximum force **P** that can be applied vertically at point A if the deflection of A is not to exceed 0.35 mm.

SOLUTION

Free body member AC:

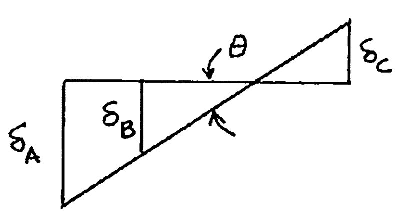


$$+\circlearrowleft \Sigma M_C = 0:\ 0.350P - 0.225F_{BD} = 0$$
$$F_{BD} = 1.55556P$$
$$+\circlearrowleft \Sigma M_B = 0:\ 0.125P - 0.225F_{CE} = 0$$
$$F_{CE} = 0.55556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.55556P)(0.225)}{(105\times10^9)(240\times10^{-6})} = 13.8889\times10^{-9}P$$
$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.55556P)(0.150)}{(72\times10^9)(300\times10^{-6})} = 3.8581\times10^{-9}P$$

<u>Deformation Diagram</u>:

From the deformation diagram,

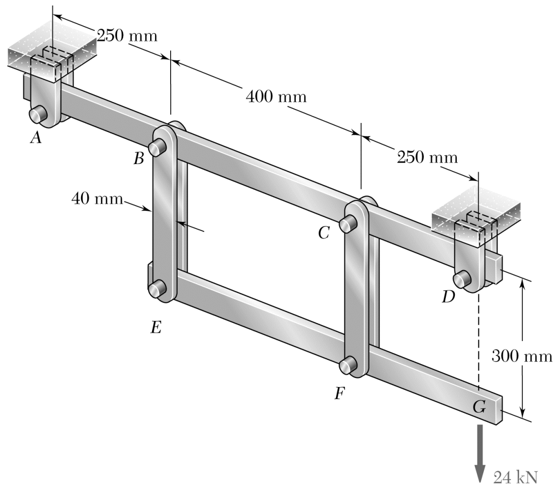


Slope, $$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{17.7470\times10^{-9}P}{0.225} = 78.876\times10^{-9}P$$
$$\delta_A = \delta_B + L_{AB}\theta$$
$$= 13.8889\times10^{-9}P + (0.125)(78.876\times10^{-9}P)$$
$$= 23.748\times10^{-9}P$$

Apply displacement limit. $\delta_A = 0.35\times10^{-3}\text{ m} = 23.748\times10^{-9}P$

$$P = 14.7381\times10^3\text{ N}$$

$P = 14.74$ kN ◀

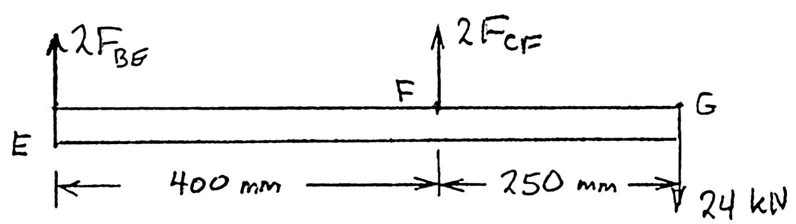


PROBLEM 2.28

Each of the four vertical links connecting the two rigid horizontal members is made of aluminum $(E = 70 \text{ GPa})$ and has a uniform rectangular cross section of 10×40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (c) point *G*.

SOLUTION

Statics. Free body *EFG*.



$$+\circlearrowleft \Sigma M_F = 0: \; -(400)(2F_{BE}) - (250)(24) = 0$$

$$F_{BE} = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

$$+\circlearrowleft \Sigma M_E = 0: \; (400)(2F_{CF}) - (650)(24) = 0$$

$$F_{CF} = 19.5 \text{ kN} = 19.5 \times 10^3 \text{ N}$$

Area of one link:

$$A = (10)(40) = 400 \text{ mm}^2$$

$$= 400 \times 10^{-6} \text{m}^2$$

Length: $L = 300 \text{ mm} = 0.300 \text{ m}$

Deformations.

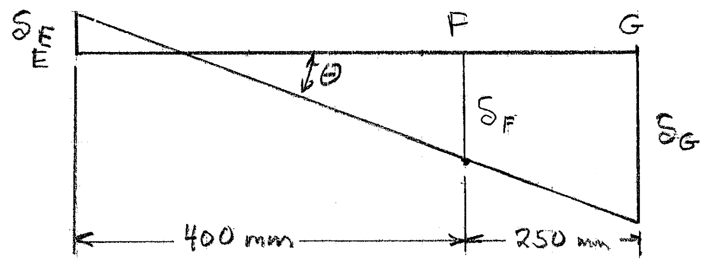
$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = -80.357 \times 10^{-6} \text{m}$$

$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = 208.93 \times 10^{-6} \text{m}$$

(a) <u>Deflection of Point E</u>. $\delta_E = |\delta_{BF}|$ $\delta_E = 80.4\ \mu\text{m} \uparrow$ ◀

(b) <u>Deflection of Point F</u>. $\delta_F = \delta_{CF}$ $\delta_F = 209\ \mu\text{m} \downarrow$ ◀

<u>Geometry change</u>.



Let θ be the small change in slope angle.

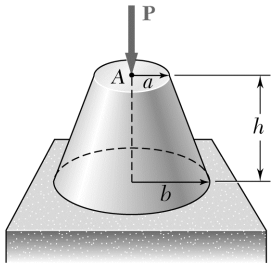
$$\theta = \frac{\delta_E + \delta_F}{L_{EF}} = \frac{80.357\times10^{-6} + 208.93\times10^{-6}}{0.400} = 723.22\times10^{-6} \text{ radians}$$

(c) <u>Deflection of Point G</u>. $\delta_G = \delta_F + L_{FG}\,\theta$

$$\delta_G = \delta_F + L_{FG}\,\theta = 208.93\times10^{-6} + (0.250)(723.22\times10^{-6})$$
$$= 389.73\times10^{-6}\,\text{m}$$

$\delta_G = 390\ \mu\text{m} \downarrow$ ◀

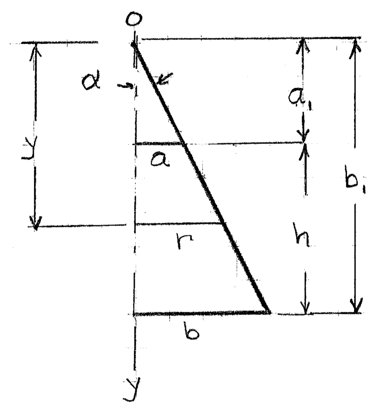
PROBLEM 2.29



A vertical load **P** is applied at the center A of the upper section of a homogeneous frustum of a circular cone of height h, minimum radius a, and maximum radius b. Denoting by E the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point A.

SOLUTION

Extend the slant sides of the cone to meet at a point O and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \dfrac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point y, $A = \pi r^2$

Deformation of element of height dy: $d\delta = \dfrac{Pdy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

Total deformation.

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1}-\frac{1}{b_1}\right)$$

$$= \frac{P}{\pi E\tan^2\alpha}\frac{b_1-a_1}{a_1b_1} = \frac{P(b_1-a_1)}{\pi Eab}$$

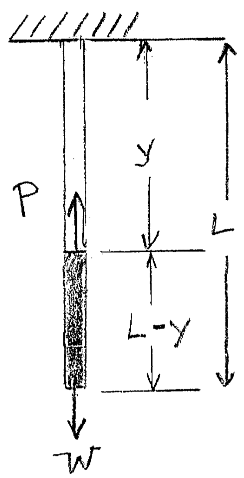
$$\delta_A = \frac{Ph}{\pi Eab}\downarrow \blacktriangleleft$$

PROBLEM 2.30

A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

SOLUTION

(*a*) For element at point identified by coordinate y,



$$P = \text{weight of portion below the point}$$
$$= \rho g\, A(L-y)$$
$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)dy}{EA} = \frac{\rho g(L-y)}{E}dy$$
$$\delta = \int_0^L \frac{\rho g(L-y)}{E}dy = \frac{\rho g}{E}\left(Ly - \frac{1}{2}y^2\right)\Bigg|_0^L$$
$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right)$$

$$\delta = \frac{1}{2}\frac{\rho g L^2}{E} \quad \blacktriangleleft$$

(*b*) Total weight: $W = \rho g A L$

$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \frac{1}{2}\rho g A L$$

$$F = \frac{1}{2}W \quad \blacktriangleleft$$

PROBLEM 2.31

The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\varepsilon_t = 2\ln(d_1/d)$.

SOLUTION

If the volume is constant, $\frac{\pi}{4}d^2 L = \frac{\pi}{4}d_1^2 L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = 2\ln\frac{d_1}{d} \blacktriangleleft$$

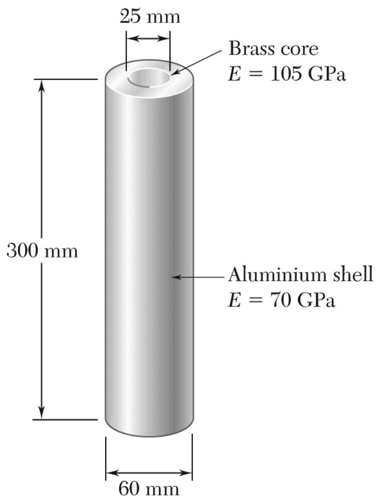
PROBLEM 2.32

Denoting by ε the "engineering strain" in a tensile specimen, show that the true strain is $\varepsilon_t = \ln(1+\varepsilon)$.

SOLUTION

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\frac{L_0+\delta}{L_0} = \ln\left(1+\frac{\delta}{L_0}\right) = \ln(1+\varepsilon)$$

Thus $\varepsilon_t = \ln(1+\varepsilon)$ ◀



PROBLEM 2.33

An axial force of 200 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the aluminum shell, (*b*) the corresponding deformation of the assembly.

SOLUTION

Let P_a = Portion of axial force carried by shell

P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus,

$$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with

$$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\text{ m}^2$$

$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\text{ m}^2$$

$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L}$$

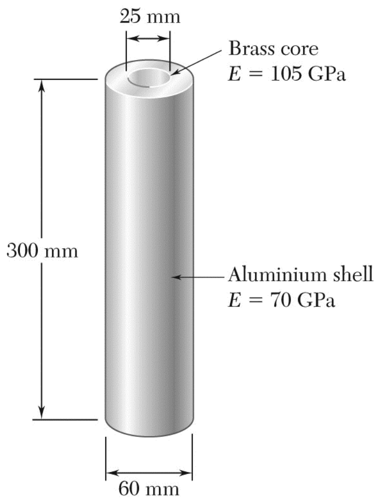
$$P = 215.10 \times 10^6 \frac{\delta}{L}$$

Strain:

$$\varepsilon = \frac{\delta}{L} = \frac{P}{215.10 \times 10^6} = \frac{200 \times 10^3}{215.10 \times 10^6} = 0.92980 \times 10^{-3}$$

(*a*) $\sigma_a = E_a \varepsilon = (70 \times 10^9)(0.92980 \times 10^{-3}) = 65.1 \times 10^6\text{ Pa}$ $\qquad \sigma_a = 65.1\text{ MPa}$ ◀

(*b*) $\delta = \varepsilon L = (0.92980 \times 10^{-3})(300\text{ mm})$ $\qquad \delta = 0.279\text{ mm}$ ◀



PROBLEM 2.34

The length of the assembly shown decreases by 0.40 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the brass core.

SOLUTION

Let P_a = Portion of axial force carried by shell and P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus, $$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with $$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\ \text{m}^2$$

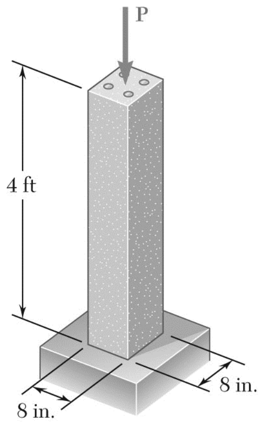
$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\ \text{m}^2$$

$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L} = 215.10 \times 10^6\,\frac{\delta}{L}$$

with $\delta = 0.40$ mm, $L = 300$ mm

(*a*) $$P = (215.10 \times 10^6)\frac{0.40}{300} = 286.8 \times 10^3\ \text{N}$$ $P = 287$ kN ◀

(*b*) $$\sigma_b = \frac{P_b}{A_b} = \frac{E_b \delta}{L} = \frac{(105 \times 10^9)(0.40 \times 10^{-3})}{300 \times 10^{-3}} = 140 \times 10^6\,\text{Pa}$$ $\sigma_b = 140.0\,\text{MPa}$ ◀



PROBLEM 2.35

A 4-ft concrete post is reinforced with four steel bars, each with a $\frac{3}{4}$-in. diameter. Knowing that $E_s = 29\times10^6$ psi and $E_c = 3.6\times10^6$ psi, determine the normal stresses in the steel and in the concrete when a 150-kip axial centric force **P** is applied to the post.

SOLUTION

$$A_s = 4\left[\frac{\pi}{4}\left(\frac{3}{4}\right)^2\right] = 1.76715 \text{ in}^2$$

$$A_c = 8^2 - A_s = 62.233 \text{ in}^2$$

$$\delta_s = \frac{P_s L}{A_s E_s} = \frac{P_s(48)}{(1.76715)(29\times10^6)} = 0.93663\times10^{-6}\ P_\text{s}$$

$$\delta_c = \frac{P_c L}{A_c E_c} = \frac{P_c(48)}{(62.233)(3.6\times10^6)} = 0.21425\times10^{-6}\ P_\text{c}$$

But $\delta_s = \delta_c$: $0.93663\times10^{-6} P_s = 0.21425\times10^{-6} P_c$

$$P_s = 0.22875 P_c \qquad (1)$$

Also: $P_s + P_c = P = 150$ kips (2)

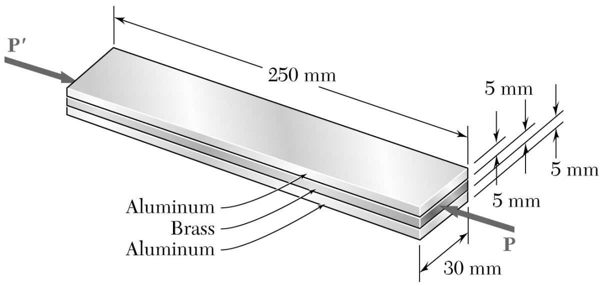
Substituting (1) into (2): $1.22875 P_c = 150$ kips

$$P_c = 122.075 \text{ kips}$$

From (1): $P_s = 0.22875(122.075) = 27.925$ kips

$$\sigma_s = -\frac{P_s}{A_s} = -\frac{27.925}{1.76715} \qquad \sigma_s = -15.80 \text{ ksi} \blacktriangleleft$$

$$\sigma_c = -\frac{P_c}{A_c} = -\frac{122.075}{62.233} \qquad \sigma_c = -1.962 \text{ ksi} \blacktriangleleft$$



PROBLEM 2.36

A 250-mm bar of 15×30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude P = 30 kN, and knowing that E_a = 70 GPa and E_b = 105 GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.

SOLUTION

For each layer,

$$A = (30)(5) = 150\ \text{mm}^2 = 150 \times 10^{-6}\,\text{m}^2$$

Let P_a = load on each aluminum layer

P_b = load on brass layer

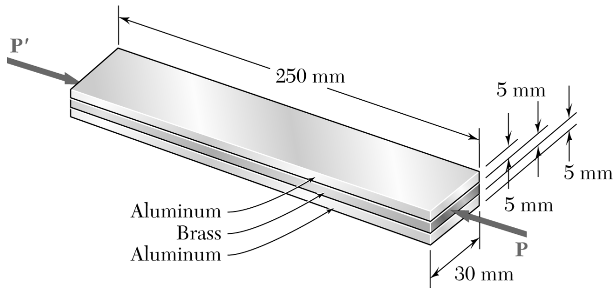
Deformation. $\delta = \dfrac{P_a L}{E_a A} = \dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5\ P_a$

Total force. $P = 2P_a + P_b = 3.5\ P_a$

Solving for P_a and P_b, $\quad P_a = \dfrac{2}{7}P \qquad P_b = \dfrac{3}{7}P$

(*a*) $\quad \sigma_a = -\dfrac{P_a}{A} = -\dfrac{2}{7}\dfrac{P}{A} = -\dfrac{2}{7}\dfrac{30\times 10^3}{150\times 10^{-6}} = -57.1\times 10^6\,\text{Pa}$ $\qquad \sigma_a = -57.1\ \text{MPa}$ ◀

(*b*) $\quad \sigma_b = -\dfrac{P_b}{A} = -\dfrac{3}{7}\dfrac{P}{A} = -\dfrac{3}{7}\dfrac{30\times 10^3}{150\times 10^{-6}} = -85.7\times 10^6\,\text{Pa}$ $\qquad \sigma_b = -85.7\ \text{MPa}$ ◀



PROBLEM 2.37

Determine the deformation of the composite bar of Prob. 2.36 if it is subjected to centric forces of magnitude $P = 45$ kN.

PROBLEM 2.36 A 250-mm bar of 15×30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.

SOLUTION

For each layer,

$$A = (30)(5) = 150\text{ mm}^2 = 150\times 10^{-6}\text{ m}^2$$

Let P_a = load on each aluminum layer

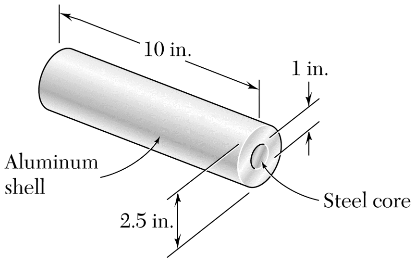
P_b = load on brass layer

<u>Deformation</u>. $\delta = -\dfrac{P_a L}{E_a A} = -\dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5\ P_a$

<u>Total force</u>. $P = 2P_a + P_b = 3.5\ \text{Pa}$ $\qquad P_a = \dfrac{2}{7} P$

$$\begin{aligned}\delta &= -\frac{P_a L}{E_a A} = -\frac{2}{7}\,\frac{PL}{E_a A}\\ &= -\frac{2}{7}\,\frac{(45\times 10^3)(250\times 10^{-3})}{(70\times 10^9)(150\times 10^{-6})}\\ &= -306\times 10^{-6}\text{ m}\end{aligned}$$

$\delta = -0.306$ mm ◀



PROBLEM 2.38

Compressive centric forces of 40 kips are applied at both ends of the assembly shown by means of rigid plates. Knowing that $E_s = 29\times10^6$ psi and $E_a = 10.1\times10^6$ psi, determine (*a*) the normal stresses in the steel core and the aluminum shell, (*b*) the deformation of the assembly.

SOLUTION

Let P_a = portion of axial force carried by shell

P_s = portion of axial force carried by core

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s}{L}\delta$$

Total force. $\quad P = P_a + P_s = (E_a A_a + E_s A_s)\dfrac{\delta}{L}$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_a A_a + E_s A_s}$$

Data: $\quad P = 40\times10^3\,\text{lb}$

$$A_a = \frac{\pi}{4}(d_0^2 - d_i^2) = \frac{\pi}{4}(2.5^2 - 1.0)^2 = 4.1233\ \text{in}^2$$

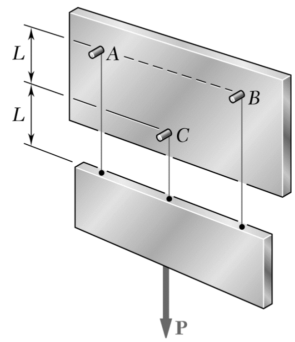
$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1)^2 = 0.7854\ \text{in}^2$$

$$\varepsilon = \frac{-40\times10^3}{(10.1\times10^6)(4.1233) + (29\times10^6)(0.7854)} = -620.91\times10^{-6}$$

(*a*) $\quad \sigma_s = E_s\varepsilon = (29\times10^6)(-620.91\times10^{-6}) = -18.01\times10^3\,\text{psi}$ $\quad -18.01$ ksi ◀

$\sigma_a = E_a\varepsilon = (10.1\times10^6)(620.91\times10^{-6}) = -6.27\times10^3\,\text{psi}$ $\quad -6.27$ ksi ◀

(*b*) $\quad \delta = L\varepsilon = (10)(620.91\times10^{-6}) = -6.21\times10^{-3}$ $\quad \delta = -6.21\times10^{-3}\,\text{in.}$ ◀



PROBLEM 2.39

Three wires are used to suspend the plate shown. Aluminum wires of $\frac{1}{8}$-in. diameter are used at A and B while a steel wire of $\frac{1}{12}$-in. diameter is used at C. Knowing that the allowable stress for aluminum $(E_a = 10.4\times10^6\,\text{psi})$ is 14 ksi and that the allowable stress for steel $(E_s = 29\times10^6\,\text{psi})$ is 18 ksi, determine the maximum load **P** that can be applied.

SOLUTION

By symmetry, $P_A = P_B$, and $\delta_A = \delta_B$

Also, $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire:

$$\varepsilon_A = \varepsilon_B = \frac{\delta}{2L}, \quad \varepsilon_C = \frac{\delta}{L} = 2\varepsilon_A$$

Determine <u>allowable strain</u>.

<u>Wires *A&B*</u>: $\varepsilon_A = \dfrac{\sigma_A}{E_A} = \dfrac{14\times10^3}{10.4\times10^6} = 1.3462\times10^{-3}$

$\varepsilon_C = 2\,\varepsilon_A = 2.6924\times10^{-4}$

<u>Wire *C*</u>: $\varepsilon_C = \dfrac{\sigma_C}{E_C} = \dfrac{18\times10^3}{29\times10^6} = 0.6207\times10^{-3}$

$\varepsilon_A = \varepsilon_B = \dfrac{1}{2}\varepsilon_C = 0.3103\times10^{-6}$

Allowable strain for wire C governs, $\quad\therefore\quad \sigma_C = 18\times10^3\,\text{psi}$

$$\sigma_A = E_A\varepsilon_A \quad P_A = A_A E_A \varepsilon_A$$

$$= \frac{\pi}{4}\left(\frac{1}{8}\right)^2 (10.4\times10^6)(0.3103\times10^{-6}) = 39.61\text{ lb}$$

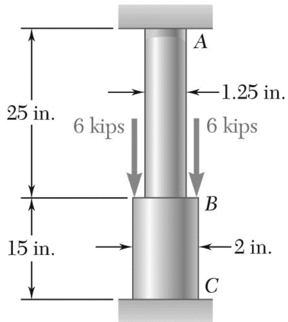
$$P_B = 39.61\text{ lb}$$

$$\sigma_C = E_C\varepsilon_C \quad P_C = A_C\sigma_C = \frac{\pi}{4}\left(\frac{1}{12}\right)^2 (18\times10^3) = 98.17\text{ lb}$$

For equilibrium of the plate,

$$P = P_A + P_B + P_C = 177.4\text{ lb}$$

P = 177.4 lb ◀

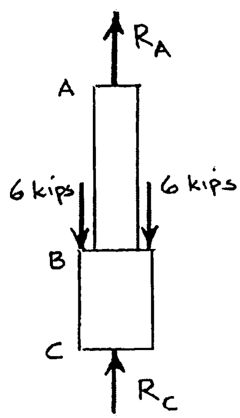


PROBLEM 2.40

A polystyrene rod consisting of two cylindrical portions AB and BC is restrained at both ends and supports two 6-kip loads as shown. Knowing that $E = 0.45\times10^6$ psi, determine (*a*) the reactions at A and C, (*b*) the normal stress in each portion of the rod.

SOLUTION

(*a*) We express that the elongation of the rod is zero:



$$\delta = \frac{P_{AB}L_{AB}}{\frac{\pi}{4}d_{AB}^2E} + \frac{P_{BC}L_{BC}}{\frac{\pi}{4}d_{BC}^2E} = 0$$

But $\quad P_{AB} = +R_A \qquad P_{BC} = -R_C$

Substituting and simplifying:

$$\frac{R_AL_{AB}}{d_{AB}^2} - \frac{R_CL_{BC}}{d_{BC}^2} = 0$$

$$R_C = \frac{L_{AB}}{L_{BC}}\left(\frac{d_{BC}}{d_{AB}}\right)^2 R_A = \frac{25}{15}\left(\frac{2}{1.25}\right)^2 R_A$$

$$R_C = 4.2667R_A \qquad (1)$$

From the free body diagram: $\quad R_A + R_C = 12$ kips $\qquad (2)$

Substituting (1) into (2): $\quad 5.2667\ R_A = 12$

$$R_A = 2.2785 \text{ kips}$$

$R_A = 2.28$ kips $\uparrow$ ◀

From (1): $\quad R_C = 4.2667(2.2785) = 9.7217$ kips

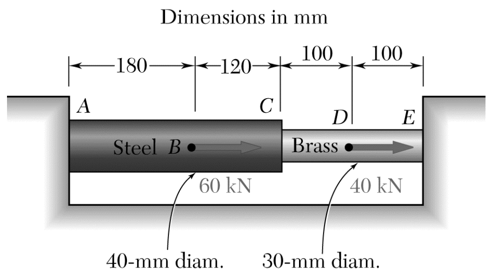
$R_C = 9.72$ kips $\uparrow$ ◀

(*b*) $$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{+R_A}{A_{AB}} = \frac{2.2785}{\frac{\pi}{4}(1.25)^2}$$

$\sigma_{AB} = +1.857$ ksi ◀

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{-R_C}{A_{BC}} = \frac{-9.7217}{\frac{\pi}{4}(2)^2}$$

$\sigma_{BC} = -3.09$ ksi ◀



PROBLEM 2.41

Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

SOLUTION

$\underline{A \text{ to } C}$: $\quad E = 200 \times 10^9 \text{ Pa}$

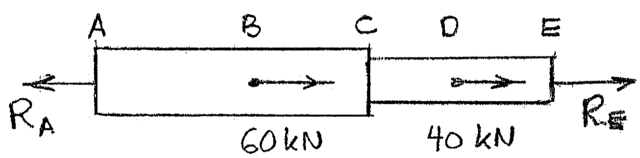
$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$EA = 251.327 \times 10^6 \text{ N}$$

$\underline{C \text{ to } E}$: $\quad E = 105 \times 10^9 \text{ Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$EA = 74.220 \times 10^6 \text{ N}$$



$\underline{A \text{ to } B}$: $\quad P = R_A$

$$L = 180 \text{ mm} = 0.180 \text{ m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6}$$

$$= 716.20 \times 10^{-12} R_A$$

$\underline{B \text{ to } C}$: $\quad P = R_A - 60 \times 10^3$

$$L = 120 \text{ mm} = 0.120 \text{ m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6}$$

$$= 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

PROBLEM 2.41 *(Continued)*

<u>C to D</u>: $P = R_A - 60\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 80.841\times10^{-6}$$

<u>D to E</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 134.735\times10^{-6}$$

<u>A to E</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837\times10^{-9} R_A - 242.424\times10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(*a*) $3.85837\times10^{-9} R_A - 242.424\times10^{-6} = 0 \quad R_A = 62.831\times10^3 \text{ N}$ $\qquad R_A = 62.8 \text{ kN} \leftarrow$ ◀

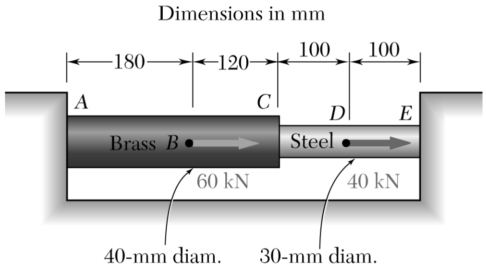
$R_E = R_A - 100\times10^3 = 62.8\times10^3 - 100\times10^3 = -37.2\times10^3 \text{ N}$ $\qquad R_E = 37.2 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times10^{-9} R_A - 26.848\times10^{-6}$

$$= (1.16369\times10^{-9})(62.831\times10^3) - 26.848\times10^{-6}$$

$$= 46.3\times10^{-6} \text{ m}$$

$\delta_C = 46.3 \ \mu\text{m} \rightarrow$ ◀



PROBLEM 2.42

Solve Prob. 2.41, assuming that rod AC is made of brass and rod CE is made of steel.

PROBLEM 2.41 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

SOLUTION

A to C: $E = 105\times 10^9\,\text{Pa}$

$$A = \frac{\pi}{4}(40)^2 = 1.25664\times 10^3\,\text{mm}^2 = 1.25664\times 10^{-3}\,\text{m}^2$$

$$EA = 131.947\times 10^6\,\text{N}$$

C to E: $E = 200\times 10^9\,\text{Pa}$

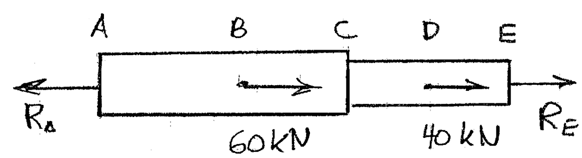
$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times 10^{-6}\,\text{m}^2$$

$$EA = 141.372\times 10^6\,\text{N}$$

A to B: $P = R_A$

$L = 180\ \text{mm} = 0.180\ \text{m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947\times 10^6} = 1.36418\times 10^{-9} R_A$$



B to C: $P = R_A - 60\times 10^3$

$L = 120\ \text{mm} = 0.120\ \text{m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times 10^3)(0.120)}{131.947\times 10^6} = 909.456\times 10^{-12} R_A - 54.567\times 10^{-6}$$

C to D: $P = R_A - 60\times 10^3$

$L = 100\ \text{mm} = 0.100\ \text{m}$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60\times 10^3)(0.100)}{141.372\times 10^6} = 707.354\times 10^{-12} R_A - 42.441\times 10^{-6}$$

PROBLEM 2.42 *(Continued)*

<u>D to E</u>: $P = R_A - 100\times 10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times 10^3)(0.100)}{141.372\times 10^6}$$

$$= 707.354\times 10^{-12} R_A - 70.735\times 10^{-6}$$

<u>A to E</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.68834\times 10^{-9} R_A - 167.743\times 10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

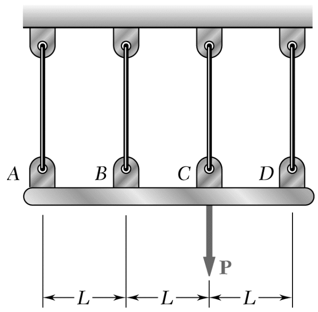
(*a*) $3.68834\times 10^{-9} R_A - 167.743\times 10^{-6} = 0 \quad R_A = 45.479\times 10^3 \text{ N}$ $\qquad R_A = 45.5 \text{ kN} \leftarrow$ ◀

$R_E = R_A - 100\times 10^3 = 45.479\times 10^3 - 100\times 10^3 = -54.521\times 10^3$ $\qquad R_E = 54.5 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364\times 10^{-9} R_A - 54.567\times 10^{-6}$

$$= (2.27364\times 10^{-9})(45.479\times 10^3) - 54.567\times 10^{-6}$$

$= 48.8\times 10^{-6} \text{ m}$ $\qquad \delta_C = 48.8\,\mu\text{m} \rightarrow$ ◀



PROBLEM 2.43

The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load **P** shown.

SOLUTION

<u>Deformations</u> Let θ be the rotation of bar *ABCD* and δ_A, δ_B, δ_C and δ_D be the deformations of wires *A*, *B*, *C*, and *D*.

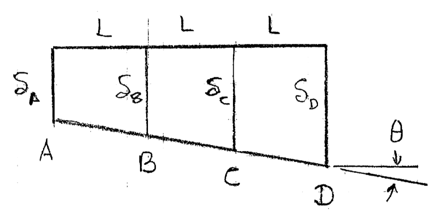
From geometry,

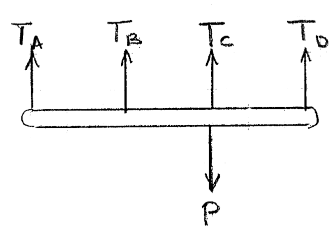
$$\theta = \frac{\delta_B - \delta_A}{L}$$

$$\delta_B = \delta_A + L\theta$$

$$\delta_C = \delta_A + 2L\theta = 2\delta_B - \delta_A \qquad (1)$$

$$\delta_D = \delta_A + 3L\theta = 3\delta_B - 2\delta_A \qquad (2)$$





Since all wires are identical, the forces in the wires are proportional to the deformations.

$$T_C = 2T_B - T_A \qquad (1')$$

$$T_D = 3T_B - 2T_A \qquad (2')$$

PROBLEM 2.43 *(Continued)*

Use bar *ABCD* as a free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -2LT_A - LT_B + LT_D = 0 \tag{3}$$

$$+\uparrow \Sigma F_y = 0: \quad T_A + T_B + T_C + T_D - P = 0 \tag{4}$$

Substituting (2′) into (3) and dividing by L,

$$-4T_A + 2T_B = 0 \qquad T_B = 2T_A \tag{3′}$$

Substituting (1′), (2′), and (3′) into (4),

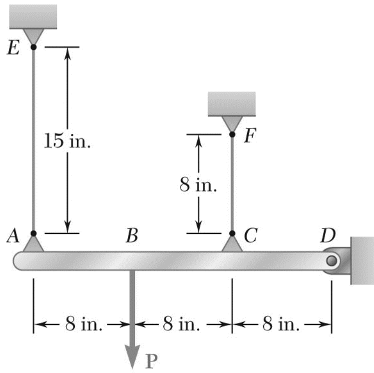
$$T_A + 2T_A + 3T_A + 4T_A - P = 0 \qquad 10T_A = P$$

$$T_A = \frac{1}{10}P \blacktriangleleft$$

$$T_B = 2T_A = (2)\left(\frac{1}{10}\right)P \qquad T_B = \frac{1}{5}P \blacktriangleleft$$

$$T_C = (2)\left(\frac{1}{5}P\right) - \left(\frac{1}{10}P\right) \qquad T_C = \frac{3}{10}P \blacktriangleleft$$

$$T_D = (3)\left(\frac{1}{5}P\right) - (2)\left(\frac{1}{10}P\right) \qquad T_D = \frac{2}{5}P \blacktriangleleft$$



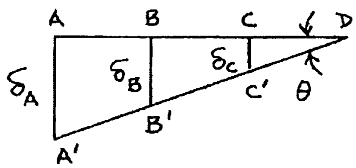
PROBLEM 2.44

The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter $(E = 29 \times 10^6 \text{ psi})$ and a pin and bracket at D. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 120-lb load **P** is applied at B, (*b*) the corresponding deflection of point B.

SOLUTION

Let θ be the rotation of bar $ABCD$.

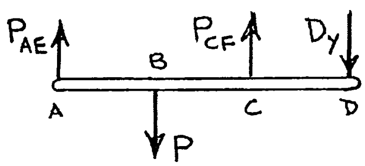
Then $\quad \delta_A = 24\theta \qquad \delta_C = 8\theta$



$$\delta_A = \frac{P_{AE}L_{AE}}{AE}$$

$$P_{AE} = \frac{EA\delta_A}{L_{AE}} = \frac{(29\times10^6)\frac{\pi}{4}(\frac{1}{16})^2(24\theta)}{15}$$

$$= 142.353\times10^3\theta$$



$$\delta_C = \frac{P_{CF}L_{CF}}{AE}$$

$$P_{CF} = \frac{EA\delta_C}{L_{CF}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(8\theta)}{8}$$

$$= 88.971\times10^3\theta$$

Using free body $ABCD$,

$$+\circlearrowleft \Sigma M_D = 0: \qquad -24P_{AE} + 16P - 8P_{CF} = 0$$

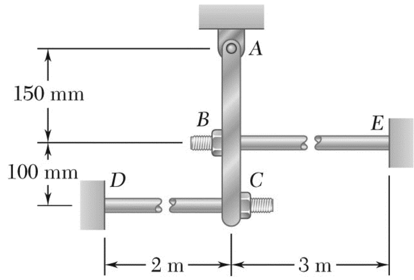
$$-24(142.353\times10^3\theta) + 16(120) - 8(88.971\times10^3\theta) = 0$$

$$\theta = 0.46510\times10^{-3} \text{ rad} \circlearrowleft$$

(*a*) $\quad P_{AE} = (142.353\times10^3)(0.46510\times10^{-3})$ $\qquad P_{AE} = 66.2 \text{ lb}$ ◀

$\quad P_{CF} = (88.971\times10^3)(0.46510\times10^{-3})$ $\qquad P_{CF} = 41.4 \text{ lb}$ ◀

(*b*) $\quad \delta_B = 16\theta = 16(0.46510\times10^{-3})$ $\qquad \delta_B = 7.44\times10^{-3} \text{ in.}\downarrow$ ◀

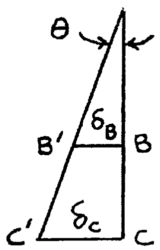


PROBLEM 2.45

The steel rods BE and CD each have a 16-mm diameter ($E = 200$ GPa); the ends of the rods are single-threaded with a pitch of 2.5 mm. Knowing that after being snugly fitted, the nut at C is tightened one full turn, determine (*a*) the tension in rod CD, (*b*) the deflection of point C of the rigid member ABC.

SOLUTION

Let θ be the rotation of bar ABC as shown.



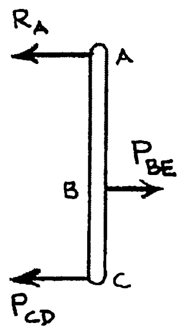
Then $\delta_B = 0.15\theta \qquad \delta_C = 0.25\theta$

But $\delta_C = \delta_{\text{turn}} - \dfrac{P_{CD}L_{CD}}{E_{CD}A_{CD}}$

$$P_{CD} = \frac{E_{CD}A_{CD}}{L_{CD}}(\delta_{\text{turn}} - \delta_C)$$

$$= \frac{(200\times 10^9 \text{ Pa})\frac{\pi}{4}(0.016 \text{ m})^2}{2\,\text{m}}(0.0025\,\text{m} - 0.25\theta)$$

$$= 50.265\times 10^3 - 5.0265\times 10^6\theta$$



$$\delta_B = \frac{P_{BE}L_{BE}}{E_{BE}A_{BE}} \quad \text{or} \quad P_{BE} = \frac{E_{BE}A_{BE}}{L_{BE}}\delta_B$$

$$P_{BE} = \frac{(200\times 10^9 \text{ Pa})\frac{\pi}{4}(0.016 \text{ m})^2}{3 \text{ m}}(0.15\theta)$$

$$= 2.0106\times 10^6\theta$$

From free body of member ABC:

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.15P_{BE} - 0.25P_{CD} = 0$$

$$0.15(2.0106\times 10^6\theta) - 0.25(50.265\times 10^3 - 5.0265\times 10^6\theta) = 0$$

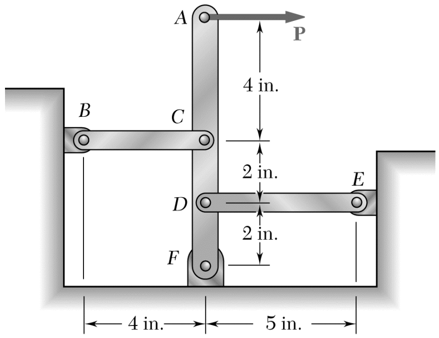
$$\theta = 8.0645\times 10^{-3} \text{ rad}$$

(*a*) $P_{CD} = 50.265\times 10^3 - 5.0265\times 10^6(8.0645\times 10^{-3})$

$= 9.7288\times 10^3$ N $\qquad P_{CD} = 9.73$ kN ◀

(*b*) $\delta_C = 0.25\theta = 0.25(8.0645\times 10^{-3})$

$= 2.0161\times 10^{-3}$ m $\qquad \delta_C = 2.02 \text{ mm} \leftarrow$ ◀



PROBLEM 2.46

Links BC and DE are both made of steel $(E = 29 \times 10^6 \text{ psi})$ and are $\frac{1}{2}$ in. wide and $\frac{1}{4}$ in. thick. Determine (*a*) the force in each link when a 600-lb force **P** is applied to the rigid member AF shown, (*b*) the corresponding deflection of point A.

SOLUTION

Let the rigid member $ACDF$ rotate through small angle θ clockwise about point F.

Then $\quad \delta_C = \delta_{BC} = 4\theta \text{ in.} \rightarrow$

$$\delta_D = -\delta_{DE} = 2\theta \text{ in.} \rightarrow$$

$$\delta = \frac{FL}{EA} \quad \text{or} \quad F = \frac{EA\delta}{L}$$

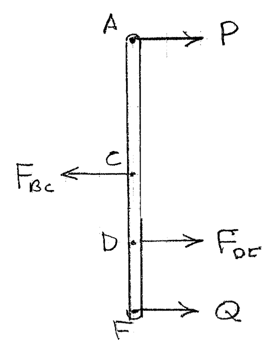
For links: $\quad A = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = 0.125 \text{ in}^2$

$$L_{BC} = 4 \text{ in.}$$

$$L_{DE} = 5 \text{ in.}$$

$$F_{BC} = \frac{EA\delta_{BC}}{L_{BC}} = \frac{(29 \times 10^6)(0.125)(4\theta)}{4} = 3.625 \times 10^6 \theta$$

$$F_{DE} = \frac{EA\delta_{DE}}{L_{DE}} = \frac{(29 \times 10^6)(0.125)(-2\theta)}{5} = -1.45 \times 10^6 \theta$$



Use member $ACDF$ as a free body.

$$+\circlearrowleft \Sigma M_F = 0: \quad 8P - 4F_{BC} + 2F_{DE} = 0$$

$$P = \frac{1}{2}F_{BC} - \frac{1}{4}F_{DE}$$

$$600 = \frac{1}{2}(3.625 \times 10^6)\theta - \frac{1}{4}(-1.45 \times 10^6)\theta = 2.175 \times 10^6 \theta$$

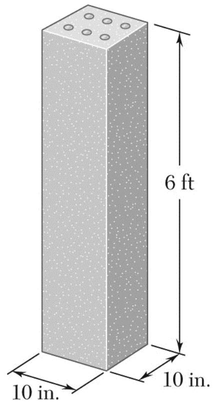
$$\theta = 0.27586 \times 10^{-3} \text{rad} \circlearrowright$$

(*a*) $\quad F_{BC} = (3.625 \times 10^6)(0.27586 \times 10^{-3}) \qquad F_{BC} = 1000 \text{ lb}$ ◀

$F_{DE} = -(1.45 \times 10^6)(0.27586 \times 10^{-3}) \qquad F_{DE} = -400 \text{ lb}$ ◀

(*b*) <u>Deflection at Point *A*</u>.

$\delta_A = 8\theta = (8)(0.27586 \times 10^{-3}) \qquad \delta_A = 2.21 \times 10^{-3} \text{in} \rightarrow$ ◀



PROBLEM 2.47

The concrete post ($E_c = 3.6\times10^6$ psi and $\alpha_c = 5.5\times10^{-6}/°\text{F}$) is reinforced with six steel bars, each of $\frac{7}{8}$-in. diameter ($E_s = 29\times10^6$ psi and $\alpha_s = 6.5\times10^{-6}/°\text{F}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65°F.

SOLUTION

$$A_s = 6\frac{\pi}{4}d^2 = 6\frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 3.6079 \text{ in}^2$$

$$A_c = 10^2 - A_s = 10^2 - 3.6079 = 96.392 \text{ in}^2$$

Let $P_c =$ tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c.

Strains: $\quad \varepsilon_s = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T) \qquad \varepsilon_c = \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s \qquad \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T) = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$

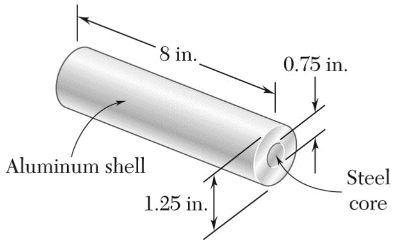
$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

$$\left[\frac{1}{(3.6\times10^6)(96.392)} + \frac{1}{(29\times10^6)(3.6079)}\right]P_c = (1.0\times10^{-6})(65)$$

$$P_c = 5.2254\times10^3 \text{ lb}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{5.2254\times10^3}{96.392} = 54.210 \text{ psi} \qquad \sigma_c = 54.2 \text{ psi}$$

$$\sigma_s = -\frac{P_c}{A_s} = -\frac{5.2254\times10^3}{3.6079} = -1448.32 \text{ psi} \qquad \sigma_s = -1.448 \text{ ksi}$$



PROBLEM 2.48

The assembly shown consists of an aluminum shell ($E_a = 10.6 \times 10^6$ psi, α_a = 12.9 × 10^{-6}/°F) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, α_s = 6.5 × 10^{-6}/°F) and is unstressed. Determine (*a*) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (*b*) the corresponding change in length of the assembly.

SOLUTION

Since $\alpha_a > \alpha_s$, the shell is in compression for a positive temperature rise.

Let
$$\sigma_a = -6 \text{ ksi} = -6\times 10^3 \text{ psi}$$

$$A_a = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(1.25^2 - 0.75^2) = 0.78540 \text{ in}^2$$

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$P = -\sigma_a A_a = \sigma_s A_s$$

where P is the tensile force in the steel core.

$$\sigma_s = -\frac{\sigma_a A_a}{A_s} = \frac{(6\times 10^3)(0.78540)}{0.44179} = 10.667\times 10^3 \text{ psi}$$

$$\varepsilon = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_a}{E_a} + \alpha_a(\Delta T)$$

$$(\alpha_a - \alpha_s)(\Delta T) = \frac{\sigma_s}{E_s} - \frac{\sigma_a}{E_a}$$

$$(6.4\times 10^{-6})(\Delta T) = \frac{10.667\times 10^3}{29\times 10^6} + \frac{6\times 10^3}{10.6\times 10^6} = 0.93385\times 10^{-3}$$

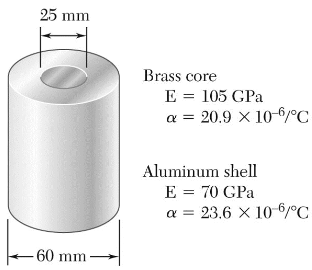
(*a*) $\Delta T = 145.91°\text{F}$ $\Delta T = 145.9°\text{F}$ ◀

(*b*) $$\varepsilon = \frac{10.667\times 10^3}{29\times 10^6} + (6.5\times 10^{-6})(145.91) = 1.3163\times 10^{-3}$$

or

$$\varepsilon = \frac{-6\times 10^3}{10.6\times 10^6} + (12.9\times 10^{-6})(145.91) = 1.3163\times 10^{-3}$$

$\delta = L\varepsilon = (8.0)(1.3163\times 10^{-3}) = 0.01053$ in. $\delta = 0.01053$ in. ◀



PROBLEM 2.49

The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Brass core:

$$E = 105 \text{ GPa}$$
$$\alpha = 20.9\times10^{-6}/^\circ\text{C}$$

Aluminum shell:

$$E = 70 \text{ GPa}$$
$$\alpha = 23.6\times10^{-6}/^\circ\text{C}$$

Let L be the length of the assembly.

Free thermal expansion:

$$\Delta T = 195 - 15 = 180\,^\circ\text{C}$$

Brass core: $(\delta_T)_b = L\alpha_b(\Delta T)$

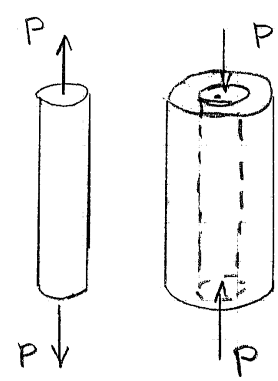
Aluminum shell: $(\delta_T) = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_b)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Brass core:

$$E_b = 105\times10^9 \text{ Pa}$$
$$A_b = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2$$
$$= 490.87\times10^{-6} \text{ m}^2$$
$$(\delta_P)_b = \frac{PL}{E_b A_b}$$



PROBLEM 2.49 *(Continued)*

Aluminum shell:

$$E_a = 70\times 10^9\,\text{Pa}$$

$$A_a = \frac{\pi}{4}(60^2 - 25^2)$$

$$= 2.3366\times 10^3\,\text{mm}^2$$

$$= 2.3366\times 10^{-3}\,\text{m}^2$$

$$\delta = (\delta_P)_b + (\delta_P)_a$$

$$L(\alpha_b - \alpha_a)(\Delta T) = \frac{PL}{E_b A_b} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_b A_b} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(105\times 10^9)(490.87\times 10^{-6})} + \frac{1}{(70\times 10^9)(2.3366\times 10^{-3})}$$

$$= 25.516\times 10^{-9}\,\text{N}^{-1}$$

Then

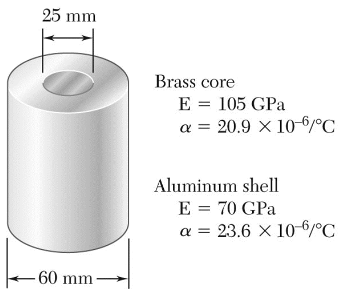
$$P = \frac{(\alpha_b - \alpha_a)(\Delta T)}{K}$$

$$= \frac{(23.6\times 10^{-6} - 20.9\times 10^{-6})(180)}{25.516\times 10^{-9}}$$

$$= 19.047\times 10^3\,\text{N}$$

Stress in aluminum:

$$\sigma_a = -\frac{P}{A_a} = -\frac{19.047\times 10^3}{2.3366\times 10^{-3}} = -8.15\times 10^6\,\text{Pa} \qquad \sigma_a = -8.15\text{ MPa} \blacktriangleleft$$



PROBLEM 2.50

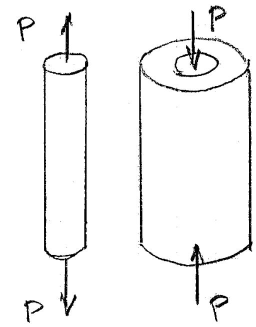
Solve Prob. 2.49, assuming that the core is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) instead of brass.

PROBLEM 2.49 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Aluminum shell: $E = 70$ GPa $\alpha = 23.6 \times 10^{-6}/°\text{C}$

Let L be the length of the assembly.



Free thermal expansion: $\Delta T = 195 - 15 = 180°\text{C}$

Steel core: $(\delta_T)_s = L\alpha_s(\Delta T)$

Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_s)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Steel core: $E_s = 200 \times 10^9\,\text{Pa}, \quad A_s = \frac{\pi}{4}(25)^2 = 490.87\ \text{mm}^2 = 490.87 \times 10^{-6}\,\text{m}^2$

$$(\delta_P)_s = \frac{PL}{E_s A_s}$$

Aluminum shell: $E_a = 70 \times 10^9\,\text{Pa}$

$$(\delta_P)_a = \frac{PL}{E_a A_a}$$

$$A_a = \frac{\pi}{4}(60^2 - 25)^2 = 2.3366 \times 10^3\,\text{mm}^2 = 2.3366 \times 10^{-3}\,\text{m}^2$$

$$\delta = (\delta_P)_s + (\delta_P)_a$$

$$L(\alpha_a - \alpha_s)(\Delta T) = \frac{PL}{E_s A_s} + \frac{PL}{E_a A_a} = KPL$$

where

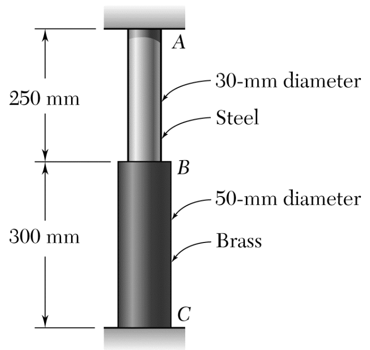
$$K = \frac{1}{E_s A_s} + \frac{1}{E_a A_a}$$
$$= \frac{1}{(200 \times 10^9)(490.87 \times 10^{-6})} + \frac{1}{(70 \times 10^9)(2.3366 \times 10^{-3})}$$
$$= 16.2999 \times 10^{-9}\,\text{N}^{-1}$$

PROBLEM 2.50 *(Continued)*

Then

$$P = \frac{(\alpha_a - \alpha_s)(\Delta T)}{K} = \frac{(23.6\times10^{-6} - 11.7\times10^{-6})(180)}{16.2999\times10^{-9}} = 131.41\times10^3\,\text{N}$$

Stress in aluminum: $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{131.19\times10^3}{2.3366\times10^{-3}} = -56.2\times10^6\,\text{Pa}$ $\qquad\sigma_a = -56.2\text{ MPa}$ ◀



PROBLEM 2.51

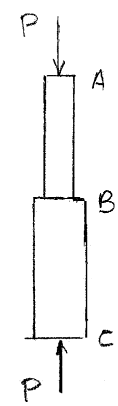
A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}/°\text{C}$) and portion BC is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°\text{C}$). Knowing that the rod is initially unstressed, determine the compressive force induced in ABC when there is a temperature rise of $50\,°\text{C}$.

SOLUTION

$$A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635\times10^3\,\text{mm}^2 = 1.9635\times10^{-3}\,\text{m}^2$$

Free thermal expansion:



$$\begin{aligned}\delta_T &= L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)\\ &= (0.250)(11.7\times10^{-6})(50) + (0.300)(20.9\times10^{-6})(50)\\ &= 459.75\times10^{-6}\,\text{m}\end{aligned}$$

Shortening due to induced compressive force P:

$$\begin{aligned}\delta_P &= \frac{PL}{E_sA_{AB}} + \frac{PL}{E_bA_{BC}}\\ &= \frac{0.250P}{(200\times10^9)(706.86\times10^{-6})} + \frac{0.300P}{(105\times10^9)(1.9635\times10^{-3})}\\ &= 3.2235\times10^{-9}P\end{aligned}$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235\times10^{-9}P = 459.75\times10^{-6}$$

$$P = 142.62\times10^3\,\text{N}$$

$P = 142.6$ kN ◀

PROBLEM 2.52

A steel railroad track $(E_s = 200\text{ GPa}, \alpha_s = 11.7\times10^{-6}/°\text{C})$ was laid out at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

SOLUTION

(*a*) $\delta_T = \alpha(\Delta T)L = (11.7\times10^{-6})(48-6)(10) = 4.914\times10^{-3}\text{ m}$

$$\delta_P = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200\times10^9} = 50\times10^{-12}\sigma$$

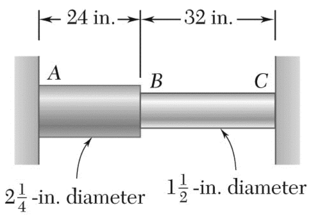
$$\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\sigma = 0$$

$\sigma = -98.3\times10^6\text{ Pa}$ $\qquad\qquad \sigma = -98.3\text{ MPa}$

(*b*) $\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\sigma = 3\times10^{-3}$

$$\sigma = \frac{3\times10^{-3} - 4.914\times10^{-3}}{50\times10^{-12}}$$

$= -38.3\times10^6\text{ Pa}$ $\qquad\qquad \sigma = -38.3\text{ MPa}$

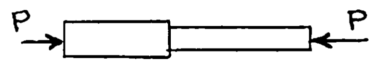


PROBLEM 2.53

A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and portion BC is made of aluminum ($E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}/°\text{F}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 70°F, (*b*) the corresponding deflection of point B.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \quad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$



<u>Free thermal expansion.</u> $\Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_s(\Delta T) + (24)(6.5\times10^{-6})(70) = 10.92\times10^{-3}\text{ in}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_a(\Delta T) = (32)(13.3\times10^{-6}(70) = 29.792\times10^{-3}\text{ in.}$$

Total: $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 40.712\times10^{-3}\text{ in.}$

<u>Shortening due to induced compressive force P.</u>

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_s A_{AB}} = \frac{24P}{(29\times10^6)(3.9761)} = 208.14\times10^{-9}P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_a A_{BC}} = \frac{32P}{(10.4\times10^6)(1.76715)} = 1741.18\times10^{-9}P$$

Total: $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1949.32\times10^{-9}P$

For zero net deflection, $\delta_P = \delta_T$ $\quad 1949.32\times10^{-9}\ P = 40.712\times10^{-3}$ $\quad P = 20.885\times10^3$ lb

(*a*) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{20.885\times10^3}{3.9761} = -5.25\times10^3$ psi $\quad \sigma_{AB} = -5.25$ ksi ◀

$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{20.885\times10^3}{1.76715} = -11.82\times10^3$ psi $\quad \sigma_{BC} = -11.82$ ksi ◀

(*b*) $(\delta_P)_{AB} = (208.14\times10^{-9})(20.885\times10^3) = 4.3470\times10^{-3}\text{ in.}$

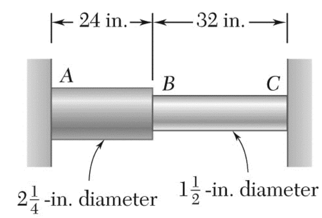
$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 10.92\times10^{-3} \rightarrow + 4.3470\times10^{-3} \leftarrow$ $\quad \delta_B = 6.57\times10^{-3}$ in. $\rightarrow$ ◀

or

$(\delta_P)_{BC} = (1741.18\times10^{-9})(20.885\times10^3) = 36.365\times10^{-3}\text{ in.}$

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 29.792\times10^{-3} \leftarrow + 36.365\times10^{-3} \rightarrow = 6.57\times10^{-3}\text{ in.} \rightarrow$ (checks)

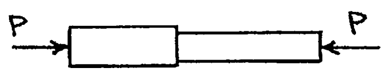
PROBLEM 2.54



Solve Prob. 2.53, assuming that portion *AB* of the composite rod is made of aluminum and portion *BC* is made of steel.

PROBLEM 2.53 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel $(E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F})$ and portion *BC* is made of aluminum $(E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}/°\text{F})$. Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 70°F, (*b*) the corresponding deflection of point *B*.

SOLUTION



$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \qquad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

<u>Free thermal expansion.</u> $\Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_a(\Delta T) = (24)(13.3\times10^{-6})(70) = 22.344\times10^{-3} \text{ in.}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_s(\Delta T) = (32)(6.5\times10^{-6})(70) = 14.56\times10^{-3} \text{ in.}$$

Total: $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 36.904\times10^{-3} \text{ in.}$

<u>Shortening due to induced compressive force *P*.</u>

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_a A_{AB}} = \frac{24\,P}{(10.4\times10^6)(3.9761)} = 580.39\times10^{-9} P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_s A_{BC}} = \frac{32\,P}{(29\times10^6)(1.76715)} = 624.42\times10^{-9} P$$

Total: $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1204.81\times10^{-9} P$

For zero net deflection, $\delta_P = \delta_T$ $\qquad 1204.81\times10^{-9} P = 36.904\times10^{-3} \qquad P = 30.631\times10^3$ lb

(*a*) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{30.631\times10^3}{3.9761} = -7.70\times10^3$ psi $\qquad \sigma_{AB} = -7.70$ ksi ◀

$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{30.631\times10^3}{1.76715} = -17.33\times10^3$ psi $\qquad \sigma_{BC} = -17.33$ ksi ◀

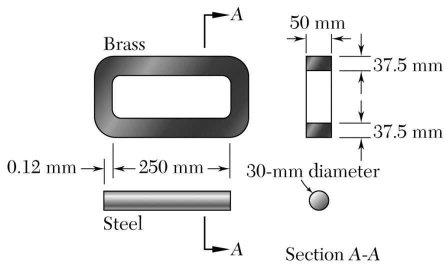
PROBLEM 2.54 *(Continued)*

(*b*) $(\delta_P)_{AB} = (580.39\times10^{-9})(30.631\times10^{3}) = 17.7779\times10^{-3}\,\text{in.}$

$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 22.344\times10^{-3} \rightarrow + 17.7779\times10^{-3} \leftarrow$ $\qquad \delta_B = 4.57\times10^{-3}\,\text{in.} \rightarrow$ ◀

or $(\delta_P)_{BC} = (624.42\times10^{-9})(30.631\times10^{3}) = 19.1266\times10^{-3}\,\text{in.}$

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 14.56\times10^{-3} \leftarrow + 19.1266\times10^{-3} \rightarrow = 4.57\times10^{-3}\,\text{in.} \rightarrow$ (checks)



PROBLEM 2.55

A brass link ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°\text{C}$) and a steel rod ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) have the dimensions shown at a temperature of 20°C. The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 45°C. Determine (*a*) the final stress in the steel rod, (*b*) the final length of the steel rod.

SOLUTION

Initial dimensions at $T = 20$ °C.

Final dimensions at $T = 45$ °C.

$$\Delta T = 45 - 20 = 25 \text{ °C}$$

<u>Free thermal expansion of each part</u>:

<u>Brass link:</u> $(\delta_T)_b = \alpha_b(\Delta T)L = (20.9 \times 10^{-6})(25)(0.250) = 130.625 \times 10^{-6}\,\text{m}$

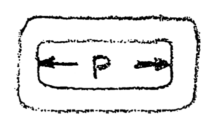
<u>Steel rod:</u> $(\delta_T)_s = \alpha_s(\Delta T)L = (11.7 \times 10^{-6})(25)(0.250) = 73.125 \times 10^{-6}\,\text{m}$

At the final temperature, the difference between the free length of the steel rod and the brass link is

$$\delta = 120 \times 10^{-6} + 73.125 \times 10^{-6} - 130.625 \times 10^{-6} = 62.5 \times 10^{-6}\,\text{m}$$

Add equal but opposite forces P to elongate the brass link and contract the steel rod.

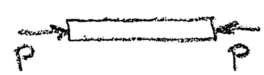
<u>Brass link</u>: $E = 105 \times 10^9\,\text{Pa}$



$$A_b = (2)(50)(37.5) = 3750 \text{ mm}^2 = 3.750 \times 10^{-3}\,\text{m}^2$$

$$(\delta_P) = \frac{PL}{EA} = \frac{P(0.250)}{(105 \times 10^9)(3.750 \times 10^{-3})} = 634.92 \times 10^{-12} P$$

<u>Steel rod</u>: $E = 200 \times 10^9\,\text{Pa}$ $\quad A_s = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6}\,\text{m}^2$



$$(\delta_P)_s = \frac{PL}{E_s A_s} = \frac{P(0.250)}{(200 \times 10^9)(706.86 \times 10^{-6})} = 1.76838 \times 10^{-9} P$$

$$(\delta_P)_b + (\delta_P)_s = \delta: \quad 2.4033 \times 10^{-9} P = 62.5 \times 10^{-6} \quad P = 26.006 \times 10^3\,\text{N}$$

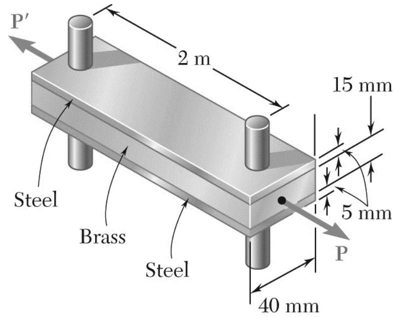
(*a*) <u>Stress in steel rod</u>: $\sigma_s = -\frac{P}{A_s} = -\frac{(26.006 \times 10^3)}{706.86 \times 10^{-6}} = -36.8 \times 10^6 \text{ Pa}$ $\quad \sigma_s = -36.8 \text{ MPa}$ ◀

(*b*) <u>Final length of steel rod</u>: $L_f = L_0 + (\delta_T)_s - (\delta_P)_s$

$$L_f = 0.250 + 120 \times 10^{-6} + 73.125 \times 10^{-6} - (1.76838 \times 10^{-9})(26.003 \times 10^3)$$

$$= 0.250147 \text{ m}$$

$L_f = 250.147 \text{ mm}$ ◀



PROBLEM 2.56

Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7\times10^{-6}/°\text{C}$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°\text{C}$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

(*a*) Required temperature change for fabrication:

$$\delta_T = 0.5 \text{ mm} = 0.5\times10^{-3}\text{ m}$$

Temperature change required to expand steel bar by this amount:

$$\delta_T = L\alpha_s\Delta T, \quad 0.5\times10^{-3} = (2.00)(11.7\times10^{-6})(\Delta T),$$

$$\Delta T = 0.5\times10^{-3} = (2)(11.7\times10^{-6})(\Delta T)$$

$$\Delta T = 21.368\,°\text{C} \qquad\qquad 21.4\,°\text{C} \blacktriangleleft$$

(*b*) Once assembled, a tensile force P^* develops in the steel, and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400\times10^{-6}\text{ m}^2$

$$(\delta_P)_s = \frac{F^*L}{A_sE_s} = \frac{P^*(2.00)}{(400\times10^{-6})(200\times10^9)} = 25\times10^{-9}P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600\times10^{-6}\text{ m}^2$

$$(\delta_P)_b = \frac{P^*L}{A_bE_b} = \frac{P^*(2.00)}{(600\times10^{-6})(105\times10^9)} = 31.746\times10^{-9}P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit:

$$(\delta_P)_s + (\delta_P)_b = 0.5\times10^{-3}, \quad 56.746\times10^{-9}P^* = 0.5\times10^{-3}$$

$$P^* = 8.811\times10^3\text{ N}$$

Stresses due to fabrication:

Steel: $$\sigma_s^* = \frac{P^*}{A_s} = \frac{8.811\times10^3}{400\times10^{-6}} = 22.03\times10^6\text{ Pa} = 22.03 \text{ MPa}$$

PROBLEM 2.56 *(Continued)*

Brass: $\sigma_b^* = -\dfrac{P^*}{A_b} = -\dfrac{8.811\times10^3}{600\times10^{-6}} = -14.68\times10^6 \text{ Pa} = -14.68 \text{ MPa}$

To these stresses must be added the stresses due to the 25 kN load.

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^9)}{2.00}\delta' = 40\times10^6\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^9)}{2.00}\delta' = 31.5\times10^6\delta'$$

Total $\qquad P = P_s + P_b = 25\times10^3 \text{ N}$

$$40\times10^6\delta' + 31.5\times10^6\delta' = 25\times10^3 \qquad \delta' = 349.65\times10^{-6} \text{ m}$$

$$P_s = (40\times10^6)(349.65\times10^{-6}) = 13.986\times10^3 \text{ N}$$

$$P_b = (31.5\times10^6)(349.65\times10^{-6}) = 11.140\times10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986\times10^3}{400\times10^{-6}} = 34.97\times10^6 \text{ Pa}$$

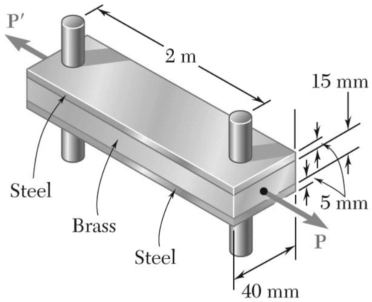
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140\times10^3}{600\times10^{-6}} = 18.36\times10^6 \text{ Pa}$$

Add stress due to fabrication.

Total stresses:

$$\sigma_s = 34.97\times10^6 + 22.03\times10^6 = 57.0\times10^6 \text{ Pa} \qquad \sigma_s = 57.0 \text{ MPa}$$

$$\sigma_b = 18.36\times10^6 - 14.68\times10^6 = 3.68\times10^6 \text{ Pa} \qquad \sigma_b = 3.68 \text{ MPa} \blacktriangleleft$$



PROBLEM 2.57

Determine the maximum load P that may be applied to the brass bar of Prob. 2.56 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

PROBLEM 2.56 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

See solution to Problem 2.56 to obtain the fabrication stresses.

$$\sigma_s^* = 22.03 \text{ MPa}$$
$$\sigma_b^* = 14.68 \text{ MPa}$$

Allowable stresses: $\sigma_{s,\text{all}} = 30$ MPa, $\sigma_{b,\text{all}} = 25$ MPa

Available stress increase from load.

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$
$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains.

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97\times10^6}{200\times10^9} = 39.85\times10^{-6}$$
$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68\times10^6}{105\times10^9} = 377.9\times10^{-6}$$

Smaller value governs $\therefore\ \varepsilon = 39.85\times10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400\times10^{-6}\,\text{m}^2$

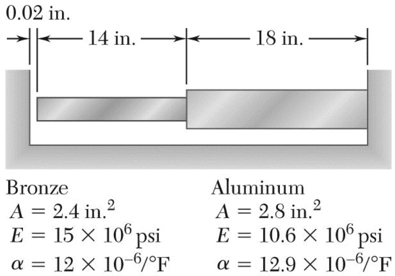
$A_b = (15)(40) = 600 \text{ mm}^2 = 600\times10^{-6}\,\text{m}^2$

Forces $P_s = E_s A_s \varepsilon = (200\times10^9)(400\times10^{-6})(39.85\times10^{-6}) = 3.188\times10^3\,\text{N}$

$P_b = E_b A_b \varepsilon = (105\times10^9)(600\times10^{-6})(39.85\times10^{-6}) = 2.511\times10^{-3}\,\text{N}$

Total allowable additional force:

$P = P_s + P_b = 3.188\times10^3 + 2.511\times10^3 = 5.70\times10^3\,\text{N}$ $\quad P = 5.70$ kN ◄



PROBLEM 2.58

Knowing that a 0.02-in. gap exists when the temperature is 75°F, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to −11 ksi, (*b*) the corresponding exact length of the aluminum bar.

SOLUTION

$$\sigma_a = -11 \text{ ksi} = -11\times 10^3 \text{ psi}$$

$$P = -\sigma_a A_a = (11\times 10^3)(2.8) = 30.8\times 10^3 \text{ lb}$$

Shortening due to *P*:

$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$

$$= \frac{(30.8\times 10^3)(14)}{(15\times 10^6)(2.4)} + \frac{(30.8\times 10^3)(18)}{(10.6\times 10^6)(2.8)}$$

$$= 30.657\times 10^{-3} \text{ in.}$$

Available elongation for thermal expansion:

$$\delta_T = 0.02 + 30.657\times 10^{-3} = 50.657\times 10^{-3} \text{ in.}$$

But $\delta_T = L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)$

$$= (14)(12\times 10^{-6})(\Delta T) + (18)(12.9\times 10^{-6})(\Delta T) = 400.2\times 10^{-6})\Delta T$$

Equating, $(400.2\times 10^{-6})\Delta T = 50.657\times 10^{-3}$ $\quad \Delta T = 126.6$ °F

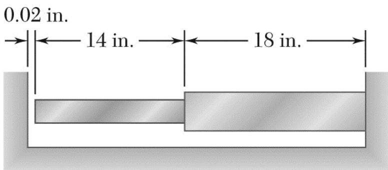
(*a*) $\quad T_{\text{hot}} = T_{\text{cold}} + \Delta T = 75 + 126.6 = 201.6$ °F $\qquad T_{\text{hot}} = 201.6°\text{F}$ ◀

(*b*) $\quad \delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (18)(12.9\times 10^{-6})(26.6) - \frac{(30.8\times 10^3)(18)}{(10.6\times 10^6)(2.8)} = 10.712\times 10^{-3} \text{ in.}$$

$$L_{\text{exact}} = 18 + 10.712\times 10^{-3} = 18.0107 \text{ in.}$$

$L = 18.0107$ in. ◀



Bronze
$A = 2.4\ \text{in.}^2$
$E = 15 \times 10^6$ psi
$\alpha = 12 \times 10^{-6}/°\text{F}$

Aluminum
$A = 2.8\ \text{in.}^2$
$E = 10.6 \times 10^6$ psi
$\alpha = 12.9 \times 10^{-6}/°\text{F}$

PROBLEM 2.59

Determine (*a*) the compressive force in the bars shown after a temperature rise of 180°F, (*b*) the corresponding change in length of the bronze bar.

SOLUTION

Thermal expansion if free of constraint:

$$\begin{aligned}\delta_T &= L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)\\ &= (14)(12\times10^{-6})(180) + (18)(12.9\times10^{-6})(180)\\ &= 72.036\times10^{-3}\ \text{in.}\end{aligned}$$

Constrained expansion: $\delta = 0.02\ \text{in.}$

Shortening due to induced compressive force P:

$$\delta_P = 72.036\times10^{-3} - 0.02 = 52.036\times10^{-3}\ \text{in.}$$

But
$$\delta_P = \frac{PL_b}{E_bA_b} + \frac{PL_a}{E_aA_a} = \left(\frac{L_b}{E_bA_b} + \frac{L_a}{E_aA_a}\right)P$$

$$= \left(\frac{14}{(15\times10^6)(2.4)} + \frac{18}{(10.6\times10^6)(2.8)}\right)P = 995.36\times10^{-9}P$$

Equating,
$$995.36\times10^{-9}P = 52.036\times10^{-3}$$
$$P = 52.279\times10^3\ \text{lb}$$

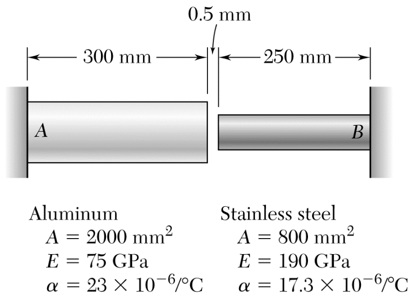
(*a*) $P = 52.3$ kips ◀

(*b*) $\delta_b = L_b\alpha_b(\Delta T) - \dfrac{PL_b}{E_bA_b}$

$$= (14)(12\times10^{-6})(180) - \frac{(52.279\times10^3)(14)}{(15\times10^6)(2.4)} = 9.91\times10^{-3}\ \text{in.}$$

$\delta_b = 9.91\times10^{-3}$ in. ◀

PROBLEM 2.60



At room temperature $(20°C)$ a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.

SOLUTION

$$\Delta T = 140 - 20 = 120\ °\text{C}$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)\\ &= (0.300)(23\times 10^{-6})(120) + (0.250)(17.3\times 10^{-6})(120)\\ &= 1.347\times 10^{-3}\ \text{m}\end{aligned}$$

Shortening due to P to meet constraint:

$$\delta_P = 1.347\times 10^{-3} - 0.5\times 10^{-3} = 0.847\times 10^{-3}\ \text{m}$$

$$\begin{aligned}\delta_P &= \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P\\ &= \left(\frac{0.300}{(75\times 10^9)(2000\times 10^{-6})} + \frac{0.250}{(190\times 10^9)(800\times 10^{-6})}\right)P\\ &= 3.6447\times 10^{-9}P\end{aligned}$$

Equating,

$$3.6447\times 10^{-9}\ P = 0.847\times 10^{-3}$$

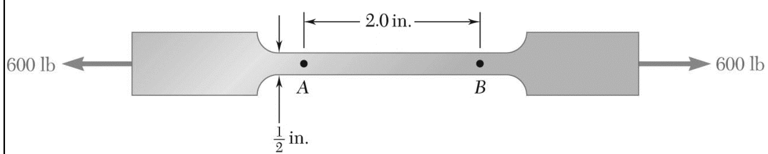
$$P = 232.39\times 10^3\ \text{N}$$

(*a*) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times 10^3}{2000\times 10^{-6}} = -116.2\times 10^6\ \text{Pa}$ $\qquad \sigma_a = -116.2\ \text{MPa}$ ◀

(*b*) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_aA_a}$

$$= (0.300)(23\times 10^{-6})(120) - \frac{(232.39\times 10^3)(0.300)}{(75\times 10^9)(2000\times 10^{-6})} = 363\times 10^{-6}\ \text{m}$$

$\delta_a = 0.363\ \text{mm}$ ◀



PROBLEM 2.61

A 600-lb tensile load is applied to a test coupon made from $\frac{1}{16}$-in. flat steel plate ($E = 29\times10^6$ psi and $\nu = 0.30$). Determine the resulting change (*a*) in the 2-in. gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.

SOLUTION

$$A = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = 0.03125 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{600}{0.03125} = 19.2\times10^3 \text{ psi}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{19.2\times10^3}{29\times10^6} = 662.07\times10^{-6}$$

(*a*) $\delta_x = L_0\varepsilon_x = (2.0)(662.07\times10^{-6})$ $\quad\quad \delta_l = 1.324\times10^{-3}$ in. ◀

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(662.07\times10^{-6}) = -198.62\times10^{-6}$$

(*b*) $\delta_{\text{width}} = w_0\varepsilon_y = \left(\frac{1}{2}\right)(-198.62\times10^{-6})$ $\quad\quad \delta_w = -99.3\times10^{-6}$ in. ◀

(*c*) $\delta_{\text{thickness}} = t_0\varepsilon_z = \left(\frac{1}{16}\right)(-198.62\times10^{-6})$ $\quad\quad \delta_t = -12.41\times10^{-6}$ in. ◀

(*d*) $A = wt = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z)$

$$= w_0t_0(1+\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

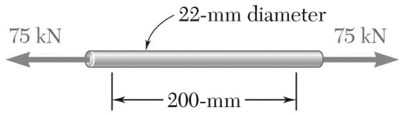
$$\Delta A = A - A_0 = w_0t_0(\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{16}\right)(-198.62\times10^{-6} - 198.62\times10^{-6} + \text{negligible term})$$

$$= -12.41\times10^{-6} \text{ in}^2$$

$\Delta A = -12.41\times10^{-6}$ in^2 ◀

PROBLEM 2.62



In a standard tensile test, a steel rod of 22-mm diameter is subjected to a tension force of 75 kN. Knowing that $v = 0.3$ and $E = 200$ GPa, determine (*a*) the elongation of the rod in a 200-mm gage length, (*b*) the change in diameter of the rod.

SOLUTION

$$P = 75\text{ kN} = 75 \times 10^3\text{ N} \qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6}\text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{75 \times 10^3}{380.13 \times 10^{-6}} = 197.301 \times 10^6\text{ Pa}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{197.301 \times 10^6}{200 \times 10^9} = 986.51 \times 10^{-6}$$

$$\delta_x = L\varepsilon_x = (200\text{ mm})(986.51 \times 10^{-6})$$

(*a*) $\delta_x = 0.1973\text{ mm}$ ◀

$$\varepsilon_y = -v\varepsilon_x = -(0.3)(986.51 \times 10^{-6}) = -295.95 \times 10^{-6}$$

$$\delta_y = d\varepsilon_y = (22\text{ mm})(-295.95 \times 10^{-6})$$

(*b*) $\delta_y = -0.00651\text{ mm}$ ◀

PROBLEM 2.63

A 20-mm-diameter rod made of an experimental plastic is subjected to a tensile force of magnitude $P = 6$ kN. Knowing that an elongation of 14 mm and a decrease in diameter of 0.85 mm are observed in a 150-mm length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio for the material.

SOLUTION

Let the y-axis be along the length of the rod and the x-axis be transverse.

$$A = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16\times10^{-6}\,\text{m}^2 \qquad P = 6\times10^3\,\text{N}$$

$$\sigma_y = \frac{P}{A} = \frac{6\times10^3}{314.16\times10^{-6}} = 19.0985\times10^6\,\text{Pa}$$

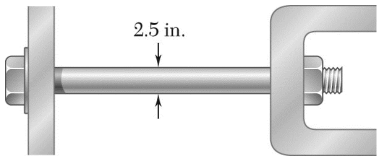
$$\varepsilon_y = \frac{\delta_y}{L} = \frac{14 \text{ mm}}{150 \text{ mm}} = 0.093333$$

Modulus of elasticity: $E = \dfrac{\sigma_y}{\varepsilon_y} = \dfrac{19.0985\times10^6}{0.093333} = 204.63\times10^6\,\text{Pa}$ $\qquad E = 205 \text{ MPa}$ ◀

$$\varepsilon_x = \frac{\delta_x}{d} = -\frac{0.85}{20} = -0.0425$$

Poisson's ratio: $v = -\dfrac{\varepsilon_x}{\varepsilon_y} = -\dfrac{-0.0425}{0.093333}$ $\qquad v = 0.455$ ◀

Modulus of rigidity: $G = \dfrac{E}{2(1+v)} = \dfrac{204.63\times10^6}{(2)(1.455)} = 70.31\times10^6\,\text{Pa}$ $\qquad G = 70.3 \text{ MPa}$ ◀



PROBLEM 2.64

The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the internal force in the bolt, if the diameter is observed to decrease by 0.5×10^{-3} in.

SOLUTION

$$\delta_y = -0.5 \times 10^{-3} \text{ in.} \qquad d = 2.5 \text{ in.}$$

$$\varepsilon_y = \frac{\varepsilon_y}{d} = -\frac{0.5 \times 10^{-3}}{2.5} = -0.2 \times 10^{-3}$$

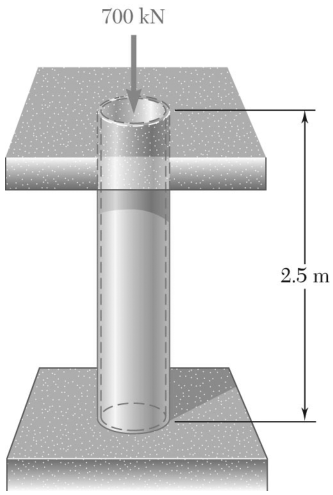
$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}: \qquad \varepsilon_x = \frac{-\varepsilon_y}{\nu} = \frac{0.2 \times 10^{-3}}{0.3} = 0.66667 \times 10^{-3}$$

$$\sigma_x = E\varepsilon_x = (29 \times 10^6)(0.66667 \times 10^{-3}) = 19.3334 \times 10^3 \text{ psi}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2.5)^2 = 4.9087 \text{ in}^2$$

$$F = \sigma_x A = (19.3334 \times 10^3)(4.9087) = 94.902 \times 10^3 \text{ lb}$$

$$F = 94.9 \text{ kips} \blacktriangleleft$$



PROBLEM 2.65

A 2.5-m length of a steel pipe of 300-mm outer diameter and 15-mm wall thickness is used as a column to carry a 700-kN centric axial load. Knowing that $E = 200$ GPa and $v = 0.30$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

SOLUTION

$$d_o = 0.3 \text{ m} \qquad t = 0.015 \text{ m} \qquad L = 2.5 \text{ m}$$

$$d_i = d_o - 2t = 0.3 - 2(0.015) = 0.27 \text{ m} \qquad P = 700\times 10^3 \text{ N}$$

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(0.3^2 - 0.27^2) = 13.4303\times 10^{-3} \text{ m}^2$$

(*a*) $$\delta = -\frac{PL}{EA} = -\frac{(700\times 10^3)(2.5)}{(200\times 10^9)(13.4303\times 10^{-3})}$$

$$= -651.51\times 10^{-6} \text{ m}$$

$\delta = -0.652$ mm ◀

$$\varepsilon = \frac{\delta}{L} = \frac{-651.51\times 10^{-6}}{2.5} = -260.60\times 10^{-6}$$

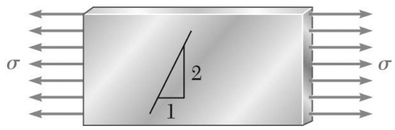
$$\varepsilon_{\text{LAT}} = -v\varepsilon = -(0.30)(-260.60\times 10^{-6})$$
$$= 78.180\times 10^{-6}$$

(*b*) $$\Delta d_o = d_o\varepsilon_{\text{LAT}} = (300 \text{ mm})(78.180\times 10^{-6})$$

$\Delta d_o = 0.0235$ mm ◀

(*c*) $$\Delta t = t\varepsilon_{\text{LAT}} = (15 \text{ mm})(78.180\times 10^{-6})$$

$\Delta t = 0.001173$ mm ◀

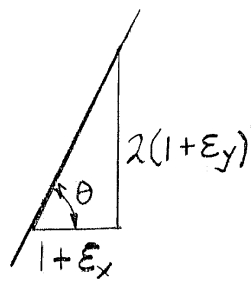


PROBLEM 2.66

An aluminum plate $(E = 74\text{ GPa}$ and $v = 0.33)$ is subjected to a centric axial load that causes a normal stress σ. Knowing that, before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when $\sigma = 125$ MPa.

SOLUTION

The slope after deformation is $\tan\theta = \dfrac{2(1+\varepsilon_y)}{1+\varepsilon_x}$

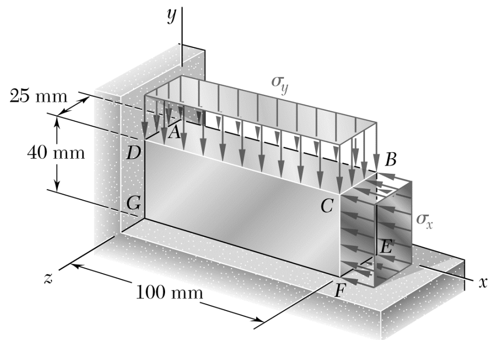


$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{125\times 10^6}{74\times 10^9} = 1.6892\times 10^{-3}$$

$$\varepsilon_y = -v\varepsilon_x = -(0.33)(1.6892\times 10^{-3}) = -0.5574\times 10^{-3}$$

$$\tan\theta = \frac{2(1-0.0005574)}{1+0.0016892} = 1.99551$$

$\tan\theta = 1.99551$ ◀



PROBLEM 2.67

The block shown is made of a magnesium alloy, for which $E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

SOLUTION

(*a*) $\delta_y = 0 \qquad \varepsilon_y = 0 \qquad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\sigma_y = \nu\sigma_x = (0.35)(-180\times10^6)$$

$$= -63\times10^6\,\text{Pa} \qquad \sigma_y = -63 \text{ MPa} \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}(\sigma_x + \sigma_y) = \frac{(0.35)(-243\times10^6)}{45\times10^9} = -1.89\times10^{-3}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_Z) = \frac{\sigma_x - \nu\sigma_y}{E} = -\frac{157.95\times10^6}{45\times10^9} = -3.51\times10^{-3}$$

(*b*) $A_0 = L_x L_z$

$$A = L_x(1+\varepsilon_x)L_z(1+\varepsilon_z) = L_xL_z(1+\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z)$$

$$\Delta A = A - A_0 = L_xL_z(\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z) \approx L_xL_z(\varepsilon_x+\varepsilon_z)$$

$$\Delta A = (100\text{ mm})(25\text{ mm})(-3.51\times10^{-3} - 1.89\times10^{-3}) \qquad \Delta A = -13.50\text{ mm}^2 \blacktriangleleft$$

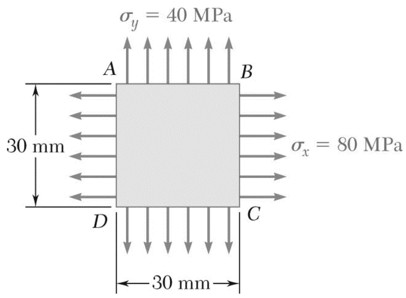
(*c*) $V_0 = L_xL_yL_z$

$$V = L_x(1+\varepsilon_x)L_y(1+\varepsilon_y)L_z(1+\varepsilon_z)$$

$$= L_xL_yL_z(1+\varepsilon_x+\varepsilon_y+\varepsilon_z+\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_z\varepsilon_x+\varepsilon_x\varepsilon_y\varepsilon_z)$$

$$\Delta V = V - V_0 = L_xL_yL_z(\varepsilon_x+\varepsilon_y+\varepsilon_z+\text{small terms})$$

$$\Delta V = (100)(40)(25)(-3.51\times10^{-3} + 0 - 1.89\times10^{-3}) \qquad \Delta V = -540\text{ mm}^3 \blacktriangleleft$$



PROBLEM 2.68

A 30-mm square was scribed on the side of a large steel pressure vessel. After pressurization, the biaxial stress condition at the square is as shown. For $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

Given: $\sigma_x = 80$ MPa $\quad \sigma_y = 40$ MPa

Using Eq's (2.28):

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{80 - 0.3(40)}{200\times 10^3} = 340\times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{40 - 0.3(80)}{200\times 10^3} = 80\times 10^{-6}$$

(*a*) Change in length of *AB*.

$$\delta_{AB} = (AB)\varepsilon_x = (30\text{ mm})(340\times 10^{-6}) = 10.20\times 10^{-3}\text{ mm}$$

$$\delta_{AB} = 10.20\ \mu\text{m} \blacktriangleleft$$

(*b*) Change in length of *BC*.

$$\delta_{BC} = (BC)\varepsilon_y = (30\text{ mm})(80\times 10^{-6}) = 2.40\times 10^{-3}\text{ mm}$$

$$\delta_{BC} = 2.40\mu\text{m} \blacktriangleleft$$

(*c*) Change in length of diagonal *AC*.

From geometry, $(AC)^2 = (AB)^2 + (BC)^2$

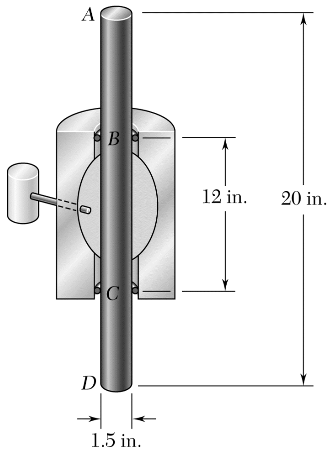
Differentiate: $2(AC)\,\Delta(AC) = 2(AB)\Delta(AB) + 2(BC)\Delta(BC)$

But $\Delta(AC) = \delta_{AC} \qquad \Delta(AB) = \delta_{AB} \qquad \Delta(BC) = \delta_{BC}$

Thus, $2(AC)\delta_{AC} = 2(AB)\delta_{AB} + 2(BC)\delta_{BC}$

$$\delta_{AC} = \frac{AB}{AC}\delta_{AB} + \frac{BC}{AC}\delta_{BC} = \frac{1}{\sqrt{2}}(10.20\ \mu\text{m}) + \frac{1}{\sqrt{2}}(2.40\mu\text{m})$$

$$\delta_{AC} = 8.91\ \mu\text{m} \blacktriangleleft$$



PROBLEM 2.69

The aluminum rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion BC of the rod. Knowing that $E = 10.1\times10^6$ psi and $v = 0.36,$ determine (*a*) the change in the total length AD, (*b*) the change in diameter at the middle of the rod.

SOLUTION

$$\sigma_x = \sigma_z = -P = -6000 \text{ psi} \qquad \sigma_y = 0$$

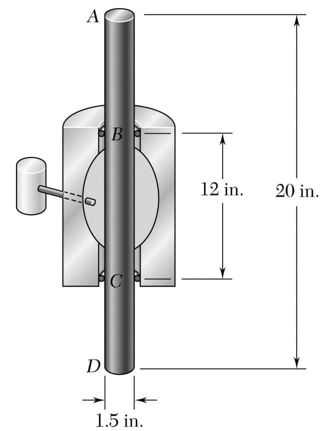
$$\begin{aligned}
\varepsilon_x &= \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z)\\
&= \frac{1}{10.1\times10^6}[-6000 - (0.36)(0) - (0.36)(-6000)]\\
&= -380.198\times10^{-6}
\end{aligned}$$

$$\begin{aligned}
\varepsilon_y &= \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z)\\
&= \frac{1}{10.1\times10^6}[-(0.36)(-6000) + 0 - (0.36)(-6000)]\\
&= 427.72\times10^{-6}
\end{aligned}$$

Length subjected to strain ε_x: $L = 12$ in.

(*a*) $\delta_y = L\varepsilon_y = (12)(427.72\times10^{-6})$ $\qquad$ $\delta_l = 5.13\times10^{-3}$ in. ◀

(*b*) $\delta_x = d\varepsilon_x = (1.5)(-380.198\times10^{-6})$ $\qquad$ $\delta_d = -0.570\times10^{-3}$ in. ◀



PROBLEM 2.70

For the rod of Prob. 2.69, determine the forces that should be applied to the ends A and D of the rod (*a*) if the axial strain in portion BC of the rod is to remain zero as the hydrostatic pressure is applied, (*b*) if the total length AD of the rod is to remain unchanged.

PROBLEM 2.69 The aluminum rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion BC of the rod. Knowing that $E = 10.1 \times 10^6$ psi and $\nu = 0.36$, determine (*a*) the change in the total length AD, (*b*) the change in diameter at the middle of the rod.

SOLUTION

Over the pressurized portion BC,

$$\sigma_x = \sigma_z = -p \qquad \sigma_y = \sigma_y$$

$$(\varepsilon_y)_{BC} = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}(2\nu p + \sigma_y)$$

(*a*) $(\varepsilon_y)_{BC} = 0 \qquad 2\nu p + \sigma_y = 0$

$$\sigma_y = -2\nu p = -(2)(0.36)(6000)$$

$$= -4320 \text{ psi}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

$$F = A\sigma_y = (1.76715)(-4320) = -7630 \text{ lb}$$

i.e., 7630 lb compression ◀

(*b*) Over unpressurized portions AB and CD, $\sigma_x = \sigma_z = 0$

$$(\varepsilon_y)_{AB} = (\varepsilon_y)_{CD} = \frac{\sigma_y}{E}$$

For no change in length,

$$\delta = L_{AB}(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} + L_{CD}(\varepsilon_y)_{CD} = 0$$

$$(L_{AB} + L_{CD})(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} = 0$$

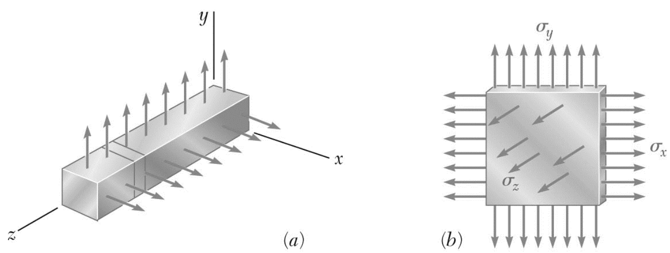
$$(20-12)\frac{\sigma_y}{E} + \frac{12}{E}(2\nu p + \sigma_y) = 0$$

$$\sigma_y = -\frac{24\nu p}{20} = -\frac{(24)(0.36)(6000)}{20} = -2592 \text{ psi}$$

$$P = A\sigma_y = (1.76715)(-2592) = -4580 \text{ lb}$$

$P = 4580$ lb compression ◀

PROBLEM 2.71



In many situations, physical constraints prevent strain from occurring in a given direction. For example, $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain,* we can express σ_z, ε_x, and ε_y as follows:

$$\sigma_z = v(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-v^2)\sigma_x - v(1+v)\sigma_y]$$

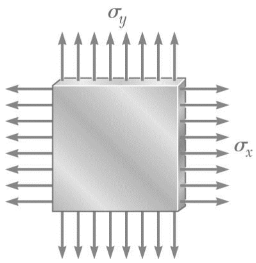
$$\varepsilon_y = \frac{1}{E}[(1-v^2)\sigma_y - v(1+v)\sigma_x]$$

SOLUTION

$$\varepsilon_z = 0 = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = v(\sigma_x + \sigma_y) \quad \blacktriangleleft$$

$$\begin{aligned}\varepsilon_x &= \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z)\\ &= \frac{1}{E}[\sigma_x - v\sigma_y - v^2(\sigma_x + \sigma_y)]\\ &= \frac{1}{E}[(1-v^2)\sigma_x - v(1+v)\sigma_y] \quad \blacktriangleleft\end{aligned}$$

$$\begin{aligned}\varepsilon_y &= \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z)\\ &= \frac{1}{E}[-v\sigma_x + \sigma_y - v^2(\sigma_x + \sigma_y)]\\ &= \frac{1}{E}[(1-v^2)\sigma_y - v(1+v)\sigma_x] \quad \blacktriangleleft\end{aligned}$$



PROBLEM 2.72

In many situations, it is known that the normal stress in a given direction is zero, for example, $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress,* show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y, and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + v\varepsilon_y}{1-v^2} \qquad \sigma_y = E\frac{\varepsilon_y + v\varepsilon_x}{1-v^2} \qquad \varepsilon_z = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y)$$

SOLUTION

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y) \qquad (2)$$

Multiplying (2) by v and adding to (1),

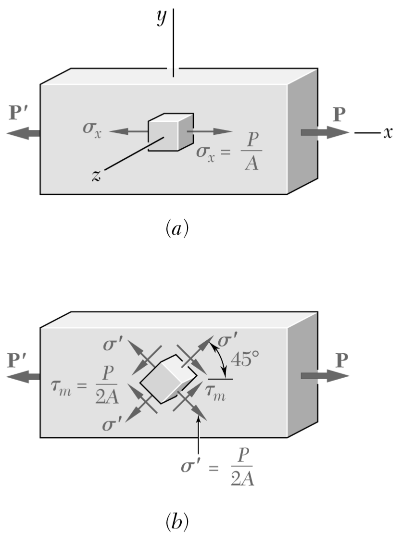
$$\varepsilon_x + v\varepsilon_y = \frac{1-v^2}{E}\sigma_x \qquad \text{or} \qquad \sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y) \quad \blacktriangleleft$$

Multiplying (1) by v and adding to (2),

$$\varepsilon_y + v\varepsilon_x = \frac{1-v^2}{E}\sigma_y \qquad \text{or} \qquad \sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x) \quad \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y) = -\frac{v}{\cancel{E}}\cdot\frac{\cancel{E}}{1-v^2}(\varepsilon_x + v\varepsilon_y + \varepsilon_y + v\varepsilon_x)$$

$$= -\frac{v(1+v)}{1-v^2}(\varepsilon_x + \varepsilon_y) = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y) \quad \blacktriangleleft$$



PROBLEM 2.73

For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypotenuses of the triangles shown in Fig. 2.49, which represent, respectively, an element before and after deformation, (*b*) using the values of the corresponding stresses of σ' and σ_x shown in Fig. 1.38, and the generalized Hooke's law.

SOLUTION

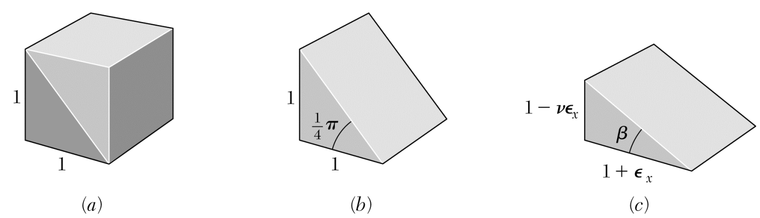


Figure 2.49

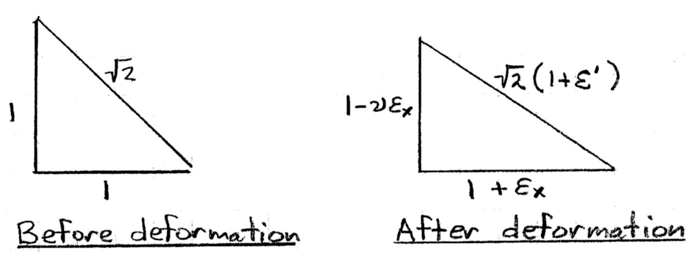
(*a*) $$[\sqrt{2}(1+\varepsilon')]^2 = (1+\varepsilon_x)^2 + (1-v\varepsilon_x)^2$$

$$2(1+2\varepsilon'+\varepsilon'^2) = 1+2\varepsilon_x+\varepsilon_x^2+1-2v\varepsilon_x+v^2\varepsilon_x^2$$

$$4\varepsilon'+2\varepsilon'^2 = 2\varepsilon_x+\varepsilon_x^2-2v\varepsilon_x+v^2\varepsilon_x^2$$

Neglect squares as small $\quad 4\varepsilon' = 2\varepsilon_x - 2v\varepsilon_x$

$$\varepsilon' = \frac{1-v}{2}\varepsilon_x \quad ◀$$



(A) **(B)**

PROBLEM 2.73 *(Continued)*

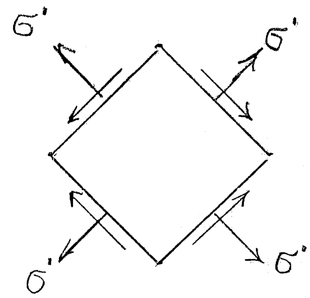
(*b*)

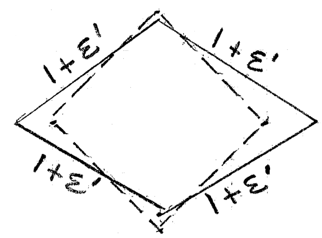
$$\varepsilon' = \frac{\sigma'}{E} - \frac{v\sigma'}{E}$$

$$= \frac{1-v}{E} \cdot \frac{P}{2A}$$

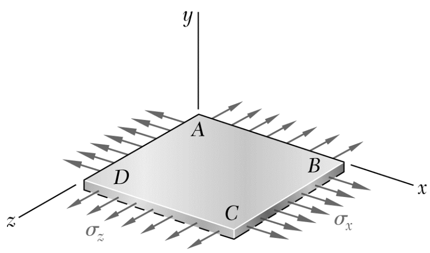
$$= \frac{1-v}{2E}\sigma_x$$

$$= \frac{1-v}{2}\varepsilon_x$$





◀



PROBLEM 2.74

The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\varepsilon_x = 0$. Denoting by E the modulus of elasticity and by v Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0 / ε_z.

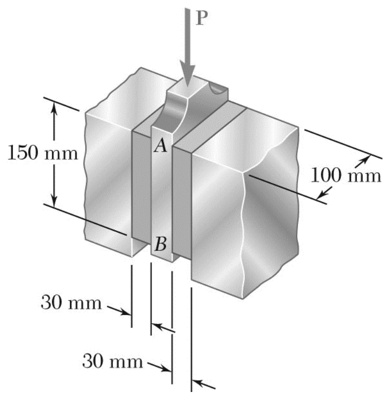
SOLUTION

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{1}{E}(\sigma_x - v\sigma_0)$$

(*a*) $\sigma_x = v\sigma_0$ ◄

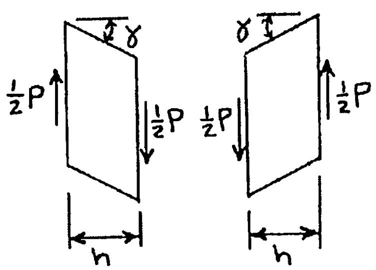
(*b*) $\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z) = \frac{1}{E}(-v^2\sigma_0 - 0 + \sigma_0) = \frac{1 - v^2}{E}\sigma_0$ $\qquad \frac{\sigma_0}{\varepsilon_z} = \frac{E}{1 - v^2}$ ◄



PROBLEM 2.75

A vibration isolation unit consists of two blocks of hard rubber bonded to a plate AB and to rigid supports as shown. Knowing that a force of magnitude $P = 25\text{ kN}$ causes a deflection $\delta = 1.5\text{ mm}$ of plate AB, determine the modulus of rigidity of the rubber used.

SOLUTION



$$F = \frac{1}{2}P = \frac{1}{2}(25\times 10^3\text{ N}) = 12.5\times 10^3\text{ N}$$

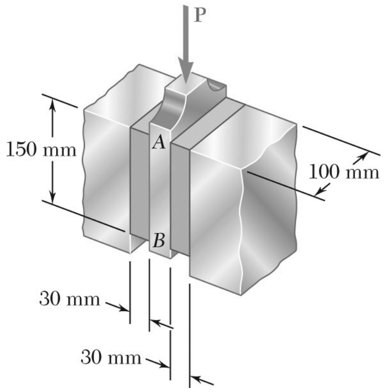
$$\tau = \frac{F}{A} = \frac{12.5\times 10^3\text{ N})}{(0.15\text{ m})(0.1\text{ m})} = 833.33\times 10^3\text{ Pa}$$

$$\delta = 1.5\times 10^{-3}\text{ m} \qquad h = 0.03\text{ m}$$

$$\gamma = \frac{\delta}{h} = \frac{1.5\times 10^{-3}}{0.03} = 0.05$$

$$G = \frac{\tau}{\gamma} = \frac{833.33\times 10^3}{0.05} = 16.67\times 10^6\text{ Pa}$$

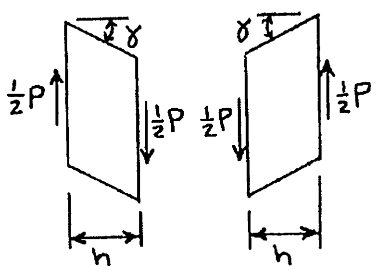
$G = 16.67\text{ MPa}$ ◀



PROBLEM 2.76

A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19$ MPa bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.

SOLUTION



Shearing strain: $\gamma = \dfrac{\delta}{h}$

Shearing stress: $\tau = G\gamma = \dfrac{G\delta}{h}$

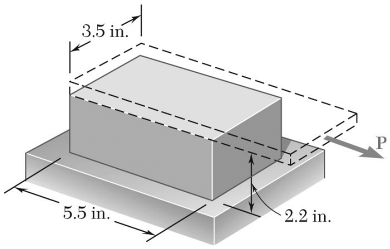
Force: $\dfrac{1}{2}P = A\tau = \dfrac{GA\delta}{h}$ or $P = \dfrac{2GA\delta}{h}$

Effective spring constant: $k = \dfrac{P}{\delta} = \dfrac{2GA}{h}$

with $A = (0.15)(0.1) = 0.015 \text{ m}^2 \quad h = 0.03 \text{ m}$

$$k = \frac{2(19 \times 10^6 \text{ Pa})(0.015 \text{ m}^2)}{0.03 \text{ m}} = 19.00 \times 10^6 \text{ N/m}$$

$k = 19.00 \times 10^3$ kN/m ◀



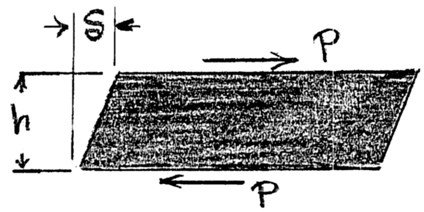
PROBLEM 2.77

The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 55$ ksi, determine the deflection of the plate when $P = 9$ kips.

SOLUTION

Consider the plastic block. The shearing force carried is $P = 9 \times 10^3$ lb

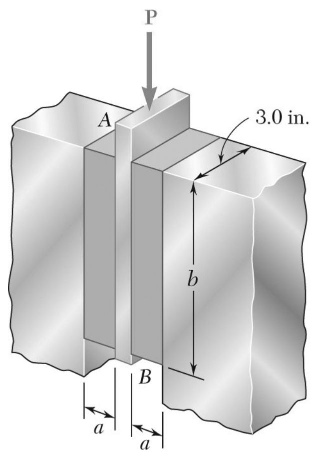
The area is $A = (3.5)(5.5) = 19.25 \text{ in}^2$



Shearing stress: $\tau = \dfrac{P}{A} = \dfrac{9 \times 10^3}{19.25} = 467.52$ psi

Shearing strain: $\gamma = \dfrac{\tau}{G} = \dfrac{467.52}{55 \times 10^3} = 0.0085006$

But $\gamma = \dfrac{\delta}{h} \quad \therefore \quad \delta = h\gamma = (2.2)(0.0085006)$ $\qquad \delta = 0.0187 \text{ in.}$ ◀



PROBLEM 2.78

A vibration isolation unit consists of two blocks of hard rubber bonded to plate AB and to rigid supports as shown. For the type and grade of rubber used $\tau_{\text{all}} = 220$ psi and $G = 1800$ psi. Knowing that a centric vertical force of magnitude $P = 3.2$ kips must cause a 0.1-in. vertical deflection of the plate AB, determine the smallest allowable dimensions a and b of the block.

SOLUTION

Consider the rubber block on the right. It carries a shearing force equal to $\frac{1}{2}P$.

The shearing stress is $\quad \tau = \dfrac{\frac{1}{2}P}{A}$

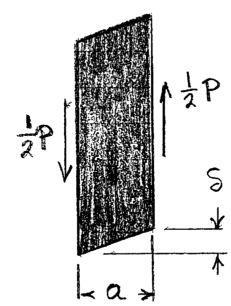
or required area $\quad A = \dfrac{P}{2\tau} = \dfrac{3.2\times 10^3}{(2)(220)} = 7.2727 \text{ in}^2$

But $\quad A = (3.0)b$

Hence, $\quad b = \dfrac{A}{3.0} = 2.42$ in. $\qquad b_{\min} = 2.42$ in. ◀

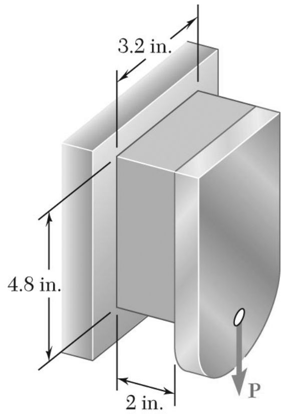
Use $\quad b = 2.42$ in and $\tau = 220$ psi

Shearing strain. $\quad \gamma = \dfrac{\tau}{G} = \dfrac{220}{1800} = 0.12222$



But $\quad \gamma = \dfrac{\delta}{a}$

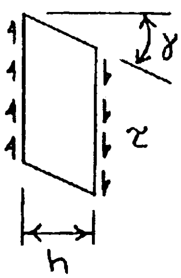
Hence, $\quad a = \dfrac{\delta}{\gamma} = \dfrac{0.1}{0.12222} = 0.818$ in. $\qquad a_{\min} = 0.818$ in. ◀



PROBLEM 2.79

The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = 55 \times 10^3 \text{ lb}$$

$$\tau = \frac{P}{A} = \frac{55 \times 10^3}{15.36} = 3580.7 \text{ psi}$$

$$G = 150 \times 10^3 \text{ psi}$$

$$\gamma = \frac{\tau}{G} = \frac{3580.7}{150 \times 10^3} = 23.871 \times 10^{-3}$$

$$h = 2 \text{ in.}$$

$$\delta = h\gamma = (2)(23.871 \times 10^{-3})$$

$$= 47.7 \times 10^{-3} \text{ in.}$$

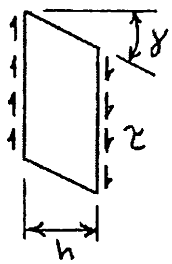
$$\delta = 0.0477 \text{ in.} \downarrow \blacktriangleleft$$

PROBLEM 2.80

What load **P** should be applied to the plate of Prob. 2.79 to produce a $\frac{1}{16}$-in. deflection?

PROBLEM 2.79 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$\delta = \frac{1}{16}\text{in.} = 0.0625 \text{ in.}$$

$$h = 2 \text{ in.}$$

$$\gamma = \frac{\delta}{h} = \frac{0.0625}{2} = 0.03125$$

$$G = 150 \times 10^3 \text{ psi}$$

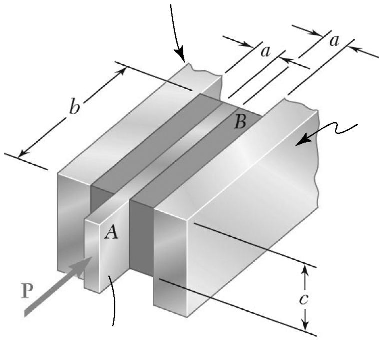
$$\tau = G\gamma = (150 \times 10^3)(0.03125)$$
$$= 4687.5 \text{ psi}$$

$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = \tau A = (4687.5)(15.36)$$
$$= 72 \times 10^3 \text{ lb}$$

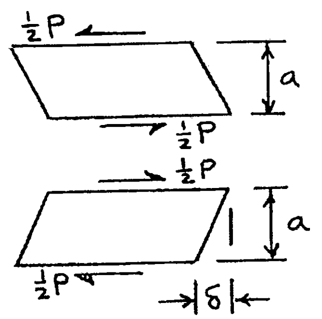
72 kips ◀

PROBLEM 2.81



Two blocks of rubber with a modulus of rigidity $G = 12$ MPa are bonded to rigid supports and to a plate AB. Knowing that $c = 100$ mm and $P = 45$ kN, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.

SOLUTION



Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

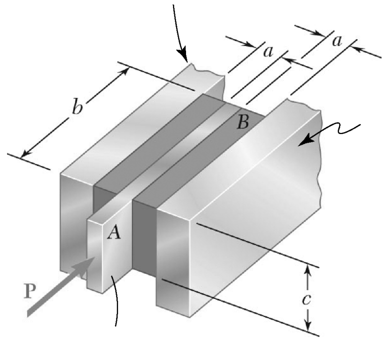
$$a = \frac{G\delta}{\tau} = \frac{(12\times10^6 \text{ Pa})(0.005 \text{ m})}{1.4\times10^6 \text{ Pa}} = 0.0429 \text{ m}$$

$a = 42.9$ mm ◀

Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{45\times10^3 \text{ N}}{2(0.1 \text{ m})\,(1.4\times10^6 \text{ Pa})} = 0.1607 \text{ m}$$

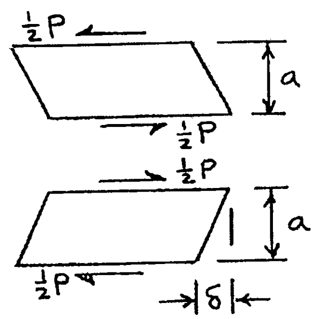
$b = 160.7$ mm ◀



PROBLEM 2.82

Two blocks of rubber with a modulus of rigidity $G = 10$ MPa are bonded to rigid supports and to a plate AB. Knowing that $b = 200$ mm and $c = 125$ mm, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm.

SOLUTION



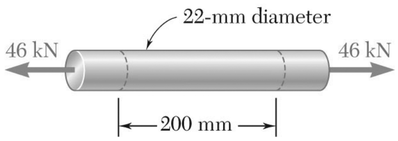
Shearing stress: $\tau = \frac{\frac{1}{2}P}{A} = \frac{P}{2bc}$

$P = 2bc\tau = 2(0.2 \text{ m})(0.125 \text{ m})(1.5\times10^3 \text{ kPa})$ $\qquad$ P = 75.0 kN ◀

Shearing strain: $\gamma = \frac{\delta}{a} = \frac{\tau}{G}$

$a = \frac{G\delta}{\tau} = \frac{(10\times10^6 \text{ Pa})(0.006 \text{ m})}{1.5\times10^6 \text{ Pa}} = 0.04 \text{ m}$ $\qquad$ a = 40.0 mm ◀

PROBLEM 2.83*



Determine the dilatation e and the change in volume of the 200-mm length of the rod shown if (a) the rod is made of steel with $E = 200$ GPa and $v = 0.30$, (b) the rod is made of aluminum with $E = 70$ GPa and $v = 0.35$.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$P = 46 \times 10^3 \text{ N}$$

$$\sigma_x = \frac{P}{A} = 121.01 \times 10^6 \text{ Pa}$$

$$\sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{\sigma_x}{E}$$

$$\varepsilon_y = \varepsilon_z = -v\varepsilon_x = -v\frac{\sigma_x}{E}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}(\sigma_x - v\sigma_x - v\sigma_x) = \frac{(1-2v)\sigma_x}{E}$$

Volume: $V = AL = (380.13 \text{ mm}^2)(200 \text{ mm}) = 76.026 \times 10^3 \text{ mm}^3$

$\Delta V = Ve$

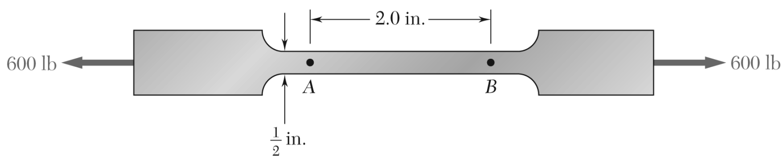
(a) Steel: $e = \dfrac{(1-0.60)(121.01 \times 10^6)}{200 \times 10^9} = 242 \times 10^{-6}$ $\qquad e = 242 \times 10^{-6}$ ◀

$\Delta V = (76.026 \times 10^3)(242 \times 10^{-6}) = 18.40 \text{ mm}^3$ $\qquad \Delta V = 18.40 \text{ mm}^3$ ◀

(b) Aluminum: $e = \dfrac{(1-0.70)(121.01 \times 10^6)}{70 \times 10^9} = 519 \times 10^{-6}$ $\qquad e = 519 \times 10^{-6}$ ◀

$\Delta V = (76.026 \times 10^3)(519 \times 10^{-6}) = 39.4 \text{ mm}^3$ $\qquad \Delta V = 39.4 \text{ mm}^3$ ◀

PROBLEM 2.84*



Determine the change in volume of the 2-in. gage length segment *AB* in Prob. 2.61 (*a*) by computing the dilatation of the material, (*b*) by subtracting the original volume of portion *AB* from its final volume.

SOLUTION

From Problem 2.61, thickness = $\frac{1}{16}$ in., $E = 29 \times 10^6$ psi, $v = 0.30$.

(*a*) $A = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = 0.03125 \text{ in}^2$

Volume: $V_0 = AL_0 = (0.03125)(2.00) = 0.0625 \text{ in}^3$

$$\sigma_x = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{\sigma_x}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -v\varepsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 264.83 \times 10^{-6}$$

$$\Delta V = V_0 e = (0.0625)(264.83 \times 10^{-6}) = 16.55 \times 10^{-6} \text{ in}^3 \quad ◀$$

(*b*) From the solution to Problem 2.61,

$\delta_x = 1.324 \times 10^{-3}$ in., $\delta_y = -99.3 \times 10^{-6}$ in., $\delta_z = -12.41 \times 10^{-6}$ in.

The dimensions when under a 600-lb tensile load are:

Length: $L = L_0 + \delta_x = 2 + 1.324 \times 10^{-3} = 2.001324$ in.

Width: $w = w_0 + \delta_y = \frac{1}{2} - 99.3 \times 10^{-6} = 0.4999007$ in.

Thickness: $t = t_0 + \delta_z = \frac{1}{16} - 12.41 \times 10^{-6} = 0.06248759$ in.

Volume: $V = Lwt = 0.062516539 \text{ in}^3$

$$\Delta V = V - V_0 = 0.062516539 - 0.0625 = 16.54 \times 10^{-6} \text{ in}^3 \quad ◀$$

PROBLEM 2.85*

A 6-in.-diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29\times10^6$ psi and $v = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

SOLUTION

For a solid sphere,

$$V_0 = \frac{\pi}{6}d_0^3$$
$$= \frac{\pi}{6}(6.00)^3$$
$$= 113.097 \text{ in.}^3$$
$$\sigma_x = \sigma_y = \sigma_z = -\text{p}$$
$$= -7.1\times10^3 \text{psi}$$
$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z)$$
$$= -\frac{(1-2v)p}{E} = -\frac{(0.4)(7.1\times10^3)}{29\times10^6}$$
$$= -97.93\times10^{-6}$$

Likewise,

$$\varepsilon_y = \varepsilon_z = -97.93\times10^{-6}$$
$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -293.79\times10^{-6}$$

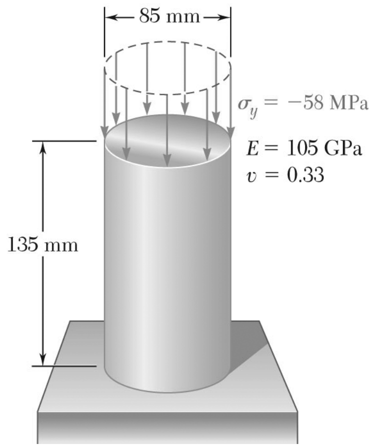
(*a*) $-\Delta d = -d_0\varepsilon_x = -(6.00)(-97.93\times10^{-6}) = 588\times10^{-6}\text{in.}$ $\qquad -\Delta d = 588\times10^{-6}\text{in.}$ ◀

(*b*) $-\Delta V = -V_0 e = -(113.097)(-293.79\times10^{-6}) = 33.2\times10^{-3}\text{in}^3$ $\qquad -\Delta V = 33.2\times10^{-3}\text{in}^3$ ◀

(*c*) Let m = mass of sphere. $\qquad m$ = constant.

$$m = \rho_0 V_0 = \rho V = \rho V_0(1+e)$$
$$\frac{\rho-\rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)}\times\frac{V_0}{m} - 1 = \frac{1}{1+e} - 1$$
$$= (1 - e + e^2 - e^3 + \cdots) - 1 = -e + e^2 - e^3 + \cdots$$
$$\approx -e = 293.79\times10^{-6}$$
$$\frac{\rho-\rho_0}{\rho_0}\times100\% = (293.79\times10^{-6})(100\%)$$

0.0294% ◀



PROBLEM 2.86*

(*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a*, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

SOLUTION

$$h_0 = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_0 = \frac{\pi}{4} d_0^2 = \frac{\pi}{4}(85)^2 = 5.6745\times 10^3 \text{ mm}^2 = 5.6745\times 10^{-3} \text{ m}^2$$

$$V_0 = A_0 h_0 = 766.06\times 10^3 \text{ mm}^3 = 766.06\times 10^{-6} \text{ m}^3$$

(*a*) $\sigma_x = 0,\ \sigma_y = -58\times 10^6 \text{ Pa},\ \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z) = \frac{\sigma_y}{E} = -\frac{58\times 10^6}{105\times 10^9} = -552.38\times 10^{-6}$$

$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-552.38\times 10^{-6})$ $\Delta h = -0.0746$ mm ◀

$$e = \frac{1-2v}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2v)\sigma_y}{E} = \frac{(0.34)(-58\times 10^6)}{105\times 10^9} = -187.81\times 10^{-6}$$

$\Delta V = V_0 e = (766.06\times 10^3 \text{ mm}^3)(-187.81\times 10^{-6})$ $\Delta V = -143.9 \text{ mm}^3$ ◀

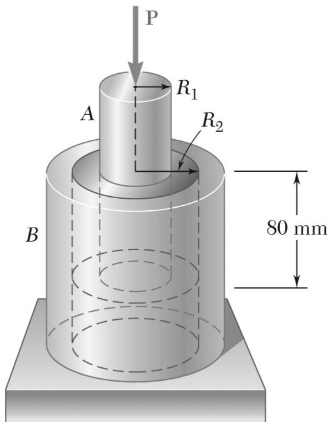
(*b*) $\sigma_x = \sigma_y = \sigma_z = -70\times 10^6 \text{ Pa}$ $\sigma_x + \sigma_y + \sigma_z = -210\times 10^6 \text{ Pa}$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z) = \frac{1-2v}{E}\sigma_y = \frac{(0.34)(-70\times 10^6)}{105\times 10^9} = -226.67\times 10^{-6}$$

$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-226.67\times 10^{-6})$ $\Delta h = -0.0306$ mm ◀

$$e = \frac{1-2v}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210\times 10^6)}{105\times 10^9} = -680\times 10^{-6}$$

$\Delta V = V_0 e = (766.06\times 10^3 \text{ mm}^3)(-680\times 10^{-6})$ $\Delta V = -521 \text{ mm}^3$ ◀

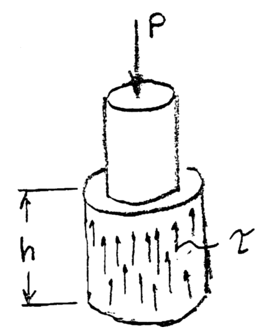


PROBLEM 2.87*

A vibration isolation support consists of a rod A of radius $R_1 = 10$ mm and a tube B of inner radius $R_2 = 25$ mm bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity G = 12 MPa. Determine the largest allowable force **P** that can be applied to rod A if its deflection is not to exceed 2.50 mm.

SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.



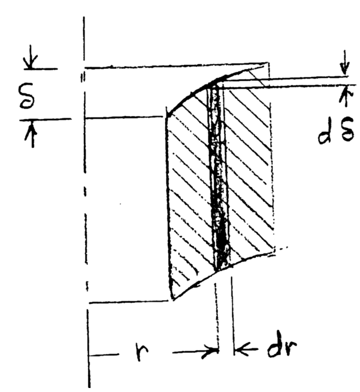
Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length dr,



$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi Gh}\frac{dr}{r}$$

Total deformation.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

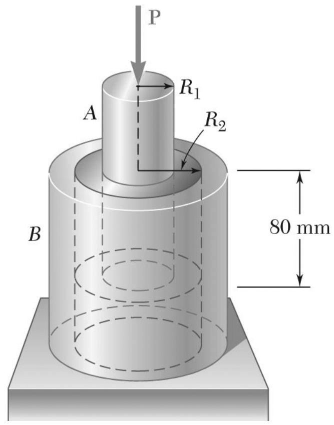
$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi Gh\delta}{\ln(R_2/R_1)}$$

Data: $R_1 = 10\text{ mm} = 0.010\text{ m}, \quad R_2 = 25\text{ mm} = 0.025\text{ m}, \quad h = 80\text{ mm} = 0.080\text{ m}$

$$G = 12\times10^6\,\text{Pa} \qquad \delta = 2.50\times10^{-3}\text{ m}$$

$$P = \frac{(2\pi)(12\times10^6)(0.080)(2.50\times10^{-3})}{\ln(0.025/0.010)} = 16.46\times10^3\,\text{N}$$

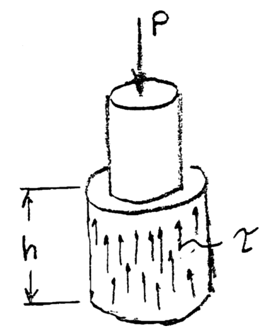
16.46 kN ◀



PROBLEM 2.88

A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93$ MPa. Determine the required value of the ratio R_2/R_1 if a 10-kN force **P** is to cause a 2-mm deflection of rod A.

SOLUTION



Let r be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.

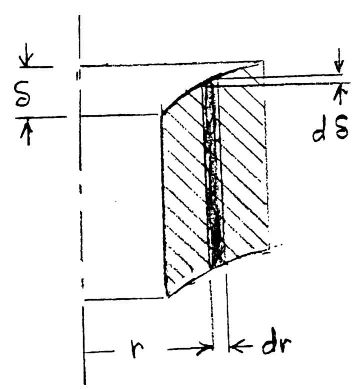
Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$

Shearing deformation over radial length dr,



$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr$$

$$dr\delta = \frac{P}{2\pi G h}\,\frac{dr}{r}$$

<u>Total deformation</u>.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi G h}\ln r\,\Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi G h\delta}{P} = \frac{(2\pi)(10.93\times 10^6)(0.080)(0.002)}{10.10^3} = 1.0988$$

$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

$R_2/R_1 = 3.00$ ◀

PROBLEM 2.89*

The material constants E, G, k, and v are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (a) $k = GE/(9G - 3E)$ and (b) $v = (3k - 2G)/(6k + 2G)$.

SOLUTION

$$k = \frac{E}{3(1-2v)} \quad \text{and} \quad G = \frac{E}{2(1+v)}$$

(a) $\quad 1 + v = \dfrac{E}{2G} \quad \text{or} \quad v = \dfrac{E}{2G} - 1$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E} \qquad k = \frac{EG}{9G - 6E} \blacktriangleleft$$

(b) $\quad \dfrac{k}{G} = \dfrac{2(1+v)}{3(1-2v)}$

$$3k - 6kv = 2G + 2Gv$$

$$3k - 2G = 2G + 6k \qquad v = \frac{3k - 2G}{6k + 2G} \blacktriangleleft$$

PROBLEM 2.90*

Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [*Hint:* Refer to Eq. (2.43) and to Sec. 2.13.]

SOLUTION

$$G = \frac{E}{2(1+v)} \qquad \text{or} \qquad \frac{E}{G} = 2(1+v)$$

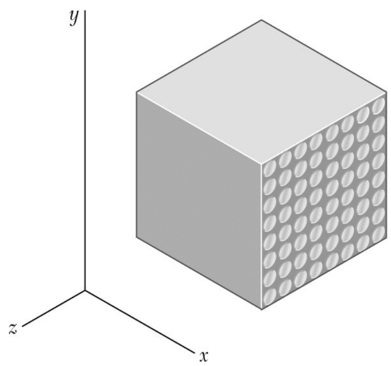
Assume $v > 0$ for almost all materials, and $v < \frac{1}{2}$ for a positive bulk modulus.

Applying the bounds, $$2 \le \frac{E}{G} < 2\left(1+\frac{1}{2}\right) = 3$$

Taking the reciprocals, $$\frac{1}{2} > \frac{G}{E} > \frac{1}{3}$$

or $$\frac{1}{3} < \frac{G}{E} < \frac{1}{2} \blacktriangleleft$$

PROBLEM 2.91*



A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (a) the change in the length of the cube in the x direction, (b) the stresses σ_x, σ_y, and σ_z.

$$E_x = 50 \text{ GPa} \quad v_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad v_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad v_{zy} = 0.428$$

SOLUTION

Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{v_{yx}\sigma_y}{E_y} - \frac{v_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{v_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{v_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{v_{xz}\sigma_x}{E_x} - \frac{v_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{v_{xy}}{E_x} = \frac{v_{yx}}{E_y} \tag{4}$$

$$\frac{v_{yz}}{E_y} = \frac{v_{zy}}{E_z} \tag{5}$$

$$\frac{v_{zx}}{E_z} = \frac{v_{xz}}{E_x} \tag{6}$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{v_{zy}}{E_z}\sigma_z = \frac{v_{xy}}{E_x}\sigma_x \tag{7}$$

$$-\frac{v_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{V_{xz}}{E_x}\sigma_x \tag{8}$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\,\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\,\sigma_y + \sigma_z = 0.077216\,\sigma_x$$

PROBLEM 2.91* *(Continued)*

Solving simultaneously, $\sigma_y = \sigma_z = 0.134993\,\sigma_x$

Using (4) and (5) in (1), $\varepsilon_x = \dfrac{1}{E_x}\sigma_x - \dfrac{v_{xy}}{E_x}\sigma_y - \dfrac{v_{xz}}{E}\sigma_z$

$$E_x = \frac{1}{E_x}[1-(0.254)(0.134993)-(0.254)(0.134993)]\sigma_x$$

$$= \frac{0.93142\sigma_x}{E_x}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600\times10^{-6} \text{ m}^2$$

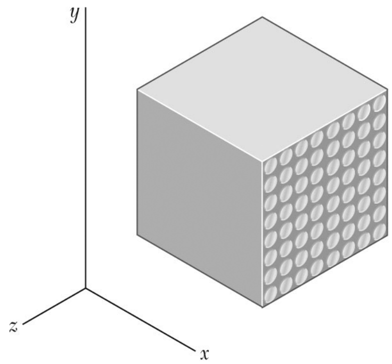
$$\sigma_x = \frac{P}{A} = \frac{65\times10^3}{1600\times10^{-6}} = 40.625\times10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{(0.93142)(40.625\times10^3)}{50\times10^9} = 756.78\times10^{-6}$$

(*a*) $\delta_x = L_x\varepsilon_x = (40 \text{ mm})(756.78\times10^{-6})$ $\delta_x = 0.0303 \text{ mm}$ ◀

(*b*) $\sigma_x = 40.625\times10^6 \text{ Pa}$ $\sigma_x = 40.6 \text{ MPa}$ ◀

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6 \text{ Pa}$ $\sigma_y = \sigma_z = 5.48 \text{ MPa}$ ◀



PROBLEM 2.92*

The composite cube of Prob. 2.91 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (*a*) the stresses σ_x, σ_y, and σ_z, (*b*) the change in the dimension in the y direction.

$$E_x = 50 \text{ GPa} \quad v_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad v_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad v_{zy} = 0.428$$

SOLUTION

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{v_{yx}\sigma_y}{E_y} - \frac{v_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{v_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{v_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{v_{xz}\sigma_x}{E_x} - \frac{v_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{v_{xy}}{E_x} = \frac{v_{yx}}{E_y} \tag{4}$$

$$\frac{v_{yz}}{E_y} = \frac{v_{zy}}{E_z} \tag{5}$$

$$\frac{v_{zx}}{E_z} = \frac{v_{xz}}{E_x} \tag{6}$$

Constraint condition: $\varepsilon_z = 0$

Load condition: $\sigma_y = 0$

From Equation (3), $$0 = -\frac{v_{xz}}{E_x}\sigma_x + \frac{1}{E_z}\sigma_z$$

$$\sigma_z = \frac{v_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\,\sigma_x$$

From Equation (1) with $\sigma_y = 0$,

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{v_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{v_{xz}}{E_x}\sigma_z$$
$$= \frac{1}{E_x}[\sigma_x - 0.254\sigma_z] = \frac{1}{E_x}[1-(0.254)(0.077216)]\sigma_x$$
$$= \frac{0.98039}{E_x}\sigma_x$$
$$\sigma_x = \frac{E_x\varepsilon_x}{0.98039}$$

But, $\varepsilon_x = \dfrac{\delta_x}{L_x} = \dfrac{0.035 \text{ mm}}{40 \text{ mm}} = 875\times 10^{-6}$

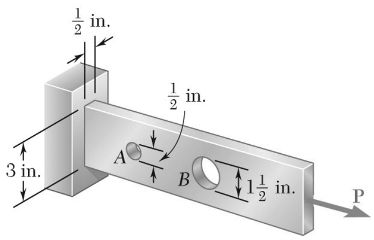
(*a*) $\sigma_x = \dfrac{(50\times 10^9)(875\times 10^{-6})}{0.98039} = 44.625\times 10^3 \text{ Pa}$ $\quad\sigma_x = 44.6 \text{ MPa}$ ◀

$\sigma_y = 0$ ◀

$\sigma_z = (0.077216)(44.625\times 10^6) = 3.446\times 10^6 \text{ Pa}$ $\quad\sigma_z = 3.45 \text{ MPa}$ ◀

From (2),
$$\varepsilon_y = \frac{v_{xy}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_y - \frac{v_{zy}}{E_z}\sigma_z$$
$$= -\frac{(0.254)(44.625\times 10^6)}{50\times 10^9} + 0 - \frac{(0.428)(3.446\times 10^6)}{15.2\times 10^9}$$
$$= -323.73\times 10^{-6}$$

(*b*) $\delta_y = L_y\varepsilon_y = (40 \text{ mm})(-323.73\times 10^{-6})$ $\quad\delta_y = -0.0129 \text{ mm}$ ◀



PROBLEM 2.93

Two holes have been drilled through a long steel bar that is subjected to a centric axial load as shown. For $P = 6.5$ kips, determine the maximum value of the stress (*a*) at *A*, (*b*) at *B*.

SOLUTION

(*a*) At hole *A*: $r = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ in.

$$d = 3 - \frac{1}{2} = 2.50 \text{ in.}$$

$$A_{\text{net}} = dt = (2.50)\left(\frac{1}{2}\right) = 1.25 \text{ in}^2$$

$$\sigma_{\text{non}} = \frac{P}{A_{\text{net}}} = \frac{6.5}{1.25} = 5.2 \text{ ksi}$$

$$\frac{2r}{D} = \frac{2\left(\frac{1}{4}\right)}{3} = 0.1667$$

From Fig. 2.60*a*, $K = 2.56$

$$\sigma_{\max} = K\sigma_{\text{non}} = (2.56)(5.2) \qquad \sigma_{\max} = 13.31 \text{ ksi} \blacktriangleleft$$

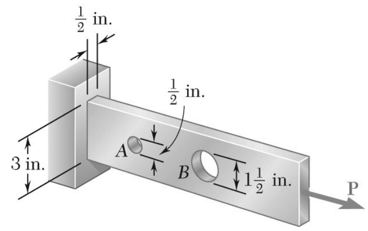
(*b*) At hole B: $r = \frac{1}{2}(1.5) = 0.75, \quad d = 3 - 1.5 = 1.5$ in.

$$A_{\text{net}} = dt = (1.5)\left(\frac{1}{2}\right) = 0.75 \text{ in}^2, \qquad \sigma_{\text{non}} = \frac{P}{A_{\text{net}}} = \frac{6.5}{0.75} = 8.667 \text{ ksi}$$

$$\frac{2r}{D} = \frac{2(0.75)}{3} = 0.5$$

From Fig. 2.60*a*, $K = 2.16$

$$\sigma_{\max} = K\sigma_{\text{non}} = (2.16)(8.667) \qquad \sigma_{\max} = 18.72 \text{ ksi} \blacktriangleleft$$



PROBLEM 2.94

Knowing that $\sigma_{\text{all}} = 16$ ksi, determine the maximum allowable value of the centric axial load **P**.

SOLUTION

At hole A:

$$r = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \text{ in.}$$

$$d = 3 - \frac{1}{2} = 2.50 \text{ in.}$$

$$A_{\text{net}} = dt = (2.50)\left(\frac{1}{2}\right) = 1.25 \text{ in}^2$$

$$\frac{2r}{D} = \frac{2\left(\frac{1}{4}\right)}{3} = 0.1667$$

From Fig. 2.60a, $K = 2.56$

$$\sigma_{\max} = \frac{KP}{A_{\text{net}}} \quad \therefore \quad P = \frac{A_{\text{net}}\sigma_{\max}}{K} = \frac{(1.25)(16)}{2.56} = 7.81 \text{ kips}$$

At hole B:

$$r = \frac{1}{2}(1.5) = 0.75 \text{ in,} \qquad d = 3 - 1.5 = 1.5 \text{ in.}$$

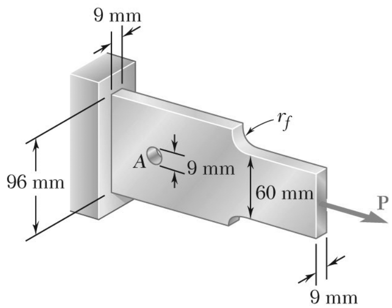
$$A_{\text{net}} = dt = (1.5)\left(\frac{1}{2}\right) = 0.75 \text{ in}^2,$$

$$\frac{2r}{D} = \frac{2(0.75)}{3} = 0.5$$

From Fig. 2.60a, $K = 2.16$

$$P = \frac{A_{\text{net}}\sigma_{\max}}{K} = \frac{(0.75)(16)}{2.16} = 5.56 \text{ kips}$$

Smaller value for P controls. $P = 5.56$ kips ◀



PROBLEM 2.95

Knowing that the hole has a diameter of 9 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 100 MPa.

SOLUTION

For the circular hole, $$r = \left(\frac{1}{2}\right)(9) = 4.5 \text{ mm}$$

$$d = 96 - 9 = 87 \text{ mm} \qquad \frac{2r}{D} = \frac{2(4.5)}{96} = 0.09375$$

$$A_{\text{net}} = dt = (0.087 \text{ m})(0.009 \text{ m}) = 783 \times 10^{-6} \text{ m}^2$$

From Fig. 2.60*a*, $$K_{\text{hole}} = 2.72$$

$$\sigma_{\max} = \frac{K_{\text{hole}} P}{A_{\text{net}}}$$

$$P = \frac{A_{\text{net}}\sigma_{\max}}{K_{\text{hole}}} = \frac{(783 \times 10^{-6})(100 \times 10^6)}{2.72} = 28.787 \times 10^3 \text{ N}$$

(*a*) For fillet, $$D = 96 \text{ mm}, d = 60 \text{ mm}$$

$$\frac{D}{d} = \frac{96}{60} = 1.60$$

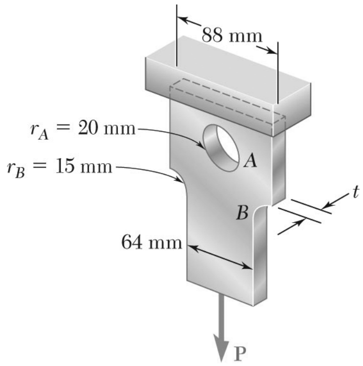
$$A_{\min} = dt = (0.060 \text{ m})(0.009 \text{ m}) = 540 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\max} = \frac{K_{\text{fillet}} P}{A_{\min}} \quad \therefore \quad K_{\text{fillet}} = \frac{A_{\min}\sigma_{\max}}{P} = \frac{(5.40 \times 10^{-6})(100 \times 10^6)}{28.787 \times 10^3}$$
$$= 1.876$$

From Fig. 2.60*b*, $$\frac{r_f}{d} \approx 0.19 \quad \therefore \quad r_f \approx 0.19d = 0.19(60)$$ $r_f = 11.4$ mm ◀

(*b*) $P = 28.8$ kN ◀

PROBLEM 2.96



For $P = 100$ kN, determine the minimum plate thickness t required if the allowable stress is 125 MPa.

SOLUTION

At the hole: $r_A = 20$ mm $\quad d_A = 88 - 40 = 48$ mm

$$\frac{2r_A}{D_A} = \frac{2(20)}{88} = 0.455$$

From Fig. 2.60*a*, $K = 2.20$

$$\sigma_{\max} = \frac{KP}{A_{\text{net}}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{\max}}$$

$$t = \frac{(2.20)(100 \times 10^3 \text{ N})}{(0.048 \text{ m})(125 \times 10^6 \text{ Pa})} = 36.7 \times 10^{-3} \text{ m} = 36.7 \text{ mm}$$

At the fillet: $D = 88$ mm, $\quad d_B = 64$ mm $\quad \dfrac{D}{d_B} = \dfrac{88}{64} = 1.375$

$$r_B = 15 \text{ mm} \qquad \frac{r_B}{d_B} = \frac{15}{64} = 0.2344$$

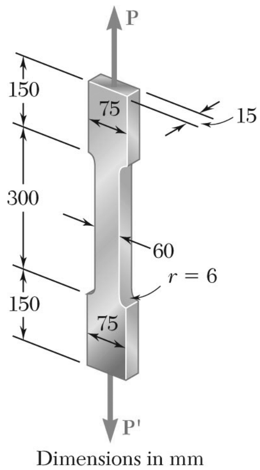
From Fig. 2.60*b*, $K = 1.70$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{KP}{d_B t}$$

$$t = \frac{KP}{d_B \sigma_{\max}} = \frac{(1.70)(100 \times 10^3 \text{ N})}{(0.064 \text{ m})(125 \times 10^6 \text{ Pa})} = 21.25 \times 10^{-3} \text{ m} = 21.25 \text{ mm}$$

The larger value is the required minimum plate thickness.

$t = 36.7$ mm ◀



PROBLEM 2.97

The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (*a*) Knowing that $E = 70$ GPa and $\sigma_{\text{all}} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60×15-mm rectangular cross section.

SOLUTION

$$\sigma_{\text{all}} = 200\times10^6 \text{ Pa} \quad E = 70\times10^9 \text{ Pa}$$

$$A_{\min} = (60 \text{ mm})(15 \text{ mm}) = 900 \text{ mm}^2 = 900\times10^{-6} \text{ m}^2$$

(*a*) Test specimen. $D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.60*b* $\quad K = 1.95 \qquad \sigma_{\max} = K\dfrac{P}{A}$

$$P = \frac{A\sigma_{\max}}{K} = \frac{(900\times10^{-6})(200\times10^6)}{1.95} = 92.308\times10^3 \text{ N} \qquad P = 92.3 \text{ kN} \blacktriangleleft$$

Wide area $A^* = (75 \text{ mm})(15 \text{ mm}) = 1125 \text{ mm}^2 = 1.125\times10^{-3} \text{m}^2$

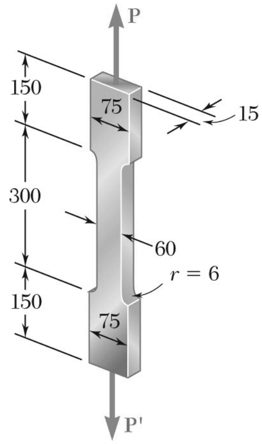
$$\delta = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{P}{E}\Sigma\frac{L_i}{A_i} = \frac{92.308\times10^3}{70\times10^9}\left[\frac{0.150}{1.125\times10^{-3}} + \frac{0.300}{900\times10^{-6}} + \frac{0.150}{1.125\times10^{-3}}\right]$$

$$= 7.91\times10^{-6} \text{m} \qquad \delta = 0.791 \text{ mm} \blacktriangleleft$$

(*b*) Uniform bar.

$$P = A\sigma_{\text{all}} = (900\times10^{-6})(200\times10^6) = 180\times10^3 \text{N} \qquad P = 180.0 \text{ kN} \blacktriangleleft$$

$$\delta = \frac{PL}{AE} = \frac{(180\times10^3)(0.600)}{(900\times10^{-6})(70\times10^9)} = 1.714\times10^{-3} \text{ m} \qquad \delta = 1.714 \text{ mm} \blacktriangleleft$$



PROBLEM 2.98

For the test specimen of Prob. 2.97, determine the maximum value of the normal stress corresponding to a total elongation of 0.75 mm.

PROBLEM 2.97 The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (a) Knowing that $E = 70$ GPa and $\sigma_{\text{all}} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (b) Solve part a, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60×15-mm rectangular cross section.

SOLUTION

$$\delta = \Sigma\frac{P_i L_i}{E_i A_i} = \frac{P}{E}\Sigma\frac{L_i}{A_i} \qquad \delta = 0.75\times10^{-3}\,\text{m}$$

$$L_1 = L_3 = 150 \text{ mm} = 0.150 \text{ m}, \quad L_2 = 300 \text{ mm} = 0.300 \text{ m}$$

$$A_1 = A_3 = (75 \text{ mm})(15 \text{ mm}) = 1125 \text{ mm}^2 = 1.125\times10^{-3}\text{ m}^2$$

$$A_2 = (60 \text{ mm})(15 \text{ mm}) = 900 \text{ mm}^2 = 900\times10^{-6}\text{ m}^2$$

$$\Sigma\frac{L_i}{A_i} = \frac{0.150}{1.125\times10^{-3}} + \frac{0.300}{900\times10^{-6}} + \frac{0.150}{1.125\times10^{-3}} = 600\text{ m}^{-1}$$

$$P = \frac{E\delta}{\Sigma\frac{L_i}{A_i}} = \frac{(70\times10^9)(0.75\times10^{-3})}{600} = 87.5\times10^3\,\text{N}$$

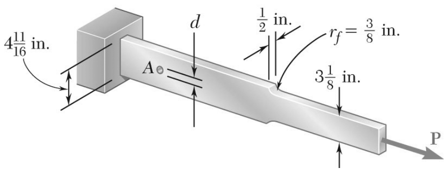
<u>Stress concentration</u>. $D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.60b $\quad K = 1.95$

$$\sigma_{\text{max}} = K\frac{P}{A_{\text{min}}} = \frac{(1.95)(87.5\times10^3)}{900\times10^{-6}} = 189.6\times10^6 \text{ Pa} \qquad \sigma_{\text{max}} = 189.6 \text{ MPa} \blacktriangleleft$$

Note that $\sigma_{\text{max}} < \sigma_{\text{all}}$.



PROBLEM 2.99

A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from $\frac{1}{2}$ to $1^1/_2$ in. in $\frac{1}{4}$-in. increments. If the allowable stress in the plate is 21 ksi, determine (*a*) the diameter d of the largest bit that can be used if the allowable load **P** at the hole is to exceed that at the fillets, (*b*) the corresponding allowable load **P**.

SOLUTION

At the fillets: $\frac{D}{d} = \frac{4.6875}{3.125} = 1.5 \qquad \frac{r}{d} = \frac{0.375}{3.125} = 0.12$

From Fig. 2.60*b*, $K = 2.10$

$$A_{\min} = (3.125)(0.5) = 1.5625 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P_{\text{all}}}{A_{\min}} = \sigma_{\text{all}}$$

$$P_{\text{all}} = \frac{A_{\min}\sigma_{\text{all}}}{K} = \frac{(1.5625)(21)}{2.10} = 15.625 \text{ kips}$$

At the hole: $A_{\text{net}} = (D - 2r)t,\ K$ from Fig. 2.60*a*

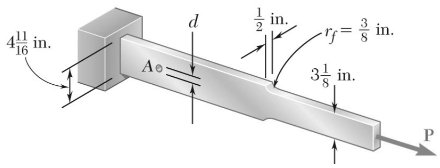
$$\sigma_{\max} = K\frac{P}{A_{\text{net}}} = \sigma_{\text{all}} \qquad \therefore \qquad P_{\text{all}} = \frac{A_{\text{net}}\sigma_{\text{all}}}{K}$$

with $D = 4.6875$ in. $\quad t = 0.5$ in. $\quad \sigma_{\text{all}} = 21$ ksi

Hole diam.	r	$d = D - 2r$	$2r/D$	K	A_{net}	P_{all}
0.5 in.	0.25 in.	4.1875 in.	0.107	2.68	2.0938 in^2	16.41 kips
0.75 in.	0.375 in.	3.9375 in.	0.16	2.58	1.96875 in^2	16.02 kips
1 in.	0.5 in.	3.6875 in.	0.213	2.49	1.84375 in^2	15.55 kips
1.25 in.	0.625 in.	3.4375 in.	0.267	2.41	1.71875 in^2	14.98 kips
1.5 in.	0.75 in.	3.1875 in.	0.32	2.34	1.59375 in^2	14.30 kips

(*a*) Largest hole with $P_{\text{all}} > 15.625$ kips is the $\frac{3}{4}$-in. diameter hole. ◀

(*b*) Allowable load $P_{\text{all}} = 15.63$ kips ◀



PROBLEM 2.100

(*a*) For $P = 13$ kips and $d = \frac{1}{2}$ in., determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.

SOLUTION

Maximum stress at hole:

Use Fig. 2.60*a* for values of *K*.

$$\frac{2r}{D} = \frac{0.5}{4.6875} = 0.017, \qquad K = 2.68$$

$$A_{\text{net}} = (0.5)(4.6875 - 0.5) = 2.0938 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P}{A_{\text{net}}} = \frac{(2.68)(13)}{2.0938} = 16.64 \text{ ksi},$$

Maximum stress at fillets:

Use Fig. 2.60*b* for values of *K*.

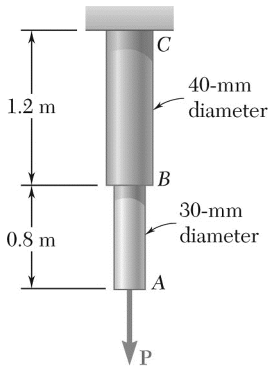
$$\frac{r}{d} = \frac{0.375}{3.125} = 0.12 \qquad \frac{D}{d} = \frac{4.6875}{3.125} = 1.5 \qquad K = 2.10$$

$$A_{\min} = (0.5)(3.125) = 1.5625 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P}{A_{\min}} = \frac{(2.10)(13)}{1.5625} = 17.47 \text{ ksi}$$

(*a*) With hole and fillets: 17.47 ksi ◀

(*b*) Without hole: 17.47 ksi ◀



PROBLEM 2.101

Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

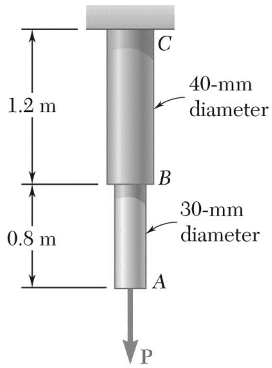
$$P_{\max} = A_{\min}\sigma_Y = (706.86 \times 10^{-6})(250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

$$P_{\max} = 176.7 \text{ kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})}$$

$$= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' \quad \text{or} \quad \delta_m = \delta_p + \delta' = 2 + 1.84375 \qquad \delta_m = 3.84 \text{ mm} \blacktriangleleft$$



PROBLEM 2.102

Rod ABC consists of two cylindrical portions AB and BC; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end A has moved down by an amount $\delta_m = 5$ mm. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664\times 10^3 \text{ mm}^2 = 1.25644\times 10^{-3} \text{ m}^2$$

$$P_{\max} = A_{\min}\sigma_Y = (706.86\times 10^{-6})(250\times 10^6) = 176.715\times 10^3 \text{ N}$$

$P_{\max} = 176.7$ kN ◀

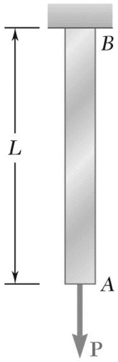
$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715\times 10^3)(0.8)}{(200\times 10^9)(706.68\times 10^{-6})} + \frac{(176.715\times 10^3)(1.2)}{(200\times 10^9)(1.25664\times 10^{-3})}$$

$$= 1.84375\times 10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 5 - 1.84375 = 3.16 \text{ mm}$$

$\delta_p = 3.16$ mm ◀

PROBLEM 2.103



The 30-mm square bar AB has a length $L = 2.2$ m; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar until end A has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing that (*a*) $\delta_m = 4.5$ mm, (*b*) $\delta_m = 8$ mm.

SOLUTION

$$A = (30)(30) = 900 \text{ mm}^2 = 900\times10^{-6}\,\text{m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.2)(345\times10^6)}{200\times10^9} = 3.795\times10^{-3} = 3.795 \text{ mm}$$

If $\delta_m \geq \delta_Y$, $P_m = A\sigma_Y = (900\times10^{-6})(345\times10^6) = 310.5\times10^3\,\text{N}$

Unloading: $\delta' = \dfrac{P_m L}{AE} = \dfrac{\sigma_Y L}{E} = \delta_Y = 3.795$ mm

$$\delta_P = \delta_m - \delta'$$

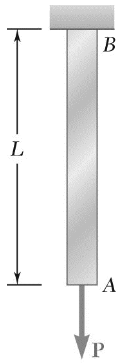
(*a*) $\delta_m = 4.5 \text{ mm} > \delta_Y$ $P_m = 310.5\times10^3$ N $\qquad \delta_m = 310.5$ kN ◀

$\delta_{\text{perm}} = 4.5 \text{ mm} - 3.795$ mm $\qquad \delta_{\text{perm}} = 0.705$ mm ◀

(*b*) $\delta_m = 8 \text{ mm} > \delta_Y$ $P_m = 310.5\times10^3$ N $\qquad \delta_m = 310.5$ kN ◀

$\delta_{\text{perm}} = 8.0 \text{ mm} - 3.795$ mm $\qquad \delta_{\text{perm}} = 4.205$ mm ◀

PROBLEM 2.104



The 30-mm square bar AB has a length $L = 2.5\,\text{m}$; it is made of mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar and then removed to give it a permanent set δ_p. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (*a*) 3.5 mm, (*b*) 6.5 mm.

SOLUTION

$$A = (30)(30) = 900\ \text{mm}^2 = 900 \times 10^{-6}\ \text{m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.5)(345\times 10^6)}{200\times 10^9} = 4.3125\times 10^3\,\text{m} = 4.3125\ \text{mm}$$

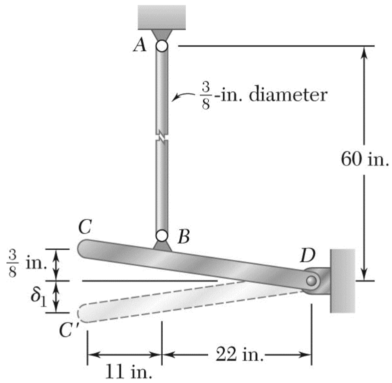
When δ_m exceeds δ_Y, thus producing a permanent stretch of δ_p, the maximum force is

$$P_m = A\sigma_Y = (900\times 10^{-6})(345\times 10^6) = 310.5\times 10^3\,\text{N} \qquad = 310.5\ \text{kN} \blacktriangleleft$$

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$$

(*a*) $\delta_p = 3.5$ mm $\quad \delta_m = 3.5\ \text{mm} + 4.3125\ \text{mm}$ $\qquad = 7.81$ mm ◀

(*b*) $\delta_p = 6.5$ mm $\quad \delta_m = 6.5\ \text{mm} + 4.3125\ \text{mm}$ $\qquad = 10.81$ mm ◀



PROBLEM 2.105

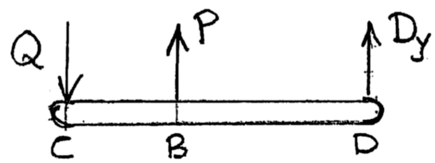
Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi. After the rod has been attached to the rigid lever CD, it is found that end C is $\frac{3}{8}$ in. too high. A vertical force **Q** is then applied at C until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

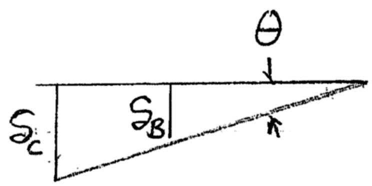
SOLUTION

Since the rod AB is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 (36) = 3.976 \text{ kips}$$

Referring to the free body diagram of lever CD,





$$\Sigma M_D = 0: \quad 33Q - 22P = 0$$

$$Q = \frac{22}{33}P = \frac{(22)(3.976)}{33} = 2.65 \text{ kips}$$

$Q = 2.65$ kips ◄

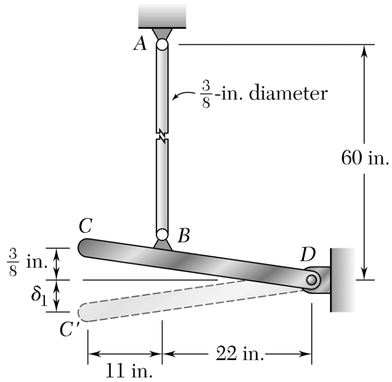
During unloading, the spring back at B is

$$\delta_B = L_{AB}\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(36\times10^3)}{29\times10^6} = 0.0745 \text{ in.}$$

From the deformation diagram,

Slope: $\theta = \dfrac{\delta_B}{22} = \dfrac{\delta_C}{33} \quad \therefore \ \delta_C = \dfrac{33}{-22}\delta_B = 0.1117$ in.

$\delta_C = 0.1117$ in. ◄



PROBLEM 2.106

Solve Prob. 2.105, assuming that the yield point of the mild steel is 50 ksi.

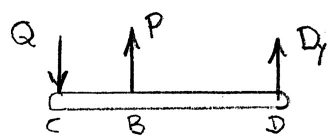
PROBLEM 2.105 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is $\frac{3}{8}$ in. too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C′*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

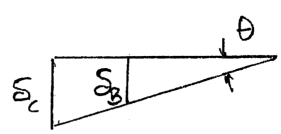
SOLUTION

Since the rod *AB* is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 (50) = 5.522 \text{ kips}$$

Referring to the free body diagram of lever *CD*,





$$\Sigma M_D = 0: \quad 33Q - 22P = 0$$

$$Q = \frac{22}{33}P = \frac{(22)(5.522)}{33} = 3.68 \text{ kips}$$

$Q = 3.68$ kips ◀

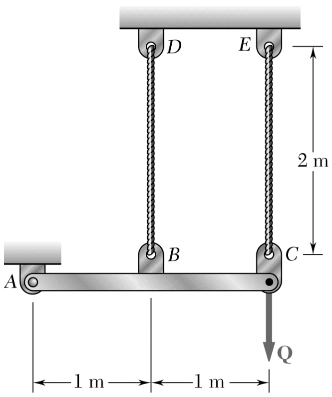
During unloading, the spring back at *B* is

$$\delta_B = L_{AB}\,\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(50\times10^3)}{29\times10^6} = 0.1034 \text{ in.}$$

From the deformation diagram,

Slope: $\theta = \dfrac{\delta_B}{22} = \dfrac{\delta_C}{33} \quad \therefore \quad \delta_C = \dfrac{33}{22}\delta_B$

$\delta_C = 0.1552$ in. ◀



PROBLEM 2.107

Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable BD, (*b*) the maximum deflection of point C, (*c*) the final displacement of point C. (*Hint:* In Part *c*, cable CE is not taut.)

SOLUTION

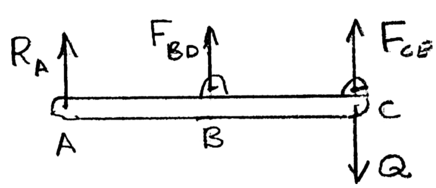
<u>Elongation constraints for taut cables</u>.

Let $\theta =$ rotation angle of rigid bar ABC.

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\ \delta_{CE} = \frac{1}{2}\delta_{CE} \tag{1}$$

<u>Equilibrium of bar ABC</u>.



$$+\circlearrowleft M_A = 0:\ L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}\ Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \tag{2}$$

Assume cable CE is yielded. $\quad F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}\,\text{N}$

From (2), $\quad F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}\,\text{N}$

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}\,\text{N},$ cable BD is elastic when $Q = 50$ kN.

PROBLEM 2.107 *(Continued)*

(a) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^3}{100\times10^{-6}} = 310\times10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa} \blacktriangleleft$$

(b) Maximum of deflection of point C.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^3)(2)}{(200\times10^9)(100\times10^{-6})} = 3.1\times10^{-3} \text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m $\qquad$ 6.20 mm $\downarrow \blacktriangleleft$

Permanent elongation of cable CE: $(\delta_{CE})_p = (\delta_{CE}) - \dfrac{\sigma_Y L_{CE}}{E}$

$$(\delta_{CE})_P = (\delta_{CE})_{\max} - \frac{F_{CE}L_{CE}}{EA} = (\delta_{CE})_{\max} - \frac{\sigma_Y L_{CE}}{E}$$

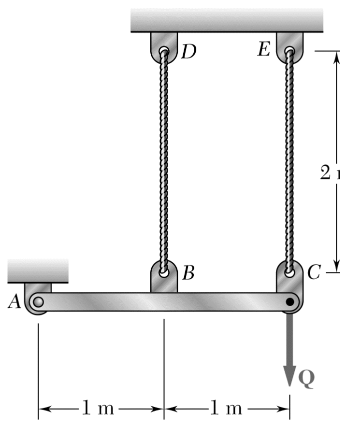
$$= 6.20\times10^{-3} - \frac{(345\times10^6)(2)}{200\times10^9} = 2.75\times10^{-3} \text{ m}$$

(c) Unloading. Cable CE is slack $(F_{CE} = 0)$ at $Q = 0$.

From (2), $F_{BD} = 2(Q - F_{CE}) = 2(0 - 0) = 0$

Since cable BD remained elastic, $\delta_{BD} = \dfrac{F_{BD}L_{BD}}{EA} = 0.$ $\blacktriangleleft$

PROBLEM 2.108



Solve Prob. 2.107, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

PROBLEM 2.107 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable BD, (*b*) the maximum deflection of point C, (*c*) the final displacement of point C. (*Hint:* In Part *c*, cable CE is not taut.)

SOLUTION

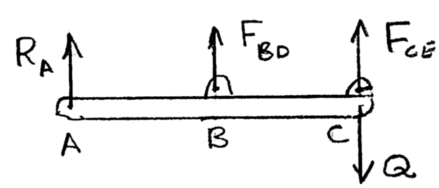
Elongation constraints.

Let $\theta =$ rotation angle of rigid bar ABC.

$$\theta = \frac{\delta_{BC}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \qquad (1)$$

Equilibrium of bar ABC.



$$+\circlearrowleft M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}\,Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \qquad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}\,\text{N}$

From (2), $\quad F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}\,\text{N}$

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}\,\text{N}$, cable BD is elastic when $Q = 50$ kN.

PROBLEM 2.108 *(Continued)*

(*a*) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345 \text{ MPa}$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^3}{100\times10^{-6}} = 310\times10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa} \blacktriangleleft$$

(*b*) Maximum of deflection of point *C*.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^3)(2)}{(200\times10^9)(100\times10^{-6})} = 3.1\times10^{-3} \text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3} \text{ m}$ $\quad 6.20 \text{ mm}\downarrow \blacktriangleleft$

Unloading. $Q' = 50\times10^3 \text{ N}$, $\delta'_{CE} = \delta'_C$

From (1), $\delta'_{BD} = \frac{1}{2}\delta'_C$

$$\text{Elastic } F''_{BD} = \frac{EA\delta'_{BD}}{L_{BD}} = \frac{(200\times10^9)(100\times10^{-6})(\frac{1}{2}\delta'_C)}{2} = 5\times10^6\,\delta'_C$$

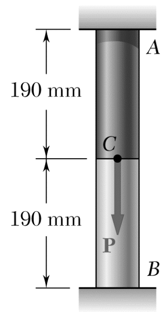
$$F'_{CE} = \frac{EA\delta'_{CE}}{L_{CE}} = \frac{(200\times10^9)(100\times10^{-6})(\delta'_C)}{2} = 10\times10^6\,\delta'_C$$

From (2), $Q' = F'_{CE} + \frac{1}{2}F'_{BD} = 12.5\times10^6\,\delta'_C$

Equating expressions for Q', $12.5\times10^6\,\delta'_C = 50\times10^3$

$$\delta'_C = 4\times10^{-3} \text{ m}$$

(*c*) Final displacement. $\delta_C = (\delta_C)_m - \delta'_C = 6.2\times10^{-3} - 4\times10^{-3} = 2.2\times10^{-3} \text{ m}$ $\quad 2.20 \text{ mm}\downarrow \blacktriangleleft$



PROBLEM 2.109

Rod *AB* consists of two cylindrical portions *AC* and *BC*, each with a cross-sectional area of 1750 mm^2. Portion *AC* is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion *CB* is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at *C* as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of *C* if *P* is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of *C*.

SOLUTION

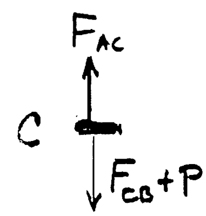
Displacement at *C* to cause yielding of *AC*.

$$\delta_{C,Y} = L_{AC}\varepsilon_{Y,AC} = \frac{L_{AC}\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\text{ m}$$

Corresponding force. $\qquad F_{AC} = A\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6) = 437.5\times10^3\text{ N}$

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\text{ N}$$

For equilibrium of element at *C*,



$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\text{ N}$$

Since applied load $P = 975\times10^3\text{ N} > 875\times10^3\text{ N}$, portion *AC* yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\text{ N} = -537.5\times10^3\text{ N}$$

(*a*) $\quad \delta_C = -\dfrac{F_{CB}L_{CD}}{EA} = \dfrac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\text{ m}$

0.292 mm ◀

(*b*) $\quad$ Maximum stresses: $\sigma_{AC} = \sigma_{Y,AC} = 250$ MPa $\qquad$ 250 MPa ◀

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\text{ Pa} = -307\text{ MPa}$$

−307 MPa ◀

(*c*) $\quad$ Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore\; P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P'_{AC}$$

$$P' = 975\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \quad P'_{AC} = 487.5\times10^{-3}\text{ N}$$

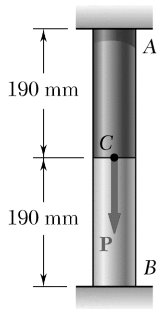
$$\delta' = \frac{(487.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.26464\times10^3\text{ m}$$

$$\delta_p = \delta_m - \delta' = 0.29179\times10^{-3} - 0.26464\times10^{-3}$$

$$= 0.02715\times10^{-3}\text{ m}$$

0.0272 mm ◀

PROBLEM 2.110



For the composite rod of Prob. 2.109, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.3$ mm and then decreased back to zero, determine (*a*) the maximum value of P, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C after the load is removed.

PROBLEM 2.109 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

SOLUTION

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789\times10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789\times10^{-3}$$

Strains at initial yielding:

$$\varepsilon_{Y,\,AC} = \frac{\sigma_{Y,\,AC}}{E} = \frac{250\times10^6}{200\times10^9} = 1.25\times10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,\,CB} = \frac{\sigma_{Y,\,BC}}{E} = -\frac{345\times10^6}{200\times10^9} = -1.725\times10^{-3} \qquad \text{(elastic)}$$

(*a*) Forces: $F_{AC} = A\sigma_Y = (1750\times10^{-6})(250\times10^6) = 437.5\times10^{-3}$ N

$$F_{CB} = EA\varepsilon_{CB} = (200\times10^9)(1750\times10^{-6})(-1.5789\times10^{-3}) = -552.6\times10^{-3} \text{ N}$$

For equilibrium of element at C, $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CD} = 437.5\times10^3 + 552.6\times10^3 = 990.1\times10^3 \text{ N} \qquad = 990 \text{ kN} ◀$$

(*b*) Stresses: AC: $\sigma_{AC} = \sigma_{Y,\,AC}$ $\qquad = 250$ MPa ◀

$$CB: \quad \sigma_{CB} = \frac{F_{CB}}{A} = -\frac{552.6\times10^3}{1750\times10^{-6}} = -316\times10^6 \text{ Pa} \qquad -316 \text{ MPa} ◀$$

PROBLEM 2.110 *(Continued)*

(c) Deflection and forces for unloading.

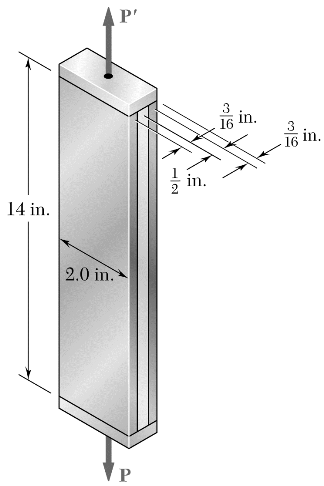
$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P_{AC}$$

$$P' = P'_{AC} - P'_{CB} = 2P'_{AC} = 990.1\times 10^3\,\text{N} \quad \therefore \quad P'_{AC} = 495.05\times 10^3\,\text{N}$$

$$\delta' = \frac{(495.05\times 10^3)(0.190)}{(200\times 10^9)(1750\times 10^{-6})} = 0.26874\times 10^{-3}\text{ m} = 0.26874\text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30\text{ mm} - 0.26874\text{ mm}$$

0.031 mm ◀



PROBLEM 2.111

Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (a) the maximum value of P, (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$ $\qquad \delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138$ in.

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$ $\qquad \delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^3} = 0.048276$ in.

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$. The mild steel yields. Tempered steel is elastic.

(a) Forces: $P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3$ lb

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^3)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb}$$

$$P = P_1 + P_2 = 112.14\times10^3 \text{ lb} = 112.1 \text{kips}$$

$P = 112.1$ kips ◀

(b) Stresses: $\sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times10^3$ psi $= 50$ ksi

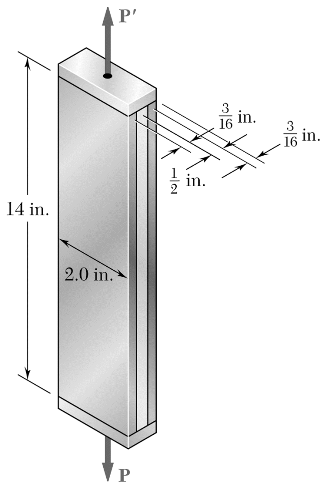
$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi} = 82.86 \text{ ksi}$$

82.86 ksi ◀

Unloading: $\delta' = \frac{PL}{EA} = \frac{(112.14\times10^3)(14)}{(29\times10^6)(1.75)} = 0.03094$ in.

(c) Permanent set: $\delta_p = \delta_m - \delta' = 0.04 - 0.03094 = 0.00906$ in.

0.00906 in. ◀



PROBLEM 2.112

For the composite bar of Prob. 2.111, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

SOLUTION

Areas: Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$

Tempered steel: $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel:

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3 \text{ lb}$$

$P > P_Y$, therefore, mild steel yields.

Let P_1 = force carried by mild steel.

P_2 = force carried by tempered steel.

$$P_1 = A_1\sigma_1 = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$$

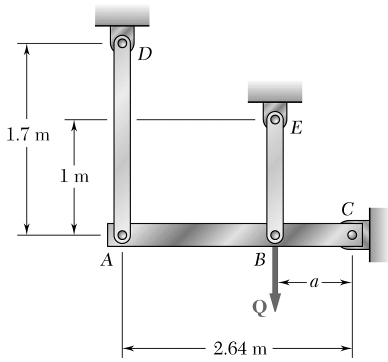
$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3 \text{ lb}$$

(*a*) $\delta_m = \frac{P_2 L}{EA_2} = \frac{(48\times10^3)(14)}{(29\times10^6)(0.75)}$ $= 0.03090 \text{ in.}$ ◀

(*b*) $\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3 \text{ psi}$ $= 64 \text{ ksi}$ ◀

<u>Unloading</u>: $\delta' = \frac{PL}{EA} = \frac{(98\times10^3)(14)}{(29\times10^6)(1.75)} = 0.02703 \text{ in.}$

(*c*) $\delta_P = \delta_m - \delta' = 0.03090 - 0.02703$ $= 0.00387 \text{ in.}$ ◀



PROBLEM 2.113

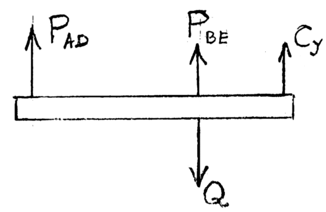
The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B.

SOLUTION

Statics: $\Sigma M_C = 0: \; 0.640(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta, \;\; \delta_B = a\theta = 0.640\theta$

Elastic analysis:

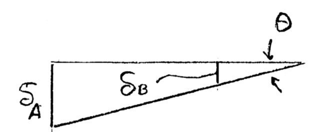


$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times 10^9)(225\times 10^{-6})}{1.7}\delta_A = 26.47\times 10^6 \delta_A$$

$$= (26.47\times 10^6)(2.64\theta) = 69.88\times 10^6 \theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times 10^9 \theta$$



$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times 10^9)(225\times 10^{-6})}{1.0}\delta_B = 45\times 10^6 \delta_B$$

$$= (45\times 10^6)(0.640\theta) = 28.80\times 10^6 \theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128\times 10^9 \theta$$

From Statics, $Q = P_{BE} + \dfrac{2.64}{0.640} P_{AD} = P_{BE} + 4.125 P_{AD}$

$$= [28.80\times 10^6 + (4.125)(69.88\times 10^6)]\,\theta = 317.06\times 10^6 \theta$$

θ_Y at yielding of link AD: $\quad \sigma_{AD} = \sigma_Y = 250\times 10^6 = 310.6\times 10^9 \theta$

$$\theta_Y = 804.89\times 10^{-6}$$

$$Q_Y = (317.06\times 10^6)(804.89\times 10^{-6}) = 255.2\times 10^3 \text{ N}$$

PROBLEM 2.113 *(Continued)*

(*a*) Since $Q = 260\times 10^3 > Q_Y$, link *AD* yields. $\sigma_{AD} = 250$ MPa ◀

$$P_{AD} = A\sigma_Y = (225\times 10^{-6})(250\times 10^{-6}) = 56.25\times 10^3\,\text{N}$$

From Statics, $P_{BE} = Q - 4.125P_{AD} = 260\times 10^3 - (4.125)(56.25\times 10^3)$

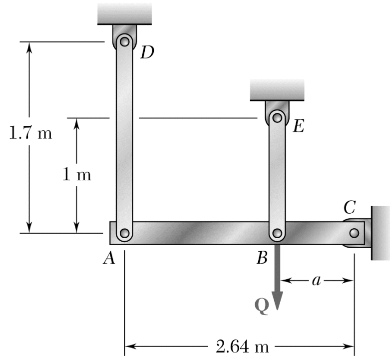
$$P_{BE} = 27.97\times 10^3\,\text{N}$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97\times 10^3}{225\times 10^{-6}} = 124.3\times 10^6\ \text{Pa}$$

$\sigma_{BE} = 124.3$ MPa ◀

(*b*) $$\delta_B = \frac{P_{BE}L_{BE}}{EA} = \frac{(27.97\times 10^3)(1.0)}{(200\times 10^9)(225\times 10^{-6})} = 621.53\times 10^{-6}\ \text{m}$$

$\delta_B = 0.622$ mm $\downarrow$ ◀



PROBLEM 2.114

Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at B is gradually increased from zero to 135 kN.

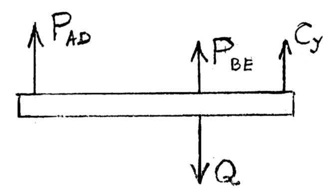
PROBLEM 2.113 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B.

SOLUTION

Statics: $\Sigma M_C = 0:\ 1.76(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta,\ \ \delta_B = 1.76\theta$

Elastic Analysis:

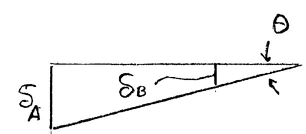


$$A = (37.5)(6) = 225 \text{ mm}^2 = 225\times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times 10^9)(225\times 10^{-6})}{1.7}\delta_A = 26.47\times 10^6\,\delta_A$$

$$= (26.47\times 10^6)(2.64\theta) = 69.88\times 10^6\,\theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times 10^9\,\theta$$



$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times 10^9)(225\times 10^{-6})}{1.0}\delta_B = 45\times 10^6\,\delta_B$$

$$= (45\times 10^6)(1.76\theta) = 79.2\times 10^6\,\theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 352\times 10^9\,\theta$$

From Statics, $Q = P_{BE} + \dfrac{2.64}{1.76}P_{AD} = P_{BE} + 1.500\,P_{AD}$

$$= [73.8\times 10^6 + (1.500)(69.88\times 10^6]\,\theta = 178.62\times 10^6\,\theta$$

θ_Y at yielding of link BE: $\quad \sigma_{BE} = \sigma_Y = 250\times 10^6 = 352\times 10^9\,\theta_Y$

$$\theta_Y = 710.23\times 10^{-6}$$

$$Q_Y = (178.62\times 10^6)(710.23\times 10^{-6}) = 126.86\times 10^3 \text{ N}$$

Since $Q = 135\times 10^3 \text{ N} > Q_Y$, link BE yields. $\quad \sigma_{BE} = \sigma_Y = 250$ MPa ◀

$$P_{BE} = A\sigma_Y = (225\times 10^{-6})(250\times 10^6) = 56.25\times 10^3 \text{ N}$$

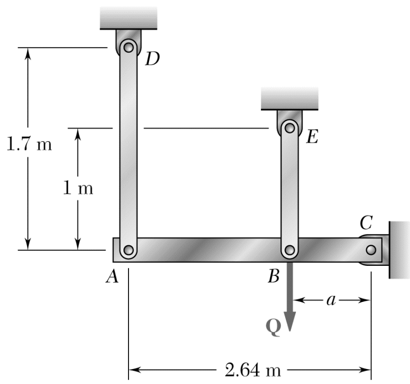
PROBLEM 2.114 *(Continued)*

From Statics, $P_{AD} = \dfrac{1}{1.500}(Q - P_{BE}) = 52.5\times10^3\,\text{N}$

(a) $$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5\times10^3}{225\times10^{-6}} = 233.3\times10^6$$ $\sigma_{AD} = 233\ \text{MPa}$ ◀

From elastic analysis of AD, $\theta = \dfrac{P_{AD}}{69.88\times10^6} = 751.29\times10^{-3}\,\text{rad}$

(b) $$\delta_B = 1.76\theta = 1.322\times10^{-3}\ \text{m}$$ $\delta_B = 1.322\ \text{mm}\downarrow$ ◀



PROBLEM 2.115*

Solve Prob. 2.113, assuming that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point *B*. Assume that the links are braced so that they can carry compressive forces without buckling.

PROBLEM 2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

See solution to Problem 2.113 for the normal stresses in each link and the deflection of Point *B* after loading.

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa}$$
$$\sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$
$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to Problem 2.113 applies to the unloading.

$$Q' = 317.06 \times 10^6 \theta'$$
$$Q' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$$\sigma'_{AD} = 310.6 \times 10^9 \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$
$$\sigma'_{BE} = 128 \times 10^9 \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$
$$\delta'_B = 0.640\theta' = 524.82 \times 10^{-6} \text{ m}$$

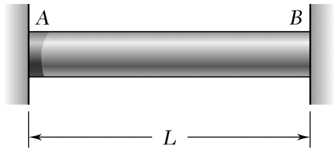
(*a*) Residual stresses.

$$\sigma_{AD,\text{ res}} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^{-6} = -4.70 \times 10^6 \text{ Pa} \qquad = -4.70 \text{ MPa} \blacktriangleleft$$

$$\sigma_{BE,\text{ res}} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} \qquad = 19.34 \text{ MPa} \blacktriangleleft$$

(*b*) $$\delta_{B,P} = \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} = 96.71 \times 10^{-6} \text{ m} \qquad = 0.0967 \text{ mm} \downarrow \blacktriangleleft$$

PROBLEM 2.116



A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of $45\,^\circ\text{F}$. The steel is assumed to be elastoplastic with $\sigma_Y = 36$ ksi and $E = 29\times10^6$ psi. Knowing that $\alpha = 6.5\times10^{-6}/^\circ\text{F}$, determine the stress in the bar (*a*) when the temperature is raised to $320\,^\circ\text{F}$, (*b*) after the temperature has returned to $45\,^\circ\text{F}$.

SOLUTION

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{36\times10^3}{(29\times10^6)(6.5\times10^{-6})} = 190.98\,^\circ\text{F}$$

But $\Delta T = 320 - 45 = 275\,^\circ\text{F} > (\Delta T_Y)$

(*a*) Yielding occurs. $\sigma = -\sigma_Y = -36 \text{ ksi}$ ◀

Cooling: $(\Delta \text{T})' = 275\,^\circ\text{F}$

$$\delta' = \delta'_P = \delta'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

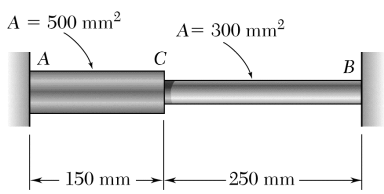
$$= -(29\times10^6)(6.5\times10^{-6})(275) = -51.8375\times10^3 \text{ psi}$$

(*b*) Residual stress:

$$\sigma_{\text{res}} = -\sigma_Y - \sigma' = -36\times10^3 + 51.8375\times10^3 = 15.84\times10 \text{ psi}$$

15.84 ksi ◀

PROBLEM 2.117



The steel rod ABC is attached to rigid supports and is unstressed at a temperature of $25°C$. The steel is assumed elastoplastic, with E = 200 GPa and $\sigma_y = 250$ MPa. The temperature of both portions of the rod is then raised to $150°C$. Knowing that $\alpha = 11.7 \times 10^{-6}/°C$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point C.

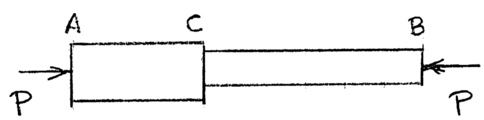
SOLUTION

$$A_{AC} = 500 \times 10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\text{ m}$$
$$A_{CB} = 300 \times 10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\text{ m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion CB.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$



$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300 \times 10^{-6})(2.50 \times 10^6) = 75 \times 10^3\,\text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$
$$= \frac{75 \times 10^3}{(0.400)(200 \times 10^9)(11.7 \times 10^{-6})}\left(\frac{0.150}{500 \times 10^{-6}} + \frac{0.250}{300 \times 10^{-6}}\right) = 90.812°\text{C}$$

Actual ΔT: $150°\text{C} - 25°\text{C} = 125°\text{C} > (\Delta T)_Y$

Yielding occurs. For $\Delta T > (\Delta T)_Y$, $\quad P = P_Y = 75 \times 10^3\,\text{N}$

(*a*) $\quad \sigma_{AC} = -\dfrac{P_Y}{A_{AC}} = -\dfrac{75 \times 10^3}{500 \times 10^{-6}} = -150 \times 10^{-6}\text{ Pa}$ $\qquad \sigma_{AC} = -150\text{ MPa}$ ◀

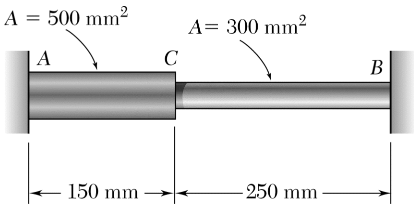
$\quad \sigma_{CB} = -\dfrac{P_Y}{A_{CB}} = -\sigma_Y$ $\qquad \sigma_{CB} = -250\text{ MPa}$ ◀

(*b*) For $\Delta T > (\Delta T)_Y$, portion AC remains elastic.

$$\delta_{C/A} = -\frac{P_Y L_{AC}}{EA_{AC}} + L_{AC}\alpha(\Delta T)$$
$$= -\frac{(75 \times 10^3)(0.150)}{(200 \times 10^9)(500 \times 10^{-6})} + (0.150)(11.7 \times 10^{-6})(125) = 106.9 \times 10^{-6}\,\text{m}$$

Since Point A is stationary, $\delta_C = \delta_{C/A} = 106.9 \times 10^{-6}\,\text{m}$ $\qquad \delta_C = 0.1069\text{ mm} \rightarrow$ ◀

PROBLEM 2.118*



Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150°C and then returned to 25°C.

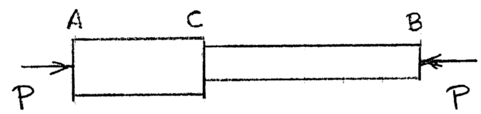
PROBLEM 2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7\times 10^{-6}/°\text{C}$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

SOLUTION

$$A_{AC} = 500\times 10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\text{ m} \qquad A_{CB} = 300\times 10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\text{ m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion *CB*.



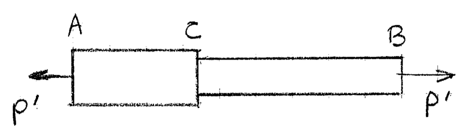
$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times 10^{-6})(250\times 10^{6}) = 75\times 10^{3}\,\text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{75\times 10^{3}}{(0.400)(200\times 10^{9})(11.7\times 10^{-6})}\left(\frac{0.150}{500\times 10^{-6}} + \frac{0.250}{300\times 10^{-6}}\right)$$
$$= 90.812\ °\text{C}$$

Actual ΔT: $150\ °\text{C} - 25\ °\text{C} = 125\ °\text{C} > (\Delta T)_Y$



Yielding occurs. For $\Delta T > (\Delta T)_Y \qquad P = P_Y = 75\times 10^{3}\,\text{N}$

Cooling: $(\Delta T)' = 125\ °\text{C} \quad P' = \dfrac{EL_{AB}\alpha(\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)} = \dfrac{(200\times 10^{9})(0.400)(11.7\times 10^{-6})(125)}{\frac{0.150}{500\times 10^{-6}} + \frac{0.250}{300\times 10^{-6}}} = 103.235\times 10^{3}\,\text{N}$

Residual force: $P_{\text{res}} = P' - P_Y = 103.235\times 10^{3} - 75\times 10^{3} = 28.235\times 10^{3}\,\text{N}$ (tension)

PROBLEM 2.118* *(Continued)*

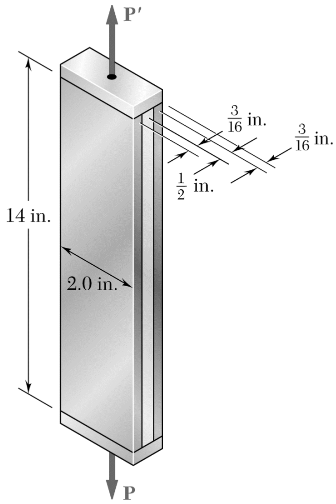
(*a*) Residual stresses.

$$\sigma_{AC} = \frac{P_{\text{res}}}{A_{AC}} = \frac{28.235\times10^{3}}{500\times10^{-6}} \qquad \sigma_{AC} = 56.5\text{ MPa} \blacktriangleleft$$

$$\sigma_{CB} = \frac{P_{\text{res}}}{A_{CB}} = \frac{28.235\times10^{3}}{300\times10^{-6}} \qquad \sigma_{CB} = 9.41\text{ MPa} \blacktriangleleft$$

(*b*) Permanent deflection of point *C*.

$$\delta_C = \frac{P_{\text{res}}L_{AC}}{EA_{AC}} \qquad \delta_C = 0.0424\text{ mm} \rightarrow \blacktriangleleft$$



PROBLEM 2.119*

For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (a) the maximum value of P, (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

SOLUTION

Areas: Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$

Tempered steel: $A_2 = (2)\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel: $\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3 \text{ lb}$

$P > P_Y$; therefore, mild steel yields.

Let P_1 = force carried by mild steel.

P_2 = force carried by tempered steel.

$$P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3 \text{ lb}$$

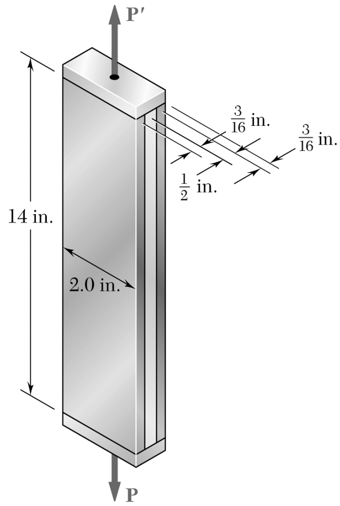
$$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3 \text{ psi}$$

Unloading: $\sigma' = \frac{P}{A} = \frac{98\times10^3}{1.75} = 56\times10^3 \text{ psi}$

Residual stresses.

Mild steel: $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 56\times10^3 = -6\times10^{-3} \text{ psi} = -6 \text{ ksi}$

Tempered steel: $\sigma_{2,\text{res}} = \sigma_2 - \sigma_1 = 64\times10^3 - 56\times10^3 = 8\times10^3 \text{ psi}$ 8.00 ksi ◀



PROBLEM 2.120*

For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2 \quad \delta_{Y1} = \frac{L\delta_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in}$

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2 \quad \delta_{Y2} = \frac{L\delta_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^6} = 0.048276 \text{ in.}$

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$$\delta_{Y1} < \delta_m < \delta_{Y2}$$

The mild steel yields. Tempered steel is elastic.

Forces: $P_1 = A_1\delta_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^6)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb}$$

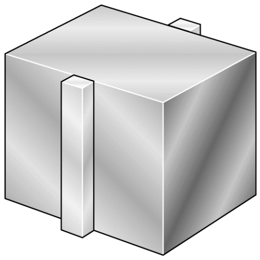
Stresses: $\sigma_1 = \frac{P_1}{A_1} = \delta_{Y1} = 50\times10^3 \text{ psi} \quad \sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi}$

Unloading: $\sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08\times10^3 \text{ psi}$

Residual stresses. $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 64.08\times10^3 = -14.08\times10^3 \text{ psi} = -14.08 \text{ ksi}$

$$\sigma_{2,\text{res}} = \sigma_2 - \sigma' = 82.86\times10^3 - 64.08\times10^3 = 18.78\times10^3 \text{ psi} = 18.78 \text{ ksi} \quad ◀$$

PROBLEM 2.121*



Narrow bars of aluminum are bonded to the two sides of a thick steel plate as shown. Initially, at $T_1 = 70°\text{F}$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 58 ksi. Assume $\alpha_a = 12.8\times10^{-6}/°\text{F}$ for the aluminum and $\alpha_s = 6.5\times10^{-6}/°\text{F}$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 10.9\times10^6$ psi and $\sigma_Y = 58$ ksi. (*Hint:* Neglect the small stresses in the plate.)

SOLUTION

Determine temperature change to cause yielding.

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{58\times10^3}{(10.9\times10^6)(12.8-6.5)(10^{-6})} = 844.62\ °\text{F}$$

(*a*) $T_{2Y} = T_1 + (\Delta T)_Y = 70 + 844.62 = 915\ °\text{F}$ 915 °F ◀

After yielding,

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling:

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{\text{res}} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

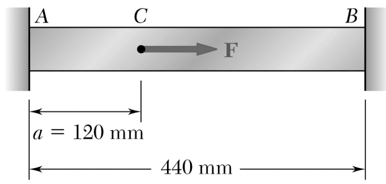
Set $\sigma_{\text{res}} = -\sigma_Y$

$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(58\times10^3)}{(10.9\times10^6)(12.8-6.5)(10^{-6})} = 1689\ °\text{F}$$

(*b*) $T_2 = T_1 + \Delta T = 70 + 1689 = 1759\ °\text{F}$ 1759 °F ◀

If $T_2 > 1759\ °\text{F}$, the aluminum bar will most likely yield in compression.



PROBLEM 2.122*

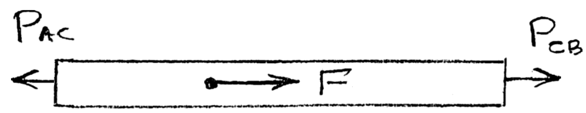
Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point C, (*b*) the residual stress in the bar.

SOLUTION

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{m}^2$$

Force to yield portion AC:

$$P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$$
$$= 300 \times 10^3 \text{N}$$



For equilibrium, $F + P_{CB} - P_{AC} = 0$.

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$
$$= -220 \times 10^3 \text{N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})}$$
$$= 0.293333 \times 10^{-3} \text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-6}}$$
$$= -183.333 \times 10^6 \text{Pa}$$

PROBLEM 2.122* *(Continued)*

Unloading:

$$\delta_C' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} = \frac{(F - P_{AC}')L_{CB}}{EA}$$

$$P_{AC}'\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P_{AC}' = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times 10^3)(0.440 - 0.120)}{0.440} = 378.182\times 10^3\,\text{N}$$

$$P_{CB}' = P_{AC}' - F = 378.182\times 10^3 - 520\times 10^3 = -141.818\times 10^3\,\text{N}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{378.182\times 10^3}{1200\times 10^{-6}} = 315.152\times 10^6\,\text{Pa}$$

$$\sigma_{BC}' = \frac{P_{BC}'}{A} = -\frac{141.818\times 10^3}{1200\times 10^{-6}} = -118.182\times 10^6\,\text{Pa}$$

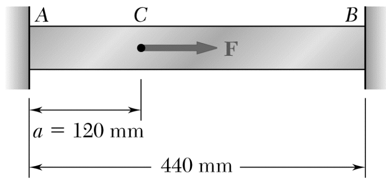
$$\delta_C' = \frac{(378.182)(0.120)}{(200\times 10^9)(1200\times 10^6)} = 0.189091\times 10^{-3}\,\text{m}$$

(*a*) $\delta_{C,p} = \delta_C - \delta_C' = 0.293333\times 10^{-3} - 0.189091\times 10^{-3} = 0.1042\times 10^{-3}\,\text{m}$ $= 0.1042$ mm ◀

(*b*) $\sigma_{AC,\,\text{res}} = \sigma_Y - \sigma_{AC}' = 250\times 10^6 - 315.152\times 10^6 = -65.2\times 10^6\,\text{Pa}$ $= -65.2$ MPa ◀

$\sigma_{CB,\,\text{res}} = \sigma_{CB} - \sigma_{CB}' = -183.333\times 10^6 + 118.182\times 10^6 = -65.2\times 10^6\,\text{Pa}$ $= -65.2$ MPa ◀

PROBLEM 2.123*



Solve Prob. 2.122, assuming that $a = 180$ mm.

PROBLEM 2.122 Bar *AB* has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

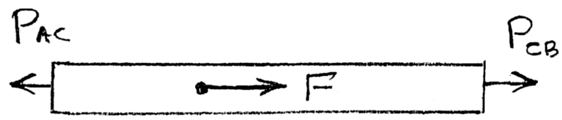
SOLUTION

$$A = 1200\ \text{mm}^2 = 1200\times 10^{-6}\,\text{m}^2$$

Force to yield portion *AC*:

$$P_{AC} = A\sigma_Y = (1200\times 10^{-6})(250\times 10^6)$$
$$= 300\times 10^3\,\text{N}$$

For equilibrium, $F + P_{CB} - P_{AC} = 0.$



$$P_{CB} = P_{AC} - F = 300\times 10^3 - 520\times 10^3$$
$$= -220\times 10^3\,\text{N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220\times 10^3)(0.440 - 0.180)}{(200\times 10^9)(1200\times 10^{-6})}$$
$$= 0.238333\times 10^{-3}\,\text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220\times 10^3}{1200\times 10^{-6}}$$
$$= -183.333\times 10^6\,\text{Pa}$$

PROBLEM 2.123* (Continued)

Unloading:

$$\delta'_C = \frac{P'_{AC} L_{AC}}{EA} = -\frac{P'_{CB} L_{CB}}{EA} = \frac{(F - P'_{AC}) L_{CB}}{EA}$$

$$= P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{F L_{CB}}{EA}$$

$$P'_{AC} = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times10^3)(0.440 - 0.180)}{0.440} = 307.273\times10^3\,\text{N}$$

$$P'_{CB} = P'_{AC} - F = 307.273\times10^3 - 520\times10^3 = -212.727\times10^3\,\text{N}$$

$$\delta'_C = \frac{(307.273\times10^3)(0.180)}{(200\times10^9)(1200\times10^{-6})} = 0.230455\times10^{-3}\,\text{m}$$

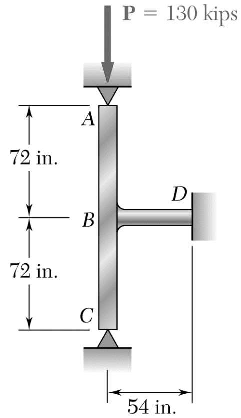
$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{307.273\times10^3}{1200\times10^{-6}} = 256.061\times10^6\,\text{Pa}$$

$$\sigma'_{CB} = \frac{P'_{CB}}{A} = \frac{-212.727\times10^3}{1200\times10^{-6}} = -177.273\times10^6\,\text{P}$$

(*a*) $\delta_{C,p} = \delta_C - \delta'_C = 0.238333\times10^{-3} - 0.230455\times10^{-3} = 0.00788\times10^{-3}\,\text{m}$ $= 0.00788\text{ mm}$ ◀

(*b*) $\sigma_{AC,\text{res}} = \sigma_{AC} - \sigma'_{AC} = 250\times10^6 - 256.061\times10^6 = -6.06\times10^6\,\text{Pa}$ $= -6.06\text{ MPa}$ ◀

$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times10^6 + 177.273\times10^6 = -6.06\times10^6\,\text{Pa}$ $= -6.06\text{ MPa}$ ◀



PROBLEM 2.124

Rod BD is made of steel $(E = 29\times10^6 \text{ psi})$ and is used to brace the axially compressed member ABC. The maximum force that can be developed in member BD is $0.02P$. If the stress must not exceed 18 ksi and the maximum change in length of BD must not exceed 0.001 times the length of ABC, determine the smallest-diameter rod that can be used for member BD.

SOLUTION

$$F_{BD} = 0.02P = (0.02)(130) = 2.6 \text{ kips} = 2.6\times10^3 \text{lb}$$

Considering stress: $\sigma = 18 \text{ ksi} = 18\times10^3 \text{ psi}$

$$\sigma = \frac{F_{BD}}{A} \quad \therefore \quad A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444 \text{ in}^2$$

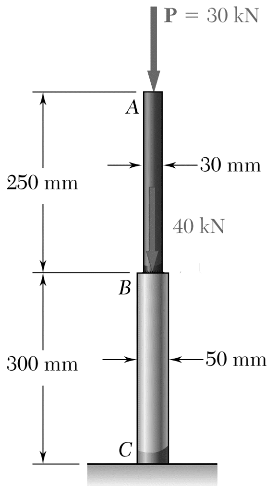
Considering deformation: $\delta = (0.001)(144) = 0.144 \text{ in.}$

$$\delta = \frac{F_{BD}L_{BD}}{AE} \quad \therefore \quad A = \frac{F_{BD}L_{BD}}{E\delta} = \frac{(2.6\times10^3)(54)}{(29\times10^6)(0.144)} = 0.03362 \text{ in}^2$$

Larger area governs. $A = 0.14444 \text{ in}^2$

$$A = \frac{\pi}{4}d^2 \quad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}}$$

$d = 0.429 \text{in.}$ ◀



PROBLEM 2.125

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel $(E = 200 \text{ GPa})$ and rod BC of brass $(E = 105 \text{ GPa})$. Determine (a) the total deformation of the composite rod ABC, (b) the deflection of point B.

SOLUTION

Rod AB:

$$F_{AB} = -P = -30 \times 10^3 \text{ N}$$

$$L_{AB} = 0.250 \text{ m}$$

$$E_{AB} = 200 \times 10^9 \text{ GPa}$$

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.85 \text{ mm}^2 = 706.85 \times 10^{-6} \text{m}^2$$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{E_{AB}A_{AB}} = -\frac{(30 \times 10^3)(0.250)}{(200 \times 10^9)(706.85 \times 10^{-6})} = -53.052 \times 10^{-6} \text{ m}$$

Rod BC:

$$F_{BC} = 30 + 40 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$L_{BC} = 0.300 \text{ m}$$

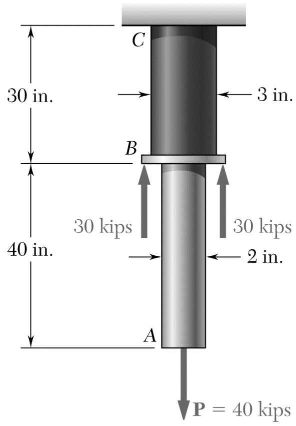
$$E_{BC} = 105 \times 10^9 \text{ Pa}$$

$$A_{BC} = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{mm}^2 = 1.9635 \times 10^{-3} \text{m}^2$$

$$\delta_{BC} = \frac{F_{BC}L_{BC}}{E_{BC}A_{BC}} = -\frac{(70 \times 10^3)(0.300)}{(105 \times 10^9)(1.9635 \times 10^{-3})} = -101.859 \times 10^{-6} \text{ m}$$

(a) Total deformation: $\delta_{\text{tot}} = \delta_{AB} + \delta_{BC} = -154.9 \times 10^{-6} \text{ m}$ $= -0.1549 \text{ mm}$ ◀

(b) Deflection of Point B. $\delta_B = \delta_{BC}$ $\delta_B = 0.1019 \text{ mm} \downarrow$ ◀



PROBLEM 2.126

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel $(E = 29\times10^6 \text{ psi})$, and rod BC of brass $(E = 15\times10^6 \text{ psi})$. Determine (*a*) the total deformation of the composite rod ABC, (*b*) the deflection of point B.

SOLUTION

<u>Portion *AB*</u>:

$$P_{AB} = 40\times10^3 \text{ lb}$$
$$L_{AB} = 40 \text{ in.}$$
$$d = 2 \text{ in.}$$
$$A_{AB} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$$
$$E_{AB} = 29\times10^6 \text{ psi}$$
$$\delta_{AB} = \frac{P_{AB}L_{AB}}{E_{AB}A_{AB}} = \frac{(40\times10^3)(40)}{(29\times10^6)(3.1416)} = 17.5619\times10^{-3} \text{ in.}$$

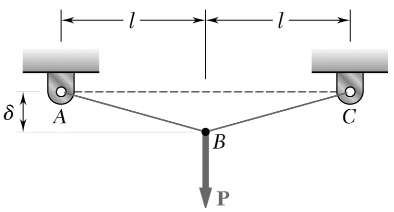
<u>Portion *BC*</u>:

$$P_{BC} = -20\times10^3 \text{ lb}$$
$$L_{BC} = 30 \text{ in.}$$
$$d = 3 \text{ in.}$$
$$A_{BC} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$$
$$E_{BC} = 15\times10^6 \text{ psi}$$
$$\delta_{BC} = \frac{P_{BC}L_{BC}}{E_{BC}A_{BC}} = \frac{(-20\times10^3)(30)}{(15\times10^6)(7.0686)} = -5.6588\times10^{-3} \text{ in.}$$

(*a*) $\delta = \delta_{AB} + \delta_{BC} = 17.5619\times10^{-6} - 5.6588\times10^{-6}$ $\qquad \delta = 11.90\times10^{-3} \text{ in.} \downarrow$ ◀

(*b*) $\delta_B = -\delta_{BC}$ $\qquad \delta_B = 5.66\times10^{-3} \text{ in.} \uparrow$ ◀

PROBLEM 2.127



The uniform wire *ABC*, of unstretched length $2l$, is attached to the supports shown and a vertical load **P** is applied at the midpoint *B*. Denoting by *A* the cross-sectional area of the wire and by *E* the modulus of elasticity, show that, for $\delta \ll l$, the deflection at the midpoint *B* is

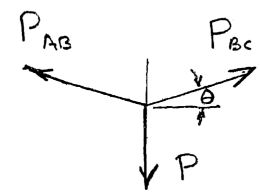
$$\delta = l\sqrt[3]{\frac{P}{AE}}$$

SOLUTION

Use approximation.

$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$

<u>Statics</u>: $+\uparrow \Sigma F_Y = 0: \quad 2P_{AB}\sin\theta - P = 0$

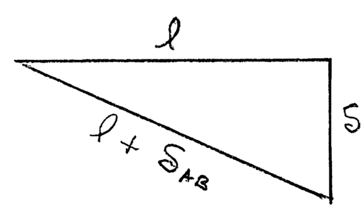


$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

<u>Elongation</u>: $\delta_{AB} = \frac{P_{AB}l}{AE} = \frac{Pl^2}{2AE\delta}$

<u>Deflection</u>:

From the right triangle,



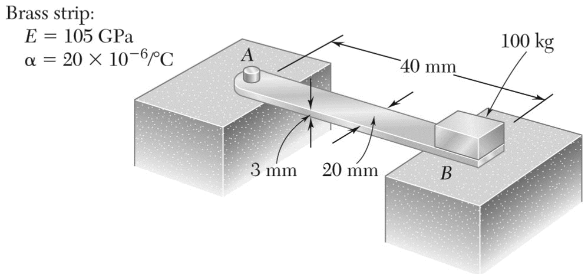
$$(l+\delta_{AB})^2 = l^2 + \delta^2$$

$$\delta^2 = \cancel{l^2} + 2l\,\delta_{AB} + \delta_{AB}^2 - \cancel{l^2}$$

$$= 2l\,\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\,\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

$$\delta^3 \approx \frac{Pl^3}{AE} \quad \therefore \quad \delta \approx l\sqrt[3]{\frac{P}{AE}} \quad ◀$$

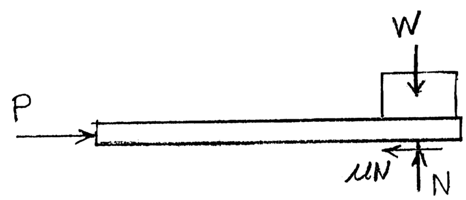


PROBLEM 2.128

The brass strip AB has been attached to a fixed support at A and rests on a rough support at B. Knowing that the coefficient of friction is 0.60 between the strip and the support at B, determine the decrease in temperature for which slipping will impend.

SOLUTION

Brass strip:



$$E = 105 \text{ GPa}$$

$$\alpha = 20\times10^{-6}/°\text{C}$$

$$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \qquad N = W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - \mu N = 0 \qquad P = \mu W = \mu mg$$

$$\delta = -\frac{PL}{EA} + L\alpha(\Delta T) = 0 \quad \Delta T = \frac{P}{EA\alpha} = \frac{\mu mg}{EA\alpha}$$

Data: $\mu = 0.60$

$A = (20)(3) = 60 \text{ mm}^2 = 60\times10^{-6}\text{ m}^2$

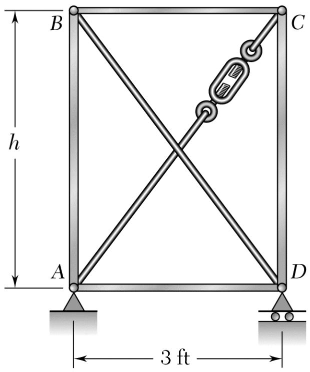
$m = 100 \text{ kg}$

$g = 9.81 \text{ m/s}^2$

$E = 105\times10^9 \text{ Pa}$

$$\Delta T = \frac{(0.60)(100)(9.81)}{(105\times10^9)(60\times10^{-6})(20\times10^6)}$$

$\Delta T = 4.67\ °\text{C}$ ◀



PROBLEM 2.129

Members AB and CD are $1\frac{1}{8}$-in.-diameter steel rods, and members BC and AD are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29\times10^6$ psi and $h = 4$ ft, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

SOLUTION

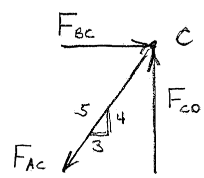
$$\delta_{AB} = \delta_{CD} = 0.04 \text{ in.}$$

$$h = 4\text{ ft} = 48 \text{ in.} = L_{CD}$$

$$A_{CD} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA_{CD}}$$

$$F_{CD} = \frac{EA_{CD}\delta_{CD}}{L_{CD}} = \frac{(29\times10^6)(0.99402)(0.04)}{48} = 24.022\times10^3 \text{ lb}$$

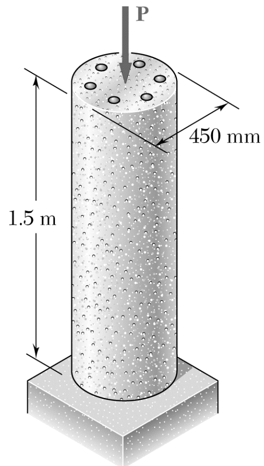


Use joint C as a free body.

$$+\uparrow \Sigma F_y = 0: \; F_{CD} - \frac{4}{5}F_{AC} = 0 \quad \therefore \quad F_{AC} = \frac{5}{4}F_{CD}$$

$$F_{AC} = \frac{5}{4}(24.022\times10^3) = 30.0\times10^3 \text{ lb}$$

$F_{AC} = 30.0$ kips ◀



PROBLEM 2.130

The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550-kN axial centric force **P** is applied to the post.

SOLUTION

Let P_c = portion of axial force carried by concrete.

P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{-P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945\times 10^3\,\text{mm}^2$$

$$= 3.6945\times 10^{-3}\,\text{m}^2$$

$$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945\times 10^3$$

$$= 155.349\times 10^3\,\text{mm}^2$$

$$= 153.349\times 10^{-3}\,\text{m}^2$$

$$L = 1.5\text{ m}$$

$$\varepsilon = \frac{1550\times 10^3}{(25\times 10^9)(155.349\times 10^{-3}) + (200\times 10^9)(3.6945\times 10^{-3})} = 335.31\times 10^{-6}$$

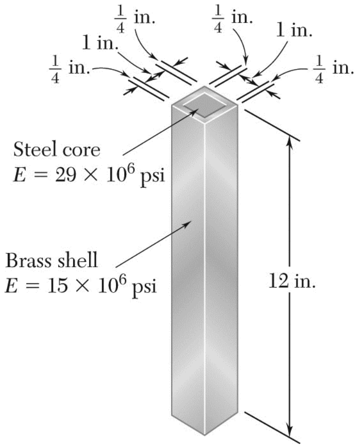
$$\sigma_s = E_s\varepsilon = (200\times 10^9)(335.31\times 10^{-6}) = 67.1\times 10^6\,\text{Pa}$$

$\sigma_s = 67.1$ MPa ◀

$$\sigma_c = E_c\varepsilon = (25\times 10^9)(-335.31\times 10^{-6}) = 8.38\times 10^6\,\text{Pa}$$

$\sigma_c = 8.38$ MPa ◀

PROBLEM 2.131



The brass shell $(\alpha_b = 11.6 \times 10^{-6}/°\text{F})$ is fully bonded to the steel core $(\alpha_s = 6.5 \times 10^{-6}/°\text{F})$. Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 8 ksi.

SOLUTION

Let $P_s =$ axial force developed in the steel core.

For equilibrium with zero total force, the compressive force in the brass shell is P_s.

Strains:

$$\varepsilon_s = \frac{P_s}{E_s A_s} + \alpha_s(\Delta T)$$

$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching:

$$\varepsilon_s = \varepsilon_b$$

$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T) \tag{1}$$

$$A_b = (1.5)(1.5) - (1.0)(1.0) = 1.25 \text{ in}^2$$

$$A_s = (1.0)(1.0) = 1.0 \text{ in}^2$$

$$\alpha_b - \alpha_s = 5.1 \times 10^{-6}/°\text{F}$$

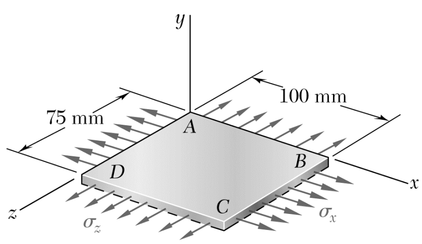
$$P_s = \sigma_s A_s = (8 \times 10^3)(1.0) = 8 \times 10^3 \text{ lb}$$

$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(29 \times 10^6)(1.0)} + \frac{1}{(15 \times 10^6)(1.25)} = 87.816 \times 10^{-9} \text{ lb}^{-1}$$

From (1),

$$(87.816 \times 10^{-9})(8 \times 10^3) = (5.1 \times 10^{-6})(\Delta T)$$

$\Delta T = 137.8°\text{F}$ ◀



PROBLEM 2.132

A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$$\sigma_x = 120 \times 10^6 \text{ Pa},$$

$$\sigma_y = 0,$$

$$\sigma_z = 160 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{87 \times 10^9}[120 \times 10^6 - (0.34)(160 \times 10^6)] = 754.02 \times 10^{-6}$$

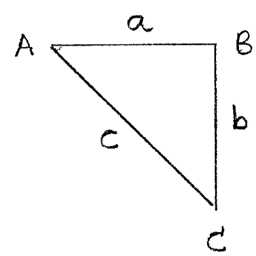
$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87 \times 10^9}[-(0.34)(120 \times 10^6) + 160 \times 10^6] = 1.3701 \times 10^{-3}$$

(*a*) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100 \text{ mm})(754.02 \times 10^{-6})$ $= 0.0754 \text{ mm}$ ◀

(*b*) $\delta_{BC} = (\overline{BC})\varepsilon_z = (75 \text{ mm})(1.3701 \times 10^{-6})$ $= 0.1028 \text{ mm}$ ◀

Label sides of right triangle *ABC* as *a*, *b*, and *c*.

$$c^2 = a^2 + b^2$$



Obtain differentials by calculus.

$$2c\ dc = 2a\ da + 2b\ db$$

$$dc = \frac{a}{c}da + \frac{b}{c}db$$

But $a = 100$ mm,

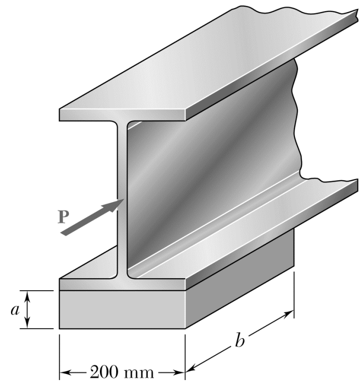
$b = 75$ mm,

$c = \sqrt{(100^2 + 75^2)} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm

$db = \delta_{BC} = 0.1370$ mm

(*c*) $\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028)$ $= 0.1220 \text{ mm}$ ◀



PROBLEM 2.133

An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22-kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine (*a*) the smallest allowable dimension *b*, (*b*) the smallest required thickness *a*.

SOLUTION

Shearing force: $P = 22 \times 10^3\ \text{N}$

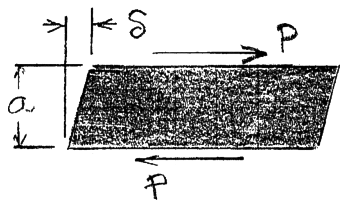
Shearing stress: $\tau = 420 \times 10^3\ \text{Pa}$

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau}$$

$$= \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3}\ \text{m}^2$$

$$= 52.381 \times 10^3\ \text{mm}^2$$

$$A = (200\ \text{mm})(b)$$

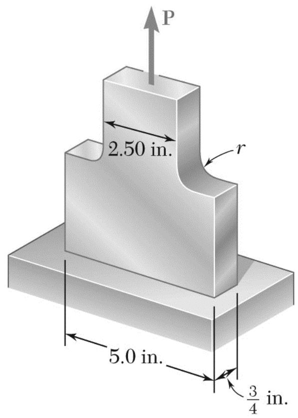


(*a*) $b = \dfrac{A}{200} = \dfrac{52.381 \times 10^3}{200} = 262\ \text{mm}$ $\qquad b = 262\ \text{mm}$ ◀

$$\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$$

(*b*) But $\gamma = \dfrac{\delta}{a} \quad \therefore \quad a = \dfrac{\delta}{\gamma} = \dfrac{10\ \text{mm}}{466.67 \times 10^{-3}} = 21.4\ \text{mm}$ $\qquad a = 21.4\ \text{mm}$ ◀

PROBLEM 2.134



Knowing that $P = 10$ kips, determine the maximum stress when (*a*) $r = 0.50$ in., (*b*) $r = 0.625$ in.

SOLUTION

$$P = 10 \times 10^3 \text{ lb} \quad D = 5.0 \text{ in.} \quad d = 2.50 \text{ in.}$$

$$\frac{D}{d} = \frac{5.0}{2.50} = 2.00$$

$$A_{\min} = dt = (2.50)\left(\frac{3}{4}\right) = 1.875 \text{ in}^2$$

(*a*) $r = 0.50$ in. $\quad \frac{r}{d} = \frac{0.50}{2.50} = 0.20$

From Fig. 2.60*b*, $\quad K = 1.94$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{(1.94)(10 \times 10^3)}{1.875} = 10.35 \times 10^3 \text{ psi}$$

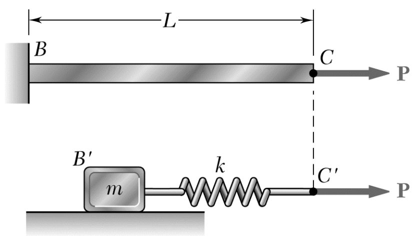
10.35 ksi ◀

(*b*) $r = 0.625$ in. $\quad \frac{r}{d} = \frac{0.625}{2.50} = 0.25 \quad K = 1.82$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{(1.82)(10 \times 10^3)}{1.875} = 9.71 \times 10^3 \text{ psi}$$

9.71 ksi ◀

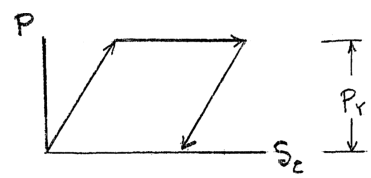
PROBLEM 2.135



The uniform rod BC has a cross-sectional area A and is made of a mild steel that can be assumed to be elastoplastic with a modulus of elasticity E and a yield strength σ_y. Using the block-and-spring system shown, it is desired to simulate the deflection of end C of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points C and C' should be the same for all values of P. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (a) the required mass m of the block, (b) the required constant k of the spring.

SOLUTION

Force-deflection diagram for Point C or rod BC.

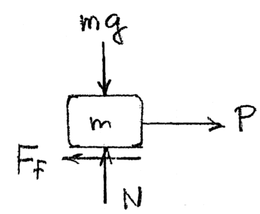


For $\qquad P < P_Y = A\sigma_Y$

$$\delta_C = \frac{PL}{EA} \qquad P = \frac{EA}{L}\delta_C$$

$$P_{\max} = P_Y = A\sigma_Y$$

Force-deflection diagram for Point C' of block-and-spring system.



$$+\uparrow \Sigma F_y = 0: \quad N - mg = 0 \qquad N = mg$$

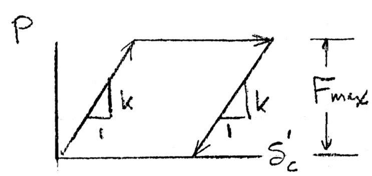
$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_f = 0 \qquad P = F_f$$

If block does not move, i.e., $F_f < \mu N = \mu mg$ or $P < \mu mg$,

then $\qquad \delta'_c = \dfrac{P}{K} \quad$ or $\quad P = k\delta'_c$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

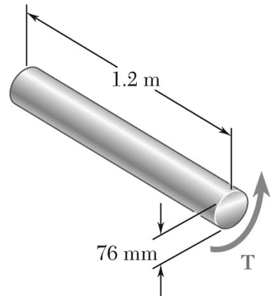
If the force P is the removed, the spring returns to its initial length.



(a) Equating P_Y and $F_{\max}$, $\qquad A\sigma_Y = \mu mg \qquad m = \dfrac{A\sigma_Y}{\mu g}$ ◀

(b) Equating slopes, $\qquad k = \dfrac{EA}{L}$ ◀

CHAPTER 3



PROBLEM 3.1

(*a*) Determine the maximum shearing stress caused by a 4.6-kN · m torque **T** in the 76-mm-diameter shaft shown. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same outer diameter and of 24-mm inner diameter.

SOLUTION

(*a*) Solid shaft:

$$c = \frac{d}{2} = 38 \text{ mm} = 0.038 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.038)^4 = 3.2753 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(4.6 \times 10^3)(0.038)}{3.2753 \times 10^{-6}} = 53.4 \times 10^6 \text{ Pa}$$

$\tau = 53.4$ MPa ◀

(*b*) Hollow shaft:

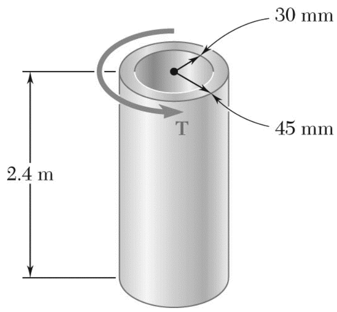
$$c_2 = \frac{d_o}{2} = 0.038 \text{ m}$$

$$c_1 = \frac{1}{2}d_i = 12 \text{ mm} = 0.012 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.038^4 - 0.012^4) = 3.2428 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(4.6 \times 10^3)(0.038)}{3.2428 \times 10^{-6}} = 53.9 \times 10^6 \text{ Pa}$$

$\tau = 53.9$ MPa ◀



PROBLEM 3.2

(*a*) Determine the torque **T** that causes a maximum shearing stress of 45 MPa in the hollow cylindrical steel shaft shown. (*b*) Determine the maximum shearing stress caused by the same torque **T** in a solid cylindrical shaft of the same cross-sectional area.

SOLUTION

(*a*) Given shaft:

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$

$$J = \frac{\pi}{2}(45^4 - 30^4) = 5.1689 \times 10^6 \text{ mm}^4 = 5.1689 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c}$$

$$T = \frac{(5.1689 \times 10^{-6})(45 \times 10^6)}{45 \times 10^{-3}} = 5.1689 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 5.17 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) Solid shaft of same area:

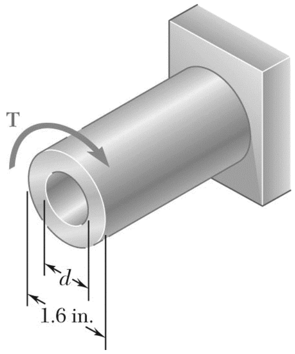
$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(45^2 - 30^2) = 3.5343 \times 10^3 \text{ mm}^2$$

$$\pi c^2 = A \quad \text{or} \quad c = \sqrt{\frac{A}{\pi}} = 33.541 \text{ mm}$$

$$J = \frac{\pi}{2}c^4, \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau = \frac{(2)(5.1689 \times 10^3)}{\pi(0.033541)^3} = 87.2 \times 10^6 \text{ Pa}$$

$$\tau = 87.2 \text{ MPa} \blacktriangleleft$$



PROBLEM 3.3

Knowing that $d = 1.2$ in., determine the torque **T** that causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in.} \qquad c = 0.8 \text{ in.}$$

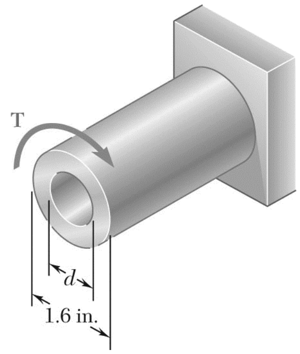
$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(1.2) = 0.6 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.4398 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{\max}}{c} = \frac{(0.4398)(7.5)}{0.8} \qquad T = 4.12 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROBLEM 3.4



Knowing that the internal diameter of the hollow shaft shown is $d = 0.9\,\text{in.}$, determine the maximum shearing stress caused by a torque of magnitude $T = 9\,\text{kip} \cdot \text{in.}$

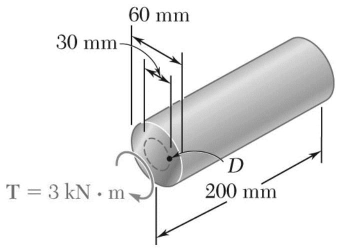
SOLUTION

$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in.} \qquad c = 0.8 \text{ in.}$$

$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(0.9) = 0.45 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{(9)(0.8)}{0.5790} \qquad \tau_{\max} = 12.44 \text{ ksi} \blacktriangleleft$$

	**PROBLEM 3.5** A torque $T = 3\ \text{kN} \cdot \text{m}$ is applied to the solid bronze cylinder shown. Determine (*a*) the maximum shearing stress, (*b*) the shearing stress at point *D* which lies on a 15-mm-radius circle drawn on the end of the cylinder, (*c*) the percent of the torque carried by the portion of the cylinder within the 15 mm radius.

SOLUTION

(*a*) $$c = \frac{1}{2}d = 30\ \text{mm} = 30 \times 10^{-3}\ \text{m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30 \times 10^{-3})^4 = 1.27235 \times 10^{-6}\ \text{m}^4$$

$$T = 3\ \text{kN} = 3 \times 10^3\ \text{N}$$

$$\tau_m = \frac{Tc}{J} = \frac{(3 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}} = 70.736 \times 10^6\ \text{Pa}$$

$\tau_m = 70.7\ \text{MPa}$ ◀

(*b*) $$\rho_D = 15\ \text{mm} = 15 \times 10^{-3}\ \text{m}$$

$$\tau_D = \frac{\rho_D}{c}\tau = \frac{(15 \times 10^{-3})(70.736 \times 10^{-6})}{(30 \times 10^{-3})}$$

$\tau_D = 35.4\ \text{MPa}$ ◀

(*c*) $$\tau_D = \frac{T_D \rho_D}{J_D} \qquad T_D = \frac{J_D \tau_D}{\rho_D} = \frac{\pi}{2}\rho_D^3\, \tau_D$$

$$T_D = \frac{\pi}{2}(15 \times 10^{-3})^3(35.368 \times 10^6) = 187.5\ \text{N} \cdot \text{m}$$

$$\frac{T_D}{T} \times 100\% = \frac{187.5}{3 \times 10^3}(100\%) = 6.25\%$$

6.25% ◀

PROBLEM 3.6

(*a*) Determine the torque that can be applied to a solid shaft of 20-mm diameter without exceeding an allowable shearing stress of 80 MPa. (*b*) Solve Part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half of its own outer diameter.

SOLUTION

(*a*) Solid shaft:

$$c = \frac{1}{2}d = \frac{1}{2}(0.020) = 0.010 \text{ m}$$

$$J = \frac{\pi}{2}c^4 - \frac{\pi}{2}(0.10)^4 = 15.7080 \times 10^{-9} \text{m}^4$$

$$T = \frac{J\tau_{\max}}{c} = \frac{(15.7080 \times 10^{-9})(80 \times 10^6)}{0.010} = 125.664 \qquad T = 125.7 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) Hollow shaft: Same area as solid shaft.

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi\left[c_2^2 - \left(\frac{1}{2}c_2\right)^2\right] = \frac{3}{4}\pi c_2^2 = \pi c^2$$

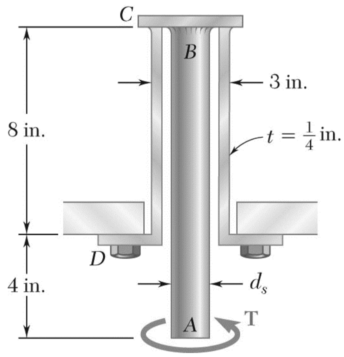
$$c_2 = \frac{2}{\sqrt{3}}c = \frac{2}{\sqrt{3}}(0.010) = 0.0115470 \text{ m}$$

$$c_1 = \frac{1}{2}c_2 = 0.0057735 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0115470^4 - 0.0057735^4) = 26.180 \times 10^{-9} \text{m}^4$$

$$T = \frac{\tau_{\max} J}{c_2} = \frac{(80 \times 10^6)(26.180 \times 10^{-9})}{0.0115470} = 181.38 \qquad T = 181.4 \text{ N} \cdot \text{m} \blacktriangleleft$$

PROBLEM 3.7



The solid spindle *AB* has a diameter $d_s = 1.5$ in. and is made of a steel with an allowable shearing stress of 12 ksi, while sleeve *CD* is made of a brass with an allowable shearing stress of 7 ksi. Determine the largest torque **T** that can be applied at *A*.

SOLUTION

Analysis of solid spindle *AB*:

$$c = \frac{1}{2} d_s = 0.75 \text{ in.}$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c} = \frac{\pi}{2} \tau c^3$$

$$T = \frac{\pi}{2}(12 \times 10^3)(0.75)^3 = 7.95 \times 10^3 \text{ lb} \cdot \text{in}$$

Analysis of sleeve *CD*:

$$c_2 = \frac{1}{2} d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

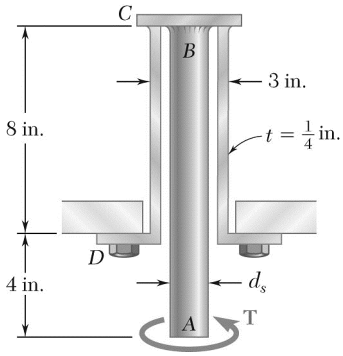
$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7 \times 10^3)}{1.5} = 19.21 \times 10^3 \text{ lb} \cdot \text{in}$$

The smaller torque governs:

$$T = 7.95 \times 10^3 \text{ lb} \cdot \text{in}$$

$$T = 7.95 \text{kip} \cdot \text{in} \blacktriangleleft$$

PROBLEM 3.8



The solid spindle AB is made of a steel with an allowable shearing stress of 12 ksi, and sleeve CD is made of a brass with an allowable shearing stress of 7 ksi. Determine (*a*) the largest torque **T** that can be applied at A if the allowable shearing stress is not to be exceeded in sleeve CD, (*b*) the corresponding required value of the diameter d_s of spindle AB.

SOLUTION

(*a*) Analysis of sleeve CD:

$$c_2 = \frac{1}{2}d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7\times 10^3)}{1.5} = 19.21\times 10^3 \text{ lb}\cdot\text{in}$$

$$T = 19.21 \text{ kip}\cdot\text{in} \blacktriangleleft$$

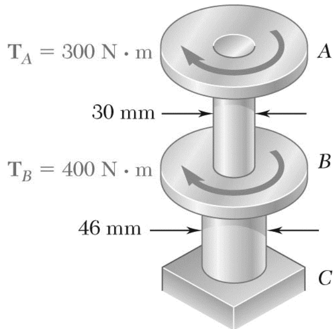
(*b*) Analysis of solid spindle AB:

$$\tau = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{T}{\tau} = \frac{19.21\times 10^3}{12\times 10^3} = 1.601 \text{ in}^3$$

$$c = \sqrt[3]{\frac{(2)(1.601)}{\pi}} = 1.006 \text{ in.} \quad d_s = 2c$$

$$d = 2.01 \text{in.} \blacktriangleleft$$



PROBLEM 3.9

The torques shown are exerted on pulleys A and B. Knowing that both shafts are solid, determine the maximum shearing stress (a) in shaft AB, (b) in shaft BC.

SOLUTION

(a) Shaft AB: $T_{AB} = 300 \text{ N} \cdot \text{m}, \; d = 0.030 \text{ m}, \; c = 0.015 \text{ m}$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

$$= 56.588 \times 10^6 \text{Pa}$$

$\tau_{\max} = 56.6 \text{ MPa}$ ◀

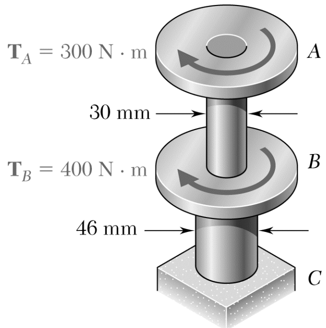
(b) Shaft BC: $T_{BC} = 300 + 400 = 700 \text{ N} \cdot \text{m}$

$$d = 0.046 \text{ m}, \; c = 0.023 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$$

$$= 36.626 \times 10^6 \text{Pa}$$

$\tau_{\max} = 36.6 \text{ MPa}$ ◀



PROBLEM 3.10

In order to reduce the total mass of the assembly of Prob. 3.9, a new design is being considered in which the diameter of shaft BC will be smaller. Determine the smallest diameter of shaft BC for which the maximum value of the shearing stress in the assembly will not increase.

SOLUTION

Shaft AB: $T_{AB} = 300 \text{ N} \cdot \text{m}, \; d = 0.030 \text{ m}, \; c = 0.015 \text{ m}$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

Shaft BC: $T_{BC} = 300 + 400 = 700 \text{ N} \cdot \text{m}$

$$d = 0.046 \text{ m}, \; c = 0.023 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$$

$$= 36.626 \times 10^6 \text{ Pa} = 36.6 \text{ MPa}$$

The largest stress $(56.588 \times 10^6 \text{ Pa})$ occurs in portion AB.

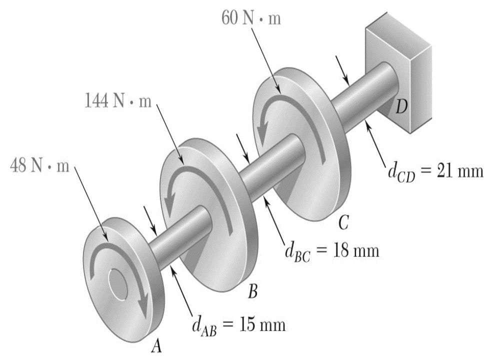
Reduce the diameter of BC to provide the same stress.

$$T_{BC} = 700 \text{N} \cdot \text{m} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau_{\max}} = \frac{(2)(700)}{\pi(56.588 \times 10^6)} = 7.875 \times 10^{-6} \text{m}^3$$

$$c = 19.895 \times 10^{-3} \text{m} \qquad d = 2c = 39.79 \times 10^{-3} \text{m}$$

$d = 39.8$ mm ◄



PROBLEM 3.11

Knowing that each portion of the shafts AB, BC, and CD consist of a solid circular rod, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Shaft *AB*:

$$T = 48 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 7.5 \text{ mm} = 0.0075 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)\,(48)}{\pi(0.0075)^3} = 72.433 \text{ MPa}$$

Shaft *BC*:

$$T = -48 + 144 = 96 \text{ N} \cdot \text{m}$$

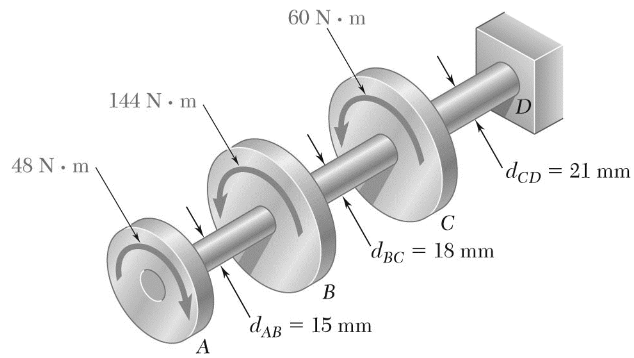
$$c = \frac{1}{2}d = 9 \text{ mm} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)\,(96)}{\pi(0.009)^3} = 83.835 \text{ MPa}$$

Shaft *CD*:

$$T = -48 + 144 + 60 = 156 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 10.5 \text{ mm} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2 \times 156)}{\pi(0.0105)^3} = 85.79 \text{ MPa}$$

Answers: (*a*) shaft *CD* (*b*) 85.8 MPa ◀



PROBLEM 3.12

Knowing that an 8-mm-diameter hole has been drilled through each of the shafts *AB, BC*, and *CD,* determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Hole: $c_1 = \frac{1}{2}d_1 = 4 \text{ mm}$

Shaft *AB*: $T = 48 \text{ N} \cdot \text{m}$

$$c_2 = \frac{1}{2}d_2 = 7.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0075^4 - 0.004^4) = 4.5679 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(48)(0.0075)}{4.5679 \times 10^{-9}} = 78.810 \text{ MPa}$$

Shaft *BC*: $T = -48 + 144 = 96 \text{ N} \cdot \text{m}$ $\quad c_2 = \frac{1}{2}d_2 = 9 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.009^4 - 0.004^4) = 9.904 \times 10^{-9} \text{ m}^4$$

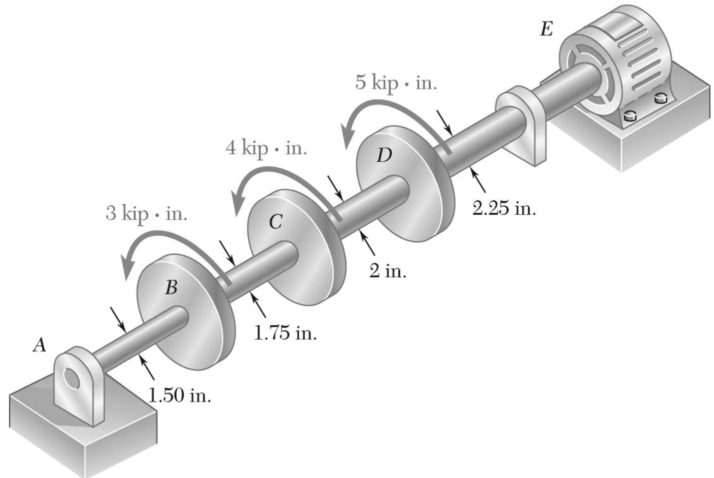
$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(96)(0.009)}{9.904 \times 10^{-9}} = 87.239 \text{ MPa}$$

Shaft *CD*: $T = -48 + 144 + 60 = 156 \text{ N} \cdot \text{m}$ $\quad c_2 = \frac{1}{2}d_2 = 10.5 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0105^4 - 0.004^4) = 18.691 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(156)(0.0105)}{18.691 \times 10^{-9}} = 87.636 \text{ MPa}$$

Answers: (*a*) shaft *CD* (*b*) 87.6 MPa ◀



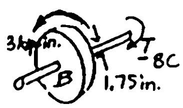
PROBLEM 3.13

Under normal operating conditions, the electric motor exerts a 12-kip · in. torque at *E*. Knowing that each shaft is solid, determine the maximum shearing in (*a*) shaft *BC*, (*b*) shaft *CD*, (*c*) shaft *DE*.

SOLUTION

(*a*) <u>Shaft *BC*</u>: From free body shown:

$$T_{BC} = 3 \text{ kip} \cdot \text{in}$$

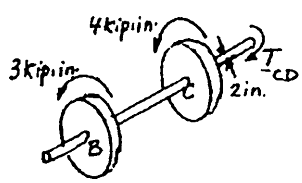


$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2}{\pi}\frac{T}{c^3} \qquad (1)$$

$$\tau = \frac{2}{\pi}\frac{3 \text{ kip} \cdot \text{in}}{\left(\frac{1}{2} \times 1.75 \text{ in.}\right)^3}$$

$\tau = 2.85 \text{ ksi}$ ◀

(*b*) <u>Shaft *CD*</u>: From free body shown:



$$T_{CD} = 3 + 4 = 7 \text{ kip} \cdot \text{in}$$

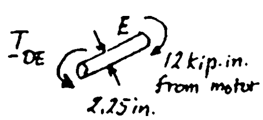
From Eq. (1):

$$\tau = \frac{2}{\pi}\frac{T}{c^3} = \frac{2}{\pi}\frac{7 \text{ kip} \cdot \text{in}}{(1 \text{ in.})^3}$$

$\tau = 4.46 \text{ ksi}$ ◀

(*c*) <u>Shaft *DE*</u>: From free body shown:

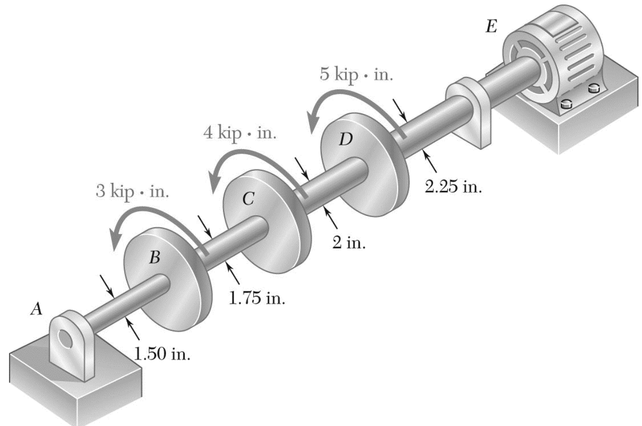
$$T_{DE} = 12 \text{ kip} \cdot \text{in}$$



From Eq. (1):

$$\tau = \frac{2}{\pi}\frac{T}{c^3} = \frac{2}{\pi}\frac{12 \text{ kip} \cdot \text{in}}{\left(\frac{1}{2} \times 2.25 \text{ in.}\right)^3}$$

$\tau = 5.37 \text{ ksi}$ ◀



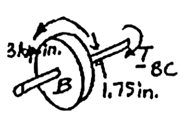
PROBLEM 3.14

Solve Prob.3.13, assuming that a 1-in.-diameter hole has been drilled into each shaft.

PROBLEM 3.13 Under normal operating conditions, the electric motor exerts a 12-kip · in. torque at *E*. Knowing that each shaft is solid, determine the maximum shearing in (*a*) shaft *BC*, (*b*) shaft *CD*, (*c*) shaft *DE*.

SOLUTION

(*a*) Shaft *BC*:



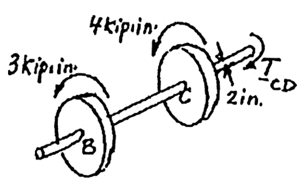
From free body shown: $T_{BC} = 3 \text{ kip} \cdot \text{in}$

$$c_2 = \frac{1}{2}(1.75) = 0.875 \text{ in.} \qquad c_1 = \frac{1}{2}(1) = 0.5 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.875^4 - 0.5^4) = 0.82260 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{(3 \text{ kip} \cdot \text{in})(0.875 \text{ in.})}{0.82260 \text{ in}^4} \qquad \tau = 3.19 \text{ ksi} \blacktriangleleft$$

(*b*) Shaft *CD*:



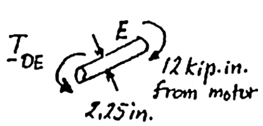
From free body shown: $T_{CD} = 3 + 4 = 7 \text{ kip} \cdot \text{in}$

$$c_2 = \frac{1}{2}(2.0) = 1.0 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.0^4 - 0.5^4) = 1.47262 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{(7 \text{ kip} \cdot \text{in})(1.0 \text{ in.})}{1.47262 \text{ in}^4} \qquad \tau = 4.75 \text{ ksi} \blacktriangleleft$$

(*c*) Shaft *DE*:

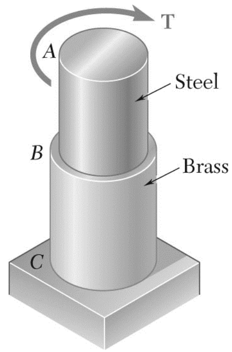


From free body shown: $T_{DE} = 12 \text{ kip} \cdot \text{in}$

$$c_2 = \frac{2.25}{2} = 1.125 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.125^4 - 0.5^4) = 2.4179 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12 \text{ kip} \cdot \text{in})(1.125 \text{ in.})}{2.4179 \text{ in}^4} \qquad \tau = 5.58 \text{ ksi} \blacktriangleleft$$



PROBLEM 3.15

The allowable shearing stress is 15 ksi in the 1.5-in.-diameter steel rod *AB* and 8 ksi in the 1.8-in.-diameter brass rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad T = \frac{\pi}{2}c^3\tau_{\max}$$

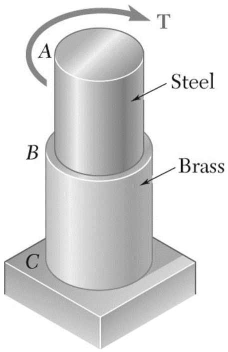
Rod *AB*: $\tau_{\max} = 15 \text{ ksi} \qquad c = \frac{1}{2}d = 0.75 \text{ in.}$

$$T = \frac{\pi}{2}(0.75)^3(15) = 9.94 \text{ kip} \cdot \text{in}$$

Rod *BC*: $\tau_{\max} = 8 \text{ ksi} \qquad c = \frac{1}{2}d = 0.90 \text{ in.}$

$$T = \frac{\pi}{2}(0.90)^3(8) = 9.16 \text{ kip} \cdot \text{in}$$

The allowable torque is the smaller value. $T = 9.16 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 3.16

The allowable shearing stress is 15 ksi in the steel rod *AB* and 8 ksi in the brass rod BC. Knowing that a torque of magnitude $T = 10$ kip · in. is applied at *A*, determine the required diameter of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}, \quad c^3 = \frac{2T}{\pi\tau_{\max}}$$

(*a*) Rod *AB*:

$T = 10$ kip · in $\qquad \tau_{\max} = 15$ ksi

$$c^3 = \frac{(2)(10)}{\pi(15)} = 0.4244 \text{ in}^3$$

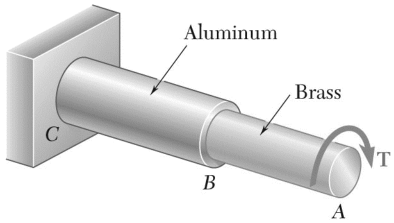
$c = 0.7515$ in. $\qquad d = 2c = 1.503$ in. ◀

(*b*) Rod *BC*:

$T = 10$ kip · in $\qquad \tau_{\max} = 8$ ksi

$$c^3 = \frac{(2)(10)}{\pi(8)} = 0.79577 \text{ in}^2$$

$c = 0.9267$ in. $\qquad d = 2c = 1.853$ in. ◀



PROBLEM 3.17

The allowable stress is 50 MPa in the brass rod *AB* and 25 MPa in the aluminum rod *BC*. Knowing that a torque of magnitude $T = 1250\ \text{N} \cdot \text{m}$ is applied at *A*, determine the required diameter of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4 \quad c^3 = \frac{2T}{\pi\tau_{\max}}$$

(*a*) Rod *AB*:

$$c^3 = \frac{(2)(1250)}{\pi(50\times 10^6)} = 15.915\times 10^{-6}\text{m}^3$$

$$c = 25.15\times 10^{-3}\text{m} = 25.15\ \text{mm}$$

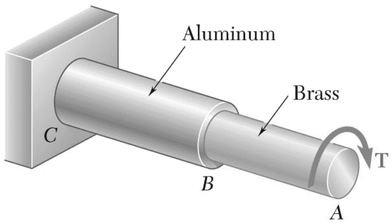
$$d_{AB} = 2c = 50.3\ \text{mm} \blacktriangleleft$$

(*b*) Rod *BC*:

$$c^3 = \frac{(2)(1250)}{\pi(25\times 10^6)} = 31.831\times 10^{-6}\text{m}^3$$

$$c = 31.69\times 10^{-3}\text{m} = 31.69\ \text{mm}$$

$$d_{BC} = 2c = 63.4\ \text{mm} \blacktriangleleft$$



PROBLEM 3.18

The *solid* rod *BC* has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is *hollow* and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (*a*) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (*b*) the largest torque that can be applied at *A*.

SOLUTION

Solid rod *BC*:

$$\tau = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4$$

$$\tau_{\text{all}} = 25 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$T_{\text{all}} = \frac{\pi}{2}c^3\tau_{\text{all}} = \frac{\pi}{2}(0.015)^3(25 \times 10^6) = 132.536 \text{ N} \cdot \text{m}$$

Hollow rod *AB*:

$$\tau_{\text{all}} = 50 \times 10^6 \text{ Pa}$$

$$T_{\text{all}} = 132.536 \text{ N} \cdot \text{m}$$

$$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125 \text{ m}$$

$$c_1 = ?$$

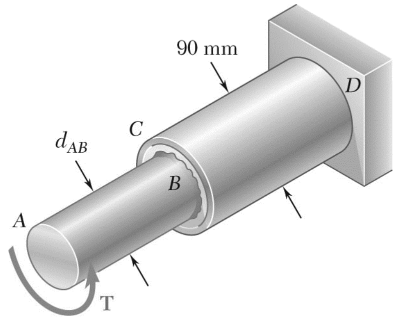
$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)\frac{\tau_{\text{all}}}{c_2}$$

$$c_1^4 = c_2^4 - \frac{2T_{\text{all}}c_2}{\pi\tau_{\text{all}}}$$

$$= 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50 \times 10^6)} = 3.3203 \times 10^{-9} \text{ m}^4$$

(*a*) $c_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm}$ $\quad d_1 = 2c_1 = 15.18 \text{ mm}$ ◀

(*b*) Allowable torque. $\quad T_{\text{all}} = 132.5 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.19

The solid rod AB has a diameter d_{AB} = 60 mm. The pipe CD has an outer diameter of 90 mm and a wall thickness of 6 mm. Knowing that both the rod and the pipe are made of a steel for which the allowable shearing stress is 75 Mpa, determine the largest torque **T** that can be applied at A.

SOLUTION

$$\tau_{\text{all}} = 75\times 10^6 \text{ Pa} \quad T_{\text{all}} = \frac{J\tau_{\text{all}}}{c}$$

Rod AB:

$$c = \frac{1}{2}d = 0.030 \text{ m} \quad J = \frac{\pi}{2}c^4$$

$$T_{\text{all}} = \frac{\pi}{2}c^3\tau_{\text{all}} = \frac{\pi}{2}(0.030)^3(75\times 10^6)$$

$$= 3.181\times 10^3 \text{ N}\cdot\text{m}$$

Pipe CD:

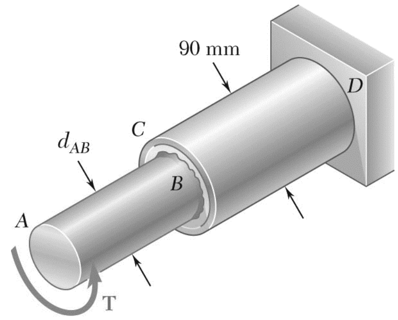
$$c_2 = \frac{1}{2}d_2 = 0.045 \text{ m} \quad c_1 = c_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073\times 10^{-6} \text{ m}^4$$

$$T_{\text{all}} = \frac{(2.8073\times 10^{-6})(75\times 10^6)}{0.045} = 4.679\times 10^3 \text{ N}\cdot\text{m}$$

Allowable torque is the smaller value. $T_{\text{all}} = 3.18\times 10^3 \text{ N}\cdot\text{m}$

3.18 kN · m ◀



PROBLEM 3.20

The solid rod AB has a diameter d_{AB} = 60 mm and is made of a steel for which the allowable shearing stress is 85 Mpa. The pipe CD, which has an outer diameter of 90 mm and a wall thickness of 6 mm, is made of an aluminum for which the allowable shearing stress is 54 MPa. Determine the largest torque **T** that can be applied at A.

SOLUTION

Rod *AB*: $\tau_{\text{all}} = 85 \times 10^6 \text{ Pa} \quad c = \frac{1}{2}d = 0.030 \text{ m}$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c} = \frac{\pi}{2}c^3\,\tau_{\text{all}}$$

$$= \frac{\pi}{2}(0.030)^3(85 \times 10^6) = 3.605 \times 10^3 \text{ N} \cdot \text{m}$$

Pipe *CD*: $\tau_{\text{all}} = 54 \times 10^6 \text{ Pa} \qquad c_2 = \frac{1}{2}d_2 = 0.045 \text{ m}$

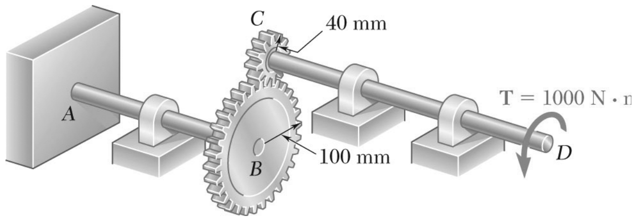
$$c_1 = c_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{(2.8073 \times 10^{-6})(54 \times 10^6)}{0.045} = 3.369 \times 10^3 \text{ N} \cdot \text{m}$$

Allowable torque is the smaller value. $T_{\text{all}} = 3.369 \times 10^3 \text{ N} \cdot \text{m}$

3.37 kN · m ◀



PROBLEM 3.21

A torque of magnitude $T = 1000\ \text{N}\cdot\text{m}$ is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and that the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.

SOLUTION

$$T_{CD} = 1000\ \text{N}\cdot\text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500\ \text{N}\cdot\text{m}$$

(*a*) <u>Shaft *AB*</u>:

$$c = \frac{1}{2}d = 0.028\ \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2500)}{\pi(0.028)^3} = 72.50\times 10^6$$

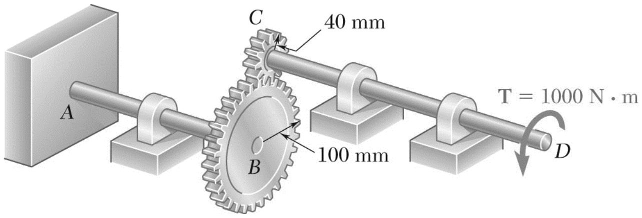
72.5 MPa ◀

(*b*) <u>Shaft *CD*</u>:

$$c = \frac{1}{2}d = 0.020\ \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1000)}{\pi(0.020)^3} = 68.7\times 10^6$$

68.7 MPa ◀



PROBLEM 3.22

A torque of magnitude $T = 1000\text{ N}\cdot\text{m}$ is applied at D as shown. Knowing that the allowable shearing stress is 60 MPa in each shaft, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*.

SOLUTION

$$T_{CD} = 1000\text{ N}\cdot\text{m}$$

$$T_{AB} = \frac{r_B}{r_C}T_{CD} = \frac{100}{40}(1000) = 2500\text{ N}\cdot\text{m}$$

(*a*) Shaft *AB*:

$$\tau_{\text{all}} = 60\times 10^6\text{ Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2500)}{\pi(60\times 10^6)} = 26.526\times 10^{-6}\text{ m}^3$$

$$c = 29.82\times 10^{-3} = 29.82\text{ mm}$$

$d = 2c = 59.6\text{ mm}$ ◀

(*b*) Shaft *CD*:

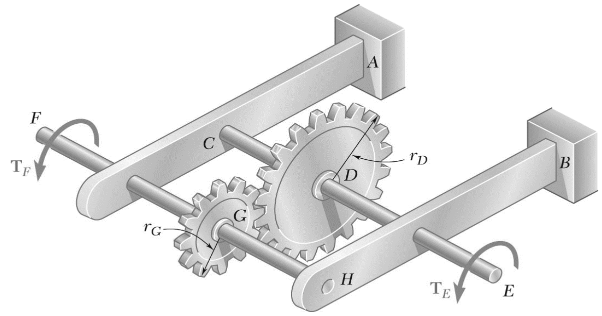
$$\tau_{\text{all}} = 60\times 10^6\text{ Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(1000)}{\pi(60\times 10^6)} = 10.610\times 10^{-6}\text{ m}^3$$

$$c = 21.97\times 10^{-3}\text{ m} = 21.97\text{ mm}$$

$d = 2c = 43.9\text{ mm}$ ◀

PROBLEM 3.23



Under normal operating conditions, a motor exerts a torque of magnitude $T_F = 1200\text{ lb}\cdot\text{in.}$ at F. Knowing that $r_D = 8\text{ in.}$, $r_G = 3\text{ in.}$, and the allowable shearing stress is 10.5 ksi · in each shaft, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.

SOLUTION

$$T_F = 1200\text{ lb}\cdot\text{in}$$

$$T_E = \frac{r_D}{r_G}T_F = \frac{8}{3}(1200) = 3200\text{ lb}\cdot\text{in}$$

$$\tau_{\text{all}} = 10.5\text{ ksi} = 10500\text{ psi}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \text{or} \quad c^3 = \frac{2T}{\pi\tau}$$

(*a*) Shaft *CDE*:

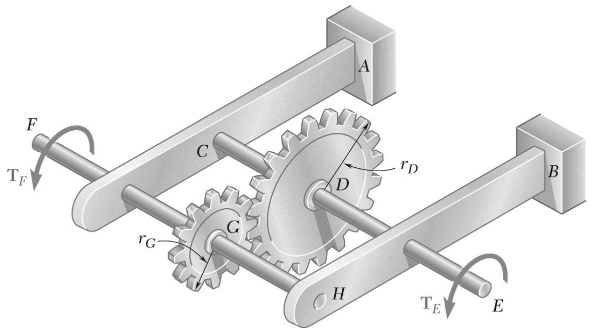
$$c^3 = \frac{(2)\,(3200)}{\pi\,(10500)} = 0.194012\text{ in}^3$$

$$c = 0.5789\text{ in.} \qquad d_{DE} = 2c \qquad d_{DE} = 1.158\text{ in.} \blacktriangleleft$$

(*b*) Shaft *FGH*:

$$c^3 = \frac{(2)\,(1200)}{\pi\,(10500)} = 0.012757\text{ in}^3$$

$$c = 0.4174\text{ in.} \qquad d_{FG} = 2c \qquad d_{FG} = 0.835\text{ in.} \blacktriangleleft$$



PROBLEM 3.24

Under normal operating conditions, a motor exerts a torque of magnitude T_F at F. The shafts are made of a steel for which the allowable shearing stress is 12 ksi and have diameters $d_{CDE} = 0.900$ in. and $d_{FGH} = 0.800$ in. Knowing that $r_D = 6.5$ in. and $r_G = 4.5$ in., determine the largest allowable value of T_F.

SOLUTION

$$\tau_{\text{all}} = 12 \text{ ksi}$$

Shaft *FG*:

$$c = \frac{1}{2}d = 0.400 \text{ in.}$$

$$T_{F,\text{all}} = \frac{J\tau_{\text{all}}}{c} = \frac{\pi}{2}c^3\tau_{\text{all}}$$

$$= \frac{\pi}{2}(0.400)^3(12) = 1.206 \text{ kip} \cdot \text{in}$$

Shaft *DE*:

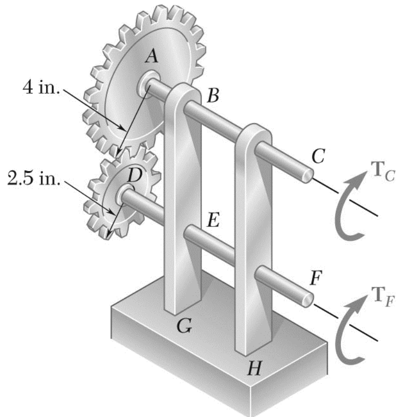
$$c = \frac{1}{2}d = 0.450 \text{ in.}$$

$$T_{E,\text{all}} = \frac{\pi}{2}c^3\tau_{all}$$

$$= \frac{\pi}{2}(0.450)^3(12) = 1.7177 \text{ kip} \cdot \text{in}$$

$$T_F = \frac{r_G}{r_D}T_E \quad T_{F,\text{all}} = \frac{4.5}{6.5}(1.7177) = 1.189 \text{ kip} \cdot \text{in}$$

Allowable value of T_F is the smaller. $T_F = 1.189 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 3.25

The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 8500 psi. Knowing that a torque of magnitude $T_C = 5$ kip · in. is applied at C and that the assembly is in equilibrium, determine the required diameter of (*a*) shaft *BC*, (*b*) shaft *EF*.

SOLUTION

$$\tau_{\max} = 8500 \text{ psi} = 8.5 \text{ ksi}$$

(*a*) Shaft *BC*:

$$T_C = 5 \text{ kip} \cdot \text{in}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}}$$

$$c = \sqrt[3]{\frac{(2)(5)}{\pi(8.5)}} = 0.7208 \text{ in.}$$

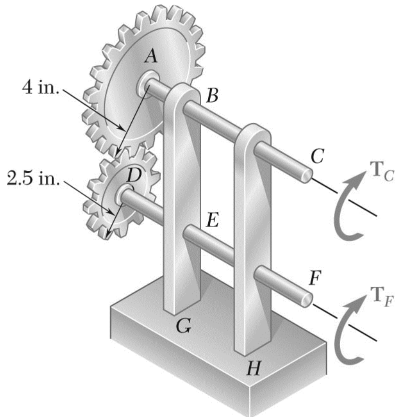
$$d_{BC} = 2c = 1.442 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *EF*:

$$T_F = \frac{r_D}{r_A} T_C = \frac{2.5}{4}(5) = 3.125 \text{ kip} \cdot \text{in}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(3.125)}{\pi(8.5)}} = 0.6163 \text{ in.}$$

$$d_{EF} = 2c = 1.233 \text{ in.} \blacktriangleleft$$



PROBLEM 3.26

The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 7000 psi. Knowing the diameters of the two shafts are, respectively, $d_{BC} = 1.6$ in. and $d_{EF} = 1.25$ in., determine the largest torque $\mathbf{T}_C$ that can be applied at C.

SOLUTION

$$\tau_{\max} = 7000 \text{ psi} = 7.0 \text{ ksi}$$

Shaft BC:

$$d_{BC} = 1.6 \text{ in.}$$

$$c = \frac{1}{2}d = 0.8 \text{ in.}$$

$$T_C = \frac{J\tau_{\max}}{c} = \frac{\pi}{2}\tau_{\max}c^3$$

$$= \frac{\pi}{2}(7.0)(0.8)^3 = 5.63 \text{ kip} \cdot \text{in}$$

Shaft EF:

$$d_{EF} = 1.25 \text{ in.}$$

$$c = \frac{1}{2}d = 0.625 \text{ in.}$$

$$T_F = \frac{J\tau_{\max}}{c} = \frac{\pi}{2}\tau_{\max}c^3$$

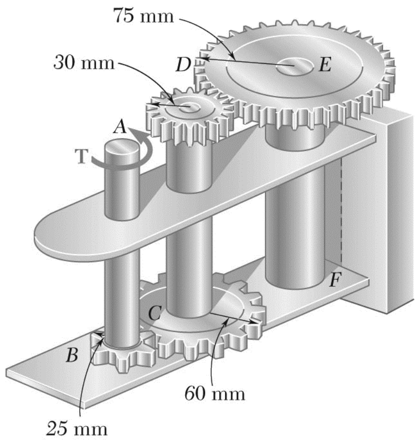
$$= \frac{\pi}{2}(7.0)(0.625)^3 = 2.684 \text{ kip} \cdot \text{in}$$

By statics,

$$T_C = \frac{r_A}{r_D}T_F = \frac{4}{2.5}(2.684) = 4.30 \text{ kip} \cdot \text{in}$$

Allowable value of T_C is the smaller.

$$T_C = 4.30 \text{ kip} \cdot \text{in} \blacktriangleleft$$



PROBLEM 3.27

A torque of magnitude $T = 100\ \text{N} \cdot \text{m}$ is applied to shaft AB of the gear train shown. Knowing that the diameters of the three solid shafts are, respectively, $d_{AB} = 21\ \text{mm}$, $d_{CD} = 30\ \text{mm}$, and $d_{EF} = 40\ \text{mm}$, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *CD*, (*c*) shaft *EF*.

SOLUTION

Statics:

Shaft *AB*: $T_{AB} = T_A = T_B = T$

Gears *B* and *C*: $r_B = 25\ \text{mm}, \quad r_C = 60\ \text{mm}$

Force on gear circles. $F_{BC} = \dfrac{T_B}{r_B} = \dfrac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4T$$

Shaft *CD*: $T_{CD} = T_C = T_D = 2.4T$

Gears *D* and *E*: $r_D = 30\ \text{mm}, \quad r_E = 75\ \text{mm}$

Force on gear circles. $F_{DE} = \dfrac{T_D}{r_D} = \dfrac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30}(2.4T) = 6T$$

Shaft *EF*: $T_{EF} = T_E = T_F = 6T$

Maximum Shearing Stresses. $\tau_{\max} = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

PROBLEM 3.27 (*Continued*)

(*a*) Shaft *AB*: $T = 100\ \text{N} \cdot \text{m}$

$$c = \frac{1}{2}d = 10.5\ \text{mm} = 10.5 \times 10^{-3}\ \text{m}$$

$$\tau_{\text{max}} = \frac{(2)(100)}{\pi(10.5 \times 10^{-3})^3} = 55.0 \times 10^6\ \text{Pa} \qquad \tau_{\text{max}} = 55.0\ \text{MPa} \blacktriangleleft$$

(*b*) Shaft *CD*: $T = (2.4)(100) = 240\ \text{N} \cdot \text{m}$

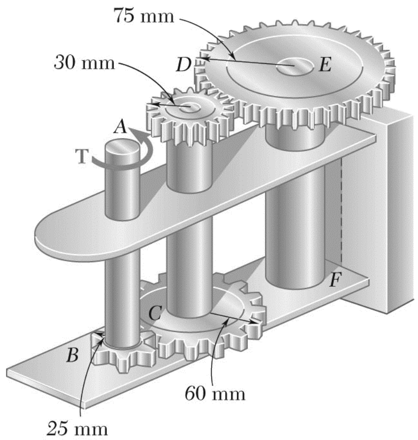
$$c = \frac{1}{2}d = 15\ \text{mm} = 15 \times 10^{-3}\ \text{m}$$

$$\tau_{\text{max}} = \frac{(2)(240)}{\pi(15 \times 10^{-3})^3} = 45.3 \times 10^6\ \text{Pa} \qquad \tau_{\text{max}} = 45.3\ \text{MPa} \blacktriangleleft$$

(*c*) Shaft *EF*: $T = (6)(100) = 600\ \text{N} \cdot \text{m}$

$$c = \frac{1}{2}d = 20\ \text{mm} = 20 \times 10^{-3}\text{m}$$

$$\tau_{\text{max}} = \frac{(2)(600)}{\pi(20 \times 10^{-3})^3} = 47.7 \times 10^6\ \text{Pa} \qquad \tau_{\text{max}} = 47.7\ \text{MPa} \blacktriangleleft$$



PROBLEM 3.28

A torque of magnitude $T = 120\,\text{N}\cdot\text{m}$ is applied to shaft AB of the gear train shown. Knowing that the allowable shearing stress is 75 MPa in each of the three solid shafts, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD, (*c*) shaft EF.

SOLUTION

Statics:

Shaft AB: $\quad T_{AB} = T_A = T_B = T$

Gears B and C: $\quad r_B = 25\text{ mm}, \quad r_C = 60\text{ mm}$

Force on gear circles. $\quad F_{BC} = \dfrac{T_B}{r_B} = \dfrac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B}T_B = \frac{60}{25}T = 2.4T$$

Shaft CD: $\quad T_{CD} = T_C = T_D = 2.4T$

Gears D and E: $\quad r_D = 30\text{ mm}, \quad r_E = 75\text{ mm}$

Force on gear circles. $\quad F_{DE} = \dfrac{T_D}{r_D} = \dfrac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D}T_D = \frac{75}{30}(2.4T) = 6T$$

Shaft EF: $\quad T_{EF} = T_E = T_F = 6T$

Required Diameters.

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}}$$

$$d = 2c = 2\sqrt[3]{\frac{2T}{\pi\tau_{\max}}}$$

$$\tau_{\max} = 75\times 10^6\text{ Pa}$$

PROBLEM 3.28 (*Continued*)

(*a*) Shaft *AB*: $T_{AB} = T = 120 \text{ N} \cdot \text{m}$

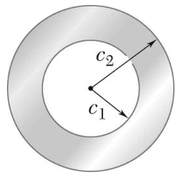
$$d_{AB} = 2\sqrt[3]{\frac{2(120)}{\pi(75\times10^6)}} = 20.1\times10^{-3}\text{ m} \qquad d_{AB} = 20.1 \text{ mm} \blacktriangleleft$$

(*b*) Shaft *CD*: $T_{CD} = (2.4)(120) = 288 \text{ N} \cdot \text{m}$

$$d_{CD} = 2\sqrt[3]{\frac{(2)(288)}{\pi(75\times10^6)}} = 26.9\times10^{-3}\text{ m} \qquad d_{CD} = 26.9 \text{ mm} \blacktriangleleft$$

(*c*) Shaft *EF*: $T_{EF} = (6)(120) = 720 \text{ N} \cdot \text{m}$

$$d_{EF} = 2\sqrt[3]{\frac{(2)(720)}{\pi(75\times10^3)}} = 36.6\times10^{-3}\text{ m} \qquad d_{EF} = 36.6 \text{ mm} \blacktriangleleft$$



PROBLEM 3.29

(*a*) For a given allowable shearing stress, determine the ratio *T*/*w* of the maximum allowable torque *T* and the weight per unit length *w* for the hollow shaft shown. (*b*) Denoting by $(T/w)_0$ the value of this ratio for a solid shaft of the same radius c_2, express the ratio *T*/*w* for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2.

SOLUTION

$$\begin{aligned} w &= \text{weight per unit length,} \\ \rho g &= \text{specific weight,} \\ W &= \text{total weight,} \\ L &= \text{length} \end{aligned}$$

$$w = \frac{W}{L} = \frac{\rho g L A}{L} = \rho g A = \rho g \pi \left(c_2^2 - c_1^2\right)$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{\pi}{2}\frac{c_2^4 - c_1^4}{c_2}\tau_{\text{all}} = \frac{\pi}{2}\frac{\left(c_2^2 + c_1^2\right)\left(c_2^2 - c_1^2\right)}{c_2}\tau_{\text{all}}$$

(*a*) $\dfrac{T}{W} = \left(c_1^2 + c_2^2\right)\tau_{\text{all}}$

$$\frac{T}{w} = \frac{\left(c_1^2 + c_2^2\right)\tau_{\text{all}}}{2\rho g c_2} \quad \text{(hollow shaft)} \blacktriangleleft$$

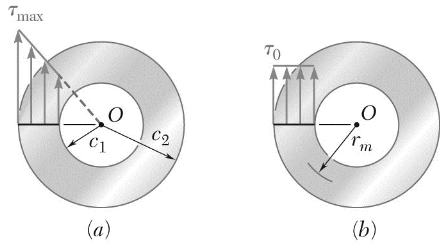
$c_1 = 0$ for solid shaft:

$$\left(\frac{T}{w}\right)_0 = \frac{c_2\tau_{\text{all}}}{2pg} \quad \text{(solid shaft)}$$

(*b*) $\dfrac{(T/w)_h}{(T/w)_0} = 1 + \dfrac{c_1^2}{c_2^2}$

$$\left(\frac{T}{w}\right) = \left(\frac{T}{w}\right)_0\left(1 + \frac{c_1^2}{c_2^2}\right) \blacktriangleleft$$

PROBLEM 3.30



While the exact distribution of the shearing stresses in a hollow-cylindrical shaft is as shown in Fig. *a*, an approximate value can be obtained for τ_{max} by assuming that the stresses are uniformly distributed over the area A of the cross section, as shown in Fig. *b*, and then further assuming that all of the elementary shearing forces act at a distance from O equal to the mean radius $\frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2, respectively equal to 1.00, 0.95, 0.75, 0.50, and 0.

SOLUTION

For a hollow shaft:

$$\tau_{max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)} = \frac{2Tc_2}{\pi\left(c_2^2 - c_1^2\right)\left(c_2^2 + c_1^2\right)} = \frac{2Tc_2}{A\left(c_2^2 + c_1^2\right)}$$

By definition,

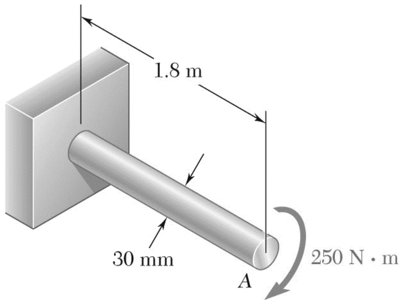
$$\tau_0 = \frac{T}{Ar_m} = \frac{2T}{A(c_2 + c_1)}$$

Dividing,

$$\frac{\tau_{max}}{\tau_0} = \frac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2} \blacktriangleleft$$

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

◀



PROBLEM 3.31

(a) For the solid steel shaft shown ($G = 77$ GPa), determine the angle of twist at A. (b) Solve part a, assuming that the steel shaft is hollow with a 30-mm-outer diameter and a 20-mm-inner diameter.

SOLUTION

(a) $c = \dfrac{1}{2}d = 0.015 \text{ m}, \quad J = \dfrac{\pi}{2}c^4 = \dfrac{\pi}{2}(0.015)^4$

$J = 79.522 \times 10^{-9}\text{m}^4, \quad L = 1.8 \text{ m}, \quad G = 77 \times 10^9 \text{Pa}$

$T = 250 \text{ N} \cdot \text{m} \qquad \varphi = \dfrac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 73.49 \times 10^{-3} \text{ rad}$$

$$\varphi = \frac{(73.49 \times 10^{-3})180}{\pi}$$

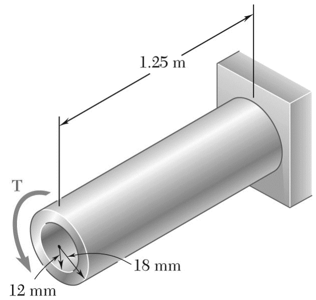
$\varphi = 4.21°$ ◀

(b) $c_2 = 0.015 \text{ m}, \quad c_1 \dfrac{1}{2}d_1 = 0.010 \text{ m}, \quad J = \dfrac{\pi}{2}\left(c_2^4 - c_1^4\right)$

$J = \dfrac{\pi}{2}(0.015^4 - 0.010^4) = 63.814 \times 10^{-9} \text{ m}^4 \quad \varphi \dfrac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77 \times 10^9)(63.814 \times 10^{-9})} = 91.58 \times 10^{-3} \text{ rad} = \frac{180}{\pi}(91.58 \times 10^{-3})$$

$\varphi = 5.25°$ ◀



PROBLEM 3.32

For the aluminum shaft shown (G = 27 GPa), determine (a) the torque **T** that causes an angle of twist of 4°, (b) the angle of twist caused by the same torque **T** in a solid cylindrical shaft of the same length and cross-sectional area.

SOLUTION

(a)

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

$$\varphi = 4° = 69.813 \times 10^{-3} \text{ rad}, \quad L = 1.25 \text{ m}$$

$$G = 27 \text{ GPa} = 27 \times 10^9 \text{ Pa}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.018^4 - 0.012^4) = 132.324 \times 10^{-9} \text{ m}^4$$

$$T = \frac{(27 \times 10^9)(132.324 \times 10^9)(69.813 \times 10^{-3})}{1.25}$$

$$= 199.539 \text{ N} \cdot \text{m} \qquad T = 199.5 \text{ N} \cdot \text{m} \blacktriangleleft$$

(b) Matching areas:

$$A = \pi c^2 = \pi\left(c_2^2 - c_1^2\right)$$

$$c = \sqrt{c_2^2 - c_1^2} = \sqrt{0.018^2 - 0.012^2} = 0.013416 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.013416)^4 = 50.894 \times 10^{-9} \text{ m}^4$$

$$\varphi = \frac{TL}{GJ} = \frac{(195.539)(1.25)}{(27 \times 10^9)(50.894 \times 10^9)} = 181.514 \times 10^{-3} \text{ rad} \qquad \varphi = 10.40° \blacktriangleleft$$

PROBLEM 3.33

Determine the largest allowable diameter of a 10-ft-long steel rod $(G = 11.2 \times 10^6 \text{ psi})$ if the rod is to be twisted through 30° without exceeding a shearing stress of 12 ksi.

SOLUTION

$$L = 10 \text{ ft} = 120 \text{ in.} \qquad \varphi = 30° = \frac{30\pi}{180} = 0.52360 \text{ rad}$$

$$\tau = 12 \text{ ksi} = 12 \times 10^3 \text{psi}$$

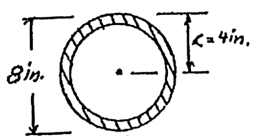
$$\varphi = \frac{TL}{GJ}, \quad T = \frac{GJ\varphi}{L}, \quad \tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}, \quad c = \frac{\tau L}{G\varphi}$$

$$c = \frac{(12 \times 10^3)(120)}{(11.2 \times 10^6)(0.52360)} = 0.24555 \text{ in.} \qquad d = 2c = 0.491 \text{ in.} \blacktriangleleft$$

PROBLEM 3.34

While an oil well is being drilled at a depth of 6000 ft, it is observed that the top of the 8-in.-diameter steel drill pipe rotates though two complete revolutions before the drilling bit starts to rotate. Using $G = 11.2 \times 10^6$ psi, determine the maximum shearing stress in the pipe caused by torsion.

SOLUTION



For outside diameter of 8 in., $c = 4$ in.

For two revolutions, $\varphi = 2(2\pi) = 4\pi$ radians.

$G = 11.2 \times 10^6$ psi

$L = 6000$ ft $= 72000$ in.

From text book,

$$\varphi = \frac{TL}{GJ} \tag{1}$$

$$\tau_m = \frac{Tc}{J} \tag{2}$$

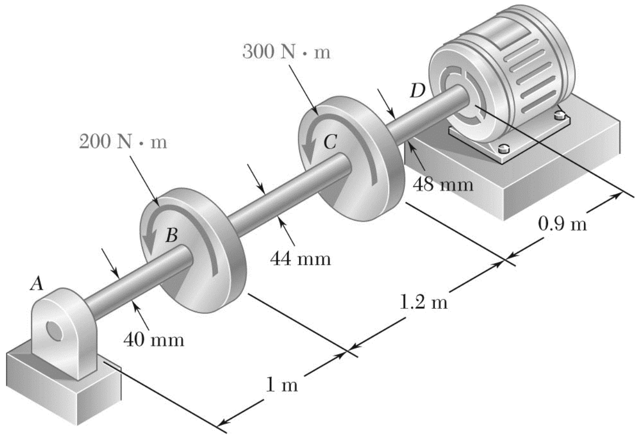
Divide (2) by (1).

$$\frac{\tau_m}{\varphi} = \frac{Gc}{L}$$

$$\tau_m = \frac{Gc\varphi}{L} = \frac{(11 \times 10^6)(4)(4\pi)}{72000} = 7679 \text{ psi}$$

$\tau_m = 7.68$ ksi ◀

PROBLEM 3.35



The electric motor exerts a 500 N · m torque on the aluminum shaft *ABCD* when it is rotating at a constant speed. Knowing that $G = 27\text{ GPa}$ and that the torques exerted on pulleys *B* and *C* are as shown, determine the angle of twist between (*a*) *B* and *C*, (*b*) *B* and *D*.

SOLUTION

(*a*) Angle of twist between *B* and *C*.

$$T_{BC} = 200\text{ N}\cdot\text{m},\quad L_{BC} = 1.2\text{ m}$$

$$c = \frac{1}{2}d = 0.022\text{ m},\quad G = 27\times 10^9\text{ Pa}$$

$$J_{BC} = \frac{\pi}{2}c^4 = 367.97\times 10^{-9}\text{ m}$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(200)(1.2)}{(27\times 10^9)(367.97\times 10^9)} = 24.157\times 10^{-3}\text{ rad}\qquad \varphi_{B/C} = 1.384° \blacktriangleleft$$

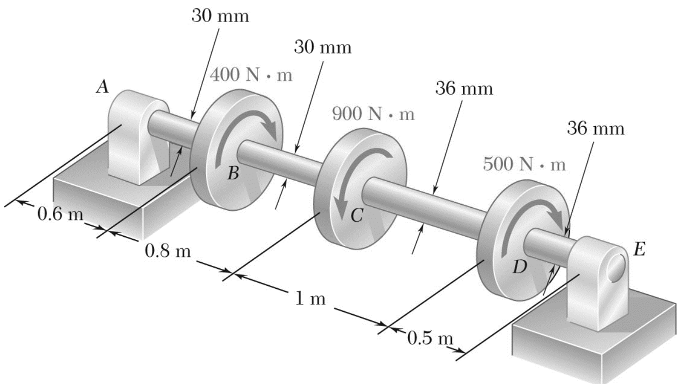
(*b*) Angle of twist between *B* and *D*.

$$T_{CD} = 500\text{ N}\cdot\text{m},\quad L_{CD} = 0.9\text{ m},\quad c = \frac{1}{2}d = 0.024\text{ m},\quad G = 27\times 10^9\text{Pa}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.024)^4 = 521.153\times 10^{-9}\text{ m}^4$$

$$\varphi_{C/D} = \frac{(500)(0.9)}{(27\times 10^9)(521.153\times 10^9)} = 31.980\times 10^{-3}\text{ rad}$$

$$\varphi_{B/D} = \varphi_{B/C} + \varphi_{C/D} = 24.157\times 10^{-3} + 31.980\times 10^{-3} = 56.137\times 10^{-3}\text{ rad}\qquad \varphi_{B/D} = 3.22° \blacktriangleleft$$



PROBLEM 3.36

The torques shown are exerted on pulleys B, C, and D. Knowing that the entire shaft is made of steel ($G = 27$ GPa), determine the angle of twist between (a) C and B, (b) D and B.

SOLUTION

(a) Shaft BC:

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$J_{BC} = \frac{\pi}{4}c^4 = 79.522 \times 10^{-9} \text{ m}^4$$

$$L_{BC} = 0.8 \text{ m}, \quad G = 27 \times 10^9 \text{ Pa}$$

$$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27 \times 10^9)(79.522 \times 10^{-9})} = 0.149904 \text{ rad}$$

$\varphi_{BC} = 8.54° \blacktriangleleft$

(b) Shaft CD:

$$c = \frac{1}{2}d = 0.018 \text{ m} \qquad J_{CD} = \frac{\pi}{4}c^4 = 164.896 \times 10^{-9} \text{ m}^4$$

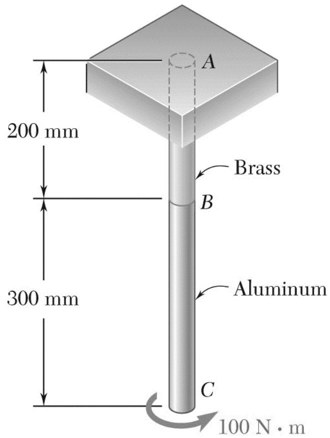
$$L_{CD} = 1.0 \text{ m} \quad T_{CD} = 400 - 900 = -500 \text{ N} \cdot \text{m}$$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27 \times 10^9)(164.896 \times 10^{-9})} = -0.11230 \text{ rad}$$

$$\varphi_{BD} = \varphi_{BC} + \varphi_{CD} = 0.14904 - 0.11230 = 0.03674 \text{ rad}$$

$\varphi_{BD} = 2.11° \blacktriangleleft$

PROBLEM 3.37



The aluminum rod BC ($G = 26$ GPa) is bonded to the brass rod AB ($G = 39$ GPa). Knowing that each rod is solid and has a diameter of 12 mm, determine the angle of twist (a) at B, (b) at C.

SOLUTION

Both portions:

$$c = \frac{1}{2}d = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(6 \times 10^{-3})^4 = 2.03575 \times 10^{-9} \text{ m}^4$$

$$T = 100 \text{ N} \cdot \text{m}$$

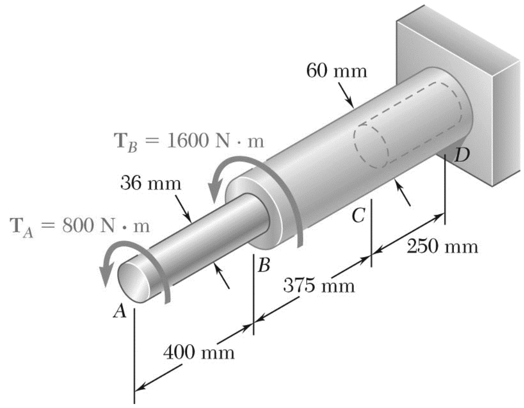
<u>Rod AB</u>: $\quad G_{AB} = 39 \times 10^9 \text{ Pa}, \quad L_{AB} = 0.200 \text{ m}$

(a) $$\varphi_B = \varphi_{AB} = \frac{TL_{AB}}{G_{AB}J} = \frac{(100)(0.200)}{(39 \times 10^9)(2.03575 \times 10^{-9})} = 0.25191 \text{ rad}$$ $\varphi_B = 14.43° ◀$

<u>Rod BC</u>: $\quad G_{BC} = 26 \times 10^9 \text{ Pa}, \quad L_{BC} = 0.300 \text{ m}$

$$\varphi_{BC} = \frac{TL_{BC}}{G_{BC}J} = \frac{(100)(0.300)}{(26 \times 10^9)(2.03575 \times 10^{-9})} = 0.56679 \text{ rad}$$

(b) $$\varphi_C = \varphi_B + \varphi_{BC} = 0.25191 + 0.56679 = 0.81870 \text{ rad}$$ $\varphi_C = 46.9° ◀$



PROBLEM 3.38

The aluminum rod AB $(G = 27\,\text{GPa})$ is bonded to the brass rod BD $(G = 39\,\text{GPa})$. Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at A.

SOLUTION

Rod AB:

$$G = 27 \times 10^9\ \text{Pa}, \quad L = 0.400\ \text{m}$$

$$T = 800\,\text{N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.018\ \text{m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9}\ \text{m}$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3}\ \text{rad}$$

Part BC:

$$G = 39 \times 10^9\ \text{Pa} \quad L = 0.375\ \text{m}, \quad c = \frac{1}{2}d = 0.030\ \text{m}$$

$$T = 800 + 1600 = 2400\ \text{N} \cdot \text{m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6}\ \text{m}^4$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3}\ \text{rad}$$

Part CD:

$$c_1 = \frac{1}{2}d_1 = 0.020\ \text{m}$$

$$c_2 = \frac{1}{2}d_2 = 0.030\ \text{m}, \quad L = 0.250\ \text{m}$$

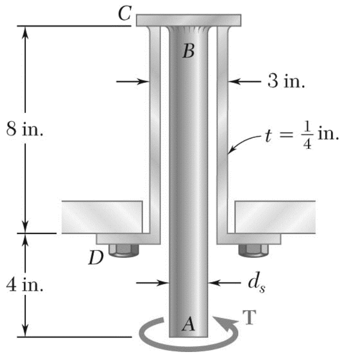
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6}\ \text{m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3}\ \text{rad}$$

Angle of twist at A.

$$\varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$$
$$= 105.080 \times 10^{-3}\ \text{rad}$$

$$\varphi_A = 6.02° \blacktriangleleft$$



PROBLEM 3.39

The solid spindle AB has a diameter d_s=1.5 in. and is made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{\text{all}} = 12$ ksi, while sleeve CD is made of a brass with $G = 5.6 \times 10^6$ psi and $\tau_{\text{all}} = 7$ ksi. Determine the angle through end A can be rotated.

SOLUTION

Stress analysis of solid spindle AB: $c = \frac{1}{2} d_s = 0.75$ in.

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3$$

$$T = \frac{\pi}{2}(12 \times 10^3)(0.75)^3 = 7.95 \times 10^3 \text{ lb} \cdot \text{in}$$

Stress analysis of sleeve CD: $c_2 = \frac{1}{2}d_o = \frac{1}{2}(3) = 1.5$ in.

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7 \times 10^{-3})}{1.5} = 19.21 \times 10^3 \text{ lb} \cdot \text{in}$$

The smaller torque governs. $T = 7.95 \times 10^3$ lb · in

Deformation of spindle AB: $c = 0.75$ in.

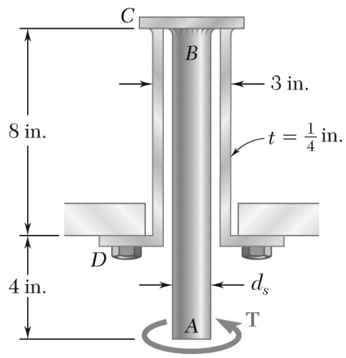
$$J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4, \quad L = 12 \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(7.95 \times 10^3)\,(12)}{(11.2 \times 10^6)(0.49701)} = 0.017138 \text{ radians}$$

Deformation of sleeve CD: $J = 4.1172$ in^4, $L = 8$ in., $G = 5.6 \times 10^6$ psi

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(7.95 \times 10^3)\,(8)}{(5.6 \times 10^6)\,(4.1172)} = 0.002758 \text{ radians}$$

Total angle of twist: $\varphi_{AD} = \varphi_{AB} + \varphi_{CD} = 0.019896$ radians $\qquad \varphi_{AD} = 1.140° \blacktriangleleft$



PROBLEM 3.40

The solid spindle AB has a diameter $d_s = 1.75$ in. and is made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{\text{all}} = 12$ ksi, while sleeve CD is made of a brass with $G = 5.6 \times 10^6$ psi and $\tau_{\text{all}} = 7$ ksi. Determine (*a*) the largest torque **T** that can be applied at A if the given allowable stresses are not to be exceeded and if the angle of twist of sleeve CD is not to exceed 0.375°, (*b*) the corresponding angle through which end A rotates.

SOLUTION

Spindle *AB*:

$$c = \frac{1}{2}(1.75\text{ in.}) = 0.875\text{ in.} \quad L = 12\text{ in.}, \quad \tau_{\text{all}} = 12\text{ ksi}, \quad G = 11.2 \times 10^6\text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.875)^4 = 0.92077\text{ in}^4$$

Sleeve *CD*:

$$c_1 = 1.25\text{ in.}, \quad c_2 = 1.5\text{ in.}, \quad L = 8\text{ in.}, \quad \tau_{\text{all}} = 7\text{ ksi}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 4.1172\text{ in}^4, \quad G = 5.6 \times 10^6\text{ psi}$$

(*a*) Largest allowable torque *T*.

Ciriterion: Stress in spindle *AB*.

$$\tau = \frac{Tc}{J} \quad T = \frac{J\tau}{c}$$

$$T = \frac{(0.92077)(12)}{0.875} = 12.63\text{ kip}\cdot\text{in}$$

Critrion: Stress in sleeve *CD*.

$$T = \frac{J\tau}{c_2} = \frac{4.1172\text{ in}^4}{1.5\text{ in.}}(7\text{ ksi}) \qquad T = 19.21\text{ kip}\cdot\text{in}$$

Criterion: Angle of twist of sleeve CD

$$\phi = 0.375° = 6.545 \times 10^{-3}\text{ rad}$$

$$\phi = \frac{TL}{JG} \quad T = \frac{JG}{L}\phi = \frac{(4.1172)(5.6 \times 10^6)}{8}(6.545 \times 10^{-3})$$

$$T = 18.86\text{ kip}\cdot\text{in}$$

The largest allowable torque is $T = 12.63\text{ kip}\cdot\text{in}$ ◀

(*b*) Angle of rotation of end A.

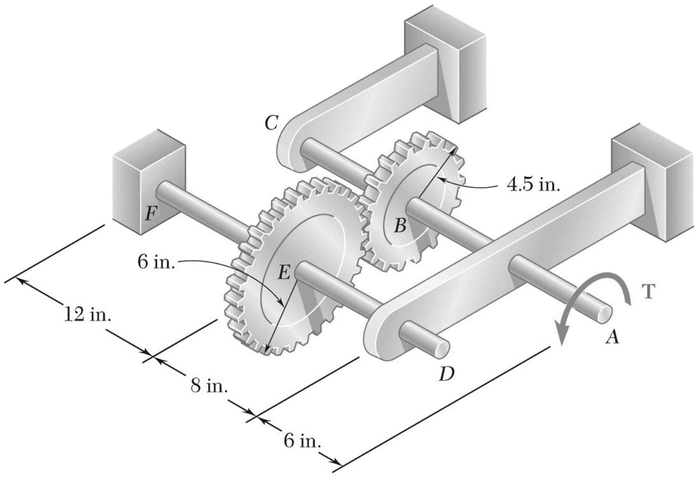
$$\phi_A = \phi_{A/D} = \phi_{A/B} + \phi_{C/D} = \sum\frac{T_i L_i}{J_i G_i} = T\sum\frac{L_i}{J_i G_i}$$

$$= (12.63 \times 10^3)\left[\frac{12}{(0.92077)(11.2 \times 10^6)} + \frac{8}{(4.1172)(5.6 \times 10^6)}\right]$$

$$= 0.01908\text{ radians}$$

$\varphi_A = 1.093°$ ◀

PROBLEM 3.41



Two shafts, each of $\frac{7}{8}$-in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 1.2 kip · in. torque is applied at A.

SOLUTION

Calculation of torques.

Circumferential contact force between gears B and E. $\quad F = \dfrac{T_{AB}}{r_B} = \dfrac{T_{EF}}{r_E} \quad T_{EF} = \dfrac{r_E}{r_B} T_{AB}$

$$T_{AB} = 1.2 \text{ kip} \cdot \text{in} = 1200 \text{ lb} \cdot \text{in}$$

$$T_{EF} = \frac{6}{4.5}(1200) = 1600 \text{ lb} \cdot \text{in}$$

Twist in shaft FE.

$$L = 12 \text{ in.}, \quad c = \frac{1}{2}d = \frac{7}{16} \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{7}{16}\right)^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$\varphi_{E/F} = \frac{TL}{GJ} = \frac{(1600)(12)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 29.789 \times 10^{-3} \text{ rad}$$

Rotation at E. $\quad \varphi_E = \varphi_{E/F} = 29.789 \times 10^{-3}$ rad

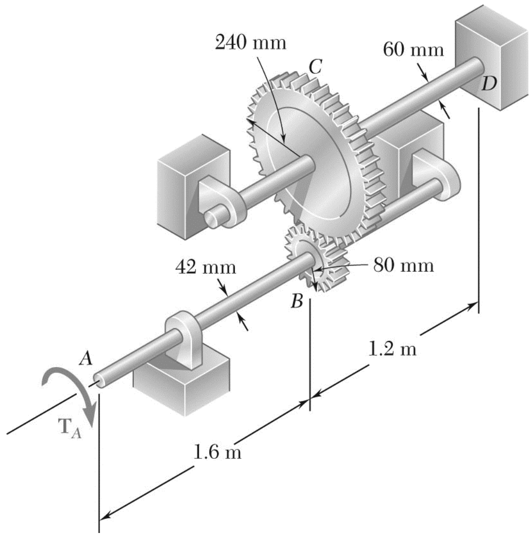
Tangential displacement at gear circle. $\quad \delta = r_E \varphi_E = r_B \varphi_B$

Rotation at B. $\quad \varphi_B = \dfrac{r_E}{r_B}\varphi_E = \dfrac{6}{4.5}(29.789 \times 10^{-3}) = 39.718 \times 10^{-3}$ rad

Twist in shaft BA. $\quad L = 8 + 6 = 14$ in. $\quad J = 57.548 \times 10^{-3}$ in^4

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(14)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 26.065 \times 10^{-3} \text{ rad}$$

Rotation at A. $\quad \varphi_A = \varphi_B + \varphi_{A/B} = 65.783 \times 10^{-3}$ rad $\qquad \varphi_A = 3.77°$ ◀



PROBLEM 3.42

Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200\,\text{N}\cdot\text{m}$.

SOLUTION

Calculation of torques:

Circumferential contact force between gears B and C. $\quad F = \dfrac{T_{AB}}{r_B} = \dfrac{T_{CD}}{r_C} \quad T_{CD} = \dfrac{r_C}{r_B}T_{AB}$

$$T_{AB} = 1200\ \text{N}\cdot\text{m} \qquad T_{CD} = \frac{240}{80}(1200) = 3600\ \text{N}\cdot\text{m}$$

Twist in shaft CD: $\quad c = \dfrac{1}{2}d = 0.030\ \text{m}, \quad L = 1.2\ \text{m}, \quad G = 77.2\times10^9\ \text{Pa}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234\times10^{-6}\ \text{m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2\times10^9)(1.27234\times10^{-9})} = 43.981\times10^{-3}\ \text{rad}$$

Rotation angle at C. $\quad \varphi_C = \varphi_{C/D} = 43.981\times10^{-3}\ \text{rad}$

Circumferential displacement at contact points of gears B and C. $\quad \delta = r_C\varphi_C = r_B\varphi_B$

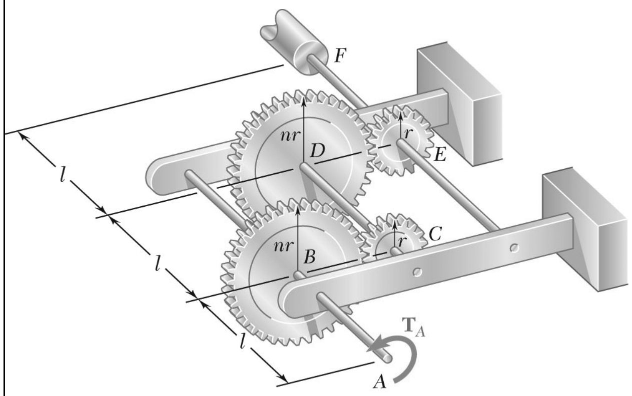
Rotation angle at B. $\quad \varphi_B = \dfrac{r_C}{r_B}\varphi_C = \dfrac{240}{80}(43.981\times10^{-3}) = 131.942\times10^{-3}\ \text{rad}$

Twist in shaft AB: $\quad c = \dfrac{1}{2}d = 0.021\text{m}, \quad L = 1.6\text{m}, \quad G = 77.2\times10^9\ \text{Pa}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.021)^4 = 305.49\times10^9\ \text{m}^4$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2\times10^9)(305.49\times10^{-9})} = 81.412\times10^{-3}\ \text{rad}$$

Rotation angle at A. $\quad \varphi_A = \varphi_B + \varphi_{A/B} = 213.354\times10^{-3}\ \text{rad} \qquad \varphi_A = 12.22° \blacktriangleleft$



PROBLEM 3.43

A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

SOLUTION

$$T_{AB} = T_A$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$$

$$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{n} = \frac{T_A}{n^2}$$

$$\varphi_E = \varphi_{EF} = \frac{T_{EF} l_{EF}}{GJ} = \frac{T_A l}{n^2 GJ}$$

$$\varphi_D = \frac{r_E}{r_D} \varphi_E = \frac{\varphi_E}{n} = \frac{T_A l}{n^3 GJ}$$

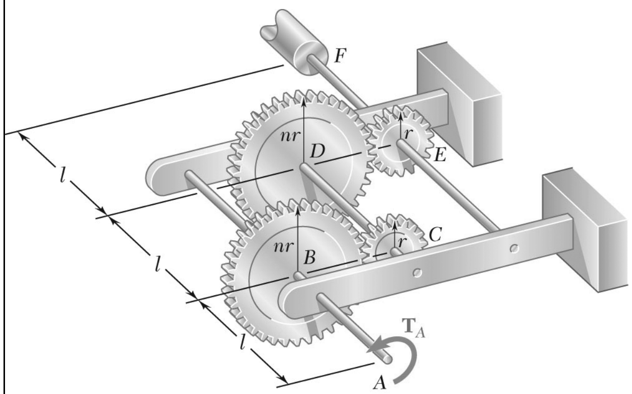
$$\varphi_{CD} = \frac{T_{CD} l_{CD}}{GJ} = \frac{T_A l}{nGJ}$$

$$\varphi_C = \varphi_D + \varphi_{CD} = \frac{T_A l}{n^3 GJ} + \frac{T_A l}{nGJ} = \frac{T_A l}{GJ}\left(\frac{1}{n^3} + \frac{1}{n}\right)$$

$$\varphi_B = \frac{r_C}{r_B} \varphi_C = \frac{\varphi_C}{n} = \frac{Tl}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2}\right)$$

$$\varphi_{AB} = \frac{T_{AB} l_{AB}}{GJ} = \frac{T_A l}{GJ}$$

$$\varphi_A = \varphi_B + \varphi_{AB} = \frac{T_A l}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2} + 1\right) \blacktriangleleft$$



PROBLEM 3.44

For the gear train described in Prob. 3.43, determine the angle through which end A rotates when $T = 5\,\text{lb}\cdot\text{in.}$, $l = 2.4\,\text{in.}$, $d = 1/16\,\text{in.}$, $G = 11.2\times10^6\,\text{psi}$, and $n = 2$.

PROBLEM 3.43 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

SOLUTION

See solution to Prob. 3.43 for development of equation for φ_A.

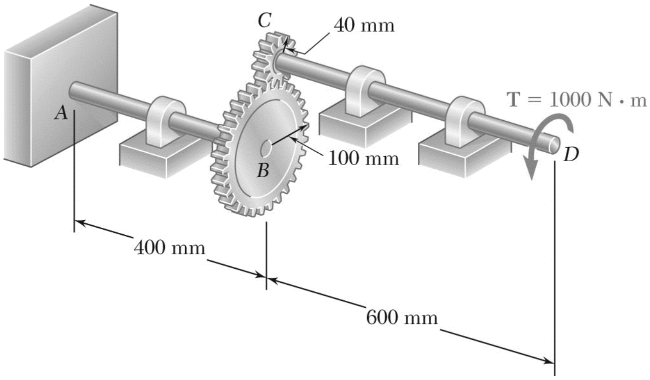
$$\varphi_A = \frac{Tl}{GJ}\left(1+\frac{1}{n^2}+\frac{1}{n^4}\right)$$

Data: $T = 5\ \text{lb}\cdot\text{in},\quad l = 2.4\ \text{in.},\quad c = \frac{1}{2}d = \frac{1}{32}\ \text{in.},\quad G = 11.2\times10^6\ \text{psi}$

$$n = 2,\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{1}{32}\right)^4 = 1.49803\times10^{-6}\,\text{in}^4$$

$$\varphi_A = \frac{(5)(2.4)}{(11.2\times10^6)(1.49803\times10^{-6})}\left(1+\frac{1}{4}+\frac{1}{16}\right) = 938.73\times10^{-3}\,\text{rad}$$

$\varphi_A = 53.8° \blacktriangleleft$



PROBLEM 3.45

The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both *AB* and *CD*. It is further required that $\tau_{max} \le 60$ MPa, and that the angle ϕ_D through which end *D* of shaft *CD* rotates not exceed 1.5°. Knowing that $G = 77$ GPa, determine the required diameter of the shafts.

SOLUTION

$$T_{CD} = T_D = 1000 \text{ N}\cdot\text{m} \qquad T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500 \text{ N}\cdot\text{m}$$

For design based on stress, use larger torque. $\quad T_{AB} = 2500 \text{ N}\cdot\text{m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2500)}{\pi(60\times 10^6)} = 26.526\times 10^{-6}\text{ m}^3$$

$$c = 29.82\times 10^{-3}\text{ m} = 29.82\text{ mm}, \quad d = 2c = 59.6\text{ mm}$$

Design based on rotation angle. $\quad \varphi_D = 1.5° = 26.18\times 10^{-3}$ rad

<u>Shaft *AB*</u>: $\quad T_{AB} = 2500 \text{ N}\cdot\text{m}, \quad L = 0.4\text{ m}$

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(2500)(0.4)}{GJ} = \frac{1000}{GJ}$$

$$Gears \begin{cases} \varphi_B = \varphi_{AB} = \dfrac{1000}{GJ} \\ \varphi_C = \dfrac{r_B}{r_C}\varphi_B = \left(\dfrac{100}{40}\right)\left(\dfrac{1000}{GJ}\right) = \dfrac{2500}{GJ} \end{cases}$$

<u>Shaft *CD*</u>: $\quad T_{CD} = 1000 \text{ N}\cdot\text{m}, \quad L = 0.6\text{ m}$

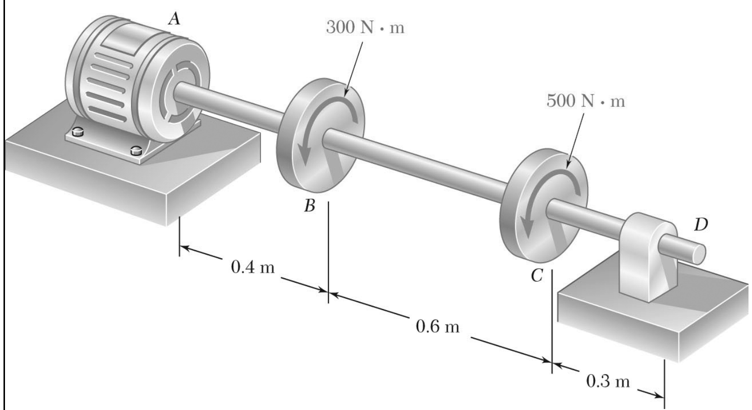
$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(1000)(0.6)}{GJ} = \frac{600}{GJ}$$

$$\varphi_D = \varphi_C + \varphi_{CD} = \frac{2500}{GJ} + \frac{600}{GJ} = \frac{3100}{GJ} = \frac{3100}{G\frac{\pi}{2}c^4}$$

$$c^4 = \frac{(2)(3100)}{\pi G\varphi_D} = \frac{(2)(3100)}{\pi(77\times 10^9)(26.18\times 10^{-3})} = 979.06\times 10^{-9}\text{ m}^4$$

$$c = 31.46\times 10^{-3}\text{ m} = 31.46\text{ mm}, \quad d = 2c = 62.9\text{ mm}$$

Design must use larger value for *d*. $\quad d = 62.9$ mm ◀



PROBLEM 3.46

The electric motor exerts a torque of $800\ \text{N}\cdot\text{m}$ on the steel shaft *ABCD* when it is rotating at constant speed. Design specifications require that the diameter of the shaft be uniform from *A* to *D* and that the angle of twist between *A* to *D* not exceed 1.5°. Knowing that $\tau_{\max} \le 60\,\text{MPa}$ and G = 77 GPa, determine the minimum diameter shaft that can be used.

SOLUTION

Torques:

$$T_{AB} = 300 + 500 = 800\ \text{N}\cdot\text{m}$$
$$T_{BC} = 500\ \text{N}\cdot\text{m}, \qquad T_{CD} = 0$$

Design based on stress. $\qquad \tau = 60\times 10^6\ \text{Pa}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(800)}{\pi(60\times 10^6)} = 8.488\times 10^{-6}\ \text{m}^3$$

$$c = 20.40\times 10^{-3}\ \text{m} = 20.40\ \text{mm}, \qquad d = 2c = 40.8\ \text{mm}$$

Design based on deformation. $\qquad \varphi_{D/A} = 1.5° = 26.18\times 10^{-3}\ \text{rad}$

$$\varphi_{D/C} = 0$$

$$\varphi_{C/B} = \frac{T_{BC}L_{BC}}{GJ} = \frac{(500)(0.6)}{GJ} = \frac{300}{GJ}$$

$$\varphi_{B/A} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(800)(0.4)}{GJ} = \frac{320}{GJ}$$

$$\varphi_{D/A} = \varphi_{D/C} + \varphi_{C/B} + \varphi_{B/A} = \frac{620}{GJ} = \frac{620}{G\frac{\pi}{2}c^4} = \frac{(2)(620)}{\pi G c^4}$$

$$c^4 = \frac{(2)(620)}{\pi G\varphi_{D/A}} = \frac{(2)(620)}{\pi(77\times 10^9)(26.18\times 10^{-3})} = 195.80\times 10^{-9}\ \text{m}^4$$

$$c = 21.04\times 10^{-3}\ \text{m} = 21.04\ \text{mm}, \qquad d = 2c = 42.1\ \text{mm}$$

Design must use larger value of *d*. $\qquad d = 42.1\ \text{mm}$ ◀

PROBLEM 3.47

The design specifications of a 2-m-long solid circular transmission shaft require that the angle of twist of the shaft not exceed 3° when a torque of $9\ \text{kN}\cdot\text{m}$ is applied. Determine the required diameter of the shaft, knowing that the shaft is made of (*a*) a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77 GPa, (*b*) a bronze with an allowable shearing of 35 MPa and a modulus of rigidity of 42 GPa.

SOLUTION

$$\varphi = 3° = 52.360\times 10^{-3}\ \text{rad},\quad T = 9\times 10^{3}\ \text{N}\cdot\text{m}\qquad L = 2.0\ \text{m}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G}\quad \therefore\quad c^4 = \frac{2TL}{\pi G\varphi}\quad \text{based on twist angle.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^2}\quad \therefore\quad c^3 = \frac{2T}{\pi\tau}\quad \text{based on shearing stress.}$$

(*a*) Steel shaft: $\tau = 90\times 10^{6}\ \text{Pa},\quad G = 77\times 10^{9}\ \text{Pa}$

Based on twist angle, $$c^4 = \frac{(2)(9\times 10^3)(2.0)}{\pi(77\times 10^9)(52.360\times 10^{-3})} = 2.842\times 10^{-6}\ \text{m}^4$$

$$c = 41.06\times 10^{-3}\ \text{m} = 41.06\ \text{mm}\qquad d = 2c = 82.1\ \text{mm}$$

Based on shearing stress, $$c^3 = \frac{(2)(9\times 10^3)}{\pi(90\times 10^6)} = 63.662\times 10^{-6}\ \text{m}^3$$

$$c = 39.93\times 10^{-3}\ \text{m} = 39.93\ \text{mm}\qquad d = 2c = 79.9\ \text{mm}$$

Required value of *d* is the larger. $d = 82.1\ \text{mm}$ ◀

(*b*) Bronze shaft: $\tau = 35\times 10^{6}\ \text{Pa},\quad G = 42\times 10^{9}\ \text{Pa}$

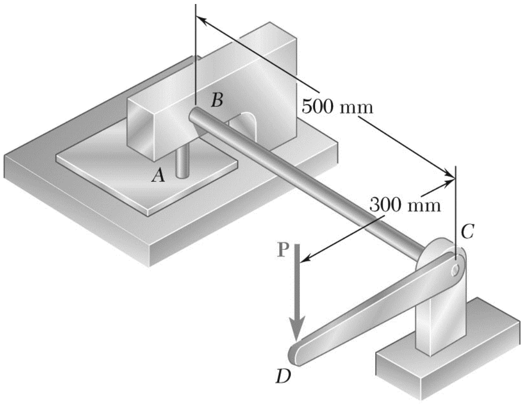
Based on twist angle, $$c^4 = \frac{(2)(9\times 10^3)(2.0)}{\pi(42\times 10^9)(52.360\times 10^{-3})} = 5.2103\times 10^{-6}\ \text{m}^4$$

$$c = 47.78\times 10^{-3}\ \text{m} = 47.78\ \text{mm}\qquad d = 2c = 95.6\ \text{mm}$$

Based on shearing stress, $$c^3 = \frac{(2)(9\times 10^3)}{\pi(35\times 10^6)} = 163.702\times 10^{-6}\ \text{m}^3$$

$$c = 54.70\times 10^{-3}\ \text{m} = 54.70\ \text{mm}\qquad d = 2c = 109.4\ \text{mm}$$

Required value of *d* is the larger. $d = 109.4\ \text{mm}$ ◀



PROBLEM 3.48

A hole is punched at A in a plastic sheet by applying a 600-N force **P** to end D of lever CD, which is rigidly attached to the solid cylindrical shaft BC. Design specifications require that the displacement of D should not exceed 15 mm from the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft BC if the shaft is made of a steel with $G = 77\,\text{GPa}$ and $\tau_{\text{all}} = 80$ MPa.

SOLUTION

Torque $\qquad T = rP = (0.300\text{ m})(600\text{ N}) = 180\text{ N}\cdot\text{m}$

Shaft diameter based on displacement limit.

$$\varphi = \frac{\delta}{r} = \frac{15\text{ mm}}{300\text{ mm}} = 0.005\text{ rad}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$c^4 = \frac{2TL}{\pi G \varphi} = \frac{(2)(180)(0.500)}{\pi(77\times 10^9)(0.05)} = 14.882\times 10^{-9}\text{ m}^4$$

$$c = 11.045\times 10^{-3}\text{ m} = 11.045\text{ m} \qquad d = 2c = 22.1\text{ mm}$$

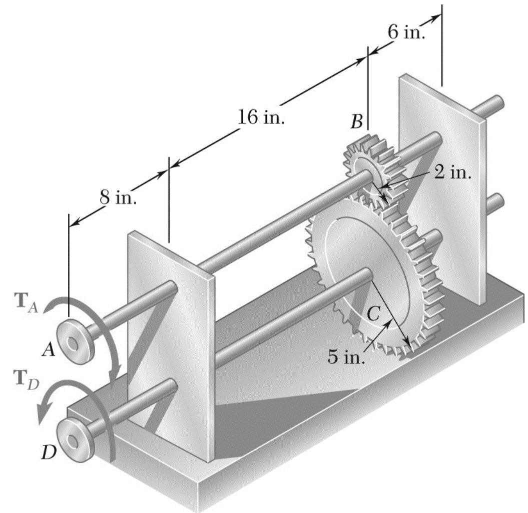
Shaft diameter based on stress.

$$\tau = 80\times 10^6\text{ Pa} \qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(180)}{\pi(80\times 10^6)} = 1.43239\times 10^{-6}\text{ m}^3$$

$$c = 11.273\times 10^{-3}\text{ m} = 11.273\text{ mm} \qquad d = 2c = 22.5\text{ mm}$$

Use the larger value to meet both limits. $\qquad d = 22.5\text{ mm}$ ◀



PROBLEM 3.49

The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts, and that the angle through which pulley A will rotate when subjected to a 2-kip · in. torque $\mathbf{T}_A$ while pulley D is held fixed will not exceed 7.5°. Determine the required diameter of the shafts if both shafts are made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{\text{all}} = 12$ ksi.

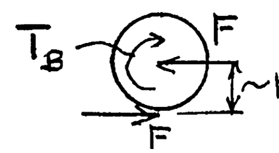
SOLUTION

Statics:

Gear B.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$r_B F - T_A = 0 \quad F = T_B / r_B$$

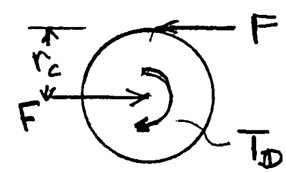


Gear C.

$$+\circlearrowleft \Sigma M_C = 0:$$

$$r_C F - T_D = 0$$

$$T_D = r_C F = \frac{r_C}{r_B} T_A = n T_B$$



$$n = \frac{r_C}{r_B} = \frac{5}{2} = 2.5$$

Torques in shafts. $\quad T_{AB} = T_A = T_B \quad T_{CD} = T_C = nT_B = nT_A$

Deformations:

$$\varphi_{C/D} = \frac{T_{CD}L}{GJ} \circlearrowleft = \frac{nT_AL}{GJ} \circlearrowleft$$

$$\varphi_{A/B} = \frac{T_{AB}L}{GJ} \circlearrowright = \frac{T_AL}{GJ} \circlearrowright$$

Kinematics:

$$\varphi_D = 0 \quad \varphi_C = \varphi_D + \varphi_{C/D} = 0 + \frac{nT_AL}{GJ} \circlearrowright$$

$$r_B\varphi_B = -r_C\varphi_B \quad \varphi_B = -\frac{r_C}{r_B}\varphi_C = -n\varphi_C \quad \varphi_B \frac{n^2T_AL}{GJ} \circlearrowright$$

$$\varphi_A = \varphi_C + \varphi_{B/C} = \frac{n^2T_AL}{GJ} + \frac{T_AL}{GJ} = \frac{(n^2+1)T_AL}{GJ} \circlearrowright$$

PROBLEM 3.49 *(Continued)*

Diameter based on stress.

Largest torque: $T_m = T_{CD} = nT_A$

$$\tau_m = \frac{T_m c}{J} = \frac{2nT_A}{\pi c^3} \qquad \tau_m = \tau_{\text{all}} = 12 \times 10^3 \text{ psi}, \quad T_A = 2 \times 10^3 \text{ lb} \cdot \text{in}$$

$$c = \sqrt[3]{\frac{2nT_A}{\pi \tau_m}} = \sqrt[3]{\frac{(2)(2.5)(2 \times 10^3)}{\pi(12 \times 10^3)}} = 0.6425 \text{ in.}, \quad d = 2c = 1.285 \text{ in.}$$

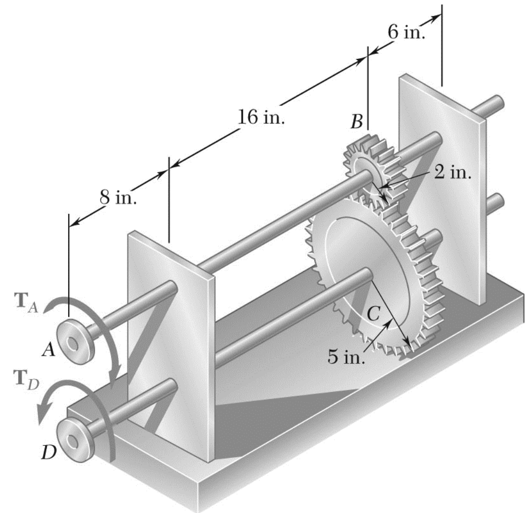
Diameter based on rotation limit.

$$\varphi = 7.5^\circ = 0.1309 \text{ rad}$$

$$\varphi = \frac{(n^2 + 1)T_A L}{GJ} = \frac{(2)(7.25)T_A L}{\pi c^4 G} \qquad L = 8 + 16 = 24 \text{ in.}$$

$$c = \sqrt[4]{\frac{(2)(7.25)T_A L}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(7.25)(2 \times 10^3)(24)}{\pi(11.2 \times 10^6)(0.1309)}} = 0.62348 \text{ in.}, \quad d = 2c = 1.247 \text{ in.}$$

Choose the larger diameter. $d = 1.285$ in. ◀



PROBLEM 3.50

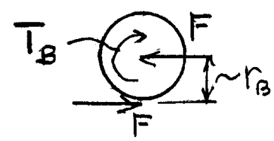
Solve Prob. 3.49, assuming that both shafts are made of a brass with $G = 5.6 \times 10^6$ psi and $\tau_{\text{all}} = 8$ ksi.

PROBLEM 3.49 The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts, and that the angle through which pulley A will rotate when subjected to a 2-kip $\cdot$ in. torque $\mathbf{T}_A$ while pulley D is held fixed will not exceed 7.5°. Determine the required diameter of the shafts if both shafts are made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{\text{all}} = 12$ ksi.

SOLUTION

<u>Statics</u>:

<u>Gear *B*</u>.

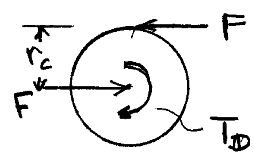


$$+\circlearrowleft \Sigma M_B = 0:$$

$$r_B F - T_A = 0 \quad F = T_B / r_B$$

<u>Gear *C*</u>.

$$+\circlearrowleft \Sigma M_C = 0:$$



$$r_C F - T_D = 0$$

$$T_D = r_C F = \frac{r_C}{r_B} T_A = n T_B$$

$$n = \frac{r_C}{r_B} = \frac{5}{2} = 2.5$$

<u>Torques in shafts</u>. $\quad T_{AB} = T_A = T_B \quad T_{CD} = T_C = nT_B = nT_A$

<u>Deformations</u>:

$$\varphi_{C/D} = \frac{T_{CD} L}{GJ} = \frac{nT_A L}{GJ}$$

$$\varphi_{A/B} = \frac{T_{AB} L}{GJ} = \frac{T_A L}{GJ}$$

<u>Kinematics</u>:

$$\varphi_D = 0 \quad \varphi_C = \varphi_D + \varphi_{C/D} = 0 + \frac{nT_A L}{GJ}$$

$$r_B \varphi_B = -r_C \varphi_B \quad \varphi_B = -\frac{r_C}{r_B}\varphi_C = -n\varphi_C \quad \varphi_B \frac{n^2 T_A L}{GJ}$$

$$\varphi_A = \varphi_C + \varphi_{B/C} = \frac{n^2 T_A L}{GJ} + \frac{T_A L}{GJ} = \frac{(n^2+1) T_A L}{GJ}$$

PROBLEM 3.50 *(Continued)*

Diameter based on stress.

Largest torque: $T_m = T_{CD} = nT_A$

$$\tau_m = \frac{T_m c}{J} = \frac{2nT_A}{\pi c^3} \quad \tau_m = \tau_{\text{all}} = 8\times 10^3 \text{ psi}, \quad T_A = 2\times 10^3 \text{ lb}\cdot\text{in}$$

$$c = \sqrt[3]{\frac{2nT_A}{\pi\tau_m}} = \sqrt[3]{\frac{(2)(2.5)(2\times 10^3)}{\pi(8\times 10^3)}} = 0.7355 \text{ in.}, \quad d = 2c = 1.471 \text{ in.}$$

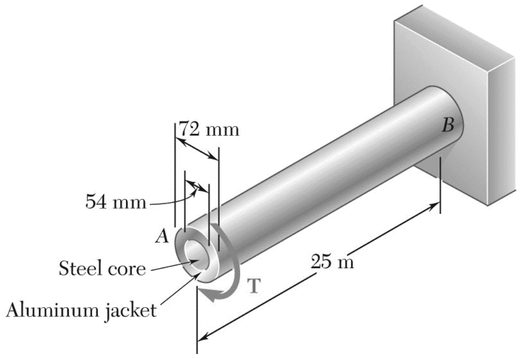
Diameter based on rotation limit.

$$\varphi = 7.5^\circ = 0.1309 \text{ rad}$$

$$\varphi = \frac{(n^2+1)T_A L}{GJ} = \frac{(2)(7.25)T_A L}{\pi c^4 G} \quad L = 8 + 16 = 24 \text{ in.}$$

$$c = \sqrt[4]{\frac{(2)(7.25)T_A L}{\pi G\varphi}} = \sqrt[4]{\frac{(2)(7.25)(2\times 10^3)(24)}{\pi(5.6\times 10^6)(0.1309)}} = 0.7415 \text{ in.}, \quad d = 2c = 1.483 \text{ in.}$$

Choose the larger diameter. $d = 1.483$ in. ◀



PROBLEM 3.51

A torque of magnitude $T = 4\text{ kN}\cdot\text{m}$ is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.

SOLUTION

Steel core: $c_1 = \frac{1}{2} d_1 = 0.027\text{ m} \quad J_1 = \frac{\pi}{2} c_1^4 = \frac{\pi}{2}(0.027)^4 = 834.79 \times 10^{-9}$

$$G_1 J_1 = (77 \times 10^9)(834.79 \times 10^{-9}) = 64.28 \times 10^3\text{ N}\cdot\text{m}^2$$

Torque carried by steel core. $T_1 = G_1 J_1 \varphi / L$

Aluminum jacket: $c_1 = \frac{1}{2} d_1 = 0.027\text{ m}, \quad c_2 = \frac{1}{2} d_2 = 0.036\text{ m}$

$$J_2 = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.036^4 - 0.027^4) = 1.80355 \times 10^{-6}\text{ m}^4$$

$$G_2 J_2 = (27 \times 10^9)(1.80355 \times 10^{-6}) = 48.70 \times 10^3\text{ N}\cdot\text{m}^2$$

Torque carried by aluminum jacket. $T_2 = G_2 J_2 \varphi / L$

Total torque: $T = T_1 + T_2 = (G_1 J_1 + G_2 J_2)\, \varphi / L$

$$\frac{\varphi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{4 \times 10^3}{64.28 \times 10^3 + 48.70 \times 10^3} = 35.406 \times 10^{-3}\text{ rad/m}$$

(*a*) <u>Maximum shearing stress in steel core.</u>

$$\tau = G_1 \gamma = G_1 c_1 \frac{\varphi}{L} = (77 \times 10^9)(0.027)(35.406 \times 10^{-3}) \qquad = 73.6 \times 10^6\text{ Pa}$$

73.6 MPa ◀

(*b*) <u>Maximum shearing stress in aluminum jacket.</u>

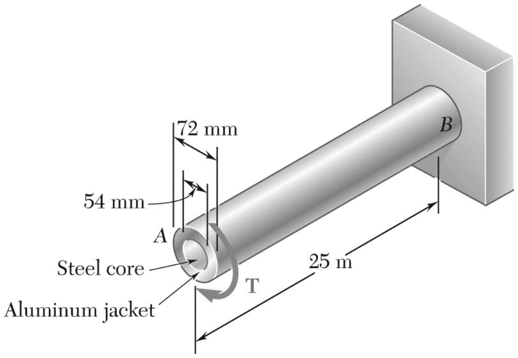
$$\tau = G_2 \gamma = G_2 c_2 \frac{\varphi}{L} = (27 \times 10^9)(0.036)(35.406 \times 10^{-3}) \qquad = 34.4 \times 10^6\text{ Pa}$$

34.4 MPa ◀

(*c*) <u>Angle of twist.</u> $\varphi = L \frac{\varphi}{L} = (2.5)(35.406 \times 10^{-3}) = 88.5 \times 10^{-3}\text{ rad}$

$\varphi = 5.07°$ ◀

PROBLEM 3.52



The composite shaft shown is to be twisted by applying a torque **T** at end *A*. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end *A* can be rotated if the following allowable stresses are not to be exceeded: $\tau_{\text{steel}} = 60\text{ MPa}$ and $\tau_{\text{aluminum}} = 45\text{ MPa}$.

SOLUTION

$$\tau_{\max} = G\gamma_{\max} = Gc_{\max}\frac{\varphi}{L}$$

$$\frac{\varphi_{\text{all}}}{L} = \frac{\tau_{\text{all}}}{Gc_{\max}} \quad \text{for each material.}$$

Steel core: $\tau_{\text{all}} = 60\times10^6\text{ Pa},\quad c_{\max} = \frac{1}{2}d = 0.027\text{ m},\quad G = 77\times10^9\text{ Pa}$

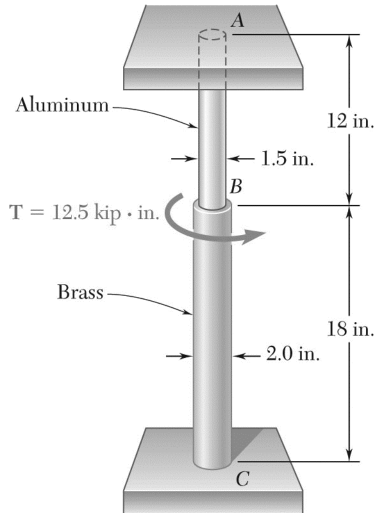
$$\frac{\varphi_{\text{all}}}{L} = \frac{60\times10^6}{(77\times10^9)(0.027)} = 28.860\times10^{-3}\text{ rad/m}$$

Aluminum Jacket: $\tau_{\text{all}} = 45\times10^6\text{ Pa},\quad c_{\max} = \frac{1}{2}d = 0.036\text{ m},\quad G = 27\times10^9\text{ Pa}$

$$\frac{\varphi_{\text{all}}}{L} = \frac{45\times10^6}{(27\times10^9)(0.036)} = 46.296\times10^{-3}\text{ rad/m}$$

Smaller value governs: $\frac{\varphi_{\text{all}}}{L} = 28.860\times10^{-3}\text{ rad/m}$

Allowable angle of twist: $\varphi_{\text{all}} = L\frac{\varphi_{\text{all}}}{L} = (2.5)\,(28.860\times10^{-3}) = 72.15\times10^{-3}\text{ rad}$ $\qquad \varphi_{\text{all}} = 4.13°$ ◀



PROBLEM 3.53

The solid cylinders AB and BC are bonded together at B and are attached to fixed supports at A and C. Knowing that the modulus of rigidity is 3.7×10^6 psi for aluminum and 5.6×10^6 psi for brass, determine the maximum shearing stress (a) in cylinder AB, (b) in cylinder BC.

SOLUTION

The torques in cylinders AB and BC are statically indeterminate. Match the rotation φ_B for each cylinder.

Cylinder AB:

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad L = 12 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$$

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(12)}{(3.7 \times 10^6)(0.49701)} = 6.5255 \times 10^{-6} T_{AB}$$

Cylinder BC:

$$c = \frac{1}{2}d = 1.0 \text{ in.} \qquad L = 18 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.0)^4 = 1.5708 \text{ in}^4$$

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(18)}{(5.6 \times 10^6)(1.5708)} = 2.0463 \times 10^{-6} T_{BC}$$

Matching expressions for φ_B

$$6.5255 \times 10^{-6} T_{AB} = 2.0463 \times 10^{-6} T_{BC}$$

$$T_{BC} = 3.1889\, T_{AB} \qquad (1)$$

Equilibrium of connection at B:

$$T_{AB} + T_{BC} - T = 0 \qquad T = 12.5 \times 10^3 \text{ lb} \cdot \text{in}$$

$$T_{AB} + T_{BC} = 12.5 \times 10^3 \qquad (2)$$

PROBLEM 3.53 *(Continued)*

Substituting (1) into (2), $4.1889\ T_{AB} = 12.5\times 10^3$

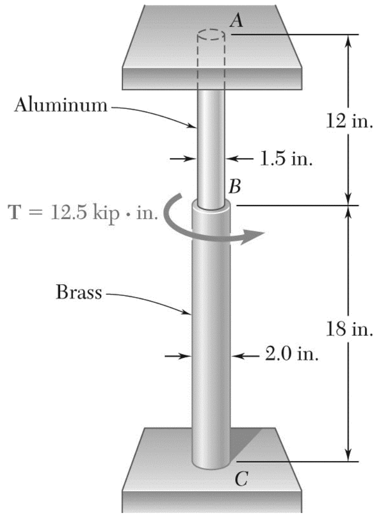
$$T_{AB} = 2.9841\times 10^3\ \text{lb}\cdot\text{in} \qquad T_{BC} = 9.5159\times 10^3\ \text{lb}\cdot\text{in}$$

(*a*) Maximum stress in cylinder *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(2.9841\times 10^3)(0.75)}{0.49701} = 4.50\times 10^3\ \text{psi} \qquad \tau_{AB} = 4.50\ \text{ksi} \blacktriangleleft$$

(*b*) Maximum stress in cylinder *BC*.

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(9.5159\times 10^3)(1.0)}{1.5708} = 6.06\times 10^3\ \text{psi} \qquad \tau_{BC} = 6.06\ \text{ksi} \blacktriangleleft$$



PROBLEM 3.54

Solve Prob. 3.53, assuming that cylinder AB is made of steel, for which $G = 11.2 \times 10^6$ psi.

PROBLEM 3.53 The solid cylinders AB and BC are bonded together at B and are attached to fixed supports at A and C. Knowing that the modulus of rigidity is 3.7×10^6 psi for aluminum and 5.6×10^6 psi for brass, determine the maximum shearing stress (a) in cylinder AB, (b) in cylinder BC.

SOLUTION

The torques in cylinders AB and BC are statically indeterminate. Match the rotation φ_B for each cylinder.

Cylinder AB: $\quad c = \frac{1}{2}d = 0.75 \text{ in.} \quad L = 12 \text{ in.} \quad J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(12)}{(11.2 \times 10^6)(0.49701)} = 2.1557 \times 10^{-6}\, T_{AB}$$

Cylinder BC. $\quad c = \frac{1}{2}d = 1.0 \text{ in.} \quad L = 18 \text{ in.} \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.0)^4 = 1.5708 \text{ in}^4$

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(18)}{(5.6 \times 10^6)(1.5708)} = 2.0463 \times 10^{-6} T_{BC}$$

Matching expressions for φ_B $\quad 2.1557 \times 10^{-6} T_{AB} = 2.0463 \times 10^{-6} T_{BC} \quad T_{BC} = 1.0535\, T_{AB}$ (1)

Equilibrium of connection at B: $\quad T_{AB} + T_{BC} - T = 0 \quad T_{AB} + T_{BC} = 12.5 \times 10^3$ (2)

Substituting (1) into (2), $\quad 2.0535\; T_{AB} = 12.5 \times 10^3$

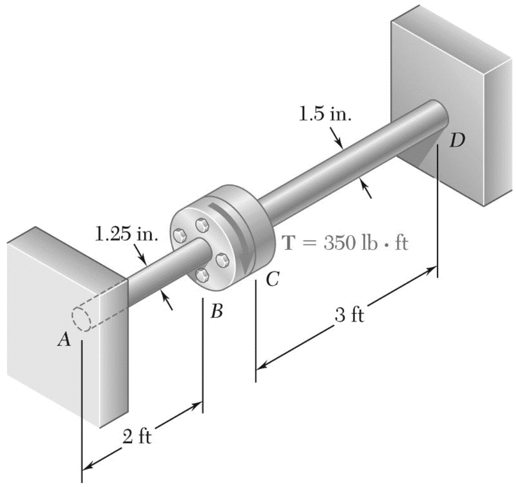
$$T_{AB} = 6.0872 \times 10^3 \text{ lb} \cdot \text{in} \qquad T_{BC} = 6.4128 \times 10^3 \text{ lb} \cdot \text{in}$$

(a) Maximum stress in cylinder AB.

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(6.0872 \times 10^3)(0.75)}{0.49701} = 9.19 \times 10^3 \text{ psi} \qquad \tau_{AB} = 9.19 \text{ ksi} \blacktriangleleft$$

(b) Maximum stress in cylinder BC.

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(6.4128 \times 10^3)(1.0)}{1.5708} = 4.08 \times 10^3 \text{ psi} \qquad \tau_{BC} = 4.08 \text{ ksi} \blacktriangleleft$$



PROBLEM 3.55

Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 11.2 \times 10^6$ psi, determine the maximum shearing stress in each shaft when a torque of **T** of magnitude 420 kip · ft is applied to the flange indicated.

PROBLEM 3.55 The torque **T** is applied to flange *B*.

SOLUTION

<u>Shaft *AB*</u>:

$$T = T_{AB}, \quad L_{AB} = 2\,\text{ft} = 24\text{ in.}, \quad c = \frac{1}{2}d = 0.625\text{ in.}$$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.625)^4 = 0.23968\text{ in}^4$$

$$\varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$$

$$T_{AB} = \frac{GJ_{AB}\varphi_B}{L_{AB}} - \frac{(11.2\times 10^6)(0.23968)}{24}\varphi_B$$

$$= 111.853\times 10^3\varphi_B$$

<u>Shaft *CD*</u>:

Applied torque: $T = 420\text{ kip}\cdot\text{ft} = 5040\text{ lb}\cdot\text{in}$

$$T = T_{CD}, \quad L_{CD} = 3\,\text{ft} = 36\text{ in.}, \quad c = \frac{1}{2}d = 0.75\text{ in.}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701\text{ in}^4$$

$$\varphi_C = \frac{T_{CD}L_{CD}}{GJ_{CD}}$$

$$T_{CD} = \frac{GJ_{CD}\,\varphi_C}{L_{CD}} = \frac{(11.2\times 10^6)(0.49701)}{36}\varphi_C = 154.625\times 10^3\varphi_C$$

PROBLEM 3.55 *(Continued)*

Clearance rotation for flange B: $\varphi'_B = 1.5° = 26.18 \times 10^{-3}$ rad

Torque to remove clearance: $T'_{AB} = (111.853 \times 10^3)(26.18 \times 10^{-3}) = 2928.3 \text{ lb} \cdot \text{in}$

Torque T'' to cause additional rotation φ'': $T'' = 5040 - 2928.3 = 2111.7 \text{ lb} \cdot \text{in}$

$$T'' = T''_{AB} + T''_{CD}$$

$$2111.7 = (111.853 \times 10^3)\varphi'' + (154.625 \times 10^3)\varphi'' \quad \therefore \quad \varphi'' = 7.923 \times 10^{-3} \text{ rad}$$

$$T''_{AB} = (111.853 \times 10^3)(7.923 \times 10^{-3}) = 886.21 \text{ lb} \cdot \text{in}$$

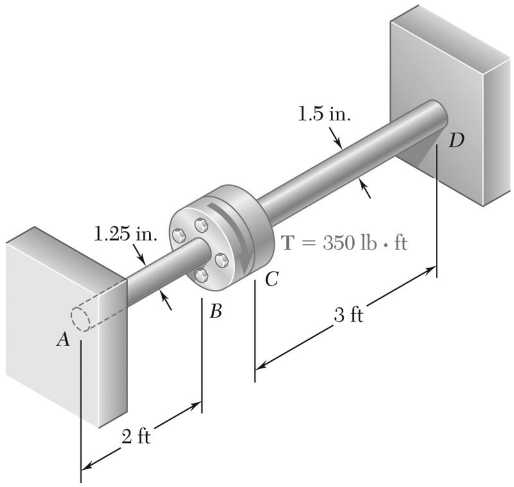
$$T''_{CD} = (154.625 \times 10^3)(7.923 \times 10^{-3}) = 1225.09 \text{ lb} \cdot \text{in}$$

Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(2928.3 + 886.21)(0.625)}{0.23968} = 9950 \text{ psi} \qquad \tau_{AB} = 9.95 \text{ ksi} \blacktriangleleft$$

Maximum shearing stress in CD.

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(1225.09)(0.75)}{0.49701} = 1849 \text{ psi} \qquad \tau_{CD} = 1.849 \text{ ksi} \blacktriangleleft$$



PROBLEM 3.56

Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 11.2 \times 10^6$ psi, determine the maximum shearing stress in each shaft when a torque of **T** of magnitude 420 kip · ft is applied to the flange indicated.

PROBLEM 3.56 The torque **T** is applied to flange *C*.

SOLUTION

Shaft *AB*:

$$T = T_{AB}, \quad L_{AB} = 2\,\text{ft} = 24\text{ in.}, \quad c = \frac{1}{2}d = 0.625\text{ in.}$$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.625)^4 = 0.23968\,\text{in}^4$$

$$\varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$$

$$T_{AB} = \frac{GJ_{AB}\varphi_B}{L_{AB}} - \frac{(11.2\times10^6)(0.23968)}{24}\varphi_B$$

$$= 111.853\times10^3\varphi_B$$

Shaft *CD*:

Applied torque: $T = 420\text{ kip}\cdot\text{ft} = 5040\text{ lb}\cdot\text{in}$

$$T = T_{CD}, \quad L_{CD} = 3\text{ ft} = 36\text{ in.}, \quad c = \frac{1}{2}d = 0.75\text{ in.}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701\,\text{in}^4$$

$$\varphi_C = \frac{T_{CD}L_{CD}}{GJ_{CD}}$$

$$T_{CD} = \frac{GJ_{CD}\,\varphi_C}{L_{CD}} = \frac{(11.2\times10^6)(0.49701)}{36}\varphi_C = 154.625\times10^3\,\varphi_C$$

Clearance rotation for flange *C*: $\varphi'_C = 1.5° = 26.18\times10^{-3}\text{ rad}$

PROBLEM 3.56 *(Continued)*

Torque to remove clearance: $T'_{CD} = (154.625\times10^3)(26.18\times10^{-3}) = 4048.1 \text{ lb}\cdot\text{in}$

Torque T'' to cause additional rotation φ'': $T'' = 5040 - 4048.1 = 991.9 \text{ lb}\cdot\text{in}$

$$T'' = T''_{AB} + T''_{CD}$$

$$991.9 = (111.853\times10^3)\varphi'' + (154.625\times10^3)\varphi'' \quad \therefore \quad \varphi'' = 3.7223\times10^{-3} \text{ rad}$$

$$T''_{AB} = (111.853\times10^{-3})(3.7223\times10^{-3}) = 416.35 \text{ lb}\cdot\text{in}$$

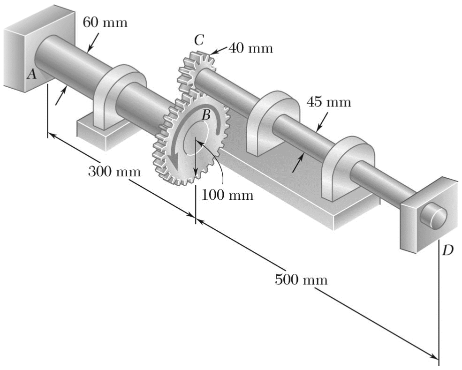
$$T''_{CD} = (154.625\times10^{-3})(3.7223\times10^{-3}) = 575.56 \text{ lb}\cdot\text{in}$$

Maximum shearing stress in *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(416.35)(0.625)}{0.23968} = 1086 \text{ psi} \qquad \tau_{AB} = 1.086 \text{ ksi} \blacktriangleleft$$

Maximum shearing stress in *CD*.

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(4048.1 + 575.56)(0.75)}{0.49701} = 6980 \text{ psi} \qquad \tau_{CD} = 6.98 \text{ ksi} \blacktriangleleft$$

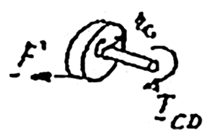


PROBLEM 3.57

Ends A and D of two solid steel shafts AB and CD are fixed, while ends B and C are connected to gears as shown. Knowing that a 4-kN · m torque **T** is applied to gear B, determine the maximum shearing stress (a) in shaft AB, (b) in shaft CD.

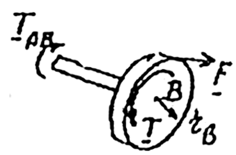
SOLUTION

Gears B and C:



$$\phi_B = \frac{r_C}{r_B}\phi_C = \frac{40}{100}\phi_C \qquad \phi_B = 0.4\,\phi_C \tag{1}$$

$$\sum M_C = 0: T_{CD} = r_C F \tag{2}$$



$$\sum M_B = 0: T - T_{AB} = r_B F \tag{3}$$

Solve (2) for F and substitute into (3):

$$T - T_{AB} = \frac{r_B}{r_C}T_{CD} \qquad T = T_{AB} + \frac{100}{40}T_{CD}$$

$$T = T_{AB} + 2.5\,T_{CD} \tag{4}$$

Shaft AB: $\quad L = 0.3 \text{ m}, \quad c = 0.030 \text{ m}$

$$\phi_B = \phi_{B/A} = \frac{T_{AB}L}{JG} = \frac{T_{AB}\,(0.3)}{\frac{\pi}{2}(0.030)^4 6} = 235.79\times 10^3\,\frac{T_{AB}}{G} \tag{5}$$

Shaft CD: $\quad L = 0.5 \text{ m}, \quad c = 0.0225 \text{ m}$

$$\phi_C = \phi_{C/D} = \frac{T_{CD}L}{JG} \quad \frac{T_{CD}\,(0.5)}{\frac{\pi}{2}(0.0225)^4 G} = 1242\times 10^3\,\frac{T_{CD}}{G} \tag{6}$$

Substitute from (5) and (6) into (1):

$$\phi_B = 0.4\phi_C: \quad 235.79\times 10^3\,\frac{T_{AB}}{G} = 0.4\times 1242\times 10^3\,\frac{T_{CD}}{G}$$

$$T_{CD} = 0.47462 = T_{AB} \tag{7}$$

PROBLEM 3.57 *(Continued)*

Substitute for T_{CD} from (7) into (4):

$$T = T_{AB} + 2.5\,(0.47462\,T_{AB}) \qquad T = 2.1865\,T_{AB} \quad (8)$$

For $T = 4\ \text{kN}\cdot\text{m}$, Eq. (8) yields

$$4000\,\text{N}\cdot\text{m} = 2.1865 T_{AB} \qquad T_{AB} = 1829.4\,\text{N}\cdot\text{m}$$

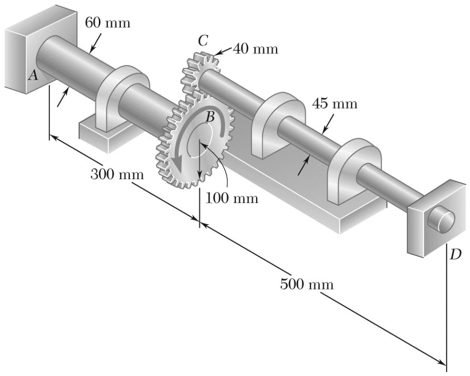
Substitute into (7): $\qquad T_{CD} = 0.47462(1829.4) = 868.3\,\text{N}\cdot\text{m}$

(*a*) <u>Stress in *AB*</u>:

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{2}{\pi}\frac{T_{AB}}{c^3} = \frac{2}{\pi}\frac{1829.4}{(0.030)^3} = 43.1\times 10^6 \qquad \tau_{AB} = 43.1\ \text{MPa} \blacktriangleleft$$

(*b*) <u>Stress in *CD*</u>:

$$\tau_{CD} = \frac{T_{CD}c}{J} = \frac{2}{\pi}\frac{T_{CD}}{c^3} = \frac{2}{\pi}\frac{868.3}{(0.0225)^3} = 48.5\times 10^6 \qquad \tau_{CD} = 48.5\ \text{MPa} \blacktriangleleft$$



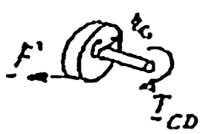
PROBLEM 3.58

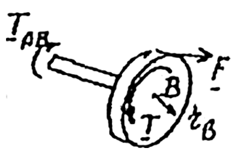
Ends A and D of the two solid steel shafts AB and CD are fixed, while ends B and C are connected to gears as shown. Knowing that the allowable shearing stress is 50 MPa in each shaft, determine the largest torque **T** that may be applied to gear B.

SOLUTION

<u>Gears B and C:</u>

$$\phi_B = \frac{r_C}{r_B}\phi_C = \frac{40}{100}\phi_C \qquad \phi_B = 0.4\,\phi_C \tag{1}$$





$$\Sigma M_C = 0:\ T_{CD} = r_C F \tag{2}$$

$$\Sigma M_B = 0:\ T - T_{AB} = r_B F \tag{3}$$

Solve (2) for F and substitute into (3):

$$T - T_{AB} = \frac{r_B}{r_C} T_{CD} \qquad T = T_{AB} + \frac{100}{40} T_{CD}$$

$$T = T_{AB} + 2.5\, T_{CD} \tag{4}$$

<u>Shaft AB:</u> $\qquad L = 0.3\ \text{m}, \quad c = 0.030\ \text{m}$

$$\phi_B = \phi_{B/A} = \frac{T_{AB}L}{JG} = \frac{T_{AB}\,(0.3)}{\frac{\pi}{2}(0.030)^4\, G} = 235.77\times 10^3\, \frac{T_{AB}}{G} \tag{5}$$

<u>Shaft CD:</u> $\qquad L = 0.5\ \text{m}, \quad c = 0.0225\ \text{m}$

$$\phi_C = \phi_{C/D} = \frac{T_{CD}L}{JG} = \frac{T_{CD}\,(0.5)}{\frac{\pi}{2}(0.0225)^4\, G} = 1242\times 10^3\, \frac{T_{CD}}{G} \tag{6}$$

Substitute from (5) and (6) into (1) :

$$\phi_B = 0.4\,\phi_C: \quad 235.79 \times 10^3 \frac{T_{AB}}{G} = 0.4 \times 1242 \times 10^3 \frac{T_{CD}}{G}$$

$$T_{CD} = 0.47462\, T_{AB} \tag{7}$$

Substitute for T_{CD} from (7) into (4):

$$T = T_{AB} + 2.5\,(0.47462\, T_{AB}) \qquad T = 2.1865\, T_{AB} \tag{8}$$

Solving (7) for T_{AB} and substituting into (8),

$$T = 2.1865\left(\frac{T_{CD}}{0.47462}\right) \qquad T = 4.6068\, T_{CD} \tag{9}$$

Stress criterion for shaft *AB*:

$$\tau_{AB} = \tau_{\text{all}} = 50 \text{ MPa}:$$

$$\tau_{AB} = \frac{T_{AB}c}{J} \quad T_{AB} = \frac{J}{c}\tau_{AB} = \frac{\pi}{2}c^3\tau_{AB}$$

$$= \frac{\pi}{2}(0.030 \text{ m})^3(50 \times 10^6 \text{ Pa}) = 2120.6 \text{ N} \cdot \text{m}$$

From (8): $\quad T = 2.1865(2120.6 \text{ N} \cdot \text{m}) = 4.64 \text{ kN} \cdot \text{m}$

Stress criterion for shaft *CD*:

$$\tau_{CD} = \tau_{\text{all}} = 50 \text{ MPa}:$$

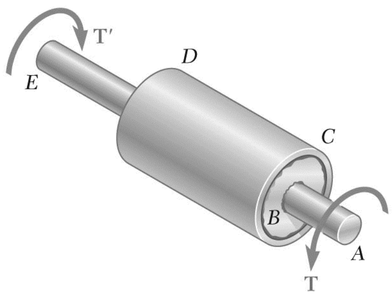
$$\tau_{CD} = \frac{T_{CD}c}{J} \quad T_{CD} = \frac{\pi}{2}c^3\tau_{CD} = \frac{\pi}{2}(0.0225 \text{ m})^3(50 \times 10^6 \text{ Pa})$$

$$= 894.62 \text{ N} \cdot \text{m}$$

From (7): $\quad T = 4.6068(894.62 \text{ N} \cdot \text{m}) = 4.12 \text{ kN} \cdot \text{m}$

The smaller value for *T* governs.

$T = 4.12 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 3.59

The steel jacket *CD* has been attached to the 40-mm-diameter steel shaft *AE* by means of *rigid* flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If 500 N · m torques are applied as shown, determine the maximum shearing stress in the jacket.

SOLUTION

Solid shaft: $$c = \frac{1}{2}d = 0.020 \text{ m}$$

$$J_S = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Jacket: $$c_2 = \frac{1}{2}d = 0.040 \text{ m}$$

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.040^4 - 0.036^4)$$

$$= 1.3829 \times 10^{-6} \text{ m}^4$$

Torque carried by shaft. $$T_S = GJ_S\,\varphi/L$$

Torque carried by jacket. $$T_J = GJ_J\,\varphi/L$$

Total torque. $$T = T_S + T_J = (J_S + J_J)G\varphi/L \quad \therefore \quad \frac{G\varphi}{L} = \frac{T}{J_S + J_J}$$

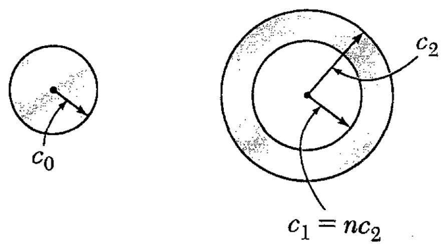
$$T_J = \frac{J_J}{J_S + J_J}T = \frac{(1.3829 \times 10^{-6})(500)}{1.3829 \times 10^{-6} + 251.33 \times 10^{-6}} = 423.1 \text{ N} \cdot \text{m}$$

Maximum shearing stress in jacket.

$$\tau = \frac{T_J c_2}{J_J} = \frac{(423.1)(0.040)}{1.3829 \times 10^{-6}} = 12.24 \times 10^6 \text{ Pa}$$

12.24 MPa ◀

PROBLEM 3.60



A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by n the ratio c_1/c_2, show that the ratio T_s/T_h of the torque T_s in the solid shaft to the torque T_h in the hollow shaft is (a) $\sqrt{(1-n^2)}/(1+n^2)$ if the maximum shearing stress is the same in each shaft, (b) $(1-n)/(1+n^2)$ if the angle of twist is the same for each shaft.

SOLUTION

For equal weight and length, the areas are equal.

$$\pi c_0^2 = \pi\left(c_2^2 - c_1^2\right) = \pi c_2^2(1-n^2) \qquad \therefore \qquad c_0 = c_2(1-n^2)^{1/2}$$

$$J_s = \frac{\pi}{2}c_0^4 = \frac{\pi}{2}c_2^4(1-n^2)^2 \qquad\qquad J_h = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}c_2^4(1-n^4)$$

(a) For equal stresses.

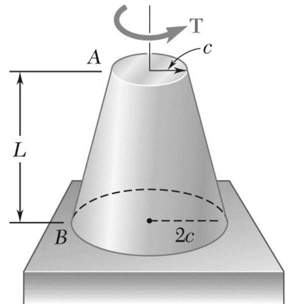
$$\tau = \frac{T_s c_0}{J_s} = \frac{T_h c_2}{J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s c_2}{J_h c_0} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2 c_2}{\frac{\pi}{2}c_2^4(1-n^4)c_2(1-n^2)^{1/2}} = \frac{1-n^2}{(1+n^2)(1-n^2)^{1/2}} = \frac{(1-n^2)^{1/2}}{1+n^2} \quad \blacktriangleleft$$

(b) For equal angles of twist.

$$\varphi = \frac{T_s L}{GJ_s} = \frac{T_h L}{GJ_h}$$

$$\frac{T_s}{T_h} = \frac{J_s}{J_h} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2}{\frac{\pi}{2}c_2^4(1-n^4)} = \frac{(1-n^2)^2}{1-n^4} = \frac{1-n^2}{1+n^2} \quad \blacktriangleleft$$



PROBLEM 3.61

A torque **T** is applied as shown to a solid tapered shaft *AB*. Show by integration that the angle of twist at *A* is

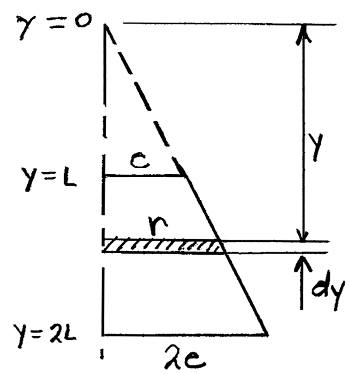
$$\phi = \frac{7TL}{12\pi Gc^4}$$

SOLUTION

Introduce coordinate y as shown.

$$r = \frac{cy}{L}$$

Twist in length dy:



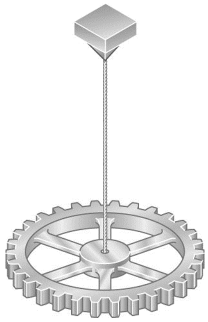
$$d\varphi = \frac{Tdy}{GJ} = \frac{Tdy}{G\frac{\pi}{2}r^4} = \frac{2TL^4dy}{\pi Gc^4y^4}$$

$$\varphi = \int_L^{2L} \frac{2TL^4}{\pi Gc^4}\frac{dy}{y^4} = \frac{2TL}{\pi Gc^4}\int_L^{2L}\frac{dy}{y^4}$$

$$= \frac{2TL^4}{\pi Gc^4}\left\{-\frac{1}{3y^3}\right\}_L^{2L} = \frac{2TL^4}{\pi Gc^4}\left\{-\frac{1}{24L^3} + \frac{1}{3L^3}\right\}$$

$$= \frac{2TL^4}{\pi Gc^4}\left\{\frac{7}{24L^3}\right\} = \frac{7TL}{12\pi Gc^4}$$

◀



PROBLEM 3.62

The mass moment of inertia of a gear is to be determined experimentally by using a torsional pendulum consisting of a 6-ft steel wire. Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of the wire for which the torsional spring constant will be 4.27 lb · ft/rad.

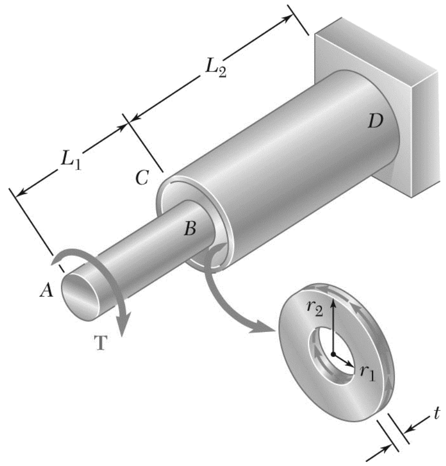
SOLUTION

Torsion spring constant $K = 4.27 \text{ lb} \cdot \text{ft/rad} = 51.24 \text{ lb} \cdot \text{in/rad}$

$$K = \frac{T}{\varphi} = \frac{T}{TL/GJ} = \frac{GJ}{L} = \frac{\pi G c^4}{2L}$$

$$c^4 = \frac{2LK}{\pi G} = \frac{(2)(72)(51.24)}{\pi(11.2 \times 10^6)} = 209.7 \times 10^{-6} \text{ in}^4$$

$c = 0.1203$ in. $\qquad d = 2c = 0.241$ in. ◀



PROBLEM 3.63

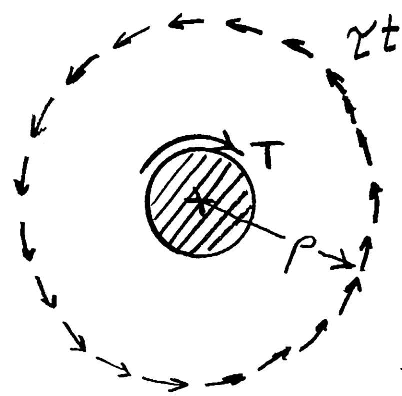
An annular plate of thickness t and modulus G is used to connect shaft AB of radius r_1 to tube CD of radius r_2. Knowing that a torque **T** is applied to end A of shaft AB and that end D of tube CD is fixed, (a) determine the magnitude and location of the maximum shearing stress in the annular plate, (b) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

$$\varphi_{BC} = \frac{T}{4\pi Gt}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

SOLUTION

Use a free body consisting of shaft AB and an inner portion of the plate BC, the outer radius of this portion being ρ.

The force per unit length of circumference is τt.



$$\Sigma M = 0$$

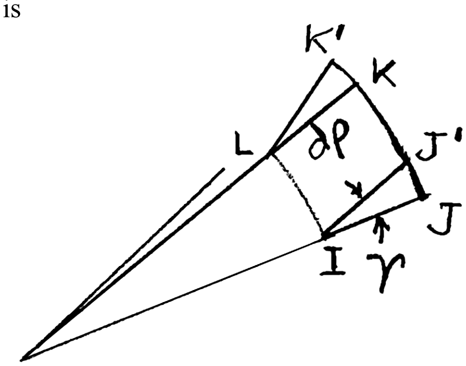
$$\tau t(2\pi\rho)\rho - T = 0$$

$$\tau = \frac{T}{2\pi t\rho^2}$$

(a) Maximum shearing stress occurs at $\rho = r_1$ $\qquad \tau_{\max} = \dfrac{T}{2\pi t r_1^2}$ ◀

Shearing strain: $\qquad \gamma = \dfrac{\tau}{G} = \dfrac{T}{2\pi G T\rho^2}$

The relative circumferential displacement in radial length $d\rho$ is



$$d\delta = \gamma d\rho = \rho\, d\varphi$$

$$d\varphi = \gamma\frac{d\rho}{\rho}$$

$$d\varphi = \frac{T}{2\pi G t\rho^2}\frac{d\rho}{\rho} = \frac{T}{2\pi G T\rho^3}d\rho$$

(b) $$\varphi_{B/C} = \int_{r_1}^{r_2}\frac{T\, d\rho}{2\pi G t\rho^3} = \frac{T}{2\pi G t}\int_{r_1}^{r_2}\frac{d\rho}{\rho^3} = \frac{T}{2\pi G t}\left\{-\frac{1}{2\rho^2}\right\}\Bigg|_{r_1}^{r_2}$$

$$= \frac{T}{2\pi G t}\left\{-\frac{1}{2r_2^2} + \frac{1}{2r_1^2}\right\} = \frac{T}{4\pi G t}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2}\right\}$$ ◀

PROBLEM 3.64

Determine the maximum shearing stress in a solid shaft of 12-mm diameter as it transmits 2.5 kW at a frequency of (*a*) 25 Hz, (*b*) 50 Hz.

SOLUTION

$$c = \frac{1}{2}d = 6 \text{ mm} = 0.006 \text{ m} \qquad P = 2.5 \text{ kW} = 2500 \text{ W}$$

(*a*) $\underline{f = 25 \text{ Hz}}$

$$T = \frac{P}{2\pi f} = \frac{2500}{2\pi(25)} = 15.9155 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(15.9155)}{\pi(0.006)^3} = 46.9 \times 10^6 \text{ Pa} \qquad \tau = 46.9 \text{ MPa} \blacktriangleleft$$

(*b*) $\underline{f = 50 \text{ Hz}}$

$$T = \frac{2500}{2\pi(50)} = 7.9577 \text{ N}\cdot\text{m}$$

$$\tau = \frac{2(7.9577)}{\pi(0.006)^3} = 23.5 \times 10^6 \text{ Pa} \qquad \tau = 23.5 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.65

Determine the maximum shearing stress in a solid shaft of 1.5-in. diameter as it transmits 75 hp at a speed of (*a*) 750 rpm, (*b*) 1500 rpm.

SOLUTION

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad P = 75 \text{ hp} = (75)(6600) = 495 \times 10^3 \text{ lb} \cdot \text{in/s}$$

(*a*) $$f = \frac{750}{60} = 12.5 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{495 \times 10^3}{2\pi(12.5)} = 6.3025 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(6.3025 \times 10^3)}{\pi (0.75)^3} = 9.51 \times 10^3 \text{ psi}$$

$\tau = 9.51$ ksi ◀

(*b*) $$f = \frac{1500}{60} = 25 \text{ Hz}$$

$$T = \frac{495 \times 10^3}{2\pi(25)} = 3.1513 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{(2)(3.1513 \times 10^3)}{\pi(0.75)^3} = 4.76 \times 10^3 \text{ psi}$$

$\tau = 4.76$ ksi ◀

PROBLEM 3.66

Design a solid steel shaft to transmit 0.375 kW at a frequency of 29 Hz, if the shearing stress in the shaft is not to exceed 35 MPa.

SOLUTION

$$\tau_{\text{all}} = 35 \times 10^6 \text{ Pa} \quad P = 0.375 \times 10^3 \text{ W} \quad f = 29 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{0.375 \times 10^3}{2\pi(29)} = 2.0580 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad \therefore \quad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2.0580)}{\pi(35 \times 10^6)} = 37.43 \times 10^{-9} \text{ m}^3$$

$$c = 3.345 \times 10^{-3} \text{ m} = 3.345 \text{ mm}$$

$$d = 2c$$

$d = 6.69$ mm ◀

PROBLEM 3.67

Design a solid steel shaft to transmit 100 hp at a speed of 1200 rpm, if the maximum shearing stress is not to exceed 7500 psi.

SOLUTION

$$\tau_{\text{all}} = 7500\,\text{psi} \quad P = 100\,\text{hp} = 660 \times 10^3\,\text{lb} \cdot \text{in/s}$$

$$f = \frac{1200}{60} = 20\,\text{Hz} \qquad T = \frac{P}{2\pi f} = \frac{660 \times 10^3}{2\pi(20)} = 5.2521 \times 10^3\,\text{lb} \cdot \text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(5.2521 \times 10^3)}{\pi(7500)} = 0.4458\,\text{in}^3$$

$$c = 0.7639\,\text{in.} \qquad d = 2c \qquad\qquad d = 1.528\,\text{in.} \blacktriangleleft$$

PROBLEM 3.68

Determine the required thickness of the 50-mm tubular shaft of Example 3.07 if is to transmit the same power while rotating at a frequency of 30 Hz.

SOLUTION

From Example 3.07, $P = 100\,\text{kW} = 100 \times 10^3\ \text{W}$

$$\tau_{\text{all}} = 60\,\text{MPa} = 60 \times 10^6\ \text{Pa} \qquad c_2 = \frac{1}{2}d = 25\ \text{mm} = 0.025\ \text{m}$$

$$f = 30\,\text{Hz}$$

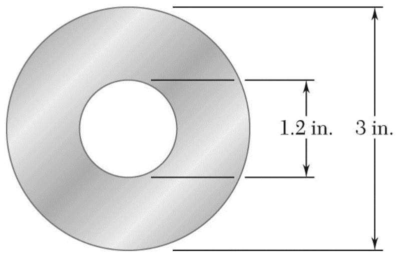
$$T = \frac{P}{2\pi f} = 530.52\ \text{N} \cdot \text{m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) \qquad \tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.025^4 - \frac{(2)(530.52)(0.025)}{\pi(60 \times 10^6)} = 249.90 \times 10^{-9}\ \text{m}^4$$

$$c_1 = 22.358 \times 10^{-3}\ \text{m} = 22.358\ \text{mm}$$

$$t = c_2 - c_1 = 25\ \text{mm} - 22.358\ \text{mm} = 2642\ \text{mm} \qquad t = 2.64\ \text{mm} \blacktriangleleft$$



PROBLEM 3.69

While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is $2°$ in a 12-ft length. Using $G = 11.2 \times 10^6$ psi, determine the power being transmitted.

SOLUTION

$$\varphi = 2° = 34.907 \times 10^{-3} \text{ rad} \qquad L = 12 \text{ ft} = 144 \text{ in.}$$

$$c_2 = \frac{1}{2} d_o = 1.5 \text{ in.} \qquad c_1 = \frac{1}{2} d_i = 0.6 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 0.6^4) = 7.7486 \text{ in}^4$$

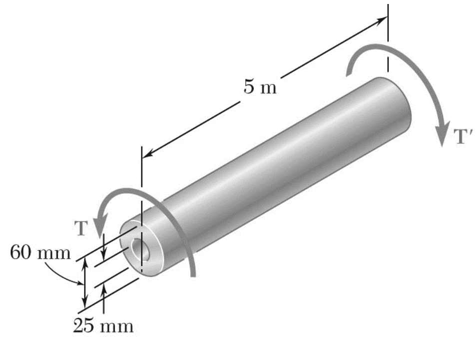
$$f = \frac{120}{60} = 2 \text{ Hz}$$

$$T = \frac{GJ\varphi}{L} = \frac{(11.2 \times 10^6)(7.7486)(34.907 \times 10^{-3})}{144} = 21.037 \times 10^3 \text{ lb} \cdot \text{in}$$

$$P = 2\pi f T = 2\pi(2)(21.037 \times 10^3) = 264.36 \times 10^3 \text{ lb} \cdot \text{in/s}$$

Since 1 hp = 6600 lb · in/s, $P = 40.1$ hp ◀

PROBLEM 3.70



The hollow steel shaft shown ($G = 77.2$ GPa, $\tau_{all} = 50$ MPa) rotates at 240 rpm. Determine (*a*) the maximum power that can be transmitted, (*b*) the corresponding angle of twist of the shaft.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4]$$

$$= 1.234 \times 10^6 \text{mm}^4 = 1.234 \times 10^{-6} \text{m}^4$$

$$\tau_m = 50 \times 10^6 \text{ Pa}$$

$$\tau_m = \frac{Tc}{J} \quad T = \frac{\tau_m J}{c} = \frac{(50 \times 10^6)(1.234 \times 10^{-6})}{30 \times 10^{-3}} = 2056.7 \text{ N} \cdot \text{m}$$

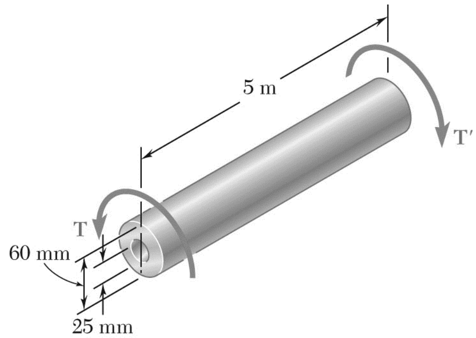
Angular speed. $f = 240 \text{ rpm} = 4 \text{ rev/sec} = 4 \text{ Hz}$

(*a*) Power being transmitted. $P = 2\pi f T = 2\pi(4)(2056.7) = 51.7 \times 10^3 \text{ W}$

$P = 51.7 \text{ kW}$ ◀

(*b*) Angle of twist. $\varphi = \dfrac{TL}{GJ} = \dfrac{(2056.7)(5)}{(77.2 \times 10^9)(1.234 \times 10^{-6})} = 0.1078 \text{ rad}$

$\varphi = 6.17°$ ◀



PROBLEM 3.71

As the hollow steel shaft shown rotates at 180 rpm, a stroboscopic measurement indicates that the angle of twist of the shaft is 3°. Knowing that $G = 77.2\text{ GPa}$, determine (*a*) the power being transmitted, (*b*) the maximum shearing stress in the shaft.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = 30\text{ mm}$$

$$c_1 = \frac{1}{2}d_1 = 12.5\text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4)]$$

$$= 1.234\times 10^6\text{ mm}^4 = 1.234\times 10^{-6}\text{ m}^4$$

$$\varphi = 3° = 0.05236\text{ rad}$$

$$\varphi = \frac{TL}{GJ}$$

$$T = \frac{GJ\varphi}{L} = \frac{(77.2\times 10^9)(1.234\times 10^{-6})(0.0536)}{5} = 997.61\text{ N}\cdot\text{m}$$

Angular speed: $f = 180\text{ rpm} = 3\text{ rev/sec} = 3\text{ Hz}$

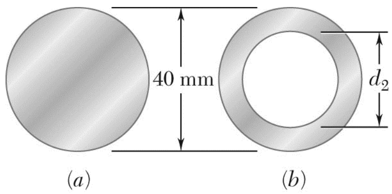
(*a*) Power being transmitted. $P = 2\pi f T = 2\pi(3)(997.61) = 18.80\times 10^3\text{ W}$

$P = 18.80\text{ kW}$ ◀

(*b*) Maximum shearing stress. $\tau_m = \dfrac{Tc_2}{J} = \dfrac{(997.61)(30\times 10^{-3})}{1.234\times 10^{-6}}$

$= 24.3\times 10^6\text{ Pa}$ $\tau_m = 24.3\text{ MPa}$ ◀

PROBLEM 3.72



The design of a machine element calls for a 40-mm-outer-diameter shaft to transmit 45 kW. (*a*) If the speed of rotation is 720 rpm, determine the maximum shearing stress in shaft *a*. (*b*) If the speed of rotation can be increased 50% to 1080 rpm, determine the largest inner diameter of shaft *b* for which the maximum shearing stress will be the same in each shaft.

SOLUTION

(*a*) $f = \dfrac{720}{60} = 12 \text{ Hz}$

$P = 45 \text{ kW} = 45 \times 10^3 \text{W}$

$T = \dfrac{P}{2\pi f} = \dfrac{45 \times 10^3}{2\pi(12)} = 596.83 \text{ N} \cdot \text{m}$

$c = \dfrac{1}{2}d = 20 \text{ mm} = 0.020 \text{ m}$

$\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} = \dfrac{(2)(596.83)}{\pi(0.020)^3} = 47.494 \times 10^6 \text{ Pa}$ $\qquad \tau_{\text{max}} = 47.5 \text{ MPa}$ ◀

(*b*) $f = \dfrac{1080}{60} = 18 \text{ Hz}$

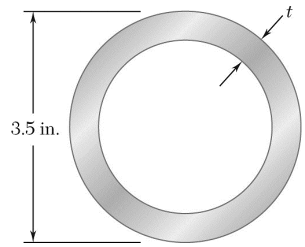
$T = \dfrac{45 \times 10^3}{2\pi(18)} = 397.89 \text{ N} \cdot \text{m}$

$\tau = \dfrac{Tc_2}{J} = \dfrac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

$c_1^4 = c_2^4 - \dfrac{2Tc_2}{\pi\tau}$

$c_1^4 = 0.020^4 - \dfrac{(2)(397.89)(0.020)}{\pi(47.494 \times 10^6)} = 53.333 \times 10^{-9}$

$c_1 = 15.20 \times 10^{-3} \text{m} = 15.20 \text{ mm}$ $\qquad d_2 = 2c_1 = 30.4 \text{ mm}$ ◀



PROBLEM 3.73

A steel pipe of 3.5-in. outer diameter is to be used to transmit a torque of 3000 lb · ft without exceeding an allowable shearing stress of 8 ksi. A series of 3.5-in.-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 0.25 in. to 0.50 in. in 0.0625-in. increments, choose the lightest pipe that can be used.

SOLUTION

$$T = 3000 \text{ lb} \cdot \text{ft} = 36 \times 10^3 \text{ lb} \cdot \text{in}$$

$$c_2 = \frac{1}{2} d_o = 1.75 \text{ in.}$$

$$\tau = \frac{Tc_2}{J} = \frac{2\text{T}c_2}{\pi\left(c_2^4 - c_1^4\right)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 1.75^4 - \frac{(2)(36 \times 10^3)(1.75)}{\pi(8 \times 10^3)} = 4.3655 \text{ in}^4$$

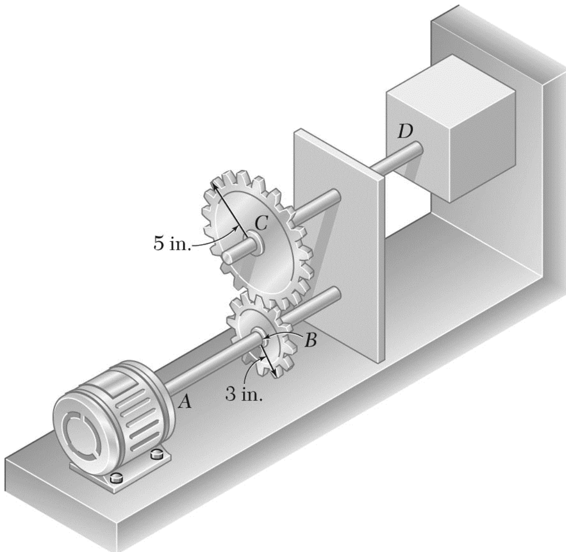
$$c_1 = 1.4455 \text{ in.}$$

Required minimum thickness: $t = c_2 - c_1$

$$t = 1.75 - 1.4455 = 0.3045 \text{ in.}$$

Available thicknesses: 0.25 in., 0.3125 in., 0.375 in., etc.

Use $t = 0.3125$ in. ◀



PROBLEM 3.74

The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A,* operating at a speed of 1260 rpm, to a machine tool at *D*. Knowing that the maximum allowable shearing stress is 8 ksi, determine the required diameter (*a*) of shaft *AB*, (*b*) of shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb} \cdot \text{in/sec}$$

$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$\tau = 8 \text{ ksi} = 8 \times 10^3 \text{ psi}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{105.6 \times 10^3}{2\pi(21)} = 800.32 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c = \sqrt[3]{\frac{2T}{\pi\tau}}$$

$$c = \sqrt[3]{\frac{(2)(800.32)}{\pi(8 \times 10^3)}} = 0.399 \text{ in.}$$

$$d_{AB} = 2c = 0.799 \text{ in.}$$

$d_{AB} = 0.799$ in. ◀

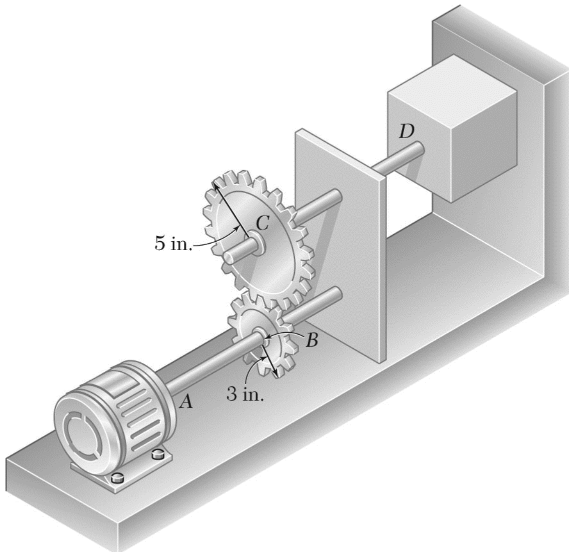
(*b*) Shaft *CD*:

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb} \cdot \text{in}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(1.33387 \times 10^3)}{\pi(8 \times 10^3)}} = 0.473 \text{ in.}$$

$$d_{CD} = 2c = 0.947 \text{ in.}$$

$d_{CD} = 0.947$ in. ◀



PROBLEM 3.75

The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A* operating at a speed of 1260 rpm to a machine tool at *D*. Knowing that each shaft has a diameter of 1 in., determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb} \cdot \text{in/sec}$$

$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{105.6 \times 10^3}{2\pi(21)} = 800.32 \text{ lb} \cdot \text{in}$$

$$c = \frac{1}{2}d = 0.5 \text{ in.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$= \frac{(2)(800.32)}{\pi(0.5)^3} = 4.08 \times 10^3 \text{ psi}$$

$\tau_{AB} = 4.08$ ksi ◀

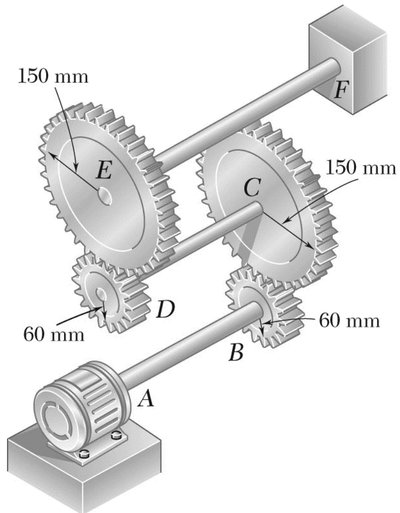
(*b*) Shaft *CD*:

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{2T}{\pi c^3} = \frac{(2)(1.33387 \times 10^3)}{\pi(0.5)^3} = 6.79 \times 10^3 \text{psi}$$

$\tau_{CD} = 6.79$ ksi ◀

PROBLEM 3.76



Three shafts and four gears are used to form a gear train that will transmit 7.5 kW from the motor at A to a machine tool at F. (Bearings for the shafts are omitted in the sketch.) Knowing that the frequency of the motor is 30 Hz and that the allowable stress for each shaft is 60 MPa, determine the required diameter of each shaft.

SOLUTION

$$P = 7.5\text{ kW} = 7.5\times 10^3\text{ W} \qquad \tau_{\text{all}} = 60\text{ MPa} = 60\times 10^6\text{ Pa}$$

Shaft AB:

$$f_{AB} = 30\text{ Hz} \qquad T_{AB} = \frac{P}{2\pi f_{AB}} = \frac{7.5\times 10^3}{2\pi(30)} = 39.789\text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc_{AB}}{J_{AB}} = \frac{2T}{\pi c_{AB}^3} \quad \therefore \quad c_{AB}^3 = \frac{2T}{\pi\tau}$$

$$c_{AB}^3 = \frac{(2)(39.789)}{\pi(60\times 10^6)} = 422.17\times 10^{-9}\text{ m}^3$$

$$c_{AB} = 7.50\times 10^{-3}\text{ m} = 7.50\text{ mm} \qquad d_{AB} = 2c_{AB} = 15.00\text{ mm} \blacktriangleleft$$

Shaft CD:

$$f_{CD} = \frac{r_B}{r_C} f_{AB} = \frac{60}{150}(30) = 12\text{ Hz} \qquad T_{CD} = \frac{P}{2\pi f_{CD}} = \frac{7.5\times 10^3}{2\pi(12)} = 99.472\text{ N}\cdot\text{m}$$

$$\tau_{CD} = \frac{Tc_{CD}}{J_{CD}} = \frac{2T}{\pi c_{CD}^3} \quad \therefore \quad c_{CD}^3 = \frac{2T_{CD}}{\pi\tau} = \frac{2(99.472)}{\pi(60\times 10^6)} = 1.05543\times 10^{-6}\text{ m}^3$$

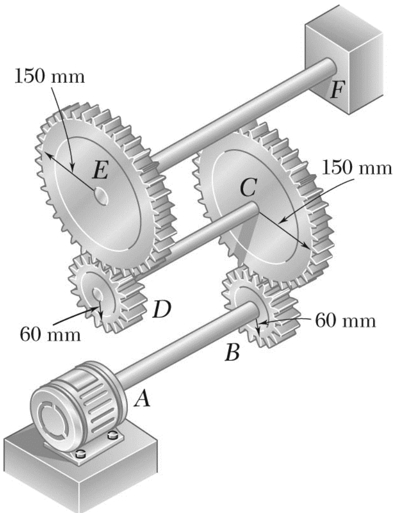
$$c_{CD} = 10.18\times 10^{-3}\text{ m} = 10.18\text{ mm} \qquad d_{CD} = 2c_{CD} = 20.4\text{ mm} \blacktriangleleft$$

Shaft EF:

$$f_{EF} = \frac{r_D}{r_E} f_{CD} = \frac{60}{150}(12) = 4.8\text{ Hz} \qquad T_{EF} = \frac{P}{2\pi f_{EF}} = \frac{7.5\times 10^3}{2\pi(4.8)} = 248.68\text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc_{EF}}{J_{EF}} = \frac{2T}{\pi c_{EF}^3} \quad \therefore \quad c_{EF}^3 = \frac{(2)(248.68)}{\pi(60\times 10^6)} = 2.6886\times 10^{-6}\text{ m}^3$$

$$c_{EF} = 13.82\times 10^{-3}\text{ m} = 13.82\text{ mm} \qquad d_{EF} = 2c_{EF} = 27.6\text{ mm} \blacktriangleleft$$



PROBLEM 3.77

Three shafts and four gears are used to form a gear train that will transmit power from the motor at A to a machine tool at F. (Bearings for the shafts are omitted in the sketch.) The diameter of each shaft is as follows: $d_{AB} = 16$ mm, $d_{CD} = 20$ mm, $d_{EF} = 28$ mm. Knowing that the frequency of the motor is 24 Hz and that the allowable shearing stress for each shaft is 75 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau_{\text{all}} = 75\text{ MPa} = 75 \times 10^6\text{ Pa}$$

Shaft AB:

$$c_{AB} = \frac{1}{2}d_{AB} = 0.008\text{ m} \qquad \tau = \frac{Tc_{AB}}{J_{AB}} = \frac{2T}{\pi c_{AB}^3}$$

$$T_{\text{all}} = \frac{\pi}{2}c_{AB}^3\tau_{\text{all}} = \frac{\pi}{2}(0.008)^3(75 \times 10^6) = 60.319\text{ N}\cdot\text{m}$$

$$f_{AB} = 24\text{ Hz} \quad P_{\text{all}} = 2\pi f_{AB}T_{\text{all}} = 2\pi(24)(60.319) = 9.10 \times 10^3\text{ W}$$

Shaft CD:

$$c_{CD} = \frac{1}{2}d_{CD} = 0.010\text{ m}$$

$$\tau = \frac{Tc_{CD}}{J_{CD}} = \frac{2T}{\pi c_{CD}^3} \quad \therefore \quad T_{\text{all}} = \frac{\pi}{2}c_{CD}^3\tau_{\text{all}} = \frac{\pi}{2}(0.010)^3(75 \times 10^6) = 117.81\text{ N}\cdot\text{m}$$

$$f_{CD} = \frac{r_B}{r_C}f_{AB} = \frac{60}{150}(24) = 9.6\text{ Hz} \quad P_{\text{all}} = 2\pi f_{CD}T_{\text{all}} = 2\pi(9.6)(117.81) = 7.11 \times 10^3\text{ W}$$

Shaft EF:

$$c_{EF} = \frac{1}{2}d_{EF} = 0.014\text{ m}$$

$$T_{\text{all}} = \frac{\pi}{2}c_{EF}^3\tau_{\text{all}} = \frac{\pi}{2}(0.014)^3(75 \times 10^6) = 323.27\text{ N}\cdot\text{m}$$

$$f_{EF} = \frac{r_D}{r_E}f_{CD} = \frac{60}{150}(9.6) = 3.84\text{ Hz}$$

$$P_{\text{all}} = 2\pi f_{EF}T_{\text{all}} = 2\pi(3.84)(323.27) = 7.80 \times 10^3\text{ W}$$

Maximum allowable power is the smallest value. $P_{\text{all}} = 7.11 \times 10^3\text{ W} = 7.11\text{ kW}$ ◄

PROBLEM 3.78

A 1.5-m-long solid steel of 48 mm diameter is to transmit 36 kW between a motor and a machine tool. Determine the lowest speed at which the shaft can rotate, knowing that $G = 77.2\ \text{GPa}$, that the maximum shearing stress must not exceed 60 MPa, and the angle of twist must not exceed 2.5°.

SOLUTION

$$P = 36 \times 10^3\ \text{W}, \quad c = \frac{1}{2}d = 0.024\ \text{m}, \quad L = 1.5\ \text{m}, \quad G = 77.2 \times 10^9\ \text{Pa}$$

Torque based on maximum stress: $\tau = 60\ \text{MPa} = 60 \times 10^6\ \text{Pa}$

$$\tau = \frac{Tc}{J} \quad T = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.024)^3(60 \times 10^6) = 1.30288 \times 10^3\ \text{N}\cdot\text{m}$$

Torque based on twist angle: $\varphi = 2.5° = 43.633 \times 10^{-3}\ \text{rad}$

$$\varphi = \frac{TL}{GJ} \quad T = \frac{GJ\varphi}{L} = \frac{\pi c^4 G\varphi}{2L} = \frac{\pi(0.024)^4(77 \times 10^9)(43.633 \times 10^{-3})}{(2)(1.5)}$$

$$= 1.17033 \times 10^3\ \text{N}\cdot\text{m}$$

Smaller torque governs, so $T = 1.17033 \times 10^3\ \text{N}\cdot\text{m}$

$$P = 2\pi fT \qquad f = \frac{P}{2\pi T} = \frac{36 \times 10^3}{2\pi(1.17033 \times 10^3)}$$

$f = 4.90\ \text{Hz}$ ◀

PROBLEM 3.79

A 2.5-m-long steel shaft of 30-mm diameter rotates at a frequency of 30 Hz. Determine the maximum power that the shaft can transmit, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 50 MPa, and that the angle of twist must not exceed 7.5°.

SOLUTION

$$c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m} \quad L = 2.5 \text{ m}$$

Stress requirement.

$$\tau = 50 \times 10^6 \text{ Pa} \quad \tau = \frac{Tc}{J}$$

$$T = \frac{\tau J}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(50 \times 10^6)(0.015)^3 = 265.07 \text{ N} \cdot \text{m}$$

Twist angle requirement.

$$\varphi = 7.5° = 130.90 \times 10^{-3} \text{ rad} \quad G = 77.2 \times 10^9 \text{ Pa}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi}{2} G c^4 \varphi = \frac{\pi}{2}(77.2 \times 10^9)(0.015)^4(130.90 \times 10^{-3}) = 803.60 \text{ N} \cdot \text{m}$$

Smaller value of T is the maximum allowable torque.

$$T = 265.07 \text{ N} \cdot \text{m}$$

Power transmitted at $f = 30$ Hz.

$$P = 2\pi f T = 2\pi(30)(265.07) = 49.96 \times 10^3 \text{ W} \qquad P = 50.0 \text{ kW} \blacktriangleleft$$

PROBLEM 3.80

A steel shaft must transmit 210 hp at a speed of 360 rpm. Knowing that $G = 11.2 \times 10^6$ psi, design a solid shaft so that the maximum shearing stress will not exceed 12 ksi, and the angle of twist in a 8.2-ft length must not exceed 3°.

SOLUTION

Power: $P = (210 \text{ hp})(6600 \text{ in} \cdot \text{lb/s/hp}) = 1.336 \times 10^6 \text{ in} \cdot \text{lb/s}$

Angular speed: $f = (360 \text{ rpm})\dfrac{1 \text{ min.}}{60 \text{ sec}} = 6 \text{ Hz}$

Torque: $T = \dfrac{P}{2\pi f} = \dfrac{1.386 \times 10^6}{(2\pi)(6)} = 36.765 \times 10^3 \text{ lb} \cdot \text{in}$

Stress requirement: $\tau = 12 \text{ ksi}, \quad \tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(36.765 \times 10^3)}{\pi(12 \times 10^3)}} = 1.2494 \text{ in.}$$

Angle of twist requirement: $\varphi = 3° = 52.36 \times 10^{-3} \text{ rad}$

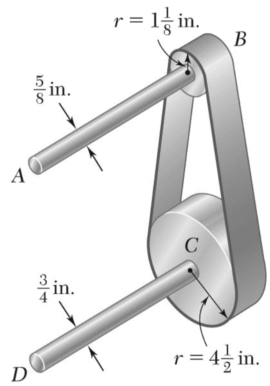
$$L = 8.2 \text{ ft} = 98.4 \text{ in.}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$c = \sqrt[4]{\frac{2TL}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(36.765 \times 10^3)(98.4)}{\pi(11.2 \times 10^6)(52.36 \times 10^{-3})}} = 1.4077 \text{ in.}$$

The larger value is the required radius. $c = 1.408 \text{ in.}$

$d = 2c$ $\qquad$ $d = 2.82 \text{ in.}$ ◀



PROBLEM 3.81

The shaft-disk-belt arrangement shown is used to transmit 3 hp from point *A* to point *D*. (*a*) Using an allowable shearing stress of 9500 psi, determine the required speed of shaft *AB*. (*b*) Solve part *a*, assuming that the diameters of shafts *AB* and *CD* are, respectively, 0.75 in. and 0.625 in.

SOLUTION

$\tau = 9500$ psi $\quad P = 3 \text{ hp} = (3)(6600) = 19800 \text{ lb} \cdot \text{in/s}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2} c^3 \tau$$

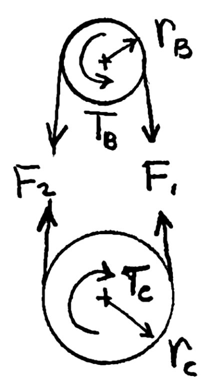
Allowable torques.

$\frac{5}{8}$- in. diameter shaft: $\quad c = \frac{5}{16}\text{in.}, \quad T_{\text{all}} = \frac{\pi}{2}\left(\frac{5}{16}\right)^3 (9500) = 455.4 \text{ lb} \cdot \text{in}$

$\frac{3}{4}$-in. diameter shaft: $\quad c = \frac{3}{8}\text{in.}, \quad T_{\text{all}} = \frac{\pi}{2}\left(\frac{3}{8}\right)^3 (9500) = 786.9 \text{ lb} \cdot \text{in}$

Statics: $\quad T_B = r_B(F_1 - F_2) \qquad T_C = r_C(F_1 - F_2)$

$$T_B = \frac{r_B}{r_C} T_C = \frac{1.125}{4.5} T_C = 0.25 T_C$$



(*a*) Allowable torques. $\quad T_{B,\text{all}} = 455.4 \text{ lb} \cdot \text{in} \qquad T_{C,\text{all}} = 786.9 \text{ lb} \cdot \text{in}$

Assume $\quad T_C = 786.9 \text{ lb} \cdot \text{in}$

Then $\quad T_B = (0.25)(786.9) = 196.73 \text{ lb} \cdot \text{in} < 455.4 \text{ lb} \cdot \text{in (okay)}$

$$P = 2\pi f T \qquad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(196.73)}$$

$f_{AB} = 16.02$ Hz ◀

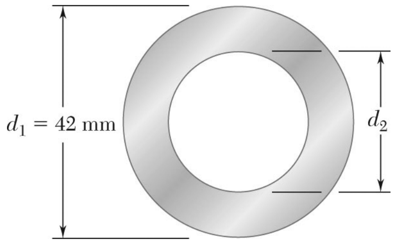
(*b*) Allowable torques. $\quad T_{B,\text{all}} = 786.9 \text{ lb} \cdot \text{in} \qquad T_{C,\text{all}} = 455.4 \text{ lb} \cdot \text{in}$

Assume $\quad T_C = 455.4 \text{ lb} \cdot \text{in}$

Then $\quad T_B = (0.25)(455.4) = 113.85 \text{ lb} \cdot \text{in} < 786.9 \text{ lb} \cdot \text{in}$

$$P = 2\pi f T \qquad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(113.85)}$$

$f_{AB} = 27.2$ Hz ◀



PROBLEM 3.82

A 1.6-m-long tubular steel shaft of 42-mm outer diameter d_1 is to be made of a steel for which $\tau_{all} = 75\,\text{MPa}$ and $G = 77.2$ GPa. Knowing that the angle of twist must not exceed 4° when the shaft is subjected to a torque of 900 N · m, determine the largest inner diameter d_2 that can be specified in the design.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.021\,\text{m} \quad L = 1.6\,\text{m}$$

Based on stress limit: $\tau = 75\,\text{MPa} = 75\times 10^6$ Pa

$$\tau = \frac{Tc_1}{J} \quad \therefore \quad J = \frac{Tc_1}{\tau} = \frac{(900)(0.021)}{75\times 10^6} = 252\times 10^{-9}\,\text{m}^4$$

Based on angle of twist limit: $\varphi = 4° = 69.813\times 10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} \quad \therefore \quad J = \frac{TL}{G\varphi} = \frac{(900)(1.6)}{(77\times 10^9)(69.813\times 10^{-3})} = 267.88\times 10^{-9}\,\text{m}^4$$

Larger value for J governs. $\quad J = 267.88\times 10^{-9}\,\text{m}^4$

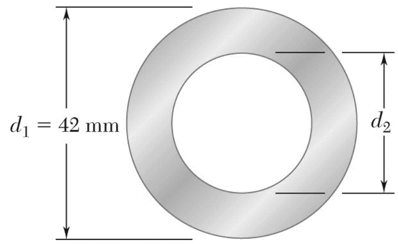
$$J = \frac{\pi}{2}\left(c_1^4 - c_2^4\right)$$

$$c_2^4 = c_1^4 - \frac{2J}{\pi} = 0.021^4 - \frac{(2)(267.88\times 10^{-9})}{\pi} = 23.943\times 10^{-9}\,\text{m}^4$$

$$c_2 = 12.44\times 10^{-3}\,\text{m} = 12.44\,\text{mm}$$

$d_2 = 2c_2 = 24.9$ mm ◀

PROBLEM 3.83



A 1.6-m-long tubular steel shaft ($G = 77.2\,\text{GPa}$) of 42-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 120 kW between a turbine and a generator. Knowing that the allowable shearing stress is 65 MPa and that the angle of twist must not exceed 3°, determine the minimum frequency at which the shaft can rotate.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.021\text{ m},\quad c_2 = \frac{1}{2}d_2 = 0.015\text{ m}$$

$$J = \frac{\pi}{2}\left(c_1^4 - c_2^4\right) = \frac{\pi}{2}(0.021^4 - 0.015^4) = 225.97 \times 10^{-9}\text{ m}^4$$

Based on stress limit: $\tau = 65\,\text{MPa} = 65 \times 10^6\text{ Pa}$

$$\tau = \frac{Tc_1}{J} \quad \text{or} \quad T = \frac{J\tau}{G} = \frac{(225.97 \times 10^{-9})(65 \times 10^6)}{0.021} = 699.43\text{ N}\cdot\text{m}$$

Based on angle of twist limit: $\varphi = 3° = 52.36{\times}10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} \quad \text{or} \quad T = \frac{GJ\varphi}{L} = \frac{(77 \times 10^9)(225.97 \times 10^{-9})(52.36 \times 10^{-3})}{1.6}$$

$$= 569.40\text{ N}\cdot\text{m}$$

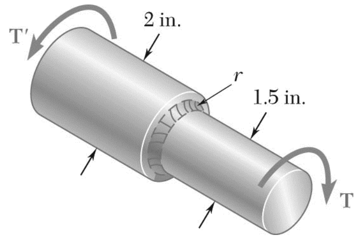
Smaller torque governs. $T = 569.40\text{ N}\cdot\text{m}$

$$P = 120\text{ kW} = 120 \times 10^3\text{ W}$$

$$P = 2\pi fT \quad \text{so} \quad f = \frac{P}{2\pi T} = \frac{120 \times 10^3}{2\pi(569.40)}$$

$f = 33.5$ Hz ◀

or 2010 rpm ◀



PROBLEM 3.84

Knowing that the stepped shaft shown transmits a torque of magnitude $T = 2.50\,\text{kip}\cdot\text{in.}$, determine the maximum shearing stress in the shaft when the radius of the fillet is (*a*) $r = \frac{1}{8}$ in., (*b*) $r = \frac{3}{16}$ in.

SOLUTION

$$D = 2\text{ in.} \quad d = 1.5\text{ in.} \qquad \frac{D}{d} = \frac{2}{1.5} = 1.33$$

$$c = \frac{1}{2}d = 0.75\text{ in.} \qquad T = 2.5\text{ kip}\cdot\text{in}$$

$$\frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.5)}{\pi(0.75)^3} = 3.773\text{ ksi}$$

(*a*) $r = \dfrac{1}{8}\text{ in.} \quad r = 0.125\text{ in.}$

$$\frac{r}{d} = \frac{0.125}{1.5} = 0.0833$$

From Fig. 3.32, $K = 1.42$

$$\tau_{max} = K\frac{Tc}{J} = (1.42)(3.773)$$

$\tau_{max} = 5.36$ ksi ◀

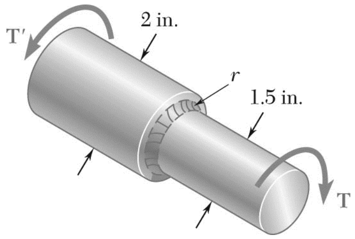
(*b*) $r = \dfrac{3}{16}\text{ in.} \qquad r = 0.1875\text{ in.}$

$$\frac{r}{d} = \frac{0.1875}{1.5} = 0.125$$

From Fig. 3.32, $K = 1.33$

$$\tau_{max} = K\frac{Tc}{J} = (1.33)(3.773)$$

$\tau_{max} = 5.02$ ksi ◀



PROBLEM 3.85

Knowing that the allowable shearing stress is 8 ksi for the stepped shaft shown, determine the magnitude T of the largest torque that can be transmitted by the shaft when the radius of the fillet is (*a*) $r = \frac{3}{16}$ in., (*b*) $r = \frac{1}{4}$ in.

SOLUTION

$$D = 2 \text{ in.} \qquad d = 1.5 \text{ in.} \qquad \frac{D}{d} = 1.33$$

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad \tau_{\max} = 8 \text{ ksi}$$

$$\tau_{\max} = K\frac{Tc}{J} \qquad \text{or} \qquad T = \frac{J\tau_{\max}}{Kc} = \frac{\pi\tau_{\max}c^3}{2K}$$

(*a*) $r = \dfrac{3}{16}$ in. $\qquad r = 0.1875$ in.

$$\frac{r}{d} = \frac{0.1875}{1.5} = 0.125$$

From Fig. 3.32, $\qquad K = 1.33$

$$T = \frac{\pi(8)(0.75)^3}{(2)(1.33)}$$

$T = 3.99 \text{ kip} \cdot \text{in}$ ◀

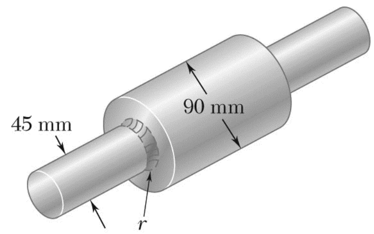
(*b*) $r = \dfrac{1}{4}$ in. $\qquad r = 0.25$ in.

$$\frac{r}{d} = \frac{0.25}{1.5} = 0.1667$$

From Fig. 3.32, $\qquad K = 1.27$

$$T = \frac{\pi(8)(0.75)^3}{(2)(1.27)}$$

$T = 4.17 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 3.86

The stepped shaft shown must transmit 40 kW at a speed of 720 rpm. Determine the minimum radius r of the fillet if an allowable stress of 36 MPa is not to be exceeded.

SOLUTION

Angular speed: $f = (720 \text{ rpm})\left(\dfrac{1 \text{ Hz}}{60 \text{ rpm}}\right) = 12 \text{ Hz}$

Power: $P = 40 \times 10^3 \text{ W}$

Torque: $T = \dfrac{P}{2\pi f} = \dfrac{40 \times 10^3}{2\pi(12)} = 530.52 \text{ N} \cdot \text{m}$

In the smaller shaft, $d = 45 \text{ mm}, \quad c = 22.5 \text{ mm} = 0.0225 \text{ m}$

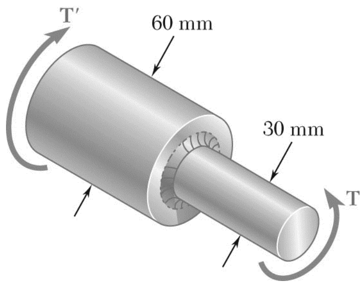
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(530.52)}{\pi(0.0225)^3} = 29.65 \times 10^6 \text{ Pa}$$

Using $\tau_{\max} = 36 \text{ MPa} = 36 \times 10^6 \text{ Pa}$ results in

$$K = \frac{\tau_{\max}}{\tau} = \frac{36 \times 10^6}{29.65 \times 10^6} = 1.214$$

From Fig 3.32 with $\dfrac{D}{d} = \dfrac{90 \text{ mm}}{45 \text{ mm}} = 2, \quad \dfrac{r}{d} = 0.24$

$r = 0.24d = (0.24)(45 \text{ mm})$ $\qquad r = 10.8 \text{ mm}$ ◀



PROBLEM 3.87

The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6$ mm, determine the smallest permissible speed of the shaft.

SOLUTION

$$\frac{r}{d} = \frac{6}{30} = 0.2$$

$$\frac{D}{d} = \frac{60}{30} = 2$$

From Fig. 3.32, $K = 1.26$

For smaller side, $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$

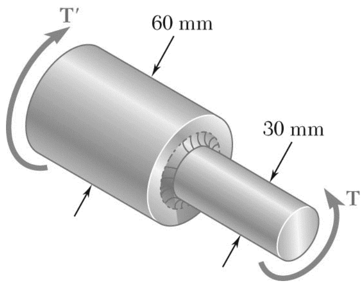
$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.015)^3(40\times 10^6)}{(2)(1.26)} = 168.30 \text{ N}\cdot\text{m}$$

$$P = 45 \text{ kW} = 45\times 10^3 \quad P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{45\times 10^3}{2\pi(168.30\times 10^3)} = 42.6 \text{ Hz}$$

$f = 42.6$ Hz ◀



PROBLEM 3.88

The stepped shaft shown must rotate at a frequency of 50 Hz. Knowing that the radius of the fillet is $r = 8$ mm and the allowable shearing stress is 45 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} \quad T = \frac{\pi c^3 \tau}{2K}$$

$$d = 30 \text{ mm} \quad c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$D = 60 \text{ mm}, \quad r = 8 \text{ mm}$$

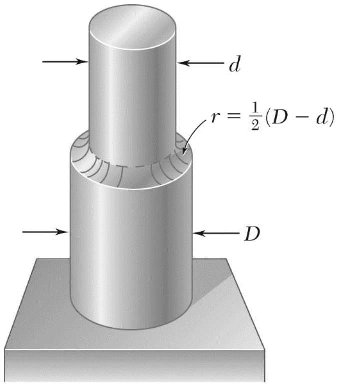
$$\frac{D}{d} = \frac{60}{30} = 2, \quad \frac{r}{d} = \frac{8}{30} = 0.26667$$

From Fig. 3.32, $K = 1.18$

Allowable torque. $$T = \frac{\pi(15 \times 10^{-3})^3 (45 \times 10^6)}{(2)(1.18)} = 202.17 \text{ N} \cdot \text{m}$$

Maximum power. $$P = 2\pi f T = (2\pi)(50)(202.17) = 63.5 \times 10^3 \text{ W}$$

$P = 63.5$ kW ◀



Full quarter-circular fillet extends to edge of larger shaft.

PROBLEM 3.89

In the stepped shaft shown, which has a full quarter-circular fillet, $D = 1.25$ in. and $d = 1$ in. Knowing that the speed of the shaft is 2400 rpm and that the allowable shearing stress is 7500 psi, determine the maximum power that can be transmitted by the shaft.

SOLUTION

$$\frac{D}{d} = \frac{1.25}{1.0} = 1.25$$

$$r = \frac{1}{2}(D - d) = 0.15 \text{ in.}$$

$$\frac{r}{d} = \frac{0.15}{1.0} = 0.15$$

From Fig. 3.32, $K = 1.31$

For smaller side, $c = \frac{1}{2}d = 0.5 \text{ in.}$

$$\tau = \frac{KTc}{J} \quad T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$$

$$T = \frac{\pi (0.5)^3 (7500)}{(2)(1.31)} = 1.1241 \times 10^3 \text{ lb} \cdot \text{in}$$

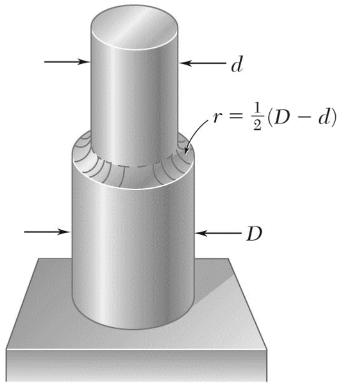
$$f = 2400 \text{ rpm} = 40 \text{ Hz}$$

$$P = 2\pi f T = 2\pi (40)(1.1241 \times 10^3)$$

$$= 282.5 \times 10^3 \text{ lb} \cdot \text{in/s}$$

$P = 42.8$ hp ◀

PROBLEM 3.90



Full quarter-circular fillet extends to edge of larger shaft.

A torque of magnitude $T = 200 \text{ lb} \cdot \text{in.}$ is applied to the stepped shaft shown, which has a full quarter-circular fillet. Knowing that $D = 1$ in., determine the maximum shearing stress in the shaft when (*a*) $d = 0.8$ in., (*b*) $d = 0.9$ in.

SOLUTION

(*a*) $\dfrac{D}{d} = \dfrac{1.0}{0.8} = 1.25$

$r = \dfrac{1}{2}(D - d) = 0.1 \text{ in.}$

$\dfrac{r}{d} = \dfrac{0.1}{0.8} = 0.125$

From Fig. 3.32, $K = 1.31$

For smaller side, $c = \dfrac{1}{2}d = 0.4 \text{ in.}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$= \frac{(2)(1.31)(200)}{\pi(0.4)^3} = 2.61 \times 10^3 \text{ psi}$$

$\tau = 2.61 \text{ ksi}$ ◀

(*b*) $\dfrac{D}{d} = \dfrac{1.0}{0.9} = 1.111$

$r = \dfrac{1}{2}(D - d) = 0.05$

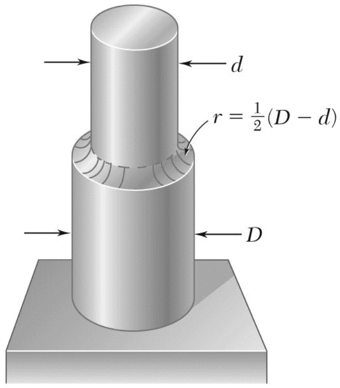
$\dfrac{r}{d} = \dfrac{0.05}{1.0} = 0.05$

From Fig. 3.32, $K = 1.44$

For smaller side, $c = \dfrac{1}{2}d = 0.45 \text{ in.}$

$$\tau = \frac{2KT}{\pi c^3} = \frac{(2)(1.44)(200)}{\pi(0.45)^3} = 2.01 \times 10^3 \text{ psi}$$

$\tau = 2.01 \text{ ksi}$ ◀



Full quarter-circular fillet extends to edge of larger shaft

PROBLEM 3.91

In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 80 MPa. Knowing that $D = 30$ mm, determine the largest allowable torque that can be applied to the shaft if (*a*) $d = 26$ mm, (*b*) $d = 24$ mm.

SOLUTION

$$\tau = 80 \times 10^6 \text{Pa}$$

(*a*) $\quad \dfrac{D}{d} = \dfrac{30}{26} = 1.154 \qquad r = \dfrac{1}{2}(D-d) = 2 \text{ mm} \qquad \dfrac{r}{d} = \dfrac{2}{26} = 0.0768$

From Fig. 3.32, $\quad K = 1.36$

Smaller side, $\quad c = \dfrac{1}{2}d = 13 \text{ mm} = 0.013 \text{ m}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.013)^3 (80 \times 10^6)}{(2)(1.36)} = 203 \text{ N} \cdot \text{m} \qquad T = 203 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $\quad \dfrac{D}{d} = \dfrac{30}{24} = 1.25 \qquad r = \dfrac{1}{2}(D-d) = 3 \text{ mm} \qquad \dfrac{r}{d} = \dfrac{3}{24} = 0.125$

From Fig. 3.32, $\quad K = 1.31$

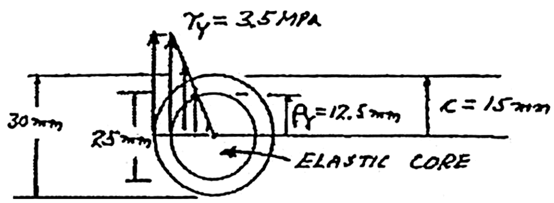
$$c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.012)^3 (80 \times 10^6)}{(2)(1.31)} = 165.8 \text{ N} \cdot \text{m} \qquad T = 165.8 \text{ N} \cdot \text{m} \blacktriangleleft$$

PROBLEM 3.92

A 30-mm diameter solid rod is made of an elastoplastic material with $\tau_Y = 3.5$ MPa. Knowing that the elastic core of the rod is 25 mm in diameter, determine the magnitude of the applied torque **T**.

SOLUTION



$$\tau_Y = 3.5 \times 10^6 \text{ Pa}, \quad c = \frac{1}{2}(30 \text{ mm}) = 15 \text{ mm} = 0.015 \text{ m}$$

$$\rho_Y = \frac{1}{2}(25 \text{ mm}) = 12.5 \text{ mm} = 0.0125 \text{ m}$$

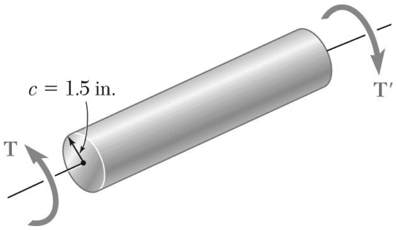
$$T_Y = \frac{J}{c}\tau_Y = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.015)^3(3.5 \times 10^6) = 18.555 \text{ N} \cdot \text{m}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(18.555)\left[1 - \frac{(0.0125)^3}{(0.015)^3}\right]$$

$$= 21.2 \text{ N} \cdot \text{m}$$

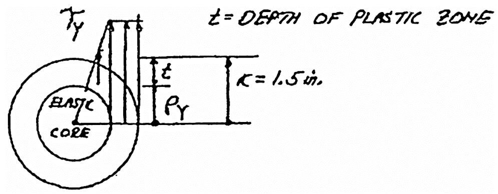
$T = 21.2$ N · m ◀

PROBLEM 3.93



The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 21\text{ ksi}$. Determine the magnitude T of the applied torques when the plastic zone is (*a*) 0.8 in. deep, (*b*) 1.2 in. deep.

SOLUTION



$$\tau_Y = 21\text{ ksi}$$

$$T_Y = \frac{\tau_Y J}{c} = \tau_Y \frac{\pi}{2} c^3$$

$$= (21\text{ ksi})\frac{\pi}{2}(1.5\text{ in.})^3$$

$$T_Y = 111.3\text{ kip}\cdot\text{in}$$

(*a*) For $t = 0.8$ in. $\rho_Y = 1.5 - 0.8 = 0.7\text{ in.}$

Eq. (3.32)

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right] = \frac{4}{3}(111.3\text{ kip}\cdot\text{in})\left[1 - \frac{1}{4}\frac{(0.7\text{ in.})^3}{(1.5\text{ in.})^3}\right]$$

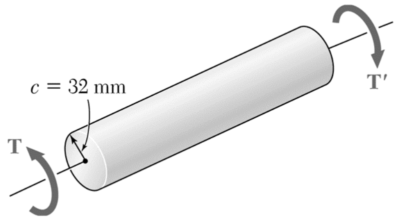
$$T = 144.7\text{ kip}\cdot\text{in} \blacktriangleleft$$

(*b*) For $t = 1.2$ in. $\rho_Y = 1.5 - 1.2 = 0.3\text{ in.}$

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right] = \frac{4}{3}(111.3\text{ kip}\cdot\text{in})\left[1 - \frac{1}{4}\frac{(0.3\text{ in.})^3}{(1.5\text{ in.})^3}\right]$$

$$T = 148.1\text{ kip}\cdot\text{in} \blacktriangleleft$$

PROBLEM 3.94



The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the magnitude T of the applied torques when the plastic zone is (*a*) 16 mm deep, (*b*) 24 mm deep.

SOLUTION

$$c = 32 \text{ mm} = 0.032 \text{ m}$$

$$\tau_Y = 145 \times 10^6 \text{Pa}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.032)^3(145 \times 10^6)$$

$$= 7.4634 \times 10^3 \text{ N} \cdot \text{m}$$

(*a*) $t_P = 16\,\text{mm} = 0.016\,\text{m}$

$$\rho_Y = c - t_P = 0.032 - 0.016 = 0.016\,\text{m}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634 \times 10^3)\left(1 - \frac{1}{4}\frac{0.016^3}{0.032^3}\right)$$

$$= 9.6402 \times 10^3 \text{ N} \cdot \text{m}$$

$T = 9.64 \text{ kN} \cdot \text{m}$ ◀

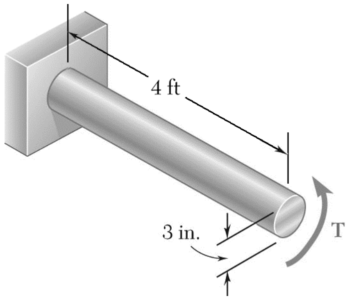
(*b*) $t_P = 24 \text{ mm} = 0.024 \text{ m}$

$$\rho_Y = c - t_P = 0.032 - 0.024 = 0.008 \text{ m}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{{\rho_Y}^3}{c^3}\right) = \frac{4}{3}(7.4634 \times 10^3)\left(1 - \frac{1}{4}\frac{0.008^3}{0.032^3}\right)$$

$$= 9.9123 \times 10^3 \text{ N} \cdot \text{m}$$

$T = 9.91 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 3.95

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 11.2 \times 10^6$ psi and $\tau_Y = 21$ ksi. Determine the maximum shearing stress and the radius of the elastic core caused by the application of torque of magnitude (*a*) $T = 100$ kip · in., (*b*) $T = 140$ kip · in.

SOLUTION

$$c = 1.5 \text{ in.}, \quad J = \frac{\pi}{2}c^4 = 7.9522 \text{ in}^4, \quad \tau_Y = 21 \text{ ksi}$$

(*a*) $\underline{T = 100 \text{ kip} \cdot \text{in}}$

$$\tau_m = \frac{Tc}{J} = \frac{(100 \text{ kip} \cdot \text{in})(1.5 \text{ in.})}{7.9522 \text{ in}^4} \qquad \tau_m = 18.86 \text{ ksi} \blacktriangleleft$$

Since $\tau_m < \tau_Y$, shaft remains elastic.

Radius of elastic core: $c = 1.500$ in. ◀

(*b*) $\underline{T = 140 \text{ kip} \cdot \text{in}}$

$$\tau_m = \frac{(140)(1.5)}{7.9522} = 26.4 \text{ ksi.}$$

<u>Impossible</u>: $\tau_m = \tau_Y = 21.0$ ksi ◀

Plastic zone has developed. Torque at onset of yield is $T_Y = \dfrac{J}{c}\tau_Y = \dfrac{7.9522}{1.5}(21 \text{ ksi}) = 111.33 \text{ kip} \cdot \text{in}$

Eq. (3.32): $T = \dfrac{4}{3}T_Y\left(1 - \dfrac{1}{4}\dfrac{\rho_Y^3}{c^3}\right)$

$$\left(\frac{\rho_Y}{c}\right)^3 = 4 - 3\frac{T}{T_Y} = 4 - 3\frac{140}{111.33} = 0.22743 \qquad \frac{\rho_Y}{c} = 0.6104$$

$$\rho_Y = 0.6104c = 0.6104(1.5 \text{ in.}) \qquad \rho_Y = 0.916 \text{ in.} \blacktriangleleft$$

PROBLEM 3.96

It is observed that a straightened paper clip can be twisted through several revolutions by the application of a torque of approximately 60 mN · m. Knowing that the diameter of the wire in the paper clip is 0.9 mm, determine the approximate value of the yield stress of the steel.

SOLUTION

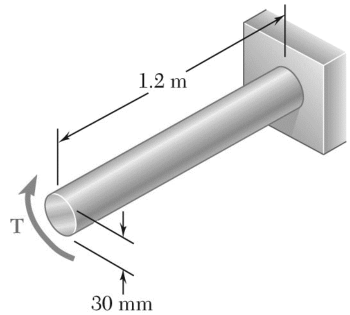
$$c = \frac{1}{2}d = 0.45 \text{ mm} = 0.45 \times 10^{-3} \text{ m}$$

$$T_P = 60 \text{ mN} \cdot \text{m} = 60 \times 10^{-3} \text{ N} \cdot \text{m}$$

$$T_P = \frac{4}{3}T_Y = \frac{4}{3}\,\frac{J\tau_Y}{c} = \frac{4}{3} \cdot \frac{\pi}{2}c^3\; T_Y = \frac{2\pi}{3}c^3\tau_Y$$

$$\tau_Y = \frac{3T_P}{2\pi c^3} = \frac{(3)(60 \times 10^{-3})}{2\pi\,(0.45 \times 10^{-3})^3} = 314 \times 10^6 \text{ Pa} \qquad \tau_Y = 314 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.97



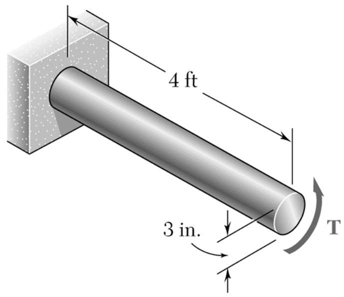
The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 145\,\text{MPa}$. Determine the radius of the elastic core caused by the application of a torque equal to 1.1 T_Y, where T_Y is the magnitude of the torque at the onset of yield.

SOLUTION

$$c = \frac{1}{2}d = 15\text{ mm} \qquad T = \frac{4}{3}T_Y\left[1 - \left(\frac{\rho_Y}{c}\right)^3\right]$$

$$\frac{\rho_Y}{c} = \sqrt[3]{4 - 3\frac{T}{T_Y}} = \sqrt{4 - (3)(1.1)} = 0.88790$$

$$\rho_Y = 0.88790\,c = (0.88790)(15\text{ mm}) \qquad \rho_Y = 13.32\text{ mm} \blacktriangleleft$$



PROBLEM 3.98

For the solid circular shaft of Prob. 3.95, determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 80\text{ kip}\cdot\text{in.}$, (*b*) $T = 130\text{ kip}\cdot\text{in.}$

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(3) = 1.5\text{ in.}\quad \tau_Y = 21\times 10^3\text{ psi}$$

$$L = 4\text{ ft} = 48\text{ in.}\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.5)^4 = 7.9522\text{ in}^4$$

Torque at onset of yielding: $\tau = \frac{Tc}{J} \qquad T = \frac{\tau J}{c}$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(21\times 10^3)(7.9522)}{1.5} = 111.330\times 10^3\text{ lb}\cdot\text{in}$$

(*a*) $\underline{\underline{T = 80\text{ kip}\cdot\text{in}}} = 80\times 10^3\text{ lb}\cdot\text{in}$

Since $T < T_Y$, the shaft is fully elastic. $\varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(80\times 10^3)(48)}{(11.2\times 10^6)(7.9522)} = 43.115\times 10^{-3}\text{ rad}$$

$\varphi = 2.47°$ ◀

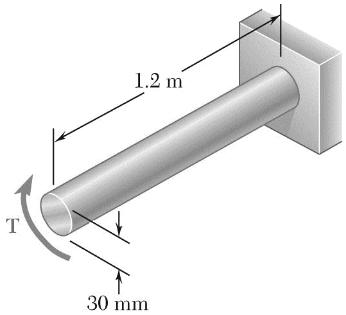
(*b*) $\underline{\underline{T = 130\text{ kip}\cdot\text{in}}} = 130\times 10^3\text{ lb}\cdot\text{in} \qquad T > T_Y \qquad T = \frac{4}{3}T_Y\left[1 - \left(\frac{\varphi_Y}{\varphi}\right)^3\right]$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(111.330\times 10^3)(48)}{(11.2\times 10^6)(7.9522)} = 60.000\times 10^{-3}\text{ rad}$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - 3\frac{T}{T_Y}} = \sqrt[3]{4 - \frac{(3)(130\times 10^3)}{111.330\times 10^3}} = 0.79205$$

$$\varphi = \frac{\varphi_Y}{0.79205} = \frac{60.000\times 10^{-3}}{0.79205} = 75.75\times 10^{-3}\text{ rad}$$

$\varphi = 4.34°$ ◀



PROBLEM 3.99

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2$ GPa and $\tau_Y = 145$ MPa. Determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 600$ N · m, (*b*) $T = 1000$ N · m.

SOLUTION

$$c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

Torque at onset of yielding: $\quad \tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi(15\times 10^{-3})^3(145\times 10^6)}{2} = 768.71 \text{ N} \cdot \text{m}$$

(*a*) $\quad T = 600$ N · m. Since $T \le T_Y$, the shaft is elastic.

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G} = \frac{(2)(600)(1.2)}{\pi(15\times 10^{-3})^4(77.2\times 10^9)} = 0.11728 \text{ rad} \qquad \varphi = 6.72° \blacktriangleleft$$

(*b*) $\quad T = 1000$ N · m. $\qquad T > T_Y$ A plastic zone has developed.

$$T = \frac{4}{3}T_Y\left[1 - \left(\frac{\varphi_Y}{\varphi}\right)^3\right] \qquad \frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - 3\left(\frac{T}{T_Y}\right)}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(768.71)(2.1)}{\pi(15\times 10^{-3})^4(77.2\times 10^9)} = 0.15026 \text{ rad}$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{(3)(1000)}{768.71}} = 0.46003$$

$$\varphi = \frac{\varphi_Y}{0.46003} = \frac{0.15026}{0.46003} = 0.32663 \text{ rad} \qquad \varphi = 18.71° \blacktriangleleft$$

PROBLEM 3.100

A 3-ft-long solid shaft has a diameter of 2.5 in. and is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi. Determine the torque required to twist the shaft through an angle of (*a*) 2.5°, (*b*) 5°.

SOLUTION

$$L = 3 \text{ ft} = 36 \text{ in.}, \quad c = \frac{1}{2}d = 1.25 \text{ in.}, \quad \tau_Y = 21 \times 10^3 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.25)^4 = 3.835 \text{ in}^4$$

$$\tau_Y = \frac{T_Y c}{J} \qquad T_Y = \frac{J\tau_Y}{c} = \frac{(3.835)(21 \times 10^3)}{1.25} = 64.427 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(64.427 \times 10^3)(36)}{(11.2 \times 10^6)(3.835)} = 53.999 \times 10^{-3} \text{ rad} = 3.0939°$$

(*a*) $\varphi = 2.5° = 43.633 \times 10^{-3}$ rad $\qquad \varphi < \varphi_Y \qquad$ The shaft remains elastic.

$$\varphi = \frac{TL}{GJ}$$

$$T = \frac{GJ\varphi}{L} = \frac{(11.2 \times 10^6)(3.835)(43.633 \times 10^{-3})}{36}$$

$$= 52.059 \times 10^3 \text{lb} \cdot \text{in}$$

$T = 52.1$ kip · in ◀

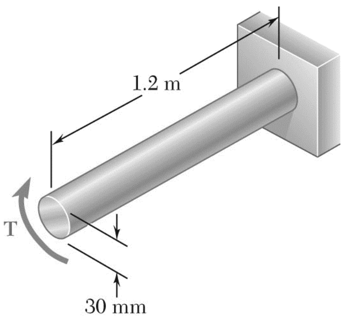
(*b*) $\varphi = 5° = 87.266 \times 10^{-3}$ rad $\qquad \varphi > \varphi_Y \qquad$ A plastic zone occurs.

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\left(\frac{\varphi_Y}{\varphi}\right)^3\right]$$

$$= \frac{4}{3}(64.427 \times 10^3)\left[1 - \frac{1}{4}\left(\frac{53.999 \times 10^{-3}}{87.266 \times 10^{-3}}\right)^3\right]$$

$$= 80.814 \times 10^3 \text{ lb} \cdot \text{in}$$

$T = 80.8$ kip · in ◀



PROBLEM 3.101

For the solid shaft of Prob. 3.99, determine (*a*) the magnitude of the torque **T** required to twist the shaft through an angle of 15°, (*b*) the radius of the corresponding elastic core.

PROBLEM 3.99 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2$ GPa and $\tau_Y = 145$ MPa. Determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 600\ \text{N}\cdot\text{m}$, (*b*) $T = 1000\ \text{N}\cdot\text{m}$.

SOLUTION

$$c = \frac{1}{2}d = 15\,\text{mm} = 15\times 10^{-3}\,\text{m}$$

$$\varphi = 15° = 0.2618\ \text{rad}$$

$$\varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(1.2)(145\times 10^6)}{(15\times 10^{-3})(77.2\times 10^9)} = 0.15026\ \text{rad}$$

(*a*) Since $\varphi > \varphi_Y$, there is a plastic zone.

$$\tau_Y = \frac{T_Y c}{J} = \frac{2T}{\pi c^3}$$

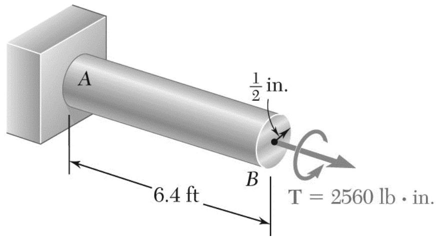
$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi(15\times 10^{-3})^3(145\times 10^6)}{2} = 768.71\ \text{N}\cdot\text{m}$$

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\left(\frac{\varphi_Y}{\varphi}\right)^3\right] = \frac{4}{3}(768.71)\left[1 - \frac{1}{4}\left(\frac{0.15026}{0.2618}\right)^3\right]$$

$$= 976.5\ \text{N}\cdot\text{m} \qquad\qquad T = 977\ \text{N}\cdot\text{m} \blacktriangleleft$$

(*b*) $L\gamma_Y = \rho_Y\varphi = c\varphi_Y$

$$\rho_Y = \frac{c\varphi_Y}{\varphi} = \frac{(15\times 10^{-3})(0.15026)}{0.2618} = 8.61\times 10^{-3}\text{m} \qquad\qquad \rho_Y = 8.61\ \text{mm} \blacktriangleleft$$



PROBLEM 3.102

The shaft AB is made of a material that is elastoplastic with $\tau_Y = 12\,\text{ksi}$ and $G = 4.5\times 10^6$ psi. For the loading shown, determine (a) the radius of the elastic core of the shaft, (b) the angle of twist at end B.

SOLUTION

(a) Radius of elastic core.

$$c = 0.5\,\text{in.} \qquad \tau_Y = 12\times 10^3\ \text{psi}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.5)^3(12\times 10^3)$$
$$= 2356.2\,\text{lb}\cdot\text{in}$$

$T = 2560\ \text{lb}\cdot\text{in} > T_Y$ (plastic region with elastic core)

$$T = \frac{4}{3}T_Y\left(1-\frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y{}^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(2560)}{2356.2} = 0.74051$$

$$\frac{\rho_Y}{c} = 0.9047 \qquad \rho_Y = (0.9047)(0.5) \qquad \rho_Y = 0.452\ \text{in.} \blacktriangleleft$$

(b) Angle of twist.

$$L = 6.4\ \text{ft} = 76.8\ \text{in.} \qquad G = 4.5\times 10^6\ \text{psi}$$

$$\varphi_Y = \frac{T_Y L}{JG} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(2356.2)(76.8)}{\pi(0.5)^4(4.5\times 10^6)} = 0.4096\ \text{radians}$$

$$\frac{\varphi_Y}{\varphi} = \frac{\rho_Y}{c} \qquad \varphi = \frac{\varphi_Y c}{\rho_Y} = \frac{0.4096}{0.9047} = 0.4527\ \ \text{radians}$$

$$\varphi = 25.9^\circ \blacktriangleleft$$

PROBLEM 3.103

A 1.25-in.-diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_Y = 18$ ksi and $G = 11.2 \times 10^6$ psi. For an 8-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 7.5 kip $\cdot$ in. torque.

SOLUTION

$$c = \frac{1}{2}d = 0.625 \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}, \quad \tau_Y = 18 \text{ ksi} = 18000 \text{ psi}$$

$$L = 8 \text{ ft} = 96 \text{ in.} \quad T = 7.5 \text{ kip} \cdot \text{in} = 7.5 \times 10^3 \text{ lb} \cdot \text{in}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.625)^3(18000) = 6.9029 \times 10^3 \text{ lb} \cdot \text{in}$$

$T > T_Y$: plastic region with elastic core $\quad \therefore \quad \tau_{\max} = \tau_Y = 18$ ksi $\qquad \tau_{\max} = 18$ ksi ◀

$$\gamma_Y = \frac{c\varphi_Y}{L} \quad \therefore \quad \varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(96)(18000)}{(0.625)(11.2 \times 10^6)} = 246.86 \times 10^{-3} \text{ rad}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right)$$

$$\frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \dfrac{3T}{T_Y}}} = \frac{1}{\sqrt[3]{4 - \dfrac{(3)(7.5 \times 10^3)}{6.9029 \times 10^3}}} = 1.10533$$

$$\varphi = 1.10533\varphi_Y = (1.10533)(246.86 \times 10^{-3}) = 272.86 \times 10^{-3} \text{ rad}$$

$\varphi = 15.63°$ ◀

PROBLEM 3.104

A 18-mm-diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_Y = 145\,\text{MPa}$ and $G = 77\,\text{GPa}$. For a 1.2-m length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 200 N · m torque.

SOLUTION

$$\tau_Y = 145 \times 10^6\ \text{Pa}, \quad c = \frac{1}{2}d = 0.009\ \text{m}, \quad L = 1.2\ \text{m}, \quad T = 200\ \text{N}\cdot\text{m}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.009)^3(145 \times 10^6) = 166.04\ \text{N}\cdot\text{m}$$

$T > T_Y$ (plastic region with elastic core) $\qquad \tau_{\max} = \tau_Y = 145\,\text{MPa}$ ◀

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(166.04)(1.2)}{\pi(0.009)^4(77 \times 10^9)} = 251.08 \times 10^{-3}\ \text{radians}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi^3}{\varphi_Y^3}\right)$$

$$\left(\frac{\varphi_Y}{\varphi}\right)^3 = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(200)}{166.04} = 0.38641 \qquad \frac{\varphi_Y}{\varphi} = 0.72837$$

$$\varphi = \frac{\varphi}{0.72837} = \frac{251.08 \times 10^{-3}}{0.72837} = 344.7 \times 10^{-3}\ \text{radians}$$

$\varphi = 19.75°$ ◀

PROBLEM 3.105

A solid circular rod is made of a material that is assumed to be elastoplastic. Denoting by T_Y and φ_Y, respectively, the torque and the angle of twist at the onset of yield, determine the angle of twist if the torque is increased to (*a*) $T = 1.1\,T_Y$, (*b*) $T = 1.25\,T_Y$, (*c*) $T = 1.3\,T_Y$.

SOLUTION

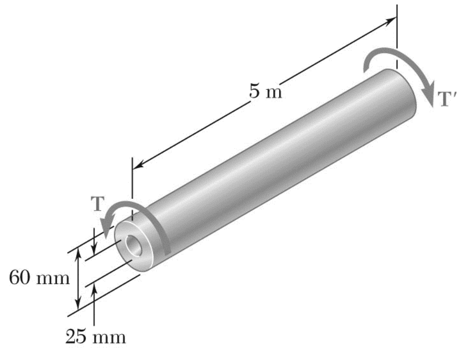
$$T = \frac{4}{3}\,T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y{}^3}{\varphi^3}\right)$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{3T}{T_Y}} \quad \text{or} \quad \frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \dfrac{3T}{T_Y}}}$$

(*a*) $\dfrac{T}{T_Y} = 1.10 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.10)}} = 1.126$ $\qquad \varphi = 1.126\,\varphi_Y$ ◀

(*b*) $\dfrac{T}{T_Y} = 1.25 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.25)}} = 1.587$ $\qquad \varphi = 1.587\,\varphi_Y$ ◀

(*c*) $\dfrac{T}{T_Y} = 1.3 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.3)}} = 2.15$ $\qquad \varphi = 2.15\,\varphi_Y$ ◀



PROBLEM 3.106

The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145\text{ MPa}$ and $G = 77.2\text{ GPa}$. Determine the magnitude T of the torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.

SOLUTION

(*a*) At the onset of yield, the stress distribution is the elastic distribution with $\tau_{\max} = \tau_Y$.

$$c_2 = \frac{1}{2}d_2 = 0.030\text{ m},\quad c_1 = \frac{1}{2}d_1 = 0.0125\text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.0125^4) = 1.2340\times 10^{-6}\text{ m}^4$$

$$\tau_{\max} = \tau_Y = \frac{T_Y c_2}{J}\quad \therefore\quad T_Y = \frac{J\tau_Y}{c_2} = \frac{(1.2340\times 10^{-6})(145\times 10^6)}{0.030} = 5.9648\times 10^3\text{ N}\cdot\text{m}$$

$T_Y = 5.96\text{ kN}\cdot\text{m}$ ◀

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(5.9643\times 10^3)(5)}{(77.2\times 10^9)(2.234010^{-6})} = 313.04\times 10^{-3}\text{ rad}$$

$\varphi_Y = 17.94°$ ◀

(*b*) $t = 0.010\text{ m}\qquad \rho_Y = c_2 - t = 0.030 - 0.010 = 0.020\text{ m}$

$$\gamma = \frac{\rho\varphi}{L} = \frac{\rho_Y\varphi}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi = \frac{\tau_Y L}{G\rho_Y} = \frac{(145\times 10^6)(5)}{(77.2\times 10^9)(0.020)} = 469.56\times 10^{-3}\text{ rad}$$

$\varphi = 26.9°$ ◀

Torque T_1 carried by elastic portion: $\quad c_1 \le \rho \le \rho_Y$

$\tau = \tau_Y$ at $\rho = \rho_Y$. $\qquad \tau_Y = \dfrac{T_1\rho_Y}{J_1}\qquad$ where $\qquad J_1 = \dfrac{\pi}{2}\left(\rho_Y^4 - c_1^4\right)$

$$J_1 = \frac{\pi}{2}(0.020^4 - 0.0125^4) = 212.978\times 10^{-9}\text{ m}^4$$

$$T_1 = \frac{J_1\tau_Y}{\rho_Y} = \frac{(212.978\times 10^{-9})(145\times 10^6)}{0.020} = 1.5441\times 10^3\text{ N}\cdot\text{m}$$

PROBLEM 3.106 (*Continued*)

Torque T_2 carried by plastic portion:

$$T_2 = 2\pi\int_{\rho_Y}^{c_2} \tau_Y \rho^2 d\rho = 2\pi\, \tau_Y \left.\frac{\rho^3}{3}\right|_{\rho_Y}^{c_2} = \frac{2\pi}{3}\tau_Y\left(c_2^3 - \rho_Y^3\right)$$

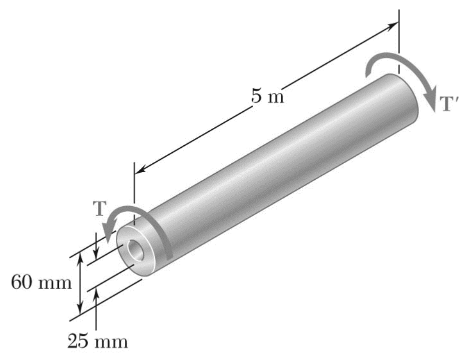
$$= \frac{2\pi}{3}(145\times 10^6)(0.030^3 - 0.020^3) = 5.7701\times 10^3 \text{ N}\cdot\text{m}$$

Total torque:

$$T = T_1 + T_2 = 1.5541\times 10^3 + 5.7701\times 10^3 = 7.3142\times 10^3 \text{ N}\cdot\text{m}$$

$T = 7.31 \text{ kN}\cdot\text{m}.$ ◀

PROBLEM 3.107



For the shaft of Prob. 3.106, determine (*a*) angle of twist at which the section first becomes fully plastic, (*b*) the corresponding magnitude T of the applied torque. Sketch the $T - \phi$ curve for the shaft.

PROBLEM 3.106 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. Determine the magnitude T of the torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.0125 \text{ m} \qquad\qquad c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}$$

(*a*) For onset of fully plastic yielding, $\rho_Y = c_1$

$$\tau = \tau_Y \quad \therefore \quad \gamma = \frac{\tau_Y}{G} = \frac{\rho_Y \varphi}{L} = \frac{c_1 \varphi}{L}$$

$$\varphi_f = \frac{L\tau_Y}{c_1 G} = \frac{(25)(145\times 10^6)}{(0.0125)(77.2\times 109)} = 751.295\times 10^{-3} \text{ rad} \qquad\qquad \varphi_f = 43.0° \blacktriangleleft$$

(*b*) $$T_P = 2\pi\int_{c_1}^{c_2} \tau_Y \rho^2 d\rho = 2\pi\, \tau_Y \left.\frac{\rho^3}{3}\right|_{c_1}^{c_2} = \frac{2\pi}{3}\tau_Y\left(c_2^3 - c_1^3\right)$$

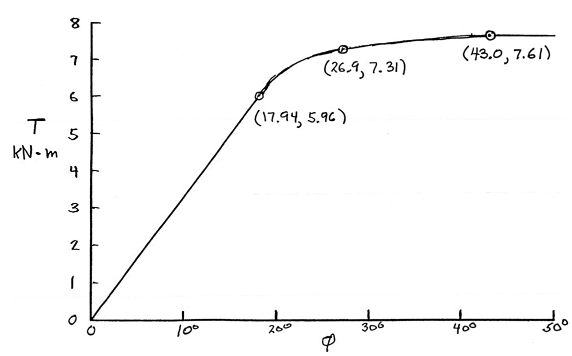
$$= \frac{2\pi}{3}(145\times 10^6)(0.030^3 - 0.0125^3) = 7.606\times 10^3 \text{ N}\cdot\text{m} \qquad\qquad T_P = 7.61 \text{ kN}\cdot\text{m} \blacktriangleleft$$

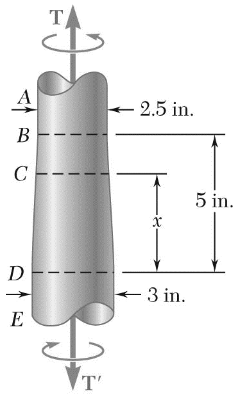
From Prob. 3.101, $\varphi_Y = 17.94°$ $\quad T_Y = 5.96$ kN · m

Also from Prob. 3.101, $\varphi = 26.9°$ $\quad T = 7.31$ kN · m

Plot T vs φ using the following data.

φ, deg	0	17.94	26.9	43.0	>43.0
T kN · m	0	5.96	7.31	7.61	7.61





PROBLEM 3.108

A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 75\text{ kip}\cdot\text{in.}$ are applied. Assuming the steel to be elastoplastic with $\tau_Y = 21\text{ ksi}$ and $G = 11.2\times10^6\text{ psi}$, determine (*a*) the radius of the elastic core in portion *AB* of the shaft, (*b*) the length of portion *CD* that remains fully elastic.

SOLUTION

(*a*) In portion *AB*. $\qquad c = \dfrac{1}{2}d = 1.25\text{ in.}$

$$T_Y = \frac{J_{AB}\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21\times10^3) = 64.427\times10^3\text{ lb}\cdot\text{in}$$

$$T = \frac{4}{3}T_Y\left(1-\frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y}{c} = \sqrt[3]{4-\frac{3T}{T_Y}} = \sqrt[3]{4-\frac{(3)(75\times10^3)}{64.427\times10^3}} = 0.79775$$

$$\rho_Y = 0.79775c = (0.79775)(1.25) = 0.99718\text{ in.}$$

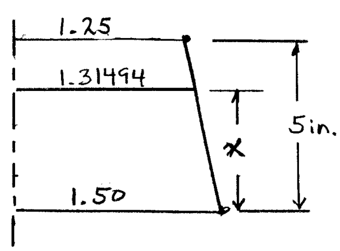
$\rho_Y = 0.997\text{ in.}$ ◀

(*b*) For yielding at point *C*. $\qquad \tau = \tau_Y,\quad c = c_x,\quad T = 75\times10^3\text{ lb}\cdot\text{in}$

$$T = \frac{J_C\tau_Y}{c_x} = \frac{\pi}{2}c_x^3\tau_Y$$

$$c_x = \sqrt[3]{\frac{2T}{\pi\tau_Y}} = \sqrt[3]{\frac{(2)(75\times10^3)}{\pi(21\times10^3)}} = 1.31494\text{ in.}$$

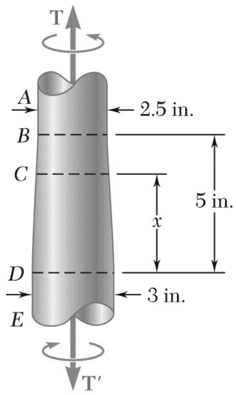
Using proportions from the sketch,



$$\frac{1.50-1.31494}{1.50-1.25} = \frac{x}{5}$$

$x = 3.70\text{ in.}$ ◀

PROBLEM 3.109



If the torques applied to the tapered shaft of Prob. 3.108 are slowly increased, determine (*a*) the magnitude *T* of the largest torques that can be applied to the shaft, (*b*) the length of the portion *CD* that remains fully elastic.

PROBLEM 3.108 A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 75$ kip · in. are applied. Assuming the steel to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi, determine (*a*) the radius of the elastic core in portion *AB* of the shaft, (*b*) the length of portion *CD* that remains fully elastic.

SOLUTION

(*a*) The largest torque that may be applied is that which makes portion *AB* fully plastic.

In portion *AB*, $c = \frac{1}{2}d = 1.25$ in.

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21 \times 10^3) = 64.427 \times 10^3 \text{ lb} \cdot \text{in}$$

For fully plastic shaft, $\rho_Y = 0$ $\quad T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}T$

$$T = \frac{4}{3}(64.427 \times 10^3) = 85.903 \times 10^3 \text{ lb} \cdot \text{in}$$

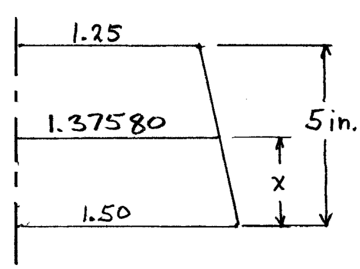
$T = 85.9$ kip · in ◀

(*b*) For yielding at point *C*, $\tau = \tau_Y$, $\quad c = c_x$, $\quad T = 85.903 \times 10^3$ lb · in

$$\tau_Y = \frac{Tc_x}{J_x} = \frac{2T}{\pi c_x^3}$$

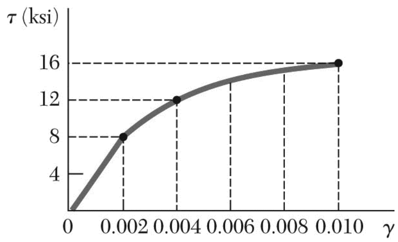
$$c_x = \sqrt[3]{\frac{2T}{\pi\tau_Y}} = \sqrt[3]{\frac{(2)(85.903 \times 10^3)}{\pi(21 \times 10^3)}} = 1.37580 \text{ in.}$$

Using proportions from the sketch,



$$\frac{1.50 - 1.37580}{1.50 - 1.25} = \frac{x}{5}$$

$x = 2.48$ in. ◀



PROBLEM 3.110

A hollow shaft of outer and inner diameters respectively equal to 0.6 in. and 0.2 in. is fabricated from an aluminum alloy for which the stress-strain diagram is given in the diagram shown. Determine the torque required to twist a 9-in. length of the shaft through 10°.

SOLUTION

$$\varphi = 10° = 174.53 \times 10^{-3} \text{ rad}$$

$$c_1 = \frac{1}{2}d_1 = 0.100 \text{ in}, \quad c_2 = \frac{1}{2}d_2 = 0.300 \text{ in.}$$

$$\gamma_{\max} = \frac{c_2\varphi}{L} = \frac{(0.300)(174.53 \times 10^{-3})}{9} = 0.00582$$

$$\gamma_{\min} = \frac{c_1\varphi}{L} = \frac{(0.100)(174.53 \times 10^{-3})}{9} = 0.00194$$

Let
$$z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c_2} \qquad z_1 = \frac{c_1}{c_2} = \frac{1}{3}$$

$$T = 2\pi\int_{c_1}^{c_2} \rho^2 \tau \, d\rho = 2\pi c_2^3 \int_{z_1}^{1} z^2 \tau \, dz = 2\pi c_2^3 I$$

where the integral I is given by
$$I = \int_{1/3}^{1} z^2 \tau \, dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \Sigma w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = \frac{1}{6}$, we get the values given in the table below.

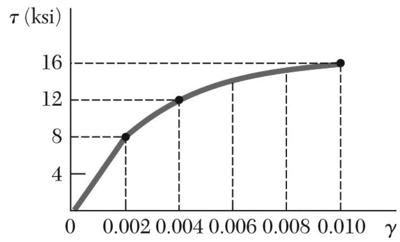
z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
1/3	0.00194	8.0	0.89	1	0.89
1/2	0.00291	10.0	2.50	4	10.00
2/3	0.00383	11.5	5.11	2	10.22
5/6	0.00485	13.0	9.03	4	36.11
1	0.00582	14.0	14.0	1	14.00
					71.22 ⟵ $\Sigma wz^2\tau$

$$I = \frac{(1/6)(71.22)}{3} = 3.96 \text{ ksi}$$

$$T = 2\pi c_2^3 I = 2\pi(0.300)^3(3.96) = 0.671 \text{ kip} \cdot \text{in}$$

$T = 671$ lb · in ◀

Note: Answer may differ slightly due to differences of opinion in reading the stress-strain curve.



PROBLEM 3.111

Using the stress-strain diagram shown, determine (*a*) the torque that causes a maximum shearing stress of 15 ksi in a 0.8-in.-diameter solid rod, (*b*) the corresponding angle of twist in a 20-in. length of the rod.

SOLUTION

(*a*) $\tau_{\max} = 15 \text{ ksi} \qquad c = \frac{1}{2}d = 0.400 \text{ in.}$

From the stress-strain diagram, $\gamma_{\max} = 0.008$

Let $$z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c}$$

$$T = 2\pi\int_0^c \rho^2 \tau \, d\rho = 2\pi c^3 \int_0^1 z^2 \tau \, dz = 2\pi c^3 I$$

where the integral I is given by $I = \int_0^1 z^2 \tau \, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \Sigma w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below.

z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
0	0.000	0	0.000	1	0.00
0.25	0.002	8	0.500	4	2.00
0.5	0.004	12	3.000	2	6.00
0.75	0.006	14	7.875	4	31.50
1.0	0.008	15	15.000	1	15.00
					54.50 ⟵ $\Sigma wz^2\tau$

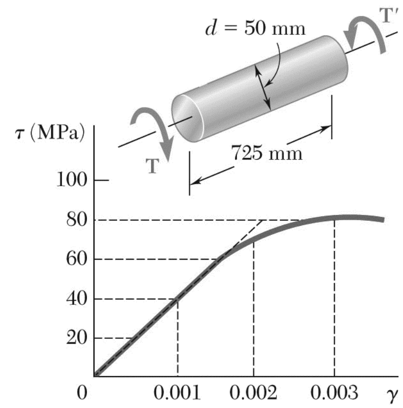
$$I = \frac{(0.25)(54.50)}{3} = 4.54 \text{ ksi}$$

$T = 2\pi c^3 I = 2\pi(0.400)^3(4.54)$ $\qquad T = 1.826 \text{ kip} \cdot \text{in}$ ◀

(*b*) $\gamma_{\max} = \frac{c\varphi}{L}$

$\varphi = \frac{L\gamma_m}{c} = \frac{(20)(0.008)}{0.400} = 400 \times 10^{-3} \text{ rad}$ $\qquad \varphi = 22.9°$ ◀

Note: Answers may differ slightly due to differences of opinion in reading the stress-strain curve.



PROBLEM 3.112

A 50-mm-diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is 5° in a 725-mm length, determine by approximate means the magnitude T of torque applied to the shaft

SOLUTION

$$\varphi = 5^\circ = 87.266 \times 10^{-3} \text{rad} \qquad c = \frac{1}{2}d = 0.025\,\text{m}, \quad L = 0.725\text{ m}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.025)(87.266 \times 10^{-3})}{0.725} = 0.00301$$

Let $$z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c}$$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c^3 \int_0^1 z^2 \tau dz = 2\pi c^3 I \qquad \text{where the integral I is given by } I = \int_0^1 z^2 \tau dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below.

z	γ	τ, MPa	$z^2\tau$, MPa	w	$wz^2\tau$, MPa
0	0	0	0	1	0
0.25	0.00075	30	1.875	4	7.5
0.5	0.0015	55	13.75	2	27.5
0.75	0.00226	75	42.19	4	168.75
1.0	0.00301	80	80	1	80
					283.75

$\leftarrow \sum w z^2 \tau = 283.75 \times 10^6$ Pa

$$I = \frac{(0.25)(283.75 \times 10^6)}{3} = 23.65 \times 10^6 \text{ Pa}$$

$$T = 2\pi c^3 I = 2\pi(0.025)^3(23.65 \times 10^6) = 2.32 \times 10^3 \text{ N} \cdot \text{m}$$

$T = 2.32 \text{ kN} \cdot \text{m}$ ◀

PROBLEM 3.113

Three points on the nonlinear stress-strain diagram used in Prob. 3.112 are (0,0), (0.0015,55 MPa), and (0.003,80 MPa). By fitting the polynomial $\tau = A + B\gamma + C\gamma^2$ through these points, the following approximate relation has been obtained.

$$T = 46.7 \times 10^9 \gamma - 6.67 \times 10^{12} \gamma^2$$

Solve Prob. 3.113 using this relation, Eq. (3.2), and Eq. (3.26).

PROBLEM 3.112 A 50-mm diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is 5° in a 725-mm length, determine by approximate means the magnitude T of torque applied to the shaft.

SOLUTION

$$\varphi = 5° = 87.266 \times 10^{-3} \text{ rad}, \quad c = \frac{1}{2}d = 0.025 \text{ m}, \quad L = 0.725 \text{ m}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.025)(87.266 \times 10^{-3})}{0.725} = 3.009 \times 10^{-3}$$

Let $\quad z = \dfrac{\gamma}{\gamma_{\max}} = \dfrac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c^3 \int_0^1 z^2 \tau dz$$

The given stress-strain curve is

$$\tau = A + B\gamma + C\gamma^2 = A + B\gamma_{\max} z + C\gamma_{\max}^2 z^2$$

$$T = 2\pi c^3 \int_0^1 z^2 \left(A + B\gamma_{\max} z + C\gamma_{\max}^2 z^2 \right) dz$$

$$= 2\pi c^3 \left\{ A\int_0^1 z^2\, dz + B\gamma_{\max} \int_0^1 z^3 dz + C\gamma_{\max}^2 \int_0^1 z^4 dz \right\}$$

$$= 2\pi c^2 \left\{ \frac{1}{3}A + \frac{1}{4}B\gamma_{\max} + \frac{1}{5}C\gamma_{\max}^2 \right\}$$

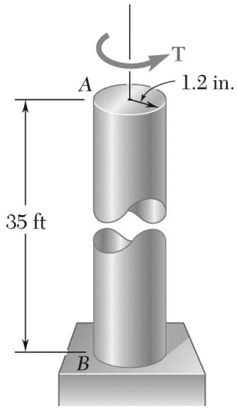
Data: $\quad A = 0, \quad B = 46.7 \times 10^9, \quad C = -6.67 \times 10^{12}$

$$\frac{1}{3}A = 0, \quad \frac{1}{4}B\gamma_{\max} = \frac{1}{4}(46.7 \times 10^9)(3.009 \times 10^{-3}) = 35.13 \times 10^3$$

$$\frac{1}{5}C\gamma_{\max}^2 = -\frac{1}{5}(6.67 \times 10^{12})(3.009 \times 10^{-3})^2 = -12.08 \times 10^3$$

$$T = 2\pi(0.025)^3(0 + (35.13 \times 10^3 - 12.08 \times 10^3) = 2.26 \times 10^3 \text{ N} \cdot \text{m}$$

$T = 2.26 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 3.114

The solid circular drill rod AB is made of a steel that is assumed to be elastoplastic with $\tau_Y = 22$ ksi and $G = 11.2 \times 10^6$ psi. Knowing that a torque $T = 75$ kip · in. is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

SOLUTION

$$c = 1.2 \text{ in.} \qquad L = 35 \text{ ft} = 420 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.2)^4 = 3.2572 \text{ in}^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{(3.2572)(22)}{1.2} = 59.715 \text{ kip} \cdot \text{in}$$

Loading: $T = 75$ kip · in

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(75)}{59.715} = 0.23213$$

$$\frac{\rho_Y}{c} = 0.61458, \qquad \rho_Y = 0.61458c = 0.73749 \text{ in.}$$

Unloading: $\tau' = \dfrac{T\rho}{J}$ where $T = 75$ kip · in

At $\rho = c$ $\qquad \tau' = \dfrac{(75)(1.2)}{3.2572} = 27.63$ ksi

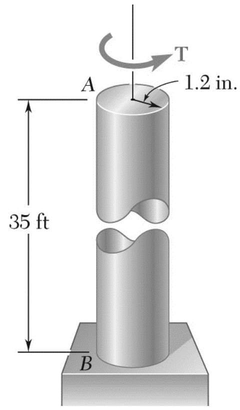
At $\rho = \rho_Y$ $\qquad \tau' = \dfrac{(75)(0.73749)}{3.2572} = 16.98$ ksi

Residual: $\tau_{\text{res}} = \tau_{\text{load}} - \tau'$

At $\rho = c$ $\qquad \tau_{\text{res}} = 22 - 27.63 = -5.63$ ksi

At $\rho = \rho_Y$ $\qquad \tau_{\text{res}} = 22 - 16.98 = 5.02$ ksi

maximum $\tau_{\text{res}} = 5.63$ ksi ◀



PROBLEM 3.115

In Prob. 3.114, determine the permanent angle of twist of the rod.

PROBLEM 3.114 The solid circular drill rod *AB* is made of steel that is assumed to be elastoplastic with $\tau_Y = 22\text{ ksi}$ and $G = 11.2 \times 10^6\text{ psi}$. Knowing that a torque $T = 75\text{kip} \cdot \text{in.}$ is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

SOLUTION

From the solution to Prob. 3.114,

$$c = 1.2\text{ in.}$$

$$J = 3.2572\text{ in}^4$$

$$\frac{\rho_Y}{c} = 0.61458$$

$$\rho_Y = 0.73749\text{ in.}$$

After loading, $\gamma = \dfrac{\rho\varphi}{L} \quad \therefore \quad \varphi = \dfrac{L\gamma}{\rho} = \dfrac{L\gamma_Y}{\rho_Y} = \dfrac{L\tau_Y}{\rho_Y G} \qquad L = 35\text{ ft} = 420\text{ in.}$

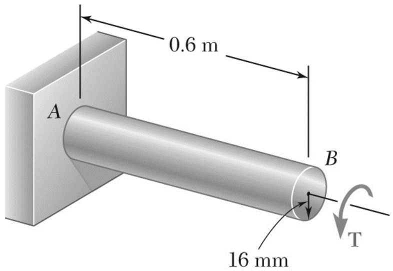
$$\varphi_{\text{load}} = \frac{(420)(22 \times 10^3)}{(0.73749)(11.2 \times 10^6)} = 1.11865\text{ rad} = 64.09°$$

During unloading, $\varphi' = \dfrac{TL}{GJ} \quad \text{(elastic)} \qquad T = 5 \times 10^3\text{N} \cdot \text{m}$

$$\varphi' = \frac{(75 \times 10^3)(420)}{(11.2 \times 10^6)(3.2572)} = 0.86347\text{ rad} = 49.47°$$

Permanent angle of twist.

$$\varphi_{\text{perm}} = \varphi_{\text{load}} - \varphi' = 1.11865 - 0.86347 = 0.25518 \qquad \varphi = 14.62° \blacktriangleleft$$



PROBLEM 3.116

The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145\text{ MPa}$ and $G = 77.2\text{ GPa}$. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.

SOLUTION

$$c = 0.016\text{ m} \qquad \varphi = 6° = 104.72 \times 10^{-3}\text{rad}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.016)(104.72 \times 10^{-3})}{0.6} = 0.0027925$$

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{145 \times 10^6}{77.2 \times 10^9} = 0.0018782$$

$$\frac{\rho_Y}{c} = \frac{\gamma_Y}{\gamma_{\max}} = \frac{0.0018}{0.0027925} = 0.67260$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.016)^4 = 102.944 \times 10^{-9}\text{m}^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.016)^3(145 \times 10^6) = 932.93\text{ N} \cdot \text{m}$$

At end of loading. $\quad T_{\text{load}} = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(932.93)\left[1 - \frac{1}{4}(0.67433)^3\right] = 1.14855 \times 10^3\text{ N} \cdot \text{m}$

Unloading: elastic $\quad T' = 1.14855 \times 10^3\text{ N} \cdot \text{m}$

At $\rho = c$ $\quad \tau' = \frac{T'c}{J} = \frac{(1.14855 \times 10^3)(0.016)}{102.944 \times 10^{-9}} = 178.52 \times 10^6\text{ Pa}$

At $\rho = \rho_Y$ $\quad \tau' = \frac{T'c}{J}\frac{\rho_Y}{c} = (178.52 \times 10^6)(0.67433) = 120.38 \times 10^6\text{ Pa}$

$$\varphi' = \frac{T'L}{GJ} = \frac{(1.14855 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 86.71 \times 10^{-3}\text{ rad} = 4.97°$$

Residual: $\quad \tau_{\text{res}} = \tau_{\text{load}} - \tau' \qquad \varphi_{\text{perm}} = \varphi_{\text{load}} - \varphi'$

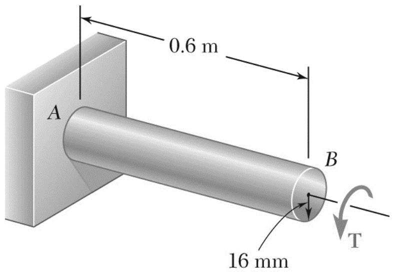
(*a*) At $\rho = c$ $\quad \tau_{\text{res}} = 145 \times 10^6 - 178.52 \times 10^6 = -33.52 \times 10^6\text{ Pa}$ $\quad \tau_{\text{res}} = -33.5\text{ MPa}$

At $\rho = \rho_Y$ $\quad \tau_{\text{res}} = 145 \times 10^6 - 120.38 \times 10^6 = 24.62 \times 10^6\text{ Pa}$ $\quad \tau_{\text{res}} = 24.6\text{ MPa}$

Maximum residual stress: $\quad$ 33.5 MPa at $\rho = 16\text{ mm}$ ◀

(*b*) $\varphi_{\text{perm}} = 104.72 \times 10^{-3} - 86.71 \times 10^{-3} = 17.78 \times 10^{-3}\text{rad}$ $\quad \varphi_{\text{perm}} = 1.032°$ ◀

PROBLEM 3.117



After the solid shaft of Prob. 3.116 has been loaded and unloaded as described in that problem, a torque $\mathbf{T}_1$ of sense opposite to the original torque $\mathbf{T}$ is applied to the shaft. Assuming no change in the value of φ_Y, determine the angle of twist φ_1 for which yield is initiated in this second loading and compare it with the angle φ_Y for which the shaft started to yield in the original loading.

PROBLEM 3.116 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145\text{ MPa}$ and $G = 77.2\text{ GPa}$. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.

SOLUTION

From the solution to Prob. 3.116,

$$c = 0.016\text{ m}, \quad L = 0.6\text{ m}$$

$$\tau_Y = 145 \times 10^6\text{ Pa},$$

$$J = 102.944 \times 10^{-9}\text{ m}^4$$

The residual stress at $\rho = c$ is $\qquad \tau_{\text{res}} = 33.5\text{ MPa}$

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_Y - \tau_{\text{res}} = 145 \times 10^6 - 33.5 \times 10^6 = 111.5 \times 10^6\text{ Pa}$$

$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J\tau_1}{c} = \frac{(102.944 \times 10^{-9})(111.5 \times 10^6)}{0.016}$$

$$= 717\text{ N} \cdot \text{m}$$

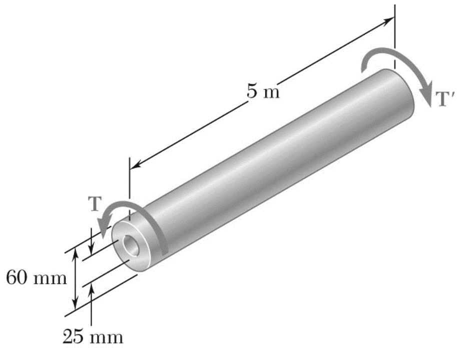
<u>Angle of twist at yielding under reversed torque</u>.

$$\varphi_1 = \frac{T_1 L}{GJ} = \frac{(717 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 54.16 \times 10^{-3}\text{ rad} \qquad \varphi_1 = 3.10° \blacktriangleleft$$

<u>Angle of twist for yielding in original loading</u>.

$$\gamma = \frac{\tau_Y}{G} = \frac{c\varphi_Y}{L}$$

$$\varphi_Y = \frac{L\tau_Y}{cG} = \frac{(0.6)(145 \times 10^6)}{(0.016)(77.2 \times 10^9)} = 70.434 \times 10^{-3}\text{ rad} \qquad \varphi_Y = 4.04° \blacktriangleleft$$



PROBLEM 3.118

The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y. The corresponding torque is calculated by integration.

$$dT = \rho\tau\, dA = \rho\tau_Y(2\pi\rho\, d\rho) = 2\pi\,\tau_Y\,\rho^2 d\rho$$

$$T = 2\pi\,\tau_Y \int_{c_1}^{c_2} \rho^2\, d\rho = \frac{2\pi}{3}\tau_Y\left(c_2^3 - c_1^3\right)$$

$$= \frac{2\pi}{3}(145\times10^6)[(30\times10^{-3})^3 - (12.5\times10^{-3})^3] = 7.6064\times10^3 \text{ N}\cdot\text{m}$$

Unloading. $\quad T' = 7.6064\times10^3 \text{ N}\cdot\text{m}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4] = 1.234\times10^6\text{mm}^4 = 1.234\times10^{-6}\text{m}^4$$

$$\tau_1' = \frac{T'c_1}{J} = \frac{(7.6064\times10^3)(12.5\times10^{-3})}{1.234\times10^{-6}} = 77.050\times10^6\text{Pa} = 77.05 \text{ MPa}$$

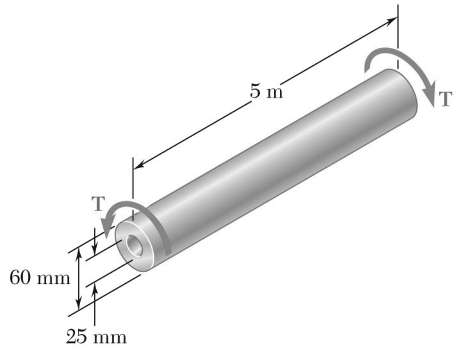
$$\tau_2' = \frac{T'c_2}{J} = \frac{(7.6064\times10^3)(30\times10^{-3})}{1.234\times10^{-6}} = 192.63\times10^6\text{Pa} = 192.63 \text{ MPa}$$

Residual stress.

Inner surface: $\quad \tau_{\text{res}} = \tau_Y - \tau_1' = 145 - 77.05 = 67.95 \text{ MPa}$

Outer surface: $\quad \tau_{\text{res}} = \tau_Y - \tau_2' = 145 - 192.63 = -47.63 \text{ MPa}$

Maximum residual stress: $\quad$ 68.0 MPa at inner surface. ◀



PROBLEM 3.119

In Prob. 3.118, determine the permanent angle of twist of the rod.

PROBLEM 3.118 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145\text{ MPa}$ and $G = 77.2\text{ GPa}$. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 12.5\text{ mm}$$

$$c_2 = \frac{1}{2}d_2 = 30\text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y. The corresponding torque is calculated by integration.

$$dT = \rho\tau\, dA = \rho\tau_Y(2\pi\rho d\rho) = 2\pi\tau_Y\,\rho^2 d\rho$$

$$T = 2\pi\tau_Y\int_{c_1}^{c_2}\rho^2\,d\rho = \frac{2\pi}{3}\tau_Y\left(c_2^3 - c_1^3\right)$$

$$= \frac{2\pi}{3}(145\times10^6)[(30\times10^{-3})^3 - (12.5\times10^{-3})^3] = 7.6064\times10^3\text{ N}\cdot\text{m}$$

Rotation angle at maximum torque.

$$\frac{c_1\varphi_{\max}}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi_{\max} = \frac{\tau_Y L}{Gc_1} = \frac{(145\times10^6)(5)}{(77.2\times10^9)(12.5\times10^{-3})} = 0.75130\text{ rad}$$

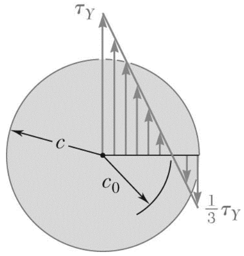
Unloading. $\quad T' = 7.6064\times10^3\text{ N}\cdot\text{m}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4] = 1.234\times10^6\text{mm}^4 = 1.234\times10^{-6}\text{m}^4$$

$$\varphi' = \frac{T'L}{GJ} = \frac{(7.6064\times10^3)(5)}{(77.2\times10^9)(1.234\times10^{-6})} = 0.39922\text{ rad}$$

Permanent angle of twist.

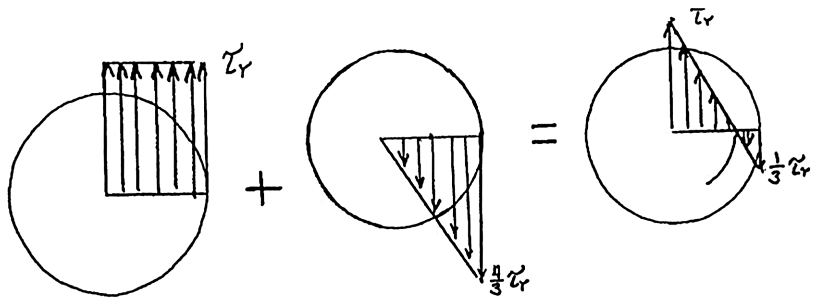
$$\varphi_{\text{perm}} = \varphi_{\max} - \varphi' = 0.75130 - 0.39922 = 0.35208\text{ rad} \qquad \varphi_{\text{perm}} = 20.2° \blacktriangleleft$$



PROBLEM 3.120

A torque **T** applied to a solid rod made of an elastoplastic material is increased until the rod is fully plastic and them removed. (*a*) Show that the distribution of residual shearing stresses is as represented in the figure. (*b*) Determine the magnitude of the torque due to the stresses acting on the portion of the rod located within a circle of radius c_0.

SOLUTION



Loading | Unloading | Residual

(*a*)

After loading: $\rho_Y = 0, \quad T_{\text{load}} = \dfrac{4}{3}T_Y = \dfrac{4}{3}\dfrac{\pi}{2}c^3\tau_Y = \dfrac{2\pi}{3}c^3\tau_Y$

Unloading: $\tau' = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} = \dfrac{2(T_{\text{load}})}{\pi c^3} = \dfrac{4}{3}\tau_Y \quad \text{at } \rho = c$

$$\tau' = \frac{4}{3}\tau_Y\frac{\rho}{c}$$

Residual: $\tau_{\text{res}} = \tau_Y - \dfrac{4}{3}\tau_Y\dfrac{\rho}{c} = \tau_Y\left(1 - \dfrac{4\rho}{3c}\right)$

To find c_0 set, $\tau_{\text{res}} = 0 \quad \text{and} \quad \rho = c_0$

$$0 = 1 - \frac{4c_0}{3c} \quad \therefore \quad c_0 = \frac{3}{4}c$$

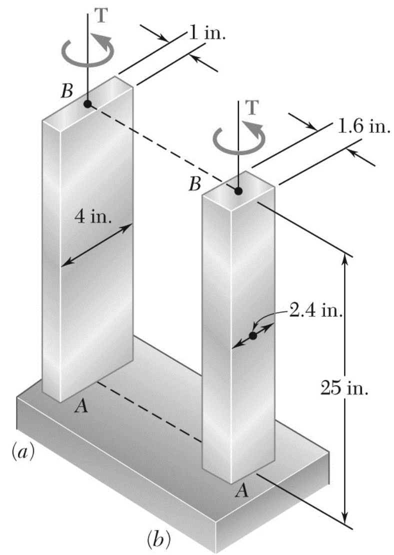
$c_0 = 0.150c$ ◀

(*b*) $T_0 = 2\pi\displaystyle\int_0^{c_0}\rho^2\tau\,d\rho = 2\pi\int_0^{(3/4)c}\rho^2\tau_Y\left(1 - \frac{4}{3}\frac{\rho}{c}\right)d\rho$

$$= 2\pi\tau_Y\left(\frac{\rho^3}{3} - \frac{4}{3}\frac{\rho^4}{4c}\right)\Bigg|_0^{(3/4)c} = 2\pi\tau_Y c^3\left\{\frac{1}{3}\left(\frac{3}{4}\right)^3 - \left(\frac{4}{3}\right)\frac{1}{4}\left(\frac{3}{4}\right)^4\right\}$$

$$= 2\pi\tau_Y c^3\left\{\frac{9}{64} - \frac{27}{256}\right\} = \frac{9\pi}{128}\tau_Y c^3 = 0.2209\,\tau_Y c^3$$

$T_0 = 0.221\tau_Y c^3$ ◀



PROBLEM 3.121

Determine the largest torque **T** that can be applied to each of the two brass bars shown and the corresponding angle of twist at B, knowing that $\tau_{\text{all}} = 12$ ksi and $G = 5.6 \times 10^6$ psi.

SOLUTION

$$L = 25 \text{ in.}, \quad G = 5.6 \times 10^6 \text{ psi}, \quad \tau_{all} = 12 \times 10^3 \text{ psi}$$

$$\tau_{\max} = \frac{T}{c_1 ab^2} \qquad \text{or} \qquad T = c_1 ab^2 \tau_{\max} \tag{1}$$

$$\varphi = \frac{TL}{c_2 ab^3 G} \qquad \text{or} \qquad \varphi = \frac{c_1 L \tau_{\max}}{c_2 bG} \tag{2}$$

(*a*) $a = 4$ in., $b = 1$ in., $\dfrac{a}{b} = 4.0$ From Table 3.1: $c_1 = 0.282, \quad c_2 = 0.281$

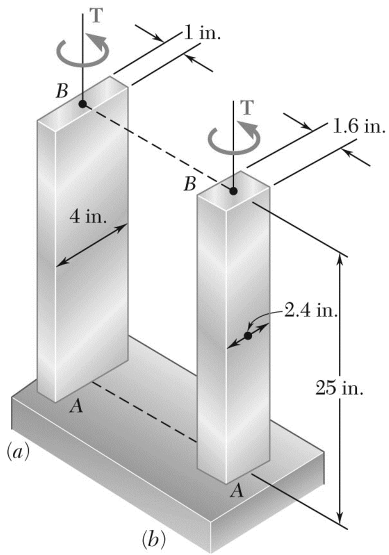
From (1): $T = (0.282)(4)(1)^2(12 \times 10^3) = 13.54 \times 10^3$ $T = 13.54$ kip · in ◀

From (2): $\varphi = \dfrac{(0.282)(25)(12 \times 10^3)}{(0.281)(1)(5.6 \times 10^6)} = 0.05376$ radians $\varphi = 3.08°$ ◀

(*b*) $a = 2.4$ in., $b = 1.6$ in., $\dfrac{a}{b} = 1.5$ From Table 3.1: $c_1 = 0.231, \quad c_2 = 0.1958$

From (1): $T = (0.231)(2.4)(1.6)^2(12 \times 10^3) = 17.03 \times 10^3$ $T = 17.03$ kip · in ◀

From (2): $\varphi = \dfrac{(0.231)(25)(12 \times 10^3)}{(0.1958)(1.6)(5.6 \times 10^6)} = 0.0395$ radians $\varphi = 2.26°$ ◀



PROBLEM 3.122

Each of the two brass bars shown is subjected to a torque of magnitude $T = 12.5\ \text{kip} \cdot \text{in.}$ Knowing that $G = 5.6 \times 10^6\ \text{psi},$ determine for each bar the maximum shearing stress and the angle of twist at B.

SOLUTION

$$L = 25\ \text{in.}, \qquad G = 5.6 \times 10^6\ \text{psi}, \qquad T = 12.5 \times 10^3\ \text{lb} \cdot \text{in}$$

(*a*) $a = 4\ \text{in}, \qquad b = 1\ \text{in}, \qquad \dfrac{a}{b} = 4.0$

From Table 3.1: $\quad c_1 = 0.282, \quad c_2 = 0.281$

$$\tau_{\max} = \frac{T}{c_1 ab^2} = \frac{12.5 \times 10^3}{(0.282)(4)(1)^2} = 11.08 \times 10^3$$

$\tau_{\max} = 11.08\ \text{ksi}$ ◀

$$\varphi = \frac{TL}{c_2 ab^3 G} = \frac{(12.5 \times 10^3)(25)}{(0.282)(4)(1)^3(5.6 \times 10^6)} = 0.04965\ \text{radians}$$

$\varphi = 2.84°$ ◀

(*b*) $a = 2.4\ \text{in.}, \qquad b = 1.6\ \text{in.}, \qquad \dfrac{a}{b} = 1.5$

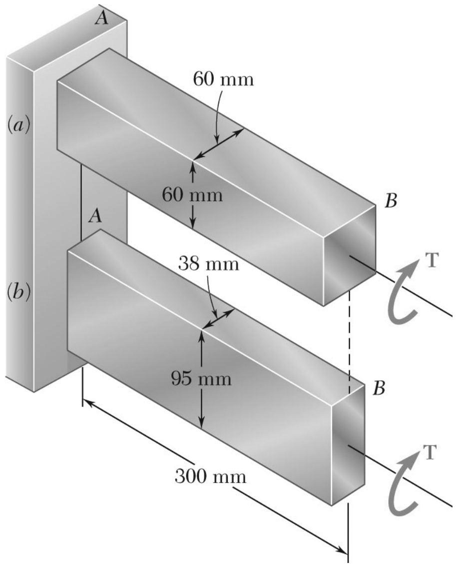
From Table 3.1: $\quad c_1 = 0.231, \quad c_2 = 0.1958$

$$\tau_{\max} = \frac{T}{c_1 ab^2} = \frac{12.5 \times 10^3}{(0.231)(2.4)(1.6)^2} = 8.81 \times 10^3$$

$\tau_{\max} = 8.81\ \text{ksi}$ ◀

$$\varphi = \frac{TL}{c_2 ab^3 G} = \frac{(12.5 \times 10^6)(25)}{(0.1958)(2.4)(1.6)^3(5.6 \times 10^6)} = 0.02899\ \text{radians}$$

$\varphi = 1.661°$ ◀



PROBLEM 3.123

Each of the two aluminium bars shown is subjected to a torque of magnitude $T = 1800\ \text{N}\cdot\text{m}$. Knowing that $G = 26\ \text{GPa}$, determine for each bar the maximum shearing stress and the angle of twist at B.

SOLUTION

$$T = 1800\ \text{N}\cdot\text{m} \qquad L = 0.300\ \text{m} \qquad G = 26\times 10^9\ \text{Pa}$$

(a) $\quad a = b = 60\,\text{mm} = 0.060\,\text{m} \qquad \dfrac{a}{b} = 1.0$

From Table 3.1: $\quad c_1 = 0.208, \quad c_2 = 0.1406$

$$\tau_{\max} = \frac{T}{c_1ab^2} = \frac{1800}{(0.208)(0.060)(0.060)^2} = 40.1\times 10^6\ \text{Pa} \qquad \tau_{\max} = 40.1\,\text{MPa} \blacktriangleleft$$

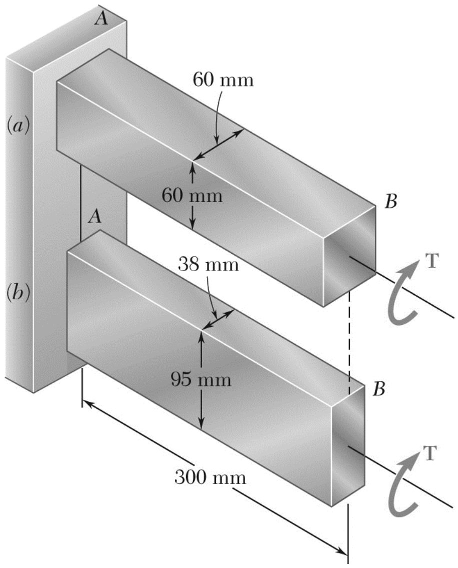
$$\varphi = \frac{TL}{c_2ab^3G} = \frac{(1800)(0.300)}{(0.1406)(0.060)(0.060)^3(26\times 10^9)} = 0.011398\ \text{radians} \qquad \varphi = 0.653^\circ \blacktriangleleft$$

(b) $\quad a = 95\ \text{mm} = 0.095\ \text{m}, \qquad b = 38\ \text{mm} = 0.038\ \text{m}, \qquad \dfrac{a}{b} = 2.5$

From Table 3.1: $\quad c_1 = 0.258, \quad c_2 = 0.249$

$$\tau_{\max} = \frac{T}{c_1ab^2} = \frac{1800}{(0.258)(0.095)(0.038)^2} = 50.9\times 10^6\ \text{Pa} \qquad \tau_{\max} = 50.9\ \text{MPa} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2ab^3G} = \frac{(1800)(0.300)}{(0.249)(0.095)(0.038)^3(26\times 10^9)} = 0.01600\ \text{radians} \qquad \varphi = 0.917^\circ \blacktriangleleft$$



PROBLEM 3.124

Determine the largest torque **T** that can be applied to each of the two aluminium bars shown and the corresponding angle of twist at B, knowing $\tau_{\text{all}} = 50\ \text{MPa}$ and $G = 26\ \text{GPa}$.

SOLUTION

$$L = 0.300\ \text{m}, \quad G = 26 \times 10^9\ \text{Pa}, \quad \tau_{\text{all}} = 50 \times 10^6\ \text{Pa}$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} \qquad \text{or} \qquad T = c_1 a b^2 \tau_{\max} \tag{1}$$

$$\varphi = \frac{TL}{c_2 a b^4 G} \qquad \text{or} \qquad \varphi = \frac{c_1 L \tau_{\max}}{c_2 b G} \tag{2}$$

(a) $a = b = 60\ \text{mm} = 0.060\ \text{m}, \quad \dfrac{a}{b} = 1.0$

From Table 3.1: $c_1 = 0.208, \quad c_2 = 0.1406$

From (1): $T = (0.208)(0.060)(0.060)^2(50 \times 10^6) = 2246\ \text{N} \cdot \text{m}$ $\quad T = 2.25\ \text{kN} \cdot \text{m}$ ◀

From (2): $\varphi = \dfrac{(0.208)(0.300)(50 \times 10^6)}{(0.1406)(0.060)(26 \times 10^9)} = 0.01422\ \text{radians}$ $\quad \varphi = 0.815°$ ◀

(b) $a = 95\ \text{mm} = 0.095\ \text{m}, \quad b = 38\ \text{mm} = 0.038\ \text{m}, \quad \dfrac{a}{b} = 2.5$

From Table 3.1: $c_1 = 0.258, \quad c_2 = 0.249$

From (1): $T = (0.258)(0.095)(0.038)^2(50 \times 10^6) = 1770\ \text{N} \cdot \text{m}$ $\quad T = 1.770\ \text{kN} \cdot \text{m}$ ◀

From (2): $\varphi = \dfrac{(0.258)(0.300)(50 \times 10^6)}{(0.249)(0.038)(26 \times 10^9)} = 0.01573\ \text{radians}$ $\quad \varphi = 0.901°$ ◀

PROBLEM 3.125

Determine the largest allowable square cross section of a steel shaft of length 20 ft if the maximum shearing stress is not to exceed 10 ksi when the shaft is twisted through one complete revolution. Use $G = 11.2 \times 10^6$ psi.

SOLUTION

$$L = 20 \text{ ft} = 240 \text{ in.}$$

$$\tau_{\max} = 10 \text{ ksi} = 10 \times 10^3 \text{ psi}$$

$$\varphi = 1 \text{ rev} = 2\pi \text{ radians}$$

$$\tau_{\max} = \frac{T}{c_1 ab^2} \qquad (1)$$

$$\varphi = \frac{TL}{c_2 ab^3 G} \qquad (2)$$

Divide (2) by (1) to eliminate T.

$$\frac{\varphi}{\tau_{\max}} = \frac{c_1 ab^2 L}{c_2 ab^3 G} = \frac{c_1 L}{c_2 bG}$$

Solve for b.

$$b = \frac{c_1 L \tau_{\max}}{c_2 G \varphi}$$

For a square section,

$$\frac{a}{b} = 1.0$$

From Table 3.1,

$$c_1 = 0.208,$$

$$c_2 = 0.1406$$

$$b = \frac{(0.208)(240)(10 \times 10^3)}{(0.1406)(11.2 \times 10^6)(2\pi)}$$

$b = 0.0505$ in. ◀

PROBLEM 3.126

Determine the largest allowable length of a stainless steel shaft of $\frac{3}{8} \times \frac{3}{4}$-in. cross section if the shearing stress is not to exceed 15 ksi when the shaft is twisted through 15°. Use $G = 11.2 \times 10^6$ psi.

SOLUTION

$$a = \frac{3}{4} \text{ in.} = 0.75 \text{ in.}$$

$$b = \frac{3}{8} \text{ in.} = 0.375 \text{ in.}$$

$$\tau_{\max} = 15 \text{ ksi} = 15 \times 10^3 \text{ psi}$$

$$\varphi = 15° = \frac{15\pi}{180} \text{ rad} = 0.26180 \text{ rad}$$

$$\tau_{\max} = \frac{T}{c_1 ab^2} \qquad (1)$$

$$\varphi = \frac{TL}{c_2 ab^3 G} \qquad (2)$$

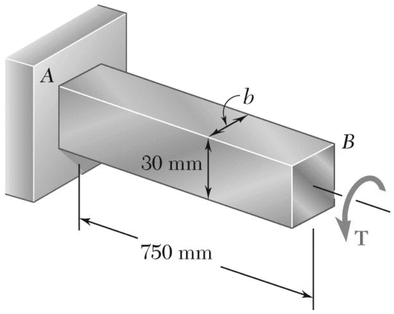
Divide (2) by (1) to eliminate T.
$$\frac{\varphi}{\tau_{\max}} = \frac{c_1 ab^2 L}{c_2 ab^3 G} = \frac{c_1 L}{c_2 bG}$$

Solve for L.
$$L = \frac{c_2 bG\varphi}{c_1 \tau_{\max}}$$

$$\frac{a}{b} = \frac{0.75}{0.375} = 2$$

Table 3.1 gives
$$c_1 = 0.246, \quad c_2 = 0.229$$

$$L = \frac{(0.229)(0.375)(11.2 \times 10^6)(0.26180)}{(0.246)(15 \times 10^3)} = 68.2 \text{ in.} \qquad L = 68.2 \text{ in.} \blacktriangleleft$$



PROBLEM 3.127

The torque **T** causes a rotation of $2°$ at end B of the stainless steel bar shown. Knowing that $b = 20\,\text{mm}$ and $G = 75\,\text{GPa}$, determine the maximum shearing stress in the bar.

SOLUTION

$$a = 30\text{ mm} = 0.030\text{ m}$$

$$b = 20\text{ mm} = 0.020\text{ m}$$

$$\varphi = 2° = 34.907 \times 10^{-3}\text{ rad}$$

$$\varphi = \frac{TL}{c_2ab^3G} \quad \therefore \quad T = \frac{c_2ab^3G\varphi}{L}$$

$$\tau_{\max} = \frac{T}{c_1ab^2} = \frac{c_2ab^3G\varphi}{c_1ab^2L} = \frac{c_2bG\varphi}{c_1L}$$

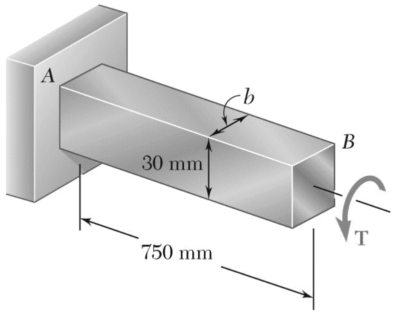
$$\frac{a}{b} = \frac{30}{20} = 1.5.$$

From Table 3.1,

$$c_1 = 0.231$$

$$c_2 = 0.1958$$

$$\tau_{\max} = \frac{(0.1958)(20\times10^{-3})(75\times10^{9})(34.907\times10^{-3})}{(0.231)(750\times10^{-3})} = 59.2\times10^{6}\text{ Pa} \qquad \tau_{\max} = 59.2\text{ MPa} \blacktriangleleft$$



PROBLEM 3.128

The torque **T** causes a rotation of $0.6°$ at end B of the aluminum bar shown. Knowing that $b = 15$ mm and $G = 26$ GPa, determine the maximum shearing stress in the bar.

SOLUTION

$$a = 30 \text{ mm} = 0.030 \text{ m}$$

$$b = 15 \text{ mm} = 0.015 \text{ m}$$

$$\varphi = 0.6° = 10.472 \times 10^{-3} \text{ rad}$$

$$\varphi = \frac{TL}{c_2 ab^3 G} \quad \therefore \quad T = \frac{c_2 ab^3 G\varphi}{c_1 L}$$

$$\tau_{\max} = \frac{T}{c_1 ab^2} = \frac{c_2 ab^3 G\varphi}{c_1 ab^2 L} = \frac{c_2 bG\varphi}{c_1 L}$$

$$\frac{a}{b} = \frac{30}{15} = 2.0$$

From Table 3.1,

$$c_1 = 0.246$$

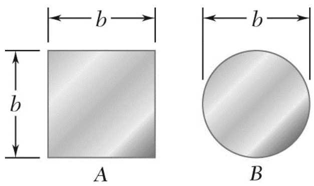
$$c_2 = 0.229$$

$$\tau_{\max} = \frac{(0.229)(15 \times 10^{-3})(26 \times 10^9)(10.472 \times 10^{-3})}{(0.246)(750 \times 10^{-3})}$$

$$= 5.07 \times 10^6 \text{ Pa} \qquad \tau_{\max} = 5.07 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.129

Two shafts are made of the same material. The cross section of shaft A is a square of side b and that of shaft B is a circle of diameter b. Knowing that the shafts are subjected to the same torque, determine the ratio τ_A/τ_B of maximum shearing stresses occurring in the shafts.



SOLUTION

A. Square: $\frac{a}{b} = 1, \quad c_1 = 0.208 \quad \text{(Table 3.1)}$

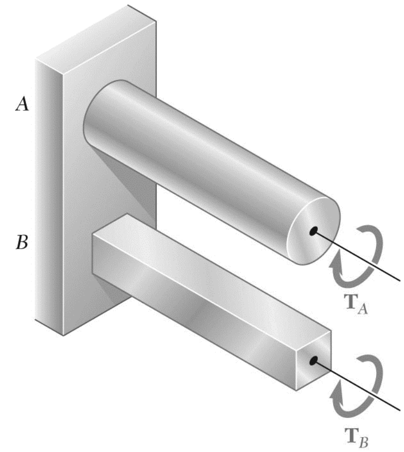
$$\tau_A = \frac{T}{c_1 ab^2} = \frac{T}{0.208b^3}$$

B. Circle: $c = \frac{1}{2}b \quad \tau_B = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$

Ratio: $\frac{\tau_A}{\tau_B} = \frac{1}{0.208} \cdot \frac{\pi}{16} = 0.3005\pi$

$$\frac{\tau_A}{\tau_B} = 0.944 \blacktriangleleft$$

PROBLEM 3.130



Shafts A and B are made of the same material and have the same cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum shearing stresses occurring in A and B, respectively, when the two shafts are subjected to the same torque $(T_A = T_B)$. Assume both deformations to be elastic.

SOLUTION

Let c be the radius of circular section A and b be the side square section B.

For equal areas, $\pi c^2 = b^2 \qquad b = c\sqrt{\pi}$

<u>Circle:</u> $\tau_A = \dfrac{T_{AC}}{J} = \dfrac{2T_A}{\pi c^3}$

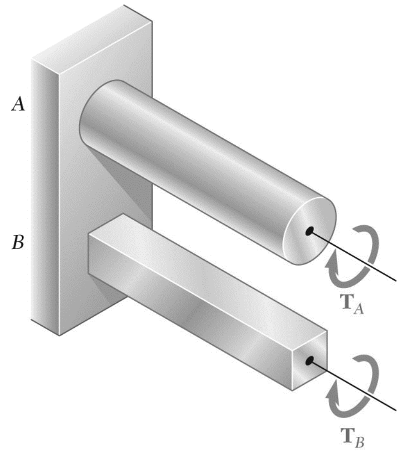
<u>Square:</u> $\dfrac{a}{b} = 1 \qquad c_1 = 0.208$ from Table 3.1

$$\tau_B = \frac{T_B}{c_1 ab^2} = \frac{T_B}{c_1 b^3}$$

Ratio: $\dfrac{\tau_A}{\tau_B} = \dfrac{2T_A}{\pi c^3}\dfrac{c_1 b^3}{T_B} = \dfrac{2c_1 b^3}{\pi c^3}\dfrac{T_A}{T_B} = 2c_1\sqrt{\pi}\,\dfrac{T_A}{T_B}$

For $T_A = T_B$, $\dfrac{\tau_A}{\tau_B} = (2)(0.208)\sqrt{\pi}$ $\qquad \dfrac{\tau_A}{\tau_B} = 0.737$ ◀

PROBLEM 3.131



Shafts A and B are made of the same material and have the same cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum torques T_A and T_B that can be safely applied to A and B, respectively.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas $\pi c^2 = b^2$,

$$c = \frac{b}{\sqrt{\pi}}$$

Circle: $$\tau_A = \frac{T_A c}{J} = \frac{2T_A}{\pi c^3} \quad \therefore \quad T_A = \frac{\pi}{2} c^3 \tau_A$$

Square:

From Table 3.1, $$c_1 = 0.208$$

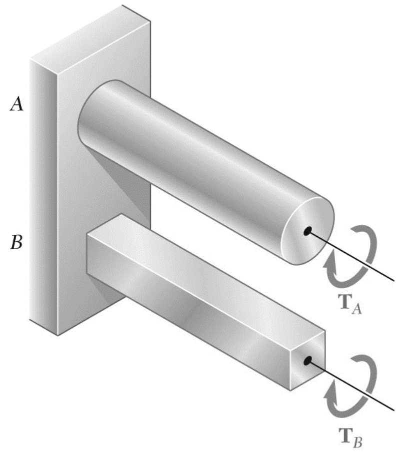
$$\tau_B = \frac{T_A}{c_1 a b^2} = \frac{T_B}{c_1 b^3} \quad \therefore \quad T_B = c_1 b^3 \tau_B$$

Ratio: $$\frac{T_A}{T_B} = \frac{\frac{\pi}{2} c^3 \tau_B}{c_1 b^3 \tau_B} = \frac{\frac{\pi}{2} \cdot \frac{b^3}{\pi^{3/2}} \tau_B}{c_1 b^3 \tau_B} = \frac{1}{2c_1\sqrt{\pi}} \frac{\tau_A}{\tau_B}$$

For the same stresses, $$\tau_B = \tau_A \quad \therefore \quad \frac{T_A}{T_B} = \frac{1}{(2)(0.208)\sqrt{\pi}}$$

$$\frac{T_A}{T_B} = 1.356 \blacktriangleleft$$

PROBLEM 3.132



Shafts A and B are made of the same material and have the same length and cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum values of the angles φ_A and φ_B through which shafts A and B, respectively, can be twisted.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas, $\quad \pi c^2 = b^2 \quad \therefore \quad b = \sqrt{\pi} c$

Circle:
$$\gamma_{\max} = \frac{\tau_A}{G} = \frac{c\varphi_A}{L} \quad \therefore \quad \varphi_A = \frac{L\tau_A}{cG}$$

Square: From Table 3.1, $\quad c_1 = 0.208, \quad c_2 = 0.1406$

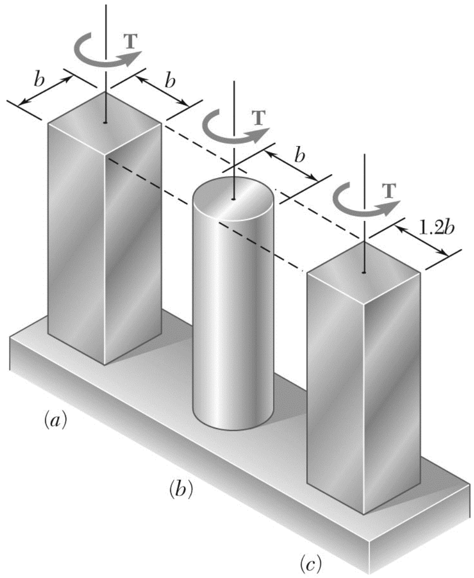
$$\tau_B = \frac{T_B}{c_1 a b^2} = \frac{T_B}{0.208 b^3} \quad \therefore \quad T_B = 0.208\, b^3 \tau_B$$

$$\varphi_B = \frac{T_B L}{c_2 a b^3 G} = \frac{0.208 b^3 \tau_B L}{0.1406 b^4 G} = \frac{1.4794\, L\tau_B}{bG}$$

Ratio:
$$\frac{\varphi_A}{\varphi_B} = \frac{L\tau_A}{cG} \cdot \frac{bG}{1.4794 L\tau_B} = 0.676\frac{b\tau_A}{c\tau_B} = 0.676\sqrt{\pi}\,\frac{\tau_A}{\tau_B}$$

For equal stresses, $\quad \tau_A = \tau_B \qquad \dfrac{\varphi_B}{\varphi_A} = 0.676\sqrt{\pi}$

$$\frac{\varphi_B}{\varphi_A} = 1.198 \blacktriangleleft$$



PROBLEM 3.133

Each of the three aluminum bars shown is to be twisted through an angle of 2°. Knowing that $b = 30$ mm, $\tau_{all} = 50$ MPa, and $G = 27$ GPa, determine the shortest allowable length of each bar.

SOLUTION

$$\varphi = 2° = 34.907 \times 10^{-3} \text{ rad}, \quad \tau = 50 \times 10^6 \text{ Pa} \quad G = 27 \times 10^9 \text{ Pa}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$$

For square and rectangle, $\tau = \dfrac{T}{c_1 ab^2} \qquad \varphi = \dfrac{TL}{c_2 ab^3 G}$

Divide to eliminate T; then solve for L. $\dfrac{\varphi}{\tau} = \dfrac{c_1 ab^2 L}{c_2 ab^3 G} \qquad L = \dfrac{c_2 bG\varphi}{c_1 \tau}$

(*a*) Square: $\dfrac{a}{b} = 1.0$ From Table 3.1, $c_1 = 0.208, \quad c_2 = 0.1406$

$$L = \frac{(0.1406)(0.030)(27 \times 10^9)(34.907 \times 10^{-3})}{(0.208)(50 \times 10^6)} = 382 \times 10^{-3} \text{ m}$$

$L = 382$ mm ◀

(*b*) Circle: $c = \dfrac{1}{2}b = 0.015 \text{ m} \quad \tau = \dfrac{Tc}{J} \quad \varphi = \dfrac{TL}{GJ}$

Divide to eliminate T; then solve for L. $\dfrac{\varphi}{\tau} = \dfrac{JL}{cGJ} = \dfrac{L}{cG}$

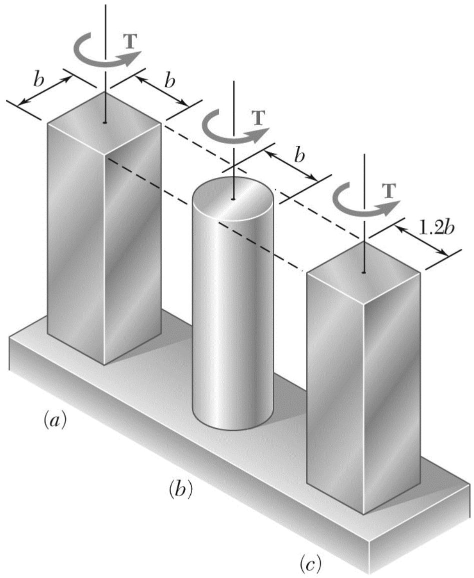
$$L = \frac{cG\varphi}{\tau} = \frac{(0.015)(27 \times 10^9)(34.907 \times 10^{-3})}{50 \times 10^6} = 283 \times 10^{-3} \text{ m}$$

$L = 283$ mm ◀

(*c*) Rectangle: $a = 1.2b \qquad \dfrac{a}{b} = 1.2$ From Table 3.1, $c_1 = 0.219, \quad c_2 = 0.1661$

$$L = \frac{(0.1661)(0.030)(27 \times 10^9)(34.907 \times 10^{-3})}{(0.219)(50 \times 10^6)} = 429 \times 10^{-3} \text{ m}$$

$L = 429$ mm ◀



PROBLEM 3.134

Each of the three steel bars is subjected to a torque as shown. Knowing that the allowable shearing stress is 8 ksi and that $b = 1.4$ in., determine the maximum torque **T** that can be applied to each bar.

SOLUTION

$$\tau_{\max} = 8 \text{ ksi}, \quad b = 1.4 \text{ in.}$$

(*a*) Square: $a = b = 1.4 \text{ in.} \quad \dfrac{a}{b} = 1.0$

From Table 3.1, $c_1 = 0.208$

$$\tau_{\max} = \frac{T}{c_1 ab^2} \qquad T = c_1 ab^2 \tau_{\max}$$

$$T = (0.208)(1.4)(1.4)(1.4)^2(8)$$

$T = 4.57 \text{ kip} \cdot \text{in}$ ◀

(*b*) Circle: $c = \dfrac{1}{2}b = 0.7 \text{ in.}$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2}c^3\tau_{\max}$$

$$T = \frac{\pi}{2}(0.7)^3(8)$$

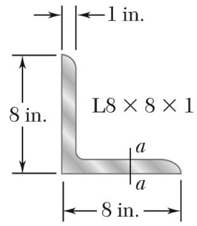
$T = 4.31 \text{ kip} \cdot \text{in}$ ◀

(*c*) Rectangle: $a = (1.2)(1.4) = 1.68 \text{ in.} \quad \dfrac{a}{b} = 1.2$

From Table 3.1, $c_1 = 0.219$

$$T = c_1 ab^2 \tau_{\max} = (0.219)(1.68)(1.4)^2(8)$$

$T = 5.77 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 3.135

A 36-kip · in. torque is applied to a 10-ft-long steel angle with an $L8 \times 8 \times 1$ cross section. From Appendix C, we find that the thickness of the section is 1 in. and that its area is, 15.00 in^2. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the angle of twist.

SOLUTION

$$a = \frac{A}{t} = \frac{15 \text{ in}^2}{1 \text{ in.}} = 15 \text{ in.}, \quad b = 1 \text{ in.}, \quad \frac{a}{b} = 15$$

Since $\quad \dfrac{a}{b} > 5, \quad c_1 = c_2 = \dfrac{1}{3}\left(1 - 0.630\dfrac{b}{a}\right)$

or $\quad c_1 = c_2 = \dfrac{1}{3}\left(1 - \dfrac{0.630}{15}\right) = 0.3193$

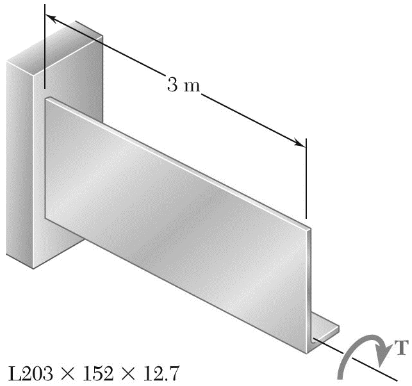
$$T = 36 \times 10^3 \text{ lb} \cdot \text{in}; \quad L = 120 \text{ in.}; \quad G = 11.2 \times 10^6 \text{ psi}$$

(*a*) Maximum shearing stress: $\quad \tau_{\max} = \dfrac{T}{c_1 a b^2}$

$$\tau_{\max} = \frac{36 \times 10^3}{(0.3193)(15)(1)^2} = 7.52 \times 10^3 \text{ psi} \qquad \tau_{\max} = 7.52 \text{ ksi} \blacktriangleleft$$

(*b*) Angle of twist: $\quad \varphi = \dfrac{TL}{c_2 a b^2 G}$

$$\varphi = \frac{(36 \times 10^3)(120)}{(0.3193)(15)(1)^3(11.2 \times 10^6)} = 0.08052 \text{ radians} \qquad \varphi = 4.61° \blacktriangleleft$$



PROBLEM 3.136

A 3-m-long steel angle has an $L203 \times 152 \times 12.7$ cross section. From Appendix C, we find that the thickness of the section is 12.7 mm and that its area is 4350 mm^2. Knowing that $\tau_{all} = 50$ MPa and that $G = 77.2$ GPa, and ignoring the effect of stress concentration, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist.

SOLUTION

$$A = 4350 \text{ mm}^2 \quad b = 12.7 \text{ mm} \quad a = ?$$

Equivalent rectangle.

$$a = \frac{A}{b} = \frac{4350}{12.7} = 342.52 \text{ mm}$$

$$\frac{a}{b} = 26.97$$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.32555$$

(*a*) $\tau_{max} = \dfrac{T}{c_1 ab^2} \quad \tau_{max} = 50 \times 10^6 \text{ Pa}$

$$T = c_1 ab^2 \tau_{max} = (0.32555)(26.97 \times 10^{-3})(12.7 \times 10^{-3})^2 (50 \times 10^6)$$

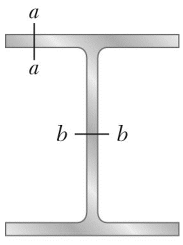
$$= 70.807 \text{ N} \cdot \text{m}$$

$T = 70.8 \text{ N} \cdot \text{m}$ ◀

(*b*) $\varphi = \dfrac{TL}{c_2 ab^3 G} = \dfrac{(70.807)(3)}{(0.32555)(26.97 \times 10^{-3})(12.7 \times 10^{-3})(77.2 \times 10^9)}$

$$= 0.15299 \text{ rad}$$

$\varphi = 8.77°$ ◀



PROBLEM 3.137

An 8-ft-long steel member with a W8 × 31 cross section is subjected to a 5-kip · in. torque. The properties of the rolled-steel section are given in Appendix C. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

SOLUTION

<u>Flange</u>: $a = 7.995 \text{ in.}, \quad b = 0.435 \text{ in.}, \quad \dfrac{a}{b} = \dfrac{7.995}{0.435} = 18.38$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3219 \qquad \varphi_f = \frac{T_f L}{c_2 ab^3 G}$$

$$T_f = c_2 ab^3 \frac{G\varphi_f}{L} = K_f \frac{G\varphi}{L} \quad \text{where} \quad K_f = c_2 ab^3$$

$$K_f = (0.3219)(7.995)(0.435)^3 = 0.2138 \text{ in}^3$$

<u>Web</u>: $a = 8.0 - (2)(0.435) = 7.13 \text{ in.}, \quad b = 0.285 \text{ in.}, \quad \dfrac{a}{b} = \dfrac{7.13}{0.285} = 25.02$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3249 \qquad \varphi_w = \frac{T_w L}{c_2 ab^3 G}$$

$$T_w = c_2 ab^3 \frac{G\varphi_w}{L} = K_w \frac{G\varphi}{L} \quad \text{where} \quad K_w = c_2 ab^3$$

$$K_w = (0.3249)(7.13)(0.285)^3 = 0.0563 \text{ in}^4$$

For matching twist angles: $\varphi_f = \varphi_w = \varphi$

<u>Total torque.</u> $T = 2T_f + T_w = (2K_f + K_w)\dfrac{G\varphi}{L}$

$$\frac{G\varphi}{L} = \frac{T}{2K_p + K_w}, \quad T_f = \frac{K_f T}{2K_f + K_w}, \quad T_w = \frac{K_w T}{2K_f + K_w}$$

$$T_f = \frac{(0.2138)(5000)}{(2)(0.2138) + 0.0563} = 2221 \text{ lb} \cdot \text{in}; \quad T_w = \frac{(0.0563)(5000)}{(2)(0.2138) + 0.0563} = 557 \text{ lb} \cdot \text{in}$$

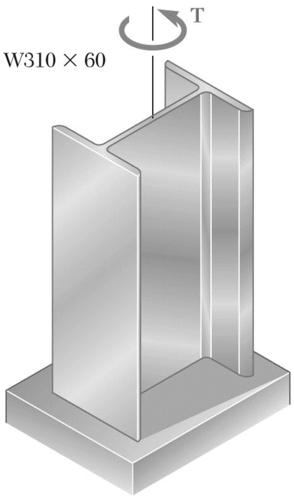
(*a*) $\tau_f = \dfrac{T_f}{c_1 ab^2} = \dfrac{2221}{(0.3219)(7.995)(0.435)^2} = 4570 \text{ psi}$ $\quad\tau_f = 4.57 \text{ ksi}$ ◀

(*b*) $\tau_w = \dfrac{T_w}{c_1 ab^2} = \dfrac{557}{(0.3249)(7.13)(0.285)^2} = 2960 \text{ psi}$ $\quad\tau_w = 2.96 \text{ ksi}$ ◀

(*c*) $\dfrac{G\varphi}{L} = \dfrac{T}{2K_f + K_w} \quad \therefore \quad \varphi = \dfrac{TL}{G(2K_f + K_w)} \quad \text{where} \quad L = 8 \text{ ft} = 96 \text{ in.}$

$$\varphi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.2138) + 0.563]} = 88.6 \times 10^{-3} \text{ rad}$$

$\varphi = 5.08°$ ◀



PROBLEM 3.138

A 4-m-long steel member has a W310 × 60 cross section. Knowing that $G = 77.2\text{ GPa}$ and that the allowable shearing stress is 40 MPa, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (See hint of Prob. 3.137.)

SOLUTION

$$\text{W}310 \times 60, \quad L = 4\text{ m}, \quad G = 77.2\text{ GPa}, \quad \tau_{\text{all}} = 40\text{ MPa}$$

<u>For one flange:</u> From App. C, $a = 203\text{ mm}, \quad b = 13.1\text{ mm}, \quad a/b = 15.50$

Eq. (3.45): $$c_1 = c_2 = \frac{1}{3}\left(1 - \frac{0.630}{15.50}\right) = 0.320$$

Eq. (3.44): $$\phi_f = \frac{T_f L}{c_2 ab^3 G} = \frac{T_f(4)}{0.320(0.203)(0.0131)^3(77.2 \times 10^9)} \tag{1}$$

$$\phi_f = 355.04 \times 10^{-6} T_f$$

<u>For web</u>: From App. C, $a = 303 - 2(13.1) = 276.8\text{ mm}, \quad b = 7.5\text{ mm}, \quad a/b = 36.9$

Eq. (3.45): $$c_1 = c_2 = \frac{1}{3}\left(1 - \frac{0.630}{36.9}\right) = 0.328$$

Eq. (3.44): $$\phi_w = \frac{T_w(4)}{0.328(0.2768)(0.0075)^3(77.2 \times 10^9)}$$

$$\phi_w = 1.354.2 \times 10^{-6} T_w \tag{2}$$

Since angle of twist is the same for flanges and web:

$$\phi_f = \phi_w: \quad 355.04 \times 10^{-6} T_f = 1354.2 \times 10^{-6} T_w$$
$$T_f = 3.814 T_w \tag{3}$$

But the sum of the torques exerted on the two flanges and on the web is equal to the torque **T** applied to the member:

$$2T_f + T_w = T \tag{4}$$

PROBLEM 3.138 (*Continued*)

Substituting for T_f from (3) into (4):

$$2(3.814T_w) + T_w = T \qquad T_w = 0.11589T \tag{5}$$

From (3): $$T_f = 3.814(0.11589T) \qquad T_f = 0.44205T \tag{6}$$

<u>For one flange:</u>

From Eq. (3.43): $$T_f = c_1 ab^2 \tau_{\max} = 0.320(0.203)(0.0131)^2(40 \times 10^6)$$
$$= 445.91 \text{ N} \cdot \text{m}$$

Eq. (6): $$445.91 = 0.44205T \qquad T = 1009 \text{ N} \cdot \text{m}$$

<u>For web</u>: $$T_w = c_1 ab^2 \tau_{\max} = 0.328(0.2768)(0.0075)^2(40 \times 10^6)$$
$$= 204.28 \text{ N} \cdot \text{m}$$

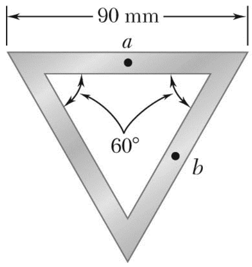
Eq. (5): $$204.28 = 0.11589T \qquad T = 1763 \text{ N} \cdot \text{m}$$

(*a*) <u>Largest allowable torque:</u> Use the smaller value. $T = 1009 \text{ N} \cdot \text{m}$ ◀

(*b*) <u>Angle of twist:</u> Use T_f, which is critical.

Eq. (1): $$\phi = \phi_f = (355.04 \times 10^{-6})(445.91) = 0.15831 \text{ rad} \qquad \phi = 9.07^\circ$$ ◀

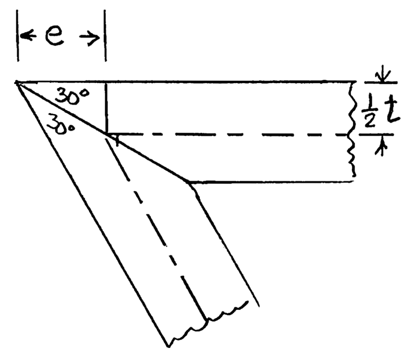
PROBLEM 3.139



A torque $T = 750$ kN · m is applied to a hollow shaft shown that has a uniform 8-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.

SOLUTION

Detail of corner.



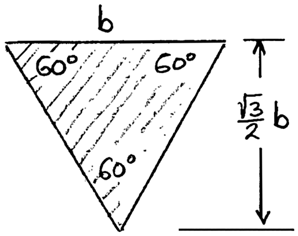
$$\frac{1}{2}t = e \tan 30°$$

$$e = \frac{t}{2 \tan 30°}$$

$$= \frac{8}{2 \tan 30°} = 6.928 \text{ mm}$$

$$b = 90 - 2e = 76.144 \text{ mm}$$

Area bounded by centerline.



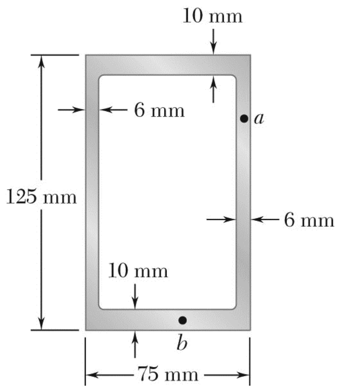
$$a = \frac{1}{2}b\frac{\sqrt{3}}{2}b = \frac{\sqrt{3}}{4}b^2 = \frac{\sqrt{3}}{4}(76.144)^2$$

$$= 2510.6 \text{ mm}^2 = 2510.6 \times 10^{-6} \text{ m}^2$$

$$t = 0.008 \text{ m}$$

$$\tau = \frac{T}{2ta} = \frac{750}{(2)(0.008)(2510 \times 10^{-6})} = 18.67 \times 10^6 \text{ Pa}$$

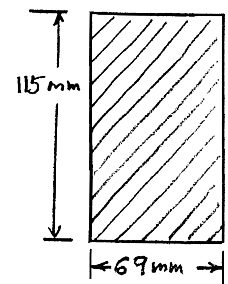
$\tau = 18.67$ MPa ◀



PROBLEM 3.140

A torque $T = 5$ kN · m is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.

SOLUTION



$$T = 5 \times 10^3 \text{ N} \cdot \text{m}$$

Area bounded by centerline.

$$a = \text{bh} = (69)(115) = 7.935 \times 10^3 \text{ mm}^2$$

$$= 7.935 \times 10^{-3} \text{ m}^2$$

At point a:

$$t = 6 \text{ mm} = 0.006 \text{ m}$$

$$\tau = \frac{T}{2ta} = \frac{5 \times 10^3}{(2)(0.006)(7.935 \times 10^{-3})}$$

$$= 52.5 \times 10^6 \text{ Pa}$$

$\tau = 52.5$ MPa ◀

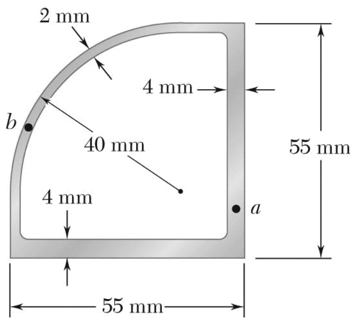
At point b:

$$t = 10 \text{ mm} = 0.010 \text{ m}$$

$$\tau = \frac{T}{2ta} = \frac{5 \times 10^3}{(2)(0.010)(7.935 \times 10^{-3})} = 31.5 \times 10^6 \text{ Pa}$$

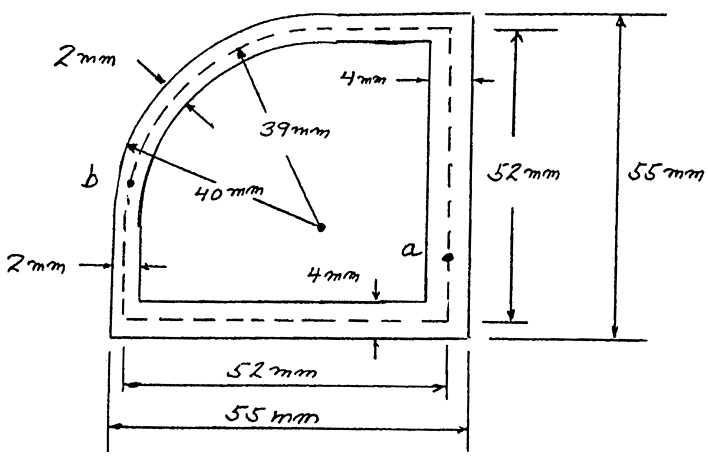
$\tau = 31.5$ MPa ◀

PROBLEM 3.141

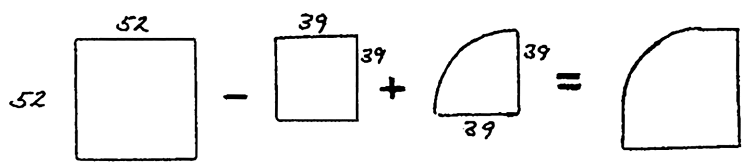


A 90-N · m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.

SOLUTION



Area bounded by centerline.



$$a = 52 \times 52 - 39 \times 39 + \frac{\pi}{4}(39)^2 = 2378 \text{ mm}^2 = 2.378 \times 10^{-3} \text{ m}^2$$

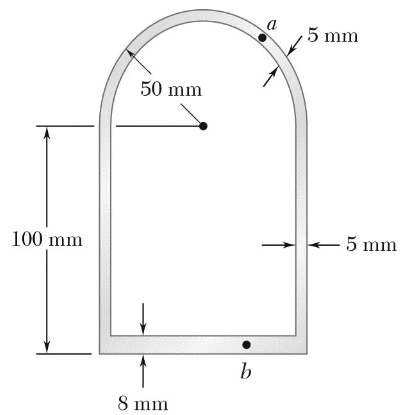
$$T = 90 \text{ N} \cdot \text{m}$$

$$\tau_a = \frac{T}{2ta} = \frac{90 \text{ N} \cdot \text{m}}{2(4 \times 10^{-3} \text{ m})(2.378 \times 10^{-3} \text{ m}^2)}$$

$\tau_a = 4.73 \text{ MPa}$ ◀

$$\tau_b = \frac{T}{2ta} = \frac{90 \text{ N} \cdot \text{m}}{2(2 \times 10^{-3} \text{ m})(2.378 \times 10^{-3} \text{m}^2)}$$

$\tau_b = 9.46 \text{ MPa}$ ◀



PROBLEM 3.142

A 5.6 kN·m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

SOLUTION

Area bounded by centerline.

$$a = (96\,\text{mm})(95\,\text{mm}) + \frac{\pi}{2}(47.5\,\text{mm})^2 = 12.664 \times 10^3\,\text{mm}^2$$

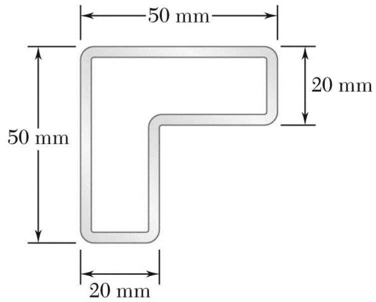
$$= 12.664 \times 10^{-3}\,\text{m}^2$$

At point *a*, $t = 5\,\text{mm} = 0.005\,\text{m}$

$$\tau = \frac{T}{2at} = \frac{5.6 \times 10^3}{(2)(12.664 \times 10^{-3})(0.005)} = 44.2 \times 10^6\,\text{Pa} \qquad \tau = 44.2\text{ MPa} \blacktriangleleft$$

At point *b*, $t = 8\,\text{mm} = 0.008\,\text{m}$

$$\tau = \frac{T}{2at} = \frac{5.6 \times 10^3}{(2)(12.664 \times 10^{-3})(0.008)} = 27.6 \times 10^6\,\text{Pa} \qquad \tau = 27.6\text{ MPa} \blacktriangleleft$$

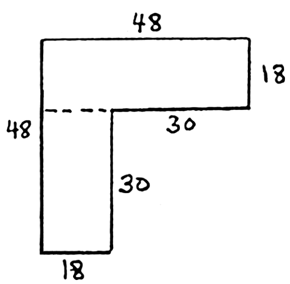


PROBLEM 3.143

A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque that can be applied to the member.

SOLUTION

Area bounded by centerline.



$$a = (48)(18) + (30)(18)$$

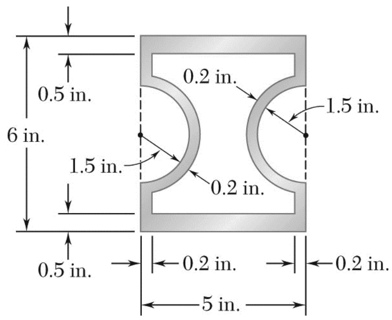
$$= 1404 \text{ mm}^2 = 1404 \times 10^{-6} \text{ m}^2$$

$$t = 0.002 \text{ m}$$

$$\tau = \frac{T}{2ta} \quad \text{or} \quad T = 2ta\tau = (2)(0.002)(1404 \times 10^{-6})(3 \times 10^6)$$

$$T = 16.85 \text{ N} \cdot \text{m} \blacktriangleleft$$

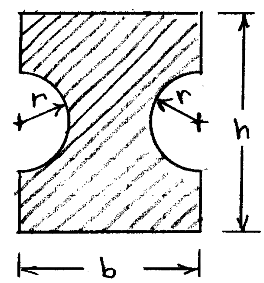
PROBLEM 3.144



A hollow brass shaft has the cross section shown. Knowing that the shearing stress must not exceed 12 ksi and neglecting the effect of stress concentrations, determine the largest torque that can be applied to the shaft.

SOLUTION

Calculate the area bounded by the center line of the wall cross section. The area is a rectangle with two semi-circular cutouts.



$$b = 5 - 0.2 = 4.8 \text{ in.}$$

$$h = 6 - 0.5 = 5.5 \text{ in.}$$

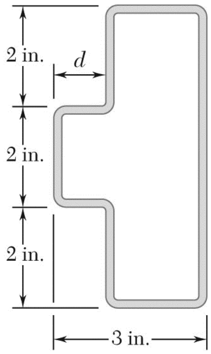
$$r = 1.5 + 0.1 = 1.6 \text{ in.}$$

$$a = \text{bh} - 2\left(\frac{\pi}{2}r^2\right) = (4.8)(5.5) - \pi(1.6)^2 = 18.3575 \text{ in}^2$$

$$\tau_{\max} = \frac{T}{2at_{\min}} \qquad \tau_{\max} = 12 \times 10^3 \text{ psi} \qquad t_{\min} = 0.2 \text{ in.}$$

$$T = 2at_{\min}\,\tau_{\max} = (2)(18.3575)(0.2)(12 \times 10^3) = 88.116 \times 10^3 \text{ lb} \cdot \text{in}$$

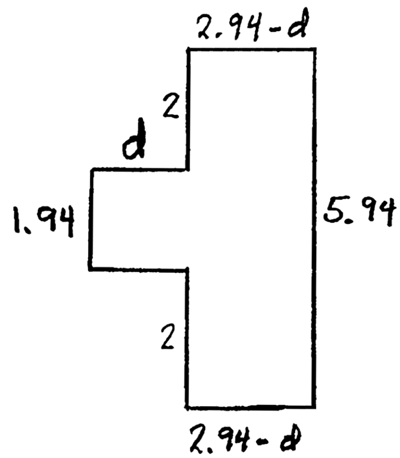
$$T = 88.1 \text{kip} \cdot \text{in} = 7.34 \text{ kip} \cdot \text{ft} \blacktriangleleft$$



PROBLEM 3.145

A hollow member having the cross section shown is to be formed from sheet metal of 0.06 in. thickness. Knowing that a 1250 lb · in.-torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

SOLUTION



<u>Area bounded by centerline.</u>

$$a = (5.94)(2.94 - d) + 1.94d = 17.4636 - 4.00d$$

$$t = 0.06 \text{ in.}, \quad \tau = 750 \text{ psi}, \quad T = 1250 \text{ lb} \cdot \text{in}$$

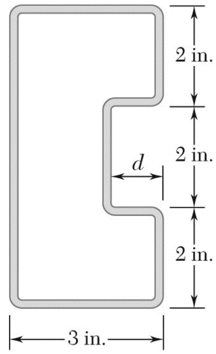
$$\tau = \frac{T}{2ta}$$

$$a = \frac{T}{2t\tau}$$

$$17.4636 - 4.00d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{4.00} = 0.894 \text{ in.}$$

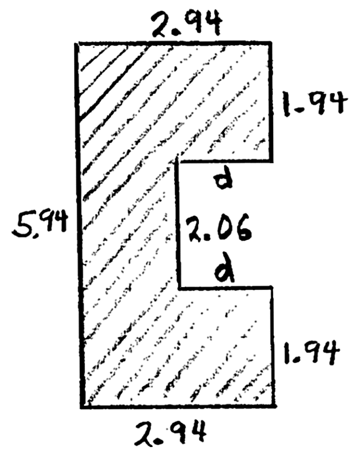
$d = 0.894$ in. ◀



PROBLEM 3.146

A hollow member having the cross section shown is to be formed from sheet metal of 0.06 in. thickness. Knowing that a 1250 lb · in.-torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

SOLUTION



Area bounded by centerline.

$$a = (5.94)(2.94) - 2.06d = 17.4636 - 2.06d$$

$$t = 0.06 \text{ in.}, \quad \tau = 750 \text{ psi}, \quad T = 1250 \text{ lb} \cdot \text{in}$$

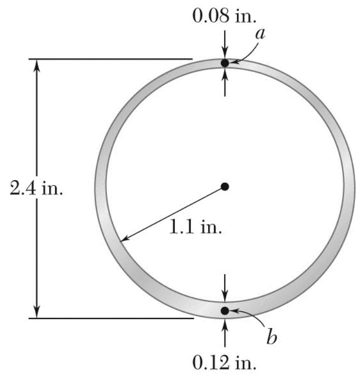
$$\tau = \frac{T}{2ta}$$

$$a = \frac{T}{2t\tau}$$

$$17.4636 - 2.06d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{2.06} = 1.735 \text{ in.}$$

$d = 1.735$ in. ◀



PROBLEM 3.147

A hollow cylindrical shaft was designed to have a uniform wall thickness of 0.1 in. Defective fabrication, however, resulted in the shaft having the cross section shown. Knowing that a 15 kip · in.-torque is applied to the shaft, determine the shearing stresses at points *a* and *b*.

SOLUTION

Radius of outer circle = 1.2 in.

Radius of inner circle = 1.1 in.

Mean radius = 1.15 in.

Area bounded by centerline.

$$a = \pi r_m^2 = \pi(1.15)^2 = 4.155 \text{ in}^2$$

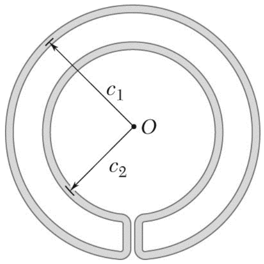
At point *a*, $t = 0.08$ in.

$$\tau = \frac{T}{2ta} = \frac{15}{(2)(0.08)(4.155)} \qquad \tau = 22.6 \text{ ksi} \blacktriangleleft$$

At point *b*, $t = 0.12$ in.

$$\tau = \frac{T}{2ta} = \frac{15}{(2)(0.12)(4.155)} \qquad \tau = 15.04 \text{ ksi} \blacktriangleleft$$

PROBLEM 3.148



A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3-mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the center line of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN · m is applied to the tube, determine (*a*) the maximum shearing stress in the tube, (*b*) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

SOLUTION

<u>Area bounded by centerline</u>.

$$a = \pi\left(c_1^2 - c_2^2\right) = \pi(150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2$$

$$= 39.27 \times 10^{-3} \text{ m}^2$$

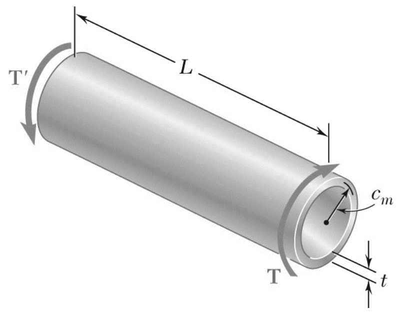
$$t = 0.003 \text{ m}$$

(*a*) $$\tau = \frac{T}{2ta} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa}$$ $\tau = 12.76$ MPa ◀

(*b*) $$T_1 = (2\pi c_1 t\ \tau c_1) = 2\pi c_1^2 t \tau$$

$$= 2\pi(0.150)^2(0.003)(12.73 \times 10^6) = 5.40 \times 10^3 \text{ N} \cdot \text{m}$$ $T_1 = 5.40$ kN · m ◀

PROBLEM 3.149



A hollow cylindrical shaft of length L, mean radius c_m, and uniform thickness t is subjected to a torque of magnitude T. Consider, on the one hand, the values of the average shearing stress τ_{ave} and the angle of twist φ obtained from the elastic torsion formulas developed in Sections 3.4 and 3.5 and, on the other hand, the corresponding values obtained from the formulas developed in Sec. 3.13 for thin-walled shafts. (*a*) Show that the relative error introduced by using the thin-walled-shaft formulas rather than the elastic torsion formulas is the same for τ_{ave} and φ and that the relative error is positive and proportional to the ratio t/c_m. (*b*) Compare the percent error corresponding to values of the ratio t/c_m of 0.1, 0.2, and 0.4.

SOLUTION

Let c_2 = outer radius $= c_m + \frac{1}{2}t$ and c_1 = inner radius $= c_m - \frac{1}{2}t$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}\left(c_2^2 + c_1^2\right)(c_2 + c_1)(c_2 - c_1)$$

$$= \frac{\pi}{2}\left(c_m^2 + c_m t + \frac{1}{4}t^2 + c_m^2 - c_m t + \frac{1}{4}t^2\right)(2c_m)t$$

$$= 2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m t$$

$$\tau_m = \frac{Tc_m}{J} = \frac{T}{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)t}$$

$$\varphi_1 = \frac{TL}{JG} = \frac{TL}{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m t\,G}$$

<u>Area bounded by centerline</u>. $\quad a = \pi c_m^2$

$$\tau_{ave} = \frac{T}{2ta} = \frac{T}{2\pi c_m^2 t}$$

$$\varphi_2 = \frac{TL}{4a^2G}\oint\frac{ds}{t} = \frac{TL(2\pi c_m/t)}{4(\pi c_m^2)^2 G} = \frac{TL}{2\pi c_m^3\,t\,G}$$

(*a*) Ratios:

$$\frac{\tau_{ave}}{\tau_m} = \frac{T}{2\pi c_m^2 t}\times\frac{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)t}{T} = 1 + \frac{1}{4}\frac{t^2}{c_m^2} \quad ◀$$

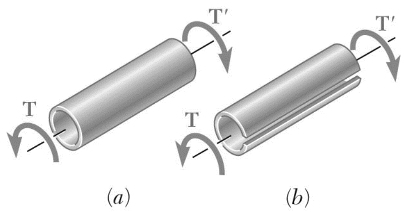
$$\frac{\varphi_2}{\varphi_1} = \frac{TL}{2\pi c_m^3 tG}\times\frac{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m tG}{TL} = 1 + \frac{1}{4}\frac{t^2}{c_m} \quad ◀$$

PROBLEM 3.149 (*Continued*)

(*b*) $$\frac{\tau_{\text{ave}}}{\tau_m} - 1 = \frac{\varphi_2}{\varphi_1} - 1 = \frac{1}{4}\frac{t^2}{c_m^2}$$

$\dfrac{t}{c_m}$	0.1	0.2	0.4
$\dfrac{1}{4}\dfrac{t^2}{c_m^2}$	0.0025	0.01	0.04
%	0.25%	1%	4%

◀



PROBLEM 3.150

Equal torques are applied to thin-walled tubes of the same length L, same thickness t, and same radius c. One of the tubes has been slit lengthwise as shown. Determine (a) the ratio τ_b / τ_a of the maximum shearing stresses in the tubes, (b) the ratio φ_b/φ_a of the angles of twist of the shafts.

SOLUTION

Without slit:

Area bounded by centerline. $a = \pi c^2$

$$\tau_a = \frac{T}{2ta} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t \qquad \varphi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t\, G}$$

With slit: $$a = 2\pi c, \quad b = t, \quad \frac{a}{b} = \frac{2\pi c}{t} >> 1$$

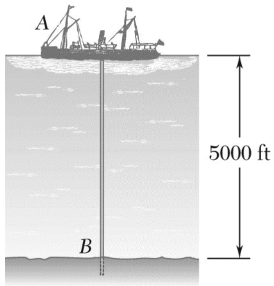
$$c_1 = c_2 = \frac{1}{3}$$

$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2}$$

$$\varphi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

(a) Stress ratio: $$\frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t} \qquad \frac{\tau_b}{\tau_a} = \frac{3c}{t} \blacktriangleleft$$

(b) Twist ratio: $$\frac{\varphi_b}{\varphi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t\, G}{TL} = \frac{3c^2}{t^2} \qquad \frac{\varphi_b}{\varphi_a} = \frac{3c^2}{t^2} \blacktriangleleft$$



PROBLEM 3.151

The ship at A has just started to drill for oil on the ocean floor at a depth of 5000 ft. Knowing that the top of the 8-in.-diameter steel drill pipe ($G = 11.2 \times 10^6$ psi) rotates through two complete revolutions before the drill bit at B starts to operate, determine the maximum shearing stress caused in the pipe by torsion.

SOLUTION

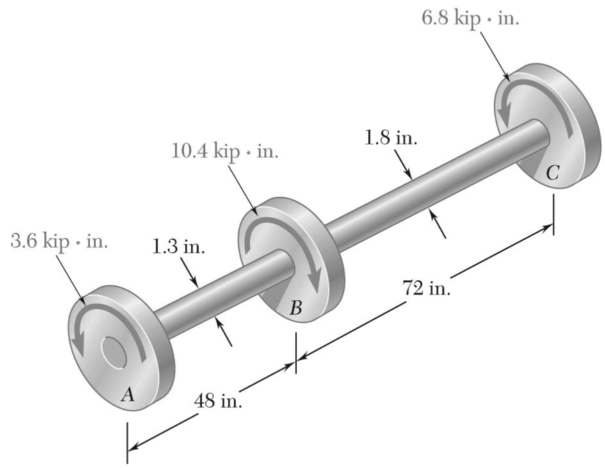
$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

$$\tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}$$

$$\varphi = 2\,\text{rev} = (2)(2\pi) = 12.566 \text{ rad}, \qquad c = \frac{1}{2}d = 4.0 \text{ in.}$$

$$L = 5000 \text{ ft} = 60000 \text{ in.}$$

$$\tau = \frac{(11.2\times 10^6)(12.566)(4.0)}{60000} = 9.3826\times 10^3 \text{ psi} \qquad \tau = 9.38 \text{ ksi} \blacktriangleleft$$



PROBLEM 3.152

The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 8.5 ksi, determine the smallest allowable diameter of (*a*) shaft *AB*, (*b*) shaft *BC*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 3.6 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau_{\max} = 8.5 \text{ ksi} = 8.5 \times 10^3 \text{ psi}$$

$$J = \frac{\pi}{2} c^4 \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T_{AB}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(3.6 \times 10^3)}{\pi (8.5 \times 10^3)}} = 0.646 \text{ in.}$$

$$d_{AB} = 2c = 1.292 \text{ in.} \blacktriangleleft$$

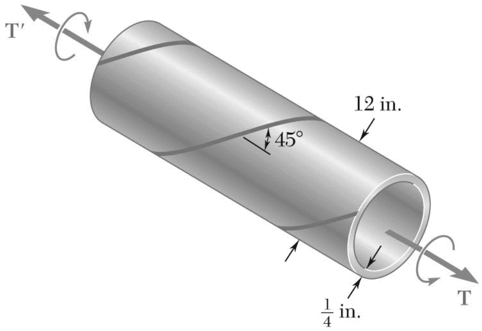
(*b*) Shaft *BC*:

$$T_{BC} = 6.8 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau_{\max} = 8.5 \times 10^3 \text{ psi}$$

$$c = \sqrt[3]{\frac{2T_{BC}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(6.8 \times 10^3)}{\pi (8.5 \times 10^3)}} = 0.7985 \text{ in.}$$

$$d_{BC} = 2c = 1.597 \text{ in.} \blacktriangleleft$$

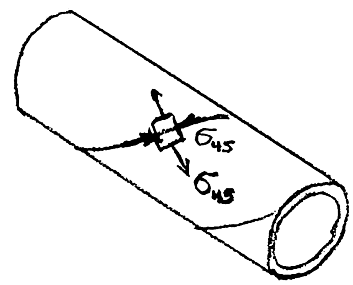


PROBLEM 3.153

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix which forms an angle of 45° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable tensile stress in the weld is 12 ksi, determine the largest torque that can be applied to the pipe.

SOLUTION

From Eq. (3.14) of the textbook,



$$\sigma_{45} = \tau_{\max}$$

hence, $$\tau_{\max} = 12\,\text{ksi} = 12 \times 10^3\ \text{psi}$$

$$c_2 = \frac{1}{2}d_0 = \frac{1}{2}(12) = 6.00\ \text{in.}$$

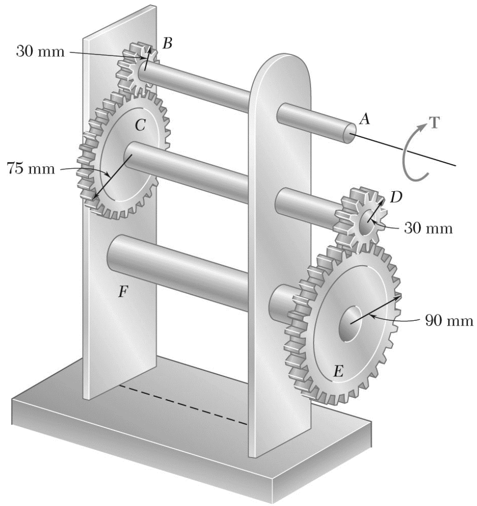
$$c_1 = c_2 - t = 6.00 - 0.25 = 5.75\ \text{in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(6.00)^4 - (5.75)^4] = 318.67\ \text{in.}$$

$$\tau_{\max} = \frac{Tc}{J} \qquad T = \frac{\tau_{\max}J}{c}$$

$$T = \frac{(12 \times 10^3)(318.67)}{6.00} = 637 \times 10^3\ \text{lb} \cdot \text{in}$$

$T = 637\ \text{kip} \cdot \text{in}$ ◀



PROBLEM 3.154

For the gear train shown, the diameters of the three solid shafts are:

$$d_{AB} = 20\,\text{mm} \quad d_{CD} = 25\,\text{mm} \quad d_{EF} = 40\,\text{mm}$$

Knowing that for each shaft the allowable shearing stress is 60 MPa, determine the largest torque T that can be applied.

SOLUTION

Statics: $T_{AB} = T$

$$\frac{T_{CD}}{r_C} = \frac{T_{AB}}{r_B} \qquad T_{CD} = \frac{r_C}{r_B}T_{AB} = \frac{75}{30}T = 2.5T$$

$$\frac{T_{EF}}{r_F} = \frac{T_{CD}}{r_D} \qquad T_{EF} = \frac{r_F}{r_D}T_{CD} = \frac{90}{30}(2.5T) = 7.5T$$

Determine the magnitude of T so that the stress is $60\text{ MPa} = 60 \times 10^6\text{ Pa}$.

$$\tau = \frac{Tc}{J} \qquad T_{\text{shaft}} = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3$$

<u>Shaft *AB*</u>: $c = \frac{1}{2}d_{AB} = 10\text{ mm} = 0.010\text{ m}$

$$T_{AB} = T = \frac{\pi}{2}(60 \times 10^6)(0.010)^3 \qquad T = 94.2\text{ N}\cdot\text{m}$$

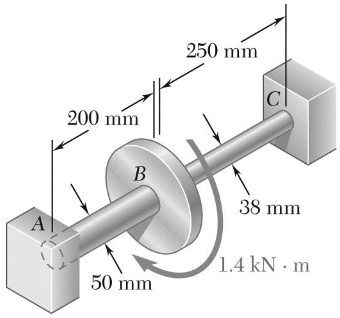
<u>Shaft *CD*</u>: $c = \frac{1}{2}d_{CD} = 12.5\text{ mm} = 0.0125\text{ m}$

$$T_{CD} = 2.5T = \frac{\pi}{2}(60 \times 10^6)(0.0125)^3 \qquad T = 73.6\text{ N}\cdot\text{m}$$

<u>Shaft *EF*</u>: $c = \frac{1}{2}d_{EF} = 20\,\text{mm} = 0.020\,\text{m}$

$$T_{EF} = 7.5T = \frac{\pi}{2}(60 \times 10^6)(0.020)^3 \qquad T = 100.5\text{ N}\cdot\text{m}$$

The smallest value of T is the largest torque that can be applied. $T = 73.6\text{ N}\cdot\text{m}$ ◀



PROBLEM 3.155

Two solid steel shafts ($G = 77.2\,\text{GPa}$) are connected to a coupling disk B and to fixed supports at A and C. For the loading shown, determine (*a*) the reaction at each support, (*b*) the maximum shearing stress in shaft AB, (*c*) the maximum shearing stress in shaft BC.

SOLUTION

Shaft AB: $\quad T = T_{AB},\quad L_{AB} = 0.200\text{ m},\quad c = \frac{1}{2}d = 25\text{ mm} = 0.025\text{ m}$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.59\times10^{-9}\text{ m}^4 \qquad \varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$$

$$T_{AB} = \frac{GJ_{AB}}{L_{AB}}\varphi_B = \frac{(77.2\times10^9)(613.59\times10^{-9})}{0.200}\varphi_B = 236.847\times10^3\,\varphi_B$$

Shaft BC: $\quad T = T_{BC},\quad L_{BC} = 0.250\text{ m},\quad c = \frac{1}{2}d = 19\text{ mm} = 0.019\text{ m}$

$$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.019)^4 = 204.71\times10^{-9}\text{ m}^4 \qquad \varphi_B = \frac{T_{BC}L_{BC}}{GJ_{BC}}$$

$$T_{BC} = \frac{GJ_{BC}}{L_{BC}}\varphi_B = \frac{(77.2\times10^9)(204.71\times10^{-9})}{0.250} = 63.214\times10^3\varphi_B$$

Equilibrium of coupling disk. $\quad T = T_{AB} + T_{BC}$

$$1.4\times10^3 = 236.847\times10^3\varphi_B + 63.214\times10^3\varphi_B \qquad \varphi_B = 4.6657\times10^{-3}\text{ rad.}$$

$$T_{AB} = (236.847\times10^3)(4.6657\times10^{-3}) = 1.10506\times10^3\text{ N}\cdot\text{m}$$

$$T_{BC} = (63.214\times10^3)(4.6657\times10^{-3}) = 294.94\text{ N}\cdot\text{m}$$

(*a*) Reactions at supports. $\qquad T_A = T_{AB} = 1105\text{ N}\cdot\text{m}$ ◀

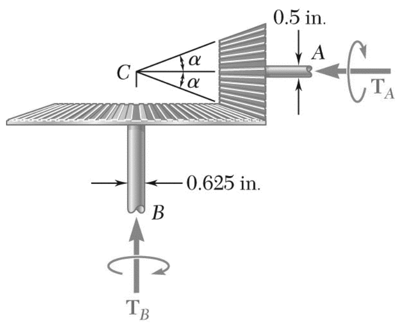
$T_C = T_{BC} = 295\text{ N}\cdot\text{m}$ ◀

(*b*) Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(1.10506\times10^3)(0.025)}{613.59\times10^{-9}} = 45.0\times10^6\text{ Pa} \qquad \tau_{AB} = 45.0\text{ MPa} ◀$$

(*c*) Maximum shearing stress in BC.

$$\tau_{BC} = \frac{T_{BC}c}{J_{BC}} = \frac{(294.94)(0.019)}{204.71\times10^{-9}} = 27.4\times10^6\text{ Pa} \qquad \tau_{BC} = 27.4\text{ MPa} ◀$$



PROBLEM 3.156

In the bevel-gear system shown, $\alpha = 18.43°$. Knowing that the allowable shearing stress is 8 ksi in each shaft and that the system is in equilibrium, determine the largest torque T_A that can be applied at A.

SOLUTION

Using stress limit for shaft A:

$$\tau = 8 \text{ ksi}, \qquad c = \frac{1}{2}d = 0.25 \text{ in.}$$

$$T_A = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(8)(0.25)^3 = 0.1963 \text{ kip} \cdot \text{in}$$

Using stress limit for shaft B:

$$\tau = 8 \text{ ksi}, \quad c = \frac{1}{2}d = 0.3125 \text{ in.}$$

$$T_B = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(8)(0.3125)^3 = 0.3835 \text{ kip} \cdot \text{in}$$

From statics,

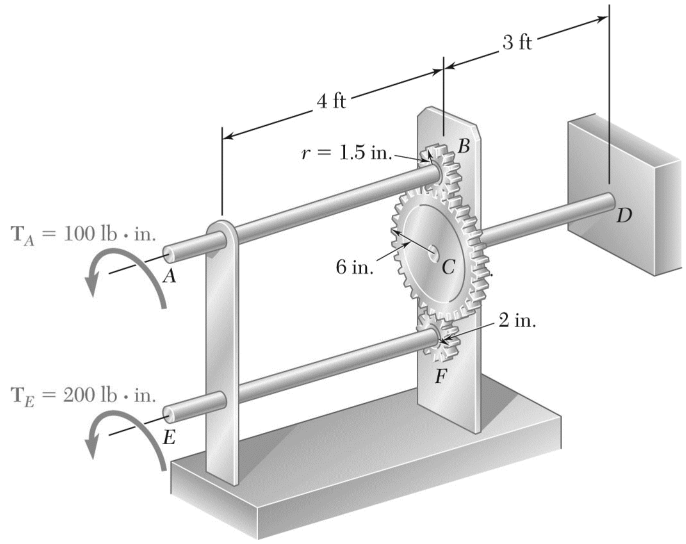
$$T_A = \frac{r_A}{r_B}T_B = (\tan\alpha)T_B$$

$$T_A = (\tan 18.43°)(0.3835) = 0.1278 \text{ kip} \cdot \text{in}$$

The allowable value of T_A is the smaller.

$$T_A = 0.1278 \text{ kip} \cdot \text{in}$$

$T_A = 127.8 \text{ lb} \cdot \text{in}$ ◀



PROBLEM 3.157

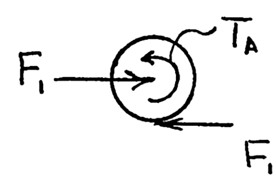
Three solid shafts, each of $\frac{3}{4}$-in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the angle through which end *A* of shaft *AB* rotates, (*b*) the angle through which end *E* of shaft *EF* rotates.

SOLUTION

<u>Geometry</u>:

$$r_B = 1.5 \text{ in.}, \quad r_C = 6 \text{ in.}, \quad r_F = 2 \text{ in.}$$

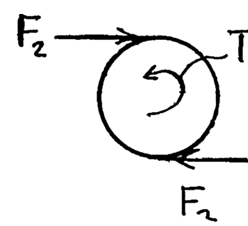
$$L_{AB} = 48 \text{ in.}, \quad L_{CD} = 36 \text{ in.}, \quad L_{EF} = 48 \text{ in.}$$



<u>Statics</u>:

$$T_A = 100 \text{ lb} \cdot \text{in} \circlearrowright$$

$$T_E = 200 \text{ lb} \cdot \text{in} \circlearrowright$$



<u>Gear *B*</u>.

$$+\circlearrowright \Sigma M_B = 0:$$

$$-r_B F_1 + T_A = 0$$

$$-1.5F_1 + 100 = 0$$

$$F_1 = 67.667 \text{ lb}$$

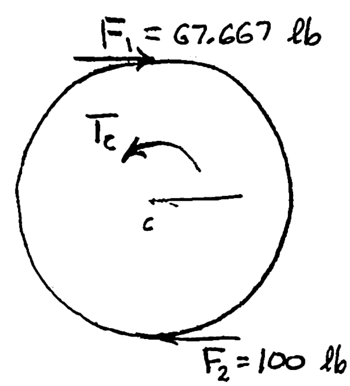
<u>Gear *F*</u>.

$$+\circlearrowright \Sigma M_F = 0:$$

$$-r_F F_2 + T_E = 0$$

$$-2F_2 + 200 = 0$$

$$F_2 = 100 \text{ lb}$$



<u>Gear *C*</u>.

$$+\circlearrowright \Sigma M_C = 0:$$

$$-r_C F_1 - r_C F_2 + T_C = 0$$

$$-(6)(66.667) - (6)(100) + T_C = 0$$

$$T_C = 1000 \text{ lb} \cdot \text{in} \circlearrowright$$

PROBLEM 3.157 (*Continued*)

Deformations:

For all shafts, $c = \frac{1}{2}d = 0.375 \text{ in.}$

$$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4$$

$$\varphi_{A/B} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(100)(48)}{(11.2 \times 10^6)(0.031063)} = 0.013797 \text{ rad}$$

$$\varphi_{E/F} = \frac{T_{EF}L_{EF}}{GJ} = \frac{(200)(48)}{(11.2 \times 10^6)(0.031063)} = 0.027594 \text{ rad}$$

$$\varphi_{C/D} = \frac{T_{CD}L_{CD}}{GJ} = \frac{(1000)(36)}{(11.2 \times 10^6)(0.031063)} = 0.103476 \text{ rad}$$

Kinematics: $\varphi_C = \varphi_{C/D} = 0.103476 \text{ rad}$

$$r_B\varphi_B = r_C\varphi_C \qquad \varphi_B = \frac{r_C}{r_B}\varphi_C \frac{6}{1.5}(0.103476) = 0.41390 \text{ rad}$$

(*a*) $\varphi_A = \varphi_B + \varphi_{A/B} = 0.41390 + 0.01397 = 0.42788 \text{ rad}$ $\qquad \varphi_A = 24.5° \blacktriangleleft$

$$r_F\varphi_F = r_C\varphi_C \qquad \varphi_F = \frac{r_C}{r_F}\varphi_C = \frac{6}{2}(0.103476) = 0.31043 \text{ rad}$$

(*b*) $\varphi_E = \varphi_F + \varphi_{E/F} = 0.31043 + 0.027594 = 0.33802 \text{ rad}$ $\qquad \varphi_E = 19.37° \blacktriangleleft$

PROBLEM 3.158

The design specifications of a 1.2-m-long solid transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of 750 N · m is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77.2 GPa.

SOLUTION

$$T = 750\ \text{N}\cdot\text{m}, \quad \varphi = 4^\circ = 69.813\times 10^{-3}\ \text{rad},$$

$$L = 1.2\ \text{m}, \quad J = \frac{\pi}{2}c^4$$

$$\tau = 90\,\text{MPa} = 90\times 10^6\ \text{Pa} \quad G = 77.2\ \text{GPa} = 77.2\times 10^9\ \text{Pa}$$

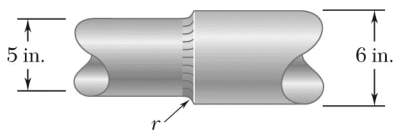
<u>Based on angle of twist</u>. $\varphi = \dfrac{TL}{GJ} = \dfrac{2TL}{\pi G c^4}$

$$c = \sqrt[4]{\frac{2TL}{\pi G\varphi}} = \sqrt[4]{\frac{(2)(750)(1.2)}{\pi(77.2\times 10^9)(69.813\times 10^{-3})}} = 18.06\times 10^{-3}\ \text{m}$$

<u>Based on shearing stress</u>. $\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(750)}{\pi(90\times 10^6)}} = 17.44\times 10^{-3}\ \text{m}$$

Use larger value. $c = 18.06\times 10^{-3}\ \text{m} = 18.06\ \text{mm}$ $\qquad d = 2c = 36.1\ \text{mm}$ ◀



PROBLEM 3.159

The stepped shaft shown rotates at 450 rpm. Knowing that $r = 0.5$ in., determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 7500 psi.

SOLUTION

$$d = 5 \text{ in.}$$
$$D = 6 \text{ in.}$$
$$r = 0.5 \text{ in.}$$
$$\frac{D}{d} = \frac{6}{5} = 1.20$$
$$\frac{r}{d} = \frac{0.5}{5} = 0.10$$

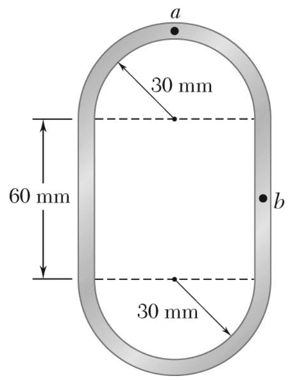
From Fig. 3.32, $K = 1.33$

For smaller side, $c = \frac{1}{2}d = 2.5$ in.

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(2.5)^3(7500)}{(2)(1.33)} = 138.404 \times 10^3 \text{ lb} \cdot \text{in}$$
$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

<u>Power.</u> $P = 2\pi f T = 2\pi(7.5)(138.404 \times 10^3) = 6.52 \times 10^6 \text{ in} \cdot \text{lb/s}$

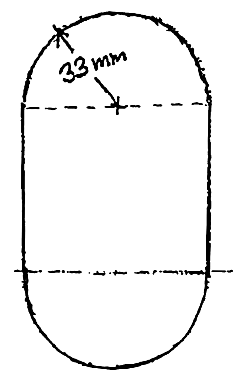
Recalling that $1 \text{ hp} = 6600 \text{ in} \cdot \text{lb/s}$, $P = 988 \text{ hp}$ ◀



PROBLEM 3.160

A 750-N · m torque is applied to a hollow shaft having the cross section shown and a uniform 6-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

SOLUTION



Area bounded by centerline.

$$a = 2\frac{\pi}{2}(33)^2 + (60)(66) = 7381 \text{ mm}^2$$

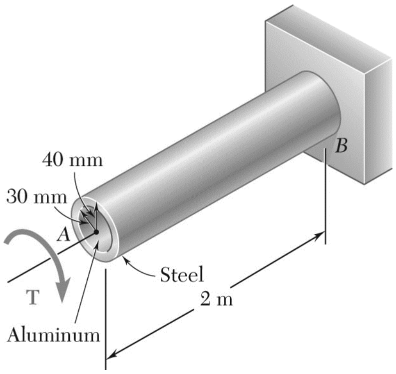
$$= 7381 \times 10^{-6} \text{ m}^2$$

$$t = 0.006 \text{ m at both } a \text{ and } b,$$

Then at points *a* and *b*,

$$\tau = \frac{T}{2ta} = \frac{750}{(2)(0.006)(7381 \times 10^{-6})} = 8.47 \times 10^6 \text{ Pa} \qquad \tau = 8.47 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.161



The composite shaft shown is twisted by applying a torque **T** at end A. Knowing that the maximum shearing stress in the steel shell is 150 MPa, determine the corresponding maximum shearing stress in the aluminum core. Use $G = 77.2\text{ GPa}$ for steel and $G = 27\text{ GPa}$ for aluminum.

SOLUTION

Let G_1, J_1, and τ_1 refer to the aluminum core and G_2, J_2, and τ_2 refer to the steel shell.

At the outer surface on the steel shell,

$$\gamma_2 = \frac{c_2\varphi}{L} \quad \therefore \quad \frac{\varphi}{L} = \frac{\gamma_2}{c_2} = \frac{\tau_2}{c_2 G_2}$$

At the outer surface of the aluminum core,

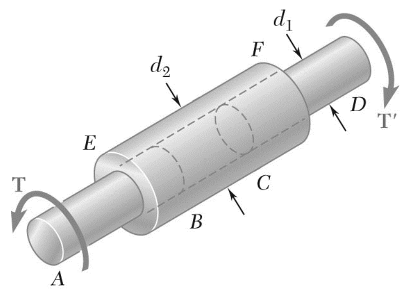
$$\gamma_1 = \frac{c_1\varphi}{L} \quad \therefore \quad \frac{\varphi}{L} = \frac{\gamma_1}{c_1} = \frac{\tau_1}{c_1 G}$$

Matching $\frac{\varphi}{L}$ for both components,

$$\frac{\tau_2}{c_2 G_2} = \frac{\tau_1}{c_1 G_1}$$

Solving for τ_2, $$\tau_2 = \frac{c_2}{c_1}\cdot\frac{G_2}{G_1}\tau_1 = \frac{0.030}{0.040}\cdot\frac{27\times 10^9}{77.2\times 10^9}\cdot 150\times 10^6 = 39.3\times 10^6\text{ Pa}$$

$\tau_2 = 39.3\text{ MPa}$ ◀



PROBLEM 3.162

Two solid brass rods AB and CD are brazed to a brass sleeve EF. Determine the ratio d_2/d_1 for which the same maximum shearing stress occurs in the rods and in the sleeve.

SOLUTION

Let $\quad c_1 = \frac{1}{2}d_1 \quad$ and $\quad c_2 = \frac{1}{2}d_2$

Shaft AB: $\quad \tau_1 = \frac{Tc_1}{J_1} = \frac{2T}{\pi c_1^3}$

Sleeve EF. $\quad \tau_2 = \frac{Tc_2}{J_2} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

For equal stresses, $\quad \frac{2T}{\pi c_1^3} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

$$c_2^4 - c_1^4 = c_1^3 c_2$$

Let $\quad x = \frac{c_2}{c_1} \qquad x^4 - 1 = x \quad$ or $\quad x = \sqrt[4]{1+x}$

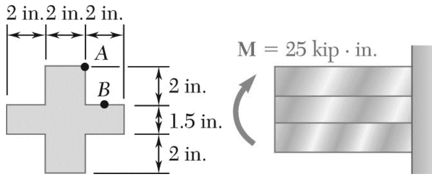
Solve by successive approximations starting with $x_0 = 1.0$.

$$x_1 = \sqrt[4]{2} = 1.189, \quad x_2 = \sqrt[4]{2.189} = 1.216, \quad x_3 = \sqrt[4]{2.216} = 1.220$$

$$x_4 = \sqrt[4]{2.220} = 1.221, \quad x_5 = \sqrt[4]{2.221} = 1.221 \quad \text{(converged)}.$$

$x = 1.221 \qquad \frac{c_2}{c_1} = 1.221 \qquad\qquad \frac{d_2}{d_1} = 1.221$ ◀

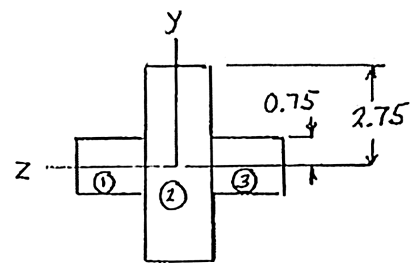
CHAPTER 4



PROBLEM 4.1

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



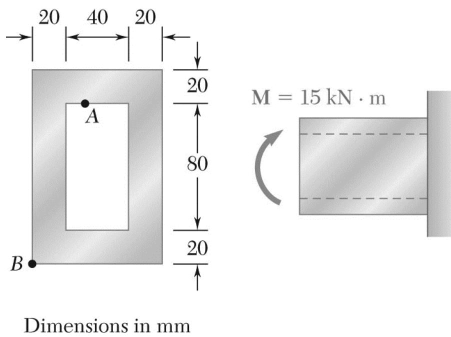
For rectangle: $I = \frac{1}{12}bh^3$

For cross sectional area:

$$I = I_1 + I_2 + I_3 = \frac{1}{12}(2)(1.5)^3 + \frac{1}{12}(2)(5.5)^3 + \frac{1}{12}(2)(1.5)^3 = 28.854 \text{ in}^4$$

(*a*) $y_A = 2.75$ in. $\qquad \sigma_A = -\frac{My_A}{I} = -\frac{(25)(2.75)}{28.854} \qquad \sigma_A = -2.38$ ksi ◀

(*b*) $y_B = 0.75$ in. $\qquad \sigma_B = -\frac{My_B}{I} = -\frac{(25)(0.75)}{28.854} \qquad \sigma_B = -0.650$ ksi ◀



PROBLEM 4.2

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

For rectangle: $I = \frac{1}{12}bh^3$

Outside rectangle: $I_1 = \frac{1}{12}(80)(120)^3$

$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$

Cutout: $I_2 = \frac{1}{12}(40)(80)^3$

$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

(*a*) $y_A = 40 \text{ mm} = 0.040 \text{ m}$

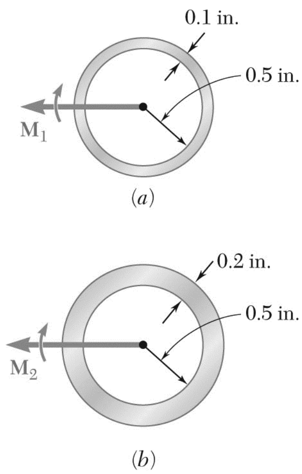
$$\sigma_A = -\frac{My_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$$

$\sigma_A = -61.6 \text{ MPa}$ ◀

(*b*) $y_B = -60 \text{ mm} = -0.060 \text{ m}$

$$\sigma_B = -\frac{My_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$$

$\sigma_B = 91.7 \text{ MPa}$ ◀



PROBLEM 4.3

Using an allowable stress of 16 ksi, determine the largest couple that can be applied to each pipe.

SOLUTION

(*a*) $I = \dfrac{\pi}{4}\left(r_o^4 - r_i^4\right) = \dfrac{\pi}{4}(0.6^4 - 0.5^4) = 52.7 \times 10^{-3}\ \text{in}^4$

$c = 0.6$ in.

$\sigma = \dfrac{Mc}{I}: \qquad M = \dfrac{\sigma I}{c} = \dfrac{(16)(52.7 \times 10^{-3})}{0.6}$

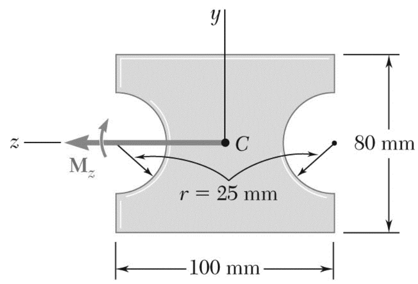
$M = 1.405$ kip · in ◀

(*b*) $I = \dfrac{\pi}{4}(0.7^4 - 0.5^4) = 139.49 \times 10^{-3}\ \text{in}^4$

$c = 0.7$ in.

$\sigma = \dfrac{Mc}{I}: \qquad M = \dfrac{\sigma I}{c} = \dfrac{(16)(139.49 \times 10^{-3})}{0.7}$

$M = 3.19$ kip · in ◀



PROBLEM 4.4

A nylon spacing bar has the cross section shown. Knowing that the allowable stress for the grade of nylon used is 24 MPa, determine the largest couple $\mathbf{M_z}$ that can be applied to the bar.

SOLUTION

$$I = I_{\text{rect}} - I_{\text{circle}} = \frac{1}{12}bh^3 - \frac{\pi}{4}r^4$$

$$= \frac{1}{12}(100)(80)^3 - \frac{\pi}{4}(25)^4 = 3.9599 \times 10^6 \text{ mm}^4$$

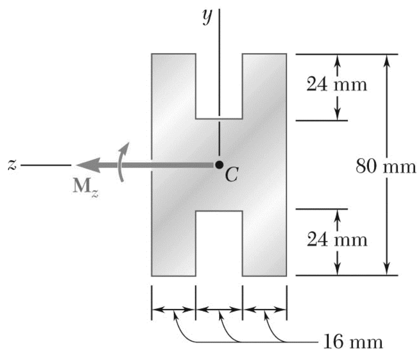
$$= 3.9599 \times 10^{-6} \text{ m}$$

$$c = \frac{80}{2} = 40 \text{ mm} = 0.040 \text{ m}$$

$$\sigma = \frac{Mc}{I}: \qquad M_z = \frac{\sigma I}{c} = \frac{(24 \times 10^6)(3.9599 \times 10^{-6})}{0.040} = 2.38 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_z = 2.38 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROBLEM 4.5

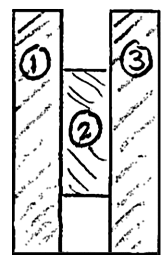


A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z-axis.

SOLUTION

Allowable stress. $= \dfrac{\sigma_U}{F.S.} = \dfrac{450}{3} = 150 \text{ MPa}$

$= 150 \times 10^6 \text{ Pa}$



<u>Moment of inertia about z-axis.</u>

$$I_1 = \frac{1}{12}(16)(80)^3 = 682.67 \times 10^3 \text{mm}^4$$

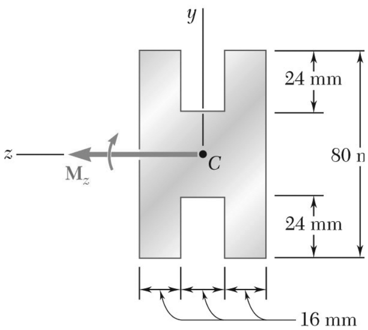
$$I_2 = \frac{1}{12}(16)(32)^3 = 43.69 \times 10^3 \text{mm}^4$$

$$I_3 = I_1 = 682.67 \times 10^3 \text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 1.40902 \times 10^6 \text{mm}^4 = 1.40902 \times 10^{-6} \text{m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(80) = 40 \text{ mm} = 0.040 \text{ m}$$

$$M = \frac{I\sigma}{c} = \frac{(1.40902 \times 10^{-6})(150 \times 10^6)}{0.040} = 5.28 \times 10^3 \text{ N} \cdot \text{m} \qquad M = 5.28 \text{ kN} \cdot \text{m} \blacktriangleleft$$

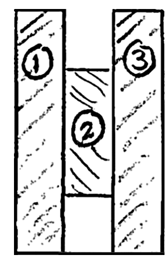


PROBLEM 4.6

Solve Prob. 4.5, assuming that the beam is bent about the y-axis.

PROBLEM 4.5 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z-axis.

SOLUTION



Allowable stress: $\quad = \dfrac{\sigma_U}{F.S.} = \dfrac{450}{3.00} = 150 \text{ MPa}$

$$= 150 \times 10^6 \text{ Pa}$$

Moment of inertia about y-axis.

$$I_1 = \frac{1}{12}(80)(16)^3 + (80)(16)(16)^2 = 354.987 \times 10^3 \text{mm}^4$$

$$I_2 = \frac{1}{12}(32)(16)^3 = 10.923 \times 10^3 \text{mm}^4$$

$$I_3 = I_1 = 354.987 \times 10^3 \text{mm}^4$$

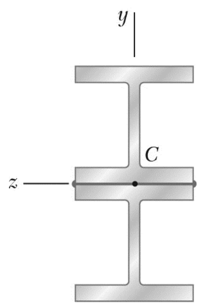
$$I = I_1 + I_2 + I_3 = 720.9 \times 10^3 \text{mm}^4 = 720.9 \times 10^{-9} \text{m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(48) = 24 \text{ mm} = 0.024 \text{ m}$$

$$M = \frac{I\sigma}{c} = \frac{(720.9 \times 10^{-9})(150 \times 10^6)}{0.024} = 4.51 \times 10^3 \text{N} \cdot \text{m}$$

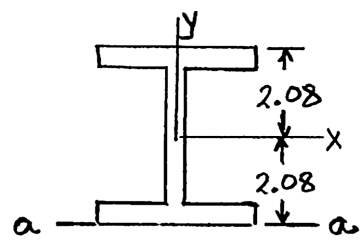
$M = 4.51 \text{ kN} \cdot \text{m}$ ◀

PROBLEM 4.7



Two W4×13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the *z* axis.

SOLUTION



Properties of W4 × 13 rolled section.

(See Appendix C.)

$\text{Area} = 3.83 \text{ in}^2$

$\text{Depth} = 4.16 \text{ in.}$

$I_x = 11.3 \text{ in}^4$

For one rolled section, moment of inertia about axis *a-a* is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

For both sections,

$$I_z = 2I_a = 55.74 \text{ in}^4$$

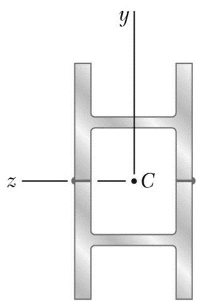
$$c = \text{depth} = 4.16 \text{ in.}$$

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{\text{all}} = \frac{\sigma_{\text{all}} I}{c} = \frac{(19.333)(55.74)}{4.16}$$

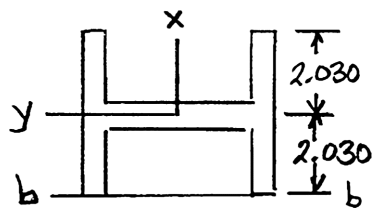
$M_{\text{all}} = 259 \text{ kip} \cdot \text{in}$ ◀

PROBLEM 4.8



Two W4×13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the *z* axis.

SOLUTION



Properties of W4 × 13 rolled section.

(See Appendix C.)

$\text{Area} = 3.83 \text{ in}^2$

$\text{Width} = 4.060 \text{ in.}$

$I_y = 3.86 \text{ in}^4$

For one rolled section, moment of inertia about axis *b-b* is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

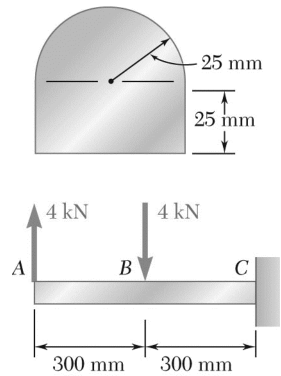
For both sections, $\quad I_z = 2I_b = 39.286 \text{ in}^4$

$$c = \text{width} = 4.060 \text{ in.}$$

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{\text{all}} = \frac{\sigma_{\text{all}} I}{c} = \frac{(19.333)(39.286)}{4.060}$$

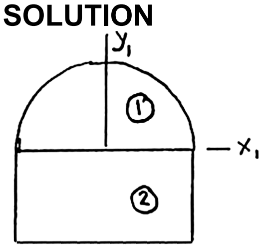
$M_{\text{all}} = 187.1 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 4.9

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

SOLUTION



$$A_1 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(25)^2 = 981.7 \text{ mm}^2 \quad \bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$$

$$A_2 = bh = (50)(25) = 1250 \text{ mm}^2 \quad \bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250} = -2.334 \text{ mm}$$

$$\bar{I}_1 = I_{x_1} - A_1\bar{y}_1^2 = \frac{\pi}{8}r^4 - A_1\bar{y}_1^2 = \frac{\pi}{8}(25)^4 - (981.7)(10.610)^2 = 42.886\times10^6 \text{ mm}^4$$

$$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$$

$$I_1 = \bar{I}_1 + A_1d_1^2 = 42.866\times10^3 + (981.7)(12.944)^2 = 207.35\times10^3 \text{ mm}^4$$

$$\bar{I}_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(25)^3 = 65.104\times10^3 \text{ mm}^4$$

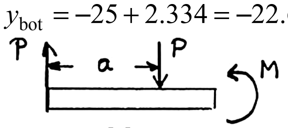
$$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5 - (-2.334)| = 10.166 \text{ mm}$$

$$I_2 = \bar{I}_2 + A_2d_2^2 = 65.104\times10^3 + (1250)(10.166)^2 = 194.288\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 401.16\times10^3 \text{ mm}^4 = 401.16\times10^{-9} \text{ m}^4$$

$$y_{\text{top}} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$$

$$y_{\text{bot}} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$$



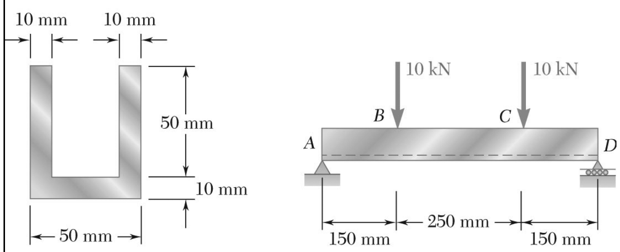
$$M - Pa = 0: \qquad M = Pa = (4\times10^3)(300\times10^{-3}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{top}} = \frac{-My_{\text{top}}}{I} = -\frac{(1200)(0.027334)}{401.16\times10^{-9}} = -81.76\times10^6 \text{ Pa}$$

$\sigma_{\text{top}} = -81.8$ MPa ◀

$$\sigma_{\text{bot}} = \frac{-My_{\text{bot}}}{I} = -\frac{(1200)(-0.022666)}{401.16\times10^{-9}} = 67.80\times10^6 \text{ Pa}$$

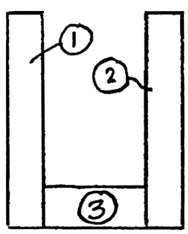
$\sigma_{\text{bot}} = 67.8$ MPa ◀



PROBLEM 4.10

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.

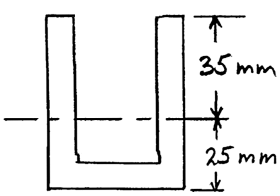
SOLUTION



	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3

$$\bar{Y}_0 = \frac{37.5\times10^3}{1500} = 25\,\text{mm}$$

Neutral axis lies 25 mm above the base.



$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3\,\text{mm}^4 \quad I_2 = I_1 = 195\ \text{mm}^4$$

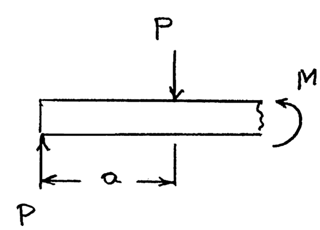
$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3\,\text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3\,\text{mm}^4 = 512.5\times10^{-9}\,\text{m}^4$$

$y_{\text{top}} = 35\ \text{mm} = 0.035\ \text{m} \quad y_{\text{bot}} = -25\ \text{mm} = -0.025\ \text{m}$

$a = 150\ \text{mm} = 0.150\ \text{m} \quad P = 10\times10^3\,\text{N}$

$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3\,\text{N}\cdot\text{m}$



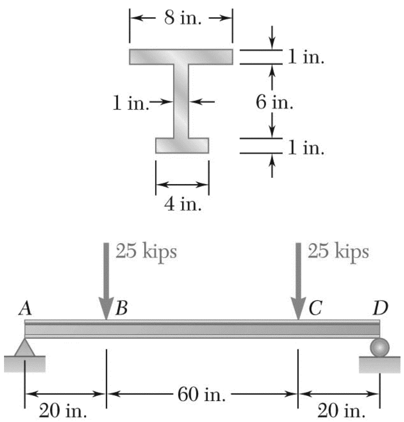
$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6\,\text{Pa}$$

$\sigma_{\text{top}} = -102.4$ MPa (compression) ◀

$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6\,\text{Pa}$$

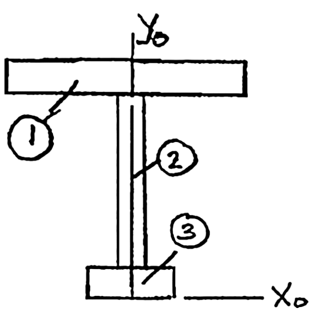
$\sigma_{\text{bot}} = 73.2$ MPa (tension) ◀

PROBLEM 4.11



Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

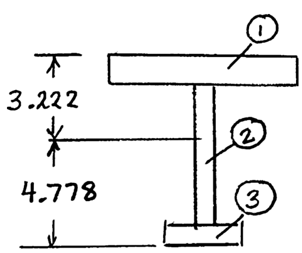
SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$$\bar{Y}_o = \frac{86}{18} = 4.778 \text{ in.}$$

Neutral axis lies 4.778 in. above the base.



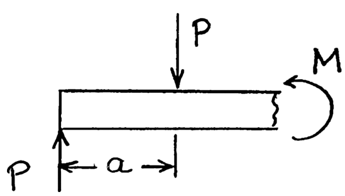
$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.772)^2 = 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2 = 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.57 = 155.16 \text{ in}^4$$

$$y_{\text{top}} = 3.222 \text{ in.} \quad y_{\text{bot}} = -4.778 \text{ in.}$$

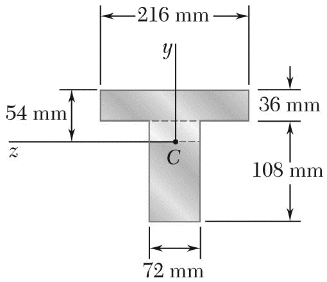


$$M - Pa = 0$$

$$M = Pa = (25)(20) = 500 \text{ kip} \cdot \text{in}$$

$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(500)(3.222)}{155.16} \qquad \sigma_{\text{top}} = -10.38 \text{ ksi (compression)} \blacktriangleleft$$

$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(500)(-4.778)}{155.16} \qquad \sigma_{\text{bot}} = 15.40 \text{ ksi (tension)} \blacktriangleleft$$



PROBLEM 4.12

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the top flange.

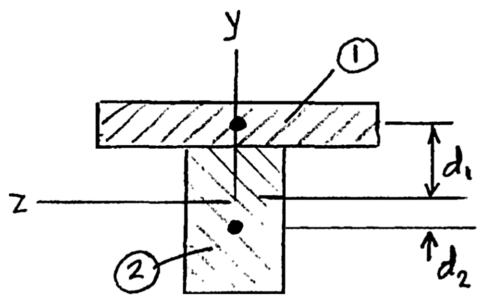
SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula:

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA, the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$



The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$d_1 = 54 - 18 = 36 \text{ mm}$$
$$d_2 = 54 + 36 - 54 = 36 \text{ mm}$$

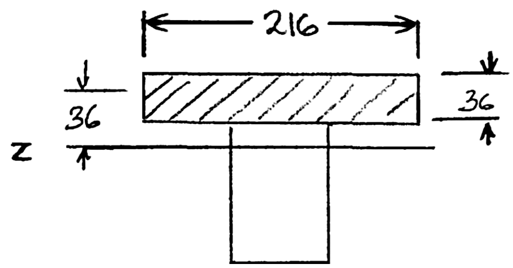
PROBLEM 4.12 *(Continued)*

Moment of inertia of entire cross section:

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175\times10^6\,\text{mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360\times10^6\,\text{mm}^4$$

$$I = I_1 + I_2 = 28.5535\times10^6\,\text{mm}^4 = 28.5535\times10^{-6}\,\text{m}^4$$



For the shaded area,

$$A^* = (216)(36) = 7776\ \text{mm}^2$$

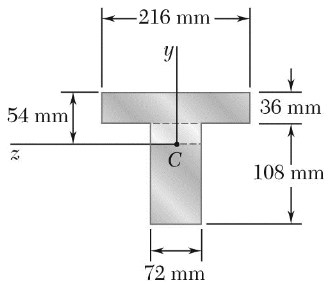
$$\overline{y}^* = 36\ \text{mm}$$

$$A^*\overline{y}^* = 279.936\times10^3\,\text{mm}^3 = 279.936\times10^{-6}\,\text{m}^3$$

$$F = -\left|\frac{MA^*\overline{y}^*}{I}\right| = \frac{(6\times10^3)(279.936\times10^{-6})}{28.5535\times10^{-6}}$$

$$= 58.8\times10^3\,\text{N}$$

$F = 58.8$ kN ◀



PROBLEM 4.13

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the shaded portion of the web.

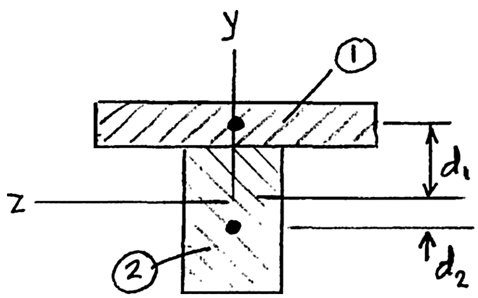
SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula:

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA, the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$



The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y dA = -\frac{M}{I}\overline{y}^* A^*$$

where $\overline{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$d_1 = 54 - 18 = 36 \text{ mm}$$
$$d_2 = 54 + 36 - 54 = 36 \text{ mm}$$

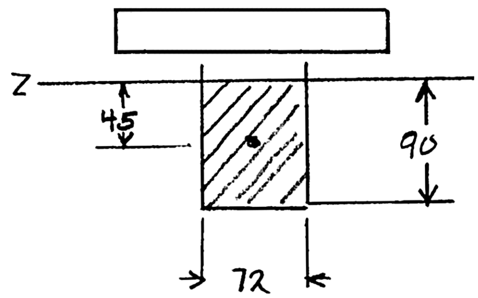
PROBLEM 4.13 *(Continued)*

Moment of inertia of entire cross section:

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175\times10^6\,\text{mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360\times10^6\,\text{mm}^4$$

$$I = I_1 + I_2 = 28.5535\times10^6\,\text{mm}^4 = 28.5535\times10^{-6}\,\text{m}^4$$



For the shaded area,

$$A^* = (72)(90) = 6480\ \text{mm}^2$$

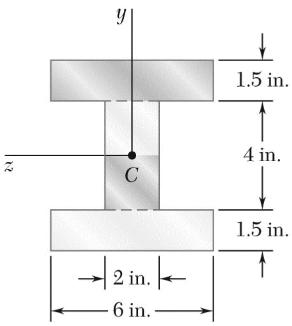
$$\overline{y}^* = 45\ \text{mm}$$

$$A^*\overline{y}^* = 291.6\times10^3\,\text{mm}^3 = 291.6\times10^{-6}\,\text{m}$$

$$F = \left|\frac{MA^*\overline{y}^*}{I}\right| = \frac{(6\times10^3)(291.6\times10^{-6})}{28.5535\times10^{-6}}$$

$$= 61.3\times10^3\,\text{N}$$

$F = 61.3$ kN ◀

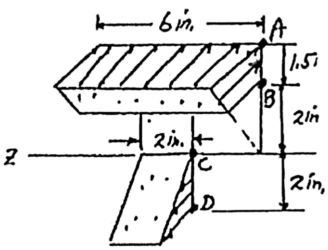


PROBLEM 4.14

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 50 kip · in., determine the total force acting (*a*) on the top flange, (*b*) on the shaded portion of the web.

SOLUTION

The stress distribution over the entire cross-section is given by the bending stress formula: $\sigma_x = -\dfrac{My}{I}$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA, the force is

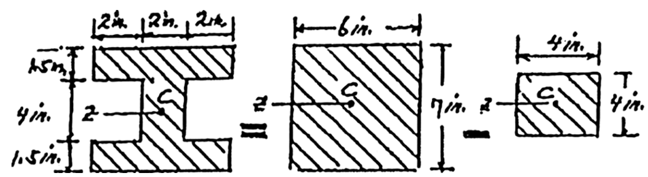
$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

Calculate the moment of inertia.



$$I = \frac{1}{12}(6 \text{ in.})(7 \text{ in.})^3 - \frac{1}{12}(4 \text{ in.})(4 \text{ in.})^3 = 150.17 \text{ in}^4$$

$$M = 50 \text{ kip} \cdot \text{in}$$

(*a*) Top flange: $A^* = (6 \text{ in.})(1.5 \text{ in.}) = 9 \text{ in}^2 \qquad \bar{y}^* = 2 \text{ in.} + 0.75 \text{ in.} = 2.75 \text{ in.}$

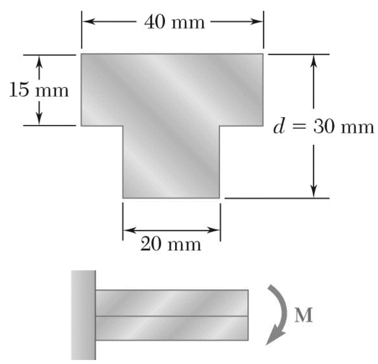
$$F = \frac{50 \text{ kip} \cdot \text{in}}{150.17 \text{ in}^4}(9 \text{ in}^2)(2.75 \text{ in.}) = 8.24 \text{ kips}$$

$F = 8.24$ kips ◀

(*b*) Half web: $A^* = (2 \text{ in.})(2 \text{ in.}) = 4 \text{ in}^2 \qquad \bar{y}^* = 1 \text{ in.}$

$$F = \frac{50 \text{ kip} \cdot \text{in}}{150.17 \text{ in}^4}(4 \text{ in}^2)(1 \text{ in.}) = 1.332 \text{ kips}$$

$F = 1.332$ kips ◀

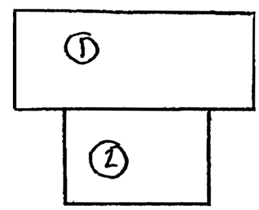


PROBLEM 4.15

The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3



$$\bar{Y}_0 = \frac{15.5\times10^3}{900} = 17.5 \text{ mm}$$ The neutral axis lies 17.5 mm above the bottom.

$$y_{\text{top}} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}$$

$$y_{\text{bot}} = -17.5 \text{ mm} = -0.0175 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(5)^2 = 26.25\times10^3\,\text{mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(20)(15)^3 + (300)(10)^2 = 35.625\times10^3\,\text{mm}^4$$

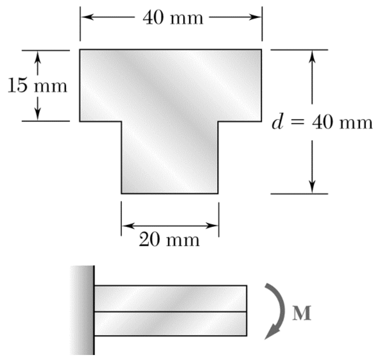
$$I = I_1 + I_2 = 61.875\times10^3\,\text{mm}^4 = 61.875\times10^{-9}\,\text{m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(24\times10^6)(61.875\times10^{-9})}{0.0125} = 118.8 \text{ N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(30\times10^6)(61.875\times10^{-9})}{0.0175} = 106.1 \text{ N}\cdot\text{m}$$

Choose smaller value. $M = 106.1 \text{ N}\cdot\text{m}$ ◀



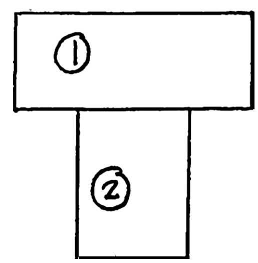
PROBLEM 4.16

Solve Prob. 4.15, assuming that $d = 40$ mm.

PROBLEM 4.15 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	32.5	19.5×10^3
②	500	12.5	6.25×10^3
Σ	1100		25.75×10^3



$\bar{Y}_0 = \dfrac{25.75\times10^3}{1100} = 23.41$ mm The neutral axis lies 23.41 mm above the bottom.

$$y_{\text{top}} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$$

$$y_{\text{bot}} = -23.41 \text{ mm} = -0.02341 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(9.09)^2 = 60.827\times10^3 \text{mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^2 + A_2d_2^2 = \frac{1}{12}(20)(25)^3 + (500)(10.91)^2 = 85.556\times10^3 \text{mm}^4$$

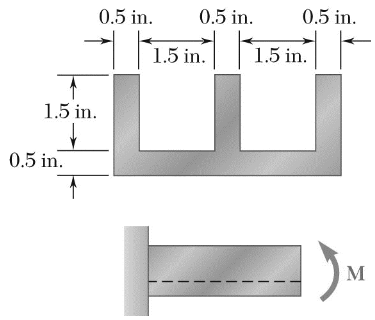
$$I = I_1 + I_2 = 146.383\times10^3 \text{mm}^4 = 146.383\times10^{-9} \text{m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $\quad M = \dfrac{(24\times10^6)(146.383\times10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$

Bottom: (compression) $\quad M = \dfrac{(30\times10^6)(146.383\times10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$

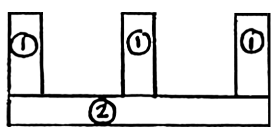
Choose smaller value. $\quad M = 187.6 \text{ N}\cdot\text{m}$ ◀



PROBLEM 4.17

Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in.}$$

The neutral axis lies 0.75 in. above bottom.

$$y_{\text{top}} = 2.0 - 0.75 = 1.25 \text{ in.}, \qquad y_{\text{bot}} = -0.75 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^2 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

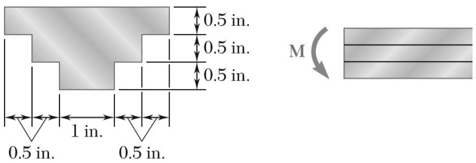
$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

Top: (compression) $\quad M = \dfrac{(16)(1.59375)}{1.25} = 20.4 \text{ kip} \cdot \text{in}$

Bottom: (tension) $\quad M = \dfrac{(12)(1.59375)}{0.75} = 25.5 \text{ kip} \cdot \text{in}$

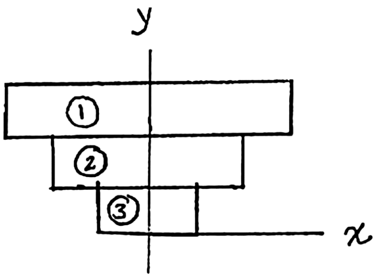
Choose the smaller as M_{all}. $\quad M_{\text{all}} = 20.4 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 4.18

Knowing that for the casting shown the allowable stress is 5 ksi in tension and 18 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION



Locate the neutral axis and compute the moment of inertia.

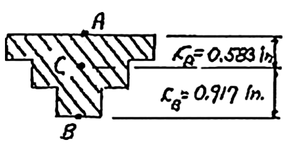
$$\overline{Y} = \frac{\sum A_i \overline{y}_i}{\sum A_i} \qquad \overline{I} = \frac{1}{12} b_i h_i^3 \quad \text{for rectangle}$$

$$d_i = \left|\overline{y}_i - \overline{Y}\right| \qquad I = \sum(A_i d_i^2 + \overline{I})$$

Part	A, in^2	$\overline{y}_i$, in.	$A_i \overline{y}_i$, in^3	d_i, in.	$A_i d_i^2$, in^4	$\overline{I}$, in^4
1	1.5	1.25	1.875	0.3333	0.1667	0.03125
2	1.0	0.75	0.75	0.1667	0.0277	0.02083
3	0.5	0.25	0.125	0.6667	0.2222	0.01042
Σ	3.0		2.75		0.4166	0.0625

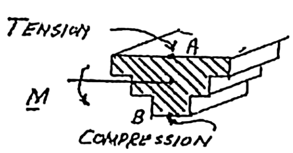
$$\overline{Y} = \frac{\sum A\overline{y}}{\sum A} = \frac{2.75}{3.0} = 0.9167 \text{ in.} \qquad I = \sum(\overline{I} + Ad^2) = 0.4166 + 0.0625 = 0.479 \text{ in}^4$$

<u>Allowable bending moment.</u>



$$\sigma = \frac{Mc}{I} \qquad \text{or} \qquad M = \frac{\sigma I}{c}$$

Tension at A: $\sigma_A \le 5$ ksi



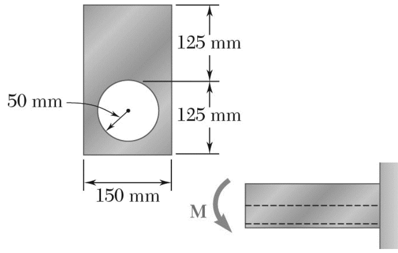
$c_A = 0.583$ in.

$$M \le \frac{(5)(0.479)}{0.583} = 4.11 \text{ kip} \cdot \text{in}$$

Compression at B: $\sigma_B \le 18$ ksi $\quad c_B = 0.917$ in.

$$M \le \frac{(18)(0.479)}{0.917} = 9.40 \text{ kip} \cdot \text{in}$$

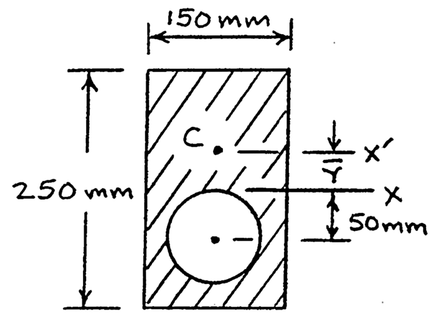
The smaller value is the allowable value of M. $\qquad M = 4.11 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 4.19

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION



① rectangle ② circular cutout

$$A_1 = (150)(250) = 37.5\times10^3\,\text{mm}^2$$

$$A_2 = -\pi(50)^2 = -7.85398\times10^3\,\text{mm}^2$$

$$A = A_1 + A_2 = 29.64602\times10^3\,\text{mm}^2$$

$$\overline{y}_1 = 0\text{ mm}$$

$$\overline{y}_2 = -50\text{ mm}$$

$$\overline{Y} = \frac{\Sigma A\overline{y}}{\Sigma A}$$

$$\overline{Y} = \frac{(37.5\times10^3)(0) + (-7.85393\times10^3)(-50)}{29.64602\times10^3}$$

$$= 13.2463\text{ mm}$$

$$I_{X'} = \Sigma(I + Ad^2) = I_1 - I_2$$

$$= \left[\frac{1}{12}(150)(250)^3 + (37.5\times10^3)(13.2463)^2\right]$$

$$-\left[\frac{\pi}{4}(50)^4 + (7.85398\times10^3)(50+13.2463)^2\right]$$

$$= 201.892\times10^6 - 36.3254\times10^6 = 165.567\times10^6\,\text{mm}^4$$

$$= 165.567\times10^{-6}\,\text{m}^4$$

Top: (tension side) $c = 125 - 13.2463 = 111.7537\text{ mm} = 0.11175\text{ m}$

$$\sigma = \frac{Mc}{I} \qquad M = \frac{I\sigma}{c} = \frac{(165.567\times10^{-6})(120\times10^6)}{0.11175}$$

$$= 177.79\times10^3\,\text{N}\cdot\text{m}$$

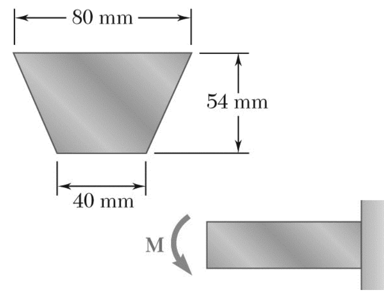
Bottom: (compression side) $c = 125 + 13.2463 = 138.2463\text{ mm}$

$$= 0.13825\text{ m}$$

$$\sigma = \frac{Mc}{I} \qquad M = \frac{I\sigma}{c} = \frac{(165.567\times10^{-6})(150\times10^6)}{0.13825}$$

$$= 179.64\times10^3\,\text{N}\cdot\text{m}$$

Choose the smaller. $M = 177.8\times10^3\,\text{N}\cdot\text{m}$ $M = 177.8\text{ kN}\cdot\text{m}$ ◀

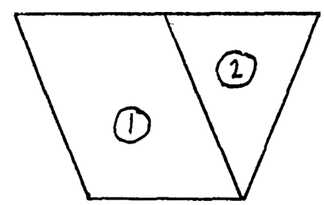


PROBLEM 4.20

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3	d, mm
①	2160	27	58320	3
②	1080	36	38880	3
Σ	3240		97200	



$\bar{Y} = \dfrac{97200}{3240} = 30$ mm The neutral axis lies 30 mm above the bottom.

$$y_{\text{top}} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m} \qquad y_{\text{bot}} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32\times10^3\,\text{mm}^4$$

$$I_2 = \frac{1}{36}b_2h_2^2 + A_2d_2^2 = \frac{1}{36}(40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84\times10^3\,\text{mm}^4$$

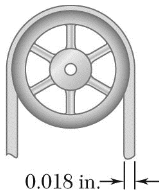
$$I = I_1 + I_2 = 758.16\times10^3\,\text{mm}^4 = 758.16\times10^{-9}\,\text{m}^4$$

$$|\sigma| = \left|\frac{M\,y}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $\quad M = \dfrac{(120\times10^6)(758.16\times10^{-9})}{0.024} = 3.7908\times10^3\,\text{N}\cdot\text{m}$

Bottom: (compression) $\quad M = \dfrac{(150\times10^6)(758.16\times10^{-9})}{0.030} = 3.7908\times10^3\,\text{N}\cdot\text{m}$

Choose the smaller as M_{all}. $\quad M_{\text{all}} = 3.7908\times10^3\,\text{N}\cdot\text{m}$ $\qquad M_{\text{all}} = 3.79\text{ kN}\cdot\text{m}$ ◀



PROBLEM 4.21

A steel band saw blade, that was originally straight, passes over 8-in.-diameter pulleys when mounted on a band saw. Determine the maximum stress in the blade, knowing that it is 0.018 in. thick and 0.625 in. wide. Use $E = 29\times10^6$ psi.

SOLUTION

Band blade thickness: $t = 0.018$ in.

Radius of pulley: $r = \frac{1}{2}d = 4.000$ in.

Radius of curvature of centerline of blade:

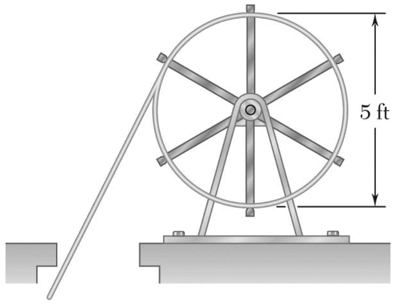
$$\rho = r + \frac{1}{2}t = 4.009 \text{ in.}$$

$$c = \frac{1}{2}t = 0.009 \text{ in.}$$

Maximum strain: $\varepsilon_m = \frac{c}{\rho} = \frac{0.009}{4.009} = 0.002245$

Maximum stress: $\sigma_m = E\varepsilon_m = (29\times10^6)(0.002245)$

$\sigma_m = 65.1\times10^3$ psi $\qquad \sigma_m = 65.1$ ksi ◀



PROBLEM 4.22

Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a rod, when the rod, which is initially straight, is wrapped on a spool, (*b*) the corresponding bending moment in the rod. Use $E = 29 \times 10^6$ psi .

SOLUTION

Radius of cross section: $r = \frac{1}{2}d = \frac{1}{2}(0.30) = 0.15 \text{ in.}$

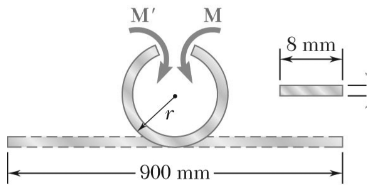
Moment of inertia: $I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6} \text{ in}^4$

$D = 5 \text{ ft} = 60 \text{ in.} \qquad \rho = \frac{1}{2}D = 30 \text{ in.}$

$c = r = 0.15 \text{ in.}$

(*a*) $\sigma_{\max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145 \times 10^3 \text{ psi}$ $\qquad \sigma_{\max} = 145 \text{ ksi}$ ◀

(*b*) $M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-6})}{30}$ $\qquad M = 384 \text{ lb} \cdot \text{in}$ ◀



PROBLEM 4.23

A 900-mm strip of steel is bent into a full circle by two couples applied as shown. Determine (*a*) the maximum thickness *t* of the strip if the allowable stress of the steel is 420 MPa, (*b*) the corresponding moment *M* of the couples. Use $E = 200$ GPa .

SOLUTION

When the rod is bent into a full circle, the circumference is 900 mm. Since the circumference is equal to 2π times ρ, the radius of curvature, we get

$$\rho = \frac{900 \text{ mm}}{2\pi} = 143.24 \text{ mm} = 0.14324 \text{ m}$$

Stress: $\sigma = E\varepsilon = \dfrac{Ec}{\rho}$ or $c = \dfrac{\rho\sigma}{E}$

For $\sigma = 420$ MPa and $E = 200$ GPa,

$$c = \frac{(0.14324)(420 \times 10^6)}{200 \times 10^9} = 0.3008 \times 10^{-3} \text{ m}$$

(*a*) Maximum thickness: $t = 2c = 0.6016 \times 10^{-3}$ m

$t = 0.602$ mm ◀

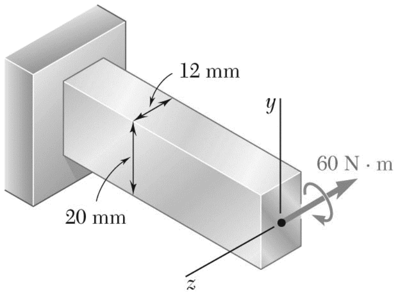
<u>Moment of inertia for a rectangular section.</u>

$$I = \frac{bt^3}{12} = \frac{(8 \times 10^{-3})(0.6016 \times 10^{-3})^3}{12} = 145.16 \times 10^{-15} \text{ m}^4$$

(*b*) Bending moment: $M = \dfrac{EI}{\rho}$

$$M = \frac{(200 \times 10^9)(145.16 \times 10^{-15})}{0.14324} = 0.203 \text{ N} \cdot \text{m}$$

$M = 0.203$ N · m ◀



PROBLEM 4.24

A 60 N · m couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.

SOLUTION

(*a*) Bending about *z*-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(20)^3 = 8\times 10^3\,\text{mm}^4 = 8\times 10^{-9}\,\text{m}^4$$

$$c = \frac{20}{2} = 10\text{ mm} = 0.010\text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.010)}{8\times 10^{-9}} = 75.0\times 10^6\,\text{Pa}$$

$\sigma = 75.0$ MPa ◀

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times 10^9)(8\times 10^{-9})} = 37.5\times 10^{-3}\,\text{m}^{-1}$$

$\rho = 26.7$ m ◀

(*b*) Bending about *y*-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(20)(12)^3 = 2.88\times 10^3\,\text{mm}^4 = 2.88\times 10^{-9}\,\text{m}^4$$

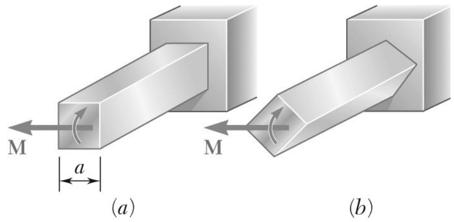
$$c = \frac{12}{2} = 6\text{ mm} = 0.006\text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.006)}{2.88\times 10^{-9}} = 125.0\times 10^6\,\text{Pa}$$

$\sigma = 125.0$ MPa ◀

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times 10^9)(2.88\times 10^{-9})} = 104.17\times 10^{-3}\,\text{m}^{-1}$$

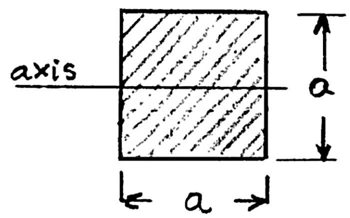
$\rho = 9.60$ m ◀



PROBLEM 4.25

A couple of magnitude **M** is applied to a square bar of side *a*. For each of the orientations shown, determine the maximum stress and the curvature of the bar.

SOLUTION



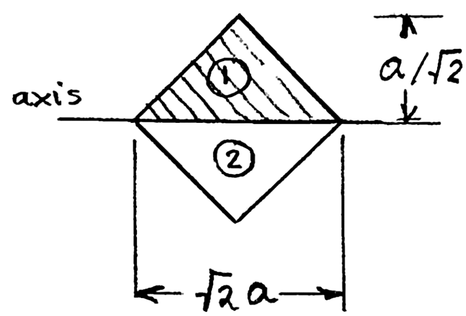
$$I = \frac{1}{12}bh^3 = \frac{1}{12}aa^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M\frac{a}{2}}{\frac{a^4}{12}} \qquad \sigma_{\max} = \frac{6M}{a^3} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4} \blacktriangleleft$$

For one triangle, the moment of inertia about its base is



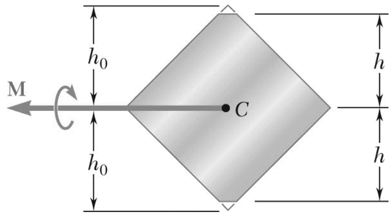
$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}\left(\sqrt{2}a\right)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

$$I_2 = I_1 = \frac{a^4}{24}$$

$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}} \qquad \sigma_{\max} = \frac{Mc}{I} = \frac{Ma/\sqrt{2}}{a^4/12} = \frac{6\sqrt{2}M}{a^3} \qquad \sigma_{\max} = \frac{8.49M}{a^3} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4} \blacktriangleleft$$



PROBLEM 4.26

A portion of a square bar is removed by milling so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

SOLUTION

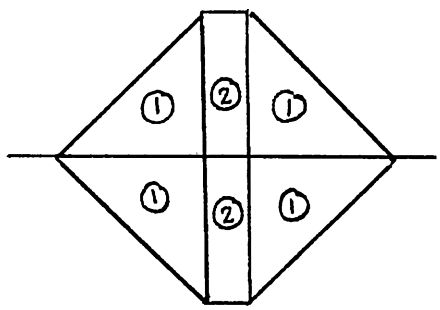
$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)h\,h^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)(h^3)$$

$$= \frac{1}{3}h^4 + \frac{4}{3}h_0h^3 - \frac{4}{3}h\;h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h$$

$$\sigma = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0h^3 - h^4} = \frac{3M}{(4h_0 - 3h)\,h^2}$$



For the original square, $h = h_0,\ c = h_0.$

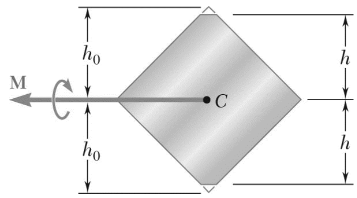
$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)\,h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9\,h_0^2)} = 0.950$$

$$\sigma = 0.950\,\sigma_0$$

$k = 0.950$ ◀

PROBLEM 4.27

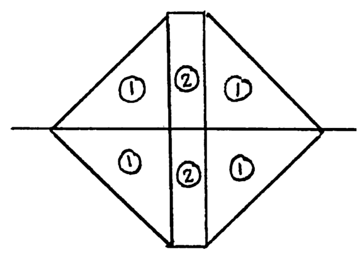


In Prob. 4.26, determine (*a*) the value of *h* for which the maximum stress σ_m is as small as possible, (*b*) the corresponding value of *k*.

PROBLEM 4.26 A portion of a square bar is removed by milling so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of *k*.

SOLUTION

$$I = 4I_1 + 2I_2$$
$$= (4)\left(\frac{1}{12}\right)hh^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)\,h^3$$
$$= \frac{1}{3}h^4 - \frac{4}{3}h_0h^3 - \frac{4}{3}h^3 = \frac{4}{3}h_0\,h^3 - h^4$$
$$c = h \qquad \frac{I}{c} = \frac{4}{3}h_0h^2 - h^3$$



$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\frac{4}{3}h_0h^2 - h^3\right] = 0.$

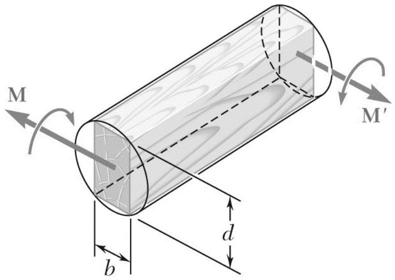
$$\frac{8}{3}h_0h - 3h^2 = 0 \qquad h = \frac{8}{9}h_0 \blacktriangleleft$$

$$\frac{I}{c} = \frac{4}{3}h_0\left(\frac{8}{9}h_0\right)^2 - \left(\frac{8}{9}h_0\right)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729M}{256h_0^3}$$

For the original square, $\quad h = h_0 \quad c = h_0 \quad \frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} = 0.949 \qquad k = 0.949 \blacktriangleleft$$



PROBLEM 4.28

A couple M will be applied to a beam of rectangular cross section that is to be sawed from a log of circular cross section. Determine the ratio d/b, for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

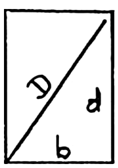
SOLUTION

Let D be the diameter of the log.

$$D^2 = b^2 + d^2 \qquad d^2 = D^2 - b^2$$

$$I = \frac{1}{12}bd^3 \qquad c = \frac{1}{2}d \qquad \frac{I}{c} = \frac{1}{6}bd^2$$

(*a*) σ_m is the minimum when $\dfrac{I}{c}$ is maximum.



$$\frac{I}{c} = \frac{1}{6}b(D^2 - b^2) = \frac{1}{6}D^2b - \frac{1}{6}b^3$$

$$\frac{d}{db}\left(\frac{I}{c}\right) = \frac{1}{6}D^2 - \frac{3}{6}b^2 = 0 \qquad b = \frac{1}{\sqrt{3}}D$$

$$d = \sqrt{D^2 - \frac{1}{3}D^2} = \sqrt{\frac{2}{3}}D \qquad \frac{d}{b} = \sqrt{2} \blacktriangleleft$$

(*b*) $\rho = \dfrac{EI}{M}$

ρ is maximum when I is maximum, $\dfrac{1}{12}bd^3$ is maximum, or b^2d^6 is maximum.

$$(D^2 - d^2)d^6 \text{ is maximum.}$$

$$6D^2d^5 - 8d^7 = 0 \qquad d = \frac{\sqrt{3}}{2}D$$

$$b = \sqrt{D^2 - \frac{3}{4}D^2} = \frac{1}{2}D \qquad \frac{d}{b} = \sqrt{3} \blacktriangleleft$$

PROBLEM 4.29

For the aluminum bar and loading of Sample Prob. 4.1, determine (*a*) the radius of curvature ρ' of a transverse cross section, (*b*) the angle between the sides of the bar that were originally vertical. Use $E = 10.6 \times 10^6$ psi and $v = 0.33$.

SOLUTION

From Sample Prob. 4.1, $\quad I = 12.97 \text{ in}^4 \quad\quad M = 103.8 \text{ kip}\cdot\text{in}$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{103.8\times 10^3}{(10.6\times 10^6)(12.97)} = 755\times 10^{-6} \text{ in}^{-1}$$

(*a*) $\quad \dfrac{1}{\rho'} = v\dfrac{1}{\rho} = (0.33)(755\times 10^{-6}) = 249\times 10^{-6} \text{ in}^{-1}$

$\rho' = 4010$ in. $\quad\quad \rho' = 334$ ft ◀

(*b*) $\quad \theta = \dfrac{\text{length of arc}}{\text{radius}} = \dfrac{b}{\rho'} = \dfrac{3.25}{4010} = 810\times 10^{-6}\,\text{rad} \quad\quad \theta = 0.0464°$ ◀

PROBLEM 4.30

For the bar and loading of Example 4.01, determine (a) the radius of curvature ρ, (b) the radius of curvature ρ' of a transverse cross section, (c) the angle between the sides of the bar that were originally vertical. Use $E = 29\times10^6$ psi and $v = 0.29$.

SOLUTION

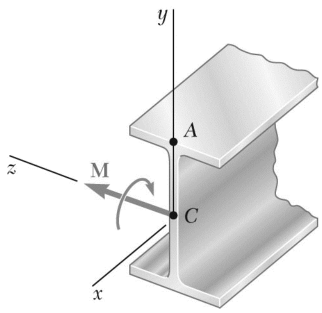
From Example 4.01, $M = 30$ kip $\cdot$ in, $I = 1.042$ in^4

(a) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{(30\times10^3)}{(29\times10^6)(1.042)} = 993\times10^{-6}\ \text{in}^{-1}$$ $\rho = 1007$ in. ◀

(b) $$\varepsilon^1 = v\varepsilon = \frac{vc}{\rho} = v\frac{c}{\rho'}$$

$$\frac{1}{\rho'} = v\frac{1}{\rho} = (0.29)(993\times10^{-6})\,\text{in.}^{-1} = 288\times10^{-6}\ \text{in.}^{-1}$$ $\rho' = 3470$ in. ◀

(c) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.8}{3470} = 230\times10^{-6}\ \text{rad}$$ $\theta = 0.01320°$ ◀



PROBLEM 4.31

A $W200\times31.3$ rolled-steel beam is subjected to a couple **M** of moment 45 kN · m. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.

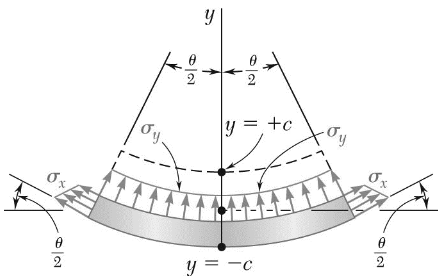
SOLUTION

For $W200\times31.3$ rolled steel section,

$$I = 31.4\times10^6\ \text{mm}^4$$
$$= 31.4\times10^{-6}\ \text{m}^4$$

(*a*) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{45\times10^3}{(200\times10^9)(31.4\times10^{-6})} = 7.17\times10^{-3}\ \text{m}^{-1}$$ $\rho = 139.6$ m ◀

(*b*) $$\frac{1}{\rho'} = \nu\frac{1}{\rho} = (0.29)(7.17\times10^{-3}) = 2.07\times10^{-3}\ \text{m}^{-1}$$ $\rho' = 481$ m ◀



PROBLEM 4.32

It was assumed in Sec. 4.3 that the normal stresses σ_y in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (*a*) derive an approximate expression for σ_y as a function of *y*, (*b*) show that $(\sigma_y)_{\max} = -(c/2\rho)(\sigma_x)_{\max}$ and, thus, that σ_y can be neglected in all practical situations. (*Hint:* Consider the free-body diagram of the portion of beam located below the surface of ordinate *y* and assume that the distribution of the stress σ_x is still linear.)

SOLUTION

Denote the width of the beam by *b* and the length by *L*.

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos\frac{\theta}{2} \approx 1$

$$\Sigma F_y = 0: \quad \sigma_y bL + 2\int_{-c}^{y} \sigma_x b\,dy \,\sin\frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L}\sin\frac{\theta}{2}\int_{-c}^{y}\sigma_x dy \approx -\frac{\theta}{L}\int_{-c}^{y}\sigma_x dy = -\frac{1}{\rho}\int_{-c}^{y}\sigma_x dy$$

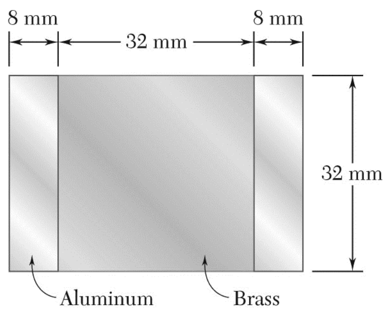
But, $\sigma_x = -(\sigma_x)_{\max}\dfrac{y}{c}$

(*a*) $\sigma_y = \dfrac{(\sigma_x)_{\max}}{\rho c}\displaystyle\int_{-c}^{y} y\,dy = \dfrac{(\sigma_x)_{\max}}{\rho c}\dfrac{y^2}{2}\Big|_{-c}^{y}$ $\qquad \sigma_y = \dfrac{(\sigma_x)_{\max}}{2\rho c}(y^2 - c^2)$ ◀

The maximum value σ_y occurs at $y = 0$.

(*b*) $(\sigma_y)_{\max} = -\dfrac{(\sigma_x)_{\max} c^2}{2\rho c} = -\dfrac{(\sigma_x)_{\max} c}{2\rho}$ ◀

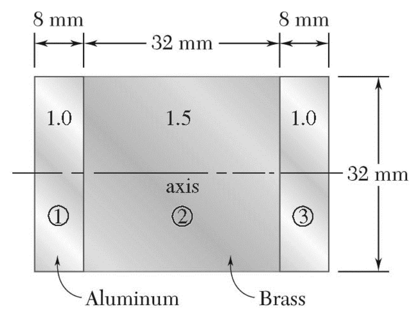
PROBLEM 4.33



A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 174.7626 \times 10^3 \text{ mm}^4 = 174.7626 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{n M y}{I}\right| \qquad M = \left|\frac{\sigma I}{n y}\right|$$

Aluminum: $n = 1.0, \; |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

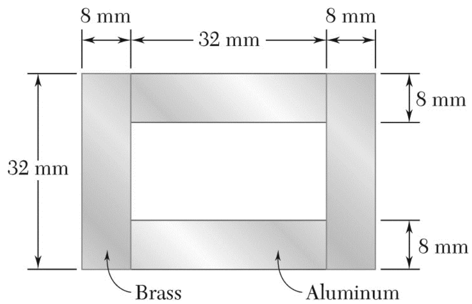
$$M = \frac{(100 \times 10^6)(174.7626 \times 10^{-9})}{(1.0)(0.016)} = 1.0923 \times 10^3 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \; |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(174.7626 \times 10^{-9})}{(1.5)(0.016)} = 1.1651 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 1.092 \times 10^3 \text{ N} \cdot \text{m}$ $\qquad M = 1.092 \text{ kN} \cdot \text{m}$ ◀

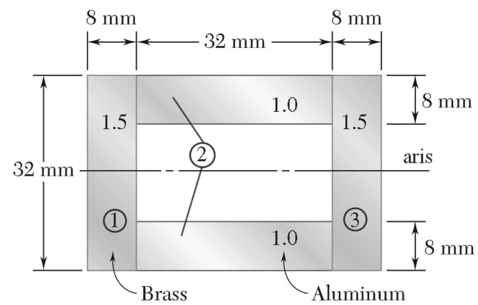
PROBLEM 4.34



A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12}(8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 \left(H_2^3 - h_2^3\right) = \frac{1.0}{12}(32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

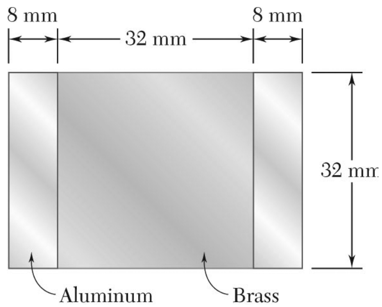
Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 887 \text{ N} \cdot \text{m}$ ◀



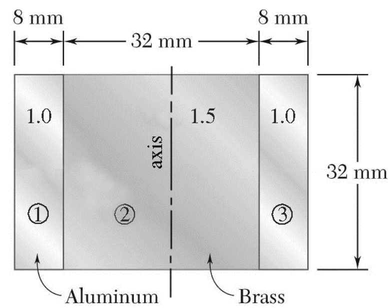
PROBLEM 4.35

For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

PROBLEM 4.35 Bar of Prob. 4.33.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} h_1 b_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(32)(8)^3 + (1.0)[(32)(8)](20)^2 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 338.58 \times 10^3 \text{ mm}^4 = 338.58 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

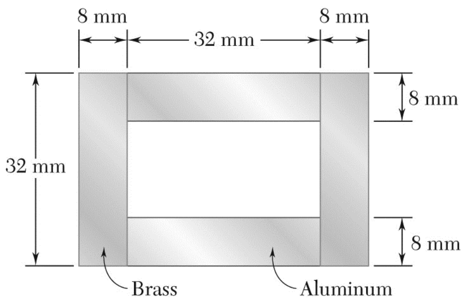
Aluminum: $n = 1.0, \; |y| = 24 \text{ mm} = 0.024 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(338.58 \times 10^{-9})}{(1.0)(0.024)} = 1.411 \times 10^3 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \; |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(338.58 \times 10^{-9})}{(1.5)(0.016)} = 2.257 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 1.411 \times 10^3 \text{ N} \cdot \text{m}$ $\quad M = 1.411 \text{ kN} \cdot \text{m}$ ◀



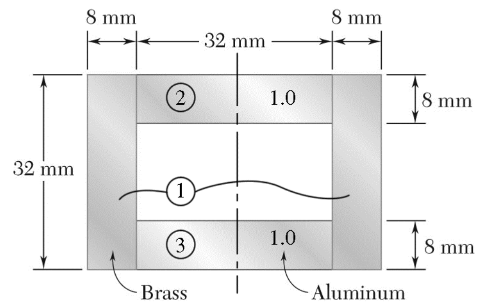
PROBLEM 4.36

For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

PROBLEM 4.36 Bar of Prob. 4.34.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} h_1 \left(B_1^3 - b_1^3\right) = \frac{1.5}{12}(32)(48^3 - 32^3) = 311.296 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 354.99 \times 10^3 \text{ mm}^4 = 354.99 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(354.99 \times 10^{-9})}{(1.0)(0.016)} = 2.2187 \times 10^3 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5 \quad |y| = 24 \text{ mm} = 0.024 \text{ m} \quad \sigma = 160 \times 10^6 \text{ Pa}$

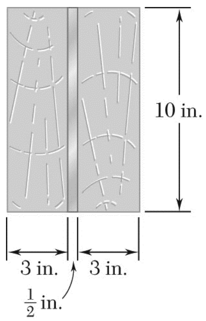
$$M = \frac{(160 \times 10^6)(354.99 \times 10^{-9})}{(1.5)(0.024)} = 1.57773 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value.

$$M = 1.57773 \times 10^3 \text{ N} \cdot \text{m}$$

$M = 1.578 \text{ kN} \cdot \text{m}$ ◀

PROBLEM 4.37



Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

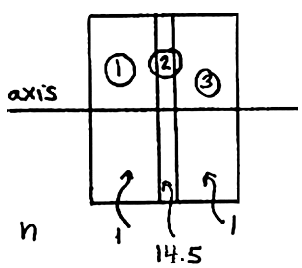
	Wood	Steel
Modulus of elasticity:	2×10^6 psi	29×10^6 psi
Allowable stress:	2000 psi	22 ksi

SOLUTION

Use wood as the reference material.

$$n = 1.0 \text{ in wood}$$

$$n = E_s/E_w = 29/2 = 14.5 \text{ in steel}$$



For the transformed section,

$$I_1 = \frac{n_1}{12}b_1h_1^3 = \frac{1.0}{12}(3)(10)^3 = 250 \text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 = \frac{14.5}{12}\left(\frac{1}{2}\right)(10)^3 = 604.17 \text{ in}^4$$

$$I_3 = I_1 = 250 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 1104.2 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

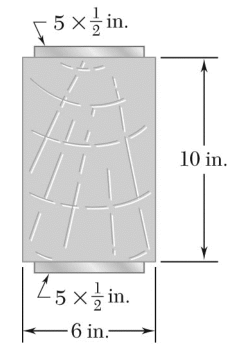
Wood: $\quad n = 1.0, \quad y = 5 \text{ in.}, \quad \sigma = 2000 \text{ psi}$

$$M = \frac{(2000)(1104.2)}{(1.0)(5)} = 441.7\times10^3 \text{ lb}\cdot\text{in}$$

Steel: $\quad n = 14.5, \quad y = 5 \text{ in.}, \quad \sigma = 22 \text{ ksi} = 22\times10^3 \text{ psi}$

$$M = \frac{(22\times10^3)(1104.2)}{(14.5)(5)} = 335.1\times10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value. $\quad M = 335\times10^3 \text{ lb}\cdot\text{in}$ $\quad M = 335 \text{ kip}\cdot\text{in}$ ◀



PROBLEM 4.38

Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

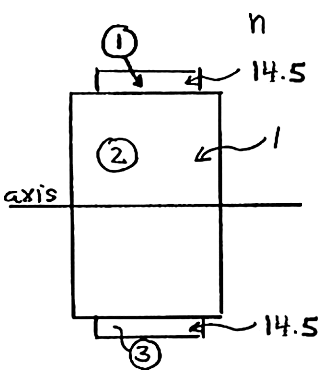
	Wood	Steel
Modulus of elasticity:	2×10^6 psi	29×10^6 psi
Allowable stress:	2000 psi	22 ksi

SOLUTION

Use wood as the reference material.

$$n = 1.0 \text{ in wood}$$

$$n = E_s/E_w = 29/2 = 14.5 \text{ in steel}$$



For the transformed section,

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2$$

$$= \frac{14.5}{12}(5)\left(\frac{1}{2}\right)^3 + (14.5)(5)\left(\frac{1}{2}\right)(5.25)^2 = 999.36 \text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^2 = \frac{1.0}{12}(6)(10)^3 = 500 \text{ in}^4$$

$$I_3 = I_1 = 999.36 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 2498.7 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

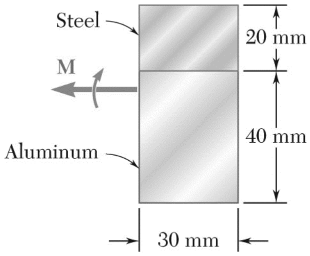
Wood: $n = 1.0, \quad y = 5 \text{ in.}, \quad \sigma = 2000 \text{ psi}$

$$M = \frac{(2000)(2499)}{(1.0)(5)} = 999.5\times10^3 \text{ lb}\cdot\text{in}$$

Steel: $n = 14.5, \quad y = 5.5 \text{ in.}, \quad \sigma = 22 \text{ ksi} = 22\times10^3 \text{ psi}$

$$M = \frac{(22\times10^3)(2499)}{(14.5)(5.5)} = 689.3\times10^3 \text{ lb}\cdot\text{in}$$

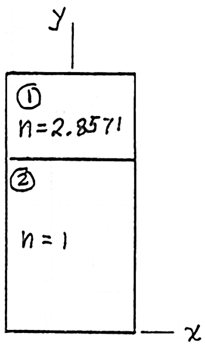
Choose the smaller value. $M = 689\times10^3 \text{ lb}\cdot\text{in}$ $\qquad M = 689 \text{ kip}\cdot\text{in}$ ◀



PROBLEM 4.39

A steel bar and an aluminum bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminum is 70 GPa and for steel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 1500\ \text{N} \cdot \text{m}$, determine the maximum stress in (*a*) the aluminum, (*b*) the steel.

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 200/70 = 2.8571$

Transformed section:

Part	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3	d, mm
1	600	1714.3	50	85714	12.35
2	1200	1200	20	24000	17.65
Σ		2914.3		109714	

$$\bar{Y}_0 = \frac{109714}{2914.3} = 37.65\ \text{mm} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{2.8571}{12}(30)(20)^3 + (1714.3)(12.35)^2 = 318.61 \times 10^3\ \text{mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(30)(40)^3 + (1200)(17.65)^2 = 533.83 \times 10^3\ \text{mm}^4$$

$$I = I_1 + I_2 = 852.44 \times 10^3\ \text{mm}^4 = 852.44 \times 10^{-9}\ \text{m}^4$$

$$M = 1500\ \text{N} \cdot \text{m}$$

Stress: $$\sigma = -\frac{nMy}{I}$$

(*a*) Aluminum: $n = 1, \quad y = -37.65\ \text{mm} = -0.03765\ \text{m}$

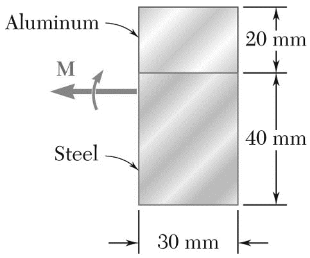
$$\sigma_a = -\frac{(1)(1500)(-0.03765)}{852.44 \times 10^{-9}} = 66.2 \times 10^6\ \text{Pa}$$

$\sigma_a = 66.2\ \text{MPa}$ ◀

(*b*) Steel: $n = 2.8571, \quad y = 60 - 37.65 = 22.35\ \text{mm} = 0.02235\ \text{m}$

$$\sigma_s = -\frac{nMy}{I} = -\frac{(2.8571)(1500)(0.02235)}{852.44 \times 10^{-9}} = -112.4 \times 10^6\ \text{Pa}$$

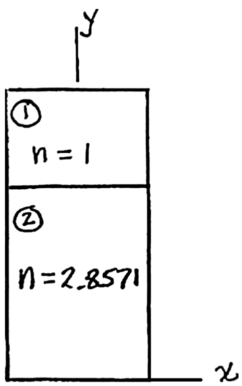
$\sigma_s = -112.4\ \text{MPa}$ ◀



PROBLEM 4.40

A steel bar and an aluminum bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminum is 70 GPa and for steel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 1500\ \text{N}\cdot\text{m}$, determine the maximum stress in (*a*) the aluminum, (*b*) the steel.

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 200/70 = 2.8571$

Transformed section:

Part	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3	d, mm
1	600	600	50	30000	25.53
2	1200	3428.5	20	68570	4.47
Σ		4028.5		98570	

$$\bar{Y}_0 = \frac{98570}{4028.5} = 24.47\ \text{mm} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(30)(20)^3 + (600)(25.53)^2 = 411.07\times10^3\ \text{mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{2.8571}{12}(30)(40)^3 + (3428.5)(4.47)^2 = 525.64\times10^3\ \text{mm}^4$$

$$I = I_1 + I_2 = 936.71\times10^3\ \text{mm}^4 = 936.71\times10^{-9}\ \text{m}^4$$

$$M = 1500\ \text{N}\cdot\text{m}$$

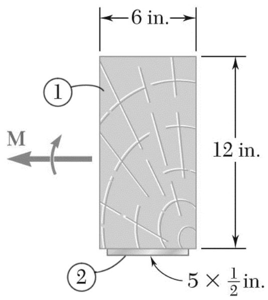
Stress: $\sigma = -\dfrac{nMy}{I}$

(*a*) Aluminum: $n = 1,\quad y = 60 - 24.47 = 35.53\ \text{mm} = 0.03553\ \text{m}$

$$\sigma_a = -\frac{(1)(1500)(0.03553)}{936.71\times10^{-9}} = -56.9\times10^6\ \text{Pa} \qquad \sigma_a = -56.9\ \text{MPa}\ \blacktriangleleft$$

(*b*) Steel: $n = 2.8571,\quad y = -24.47\ \text{mm} = -0.02447\ \text{m}$

$$\sigma_s = -\frac{(2.8571)(1500)(-0.02447)}{936.71\times10^{-9}} = 111.9\times10^6\ \text{Pa} \qquad \sigma_s = 111.9\ \text{MPa}\ \blacktriangleleft$$



PROBLEM 4.41

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel, 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

SOLUTION

Use wood as the reference material.

For wood, $n = 1$

For steel, $n = E_s / E_w = 29/1.8 = 16.1111$

Transformed section: ① = wood ② = steel

$$\overline{Y}_o = \frac{421.931}{112.278} = 3.758 \text{ in.}$$

	A, in^2	nA, in^2	$\overline{y}_o$	$nA\overline{y}_o$, in^3
①	72	72	6	432
②	2.5	40.278	−0.25	−10.069
		112.278		421.931

The neutral axis lies 3.758 in. above the wood-steel interface.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(6)(12)^3 + (72)(6 - 3.758)^2 = 1225.91 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{16.1111}{12}(5)(0.5)^3 + (40.278)(3.578 + 0.25)^2 = 647.87 \text{ in}^4$$

$$I = I_1 + I_2 = 1873.77 \text{ in}^4$$

$$M = 450 \text{ kip} \cdot \text{in} \qquad \sigma = -\frac{nMy}{I}$$

(*a*) Wood: $n = 1, \quad y = 12 - 3.758 = 8.242$ in

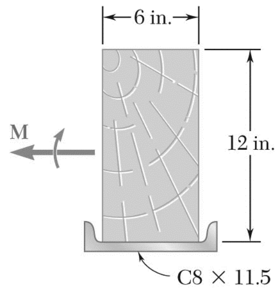
$$\sigma_w = -\frac{(1)(450)(8.242)}{1873.77} = -1.979 \text{ ksi}$$

$\sigma_w = -1.979$ ksi ◀

(*b*) Steel: $n = 16.1111, \quad y = -3.758 - 0.5 = -4.258$ in

$$\sigma_s = -\frac{(16.1111)(450)(-4.258)}{1873.77} = 16.48 \text{ ksi}$$

$\sigma_s = 16.48$ ksi ◀

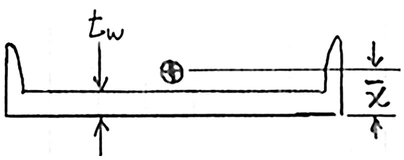


PROBLEM 4.42

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel, 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450\text{ kip}\cdot\text{in.}$, determine the maximum stress in (*a*) the wood, (*b*) the steel.

SOLUTION

Use wood as the reference material.



For wood, $n = 1$

For steel, $n = \dfrac{E_s}{E_w} = \dfrac{29\times10^6}{1.8\times10^6} = 16.1111$

For C8 $\times 11.5$ channel section,

$A = 3.38\text{ in}^2,\quad t_w = 0.220\text{ in.},\quad \bar{x} = 0.571\text{ in.},\quad I_y = 1.32\text{ in}^4$

For the composite section, the centroid of the channel (part 1) lies 0.571 in. above the bottom of the section. The centroid of the wood (part 2) lies $0.220 + 6.00 = 6.22$ in. above the bottom.

Transformed section:

Part	A, in^2	nA, in^2	$\bar{y}$, in.	$nA\bar{y}$, in^3	d, in.
1	3.38	54.456	0.571	31.091	3.216
2	72	72	6.22	447.84	2.433
Σ		126.456		478.93	

$$\bar{Y}_0 = \frac{478.93\text{ in}^3}{126.456\text{ in}^2} = 3.787\text{ in.} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

The neutral axis lies 3.787 in. above the bottom of the section.

$$I_1 = n_1\bar{I}_1 + n_1A_1d_1^2 = (16.1111)(1.32) + (54.456)(3.216)^2 = 584.49\text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1}{12}(6)(12)^3 + (72)(2.433)^2 = 1290.20\text{ in}^4$$

$$I = I_1 + I_2 = 1874.69\text{ in}^4$$

$$M = 450\text{ kip}\cdot\text{in} \qquad \sigma = -\frac{nMy}{I}$$

(*a*) Wood: $n = 1,\quad y = 12 + 0.220 - 3.787 = 8.433\text{ in.}$

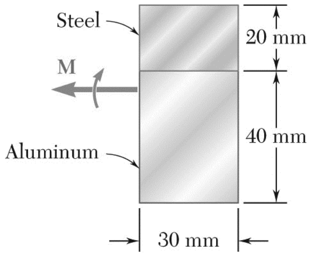
$$\sigma_w = -\frac{(1)(450)(8.433)}{1874.69} = -2.02\text{ ksi}$$

$\sigma_w = -2.02\text{ ksi}$ ◀

(*b*) Steel: $n = 16.1111,\quad y = -3.787\text{ in.}$

$$\sigma_s = -\frac{(16.1111)(450)(-3.787)}{1874.67} = 14.65\text{ ksi}$$

$\sigma_s = 14.65\text{ ksi}$ ◀



PROBLEM 4.43

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 1500 N · m.

Beam of Prob. 4.39.

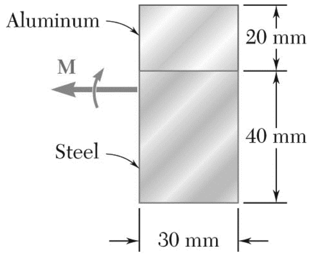
SOLUTION

See solution to Prob. 4.39 for the calculation of I.

$$I = 852.44 \times 10^{-9}\ \text{m}^4 \qquad E_a = 70 \times 10^9\ \text{Pa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{1500}{(70 \times 10^9)(852.44 \times 10^{-9})} = 0.02513\ \text{m}^{-1}$$

$$\rho = 39.8\ \text{m} \blacktriangleleft$$



PROBLEM 4.44

For the composite bar indicated, determine the radius of curvature caused by the couple of moment 1500 N · m.

Beam of Prob. 4.40.

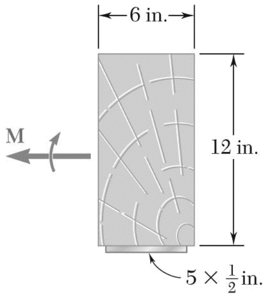
SOLUTION

See solution to Prob. 4.40 for calculation of *I*.

$$I = 936.71\times10^{-9}\ \text{m}^4 \quad E_a = 70\times10^9\ \text{Pa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{1500}{(70\times10^9)(936.71\times10^{-9})} = 0.02288\ \text{m}^{-1}$$

$\rho = 43.7$ m ◀



PROBLEM 4.45

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

Beam of Prob. 4.41.

SOLUTION

See solution to Prob. 4.41 for calculation of *I*.

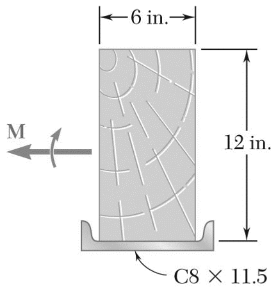
$$I = 1873.77 \text{ in}^4 \qquad E_w = 1.8\times10^6 \text{ psi}$$

$$M = 450 \text{ kip}\cdot\text{in} = 450\times10^3 \text{ lb}\cdot\text{in}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450\times10^3}{(1.8\times10^6)(1873.77)} = 133.42\times10^{-6} \text{ in}^{-1}$$

$$\rho = 7495 \text{ in.} = 625 \text{ ft} \blacktriangleleft$$

PROBLEM 4.46



For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

Beam of Prob. 4.42.

SOLUTION

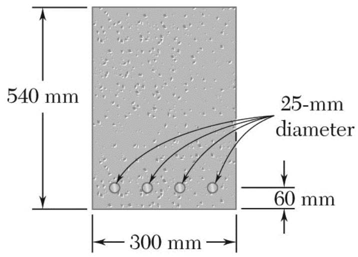
See solution to Prob. 4.42 for calculation of I.

$$I = 1874.69 \text{ in}^4 \qquad E_w = 1.8 \times 10^6 \text{ psi}$$

$$M = 450 \text{ kip} \cdot \text{in} = 450 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1874.69)} = 133.36 \times 10^{-6} \text{ in}^{-1}$$

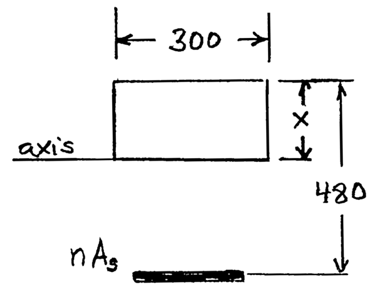
$$\rho = 7499 \text{ in.} = 625 \text{ ft} \blacktriangleleft$$



PROBLEM 4.47

The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$

Locate the neutral axis:

$$300\, x \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$150x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x:

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(150)(7.5398 \times 10^6)}}{(2)(150)}$$

$$x = 177.87 \text{ mm}, \qquad 480 - x = 302.13 \text{ mm}$$

$$I = \frac{1}{3}(300)x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2$$

$$= 1.9966 \times 10^9 \text{ mm}^4 = 1.9966 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

(*a*) Steel: $y = -302.45 \text{ mm} = -0.30245 \text{ m}$

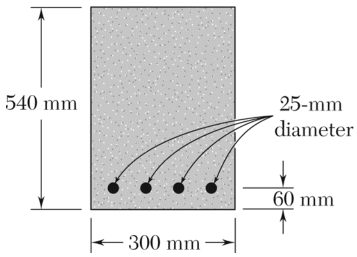
$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6 \text{ Pa}$$

$\sigma = 212$ MPa ◀

(*b*) Concrete: $y = 177.87 \text{ mm} = 0.17787 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6 \text{ Pa}$$

$\sigma = -15.59$ MPa ◀

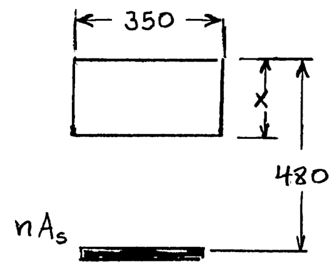


PROBLEM 4.48

Solve Prob. 4.47, assuming that the 300-mm width is increased to 350 mm.

PROBLEM 4.47 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4\frac{\pi}{4}d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2$$

$$= 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$

Locate the neutral axis:

$$350\, x\frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$175x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x:

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(175)(7.5398 \times 10^6)}}{(2)(175)}$$

$$x = 167.48 \text{ mm}, \quad 480 - x = 312.52 \text{ mm}$$

$$I = \frac{1}{3}(350)x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(350)(167.48)^3 + (15.708 \times 10^3)(312.52)^2$$

$$= 2.0823 \times 10^9 \text{ mm}^4 = 2.0823 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

(*a*) Steel: $y = -312.52 \text{ mm} = -0.31252 \text{ m}$

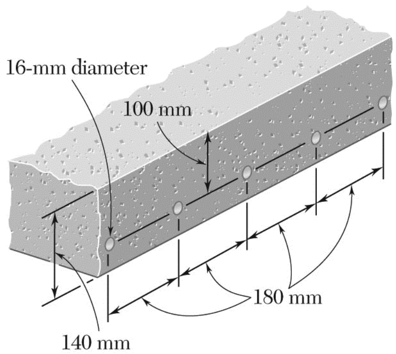
$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.31252)}{2.0823 \times 10^{-3}} = 210 \times 10^6 \text{ Pa}$$

$\sigma = 210 \text{ MPa}$ ◀

(*b*) Concrete: $y = 167.48 \text{ mm} = 0.16748 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.16748)}{2.0823 \times 10^{-3}} = -14.08 \times 10^6 \text{ Pa}$$

$\sigma = -14.08 \text{ MPa}$ ◀



PROBLEM 4.49

A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPa for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPa for the steel, determine the largest bending moment in a portion of slab 1 m wide.

SOLUTION

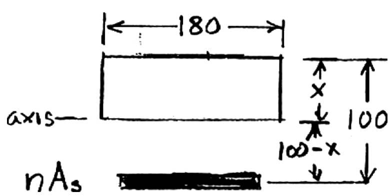
$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{20 \text{ GPa}} = 10$$

Consider a section 180-mm wide with one steel rod.

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2$$

$$nA_s = 2.0106\times10^3 \text{ mm}^2$$

Locate the neutral axis:



$$180\, x\frac{x}{2} - (100-x)(2.0106\times10^3) = 0$$

$$90x^2 + 2.0106\times10^3 x - 201.06\times10^3 = 0$$

Solving for x:

$$x = \frac{-2.0106\times10^3 + \sqrt{(2.0106\times10^3)^2 + (4)(90)(201.06\times10^3)}}{(2)(90)}$$

$$x = 37.397 \text{ mm} \qquad 100 - x = 62.603 \text{ mm}$$

$$I = \frac{1}{3}(180)x^3 + (2.0106\times10^3)(100-x)^2$$

$$= \frac{1}{3}(180)(37.397)^3 + (2.0106\times10^3)(62.603)^2$$

$$= 11.018\times10^6 \text{ mm}^4 = 11.018\times10^{-6}\text{m}^4$$

$$|\sigma| = \left|-\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1, \quad y = 37.397 \text{ mm} = 0.037397 \text{ m}, \qquad \sigma = 9\times10^6 \text{ Pa}$

$$M = \frac{(9\times10^6)(11.018\times10^{-6})}{(1.0)(0.037397)} = 2.6516\times10^3 \text{ N}\cdot\text{m}$$

PROBLEM 4.49 *(Continued)*

Steel: $n = 10, \quad y = 62.603 \text{ mm} = 0.062603 \text{ m}, \quad \sigma = 120 \times 10^6 \text{ Pa}$

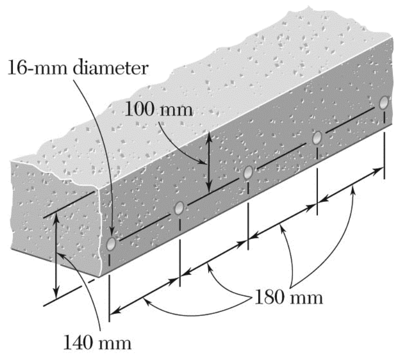
$$M = \frac{(120 \times 10^6)(11.018 \times 10^{-6})}{(10)(0.062603)} = 2.1120 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 2.1120 \times 10^3 \text{ N} \cdot \text{m}$

The above is the allowable positive moment for a 180-mm wide section. For a 1-m = 1000-mm width, mutiply by $\frac{1000}{180} = 5.556$

$$M = (5.556)(2.1120 \times 10^3) = 11.73 \times 10^3 \text{ N} \cdot \text{m}$$

$M = 11.73 \text{ kN} \cdot \text{m}$ ◀



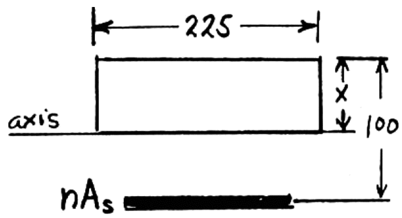
PROBLEM 4.50

A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPa for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPa for the steel, determine the largest allowable positive bending moment in a portion of slab 1 m wide.

Solve Prob. 4.49, assuming that the spacing of the 16-mm-diameter rods is increased to 225 mm on centers.

SOLUTION

$$n = \frac{E_s}{E_a} = \frac{200\text{ GPa}}{20\text{ GPa}} = 10$$



Consider a section 225-mm wide with one steel rod.

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(16)^2 = 201.06\text{ mm}^2$$

$$nA_s = 2.0106\times 10^3\text{ mm}^2$$

Locate the neutral axis:

$$225x\frac{x}{2} - (100-x)(2.0106\times 10^3) = 0$$

$$112.5x^2 + 2.0106x - 201.06\times 10^3 = 0$$

Solving for x:

$$x = \frac{-2.0106\times 10^3 + \sqrt{(2.0106\times 10^3)^2 + (4)(112.5)(201.06\times 10^3)}}{(2)(112.5)}$$

$$x = 34.273\text{ mm} \qquad 100 - x = 65.727$$

$$I = \frac{1}{3}(225)x^3 + 2.0106\times 10^3(100-x)^2$$

$$= \frac{1}{3}(225)(34.273)^3 + (2.0106\times 10^3)(65.727)^2$$

$$= 11.705\times 10^6\text{ mm}^4 = 11.705\times 10^{-6}\text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore\quad M = \frac{\sigma I}{ny}$$

Concrete:

$$n = 1,\quad y = 34.273\text{ mm} = 0.034273\text{ m},\quad \sigma = 9\times 10^6\text{ Pa}$$

$$M = \frac{(9\times 10^6)(11.705\times 10^{-6})}{(1)(0.034273)} = 3.0738\times 10^3\text{ N}\cdot\text{m}$$

PROBLEM 4.50 *(Continued)*

Steel: $n = 10, \quad y = 65.727 \text{ mm} = 0.065727 \text{ m}, \quad \sigma = 120\times 10^6 \text{ Pa}$

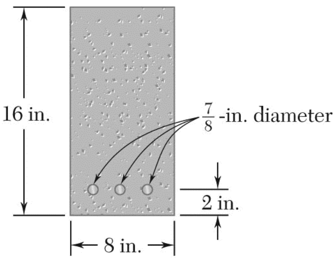
$$M = \frac{(120\times 10^6)(11.705\times 10^{-6})}{(10)(0.065727)} = 2.1370\times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 2.1370 \times 10^3 \text{ N} \cdot \text{m}$

The above is the allowable positive moment for a 225-mm-wide section. For a 1-m = 1000-mm section, multiply by $\frac{1000}{225} = 4.4444$

$$M = (4.4444)(2.1370\times 10^3) = 9.50\times 10^3 \text{ N}\cdot\text{m}$$

$M = 9.50 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 4.51

A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

SOLUTION

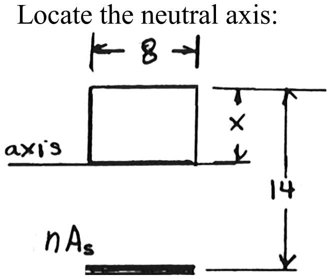
$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{3\times10^6} = 9.67$$

$$A_s = 3\frac{\pi}{4}d^2 = (3)\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2 \qquad nA_s = 17.438 \text{ in}^2$$

Locate the neutral axis:

$$8x\frac{x}{2} - (17.438)(14 - x) = 0$$

$$4x^2 + 17.438x - 244.14 = 0$$



Solve for x. $\quad x = \dfrac{-17.438 + \sqrt{17.438^2 + (4)(4)(244.14)}}{(2)(4)} = 5.6326$ in.

$$14 - x = 8.3674 \text{ in.}$$

$$I = \frac{1}{3}8x^3 + nA_s(14 - x)^2 = \frac{1}{3}(8)(5.6326)^3 + (17.438)(8.3674)^2 = 1697.45 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

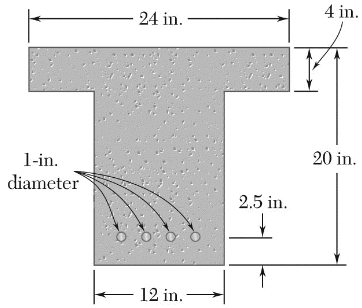
Concrete: $\quad n = 1.0, \quad |y| = 5.6326$ in., $\quad |\sigma| = 1350$ psi

$$M = \frac{(1350)(1697.45)}{(1.0)(5.6326)} = 406.835\times10^3 \text{ lb}\cdot\text{in} = 407 \text{ kip}\cdot\text{in}$$

Steel: $\quad n = 9.67, \quad |y| = 8.3674$ in., $\quad \sigma = 20\times10^3$ psi

$$M = \frac{(20\times10^3)(1697.45)}{(9.67)(8.3674)} = 419.72 \text{ lb}\cdot\text{in} = 420 \text{ kip}\cdot\text{in}$$

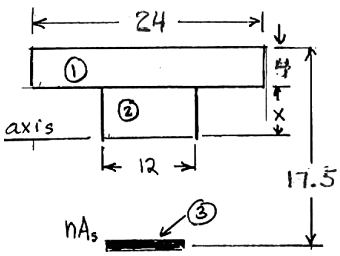
Choose the smaller value. $\quad M = 407$ kip · in $\qquad M = 33.9$ kip · ft ◀



PROBLEM 4.52

Knowing that the bending moment in the reinforced concrete beam is +100 kip · ft and that the modulus of elasticity is 3.625×10^6 psi for the concrete and 29×10^6 psi for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.625 \times 10^6} = 8.0$$

$$A_s = (4)\left(\frac{\pi}{4}\right)(1)^2 = 3.1416 \text{ in}^2 \qquad nA_s = 25.133 \text{ in}^2$$

Locate the neutral axis.

$$(24)(4)(x + 2) + (12x)\left(\frac{x}{2}\right) - (25.133)(17.5 - 4 - x) = 0$$

$$96x + 192 + 6x^2 - 339.3 + 25.133x = 0 \quad \text{or} \quad 6x^2 + 121.133x - 147.3 = 0$$

Solve for x.

$$x = \frac{-121.133 + \sqrt{(121.133)^2 + (4)(6)(147.3)}}{(2)(6)} = 1.150 \text{ in.}$$

$$d_3 = 17.5 - 4 - x = 12.350 \text{ in.}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(24)(4)^3 + (24)(4)(3.150)^2 = 1080.6 \text{ in}^4$$

$$I_2 = \frac{1}{3}b_2x^3 = \frac{1}{3}(12)(1.150)^3 = 6.1 \text{ in}^4$$

$$I_3 = nA_3d_3^2 = (25.133)(12.350)^2 = 3833.3 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 4920 \text{ in}^4$$

$$\sigma = -\frac{nMy}{I} \qquad \text{where } M = 100 \text{ kip} \cdot \text{ft} = 1200 \text{ kip} \cdot \text{in.}$$

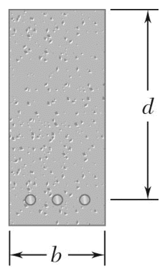
(*a*) Steel: $n = 8.0$ $\qquad y = -12.350$ in.

$$\sigma_s = -\frac{(8.0)(1200)(-12.350)}{4920} \qquad \sigma_s = 24.1 \text{ ksi} \blacktriangleleft$$

(*b*) Concrete: $n = 1.0$, $\quad y = 4 + 1.150 = 5.150$ in.

$$\sigma_c = -\frac{(1.0)(1200)(5.150)}{4920} \qquad \sigma_c = -1.256 \text{ ksi} \blacktriangleleft$$

PROBLEM 4.53



The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

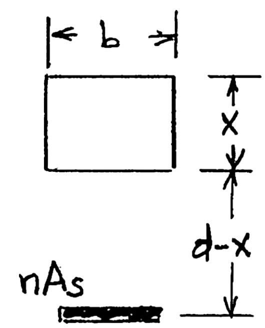
SOLUTION

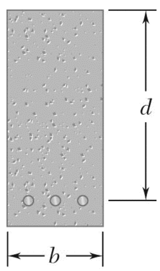
$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} \quad = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \dfrac{E_c \sigma_s}{E_s \sigma_c}}$$



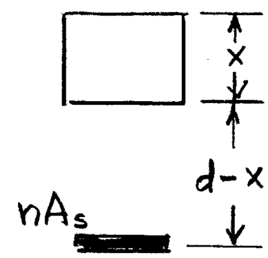


PROBLEM 4.54

For the concrete beam shown, the modulus of elasticity is 3.5×10^6 psi for the concrete and 29×10^6 psi for the steel. Knowing that $b = 8$ in. and $d = 22$ in., and using an allowable stress of 1800 psi for the concrete and 20 ksi for the steel, determine (*a*) the required area A_s of the steel reinforcement if the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.53 for definition of a balanced beam.)

The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{3.5\times10^6} = 8.2857$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.2857}\cdot\frac{20\times10^3}{1800} = 2.3410$$

$$x = 0.42717\,d = (0.42717)(22) = 9.398 \text{ in.} \qquad d - x = 22 - 9.398 = 12.602 \text{ in.}$$

Locate neutral axis: $\quad bx\frac{x}{2} - nA_s(d-x) = 0$

(*a*) $\quad A_s = \dfrac{bx^2}{2n(d-x)} = \dfrac{(8)(9.398)^2}{(2)(8.2857)(12.602)} = 3.3835 \text{ in}^2$ $\qquad A_s = 3.38 \text{ in}^2$ ◀

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(8)(9.398)^3 + (8.2857)(3.3835)(12.602)^2 = 6665.6 \text{ in}^4$$

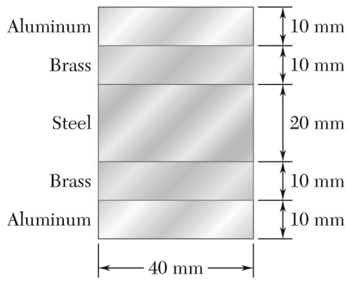
$$\sigma = \frac{nMy}{I} \qquad M = \frac{\sigma I}{ny}$$

(*b*) Concrete: $\quad n = 1.0 \quad y = 9.398$ in. $\quad \sigma_c = 1800$ psi $\quad M = \dfrac{(1800)(6665.6)}{(1.0)(9.398)} = 1.277\times10^6 \text{ lb}\cdot\text{in}$

Steel: $\quad n = 8.2857 \quad |y| = 12.602$ in. $\quad \sigma_s = 20\times10^3$ psi

$$M = \frac{(20\times10^3)(6665.6)}{(8.2857)(12.602)} = 1.277\times10^6 \text{ lb}\cdot\text{in}$$

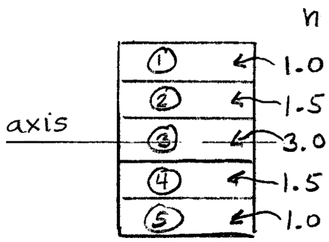
Note that both values are the same for balanced design. $\qquad M = 106.4 \text{ kip}\cdot\text{ft}$ ◀



PROBLEM 4.55

Five metal strips, each 40 mm wide, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 1800 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

SOLUTION



Use aluminum as the reference material.

$n = 1$ in aluminum.

$n = E_s / E_a = 210/70 = 3$ in steel.

$n = E_b / E_a = 105/70 = 1.5$ in brass.

Due to symmetry of both the material arrangement and the geometry, the neutral axis passes through the center of the steel portion.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(40)(10)^3 + (40)(10)(25)^2 = 253.33 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(40)(10)^3 + (1.5)(40)(10)(15)^2 = 140 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3.0}{12}(40)(20)^3 = 80 \times 10^3 \text{ mm}^4$$

$$I_4 = I_2 = 140 \times 10^3 \text{ mm}^4 \qquad I_5 = I_1 = 253.33 \times 10^3 \text{ mm}^4$$

$$I = \sum I \qquad = 866.66 \times 10^3 \text{ mm}^4 = 866.66 \times 10^{-9} \text{ m}^4$$

(*a*) $\sigma = -\dfrac{nMy}{I}$ where $M = 1800$ N · m

Aluminum: $n = 1.0$ $y = -30$ mm $= 0.030$ m

$$\sigma_a = \frac{(1.0)(1800)(0.030)}{866.66 \times 10^{-9}} = 62.3 \times 10^6 \text{ Pa}$$

$\sigma_a = 62.3$ MPa ◀

Brass: $n = 1.5$ $y = -20$ mm $= -0.020$ m

$$\sigma_b = \frac{(1.5)(1800)(0.020)}{866.66 \times 10^{-9}} = 62.3 \times 10^6 \text{ Pa}$$

$\sigma_b = 62.3$ MPa ◀

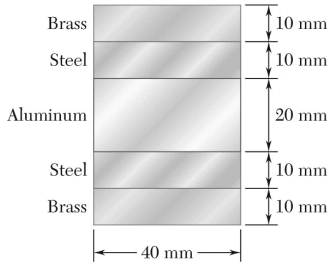
Steel: $n = 3.0$ $y = -10$ mm $= -0.010$ m

$$\sigma_s = \frac{(3.0)(1800)(0.010)}{866.66 \times 10^{-9}} = 62.3 \times 10^6 \text{ Pa}$$

$\sigma_s = 62.3$ MPa ◀

(*b*) <u>Radius of curvature.</u> $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{1800}{(70 \times 10^9)(866.66 \times 10^{-9})} = 0.02967 \text{ m}^{-1}$

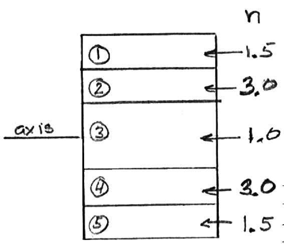
$\rho = 33.7$ m ◀



PROBLEM 4.56

Five metal strips, each of 40 mm wide, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 1800 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

SOLUTION



Use aluminum as the reference material.

$n = 1.0$ in aluminum.

$n = E_s / E_a = 210/70 = 3$ in steel.

$n = E_b / E_a = 105/70 = 1.5$ in brass.

Due to symmetry of both the material arrangement and the geometry, the neutral axis passes through the center of the aluminum portion.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.5}{12}(40)(10)^3 + (1.5)(40)(10)(25)^2 = 380 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{3.0}{12}(40)(10)^3 + (3.0)(40)(10)(15)^2 = 280 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.0}{12}(40)(20)^3 = 26.67 \times 10^3 \text{ mm}^4$$

$$I_4 = I_2 = 280 \times 10^3 \text{ mm}^4 \qquad I_5 = I_1 = 380 \times 10^3 \text{ mm}^4$$

$$I = \sum I = 1.3467 \times 10^6 \text{ mm}^4 = 1.3467 \times 10^{-6} \text{ m}^4$$

(*a*) $\sigma = -\dfrac{nMy}{I}$ where $M = 1800 \text{ N} \cdot \text{m}$

Aluminum: $n = 1, \quad y = -10 \text{ mm} = -0.010 \text{ m}$

$$\sigma_a = \frac{(1.0)(1800)(0.010)}{1.3467 \times 10^{-6}} = 13.37 \times 10^6 \text{ Pa}$$

$\sigma_a = 13.37 \text{ MPa}$ ◀

Brass: $n = 1.5, \quad y = -30 \text{ mm} = -0.030 \text{ m}$

$$\sigma_b = \frac{(1.5)(1800)(0.030)}{1.3467 \times 10^{-6}} = 60.1 \times 10^6 \text{ Pa}$$

$\sigma_b = 60.1 \text{ MPa}$ ◀

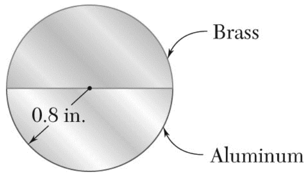
Steel: $n = 3.0, \quad y = -20 \text{ mm} = -0.020 \text{ m}$

$$\sigma_s = \frac{(3.0)(1800)(0.020)}{1.3467 \times 10^{-6}} = 80.1 \times 10^6 \text{ Pa}$$

$\sigma_s = 80.1 \text{ MPa}$ ◀

(*b*) Radius of curvature. $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{1800}{(70 \times 10^9)(1.3467 \times 10^{-6})} = 0.01909 \text{ m}^{-1}$

$\rho = 52.4 \text{ m}$ ◀



PROBLEM 4.57

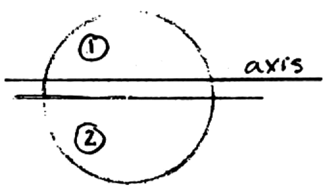
The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is 15×10^6 psi for the brass and 10×10^6 psi for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment $8 \text{ kip} \cdot \text{in.}$, determine the maximum stress (*a*) in the brass, (*b*) in the aluminum.

SOLUTION

For each semicircle, $\quad r = 0.8 \text{ in.} \quad A = \dfrac{\pi}{2} r^2 = 1.00531 \text{ in}^2,$

$$\bar{y}_o = \frac{4r}{3\pi} = \frac{(4)(0.8)}{3\pi} = 0.33953 \text{ in.} \qquad I_{\text{base}} = \frac{\pi}{8} r^4 = 0.160850 \text{ in}^4$$

$$\bar{I} = I_{\text{base}} - A\bar{y}_o^2 = 0.160850 - (1.00531)(0.33953)^2 = 0.044953 \text{ in}^4$$



Use aluminum as the reference material.

$n = 1.0$ in aluminum

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

Locate the neutral axis.

	A, in^2	nA, in^2	$\bar{y}_o$, in.	$nA\bar{y}_o$, in^3
①	1.00531	1.50796	0.33953	0.51200
②	1.00531	1.00531	−0.33953	−0.34133
Σ		2.51327		0.17067

$$\bar{Y}_o = \frac{0.17067}{2.51327} = 0.06791 \text{ in.}$$

The neutral axis lies 0.06791 in. above the material interface.

$$d_1 = 0.33953 - 0.06791 = 0.27162 \text{ in.}, \quad d_2 = 0.33953 + 0.06791 = 0.40744 \text{ in.}$$

$$I_1 = n_1\bar{I} + n_1 A d_1^2 = (1.5)(0.044957) + (1.5)(1.00531)(0.27162)^2 = 0.17869 \text{ in}^4$$

$$I_2 = n_2\bar{I} + n_2 A d_2^2 = (1.0)(0.044957) + (1.0)(1.00531)(0.40744)^2 = 0.21185 \text{ in}^4$$

$$I = I_1 + I_2 = 0.39054 \text{ in}^4$$

(*a*) Brass: $\quad n = 1.5, \quad y = 0.8 - 0.06791 = 0.73209 \text{ in.}$

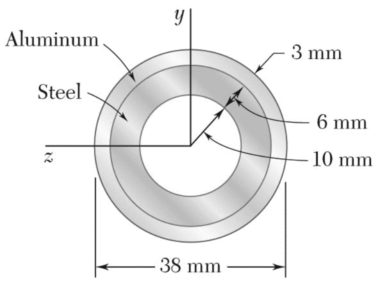
$$\sigma = -\frac{nMy}{I} = -\frac{(1.5)(8)(0.73209)}{0.39054}$$

$\sigma = -22.5 \text{ ksi}$ ◀

(*b*) Aluminium: $\quad n = 1.0, \quad y = -0.8 - 0.06791 = -0.86791 \text{ in.}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(8)(-0.86791)}{0.39054}$$

$\sigma = 17.78 \text{ ksi}$ ◀



PROBLEM 4.58

A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by a couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

SOLUTION

Use aluminum as the reference material.

$$n = 1.0 \text{ in aluminum}$$

$$n = E_s / E_a = 200/70 = 2.857 \text{ in steel}$$

For the transformed section,

Steel: $$I_s = n_s \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = (2.857)\left(\frac{\pi}{4}\right)(16^4 - 10^4) = 124.62 \times 10^3 \text{ mm}^4$$

Aluminium: $$I_a = n_a \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = (1.0)\left(\frac{\pi}{4}\right)(19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$$

$$I = I_s + I_a = 175.50 \times 10^3 \text{ mm}^4 = 175.5 \times 10^{-9} \text{ m}^4$$

(*a*) Aluminum: $c = 19 \text{ mm} = 0.019 \text{ m}$

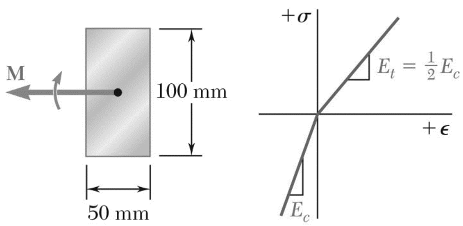
$$\sigma_a = \frac{n_a Mc}{I} = \frac{(1.0)(500)(0.019)}{175.5 \times 10^{-9}} = 54.1 \times 10^6 \text{ Pa}$$

$\sigma_a = 54.1 \text{ MPa}$ ◀

(*b*) Steel: $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\sigma_s = \frac{n_s Mc}{I} = \frac{(2.857)(500)(0.016)}{175.5 \times 10^{-9}} = 130.2 \times 10^6 \text{ Pa}$$

$\sigma_s = 130.2 \text{ MPa}$ ◀



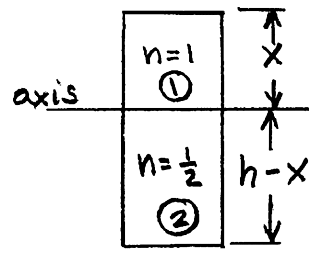
PROBLEM 4.59

The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600\ \text{N}\cdot\text{m}$, determine the maximum (*a*) tensile stress, (*b*) compressive stress.

SOLUTION

$n = \dfrac{1}{2}$ on the tension side of neutral axis.

$n = 1$ on the compression side.



Locate neutral axis.

$$n_1 bx\frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \qquad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421h = 41.421\text{ mm}$$

$$h - x = 58.579\text{ mm}$$

$$I_1 = n_1\frac{1}{3}bx^3 = (1)\left(\frac{1}{3}\right)(50)(41.421)^3 = 1.1844\times10^6\text{ mm}^4$$

$$I_2 = n_2\frac{1}{3}b(h-x)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(50)(58.579)^3 = 1.6751\times10^6\text{ mm}^4$$

$$I = I_1 + I_2 = 2.8595\times10^6\text{ mm}^4 = 2.8595\times10^{-6}\text{ m}^4$$

(*a*) Tensile stress: $n = \dfrac{1}{2}, \quad y = -58.579\text{ mm} = -0.058579\text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15\times10^6\text{ Pa}$$

$\sigma_t = 6.15$ MPa ◀

(*b*) Compressive stress: $n = 1, \quad y = 41.421\text{ mm} = 0.041421\text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69\times10^6\text{ Pa}$$

$\sigma_c = -8.69$ MPa ◀

PROBLEM 4.60*

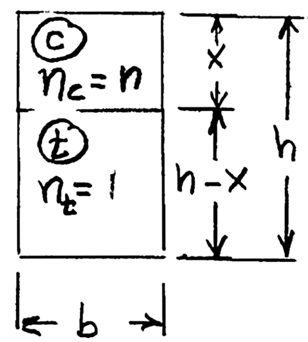
A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$

SOLUTION



Use E_t as the reference modulus.

Then $E_c = nE_t$.

Locate neutral axis.

$$nbx\frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$nx^2 - (h-x)^2 = 0 \quad \sqrt{n}x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \quad h - x = \frac{\sqrt{n}h}{\sqrt{n}+1}$$

$$I_{\text{trans}} = \frac{n}{3}bx^3 + \frac{1}{3}b(h-x)^3 = \left[\frac{h}{3}\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right]bh^3$$

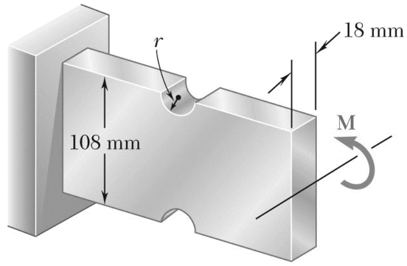
$$= \frac{1}{3}\frac{n+n^{3/2}}{\left(\sqrt{n}+1\right)^3}bh^3 = \frac{1}{3}\frac{n\left(1+\sqrt{n}\right)}{\left(\sqrt{n}+1\right)^3}bh^3 = \frac{1}{3}\times\frac{n}{\left(\sqrt{n}+1\right)^2}bh^3$$

$$\frac{1}{\rho} = \frac{M}{E_t I_{\text{trans}}} = \frac{M}{E_r I} \quad \text{where} \quad I = \frac{1}{12}bh^3$$

$$E_r I = E_t I_{\text{trans}}$$

$$E_r = \frac{E_t I_{\text{trans}}}{I} = \frac{12}{bh^3} \times E_t \times \frac{n}{3\left(\sqrt{n}+1\right)^2}bh^3$$

$$= \frac{4E_t E_c/E_t}{\left(\sqrt{E_c/E_t}+1\right)^2} = \frac{4E_t E_c}{\left(\sqrt{E_c}+\sqrt{E_t}\right)^2}$$



PROBLEM 4.61

Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 60 MPa, determine the largest bending moment that can be applied to the member when (*a*) $r = 9$ mm, (*b*) $r = 18$ mm.

SOLUTION

(*a*) $d = D - 2r = 108 - (2)(9) = 90$ mm

$$\frac{D}{d} = \frac{108}{90} = 1.20 \qquad \frac{r}{d} = \frac{9}{90} = 0.1$$

From Fig. 4.32, $K = 2.07$

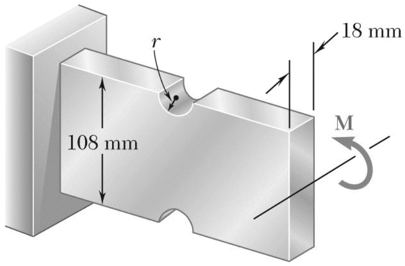
$$I = \frac{1}{12}(18)(90)^3$$
$$= 1.0935 \times 10^6 \text{ mm}^4$$
$$= 1.0935 \times 10^{-6} \text{ m}^4$$
$$c = \frac{1}{2}d = 45 \text{ mm} = 0.045 \text{ m}$$
$$\sigma = \frac{KMc}{I}$$
$$M = \frac{\sigma I}{Kc} = \frac{(60 \times 10^6)(1.0935 \times 10^{-6})}{(2.07)(0.045)} = 704 \qquad M = 704 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $d = 108 - (2)(18) = 72$ mm

$$\frac{D}{d} = \frac{108}{72} = 1.5 \qquad \frac{r}{d} = \frac{18}{72} = 0.25$$
$$c = \frac{1}{2}d = 36 \text{ mm} = 0.036 \text{ m}$$

From Fig. 4.32, $K = 1.61$

$$I = \frac{1}{12}(18)(72)^3 = 559.87 \times 10^3 \text{ mm}^4 = 559.87 \times 10^{-9} \text{m}^4$$
$$M = \frac{\sigma I}{Kc} = \frac{(60 \times 10^6)(559.87 \times 10^{-9})}{(1.61)(0.036)} = 580 \qquad M = 580 \text{ N} \cdot \text{m} \blacktriangleleft$$



PROBLEM 4.62

Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 450\ \text{N}\cdot\text{m}$, determine the maximum stress in the member when the radius r of the semicircular grooves is (*a*) $r = 9$ mm, (*b*) $r = 18$ mm.

SOLUTION

(*a*) $d = D - 2r = 108 - (2)(9) = 90$ mm

$$\frac{D}{d} = \frac{108}{90} = 1.20 \qquad \frac{r}{d} = \frac{9}{90} = 0.1$$

From Fig. 4.32, $K = 2.07$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(90)^3$$
$$= 1.0935 \times 10^6\ \text{mm}^4$$
$$= 1.0935 \times 10^{-6}\ \text{m}^4$$
$$c = \frac{1}{2}d = 45\ \text{mm} = 0.045\ \text{m}$$
$$\sigma_{\max} = \frac{KMc}{I} = \frac{(2.07)(450)(0.045)}{1.0935 \times 10^{-6}}$$
$$= 38.3 \times 10^6\ \text{Pa}$$

$\sigma_{\max} = 38.3$ MPa ◀

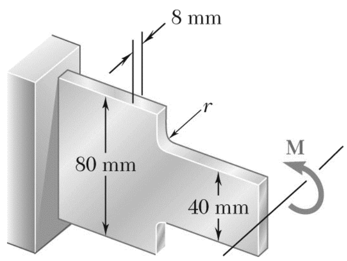
(*b*) $d = D - 2r = 108 - (2)(18) = 72$ mm

$$\frac{D}{d} = \frac{108}{72} = 1.5 \qquad \frac{r}{d} = \frac{18}{72} = 0.25$$

From Fig. 4.32, $K = 1.61$

$$c = \frac{1}{2}d = 72\ \text{mm} = 0.036\ \text{m}$$
$$I = \frac{1}{12}(18)(72)^3 = 559.87 \times 10^3\ \text{mm}^4 = 559.87 \times 10^{-9}\ \text{m}^4$$
$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.61)(450)(0.036)}{559.87 \times 10^{-9}} = 46.6 \times 10^6\ \text{Pa}$$

$\sigma_{\max} = 46.6$ MPa ◀



PROBLEM 4.63

Knowing that the allowable stress for the beam shown is 90 MPa, determine the allowable bending moment M when the radius r of the fillets is (*a*) 8 mm, (*b*) 12 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

(*a*) $\frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.2$ From Fig. 4.31, $K = 1.50$

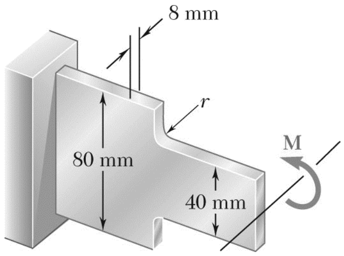
$$\sigma_{\max} = K\frac{Mc}{I} \qquad M = \frac{\sigma_{\max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)}$$

$M = 128 \text{ N} \cdot \text{m}$ ◀

(*b*) $\frac{r}{d} = \frac{12 \text{ mm}}{40 \text{ mm}} = 0.3$ From Fig. 4.31, $K = 1.35$

$$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)}$$

$M = 142 \text{ N} \cdot \text{m}$ ◀



PROBLEM 4.64

Knowing that $M = 250\ \text{N}\cdot\text{m}$, determine the maximum stress in the beam shown when the radius r of the fillets is (*a*) 4 mm, (*b*) 8 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times 10^3\ \text{mm}^4 = 42.667\times 10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

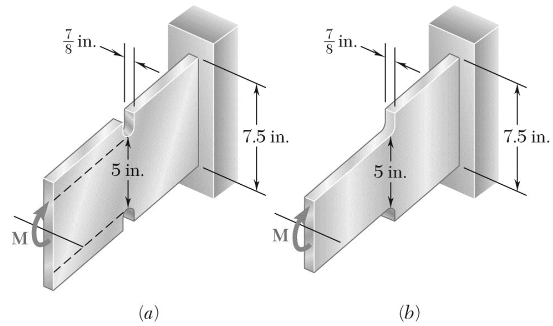
$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

(*a*) $\quad \dfrac{r}{d} = \dfrac{4\ \text{mm}}{40\ \text{mm}} = 0.10 \qquad$ From Fig. 4.31, $\qquad K = 1.87$

$$\sigma_{\max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667\times 10^{-9}} = 219\times 10^6\ \text{Pa} \qquad \sigma_{\max} = 219\ \text{MPa} \blacktriangleleft$$

(*b*) $\quad \dfrac{r}{d} = \dfrac{8\ \text{mm}}{40\ \text{mm}} = 0.20 \qquad$ From Fig. 4.31, $\qquad K = 1.50$

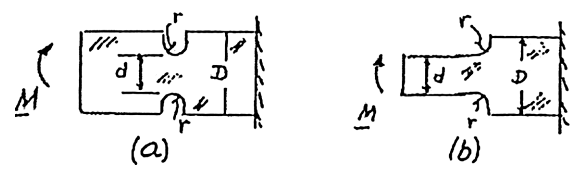
$$\sigma_{\max} = K\frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667\times 10^{-9}} = 176\times 10^6\ \text{Pa} \qquad \sigma_{\max} = 176\ \text{MPa} \blacktriangleleft$$



PROBLEM 4.65

The allowable stress used in the design of a steel bar is 12 ksi. Determine the largest couple **M** that can be applied to the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = \frac{3}{4}$ in. as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material above and below the dashed lines as shown in Fig. *b*.

SOLUTION



Dimensions: $t = \dfrac{7}{8}$ in. $= 0.875$ in. $\quad r = \dfrac{3}{4}$ in. $= 0.75$ in.

$D = 7.5$ in. $\quad d = 5$ in.

$$\frac{r}{d} = \frac{0.75}{5} = 0.15 \qquad \frac{D}{d} = \frac{7.5}{5} = 1.5$$

Stress concentration factors: Figs. 4.32 and 4.31

Configuration (*a*): $K = 1.92$

Configuration (*b*): $K = 1.58$

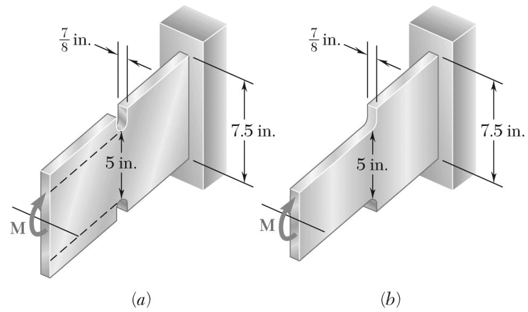
Moment of inertia: $I = \dfrac{1}{12}td^3 = \dfrac{1}{12}(0.875)(5)^3 = 9.115 \text{ in}^4$

$$c = \frac{1}{2}d = 2.5 \text{ in.} \qquad \sigma_m = 12 \text{ ksi}$$

$$\sigma_m = \frac{KMc}{I} \qquad M = \frac{I\sigma_m}{Kc}$$

(*a*) $M = \dfrac{(9.115)(12)}{(1.92)(2.5)}$ $\quad M = 22.8 \text{ kip} \cdot \text{in}$ ◀

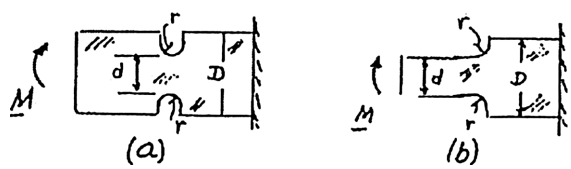
(*b*) $M = \dfrac{(9.115)(12)}{(1.58)(2.5)}$ $\quad M = 27.7 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 4.66

A couple of moment $M = 20$ kip · in. is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = \frac{1}{2}$ in., as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material above and below the dashed line as shown in Fig. *b*.

SOLUTION



Dimensions: $t = \frac{7}{8}$ in. $= 0.875$ in. $\quad r = \frac{1}{2}$ in. $= 0.5$ in.

$D = 7.5$ in. $\quad d = 5$ in.

$$\frac{r}{d} = \frac{0.5}{5} = 0.10 \qquad \frac{D}{d} = \frac{7.5}{5} = 1.5$$

Stress concentration factors: Figs. 4.32 and 4.31

Configuration (*a*): $K = 2.22$

Configuration (*b*): $K = 1.80$

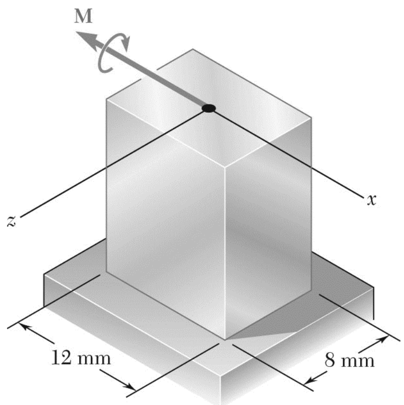
Moment of inertia: $I = \frac{1}{12}td^3 = \frac{1}{12}(0.875)(5)^3 = 9.115 \text{ in}^4$

$$c = \frac{1}{2}d = 2.5 \text{ in.} \qquad M = 20 \text{ kip} \cdot \text{in}$$

Maximum stress: $\sigma_m = \dfrac{KMc}{I}$

(*a*) $\sigma_m = \dfrac{(2.22)(20)(2.5)}{9.115}$ $\quad\sigma_m = 12.2$ ksi ◀

(*b*) $\sigma_m = \dfrac{(1.80)(20)(2.5)}{9.115}$ $\quad\sigma_m = 9.9$ ksi ◀



PROBLEM 4.67

The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (a) yield first occurs, (b) the elastic core of the bar is 4 mm thick.

SOLUTION

(a) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(8)^3 = 512 \text{ mm}^4$$

$$= 512\times 10^{-12}\,\text{m}^4$$

$$c = \frac{1}{2}h = 4 \text{ mm } = \text{ } 0.004 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times 10^6)(512\times 10^{-12})}{0.004}$$

$$= 38.4 \text{ N}\cdot\text{m}$$

$M_Y = 38.4 \text{ N}\cdot\text{m}$ ◀

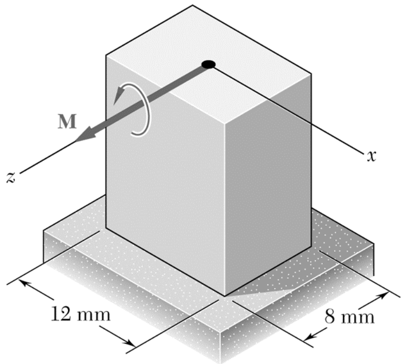
(b) $$y_Y = \frac{1}{2}(4) = 2 \text{ mm} \qquad \frac{y_Y}{c} = \frac{2}{4} = 0.5$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(38.4)\left[1 - \frac{1}{3}(0.5)^2\right]$$

$$= 52.8 \text{ N}\cdot\text{m}$$

$M = 52.8 \text{ N}\cdot\text{m}$ ◀



PROBLEM 4.68

Solve Prob. 4.67, assuming that the couple **M** is parallel to the *z* axis.

PROBLEM 4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

SOLUTION

(*a*) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(12)^3 = 1.152\times 10^3 \text{mm}^4$$

$$= 1.152\times 10^{-9}\text{ m}^4$$

$$c = \frac{1}{2}h = 6\text{ mm} = 0.006\text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times 10^6)(1.152\times 10^{-9})}{0.006}$$

$$= 57.6\text{ N}\cdot\text{m}$$

$M_Y = 57.6\text{ N}\cdot\text{m}$ ◀

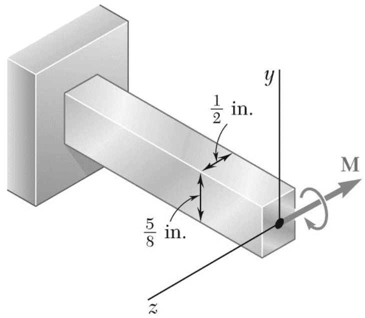
(*b*) $$y_Y = \frac{1}{2}(4) = 2\text{ mm} \qquad \frac{y_Y}{c} = \frac{2}{6} = \frac{1}{3}$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(57.6)\left[1 - \frac{1}{3}\left(\frac{1}{3}\right)^2\right]$$

$$= 83.2\text{ N}\cdot\text{m}$$

$M = 83.2\text{ N}\cdot\text{m}$ ◀



PROBLEM 4.69

The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi, is subjected to a couple of 1350 lb · in. parallel to the z axis. Determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

SOLUTION

(a) $$I = \frac{1}{12}\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)^3 = 10.1725\times10^{-3}\text{ in}^4$$

$$c = \frac{1}{2}\left(\frac{5}{8}\right) = 0.3125\text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(36\times10^3)(10.1725\times10^{-3})}{0.3125}$$
$$= 1171.872\text{ lb}\cdot\text{in}$$

$$M = \frac{3}{2}M_Y\left[1-\frac{1}{3}\left(\frac{y_Y}{c}\right)^3\right]$$

$$\frac{y_Y}{c} = \sqrt{3-2\frac{M}{M_Y}} = \sqrt{3-\frac{(2)(1350)}{1171.872}} = 0.83426$$

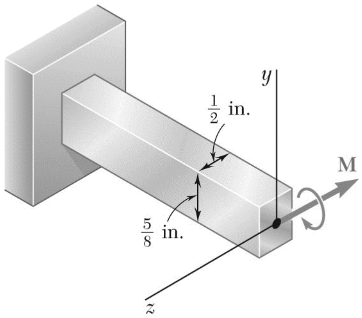
$$y_Y = (0.83426)(0.3125) = 0.26071\text{ in.}$$

Thickness of elastic core: $2y_Y = 0.521$ in. ◀

(b) $$y_Y = \varepsilon_Y\rho = \frac{\sigma_Y}{E}\rho$$

$$\rho = \frac{y_Y E}{\sigma_Y} = \frac{(0.26071)(29\times10^6)}{36\times10^3} = 210.02\text{ in.}$$

$\rho = 17.50$ ft ◀



PROBLEM 4.70

Solve Prob. 4.69, assuming that the 1350-lb $\cdot$ in. couple is parallel to the y axis.

PROBLEM 4.69 The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi, is subjected to a couple of 1350 lb $\cdot$ in parallel to the z axis. Determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

SOLUTION

(a) $$I = \frac{1}{12}\left(\frac{5}{8}\right)\left(\frac{1}{2}\right)^3 = 6.5104\times10^{-3}\,\text{in}^4$$

$$c = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 0.25 \text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(36\times10^3)(6.5104\times10^{-3})}{0.25}$$
$$= 937.5 \text{ lb}\cdot\text{in}$$

$$M = \frac{3}{2}\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(1350)}{937.5}} = 0.34641$$

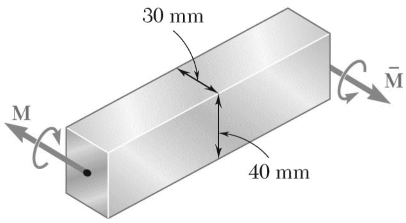
$$y_Y = (0.34641)(0.25) = 0.086603 \text{ in.}$$

Thickness of elastic core: $2y_Y = 0.1732$ in. ◀

(b) $$y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho$$

$$\rho = \frac{y_Y E}{\sigma_Y} = \frac{(0.086603)(29\times10^6)}{36\times10^3} = 69.763 \text{ in.}$$

$\rho = 5.81$ ft ◀



PROBLEM 4.71

A bar of rectangular cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa. Determine the bending moment M for which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 12 mm thick.

SOLUTION

$$I = \frac{1}{12}(30)(40)^3 = 160\times 10^3 \text{ mm}^4 = 160\times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

(a) First yielding: $\sigma = \dfrac{Mc}{I} = \sigma_Y$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(160)(10^{-9})(300\times 10^6)}{0.020} = 2400 \text{ N}\cdot\text{m}$$

$M_Y = 2.40 \text{ kN}\cdot\text{m}$ ◀

(b) Plastic zones are 12 mm thick:

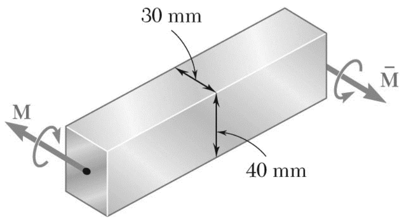
$$y_Y = 20 \text{ mm} - 12 \text{ mm} = 8 \text{ mm}$$

$$\frac{y_Y}{c} = \frac{8 \text{ mm}}{20 \text{ mm}} = 0.4$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(2400)\left[1 - \frac{1}{3}(0.4)^2\right] = 3408 \text{ N}\cdot\text{m}$$

$M = 3.41 \text{ kN}\cdot\text{m}$ ◀



PROBLEM 4.72

Bar AB is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. Determine the bending moment M for which the radius of curvature of the bar will be (*a*) 18 m, (*b*) 9 m.

SOLUTION

$$I = \frac{1}{12}(30)(40)^3 = 160 \times 10^3 \text{ mm}^4 = 160 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(160 \times 10^{-9})(240 \times 10^6)}{0.020} = 1920 \text{ N} \cdot \text{m}$$

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI} = \frac{1920}{(200 \times 10^9)(160 \times 10^{-9})} = 0.0600 \text{ m}^{-1}$$

(*a*) $\rho = 18$ m: $\frac{1}{\rho} = 0.05556 \text{ m}^{-1} < 0.0600 \text{ m}^{-1}$

The bar is fully elastic.

$$M = \frac{EI}{\rho} = \frac{(200 \times 10^9)(160 \times 10^{-9})}{18} = 1778 \text{ N} \cdot \text{m}$$

$M = 1.778 \text{ kN} \cdot \text{m}$ ◀

(*b*) $\rho = 9$ m: $\frac{1}{\rho} = 0.11111 \text{ m}^{-1} > 0.0600 \text{ m}^{-1}$

The bar is partially plastic.

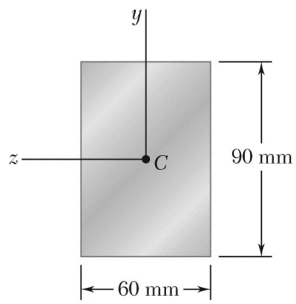
$$\sigma_Y = \frac{Ey_Y}{\rho} \qquad y_Y = \frac{\rho\sigma_Y}{E}$$

$$y_Y = \frac{(9)(240 \times 10^6)}{200 \times 10^9} = 0.0108 \text{ m} = 10.8 \text{ mm}$$

$$\frac{y_Y}{c} = \frac{10.8 \text{ mm}}{20 \text{ mm}} = 0.54$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(1920)\left[1 - \frac{1}{3}(0.54)^2\right] = 2600 \text{ N} \cdot \text{m}$$

$M = 2.60 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 4.73

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z-axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 30-mm thick.

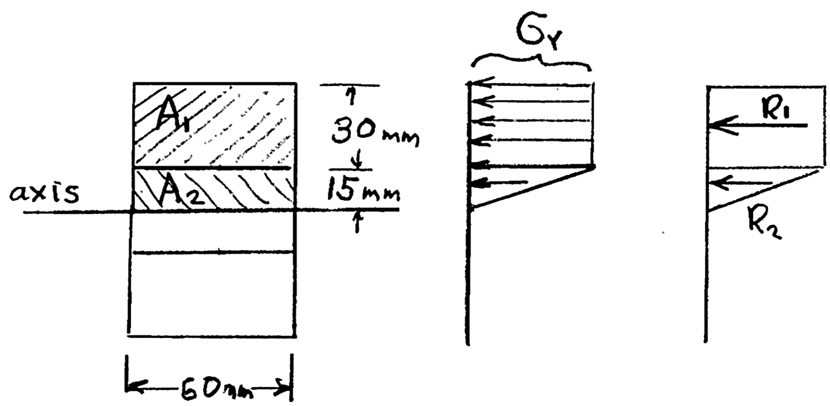
SOLUTION

(a) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times 10^6\,\text{mm}^4 = 3.645\times 10^{-6}\,\text{m}^4$$

$$c = \frac{1}{2}h = 45\text{ mm} = 0.045\text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times 10^6)(3.645\times 10^{-6})}{0.045} = 19.44\times 10\,\text{N}\cdot\text{m}$$

$M_Y = 19.44\text{ kN}\cdot\text{m}$ ◀

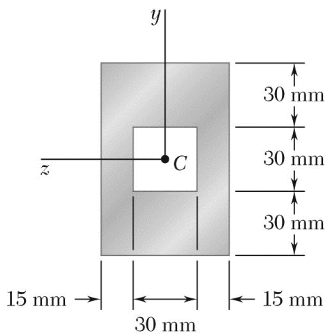


$$R_1 = \sigma_Y A_1 = (240\times 10^6)(0.060)(0.030)$$
$$= 432\times 10^3\text{ N}$$
$$y_1 = 15\text{ mm} + 15\text{ mm} = 0.030\text{ m}$$
$$R_2 = \frac{1}{2}\sigma_Y A_2 = \left(\frac{1}{2}\right)(240\times 10^6)(0.060)(0.015)$$
$$= 108\times 10^3\text{ N}$$
$$y_2 = \frac{2}{3}(15\text{ mm}) = 10\text{ mm} = 0.010\text{ m}$$

(b) $$M = 2(R_1 y_1 + R_2 y_2) = 2[(432\times 10^3)(0.030) + (108\times 10^3)(0.010)]$$

$$= 28.08\times 10^3\text{ N}\cdot\text{m}$$

$M = 28.1\text{ kN}\cdot\text{m}$ ◀



PROBLEM 4.74

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30-mm thick.

SOLUTION

(*a*) $I_{\text{rect}} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6 \text{ mm}^4$

$I_{\text{cutout}} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3 \text{ mm}^4$

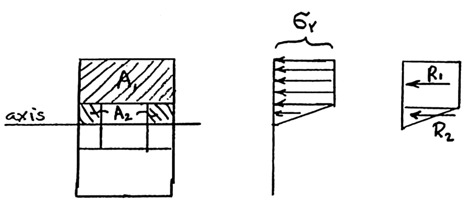
$I = 3.645\times10^6 - 67.5\times10^3 = 3.5775\times10^6 \text{ mm}^4$

$= 3.5775\times10^{-6} \text{ mm}^4$

$c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.5775\times10^{-6})}{0.045}$

$= 19.08\times10^3 \text{ N}\cdot\text{m}$ $\qquad M_Y = 19.08 \text{ kN}\cdot\text{m}$ ◀



$R_1 = \sigma_Y A_1 = (240\times10^6)(0.060)(0.030) = 432\times10^3 \text{ N}$

$y_1 = 15 \text{ mm} + 15 \text{ mm} = 30 \text{ mm} = 0.030 \text{ m}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(240\times10^6)(0.030)(0.015) = 54\times10^3 \text{ N}$

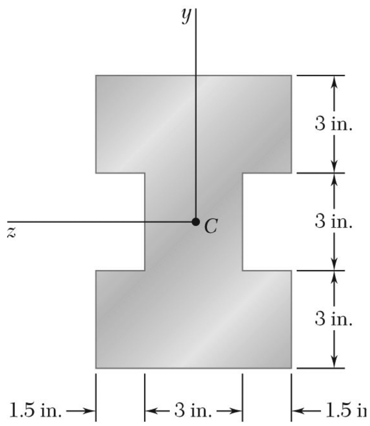
$y_2 = \frac{2}{3}(15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$

(*b*) $M = 2(R_1y_1 + R_2y_2)$

$= 2[(432\times10^3)(0.030) + (54\times10^3)(0.010)]$

$= 27.00\times10^3 \text{ N}\cdot\text{m}$ $\qquad M = 27.0 \text{ kN}\cdot\text{m}$ ◀

PROBLEM 4.75



A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

(a) $I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$

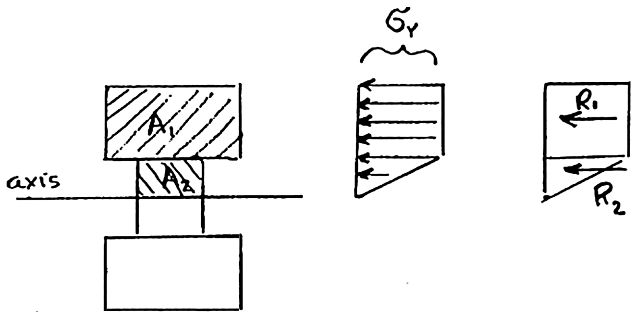
$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$

$I_3 = I_1 = 175.5 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$

$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5}$ $\qquad M_Y = 3339 \text{ kip}\cdot\text{in}$ ◀



$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756$ kip

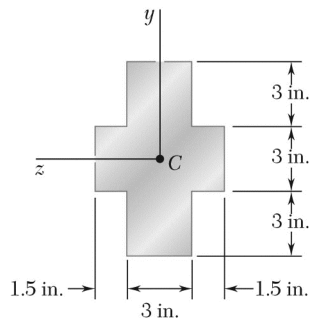
$y_1 = 1.5 + 1.5 = 3$ in.

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(3)(1.5)$

$= 94.5$ kip

$y_2 = \frac{2}{3}(1.5) = 1.0$ in.

(b) $M = 2(R_1y_1 + R_2y_2) = 2[(756)(3) + (94.5)(1.0)]$ $\qquad M = 4725 \text{ kip}\cdot\text{in}$ ◀



PROBLEM 4.76

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29\times 10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$

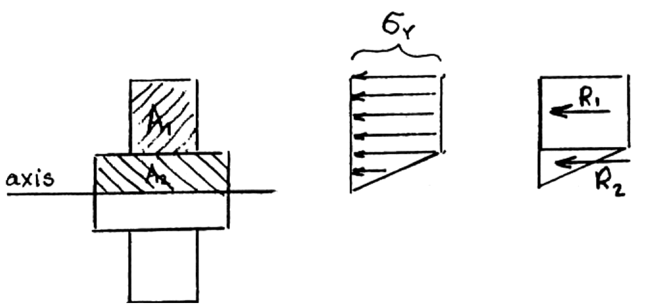
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$

$I_3 = I_1 = 87.75 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$

$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5}$ $\qquad M_Y = 1759 \text{ kip}\cdot\text{in}$ ◀



$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

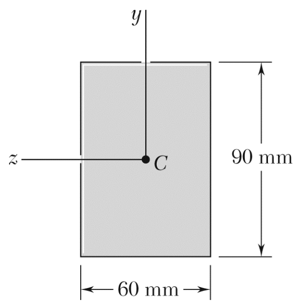
$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(6)(1.5)$

$= 189$ kip

$y_2 = \frac{2}{3}(1.5) = 1.0$ in.

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (189)(1.0)]$ $\qquad M = 2646 \text{ kip}\cdot\text{in}$ ◀



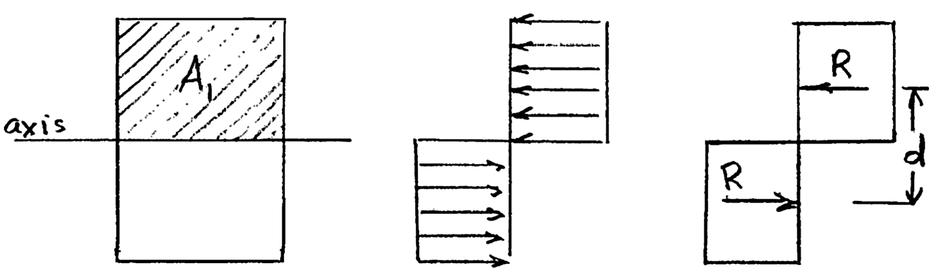
PROBLEM 4.77

For the beam indicated (of Prob. 4.73), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.73, $E = 200$ GPa and $\sigma_Y = 240$ MPa.

$$A_1 = (60)(45) = 2700 \text{ mm}^2$$
$$= 2700 \times 10^{-6} \text{ m}^2$$
$$R = \sigma_Y A_1$$
$$= (240 \times 10^6)(2700 \times 10^{-6})$$
$$= 648 \times 10^3 \text{ N}$$
$$d = 45 \text{ mm} = 0.045 \text{ m}$$



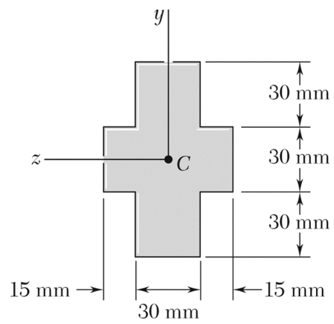
(*a*) $M_p = Rd = (648 \times 10^3)(0.045) = 29.16 \times 10^3 \text{ N} \cdot \text{m}$ $\qquad M_p = 29.2 \text{ kN} \cdot \text{m}$ ◀

(*b*) $I = \dfrac{1}{12} bh^3 = \dfrac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$

$c = 45 \text{ mm} = 0.045 \text{ m}$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3 \text{ N} \cdot \text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{29.16}{19.44}$$ $\qquad k = 1.500$ ◀



PROBLEM 4.78

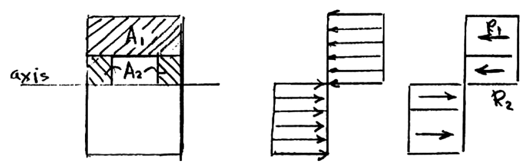
For the beam indicated (of Prob. 4.74), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.74, $E = 200$ GPa and $\sigma_Y = 240$ MPa.

(*a*)

$$R_1 = \sigma_Y A_1$$
$$= (240\times10^6)(0.060)(0.030)$$
$$= 432\times10^3 \text{ N}$$



$$y_1 = 15 \text{ mm} + 15 \text{ mm} = 30 \text{ mm}$$
$$= 0.030 \text{ m}$$

$$R_2 = \sigma_Y A_2$$
$$= (240\times10^6)(0.030)(0.015)$$
$$= 108\times10^3 \text{ N}$$
$$y_2 = \frac{1}{2}(15) = 7.5 \text{ mm} = 0.0075 \text{ m}$$

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(432\times10^3)(0.030) + (108\times10^3)(0.0075)]$$
$$= 27.54\times10^3 \text{ N}\cdot\text{m}$$

$M_p = 27.5 \text{ kN}\cdot\text{m}$ ◀

(*b*)

$$I_{\text{rect}} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6 \text{ mm}^4$$
$$I_{\text{cutout}} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3 \text{ mm}^4$$
$$I = I_{\text{rect}} - I_{\text{cutout}} = 3.645\times10^6 - 67.5\times10^3 = 3.5775\times10^3 \text{ mm}^4$$
$$= 3.5775\times10^{-9} \text{ m}^4$$

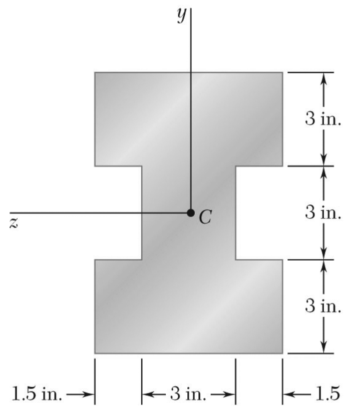
$$c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.5775\times10^{-9})}{0.045} = 19.08\times10^3 \text{ N}\cdot\text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{27.54}{19.08}$$

$k = 1.443$ ◀

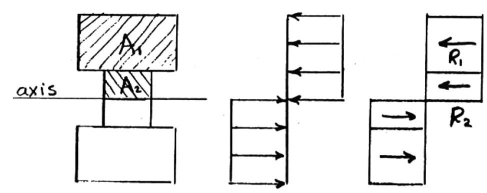
PROBLEM 4.79



For the beam indicated (of Prob. 4.75), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.75, $E = 29\times10^6$ and $\sigma_Y = 42$ ksi.



(*a*) $R_1 = \sigma_Y A_1 = (42)(6)(3) = 756$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \sigma_Y A_2 = (42)(3)(1.5) = 189$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.

$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3.0) + (189)(0.75)]$ $\quad M_p = 4819.5$ kip · in ◀

(*b*) $I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$

$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$

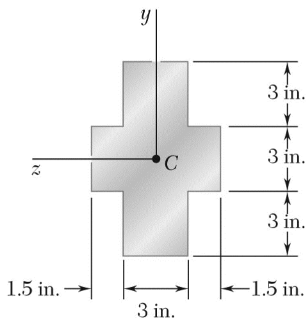
$I_3 = I_1 = 175.5 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$

$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339$ kip · in

$k = \frac{M_p}{M_Y} = \frac{4819.5}{3339}$ $\quad k = 1.443$ ◀



PROBLEM 4.80

For the beam indicated (of Prob. 4.76), determine (a) the fully plastic moment M_p, (b) the shape factor of the cross section.

SOLUTION

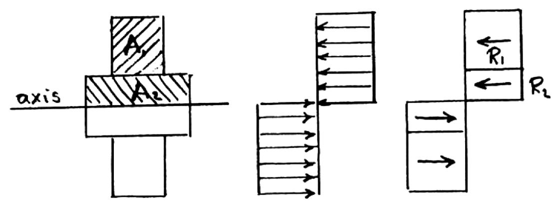
From Problem 4.76, $E = 29\times10^6\,\text{psi}$ and $\sigma_Y = 42$ ksi.

(a) $R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \sigma_Y A_2 = (42)(6)(1.5) = 378$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.



$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (378)(0.75)]$ $\qquad M_p = 2835\ \text{kip}\cdot\text{in}$ ◀

(b) $I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75\ \text{in}^4$

$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5\ \text{in}^4$

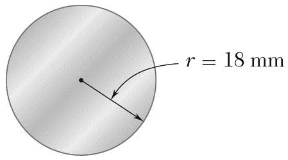
$I_3 = I_1 = 87.75\ \text{in}^4$

$I = I_1 + I_2 + I_3 = 188.5\ \text{in}^4$

$c = 4.5$ in.

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759.3\ \text{kip}\cdot\text{in}$$

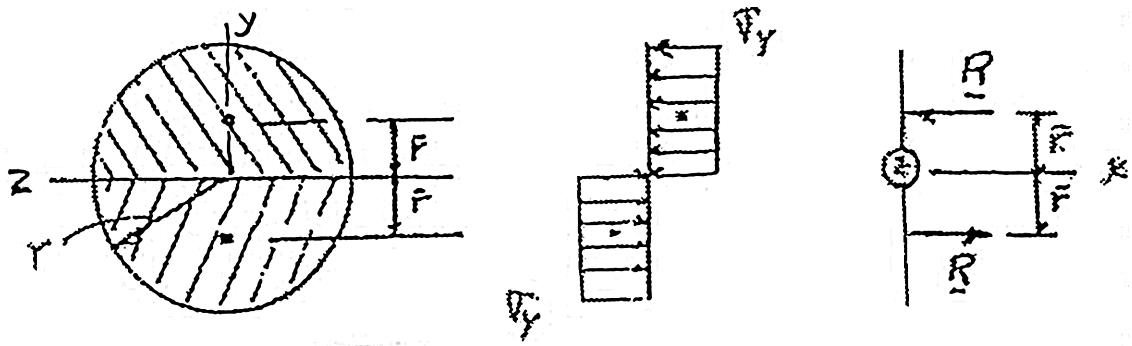
$$k = \frac{M_p}{M_Y} = \frac{2835}{1759.3}$$ $\qquad k = 1.611$ ◀



PROBLEM 4.81

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION



For a semicircle: $A = \frac{\pi}{2} r^2; \qquad \overline{r} = \frac{4r}{3\pi}$

Resultant force on semicircular section: $R = \sigma_Y A$

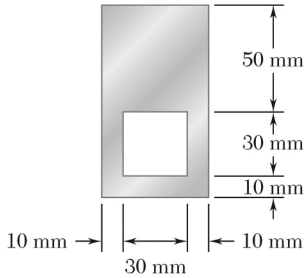
Resultant moment on entire cross section:

$$M_p = 2R\overline{r} = \frac{4}{3}\sigma_Y r^3$$

Data: $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}, \quad r = 18 \text{ mm} = 0.018 \text{ m}$

$$M_p = \frac{4}{3}(240 \times 10^6)(0.018)^3 = 1866 \text{ N} \cdot \text{m}$$

$M_p = 1.866 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 4.82

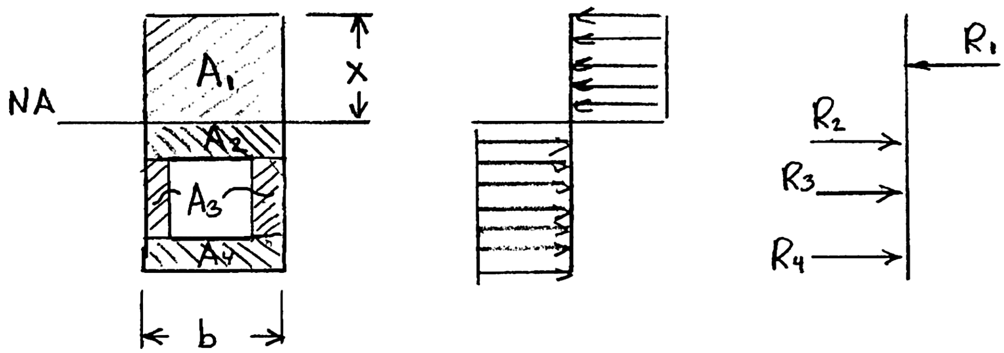
Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area: $$A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$$

$$\frac{1}{2}A = 1800 \text{ mm}^2$$

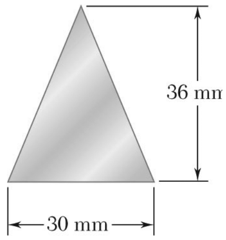
$$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$$



$$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm}, \quad A_1\bar{y}_1 = 32.4\times10^3 \text{ mm}^3$$
$$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm}, \quad A_2\bar{y}_2 = 4.9\times10^3 \text{ mm}^3$$
$$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm}, \quad A_3\bar{y}_3 = 17.4\times10^3 \text{ mm}^3$$
$$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm}, \quad A_4\bar{y}_4 = 24.5\times10^3 \text{ mm}^3$$

$$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 = 79.2\times10^3 \text{ mm}^3 = 79.2\times10^{-6} \text{ m}^3$$
$$M_p = \sigma_Y \Sigma A_i \bar{y}_i = (240\times10^6)(79.2\times10^{-6}) = 19.008\times10^3 \text{ N}\cdot\text{m}$$

$$M_p = 19.01 \text{ kN}\cdot\text{m} \blacktriangleleft$$

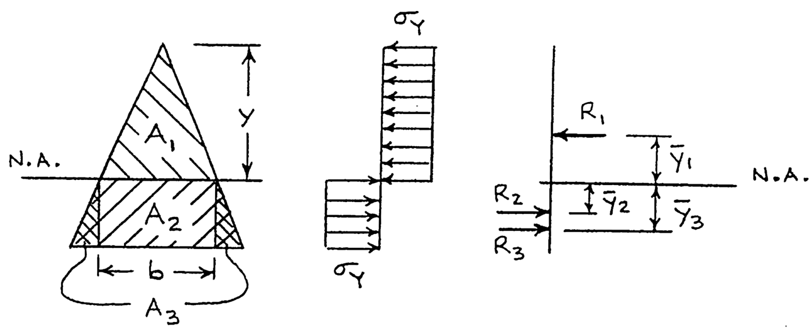


PROBLEM 4.83

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area: $A = \frac{1}{2}(30)(36) = 540 \text{ mm}^2$ Half area: $\frac{1}{2}A = 270 \text{ mm}^2 = A_1$



By similar triangles, $\frac{b}{y} = \frac{30}{36}$ $b = \frac{5}{6}y$

Since $A_1 = \frac{1}{2}by = \frac{5}{12}y^2, \quad y^2 = \frac{12}{5}A_1$

$$y = \sqrt{\frac{12}{5}(270)} = 25.4558 \text{ mm}$$

$$b = 21.2132 \text{ mm}$$

$$A_1 = \frac{1}{2}(21.2132)(25.4558) = 270 \text{ mm}^2 = 270 \times 10^{-6} \text{ m}^2$$

$$A_2 = (21.2132)(36 - 25.4558) = 223.676 \text{ mm}^2 = 223.676 \times 10^{-6} \text{ m}^2$$

$$A_3 = A - A_1 - A_2 = 46.324 \text{ mm}^2 = 46.324 \times 10^{-6} \text{ m}^2$$

$$R_i = \sigma_Y A_i = 240 \times 10^6 A_i$$

$$R_1 = 64.8 \times 10^3 \text{ N}, \quad R_2 = 53.6822 \times 10^3 \text{ N}, \quad R_3 = 11.1178 \times 10^3 \text{ N}$$

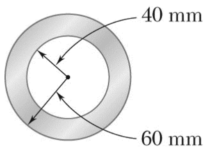
$$\bar{y}_1 = \frac{1}{3}y = 8.4853 \text{ mm} = 8.4853 \times 10^{-3} \text{ m}$$

$$\bar{y}_2 = \frac{1}{2}(36 - 25.4558) = 5.2721 \text{ mm} = 5.2721 \times 10^{-3} \text{ m}$$

$$\bar{y}_3 = \frac{2}{3}(36 - 25.4558) = 7.0295 \text{ mm} = 7.0295 \times 10^{-3} \text{ m}$$

$$M_p = R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3 = 911 \text{ N} \cdot \text{m}$$

$M_p = 911 \text{ N} \cdot \text{m}$ ◀

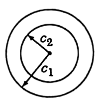


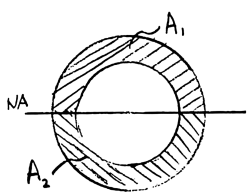
PROBLEM 4.84

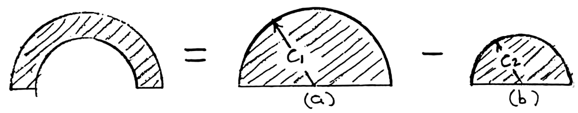
Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Let c_1 be the outer radius and c_2 the inner radius.







$$A_1\bar{y}_1 = A_a\bar{y}_a - A_b\bar{y}_b$$

$$= \left(\frac{\pi}{2}c_1^2\right)\left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2}c_2^2\right)\left(\frac{4c_2}{3\pi}\right)$$

$$= \frac{2}{3}\left(c_1^3 - c_2\right)^3$$

$$A_2\bar{y}_2 = A_1\bar{y}_1 = \frac{2}{3}\left(c_1^3 - c_2^2\right)$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2) = \frac{4}{3}\sigma_Y\left(c_1^3 - c_2^3\right)$$

Data: $\sigma_Y = 240 \text{ MPa} = 240\times10^6 \text{ Pa}$

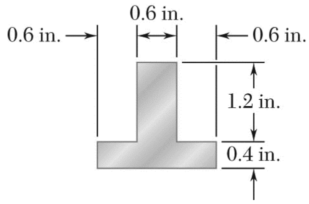
$$c_1 = 60 \text{ mm} = 0.060 \text{ m}$$

$$c_2 = 40 \text{ mm} = 0.040 \text{ m}$$

$$M_p = \frac{4}{3}(240\times10^6)(0.060^3 - 0.040^3)$$

$$= 48.64\times10^3 \text{ N}\cdot\text{m}$$

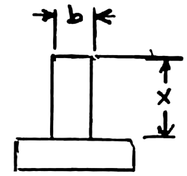
$M_p = 48.6 \text{ kN}\cdot\text{m}$ ◀



PROBLEM 4.85

Determine the plastic moment M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.

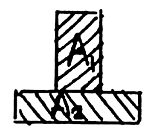
SOLUTION



Total area: $A = (1.8)(0.4) + (0.6)(1.2) = 1.44 \text{ in}^2$

$$\frac{1}{2}A = 0.72 \text{ in}^2$$

$$x = \frac{\frac{1}{2}A}{b} = \frac{0.72}{0.6} = 1.2 \text{ in.}$$



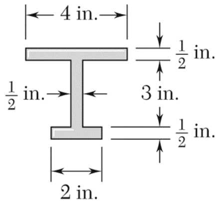
Neutral axis lies 1.2 in. below the top.

$$A_1 = \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_1 = \frac{1}{2}(1.2) = 0.6 \text{ in.}$$

$$A_2 = \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_2 = \frac{1}{2}(0.4) = 0.2 \text{ in.}$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2)$$
$$= (36)[(0.72)(0.6) + (0.72)(0.2)] = 20.7 \text{ kip} \cdot \text{in}$$

$M_p = 20.7 \text{ kip} \cdot \text{in}$ ◀



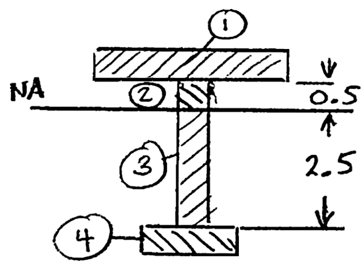
PROBLEM 4.86

Determine the plastic moment M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.

SOLUTION

Total area: $$A = (4)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)(3) + (2)\left(\frac{1}{2}\right) = 4.5 \text{ in}^2$$

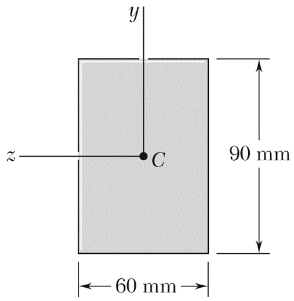
$$\frac{1}{2}A = 2.25 \text{ in}^2$$



$$A_1 = 2.00 \text{ in}^2, \quad \bar{y}_1 = 0.75, \quad A_1 y_1 = 1.50 \text{ in}^3$$
$$A_2 = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.25, \quad A_2 \bar{y}_2 = 0.0625 \text{ in}^3$$
$$A_3 = 1.25 \text{ in}^2, \quad \bar{y}_3 = 1.25, \quad A_3 \bar{y}_3 = 1.5625 \text{ in}^3$$
$$A_4 = 1.00 \text{ in}^2, \quad \bar{y}_4 = 2.75, \quad A_4 \bar{y}_4 = 2.75 \text{ in}^3$$

$$M_p = \sigma_Y (A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$$
$$= (36)(1.50 + 0.0625 + 1.5625 + 2.75)$$

$M_p = 212 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 4.87

For the beam indicated (of Prob. 4.73), a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45\,\text{mm}$.

SOLUTION

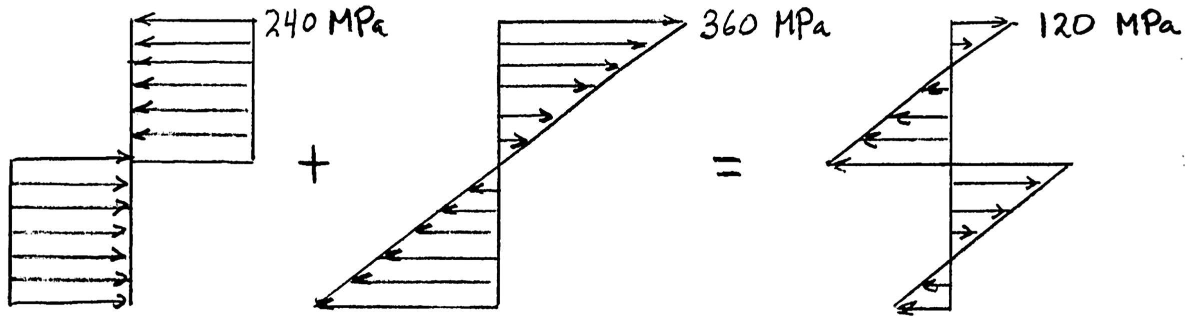
$$M_p = 29.16 \times 10^3\ \text{N} \cdot \text{m}$$

See solutions to Problems 4.73 and 4.77.

$$I = 3.645 \times 10^{-6}\ \text{m}^4$$

$$c = 0.045\ \text{m}$$

$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \qquad \text{at } y = c = 45\ \text{mm}$$

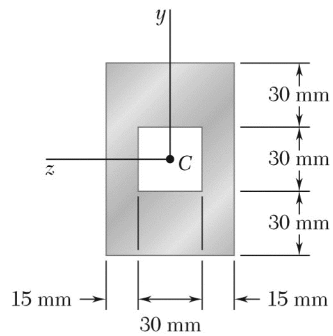


$$\sigma' = \frac{(29.16 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 360 \times 10^6\ \text{Pa}$$

$$\sigma_{\text{res}} = \sigma' - \sigma_Y = 360 \times 10^6 - 240 \times 10^6$$

$$= 120 \times 10^6\ \text{Pa}$$

$\sigma_{\text{res}} = 120.0\ \text{MPa}$ ◀



PROBLEM 4.88

For the beam indicated (of Prob. 4.74), a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45\,\text{mm}$.

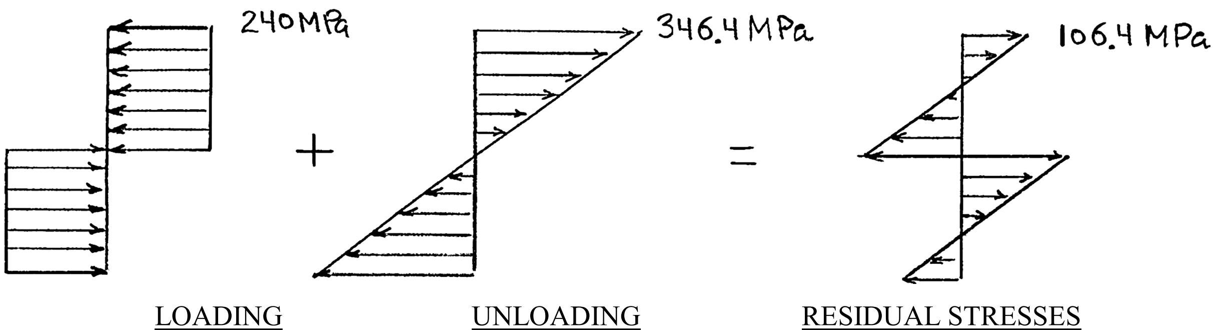
SOLUTION

$$M_p = 27.54 \times 10^3\ \text{N} \cdot \text{m} \ \text{(See solutions to Problems 4.74 and 4.78.)}$$

$$I = 3.5775 \times 10^{-6}\ \text{m}^4, \qquad c = 0.045\ \text{m}$$

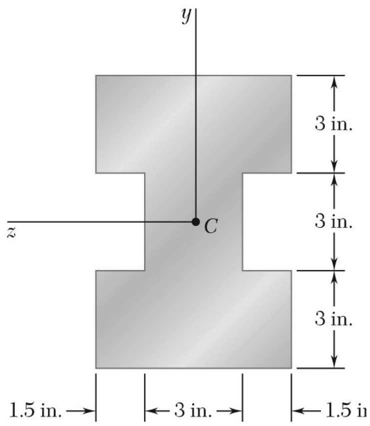
$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \quad \text{at} \quad y = c$$

$$\sigma' = \frac{(27.54 \times 10^3)(0.045)}{3.5775 \times 10^{-6}} = 346.4 \times 10^6\ \text{Pa}$$



$$\sigma_{\text{res}} = \sigma' - \sigma_Y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6\ \text{Pa}$$

$$\sigma_{\text{res}} = 106.4\ \text{MPa} \blacktriangleleft$$



PROBLEM 4.89

A bending couple is applied to the bar indicated, causing plastic zones 3 in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5\,\text{in.}$, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

SOLUTION

See solution to Problem 4.75 for bending couple and stress distribution.

$$M = 4725\ \text{kip}\cdot\text{in} \qquad y_Y = 1.5\ \text{in.} \qquad E = 29\times 10^6\ \text{psi} = 29\times 10^3\ \text{ksi}$$

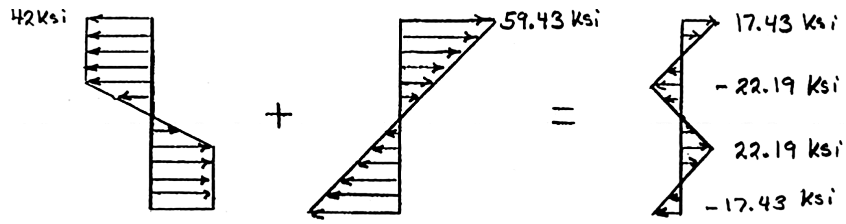
$$\sigma_Y = 42\ \text{ksi} \qquad I = 357.75\ \text{in}^4 \qquad c = 4.5\ \text{in.}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(4725)(4.5)}{357.75} = 59.43\ \text{ksi} \quad ◀$$

$$\sigma'' = \frac{My_Y}{I} = \frac{(4725)(1.5)}{357.75} = 19.81\ \text{ksi}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 59.43 - 42 = 17.43\ \text{ksi}$

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 19.81 - 42 = -22.19\ \text{ksi}$



(*b*) $$\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_Y = 0$$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(357.75)(42)}{4725} = 3.18\ \text{in.}$$

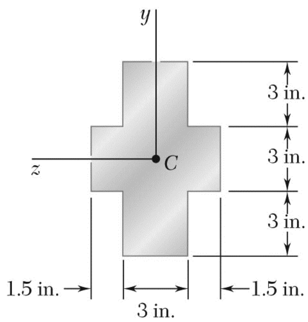
Answer: $y_0 = -3.18\,\text{in.},\ 0,\ 3.18\ \text{in.}$ ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -22.19\,\text{ksi}$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29\times 10^3)(1.5)}{22.19} = 1960\ \text{in.}$$

$\rho = 163.4$ ft. ◀

PROBLEM 4.90



A bending couple is applied to the bar indicated, causing plastic zones 3 in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

SOLUTION

See solution to Problem 4.76 for bending couple and stress distribution during loading.

$$M = 2646 \text{ kip} \cdot \text{in} \quad y_Y = 1.5 \text{in.} \quad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

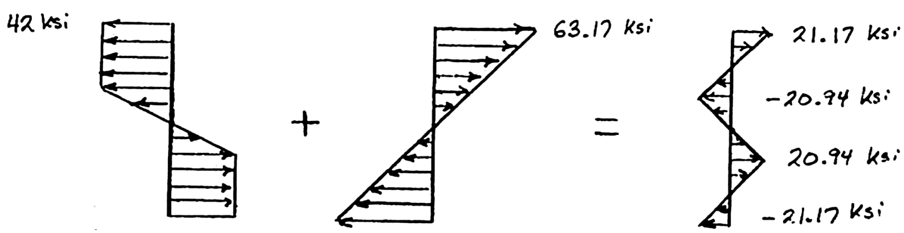
$$\sigma_Y = 42 \text{ ksi} \quad I = 188.5 \text{ in}^4 \quad c = 4.5 \text{ in.}$$

(*a*) $\sigma' = \dfrac{Mc}{I} = \dfrac{(2646)(4.5)}{188.5} = 63.17 \text{ ksi}$

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(2646)(1.5)}{188.5} = 21.06 \text{ ksi}$

At $y = c, \quad \sigma_{res} = \sigma' - \sigma_Y = 63.17 - 42 = 21.17 \text{ ksi}$

At $y = y_Y, \quad \sigma_{res} = \sigma'' - \sigma_Y = 21.06 - 42$ $\qquad \sigma_{res} = -20.94 \text{ ksi}$ ◀



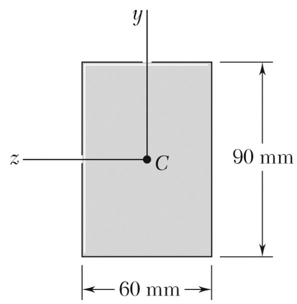
LOADING $\qquad$ UNLOADING $\qquad$ RESIDUAL STRESSES

(*b*) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} = \sigma_Y$

$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(188.5)(42)}{2646} = 2.992 \text{ in.}$ $\qquad$ **Answer:** $y_0 = -2.992 \text{ in.}, \quad 0, \quad 2.992 \text{ in.}$ ◀

(*c*) At $y = y_Y, \qquad \sigma_{res} = -20.94 \text{ ksi}$

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(29 \times 10^3)(1.5)}{20.94} = 2077 \text{ in.}$ $\qquad \rho = 173.1 \text{ ft}$ ◀



PROBLEM 4.91

A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 45$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

SOLUTION

See solution to Problem 4.73 for bending couple and stress distribution during loading:

$$M = 28.08 \times 10^3\ \text{N} \cdot \text{m} \quad y_Y = 15\ \text{mm} = 0.015\ \text{m} \quad E = 200\ \text{GPa}$$

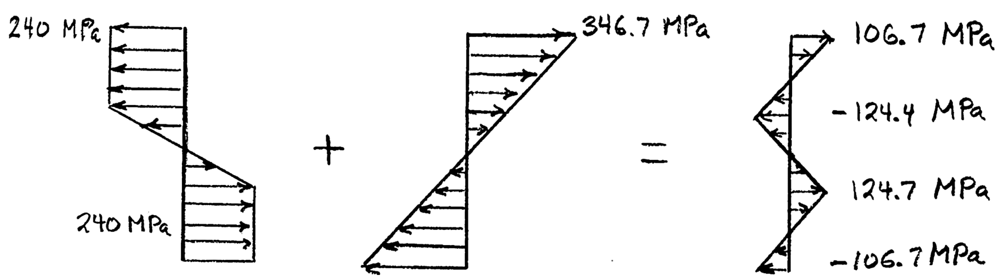
$$\sigma_Y = 240\ \text{MPa} \quad I = 3.645 \times 10^{-6}\ \text{m}^4 \quad c = 0.045\ \text{m}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(28.08 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 346.7 \times 10^6\ \text{Pa} = 346.7\ \text{MPa}$$

$$\sigma'' = \frac{M y_Y}{I} = \frac{(28.08 \times 10^3)(0.015)}{3.645 \times 10^{-6}} = 115.6 \times 10^6\ \text{Pa} = 115.6\ \text{MPa}$$

At $y = c, \quad \sigma_{res} = \sigma' - \sigma_Y = 346.7 - 240$ $\qquad \sigma_{res} = 106.7\ \text{MPa}$ ◀

At $y = y_Y, \quad \sigma_{res} = \sigma'' - \sigma_Y = 115.6 - 240$ $\qquad \sigma_{res} = -124.4\ \text{MPa}$



LOADING UNLOADING RESIDUAL STRESSES

(*b*) $$\sigma_{res} = 0 \quad \therefore \quad \frac{M y_0}{I} - \sigma_Y = 0$$

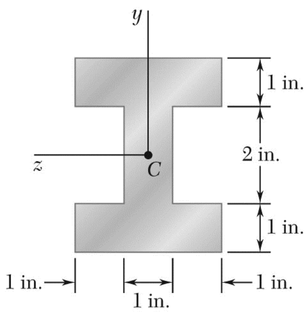
$$y_0 = \frac{I\sigma_Y}{M} = \frac{(3.645 \times 10^{-6})(240 \times 10^6)}{28.08 \times 10^3} = 31.15 \times 10^{-3}\ \text{m} = 31.15\ \text{mm}$$

Answer: $y_0 = -31.15\ \text{mm}, 0, 31.15\ \text{mm}$ ◀

(*c*) At $y = y_Y, \quad \sigma_{res} = -124.4 \times 10^6\ \text{Pa}$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.015)}{-124.4 \times 10^6}$$

$\rho = 24.1\ \text{m}$ ◀



PROBLEM 4.92

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. A bending couple is applied to the beam about z axis, causing plastic zones 2 in. thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 2$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

SOLUTION

See solution to Problem 4.76 for bending couple and stress distribution during loading.

$$M = 406 \text{ kip} \cdot \text{in} \qquad y_Y = 1.0 \text{ in.} \qquad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

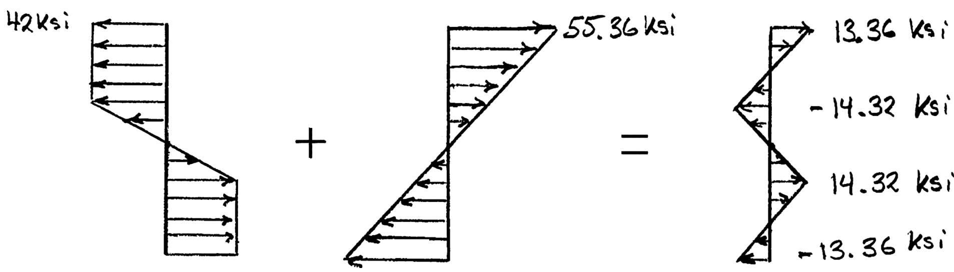
$$\sigma_Y = 42 \text{ ksi} \qquad I = 14.6667 \text{ in}^4 \qquad c = 2 \text{ in.}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(406)(2)}{14.6667} = 55.36 \text{ ksi}$$

$$\sigma'' = \frac{My_Y}{I} = \frac{(406)(1.0)}{14.6667} = 27.68 \text{ ksi}$$

At $y = c$, $\quad \sigma_{res} = \sigma' - \sigma_Y = 55.36 - 42$ $\qquad \sigma_{res} = 13.36 \text{ ksi}$ ◀

At $y = y_Y$, $\quad \sigma_{res} = \sigma'' - \sigma_Y = 27.68 - 42$ $\qquad \sigma_{res} = -14.32 \text{ ksi}$ ◀



(*b*) $$\sigma_{res} = 0 \quad \therefore \quad \frac{M y_0}{I} - \sigma_Y = 0$$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(14.6667)(42)}{406} = 1.517 \text{ in.}$$

Answer: $y_0 = -1.517 \text{ in.}, 0, 1.517 \text{ in.}$ ◀

(*c*) At $y = y_Y$, $\qquad \sigma_{res} = -14.32$ ksi

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.0)}{14.32} = 2025 \text{ in.}$$

$\rho = 168.8$ ft ◀

PROBLEM 4.93*

A rectangular bar that is straight and unstressed is bent into an arc of circle of radius by two couples of moment M. After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R. Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation:

$$\frac{1}{\rho_R} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]\right\}$$

SOLUTION

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right),$$

Let m denote $\frac{M}{M_Y}$:

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{\rho^2}{\rho_Y^2}\right) \quad \therefore \quad \frac{\rho^2}{\rho_Y^2} = 3 - 2\,m$$

$$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y}$$

$$= \frac{1}{\rho}\left\{1 - \frac{\rho}{\rho_Y}m\right\} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)\right\} \quad ◀$$

PROBLEM 4.94

A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25 M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.93.

SOLUTION

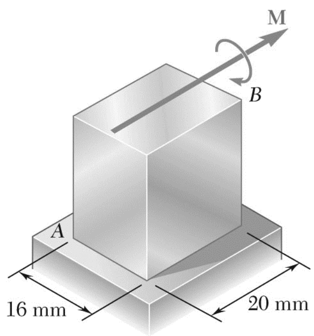
(*a*) $$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2} M_Y \left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m = \frac{M}{M_Y} = 1.25$$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

$$\rho = 0.70711\rho_Y \blacktriangleleft$$

(*b*) $$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711\rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y} \qquad \rho_R = 6.09\rho_Y \blacktriangleleft$$



PROBLEM 4.95

The prismatic bar AB is made of a steel that is assumed to be elastoplastic and for which $E = 200\text{ GPa}$. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350\text{ N}\cdot\text{m}$ is applied as shown, determine (*a*) the yield strength of the steel, (*b*) the thickness of the elastic core of the bar.

SOLUTION

$$M = \frac{3}{2}M_Y\left(1-\frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)$$

$$= \frac{3}{2}\frac{\sigma_Y I}{c}\left(1-\frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

$$= \frac{3}{2}\frac{\sigma_Y b(2c)^3}{12c}\left(1-\frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

$$= \sigma_Y bc^2\left(1-\frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

(*a*) $bc^2\sigma_y\left(1-\dfrac{\rho^2\sigma_Y^2}{3E^2c^2}\right) = M$ Cubic equation for σ_Y

Data:

$$E = 200\times 10^9\text{ Pa}$$
$$M = 420\text{ N}\cdot\text{m}$$
$$\rho = 2.4\text{ m}$$
$$b = 20\text{ mm} = 0.020\text{ m}$$
$$c = \frac{1}{2}h = 8\text{ mm} = 0.008\text{ m}$$

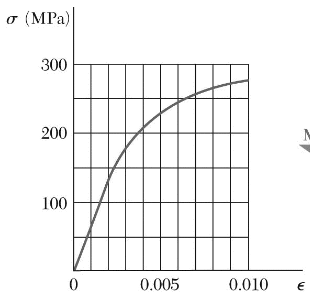
$$(1.28\times 10^{-6})\,\sigma_Y\left[1-750\times 10^{-21}\sigma_Y^2\right] = 350$$

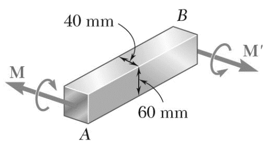
$$\sigma_Y\left[1-750\times 10^{-21}\sigma_Y^2\right] = 273.44\times 10^6$$

Solving by trial, $\sigma_Y = 292\times 10^6\text{ Pa}$ $\sigma_Y = 292\text{ MPa}$ ◀

(*b*) $y_Y = \dfrac{\sigma_Y\rho}{E} = \dfrac{(292\times 10^6)(2.4)}{200\times 10^9} = 3.504\times 10^{-3}\text{ m} = 3.504\text{ mm}$

thickness of elastic core $= 2y_Y = 7.01\text{ mm}$ ◀





PROBLEM 4.96

The prismatic bar AB is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (a) the radius of curvature of the bar when the maximum stress is 250 MPa, (b) the corresponding value of the bending moment. (*Hint:* For part b, plot σ versus y and use an approximate method of integration.)

SOLUTION

(a) $\sigma_m = 250\text{ MPa} = 250 \times 10^6\text{ Pa}$

$\varepsilon_m = 0.0064$ from curve

$$c = \frac{1}{2}\text{h} = 30\text{ mm} = 0.030\text{ m}$$

$$b = 40\text{ mm} = 0.040\text{ m}$$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333\text{ m}^{-1}$$

$\rho = 4.69\text{ m}$ ◀

(b) Strain distribution: $\varepsilon = -\varepsilon_m \dfrac{y}{c} = -\varepsilon_m u$ where $u = \dfrac{y}{\varepsilon}$

Bending couple: $M = -\int_{-c}^{c} y\sigma b dy = 2b\int_0^c y\,|\sigma|\,d y = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\Sigma wu|\sigma|$$

where w is a weighting factor.

Using $\Delta u = 0.25,$ we get the values given in the table below:

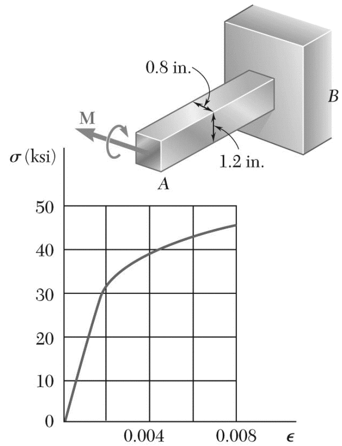
u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\,\lvert\sigma\rvert$, (MPa)	w	$wu\,\lvert\sigma\rvert$, (MPa)	
0	0	0	0	1	0	
0.25	0.0016	110	27.5	4	110	
0.5	0.0032	180	90	2	180	
0.75	0.0048	225	168.75	4	675	
1.00	0.0064	250	250	1	250	
					1215	← $\Sigma wu\,\lvert\sigma\rvert$

$$J = \frac{(0.25)(1215)}{3} = 101.25\text{ MPa} = 101.25 \times 10^6\text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(101.25 \times 10^6) = 7.29 \times 10^3\text{ N}\cdot\text{m}$$

$M = 7.29\text{ kN}\cdot\text{m}$ ◀

PROBLEM 4.97



The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the $\sigma - \varepsilon$ diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)

SOLUTION

(*a*) $\rho = 100$ in., $b = 0.8$ in., $c = 0.6$ in.

$$\varepsilon_m = \frac{c}{\rho} = \frac{0.6}{100} = 0.006$$

From the curve, $\sigma_m = 43$ ksi ◀

(*b*) Strain distribution: $\varepsilon = -\varepsilon_m \dfrac{y}{c} = -\varepsilon_m u$ where $u = \dfrac{y}{\varepsilon}$

Bending couple: $M = -\int_{-c}^{c} y\sigma b dy = 2b\int_0^c y\,|\sigma| dy = 2bc^2\int_0^1 u\,|\sigma|\,du = 2bc^2 J$

where the integral J is given by $\int_0^1 u\,|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\Sigma wu|\sigma|$$

where w is a weighting factor.

Using $\Delta u = 0.25$, we get the values given the table below:

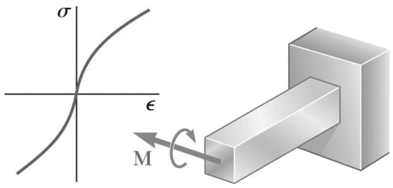
u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\,\lvert\sigma\rvert$, ksi	w	$wu\,\lvert\sigma\rvert$, ksi
0	0	0	0	1	0
0.25	0.0015	25	6.25	4	25
0.5	0.003	36	18	2	36
0.75	0.0045	40	30	4	120
1.00	0.006	43	43	1	43
					224 ← $\Sigma wu\,\lvert\sigma\rvert$

$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67)$$

$M = 10.75$ kip · in ◀

PROBLEM 4.98



A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\varepsilon = k\sigma^n$ for $\sigma > 0$ and $\varepsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n}\frac{Mc}{I}$$

SOLUTION

Strain distribution: $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u \quad \text{where} \quad u = \frac{y}{c}$

Bending couple:

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y\,|\sigma|\,dy = 2bc^2\int_0^c \frac{y}{c}\,|\sigma|\,\frac{dy}{c}$$

$$= 2bc^2\int_0^1 u\,|\sigma|\,du$$

For $\varepsilon = K\sigma^n, \quad \varepsilon_m = K\sigma_m$

$$\frac{\varepsilon}{\varepsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \quad \therefore \quad |\sigma| = \sigma_m u^{\frac{1}{n}}$$

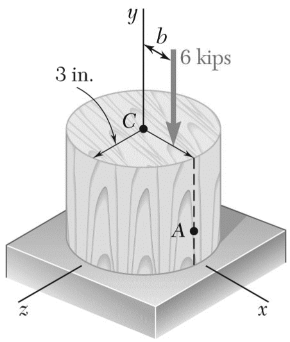
Then $M = 2bc^2\int_0^1 u\sigma_m u^{\frac{1}{n}}\,du = 2bc^2\sigma_m\int_0^1 u^{1+\frac{1}{n}}du$

$$= 2bc^2\sigma_m \frac{u^{2+\frac{1}{n}}}{2+\frac{1}{n}}\Big|_0^1 = \frac{2n}{2n+1}bc^2\sigma_m$$

$$\sigma_m = \frac{2n+1}{2}\frac{M}{bc^2}$$

Recall: that $\frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}\,bc^2 \quad \therefore \quad \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

Then $\sigma_m = \frac{2n+1}{3n}\frac{Mc}{I}$



PROBLEM 4.99

A short wooden post supports a 6-kip axial load as shown. Determine the stress at point A when (*a*) $b = 0$, (*b*) $b = 1.5\,\text{in.}$, (*c*) $b = 3\,\text{in.}$

SOLUTION

$$A = \pi r^2 = \pi(3)^2 = 28.27\ \text{in}^2$$

$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(3)^4 = 63.62\ \text{in}^4$$

$$S = \frac{I}{c} = \frac{63.62}{3} = 21.206\ \text{in}^3$$

$$P = 6\ \text{kips} \qquad M = Pb$$

(*a*) $b = 0 \qquad M = 0$

$$\sigma = -\frac{P}{A} = -\frac{6}{28.27} = -0.212\ \text{ksi}$$

$\sigma = -212\ \text{psi}$ ◀

(*b*) $b = 1.5\ \text{in.} \quad M = (6)(1.5) = 9\ \text{kip}\cdot\text{in}$

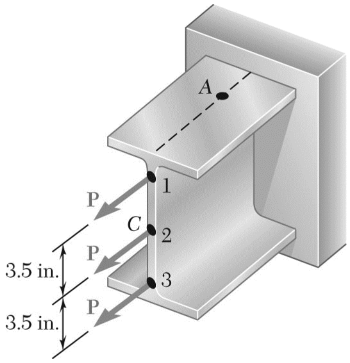
$$\sigma = -\frac{P}{A} - \frac{M}{S} = -\frac{6}{28.27} - \frac{9}{21.206} = -0.637\ \text{ksi}$$

$\sigma = -637\ \text{psi}$ ◀

(*c*) $b = 3\ \text{in.} \quad M = (6)(3) = 18\ \text{kip}\cdot\text{in}$

$$\sigma = -\frac{P}{A} - \frac{M}{S} = -\frac{6}{28.27} - \frac{18}{21.206} = -1.061\ \text{ksi}$$

$\sigma = -1061\ \text{psi}$ ◀



PROBLEM 4.100

As many as three axial loads, each of magnitude $P = 10$ kips, can be applied to the end of a $W8 \times 21$ rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 1 and 2 only.

SOLUTION

For $W8 \times 21$ Appendix C gives

$$A = 6.16 \text{ in}^2, \quad d = 8.28 \text{ in.}, \quad I_x = 75.3 \text{ in}^4$$

At point A,

$$y = \frac{1}{2}d = 4.14 \text{ in.}$$

$$\sigma = \frac{F}{A} - \frac{My}{I}$$

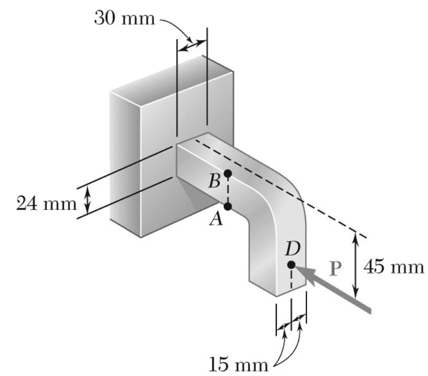
(a) Centric loading: $F = 30$ kips, $M = 0$

$$\sigma = \frac{30}{6.16} \qquad \sigma = 4.87 \text{ ksi} \blacktriangleleft$$

(b) Eccentric loading: $F = 2P = 20$ kips

$$M = -(10)(3.5) = -35 \text{ kip} \cdot \text{in}$$

$$\sigma = \frac{20}{6.16} - \frac{(-35)(4.14)}{75.3} \qquad \sigma = 5.17 \text{ ksi} \blacktriangleleft$$



PROBLEM 4.101

Knowing that the magnitude of the horizontal force **P** is 8 kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = (30)(24) = 720\text{ mm}^2 = 720\times 10^{-6}\text{ m}^2$$

$$e = 45 - 12 = 33\text{ mm} = 0.033\text{ m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(30)(24)^3 = 34.56\times 10^3\text{ mm}^4 = 34.56\times 10^{-9}\text{ m}^4$$

$$c = \frac{1}{2}(24\text{ mm}) = 12\text{ mm} = 0.012\text{ m} \qquad P = 8\times 10^3\text{ N}$$

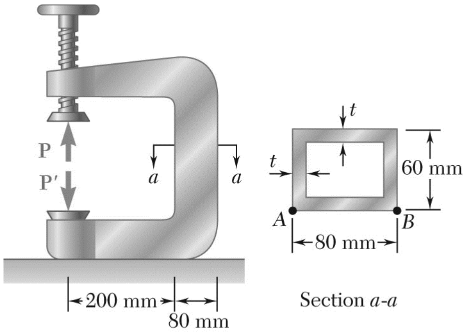
$$M = Pe = (8\times 10^3)(0.033) = 264\text{ N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8\times 10^3}{720\times 10^{-6}} = \frac{(264)(0.012)}{34.56\times 10^{-9}} = -102.8\times 10^6\text{ Pa}$$

$$\sigma_A = -102.8\text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8\times 10^3}{720\times 10^{-6}} + \frac{(264)(0.012)}{34.56\times 10^{-9}} = 80.6\times 10^6\text{ Pa}$$

$$\sigma_B = 80.6\text{ MPa} \blacktriangleleft$$



PROBLEM 4.102

The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (a) point A, (b) point B.

SOLUTION

Rectangular cutout is $60 \text{ mm} \times 40 \text{ mm}$.

$$A = (80)(60) - (60)(40) = 2.4 \times 10^3 \text{ mm}^2 = 2.4 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84 \times 10^6 \text{ mm}^4$$

$$= 1.84 \times 10^{-6} \text{ m}^4$$

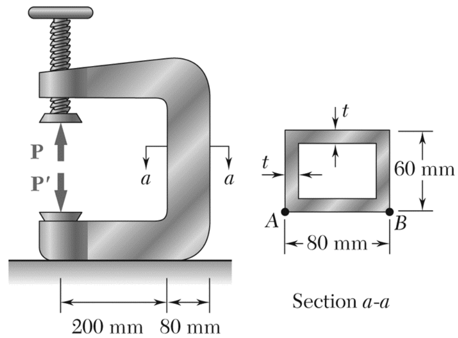
$$c = 40 \text{ mm} = 0.040 \text{ m} \quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$

$$P = 20 \times 10^3 \text{ N}$$

$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N} \cdot \text{m}$$

(a) $\sigma_A = \dfrac{P}{A} + \dfrac{Mc}{I} = \dfrac{20 \times 10^3}{2.4 \times 10^{-3}} + \dfrac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = 112.7 \times 10^6 \text{ Pa}$ $\qquad \sigma_A = 112.7 \text{ MPa}$ ◀

(b) $\sigma_B = \dfrac{P}{A} - \dfrac{Mc}{I} = \dfrac{20 \times 10^3}{2.4 \times 10^{-3}} - \dfrac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = -96.0 \times 10^6 \text{ Pa}$ $\qquad \sigma_B = -96.0 \text{ MPa}$ ◀



Section a-a

PROBLEM 4.103

Solve Prob. 4.102, assuming that $t = 8$ mm.

PROBLEM 4.102 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Rectangular cutout is $64 \text{ mm} \times 44 \text{ mm}$.

$$A = (80)(60) - (64)(44) = 1.984 \times 10^3 \text{ mm}^2$$

$$= 1.984 \times 10^{-3} \text{ mm}^2$$

$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(44)(64)^3 = 1.59881 \times 10^6 \text{ mm}^2$$

$$= 1.59881 \times 10^{-6} \text{ m}^4$$

$$c = 40 \text{ mm} = 0.004 \qquad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$

$$P = 20 \times 10^3 \text{ N}$$

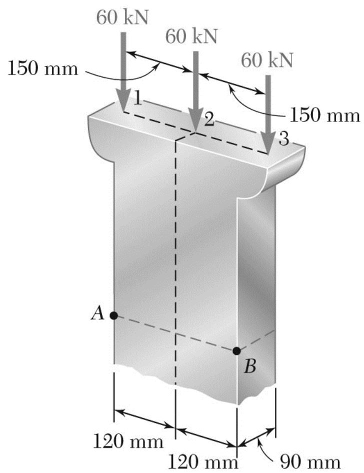
$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N} \cdot \text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = 130.2 \times 10^6 \text{ Pa}$$

$\sigma_A = 130.2$ MPa ◀

(*b*) $$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = -110.0 \times 10^6 \text{ Pa}$$

$\sigma_B = -110.0$ MPa ◀



PROBLEM 4.104

Determine the stress at points A and B, (a) for the loading shown, (b) if the 60-kN loads are applied at points 1 and 2 only.

SOLUTION

(a) Loading is centric.

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-6} \text{ m}^2$$

At A and B:

$$\sigma = -\frac{P}{A} = \frac{180 \times 10^3}{21.6 \times 10^{-3}} = -8.33 \times 10^6 \text{ Pa}$$

$$\sigma_A = \sigma_B = -8.33 \text{ MPa} \blacktriangleleft$$

(b) Eccentric loading.

$$P = 120 \text{ kN} = 120 \times 10^3 \text{ N}$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N} \cdot \text{m}$$

$$I = \frac{1}{12} bh^3 = \frac{1}{12}(90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

$$c = 120 \text{ mm} = 0.120 \text{ m}$$

At A:

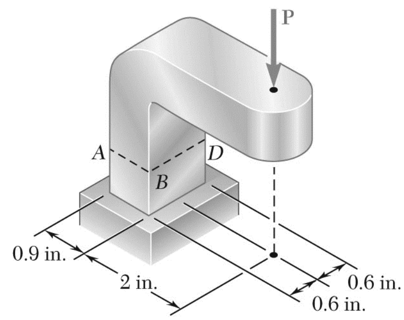
$$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -15.97 \times 10^6 \text{ Pa}$$

At B:

$$\sigma_B - \frac{P}{A} + \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 4.86 \times 10^6 \text{ Pa}$$

$$\sigma_A = -15.97 \text{ MPa} \blacktriangleleft$$

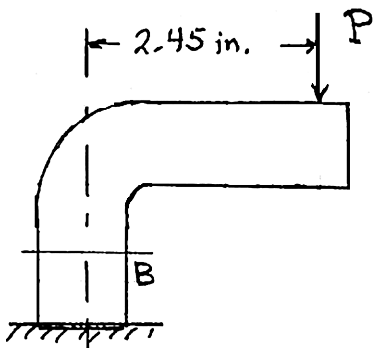
$$\sigma_B = 4.86 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.105

Knowing that the allowable stress in section *ABD* is 10 ksi, determine the largest force **P** that can be applied to the bracket shown.

SOLUTION

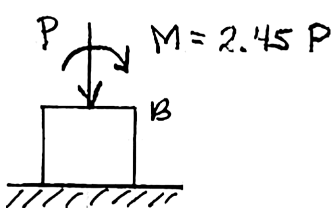


Statics: $M = 2.45P$

Cross section: $A = (0.9)(1.2) = 1.08 \text{ in}^2$

$$c = \frac{1}{2}(0.9) = 0.45 \text{ in.}$$

$$I = \frac{1}{12}(1.2)(0.9)^3 = 0.0729 \text{ in}^4$$

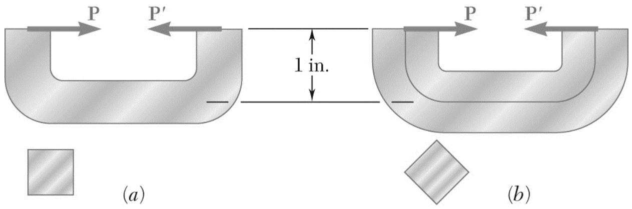


At point *B*: $\sigma = -10$ ksi

$$\sigma = -\frac{P}{A} - \frac{Mc}{I}$$

$$-10 = -\frac{P}{1.08} - \frac{(2.45P)(0.45)}{0.0729} = -16.049P$$

$P = 0.623$ kips $\qquad P = 623 \text{ lb}$ ◀

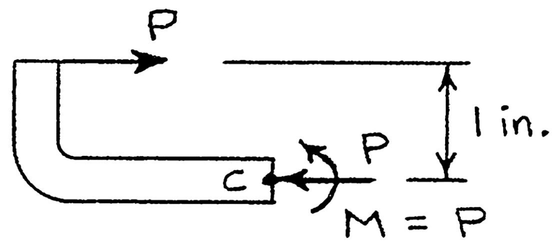


PROBLEM 4.106

Portions of a $\frac{1}{2} \times \frac{1}{2}$-in. square bar have been bent to form the two machine components shown. Knowing that the allowable stress is 15 ksi, determine the maximum load that can be applied to each component.

SOLUTION

The maximum stress occurs at point *B*.



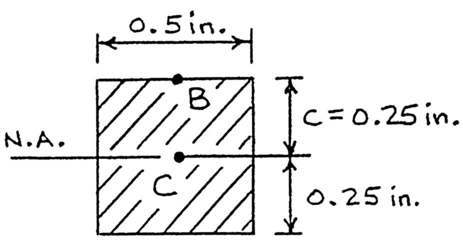
$$\sigma_B = -15 \text{ ksi} = -15 \times 10^3 \text{ psi}$$

$$\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP$$

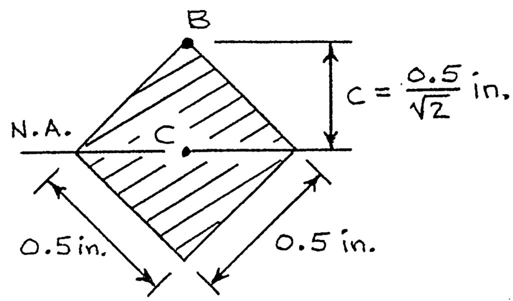
where $\quad K = \frac{1}{A} + \frac{ec}{I} \qquad e = 1.0 \text{ in.}$

$$A = (0.5)(0.5) = 0.25 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(0.5)^3 = 5.2083 \times 10^{-3} \text{ in}^4 \text{ for all centroidal axes.}$$



(*a*)



(*b*)

(*a*) $\quad c = 0.25$ in.

$$K = \frac{1}{0.25} + \frac{(1.0)(0.25)}{5.2083 \times 10^{-3}} = 52 \text{ in}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-15 \times 10^3)}{52}$$

$P = 288$ lb ◀

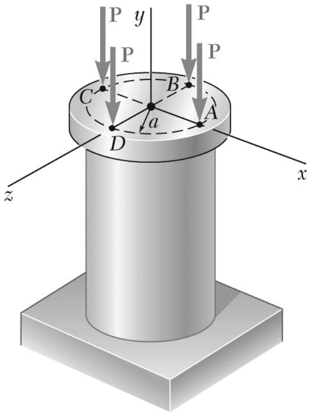
(*b*) $\quad c = \frac{0.5}{\sqrt{2}} = 0.35355$ in.

$$K = \frac{1}{0.25} + \frac{(1.0)(0.35355)}{5.2083 \times 10^{-3}} = 71.882 \text{ in}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-15 \times 10^3)}{71.882}$$

$P = 209$ lb ◀

PROBLEM 4.107



The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Knowing that $P = 100$ kN and $a = 40$ mm, determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.

SOLUTION

For a solid circular section of radius a,

$$A = \pi a^2 \qquad I = \frac{\pi}{4}a^4$$

Centric force: $\quad F = 4P, \; M_x = M_z = 0 \qquad \sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2}$

(a) Force at D is removed:

$$F = 3P, \quad M_x = -Pa, \quad M_z = 0$$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4}a^2} = -\frac{7P}{\pi a^2}$$

(b) Forces at C and D are removed:

$$F = 2P, \quad M_x = -Pa, \quad M_z = -Pa$$

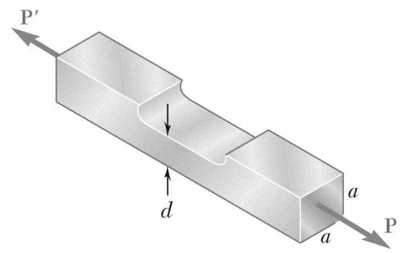
Resultant bending couple: $\quad M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}\,Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2}\,Pa\,a}{\frac{\pi}{4}a^2} = -\frac{2+4\sqrt{2}}{\pi}\frac{P}{a^2} = -2.437\,P/a^2$$

Numerical data: $\quad P = 100 \times 10^3$ N, $\quad a = 0.040$ m

Answers: (a) $\quad \sigma = -\frac{(7)(100 \times 10^3)}{\pi(0.040)^2} = -139.3 \times 10^6$ Pa $\qquad \sigma = -139.3$ MPa ◀

(b) $\quad \sigma = -\frac{(2.437)(100 \times 10^3)}{(0.040)^2} = -152.3 \times 10^6$ Pa $\qquad \sigma = -152.3$ MPa ◀



PROBLEM 4.108

A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30$ mm, $d = 20$ mm, and $\sigma_{\text{all}} = 60\text{ MPa}$, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

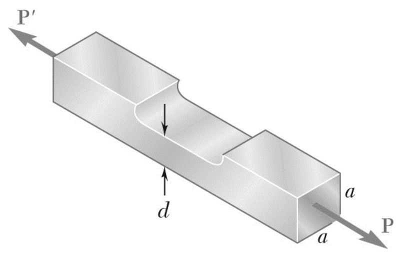
$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \quad \text{where} \quad K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

Data: $a = 30\text{ mm} = 0.030\text{ m} \quad d = 20\text{ mm} = 0.020\text{ m}$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667 \times 10^3\text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60 \times 10^6}{4.1667 \times 10^3} = 14.40 \times 10^3\text{ N}$$

$P = 14.40\text{ kN}$ ◀



PROBLEM 4.109

A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude $P = 18$ kN are applied at the centers of the ends of the bar. Knowing that $a = 30$ mm and $\sigma_{\text{all}} = 135$ MPa, determine the smallest allowable depth d of the milled portion of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P\frac{1}{2}(a-d)\frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \qquad \text{or} \qquad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

Solving for d,

$$d = \frac{1}{2\sigma}\left\{\sqrt{\left(\frac{2P}{a}\right)^2 + 12P\sigma} - \frac{2P}{a}\right\}$$

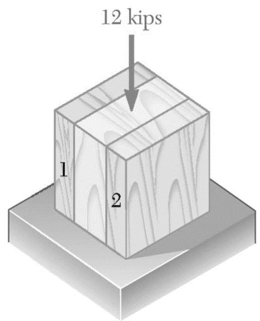
Data:

$$a = 0.030 \text{ m}, \quad P = 18\times 10^3 \text{ N}, \quad \sigma = 135\times 10^6 \text{ Pa}$$

$$d = \frac{1}{(2)(135\times 10^6)}\left\{\sqrt{\left[\frac{(2)(18\times 10^3)}{0.030}\right]^2 + 12(18\times 10^3)(135\times 10^6)} - \frac{(2)(18\times 10^3)}{0.030}\right\}$$

$$= 16.04\times 10^{-3} \text{ m}$$

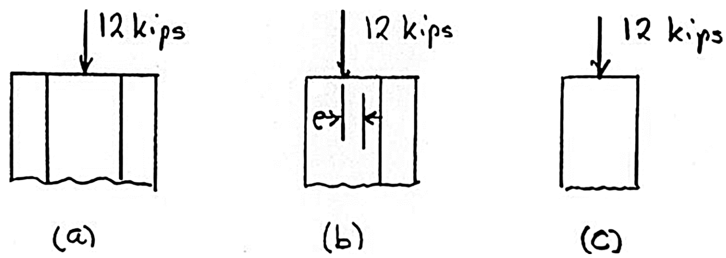
$d = 16.04$ mm ◀



PROBLEM 4.110

A short column is made by nailing two 1×4-in. planks to a 2×4-in. timber. Determine the largest compressive stress created in the column by a 12-kip load applied as shown in the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) both planks are removed.

SOLUTION



(*a*) Centric loading: 4 in. × 4 in. cross section $\quad A = (4)(4) = 16 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{16} \qquad \sigma = -0.75 \text{ ksi} \blacktriangleleft$$

(*b*) Eccentric loading: 4 in. × 3 in. cross section $\quad A = (4)(3) = 12 \text{ in}^2$

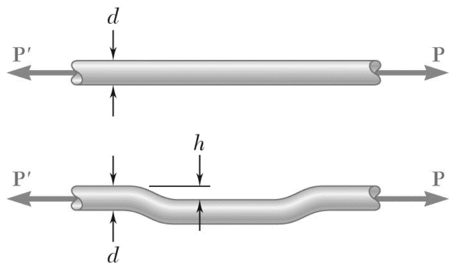
$$c = \left(\frac{1}{2}\right)(3) = 1.5 \text{ in.} \qquad e = 1.5 - 1.0 = 0.5 \text{ in.}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(4)(3)^3 = 9 \text{ in}^4$$

$$\sigma = -\frac{P}{A} - \frac{Pec}{I} = -\frac{12}{12} - \frac{(12)(0.5)(1.5)}{9} \qquad \sigma = -2.00 \text{ ksi} \blacktriangleleft$$

(*c*) Centric loading: 4 in. × 2 in. cross section $\quad A = (4)(2) = 8 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{8} \qquad \sigma = -1.50 \text{ ksi} \blacktriangleleft$$



PROBLEM 4.111

An offset h must be introduced into a solid circular rod of diameter d. Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.

SOLUTION

For centric loading, $\sigma_c = \dfrac{P}{A}$

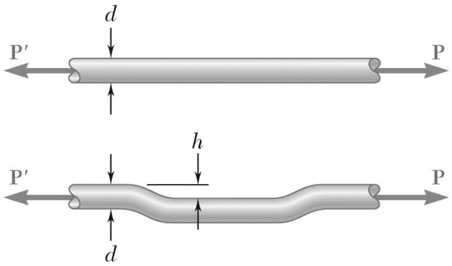
For eccentric loading, $\sigma_e = \dfrac{P}{A} + \dfrac{Phc}{I}$

Given $\sigma_e = 5\,\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 5\frac{P}{A}$$

$$\frac{Phc}{I} = 4\frac{P}{A} \qquad \therefore \quad h = \frac{4I}{cA} = \frac{(4)\left(\frac{\pi}{64}d^4\right)}{\left(\frac{d}{2}\right)\left(\frac{\pi}{4}d^2\right)} = \frac{1}{2}d$$

$h = 0.500\,d$ ◀



PROBLEM 4.112

An offset h must be introduced into a metal tube of 0.75-in. outer diameter and 0.08-in. wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the tube when it is straight, determine the largest offset that can be used.

SOLUTION

$$c = \frac{1}{2}d = 0.375 \text{ in.}$$

$$c_1 = c - t = 0.375 - 0.08 = 0.295 \text{ in.}$$

$$A = \pi\left(c^2 - c_1^2\right) = \pi(0.375^2 - 0.295^2)$$

$$= 0.168389 \text{ in}^2$$

$$I = \frac{\pi}{4}\left(c^4 - c_1^4\right) = \frac{\pi}{4}(0.375^4 - 0.295^4)$$

$$= 9.5835 \times 10^{-3} \text{ in}^4$$

For centric loading, $\sigma_{\text{cen}} = \dfrac{P}{A}$

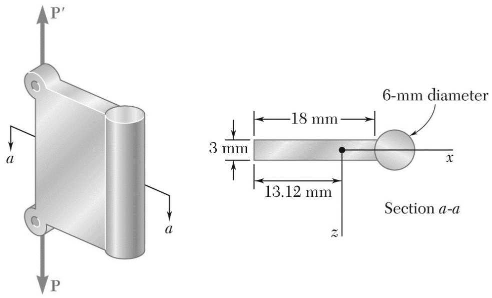
For eccentric loading, $\sigma_{\text{ecc}} = \dfrac{P}{A} + \dfrac{Phc}{I}$

$$\sigma_{\text{ecc}} = 4\sigma_{\text{cen}} \quad \text{or} \quad \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

$$\frac{hc}{I} = \frac{3}{A} \qquad h = \frac{3I}{Ac} = \frac{(3)(9.5835 \times 10^{-3})}{(0.168389)(0.375)}$$

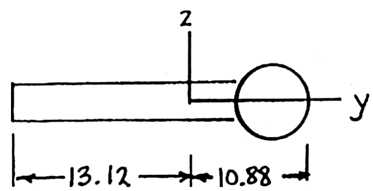
$h = 0.455$ in. ◀

PROBLEM 4.113



A steel rod is welded to a steel plate to form the machine element shown. Knowing that the allowable stress is 135 MPa, determine (*a*) the largest force **P** that can be applied to the element, (*b*) the corresponding location of the neutral axis. *Given:* The centroid of the cross section is at *C* and $I_z = 4195\ \text{mm}^4$.

SOLUTION



(*a*) $A = (3)(18) + \dfrac{\pi}{4}(6)^2 = 82.27\ \text{mm}^2 = 82.27 \times 10^{-6}\ \text{m}^2$

$I = 4195\ \text{mm}^4 = 4195 \times 10^{-12}\ \text{m}^4$

$e = 13.12\ \text{mm} = 0.01312\ \text{m}$

Based on tensile stress at $y = -13.12\ \text{mm} = -0.01312\ \text{m}$

$$\sigma = \frac{P}{A} + \frac{Pec}{I} = \left(\frac{1}{A} + \frac{ec}{I}\right)P = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{82.27 \times 10^{-6}} + \frac{(0.01312)(0.01312)}{4195 \times 10^{-12}} = 53.188 \times 10^3\ \text{m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{135 \times 10^6}{53.188 \times 10^3} = 2.538 \times 10^3\ \text{N}$$

$P = 2.54\ \text{kN}$ ◀

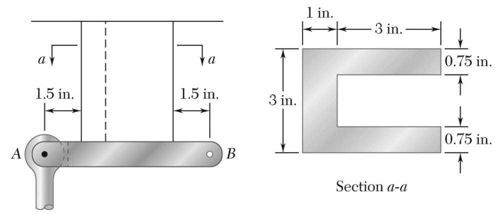
(*b*) Location of neutral axis. $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{4195 \times 10^{-12}}{(82.27 \times 10^{-6})(0.01312)} = 3.89 \times 10^{-3}\ \text{m}$$

$y = 3.89\ \text{mm}$ ◀

The neutral axis lies 3.89 mm to the right of the centroid or 17.01 mm to the right of the line of action of the loads.

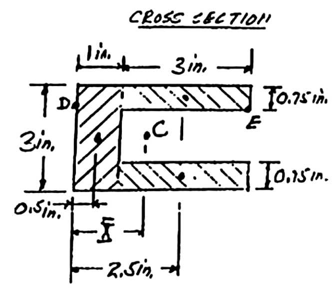


Section a-a

PROBLEM 4.114

A vertical rod is attached at point A to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are $\sigma_{\text{all}} = +5$ ksi and $\sigma_{\text{all}} = -12$ ksi, determine the largest downward force and the largest upward force that can be exerted by the rod.

SOLUTION



$$\bar{X} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1\times 3)(0.5) + 2(3\times 0.25)(2.5)}{(1\times 3) + 2(3\times 0.75)}$$

$$\bar{X} = \frac{12.75 \text{ in}^3}{7.5 \text{ in}^2} = 1.700 \text{ in.}$$

$$A = 7.5 \text{ in}^2$$

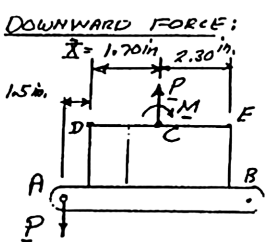
$$\sigma_{\text{all}} = +5 \text{ ksi} \qquad \sigma_{\text{all}} = -12 \text{ ksi}$$

$$I_c = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(3)(1)^3 + (3\times 1)(1.70-0.5)^2 + \frac{1}{12}(1.5)(3)^3 + (1.5\times 3)(2.5-1.70)^2$$

$$I_c = 10.825 \text{ in}^4$$

<u>Downward Force.</u>



$$M = P(1.5 \text{ in.}+1.70 \text{ in.}) = (3.20 \text{ in.})P$$

<u>At D:</u> $\sigma_D = +\dfrac{P}{A} + \dfrac{Mc}{I}$

$$+5 \text{ ksi} = \frac{P}{7.5} + \frac{(3.20)P(1.70)}{10.825}$$

$$+5 = P(+0.6359) \qquad \boldsymbol{P} = 7.86 \text{ kips} \downarrow$$

<u>At E:</u> $\sigma_E = +\dfrac{P}{A} - \dfrac{Mc}{I}$

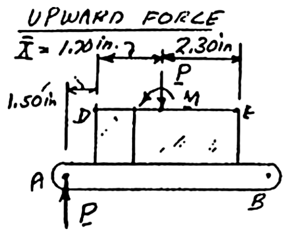
$$-12 \text{ ksi} = \frac{P}{7.5} - \frac{(3.20)P(2.30)}{10.825}$$

$$-12 = P(-0.5466) \qquad \boldsymbol{P} = 21.95 \text{ kips} \downarrow$$

We choose the smaller value. $\boldsymbol{P} = 7.96 \text{ kips} \downarrow$ ◀

PROBLEM 4.114 *(Continued)*

Upward Force.



$$M = P(1.5 \text{ in.} + 1.70 \text{ in.}) = (3.20 \text{ in.})P$$

At D: $\sigma_D = +\dfrac{P}{A} - \dfrac{Mc}{I}$

$$-12 \text{ ksi} = -\frac{P}{7.5} - \frac{(3.20)P(1.70)}{10.825}$$

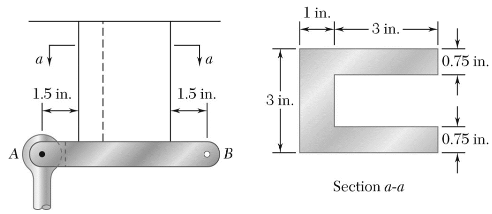
$$-12 = P(-0.6359) \qquad \boldsymbol{P} = 18.87 \text{ kips} \uparrow$$

At E: $\sigma_E = +\dfrac{P}{A} + \dfrac{Mc}{I}$

$$+5 \text{ ksi} = \frac{P}{7.5} + \frac{(3.20)P(2.30)}{10.825}$$

$$+5 = P(+0.5466) \qquad \boldsymbol{P} = 9.15 \text{ kips} \uparrow$$

We choose the smaller value. $\boldsymbol{P} = 9.15 \text{ kips} \uparrow$ ◀



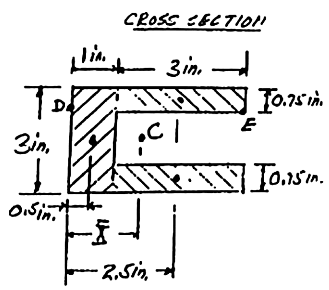
Section *a-a*

PROBLEM 4.115

Solve Prob. 4.114, assuming that the vertical rod is attached at point *B* instead of point *A*.

PROBLEM 4.114 A vertical rod is attached at point *A* to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are $\sigma_{\text{all}} = +5\text{ ksi}$ and $\sigma_{\text{all}} = -12\text{ ksi}$, determine the largest downward force and the largest upward force that can be exerted by the rod.

SOLUTION



$$\bar{X} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1\times 3)(0.5) + 2(3\times 0.25)(2.5)}{(1\times 3) + 2(3\times 0.75)}$$

$$\bar{X} = \frac{12.75\text{ in}^3}{7.5\text{ in}^2} = 1.700\text{ in.}$$

$$A = 7.5\text{ in}^2$$

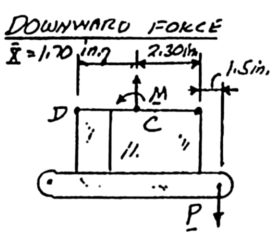
$$\sigma_{\text{all}} = +5\text{ ksi} \qquad \sigma_{\text{all}} = -12\text{ ksi}$$

$$I_c = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(3)(1)^3 + (3\times 1)(1.70 - 0.5)^2 + \frac{1}{12}(1.5)(3)^3 + (1.5\times 3)(2.5 - 1.70)^2$$

$$I_c = 10.825\text{ in}^4$$

<u>Downward Force.</u>



$$\sigma_{\text{all}} = +5\text{ ksi} \qquad \sigma_{\text{all}} = -12\text{ ksi}$$

$$M = (2.30\text{ in.} + 1.5\text{ in.}) = (3.80\text{ in.})P$$

<u>At *D*:</u> $\sigma_D = +\frac{P}{A} - \frac{Mc}{I}$

$$-12\text{ ksi} = +\frac{P}{7.5} - \frac{(3.80)P(1.70)}{10.825}$$

$$-12 = P(-0.4634) \qquad \boldsymbol{P} = 25.9\text{ kips}\downarrow$$

<u>At *E*:</u> $\sigma_E = +\frac{P}{A} + \frac{Mc}{I}$

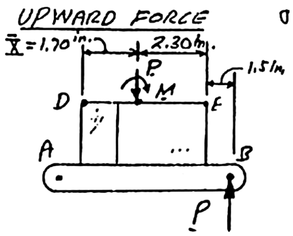
$$+5\text{ ksi} = +\frac{P}{7.5} + \frac{(3.80)P(2.30)}{10.825}$$

$$+5 = P(+0.9407) \qquad \boldsymbol{P} = 5.32\text{ kips}\downarrow$$

We choose the smaller value. $\boldsymbol{P} = 5.32\text{ kips}\downarrow$ ◀

PROBLEM 4.115 *(Continued)*

Upward Force.



$$\sigma_{\text{all}} = +5 \text{ ksi} \qquad \sigma_{\text{all}} = -12 \text{ ksi}$$

$$M = (2.30 \text{ in.} + 1.5 \text{ in.})P = (3.80 \text{ in.})P$$

At D: $\sigma_D = -\dfrac{P}{A} + \dfrac{Mc}{I}$

$$5 \text{ ksi} = -\frac{P}{7.5} + \frac{(3.80)P(1.70)}{10.825}$$

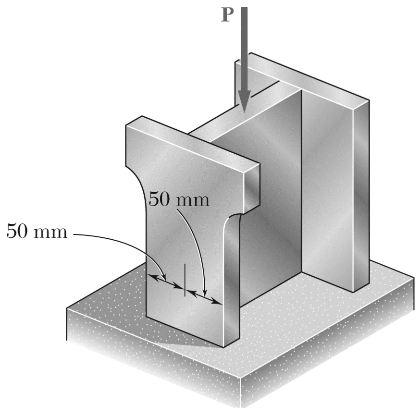
$$5 = P(+0.4634) \qquad \boldsymbol{P} = 10.79 \text{ kips} \uparrow$$

At E: $\sigma_E = -\dfrac{P}{A} - \dfrac{Mc}{I}$

$$-12 \text{ ksi} = -\frac{P}{7.5} - \frac{(3.80)P(2.30)}{10.825}$$

$$-12 = P(-0.9407) \qquad \boldsymbol{P} = 12.76 \text{ kips} \uparrow$$

We choose the smaller value. $\boldsymbol{P} = 10.79 \text{ kips} \uparrow$ ◀



PROBLEM 4.116

Three steel plates, each of 25×150-mm cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 25-mm strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 100 MPa, determine the largest force **P** (*a*) that could be applied to the original column, (*b*) that can be applied to the modified column.

SOLUTION

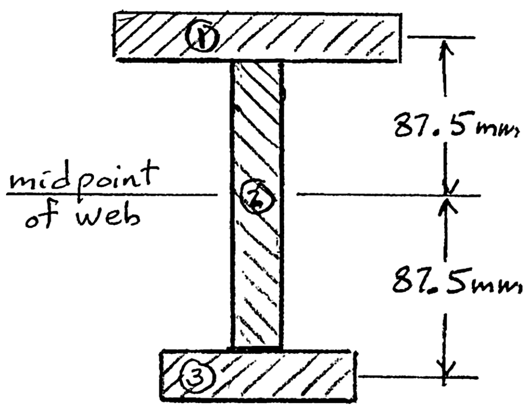
(*a*) Centric loading: $\sigma = -\dfrac{P}{A}$

$$A = (3)(150)(25) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$P = -\sigma A = -(-100 \times 10^6)(11.25 \times 10^{-3})$$

$$= 1.125 \times 10^6 \text{N} \qquad P = 1125 \text{ kN} \blacktriangleleft$$

(*b*) Eccentric loading (reduced cross section):



	A, 10^3 mm^2	$\bar{y}$, mm	$A\bar{y}$ (10^3 mm^3)	d, mm
①	3.75	87.5	328.125	76.5625
②	3.75	0	0	10.9375
③	2.50	−87.5	−218.75	98.4375
Σ	10.00		109.375	

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{109.375 \times 10^3}{10.00 \times 10^3} = 10.9375 \text{ mm}$$

PROBLEM 4.116 *(Continued)*

The centroid lies 10.9375 mm from the midpoint of the web.

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(150)(25)^3 + (3.75\times 10^3)(76.5625)^2 = 22.177\times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(25)(150)^3 + (3.75\times 10^3)(10.9375)^2 = 7.480\times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(100)(25)^3 + (2.50\times 10^3)(98.4375)^2 = 24.355\times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 54.012\times 10^6 \text{ mm}^4 = 54.012\times 10^{-6} \text{ m}^4$$

$$c = 10.9375 + 75 + 25 = 110.9375 \text{ mm} = 0.1109375 \text{ m}$$

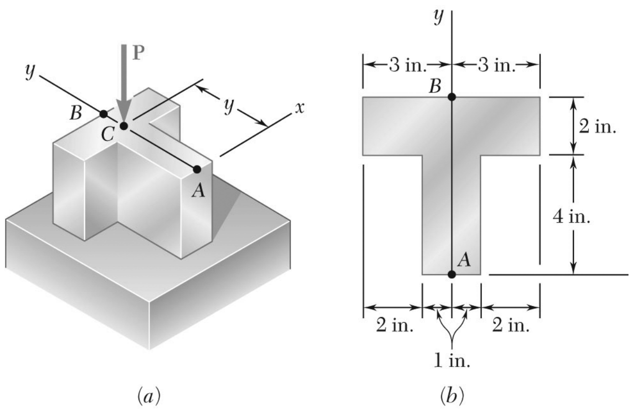
$$M = Pe \qquad \text{where} \qquad e = 10.4375 \text{ mm} = 10.4375\times 10^{-3} \text{ m}$$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP \qquad A = 10.00\times 10^{-3} \text{ m}^2$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{10.00\times 10^{-3}} + \frac{(101.9375\times 10^{-3})(0.1109375)}{54.012\times 10^{-6}} = 122.465 \text{ m}^{-2}$$

$$P = -\frac{\sigma}{K} = -\frac{(-100\times 10^6)}{122.465} = 817\times 10^3 \text{ N}$$

$P = 817$ kN ◀



PROBLEM 4.117

A vertical force **P** of magnitude 20 kips is applied at point C located on the axis of symmetry of the cross section of a short column. Knowing that $y = 5$ in., determine (a) the stress at point A, (b) the stress at point B, (c) the location of the neutral axis.

SOLUTION

Locate centroid.

Part	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{76}{20} = 3.8 \text{ in.}$$

Eccentricity of load: $e = 5 - 3.8 = 1.2$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4 \qquad I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(a) Stress at A: $c_A = 3.8$ in.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{20(1.2)(3.8)}{57.867}$$

$\sigma_A = 0.576$ ksi ◀

(b) Stress at B: $c_B = 6 - 3.8 = 2.2$ in.

$$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = -\frac{20}{20} - \frac{20(1.2)(2.2)}{57.867}$$

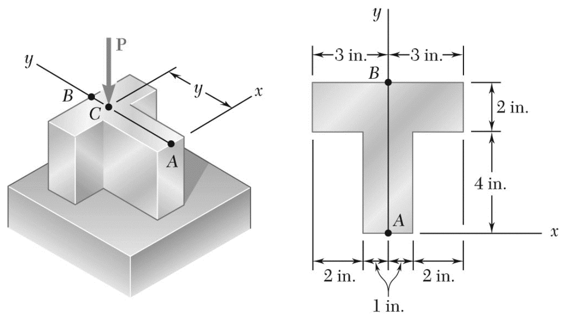
$\sigma_B = -1.912$ ksi ◀

(c) Location of neutral axis: $\sigma = 0 \qquad \sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$

$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in.}$$

Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411 = 1.389$ in. above point A.

Answer: 1.389 in. from point A. ◀



PROBLEM 4.118

A vertical force **P** is applied at point C located on the axis of symmetry of the cross section of a short column. Determine the range of values of y for which tensile stresses do not occur in the column.

SOLUTION

Locate centroid.

	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in.}$$

Eccentricity of load: $e = y - 3.8$ in. $\quad y = e + 3.8$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4 \qquad I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero, $c_A = 3.8$ in.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = 0 \qquad \therefore \qquad \frac{ec_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{Ac_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in.} \qquad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero. $c_B = 6 - 3.8 = 2.2$ in.

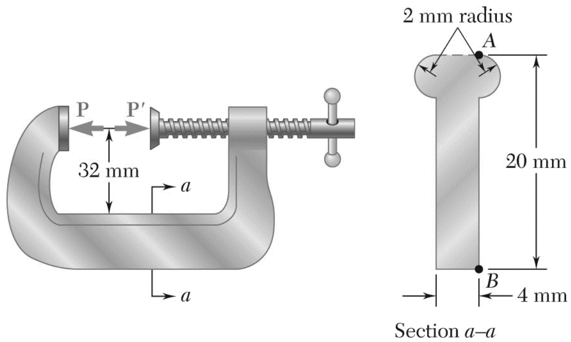
$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = 0 \qquad \therefore \qquad \frac{ec_B}{I} = -\frac{1}{A}$$

$$e = -\frac{I}{Ac_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in.}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer: $2.485 \text{ in.} < y < 4.561 \text{ in.}$ ◀

![clamp figure]

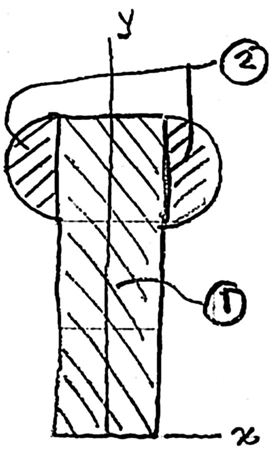


PROBLEM 4.119

Knowing that the clamp shown has been tightened until $P = 400\text{ N}$, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis of section $a-a$.

SOLUTION

Cross section: Rectangle ① + Circle ②



$$A_1 = (20\text{ mm})(4\text{ mm}) = 80\text{ mm}^2$$

$$\bar{y}_1 = \frac{1}{2}(20\text{ mm}) = 10\text{ mm}$$

$$A_2 = \pi(2\text{ mm})^2 = 4\pi\text{ mm}^2$$

$$\bar{y}_2 = 20 - 2 = 18\text{ mm}$$

$$c_B = \bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{(80)(10) + (4\pi)(18)}{80 + 4\pi} = 11.086\text{ mm}$$

$$c_A = 20 - \bar{y} = 8.914\text{ mm}$$

$$d_1 = 11.086 - 10 = 1.086\text{ mm}$$

$$d_2 = 18 - 11.086 = 6.914\text{ mm}$$

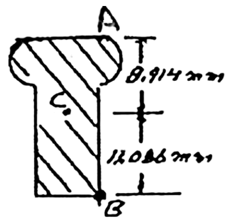
$$I_1 = \bar{I}_1 + A_1 d_1^2 = \frac{1}{12}(4)(20)^3 + (80)(1.086)^2 = 2.761\times 10^3\text{ mm}^4$$

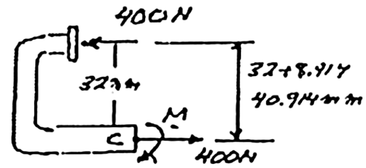
$$I_2 = \bar{I}_2 + A_2 d_2^2 = \frac{\pi}{4}(2)^4 + (4\pi)(6.914)^2 = 0.613\times 10^3\text{ mm}^4$$

$$I = I_1 + I_2 = 3.374\times 10^3\text{ mm}^4 = 3.374\times 10^{-9}\text{ m}^4$$

$$A = A_1 + A_2 = 92.566\text{ mm}^2 = 92.566\times 10^{-6}\text{ m}^2$$

PROBLEM 4.119 *(Continued)*





$$e = 32 + 8.914 = 40.914 \text{ mm} = 0.040914 \text{ m}$$

$$M = Pe = (400 \text{ N})(0.040914 \text{ m}) = 16.3656 \text{ N} \cdot \text{m}$$

(*a*) Point *A*:

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{400}{92.566 \times 10^{-6}} + \frac{(16.3656)(8.914 \times 10^{-3})}{3.374 \times 10^{-9}}$$

$$= 4.321 \times 10^6 + 43.23 \times 10^6 = 47.55 \times 10^6 \text{ Pa}$$

$\sigma_A = 47.6 \text{ MPa}$

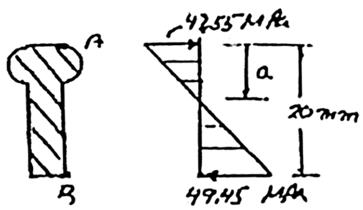
(*b*) Point *B*:

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{92.566 \times 10^{-6}} - \frac{(16.3656)(11.086)}{3.374 \times 10^{-9}}$$

$$= 4.321 \times 10^6 - 53.72 \times 10^6 = -49.45 \times 10^6 \text{ Pa}$$

$\sigma_B = -49.4 \text{ MPa}$

(*c*) Neutral axis:



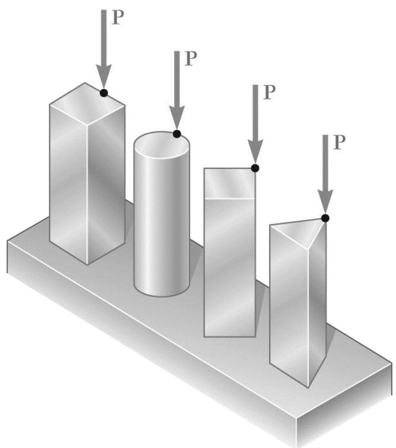
By proportions,

$$\frac{a}{47.55} = \frac{20}{47.55 + 49.45}$$

$$a = 9.80 \text{ mm}$$

9.80 mm below top of section

PROBLEM 4.120



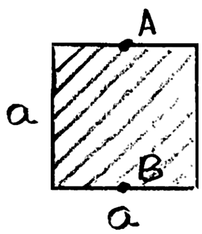
The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note:* the cross section of the triangular bar is an equilateral triangle.)

SOLUTION

Stresses:

At A, $$\sigma_A = -\frac{P}{A} - \frac{Pec_A}{I} = -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$$

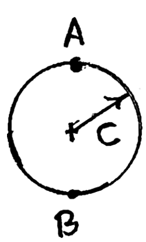
At B, $$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$$



$$A_1 = a^2, \quad I_1 = \frac{1}{12}a^4, \quad c_A = c_B = \frac{1}{2}a, \quad e = \frac{1}{2}a$$

$$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)\left(\frac{1}{2}a\right)\left(\frac{1}{2}a\right)}{\frac{1}{12}a^2}\right) \qquad \sigma_A = -4\frac{P}{A_1} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A}\left(\frac{(a^2)\left(\frac{1}{2}a\right)\left(\frac{1}{2}a\right)}{\frac{1}{12}a^2} - 1\right) \qquad \sigma_B = 2\frac{P}{A_1} \blacktriangleleft$$

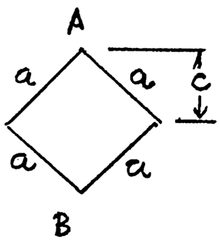


$$A_2 = \pi c^2 = a^2 \quad \therefore \quad c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4, \quad e = c$$

$$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) \qquad \sigma_A = -5\frac{P}{A_2} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) \qquad \sigma_B = 3\frac{P}{A_2} \blacktriangleleft$$

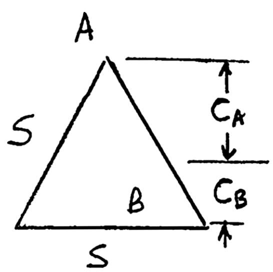
PROBLEM 4.120 *(Continued)*



$$\left\{\begin{aligned}
&A_3 = a^2 \quad c = \frac{\sqrt{2}}{2}a \quad I_3 = \frac{1}{12}a^4 \quad e = c\\
&\sigma_A = -\frac{P}{A_3}\left(1 + \frac{(a^2)\left(\frac{\sqrt{2}}{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)}{\frac{1}{12}a^4}\right)\\
&\sigma_B = \frac{P}{A_3}\left(\frac{(a^2)\left(\frac{\sqrt{2}}{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)}{\frac{1}{12}a^4} - 1\right)
\end{aligned}\right.$$

$$\sigma_A = -7\frac{P}{A_3} \blacktriangleleft$$

$$\sigma_B = 5\frac{P}{A_3} \blacktriangleleft$$



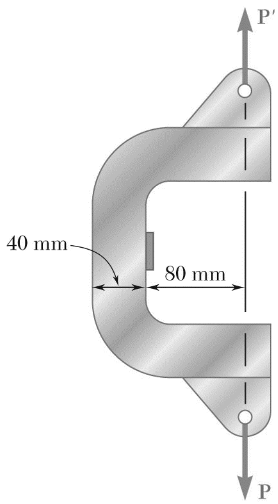
$$A_4 = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{\sqrt{3}}{4}s^2$$

$$I_4 = \frac{1}{36}s\left(\frac{\sqrt{3}}{2}s\right)^3 = \frac{\sqrt{3}}{96}s^4$$

$$c_A = \frac{2}{3}\frac{\sqrt{3}}{2}s = \frac{s}{\sqrt{3}} = e \qquad c_B = s$$

$$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s}{\sqrt{3}}\right)\left(\frac{s}{\sqrt{3}}\right)}{\frac{\sqrt{3}}{96}s^4}\right) \qquad \sigma_A = -9\frac{P}{A_4} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_4}\left(\frac{\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s}{\sqrt{3}}\right)\left(\frac{s}{2\sqrt{3}}\right)}{\frac{\sqrt{3}}{96}s^4} - 1\right) \qquad \sigma_B = 3\frac{P}{A_4} \blacktriangleleft$$



PROBLEM 4.121

The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and that strain on the inner edge was measured and found to be 450 μ, determine the magnitude P of the forces. Use $E = 200$ GPa.

SOLUTION

At the strain gage location,

$$\sigma = E\varepsilon = (200\times 10^9)(450\times 10^{-6}) = 90\times 10^6 \text{ Pa}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600\times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(40)(40)^3 = 213.33\times 10^3 \text{ mm}^4 = 213.33\times 10^{-9} \text{ m}^4$$

$$e = 80 + 20 = 100 \text{ mm} = 0.100 \text{ m}$$

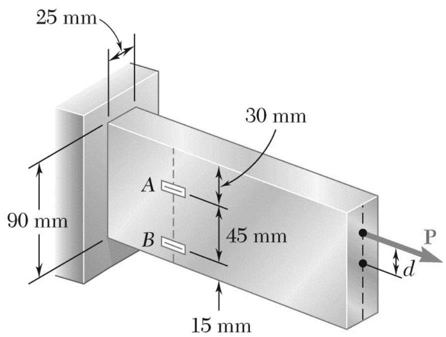
$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600\times 10^{-6}} + \frac{(0.100)(0.020)}{213.33\times 10^{-9}} = 10.00\times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90\times 10^6}{10.00\times 10^3} = 9.00\times 10^3 \text{ N}$$

$P = 9.00$ kN ◀



PROBLEM 4.122

An eccentric force **P** is applied as shown to a steel bar of 25×90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\,\mu \qquad \varepsilon_B = -70\,\mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm} \qquad b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh - (25)(90) = 2.25\times 10^3 \text{ mm}^2 = 2.25\times 10^{-3}\text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times 10^6 \text{ mm}^4 = 1.51875\times 10^{-6}\text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m} \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B: $\sigma_A = E\varepsilon_A = (200\times 10^9)(350\times 10^{-6}) = 70\times 10^6\,\text{Pa}$

$$\sigma_B = E\varepsilon_B = (200\times 10^9)(-70\times 10^{-6}) = -14\times 10^6\,\text{Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting, $\sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

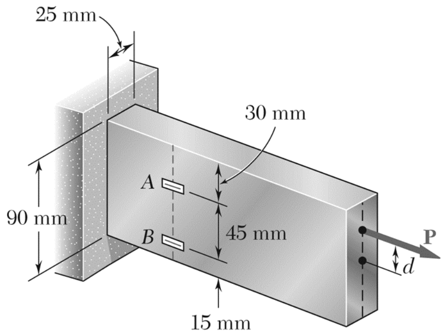
$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times 10^{-6})(84\times 10^6)}{0.045} = -2835 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting, $y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\dfrac{P}{A}$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times 10^{-3})[(0.015)(-14\times 10^6) - (-0.030)(70\times 10^6)]}{0.045} = 94.5\times 10^3\,\text{N}$$

(*a*) $M = -Pd \quad \therefore \quad d = -\dfrac{M}{P} = -\dfrac{-2835}{94.5\times 10^3} = 0.030$ m $\qquad d = 30.0$ mm ◀

(*b*) $\qquad P = 94.5$ kN ◀



PROBLEM 4.123

Solve Prob. 4.122, assuming that the measured strains are

$$\varepsilon_A = +600\,\mu \qquad \varepsilon_B = +420\,\mu$$

PROBLEM 4.122 An eccentric force **P** is applied as shown to a steel bar of 25×90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\mu \qquad \varepsilon_B = -70\mu$$

Knowing that $E = 200$ GPa, determine (a) the distance d, (b) the magnitude of the force **P**.

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm} \qquad b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25\times 10^3\,\text{mm}^2 = 2.25\times 10^{-3}\,\text{m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times 10^6\,\text{mm}^4 = 1.51875\times 10^{-6}\,\text{m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m} \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B: $\sigma_A = E\varepsilon_A = (200\times 10^9)(600\times 10^{-6}) = 120\times 10^6\,\text{Pa}$

$$\sigma_B = E\varepsilon_B = (200\times 10^9)(420\times 10^{-6}) = 84\times 10^6\,\text{Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \tag{1}$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \tag{2}$$

Subtracting, $\sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times 10^{-6})(36\times 10^6)}{0.045} = -1215\ \text{N}\cdot\text{m}$$

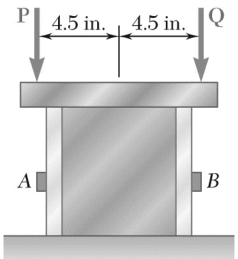
Multiplying (2) by y_A and (1) by y_B and subtracting, $y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\dfrac{P}{A}$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times 10^{-3})[(0.015)(84\times 10^6) - (-0.030)(120\times 10^6)]}{0.045} = 243\times 10^3\,\text{N}$$

$$M = -Pd$$

(a) $\therefore\ d = -\dfrac{M}{P} = -\dfrac{-1215}{243\times 10^3} = 5\times 10^{-3}\,\text{m}$ $\qquad d = 5.00$ mm ◀

(b) $\qquad P = 243$ kN ◀



PROBLEM 4.124

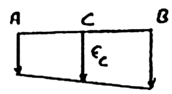
A short length of a W8 × 31 rolled-steel shape supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points A and B on the centerline of the outer faces of the flanges have been measured and found to be

$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION

Strains:



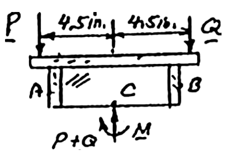
$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

$$\varepsilon_C = \frac{1}{2}(\varepsilon_A + \varepsilon_B) = \frac{1}{2}(-550 - 680)10^{-6} = -615 \times 10^{-6} \text{ in./in.}$$

Stresses:

$$\sigma_A = E\varepsilon_A = (29 \times 10^6 \text{ psi})(-550 \times 10^{-6} \text{ in./in.}) = -15.95 \text{ ksi}$$

$$\sigma_C = E\varepsilon_C = (29 \times 10^6 \text{ psi})(-615 \times 10^{-6} \text{ in./in.}) = -17.935 \text{ ksi}$$



W8 × 31 $\qquad A = 9.13 \text{ in}^2$

$$S = 27.5 \text{ in}^3$$

$$M = (4.5 \text{ in.})(P - Q)$$

At point C:

$$\sigma_C = -\frac{P+Q}{A}; \quad -17.835 \text{ ksi} = -\frac{P+Q}{9.13 \text{ in}^2}$$

$$P + Q = 162.83 \text{ kips} \quad (1)$$

At point A:

$$\sigma_A = -\frac{P+Q}{A} - \frac{M}{S}$$

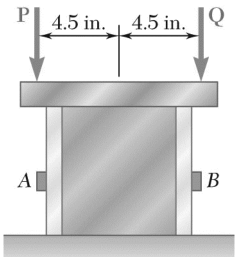
$$-15.95 \text{ ksi} = -17.835 \text{ ksi} - \frac{(4.5 \text{ in.})(P-Q)}{27.5 \text{ in}^3}; \qquad P - Q = -11.52 \text{ kips} \ (2)$$

Solve simultaneously, $\qquad P = 25.7$ kips $\quad Q = 87.2$ kips

$$\boldsymbol{P} = 25.7 \text{ kips} \downarrow \blacktriangleleft$$

$$\boldsymbol{Q} = 87.2 \text{ kips} \downarrow \blacktriangleleft$$

PROBLEM 4.125



Solve Prob. 4.124, assuming that the measured strains are

$$\varepsilon_A = +35 \times 10^{-6} \text{ in./in.} \quad \varepsilon_B = -450 \times 10^{-6} \text{ in./in.}$$

PROBLEM 4.124 A short length of a W8 × 31 rolled-steel shape supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points *A* and *B* on the centerline of the outer faces of the flanges have been measured and found to be

$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION

See solution and figures of Prob. 4.124.

$$\varepsilon_A = +35 \times 10^{-6} \text{ in./in.}; \qquad \varepsilon_B = -450 \times 10^{-6} \text{ in./in.}$$

$$\varepsilon_C = \frac{1}{2}(\varepsilon_A + \varepsilon_B) = \frac{1}{2}(35 - 450)10^{-6} \text{ in./in.} = -207.5 \times 10^{-6} \text{ in./in.}$$

Stresses: $\sigma_A = E\varepsilon_A = (29 \times 10^6 \text{ psi})(+35 \times 10^{-6} \text{ in./in.}) = +1.015 \text{ ksi}$

$\sigma_C = E\varepsilon_C = (29 \times 10^6 \text{ psi})(-207.5 \times 10^{-6} \text{ in./in.}) = -6.0175 \text{ ksi}$

At point *C*: $\sigma_C = -\dfrac{P+Q}{A}; \quad -6.0175 \text{ ksi} = -\dfrac{P+Q}{9.13 \text{ in}^2}$

$$P + Q = 54.94 \text{ kips} \quad (1)$$

At point *A*: $\sigma_A = -\dfrac{P+Q}{A} - \dfrac{M}{S}$

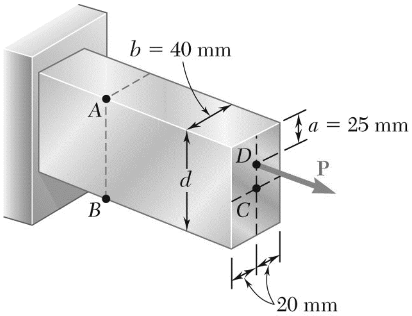
$$+1.015 \text{ ksi} = -6.0175 - \frac{(4.5 \text{ in.})(P - Q)}{27.5 \text{ in}^3}$$

$$P - Q = -42.98 \text{ kips} \quad (2)$$

Solve simultaneously, $P = 5.98$ kips $\quad Q = 49.0$ kips

$\boldsymbol{P} = 5.98 \text{ kips} \downarrow$ ◀

$\boldsymbol{Q} = 49.0 \text{ kips} \downarrow$ ◀



PROBLEM 4.126

The eccentric axial force **P** acts at point D, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (a) the depth d of the bar for which the tensile stress at point A is maximum, (b) the corresponding stress at point A.

SOLUTION

$$A = bd \qquad I = \frac{1}{12}bd^3$$

$$c = \frac{1}{2}d \qquad e = \frac{1}{2}d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

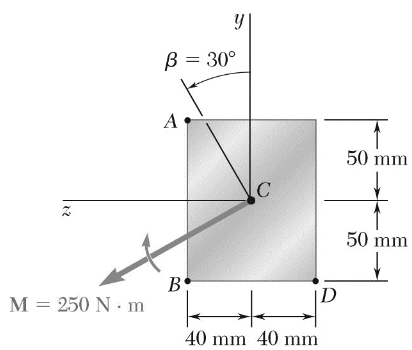
$$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12\left(\frac{1}{2}d - a\right)\left(\frac{1}{2}d\right)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$$

(a) Depth d for maximum σ_A: Differentiate with respect to d.

$$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a \qquad d = 75 \text{ mm} \blacktriangleleft$$

(b) $$\sigma_A = \frac{60\times10^3}{40\times10^{-3}}\left\{\frac{4}{75\times10^{-3}} - \frac{(6)(25\times10^{-3})}{(75\times10^{-3})^2}\right\} = 40\times10^6 \text{ Pa} \qquad \sigma_A = 40 \text{ MPa} \blacktriangleleft$$

PROBLEM 4.127



The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(80)(100)^3 = 6.6667\times 10^6\,\text{mm}^4 = 6.6667\times 10^{-6}\,\text{m}^4$$

$$I_y = \frac{1}{12}(100)(80)^3 = 4.2667\times 10^6\,\text{mm}^4 = 4.2667\times 10^{-6}\,\text{m}^4$$

$$y_A = -y_B = -y_D = 50\text{ mm}$$

$$z_A = z_B = -z_D = 40\text{ mm}$$

$$M_y = -250\sin 30° = -125\text{ N}\cdot\text{m}$$

$$M_z = 250\cos 30° = 216.51\text{ N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(0.050)}{6.6667\times 10^{-6}} + \frac{(-125)(0.040)}{4.2667\times 10^{-6}}$$

$$= -2.80\times 10^6\,\text{Pa}$$

$\sigma_A = -2.80$ MPa ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(-0.050)}{6.6667\times 10^{-6}} + \frac{(-125)(0.040)}{4.2667\times 10^{-6}}$$

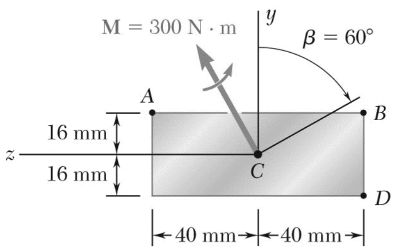
$$= 0.452\times 10^3\,\text{Pa}$$

$\sigma_B = 0.452$ MPa ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-0.050)}{6.6667\times 10^{-6}} + \frac{(-125)(-0.040)}{4.2667\times 10^{-6}}$$

$$= 2.80\times 10^6\,\text{Pa}$$

$\sigma_D = 2.80$ MPa ◀



PROBLEM 4.128

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(80)(32)^3 = 218.45\times 10^3 \text{ mm}^4 = 218.45\times 10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12}(32)(80)^3 = 1.36533\times 10^6 \text{ mm}^4 = 1.36533\times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 16 \text{ mm}$$

$$z_A = -z_B = -z_D = 40 \text{ mm}$$

$$M_y = 300\cos 30° = 259.81 \text{ N}\cdot\text{m} \qquad M_z = 300\sin 30° = 150 \text{ N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16\times 10^{-3})}{218.45\times 10^{-9}} + \frac{(259.81)(40\times 10^{-3})}{1.36533\times 10^{-6}}$$

$$= -3.37\times 10^6 \text{ Pa}$$

$\sigma_A = -3.37$ MPa ◄

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16\times 10^{-3})}{218.45\times 10^{-9}} + \frac{(259.81)(-40\times 10^{-3})}{1.36533\times 10^{-6}}$$

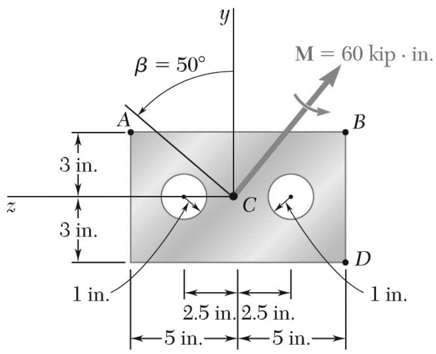
$$= -18.60\times 10^6 \text{ Pa}$$

$\sigma_B = -18.60$ MPa ◄

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16\times 10^{-3})}{218.45\times 10^{-9}} + \frac{(259.81)(-40\times 10^{-3})}{1.36533\times 10^{-6}}$$

$$= 3.37\times 10^6 \text{ Pa}$$

$\sigma_D = 3.37$ MPa ◄



PROBLEM 4.129

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$M_z = -60 \sin 40° = -38.567 \text{ kip} \cdot \text{in}$$

$$M_y = 60 \cos 40° = 45.963 \text{ kip} \cdot \text{in}$$

$$y_A = y_B = -y_D = 3 \text{ in.}$$

$$z_A = -z_B = -z_D = 5 \text{ in.}$$

$$I_z = \frac{1}{12}(10)(6)^3 - 2\left[\frac{\pi}{4}(1)^2\right] = 178.429 \text{ in}^4$$

$$I_y = \frac{1}{12}(6)(10)^3 - 2\left[\frac{\pi}{4}(1)^4 + \pi(1)^2(2.5)^2\right] = 459.16 \text{ in}^4$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-38.567)(3)}{178.429} + \frac{(45.963)(5)}{459.16}$$

$$= 1.149 \text{ ksi}$$

$\sigma_A = 1.149$ ksi ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-38.567)(3)}{178.429} + \frac{(45.963)(-5)}{459.16}$$

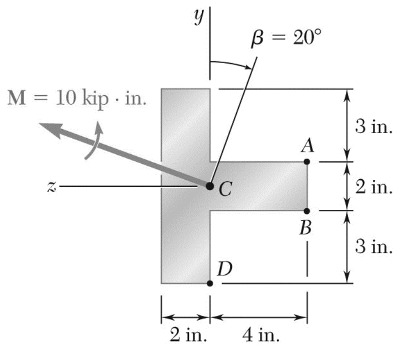
$$= 0.1479$$

$\sigma_B = 0.1479$ ksi ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-38.567)(-3)}{178.429} + \frac{(45.963)(-5)}{459.16}$$

$$= -1.149 \text{ ksi}$$

$\sigma_D = -1.149$ ksi ◀



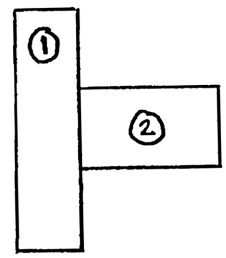
PROBLEM 4.130

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

Locate centroid.

	A, in^2	$\bar{z}$, in.	$A\bar{z}$, in^3
①	16	−1	−16
②	8	2	16
Σ	24		0



The centroid lies at point *C*.

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in.,} \qquad y_D = -4 \text{ in.}$$

$$z_A = z_B = -4 \text{ in.,} \qquad z_D = 0$$

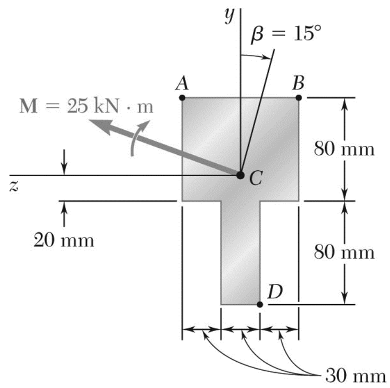
$$M_z = 10\cos 20° = 9.3969 \text{ kip}\cdot\text{in}$$

$$M_y = 10\sin 20° = 3.4202 \text{ kip}\cdot\text{in}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(9.3969)(1)}{88} + \frac{(3.4202)(-4)}{64}$$ $\sigma_A = 0.321$ ksi ◄

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(9.3969)(-1)}{88} + \frac{(3.4202)(-4)}{64}$$ $\sigma_B = -0.107$ ksi ◄

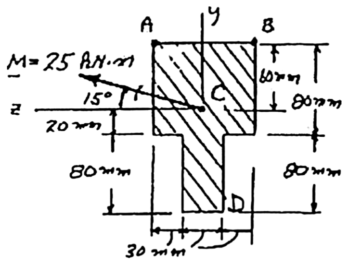
(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64}$$ $\sigma_D = 0.427$ ksi ◄



PROBLEM 4.131

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION



$$M_y = 25\sin 15° = 6.4705 \text{ kN}\cdot\text{m}$$

$$M_z = 25\cos 15° = 24.148 \text{ kN}\cdot\text{m}$$

$$I_y = \frac{1}{12}(80)(90)^3 + \frac{1}{12}(80)(30)^3 = 5.04\times10^6 \text{ mm}^4$$

$$I_y = 5.04\times10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{3}(90)(60)^3 + \frac{1}{3}(60)(20)^3 + \frac{1}{3}(30)(100)^3 = 16.64\times10^6 \text{mm}^4 = 16.64\times10^{-6} \text{m}^4$$

Stress: $\quad \sigma = \dfrac{M_y z}{I_y} - \dfrac{M_z y}{I_z}$

(*a*) $$\sigma_A = \frac{(6.4705 \text{ kN}\cdot\text{m})(0.045 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(0.060 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

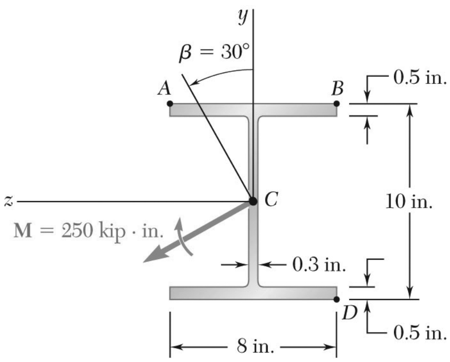
$$= 57.772 \text{ MPa} - 87.072 \text{ MPa} \qquad \sigma_A = -29.3 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = \frac{(6.4705 \text{ kN}\cdot\text{m})(-0.045 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(0.060 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

$$= -57.772 \text{ MPa} - 87.072 \text{ MPa} \qquad \sigma_B = -144.8 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma_D = \frac{(6.4705 \text{ kN}\cdot\text{m})(-0.015 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(-0.100 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

$$= -19.257 \text{ MPa} + 145.12 \text{ MPa} \qquad \sigma_D = -125.9 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.132

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

Flange:

$$I_z = \frac{1}{12}(8)(0.5)^3 + (8)(0.5)(4.75)^2$$
$$= 90.333 \text{ in}^4$$
$$I_y = \frac{1}{12}(0.5)(8)^3 = 21.333 \text{ in}^4$$

Web:

$$I_z = \frac{1}{12}(0.3)(9)^3 = 18.225 \text{ in}^4$$
$$I_y = \frac{1}{12}(9)(0.3)^3 = 0.02025 \text{ in}^4$$

Total:

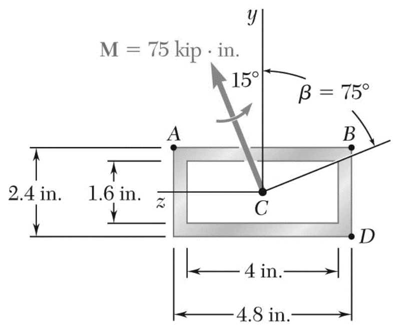
$$I_z = (2)(90.333) + 18.225 = 198.89 \text{ in}^4$$
$$I_y = (2)(21.333) + 0.02025 = 42.687 \text{ in}^4$$
$$y_A = y_B = -y_D = 5 \text{ in.}$$
$$z_A = -z_B = -z_C = 4 \text{ in.}$$
$$M_z = 250\cos 30° = 216.51 \text{ kip} \cdot \text{in}$$
$$M_y = -250\sin 30° = -125 \text{ kip} \cdot \text{in}$$

(*a*) $\sigma_A = -\dfrac{M_z y_A}{I_z} + \dfrac{M_y z_A}{I_y} = -\dfrac{(216.51)(5)}{198.89} + \dfrac{(-125)(4)}{42.687}$ $\qquad \sigma_A = -17.16 \text{ ksi}$ ◀

(*b*) $\sigma_B = -\dfrac{M_z y_B}{I_z} + \dfrac{M_y z_B}{I_y} = -\dfrac{(216.51)(5)}{198.89} + \dfrac{(-125)(-4)}{42.687}$ $\qquad \sigma_B = 6.27 \text{ ksi}$ ◀

(*c*) $\sigma_D = -\dfrac{M_z y_D}{I_z} + \dfrac{M_y z_D}{I_y} = -\dfrac{(216.51)(-5)}{198.89} + \dfrac{(-125)(-4)}{42.687}$ $\qquad \sigma_D = 17.16 \text{ ksi}$ ◀

PROBLEM 4.133



The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(4.8)(2.4)^3 - \frac{1}{12}(4)(1.6)^3 = 4.1643 \text{ in}^4$$

$$I_y = \frac{1}{12}(2.4)(4.8)^3 - \frac{1}{12}(1.6)(4)^3 = 13.5851 \text{ in}^4$$

$$y_A = y_B = -y_D = 1.2 \text{ in.}$$

$$z_A = -z_B = -z_D = 2.4 \text{ in.}$$

$$M_z = 75\sin 15° = 19.4114 \text{ kip} \cdot \text{in}$$

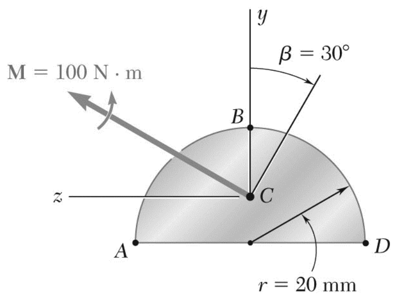
$$M_y = 75\cos 15° = 72.444 \text{ kip} \cdot \text{in}$$

(*a*) $\sigma_A = -\dfrac{M_z y_A}{I_z} + \dfrac{M_y z_A}{I_y} = -\dfrac{(19.4114)(1.2)}{4.1643} + \dfrac{(72.444)(2.4)}{13.5851}$ $\sigma_A = 7.20 \text{ ksi}$ ◀

(*b*) $\sigma_B = -\dfrac{M_z y_B}{I_z} + \dfrac{M_y z_B}{I_y} = -\dfrac{(19.4114)(1.2)}{4.1643} + \dfrac{(72.444)(-2.4)}{13.5851}$ $\sigma_B = -18.39 \text{ ksi}$ ◀

(*c*) $\sigma_D = -\dfrac{M_z y_D}{I_z} + \dfrac{M_y z_D}{I_y} = -\dfrac{(19.4114)(-1.2)}{4.1643} + \dfrac{(72.444)(-2.4)}{13.5851}$ $\sigma_D = -7.20 \text{ ksi}$ ◀

PROBLEM 4.134



The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(20)^4 = 17.5611\times 10^{-3}\ \text{mm}^4 = 17.5611\times 10^{-9}\ \text{m}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi(20)^4}{8} = 62.832\times 10^{-3}\ \text{mm}^4 = 62.832\times 10^{-9}\ \text{m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883\ \text{mm}$$

$$y_B = 20 - 8.4883 = 11.5117\ \text{mm}$$

$$z_A = -z_D = 20\ \text{mm} \qquad z_B = 0$$

$$M_z = 100\cos 30^\circ = 86.603\ \text{N}\cdot\text{m}$$

$$M_y = 100\sin 30^\circ = 50\ \text{N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883\times 10^{-3})}{17.5611\times 10^{-9}} + \frac{(50)(20\times 10^{-3})}{62.832\times 10^{-9}}$$

$$= 57.8\times 10^6\ \text{Pa}$$

$\sigma_A = 57.8$ MPa ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117\times 10^{-3})}{17.5611\times 10^{-9}} + \frac{(50)(0)}{62.832\times 10^{-9}}$$

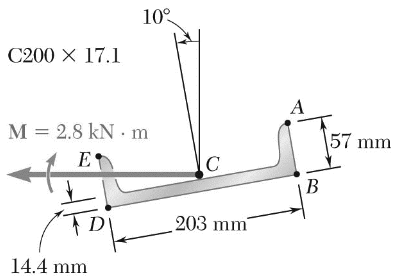
$$= -56.8\times 10^6\ \text{Pa}$$

$\sigma_B = -56.8$ MPa ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883\times 10^{-3})}{17.5611\times 10^{-9}} + \frac{(50)(-20\times 10^{3})}{62.832\times 10^{-9}}$$

$$= 25.9\times 10^6\ \text{Pa}$$

σ_D=25.9 MPa ◀



PROBLEM 4.135

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

For $C200\times17.1$ rolled steel shape, $I_z = 0.538\times10^6\,\text{mm}^4 = 0.538\times10^{-6}\,\text{m}^4$

$$I_y = 13.4\times10^6\,\text{mm}^4 = 13.4\times10^{-6}\,\text{m}^4$$

$$z_E = z_D = -z_A = -z_B = \frac{1}{2}(203) = 101.5 \text{ mm}$$

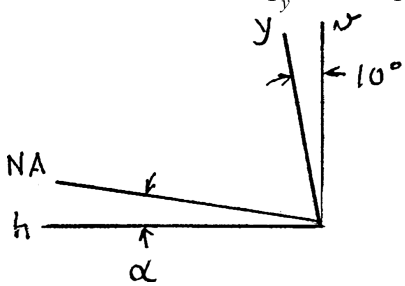
$$y_D = y_B = -14.4 \text{ mm} \qquad y_E = y_A = 57 - 14.4 = 42.6 \text{ mm}$$

$$M_z = (2.8\times10^3)\cos 10° = 2.7575\times10^3 \text{ N}\cdot\text{m}$$

$$M_y = (2.8\times10^3)\sin 10° = 486.21 \text{ N}\cdot\text{m}$$

(*a*) Angle of neutral axis.

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{0.538}{13.4}\tan 10° = 0.007079 \qquad \varphi = 0.4056°$$

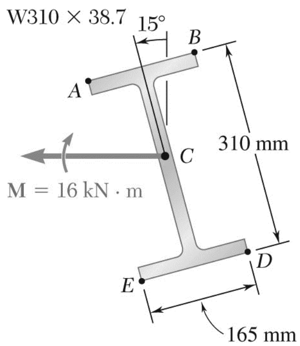


$$\alpha = 10° - 0.4056° \qquad\qquad \alpha = 9.59° \blacktriangleleft$$

(*b*) Maximum tensile stress occurs at point *D*.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(2.7575\times10^3)(-14.4\times10^{-3})}{0.538\times10^{-6}} + \frac{(486.21)(0.1015)}{13.4\times10^{-6}}$$

$$= 73.807\times10^6 + 3.682\times10^6 = 77.5\times10^6 \text{ Pa} \qquad\qquad \sigma_D = 77.5 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.136

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

For $W310\times38.7$ rolled steel shape,

$$I_z = 85.1\times10^6\,\text{mm}^4 = 85.1\times10^{-6}\,\text{m}^4$$

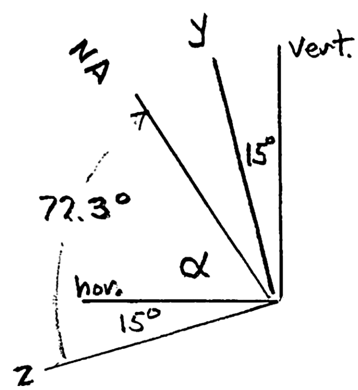
$$I_y = 7.27\times10^6\,\text{mm}^4 = 7.27\times10^{-6}\,\text{m}^4$$

$$y_A = y_B = -y_D = -y_E = \left(\frac{1}{2}\right)(310) = 155\text{ mm}$$

$$z_A = z_E = -z_B = -z_D = \left(\frac{1}{2}\right)(165) = 82.5\text{ mm}$$

$$M_z = (16\times10^3)\cos15° = 15.455\times10^3\,\text{N}\cdot\text{m}$$

$$M_y = (16\times10^3)\sin15° = 4.1411\times10^3\,\text{N}\cdot\text{m}$$



(*a*) Angle of neutral axis.

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{85.1\times10^{-6}}{7.27\times10^{-6}}\tan15° = 3.1365$$

$$\varphi = 72.3°$$

$$\alpha = 72.3° - 15°$$

$\alpha = 57.3°$ ◀

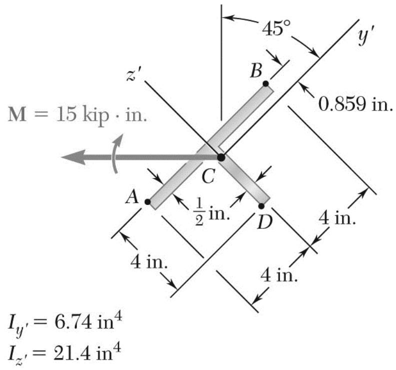
(*b*) Maximum tensile stress occurs at point *E*.

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y}$$

$$= -\frac{(15.455\times10^3)(-155\times10^{-3})}{85.1\times10^{-6}} + \frac{(4.1411\times10^3)(82.5\times10^{-3})}{7.27\times10^{-6}}$$

$$= 75.1\times10^6\,\text{Pa}$$

$\sigma_E = 75.1\text{ MPa}$ ◀



PROBLEM 4.137

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 21.4 \text{ in}^4 \qquad I_{y'} = 6.74 \text{ in}^4$$

$$z'_A = z'_B = 0.859 \text{ in.} \qquad z'_D = -4 + 0.859 \text{ in.} = -3.141 \text{ in.}$$

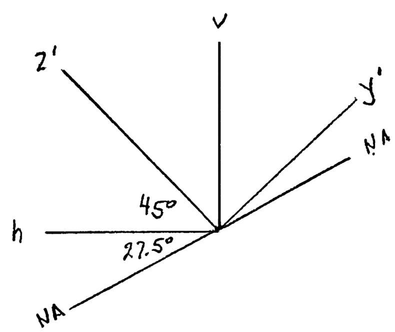
$$y_A = -4 \text{ in.} \qquad y_{B'} = 4 \text{ in.} \qquad y_{D'} = -0.25 \text{ in.}$$

$$M_{y'} = -15 \sin 45° = -10.6066 \text{ kip} \cdot \text{in}$$

$$M_{z'} = 15 \cos 45° = 10.6066 \text{ kip} \cdot \text{in}$$

(*a*) Angle of neutral axis.

$$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{21.4}{6.74} \tan(-45°) = 3.1751$$



$$\varphi = -72.5°$$

$$\alpha = 72.5° - 45°$$

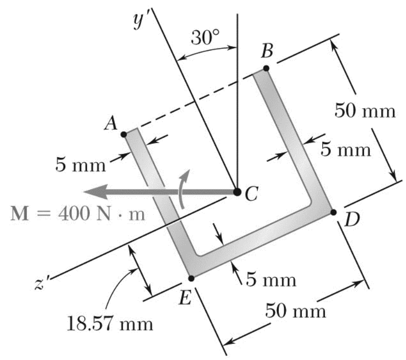
$\alpha = 27.5°$ ◀

(*b*) The maximum tensile stress occurs at point *D*.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(10.6066)(-0.25)}{21.4} + \frac{(-10.6066)(-3.141)}{6.74}$$

$$= 0.12391 + 4.9429$$

$\sigma_D = 5.07$ ksi ◀



$I_{y'} = 281 \times 10^3\ \text{mm}^4$
$I_{z'} = 176.9 \times 10^3\ \text{mm}^4$

PROBLEM 4.138

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 176.9 \times 10^3\ \text{mm}^4 = 176.9 \times 10^{-9}\ \text{m}^4$$
$$I_{y'} = 281 \times 10^3\ \text{mm}^4 = 281 \times 10^{-9}\ \text{m}^4$$
$$y'_E = -18.57\ \text{mm}, \quad z_E = 25\ \text{mm}$$
$$M_{z'} = 400 \cos 30° = 346.41\ \text{N} \cdot \text{m}$$
$$M_{y'} = 400 \sin 30° = 200\ \text{N} \cdot \text{m}$$

(*a*) $$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30° = 0.36346$$

$$\varphi = 19.97°$$

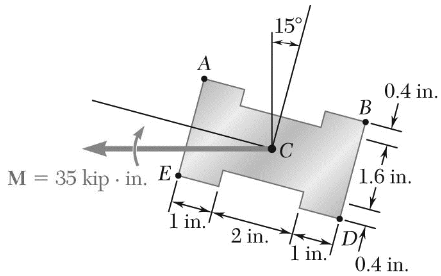
$$\alpha = 30° - 19.97°$$

$\alpha = 10.03°$ ◀

(*b*) Maximum tensile stress occurs at point *E*.

$$\sigma_E = -\frac{M_{z'} y'_E}{I_{z'}} + \frac{M_{y'} z'_E}{I_{y'}} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$
$$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6\ \text{Pa}$$

$\sigma_E = 54.2\ \text{MPa}$ ◀

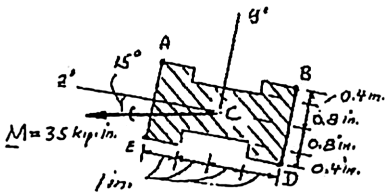


PROBLEM 4.139

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

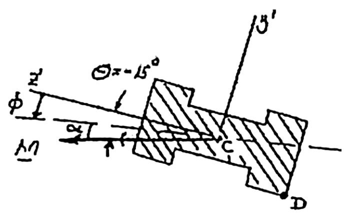
SOLUTION

Bending moments:



$$M_{y'} = -35\sin 15° = -9.059 \text{ kip} \cdot \text{in}$$
$$M_{z'} = -35\cos 15° = 33.807 \text{ kip} \cdot \text{in}$$

Moments of inertia:



$$I_{y'} = \frac{1}{12}(2.4)(4)^3 - \frac{1}{12}(2 \times 0.4)(2)^3 = 12.267 \text{ in}^4$$
$$I_{z'} = \frac{1}{12}(2)(2.4)^3 + \frac{1}{12}(2)(1.6)^3 = 2.987 \text{ in}^4$$

(*a*) Neutral axis:

$$\tan\phi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{2.987 \text{ in}^4}{12.267 \text{ in}^4}\tan(-15°) = -0.06525$$

$\phi = 3.73°$

$\alpha = 15° - \phi = 15° - 3.73° = 11.27°$

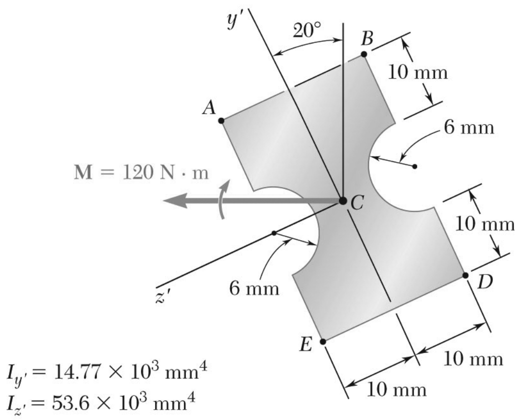
$\alpha = 11.3°$ ◀

(*b*) Maximum tensile stress at *D*:

$y'_D = -1.2$ in. $\quad z'_D = -2$ in.

$$\sigma_D = \frac{M_{y'}z_D}{I_{y'}} - \frac{M_{z'}y'_D}{I_{z'}} = \frac{(-9.059 \text{ kip} \cdot \text{in})(-2 \text{ in.})}{12.267 \text{ in}^4} - \frac{(33.807 \text{ kip} \cdot \text{in})(-1.2 \text{ in.})}{2.987 \text{ in}^4}$$
$$= 1.477 \text{ ksi} + 13.582 \text{ ksi} = 15.059 \text{ ksi}$$

$\sigma_D = 15.06$ ksi ◀



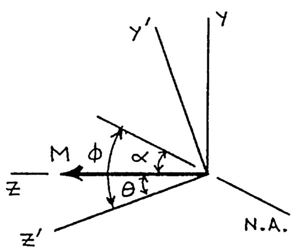
PROBLEM 4.140

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 53.6\times10^3\,\text{mm}^4 = 53.6\times10^{-9}\,\text{m}^4$$
$$I_{y'} = 14.77\times10^3\,\text{mm}^4 = 14.77\times10^{-9}\,\text{m}^4$$
$$M_{z'} = 120\sin 70° = 112.763\ \text{N}\cdot\text{m}$$
$$M_{y'} = 120\cos 70° = 41.042\ \text{N}\cdot\text{m}$$

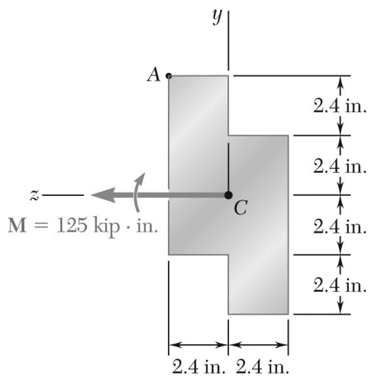
(*a*) Angle of neutral axis. $\theta = 20°$.



$$\tan\varphi = \frac{I_{z'}}{I_{y'}} \quad \tan\theta = \frac{53.6\times10^{-9}}{14.77\times10^{-9}}\tan 20° = 1.32084$$
$$\varphi = 52.871°$$
$$\alpha = 52.871° - 20° \qquad \alpha = 32.9° \blacktriangleleft$$

(*b*) The maximum tensile stress occurs at point *E*.

$$y'_E = -16\ \text{mm} = -0.016\ \text{m}$$
$$z_E = 10\ \text{mm} = 0.010\ \text{m}$$
$$\sigma_E = -\frac{M_{z'}y'_E}{I_{z'}} + \frac{M_{y'}z'_E}{I_{y'}}$$
$$= -\frac{(112.763)(-0.016)}{53.6\times10^{-9}} + \frac{(41.042)(0.010)}{14.77\times10^{-9}}$$
$$= 61.448\times10^6\ \text{Pa} \qquad \sigma_E = 61.4\ \text{MPa} \blacktriangleleft$$



PROBLEM 4.141

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.

SOLUTION

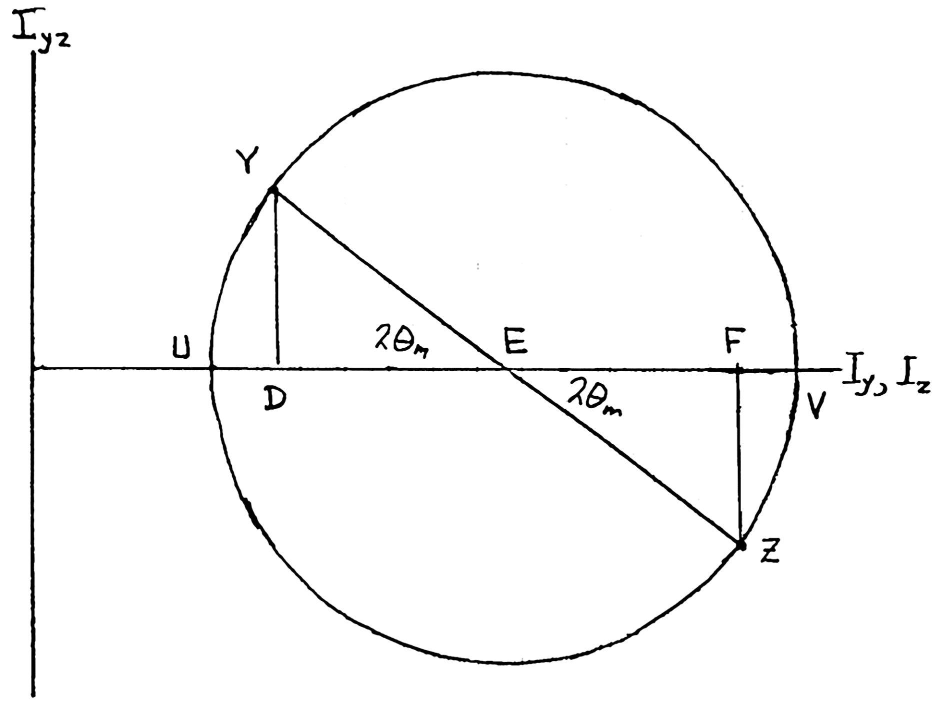
$$I_y = 2\left\{\frac{1}{3}(7.2)(2.4)^3\right\} = 66.355 \text{ in}^4$$

$$I_z = 2\left\{\frac{1}{12}(2.4)(7.2)^3 + (2.4)(7.2)(1.2)^2\right\} = 199.066 \text{ in}^4$$

$$I_{yz} = 2\{(2.4)(7.2)(1.2)(1.2)\} = 49.766 \text{ in}^4$$

Using Mohr's circle determine the principal axes and principal moments of inertia.

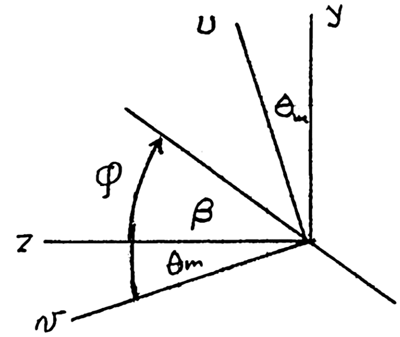
$$Y: (66.355 \text{ in}^4,\ 49.766 \text{ in}^4)$$
$$Z: (199.066 \text{ in}^4,\ -49.766 \text{ in}^4)$$
$$E: (132.710 \text{ in}^4,\ 0)$$



PROBLEM 4.141 *(Continued)*

$$\tan 2\theta_m = \frac{DY}{DE} = \frac{49.766}{66.355}$$

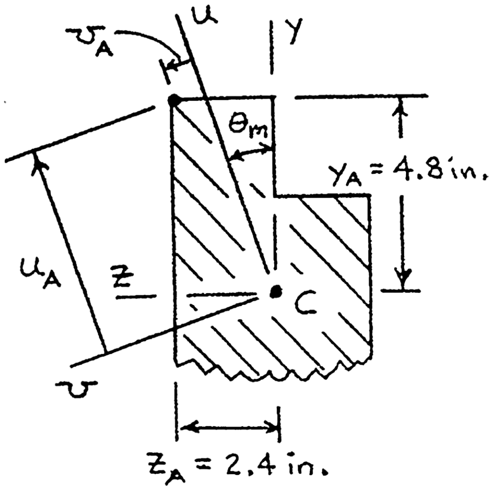
$$2\theta_m = 36.87° \quad \theta_m = 18.435°$$



$$R = \sqrt{\overline{DE}^2 + \overline{DY}^2} = 82.944 \text{ in}^4$$

$$I_u = 132.710 - 82.944 = 49.766 \text{ in}^4$$

$$I_v = 132.710 + 82.944 = 215.654 \text{ in}^4$$



$$M_u = 125 \sin 18.435° = 39.529 \text{ kip} \cdot \text{in}$$

$$M_v = 125 \cos 18.435° = 118.585 \text{ kip} \cdot \text{in}$$

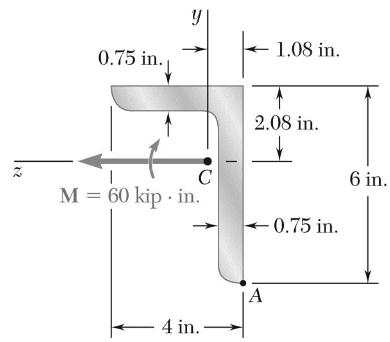
$$u_A = 4.8 \cos 18.435° + 2.4 \sin 18.435° = 5.3126 \text{ in.}$$

$$v_A = -4.8 \sin 18.435° + 2.4 \cos 18.435° = 0.7589 \text{ in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u}$$

$$= -\frac{(118.585)(5.3126)}{215.654} + \frac{(39.529)(0.7589)}{49.766}$$

$$= -2.32 \text{ ksi} \qquad \sigma_A = -2.32 \text{ ksi} \blacktriangleleft$$



$I_y = 8.7\ \text{in}^4$
$I_z = 24.5\ \text{in}^4$
$I_{yz} = +8.3\ \text{in}^4$

PROBLEM 4.142

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.

SOLUTION

Using Mohr's circle determine the principal axes and principal moments of inertia.

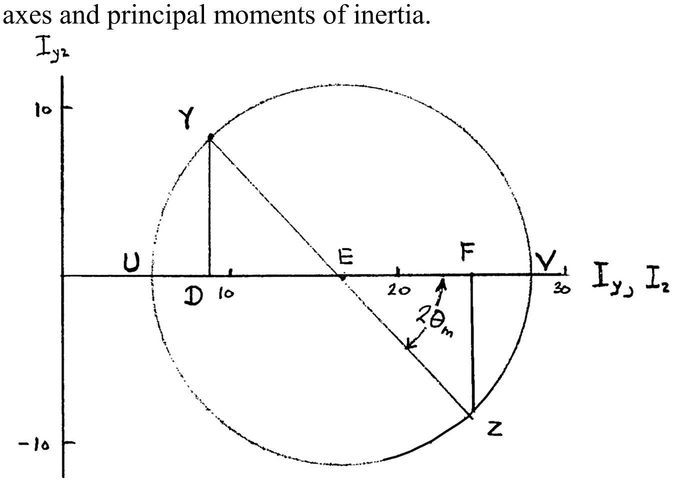
$Y: (8.7,\ 8.3)\ \text{in}^4$

$Z: (24.5,\ -8.3)\ \text{in}^4$

$E: (16.6,\ 0)\ \text{in}^4$

$EF = 7.9\ \text{in}^4$

$FZ = 8.3\ \text{in}^4$



$$R = \sqrt{7.9^2 + 8.3^2} = 11.46\ \text{in}^4 \qquad \tan 2\theta_m = \frac{FZ}{EF} = \frac{8.3}{7.9} = 1.0506$$

$$\theta_m = 23.2° \quad I_u = 16.6 - 11.46 = 5.14\ \text{in}^4 \quad I_v = 16.6 + 11.46 = 28.06\ \text{in}^4$$

$$M_u = M \sin\theta_m = (60)\sin 23.2° = 23.64\ \text{kip}\cdot\text{in}$$

$$M_v = M \cos\theta_m = (60)\cos 23.2° = 55.15\ \text{kip}\cdot\text{in}$$

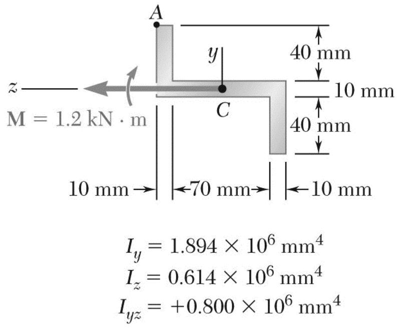
$$u_A = y_A \cos\theta_m + z_A \sin\theta_m = -3.92\cos 23.2° - 1.08\sin 23.2° = -4.03\ \text{in.}$$

$$v_A = z_A \cos\theta_m - y_A \sin\theta_m = -1.08\cos 23.2° + 3.92\sin 23.2° = 0.552\ \text{in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$$

$\sigma_A = 10.46\ \text{ksi}$ ◀

PROBLEM 4.143



The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.

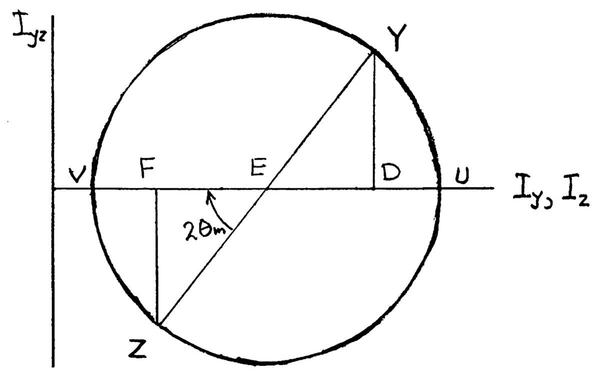
SOLUTION

Using Mohr's circle determine the principal axes and principal moments of inertia.

$Y: (1.894, 0.800) \times 10^6 \text{ mm}^4$

$Z: (0.614, 0.800) \times 10^6 \text{ mm}^4$

$E: (1.254, 0) \times 10^6 \text{ mm}^4$



$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6 \text{ mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6 \text{ mm}^4 = 0.2295 \times 10^6 \text{ mm}^4 = 0.2295 \times 10^{-6} \text{ m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6 \text{ mm}^4 = 2.2785 \times 10^6 \text{ mm}^4 = 2.2785 \times 10^{-6} \text{ m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \qquad \theta_m = 25.67^\circ$$

$$M_v = M \cos \theta_m = (1.2 \times 10^3) \cos 25.67^\circ = 1.0816 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_u = -M \sin \theta_m = -(1.2 \times 10^3) \sin 25.67^\circ = -0.5198 \times 10^3 \text{ N} \cdot \text{m}$$

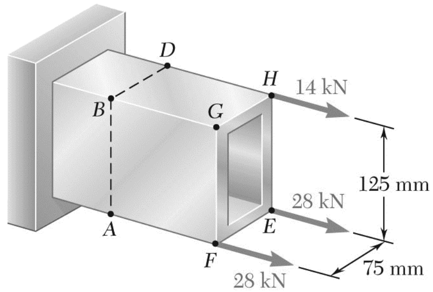
$$u_A = y_A \cos \theta_m - z_A \sin \theta_m = 45 \cos 25.67^\circ - 45 \sin 25.67^\circ = 21.07 \text{ mm}$$

$$v_A = z_A \cos \theta_m + y_A \sin \theta_m = 45 \cos 25.67^\circ + 45 \sin 25.67^\circ = 60.05 \text{ mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6 \text{ Pa}$$

$\sigma_A = 113.0 \text{ MPa}$ ◀

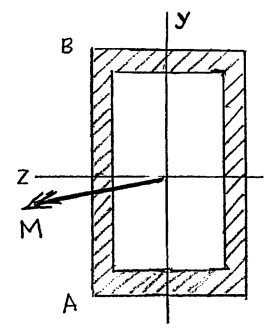


PROBLEM 4.144

The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

SOLUTION

Add *y*- and *z*-axes as shown. Cross section is a 75 mm × 125 mm rectangle with a 51 mm × 101 mm rectangular cutout.



$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283\times10^6\,\text{mm}^4 = 7.8283\times10^{-6}\,\text{m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781\times10^3\,\text{mm}^4 = 3.2781\times10^{-6}\,\text{m}^4$$

$$A = (75)(125) - (51)(101) = 4.224\times10^3\,\text{mm}^2 = 4.224\times10^{-3}\,\text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 + 28 = 70\text{ kN} = 70\times10^3\text{ N}$$

$$M_z = -(62.5\text{ mm})(14\text{ kN}) + (62.5\text{ mm})(28\text{kN}) + (62.5\text{ mm})(28\text{ kN}) = 2625\text{ N}\cdot\text{m}$$

$$M_y = -(37.5\text{ mm})(14\text{ kN}) + (37.5\text{ mm})(28\text{ kN}) + (37.5\text{ mm})(28\text{ kN}) = -525\text{ N}\cdot\text{m}$$

(*a*)
$$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{70\times10^3}{4.224\times10^{-3}} - \frac{(2625)(-0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 31.524\times10^6\,\text{Pa}$$

$\sigma_A = 31.5$ MPa ◀

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{70\times10^3}{4.224\times10^{-3}} - \frac{(2625)(0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= -10.39\times10^6\,\text{Pa}$$

$\sigma_B = -10.39$ MPa ◀

(*b*) Let point *H* be the point where the neutral axis intersects *AB*.

$$z_H = 0.0375\text{ m},\quad y_H = ?,\quad \sigma_H = 0$$

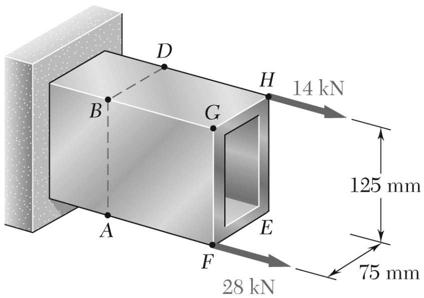
$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{Mz_H}{I_y}\right) = \frac{7.8283\times10^{-6}}{2625}\left[\frac{70\times10^3}{4.224\times10^{-3}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}\right]$$

$$= 0.03151\text{ m} = 31.51\text{ mm}$$

$$31.51 + 62.5 = 94.0\text{ mm}$$

Answer: 94.0 mm above point *A*. ◀



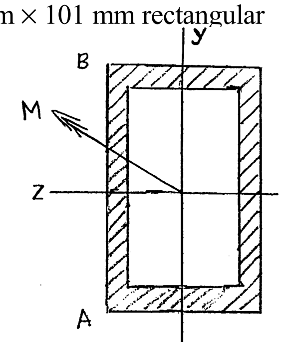
PROBLEM 4.145

Solve Prob. 4.144, assuming that the 28-kN force at point E is removed.

PROBLEM 4.144 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points A and B, (*b*) the point where the neutral axis intersects line ABD.

SOLUTION

Add y- and z-axes as shown. Cross section is a 75 mm × 125 mm rectangle with a 51 mm × 101 mm rectangular cutout.



$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283\times10^6\,\text{mm}^4 = 7.8283\times10^{-6}\,\text{m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781\times10^6\,\text{mm}^4 = 3.2781\times10^{-6}\,\text{m}^4$$

$$A = (75)(125) - (51)(101) = 4.224\times10^3\,\text{mm}^2 = 4.224\times10^{-3}\,\text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 = 42\text{ kN} = 42\times10^3\text{ N}$$

$$M_z = -(62.5\text{ mm})(14\text{ kN}) + (62.5\text{ mm})(28\text{ kN}) = 875\text{ N}\cdot\text{m}$$

$$M_y = -(37.5\text{ mm})(14\text{ kN}) + (37.5\text{ mm})(28\text{ kN}) = 525\text{ N}\cdot\text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{42\times10^3}{4.224\times10^{-3}} - \frac{(875)(-0.0625)}{7.8283\times10^{-6}} + \frac{(525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 22.935\times10^6\,\text{Pa}$$

$\sigma_A = 22.9$ MPa ◀

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{42\times10^3}{4.224\times10^{-3}} - \frac{(875)(0.0625)}{7.8283\times10^{-6}} + \frac{(525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 8.9631\times10^6\,\text{Pa}$$

$\sigma_B = 8.96$ MPa ◀

(*b*) Let point K be the point where the neutral axis intersects BD.

$$z_K = ?,\quad y_K = 0.0625\text{ m},\quad \sigma_H = 0$$

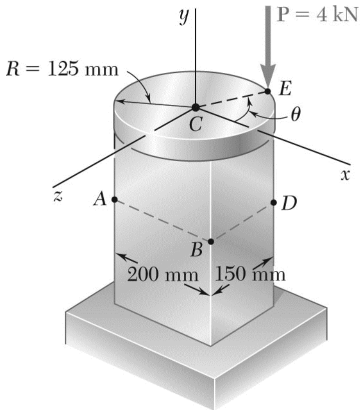
$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$z_H = \frac{I_y}{M_y}\left(\frac{M_z y_H}{I_z} - \frac{P}{A}\right) = \frac{3.2781\times10^{-6}}{525}\left[\frac{(875)(0.0625)}{7.8283\times10^{-6}} - \frac{42\times10^3}{4.224\times10^{-3}}\right]$$

$$= -0.018465\text{ m} = -18.465\text{ mm}$$

$$37.5 + 18.465 = 56.0\text{ mm}$$

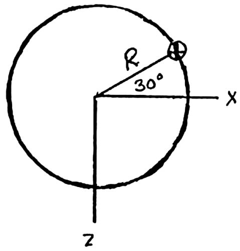
Answer: 56.0 mm to the right of point B. ◀



PROBLEM 4.146

A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with $\theta = 30°$, determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the point where the neutral axis intersects line ABD.

SOLUTION

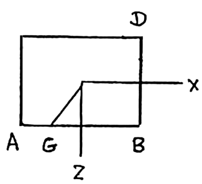


$$P = 4\times 10^3 \text{ N (compression)}$$

$$M_x = -PR\sin 30° = -(4\times 10^3)(125\times 10^{-3})\sin 30° = -250 \text{ N}\cdot\text{m}$$

$$M_z = -PR\cos 30° = -(4\times 10^3)(125\times 10^{-3})\cos 30° = -433 \text{ N}\cdot\text{m}$$

$$I_x = \frac{1}{12}(200)(150)^3 = 56.25\times 10^6 \text{ mm}^4 = 56.25\times 10^{-6} \text{ m}^4$$



$$I_z = \frac{1}{12}(150)(200)^3 = 100\times 10^6 \text{ mm}^4 = 100\times 10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm} \qquad z_A = z_B = 75 \text{ mm}$$

$$A = (200)(150) = 30\times 10^3 \text{ mm}^2 = 30\times 10^{-3} \text{ m}^2$$

(*a*) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4\times 10^3}{30\times 10^{-3}} - \frac{(-250)(75\times 10^{-3})}{56.25\times 10^{-6}} + \frac{(-433)(-100\times 10^{-3})}{100\times 10^{-6}}$$

$$\sigma_A = 633\times 10^3 \text{ Pa} = 633 \text{ kPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4\times 10^3}{30\times 10^{-3}} - \frac{(-250)(75\times 10^{-3})}{56.25\times 10^{-6}} + \frac{(-433)(100\times 10^{-3})}{100\times 10^{-6}}$$

$$\sigma_B = -233\times 10^3 \text{ Pa} = -233 \text{ kPa} \blacktriangleleft$$

(*c*) Let G be the point on AB where the neutral axis intersects.

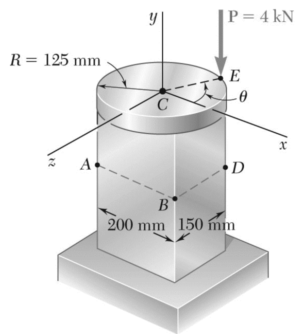
$$\sigma_G = 0 \qquad z_G = 75 \text{ mm} \qquad x_G = ?$$

$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z}\left\{\frac{P}{A} + \frac{M_x Z_G}{I_x}\right\} = \frac{100\times 10^{-6}}{-433}\left\{\frac{4\times 10^3}{30\times 10^{-3}} + \frac{(-250)(75\times 10^{-3})}{56.25\times 10^{-6}}\right\}$$

$$= 46.2\times 10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 146.2 mm from point A ◀



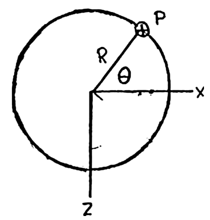
PROBLEM 4.147

4.147 In Prob. 4.146, determine (*a*) the value of θ for which the stress at D reaches it largest value, (*b*) the corresponding values of the stress, at A, B, C, and D.

4.146 A rigid circular plate of 125-mm radius is attached to a solid 150×200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with $\theta = 30°$, determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the point where the neutral axis intersects line ABD.

SOLUTION

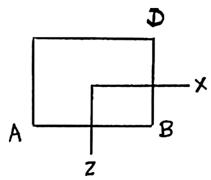
(*a*)



$$P = 4 \times 10^3\ \text{N} \qquad PR = (4 \times 10^3)(125 \times 10^{-3}) = 500\ \text{N} \cdot \text{m}$$

$$M_x = -PR\sin\theta = -500\sin\theta \qquad M_x = -PR\cos\theta = -500\cos\theta$$

$$I_x = \frac{1}{2}(200)(150)^3 = 56.25 \times 10^6\text{mm}^4 = 56.25 \times 10^{-6}\text{m}^4$$



$$I_z = \frac{1}{2}(150)(200)^3 = 100 \times 10^6\text{mm}^4 = 100 \times 10^{-6}\text{m}^4$$

$$x_D = 100\ \text{mm} \qquad z_D = -75\ \text{mm}$$

$$A = (200)(150) = 30 \times 10^3\ \text{mm}^2 = 30 \times 10^{-3}\ \text{m}^2$$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P\left\{\frac{1}{A} - \frac{Rz\sin\theta}{I_x} + \frac{Rx\cos\theta}{I_z}\right\}$$

For σ to be a maximum, $\quad \dfrac{d\sigma}{d\theta} = 0 \quad$ with $z = z_D,\ \ x = x_D$

$$\frac{d\sigma_D}{d\theta} = -P\left\{0 + \frac{Rz_D\cos\theta}{I_x} + \frac{Rx_D\sin\theta}{I_Z}\right\} = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100 \times 10^{-6})(-75 \times 10^{-3})}{(56.25 \times 10^{-6})(100 \times 10^{-3})} = \frac{4}{3}$$

$$\sin\theta = 0.8, \qquad \cos\theta = 0.6, \qquad\qquad \theta = 53.1° \ \blacktriangleleft$$

(*b*)

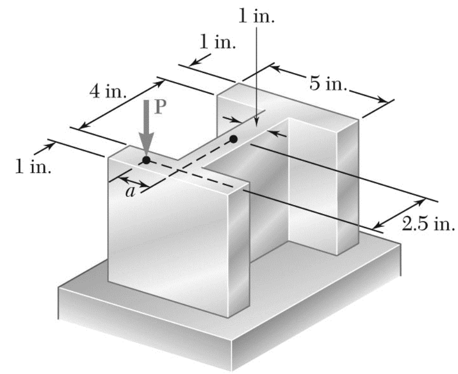
$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(500)(0.8)(75 \times 10^{-3})}{56.25 \times 10^{-6}} - \frac{(500)(0.6)(-100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$= (-0.13333 + 0.53333 + 0.300) \times 10^6\ \text{Pa} = 0.700 \times 10^6\ \text{Pa} \qquad \sigma_A = 700\,\text{kPa} \ \blacktriangleleft$$

$$\sigma_B = (-0.13333 + 0.53333 - 0.300) \times 10^6\ \text{Pa} = 0.100 \times 10^6\ \text{Pa} \qquad \sigma_B = 100\,\text{kPa} \ \blacktriangleleft$$

$$\sigma_C = (-0.13333 + 0 + 0) \times 10^6\ \text{Pa} \qquad \sigma_C = -133.3\,\text{kPa} \ \blacktriangleleft$$

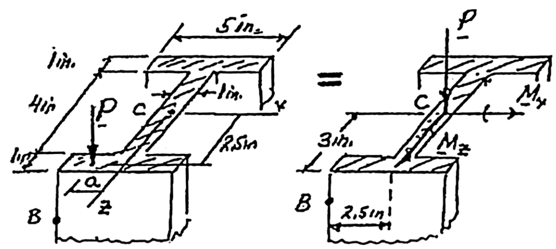
$$\sigma_D = (-0.13333 - 0.53333 - 0.300) \times 10^6\ \text{Pa} = -0.967 \times 10^6\ \text{Pa} \qquad \sigma_D = -967\,\text{kPa} \ \blacktriangleleft$$



PROBLEM 4.148

Knowing that $P = 90$ kips, determine the largest distance a for which the maximum compressive stress dose not exceed 18 ksi.

SOLUTION



$$A = (5\text{ in.})(6\text{ in.}) - 2(2\text{ in.})(4\text{ in.}) = 14\text{ in}^2$$

$$I_x = \frac{1}{12}(5\text{ in.})(6\text{ in.})^3 - 2\frac{1}{12}(2\text{ in.})(4\text{ in.})^3 = 68.67\text{ in}^4$$

$$I_z = 2\frac{1}{12}(1\text{ in.})(5\text{ in.})^3 + \frac{1}{12}(4\text{ in.})(1\text{ in.})^3 = 21.17\text{ in}^4$$

Force-couple system at C: $\quad P = P \quad M_x = P(2.5\text{ in.}) \quad M_z = Pa$

For $P = 90$ kips: $\quad P = 90\text{ kips} \quad M_x = (90\text{ kips})(2.5\text{ in.}) = 225\text{ kip}\cdot\text{in} \quad M_z = (90\text{ kips})a$

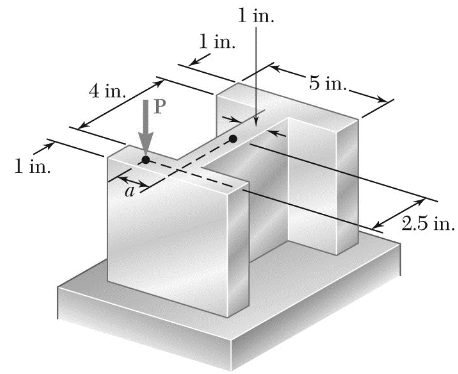
Maximum compressive stress at B: $\quad \sigma_B = -18\text{ ksi}$

$$\sigma_B = -\frac{P}{A} - \frac{M_x(3\text{ in.})}{I_x} - \frac{M_z(2.5\text{ in.})}{I_z}$$

$$-18\text{ ksi} = -\frac{90\text{ kips}}{14\text{ in}^2} - \frac{(225\text{ kip}\cdot\text{in})(3\text{ in.})}{68.67\text{ in}^4} - \frac{(90\text{ kips})a(2.5\text{ in.})}{21.17\text{ in}^4}$$

$$-18 = -6.429 - 9.830 - 10.628a$$

$$-1.741 = -10.628\,a \qquad a = 0.1638\text{ in.} \blacktriangleleft$$



PROBLEM 4.149

Knowing that $a = 1.25$ in., determine the largest value of **P** that can be applied without exceeding either of the following allowable stresses:

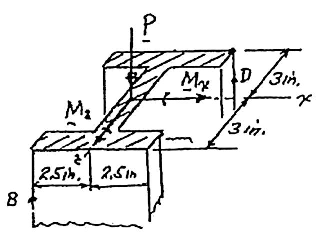
$$\sigma_{\text{ten}} = 10 \text{ ksi} \qquad \sigma_{\text{comp}} = 18 \text{ ksi}$$

SOLUTION

$$A = (5 \text{ in.})(6 \text{ in.}) - (2)(2 \text{ in.})(4 \text{ in.}) = 14 \text{ in}^2$$

$$I_x = \frac{1}{12}(5 \text{ in.})(6 \text{ in.})^3 - 2\frac{1}{12}(2 \text{ in.})(4 \text{ in.})^3 = 68.67 \text{ in}^4$$

$$I_z = 2\frac{1}{12}(1 \text{ in.})(5 \text{ in.})^3 + \frac{1}{12}(4 \text{ in.})(1 \text{ in.})^3 = 21.17 \text{ in}^4$$



Force-couple system at C: For $a = 1.25$ in.,

$$P = P \quad M_x = P(2.5 \text{ in.})$$

$$M_y = Pa = (1.25 \text{ in.})$$

Maximum compressive stress at B: $\sigma_B = -18$ ksi

$$\sigma_B = -\frac{P}{A} - \frac{M_x(3 \text{ in.})}{I_x} - \frac{M_z(2.5 \text{ in.})}{I_z}$$

$$-18 \text{ ksi} = -\frac{P}{14 \text{ in}^2} - \frac{P(2.5 \text{ in.})(3 \text{ in.})}{68.67 \text{ in}^4} - \frac{P(1.25 \text{ in.})(2.5 \text{ in.})}{21.17 \text{ in}^4}$$

$$-18 = -0.0714P - 0.1092P - 0.1476P$$

$$-18 = 0.3282P \qquad P = 54.8 \text{ kips}$$

Maximum tensile stress at D: $\sigma_D = +10$ ksi

$$\sigma_D = -\frac{P}{A} + \frac{M_x(3 \text{ in.})}{I_x} + \frac{M_z(2.5 \text{ in.})}{I_z}$$

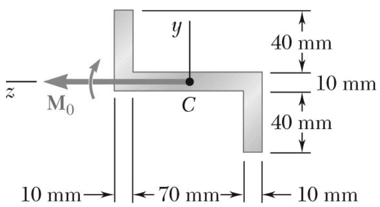
$$+10 \text{ ksi} = -0.0714P + 0.1092P + 0.1476P$$

$$10 = 0.1854P \qquad P = 53.9 \text{ kips}$$

The smaller value of **P** is the largest allowable value.

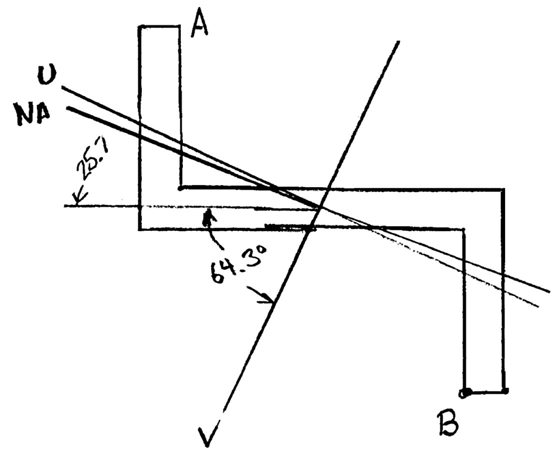
$P = 53.9$ kips ◀

PROBLEM 4.150



The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes $25.7°$ and $64.3°$.

SOLUTION



$$I_v = I_{max} = 2.28 \times 10^6\ \text{mm}^4 = 2.28 \times 10^{-6}\ \text{m}^4$$

$$I_u = I_{min} = 0.23 \times 10^6\ \text{mm}^4 = 0.23 \times 10^{-6}\ \text{m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan \varphi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3°$$

$$= 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$u_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45) \cos 64.3° + (-35) \sin 64.3°$$

$$= -51.05\ \text{mm}$$

$$v_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35) \cos 64.3° - (-45) \sin 64.3°$$

$$= +25.37\ \text{mm}$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$

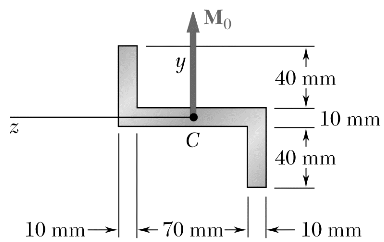
$$80 \times 10^6 = -\frac{(M_0 \cos 64.3°)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

$$= 109.1 \times 10^3 M_0$$

$$M_0 = \frac{80 \times 10^6}{109.1 \times 10^3}$$

$M_0 = 733\ \text{N} \cdot \text{m}$ ◀

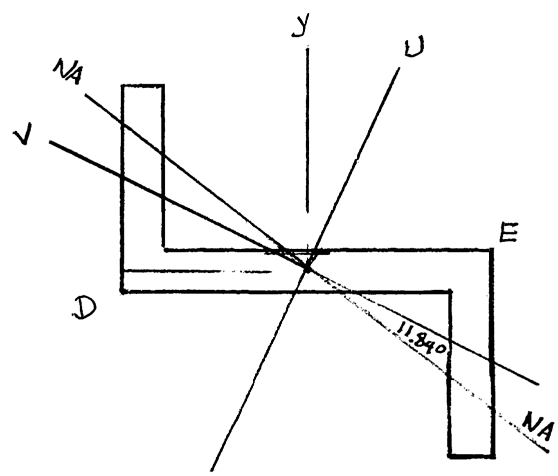
PROBLEM 4.151



Solve Prob. 4.150 assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

PROBLEM 4.150 The *Z* section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{\max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{\min} = 0.23 \times 10^{-6}\text{mm}^4$, principal axes 25.7° and 64.3°.

SOLUTION



$$I_v = I_{\min} = 0.23 \times 10^6\ \text{mm}^4 = 0.23 \times 10^6\ \text{m}^4$$

$$I_u = I_{\max} = 2.28 \times 10^6\ \text{mm}^4 = 2.28 \times 10^6\ \text{m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$

$$= \frac{0.23 \times 10^{-6}}{2.28 \times 10^{-6}}\tan 64.3°$$

$$= 0.20961$$

$$\varphi = 11.84°$$

Points *D* and *E* are farthest from the neutral axis.

$$u_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5)\cos 25.7° - 45\sin 25.7°$$

$$= -24.02\ \text{mm}$$

$$v_D = z_D \cos 25.7° + y_D \sin 25.7° = 45\cos 25.7° + (-5)\sin 25.7°$$

$$= 38.38\ \text{mm}$$

$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -\frac{(M_D \cos 64.3°)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}}$$

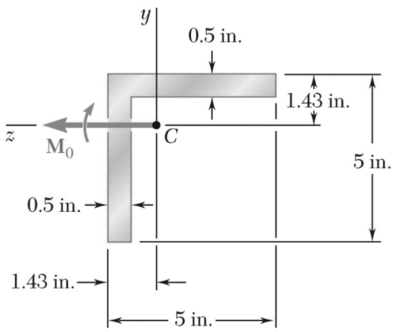
$$+\frac{(M_0 \sin 64.3°)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$

$$80 \times 10^6 = 60.48 \times 10^3 M_0$$

$$M_0 = 1.323 \times 10^3\ \text{N} \cdot \text{m}$$

$M_0 = 1.323\ \text{kN} \cdot \text{m}$ ◀

PROBLEM 4.152



A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not |to exceed 12 ksi. *Given:* $I_y = I_z = 11.3 \text{ in.}^4$, $A = 4.75 \text{ in.}^2$, $k_{\min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak_{\min}^2$ and $I_{\min} + I_{\max} = I_y + I_z$)

SOLUTION

$$M_u = M_0 \sin 45° = 0.70711\, M_0$$

$$M_v = M_0 \cos 45° = 0.7071\, M_0$$

$$I_{\min} = Ak_{\min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$u_B = y_B \cos 45° + z_B \sin 45° = -3.57 \cos 45° + 0.93 \sin 45° = -1.866 \text{ in.}$$

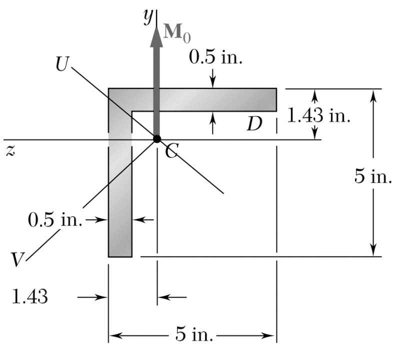
$$v_B = z_B \cos 45° - y_B \sin 45° = 0.93 \cos 45° - (-3.57) \sin 45° = 3.182 \text{ in.}$$

$$\sigma_B = -\frac{M_v u_B}{Iv} + \frac{M_u v_B}{Iu} = -0.70711\, M_0 \left[-\frac{u_B}{I_{\min}} + \frac{v_B}{I_{\max}} \right]$$

$$= 0.70711\, M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124\, M_0$$

$$M_0 = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124}$$

$M_0 = 29.1 \text{ kip} \cdot \text{in}$ ◀



PROBLEM 4.153

Solve Prob. 4.152, assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

PROBLEM 4.152 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3\text{ in.}^4$, $A = 4.75\text{ in.}^2$, $k_{\min} = 0.983\text{ in.}$ (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak_{\min}^2$ and $I_{\min} + I_{\max} = I_y + I_z$)

SOLUTION

$$M_u = M_0 \cos 45° = 0.70711\, M_0$$

$$M_v = -M_0 \sin 45° = -0.70711\, M_0$$

$$I_{\min} = Ak_{\min}^2 = (4.75)(0.983)^2 = 4.59\text{ in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01\text{ in}^4$$

$$u_D = y_D \cos 45° + z_D \sin 45° = -0.93 \cos 45° + (-3.57 \sin 45°) = -1.866\text{ in.}$$

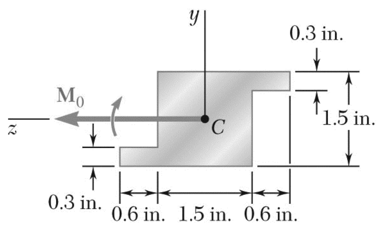
$$v_D = z_D \cos 45° - y_D \sin 45° = (-3.57)\cos 45° - (0.93)\sin 45° = 3.182\text{ in.}$$

$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -0.70711\,\mathrm{M}_0\left[-\frac{u_D}{I_{\min}} + \frac{v_D}{I_{\max}}\right]$$

$$= 0.70711\, M_0\left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01}\right] = 0.4124\, M_0$$

$$M_0 = \frac{\sigma_D}{0.4124} = \frac{12}{0.4124}$$

$M_0 = 29.1\text{ kip}\cdot\text{in}$ ◀



PROBLEM 4.154

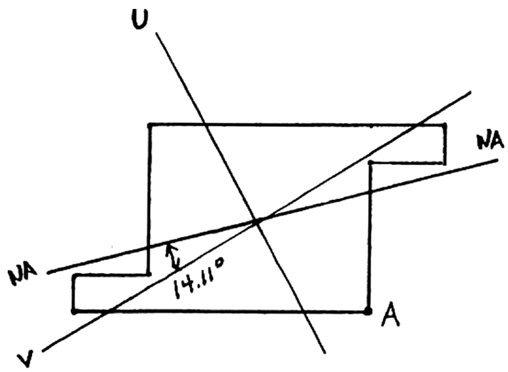
An extruded aluminum member having the cross section shown is subjected to a couple acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 12 ksi. *Given:* $I_{max} = 0.957\text{ in}^4$, $I_{min} = 0.427\text{ in}^4$, principal axes 29.4° and 60.6°.

SOLUTION

$$I_u = I_{max} = 0.957\,\text{in}^4$$

$$I_v = I_{min} = 0.427\,\text{in}^4$$

$$M_u = M_0 \sin 29.4°, \quad M_v = M_0 \cos 29.4°$$



$$\theta = 29.4°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.427}{0.957}\tan 29.4°$$

$$= 0.2514 \qquad \varphi = 14.11°$$

Point A is farthest from the neutral axis.

$$y_A = -0.75\text{ in.}, \quad z_A = -0.75\text{ in.}$$

$$u_A = y_A\cos 29.4° + z_A \sin 29.4° = -1.0216\text{ in.}$$

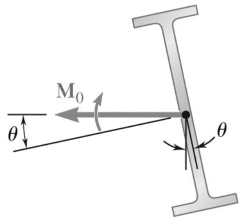
$$v_A = z_A\cos 29.4° - y_A \sin 29.4° = -0.2852\text{ in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u V_A}{I_u} = -\frac{(M_0\cos 29.4°)(-1.0216)}{0.427} + \frac{(M_0 \sin 29.4°)(-0.2852)}{0.957}$$

$$= 1.9381 M_0$$

$$M_0 = \frac{\sigma_A}{1.9381} = \frac{12}{1.9381}$$

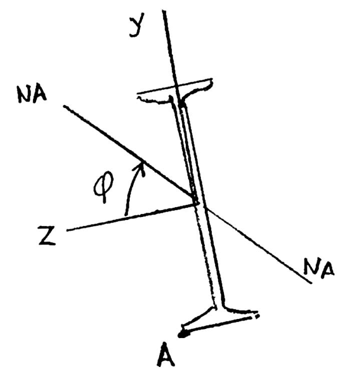
$M_0 = 6.19\text{ kip}\cdot\text{in}$ ◀



PROBLEM 4.155

A couple $\mathbf{M}_0$ acting in a vertical plane is applied to a W12 $\times$ 16 rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.

SOLUTION



For W12 $\times$ 16 rolled steel section,

$$I_z = 103 \text{ in}^4 \qquad I_y = 2.82 \text{ in}^4$$

$$d = 11.99 \text{ in.} \qquad b_f = 3.990 \text{ in.}$$

$$y_A = -\frac{d}{2} \qquad z_A = \frac{b_f}{2}$$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{103}{2.82}\tan\theta = 36.52\tan\theta$$

Point A is farthest from the neutral axis.

$$M_y = M_0\sin\theta \qquad M_z = M_0\cos\theta$$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z}\cos\theta + \frac{M_0 b_f}{2I_y}\sin\theta = \frac{M_0 d}{2I_z}\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)\cos\theta$$

For $\theta = 0$, $\sigma_0 = \dfrac{M_0 d}{2I_z}$

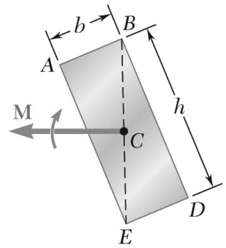
$$\sigma_A = \sigma_0\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)\cos\theta = 2\sigma_0$$

$$\frac{I_2 b_f}{I_y d}\tan\theta = \frac{(103)(3.990)}{(2.82)(11.99)}\left(\frac{2}{\cos\theta} - 1\right)$$

$$\tan\theta = 0.082273\left(\frac{2}{\cos\theta} - 1\right)$$

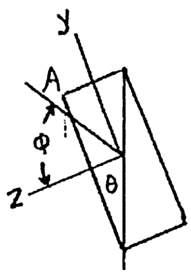
Assuming $\cos\theta \approx 1$, we get $\theta = 4.70°$ ◀

PROBLEM 4.156



Show that, if a solid rectangular beam is bent by a couple applied in a plane containing one diagonal of a rectangular cross section, the neutral axis will lie along the other diagonal.

SOLUTION



$$\tan\theta = \frac{b}{h}$$

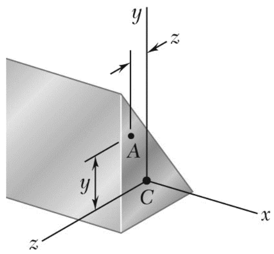
$$M_z = M\cos\theta, \quad M_z = M\sin\theta$$

$$I_z = \frac{1}{12}bh^3 \qquad I_y = \frac{1}{12}hb^3$$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{\frac{1}{12}bh^3}{\frac{1}{12}hb^3}\cdot\frac{b}{h} = \frac{h}{b}$$

The neutral axis passes through corner A of the diagonal AD. ◀

PROBLEM 4.157



A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the horizontal plane xz. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = \frac{zI_z - yI_{yz}}{I_y I_z - I_{yz}^2} M_y$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.

SOLUTION

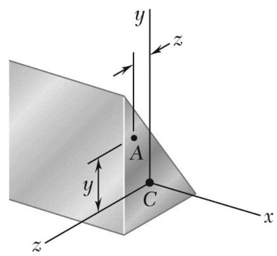
The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes:

$$\sigma_A = C_1 y + C_2 z$$

where C_1 and C_2 are constants.

$$M_z = -\int y\sigma_A dA = -C_1\int y^2 dA - C_2\int yz dA$$
$$= -I_z C_1 - I_{yz} C_2 = 0$$
$$C_1 = -\frac{I_{yz}}{I_z} C_2$$
$$M_y = \int z\sigma_A dA = C_1\int yz\, dA + C_2\int z^2 dA$$
$$= I_{yz} C_1 + I_y C_2$$
$$- I_{yz}\frac{I_{yz}}{I_z} C_2 + I_y C_2$$
$$I_z M_y = \left(I_y I_z - I_{yz}^2\right) C_2$$
$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2}$$
$$C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$
$$\sigma_A = \frac{I_z z - I_{yz} y}{I_y I_z - I_{yz}^2} M_y \quad ◀$$

PROBLEM 4.158



A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the vertical plane *xy*. Show that the stress at point *A*, of coordinates *y* and *z*, is

$$\sigma_A = \frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_z the moment of the couple.

SOLUTION

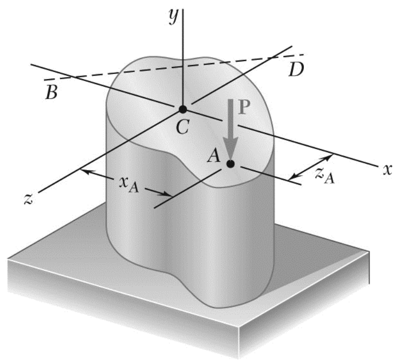
The stress σ_A varies linearly with the coordinates *y* and *z*. Since the axial force is zero, the *y*- and *z*-axes are centroidal axes:

$$\sigma_A = C_1 y + C_2 z$$

where C_1 and C_2 are constants.

$$M_y = \int z\sigma_A dA = C_1 \int yz dA + C_2 \int z^2 dA$$
$$= I_{yz} C_1 + I_y C_2 = 0$$
$$C_2 = -\frac{I_{yz}}{I_y} C_1$$
$$M_z = -\int y\sigma_A dz = -C_1 \int y^2 dA + C_2 \int yz dA$$
$$= -I_Z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1$$
$$I_y M_z = -\left(I_y I_z - I_{yz}^2\right) C_1$$
$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2}$$
$$C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$
$$\sigma_A = -\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2} M_z \quad \blacktriangleleft$$

PROBLEM 4.159



(a) Show that, if a vertical force **P** is applied at point A of the section shown, the equation of the neutral axis BD is

$$\left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1$$

where r_z and r_x denote the radius of gyration of the cross section with respect to the z axis and the x axis, respectively. (b) Further show that, if a vertical force **Q** is applied at any point located on line BD, the stress at point A will be zero.

SOLUTION

Definitions:

$$r_x^2 = \frac{I_x}{A}, \quad r_z^2 = \frac{I_z}{A}$$

(a) $\quad M_x = Pz_A \qquad M_z = -Px_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ar_z^2} - \frac{Pz_A z_E}{Ar_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{r_z^2}\right)x_E + \left(\frac{z_A}{r_x^2}\right)z_E\right] = 0$$

if E lies on neutral axis.

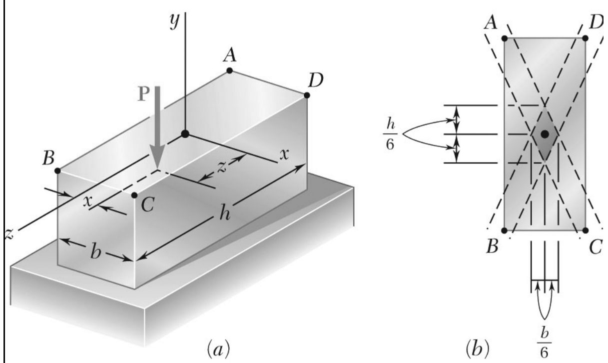
$$1 + \left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = 0, \qquad \left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1 \quad ◀$$

(b) $\quad M_x = Pz_E \quad M_z = -Px_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_y} = -\frac{P}{A} - \frac{Px_E x_A}{Ar_z^2} - \frac{Pz_E z_A}{Ar_x^2}$$

$= 0$ by equation from part (a). ◀

PROBLEM 4.160



(a) Show that the stress at corner A of the prismatic member shown in part a of the figure will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6} + \frac{z}{h/6} = 1$$

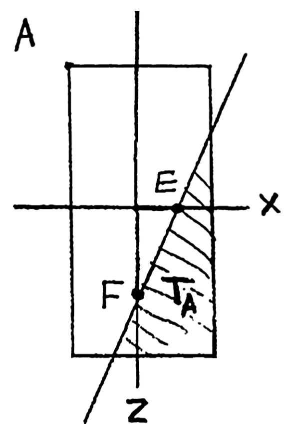
(b) Further show that, if no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part a and three similar lines corresponding to the condition of zero stress at B, C, and D, respectively. This area, shown in part b of the figure, is known as the *kern* of the cross section.

SOLUTION

$$I_z = \frac{1}{12}hb^3 \quad I_x = \frac{1}{12}bh^3 \quad A = bh$$

$$z_A = -\frac{h}{2} \quad x_A = -\frac{b}{2}$$

Let P be the load point.



$$M_z = -Px_P \quad M_x = Pz_P$$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

$$= -\frac{P}{bh} + \frac{(-Px_P)\left(-\frac{b}{2}\right)}{\frac{1}{12}hb^3} - \frac{(Pz_P - \frac{h}{2})}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$

(a) For $\sigma_A = 0, \quad 1 - \frac{x}{b/6} - \frac{z}{h/6} = 0 \qquad \frac{x}{b/6} + \frac{z}{h/6} = 1$

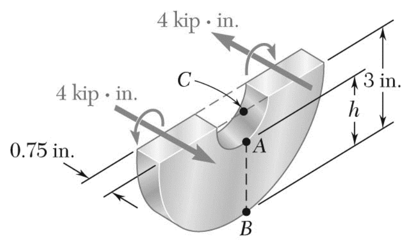
(b) At point E: $z = 0 \quad \therefore \quad x_E = b/6$

At point F: $x = 0 \quad \therefore \quad z_F = h/6$

If the line of action $(x_P,\ z_P)$ lies within the portion marked T_A, a tensile stress will occur at corner A.

By considering $\sigma_B = 0,\ \sigma_C = 0,$ and $\sigma_D = 0,$ the other portions producing tensile stresses are identified.

PROBLEM 4.161



For the machine component and loading shown, determine the stress at point A when (*a*) $h = 2$ in., (*b*) $h = 2.6$ in.

SOLUTION

$$M = -4 \text{ kip} \cdot \text{in}$$

Rectangular cross section: $A = bh \quad r_2 = 3 \text{ in.} \quad r_1 = r_2 - h$

$$\bar{r} = \frac{1}{2}(r_1 + r_2), \quad R = \frac{h}{\ln \frac{r_2}{r_1}}, \quad e = \bar{r} - R$$

(*a*) $h = 2$ in. $\quad A = (0.75)(2) = 1.5 \text{ in}^2$

$$r_1 = 3 - 2 = 1 \text{ in.} \qquad \bar{r} = \frac{1}{2}(3 + 1) = 2 \text{ in.}$$

$$R = \frac{2}{\ln \frac{3}{1}} = 1.8205 \text{ in.} \qquad e = 2 - 1.8205 = 0.1795 \text{ in.}$$

At point A: $\quad r = r_1 = 1$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4)(1 - 1.8205)}{(1.5)(0.1795)(1)} = 12.19 \text{ ksi}$$

$\sigma_A = 12.19$ ksi ◀

(*b*) $h = 2.6$ in. $\quad A = (0.75)(2.6) = 1.95 \text{ in}^2$

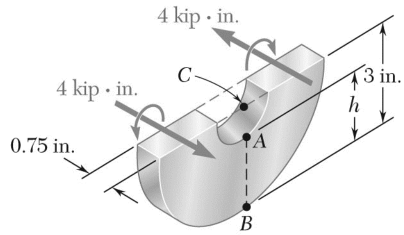
$$r_1 = 3 - 2.6 = 0.4 \text{ in.} \qquad \bar{r} = \frac{1}{2}(3 + 0.4) = 1.7 \text{ in.}$$

$$R = \frac{2.6}{\ln \frac{3}{0.4}} = 1.2904 \text{ in.} \qquad e = 1.7 - 1.2904 = 0.4906 \text{ in.}$$

At point A: $\quad r = r_1 = 0.4$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4)(0.4 - 1.2904)}{(1.95)(0.4096)(0.4)} = 11.15 \text{ ksi}$$

$\sigma_A = 11.15$ ksi ◀



PROBLEM 4.162

For the machine component and loading shown, determine the stress at points A and B when $h = 2.5$ in.

SOLUTION

$$M = -4 \text{ kip} \cdot \text{in}$$

Rectangular cross section: $h = 2.5$ in. $b = 0.75$ in. $A = 1.875$ in.^{2}

$$r_2 = 3 \text{ in.} \quad r_1 = r_2 - h = 0.5 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = \frac{1}{2}(0.5 + 3.0) = 1.75 \text{ in.}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.5}{\ln \frac{3.0}{0.5}} = 1.3953 \text{ in.}$$

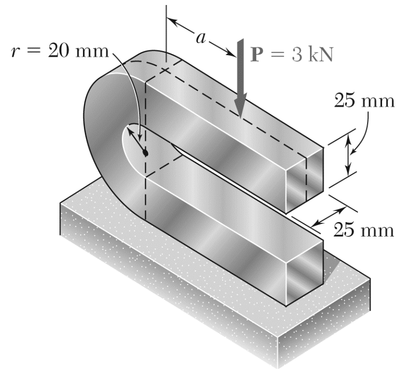
$$e = \bar{r} - R = 1.75 - 1.3953 = 0.3547 \text{ in.}$$

<u>At point A</u>: $r = r_1 = 0.5$ in.

$$\sigma_A = \frac{M(r-R)}{Aer} = \frac{(-4 \text{ kip} \cdot \text{in})(0.5 \text{ in.} - 1.3953 \text{ in.})}{(0.75 \text{ in.})(2.5 \text{ in.})(0.3547 \text{ in.})(0.5 \text{ in.})} \qquad \sigma_A = 10.77 \text{ ksi} \blacktriangleleft$$

<u>At point B</u>: $r = r_2 = 3$ in.

$$\sigma_B = \frac{M(r-R)}{Aer} = \frac{(-4 \text{ kip} \cdot \text{in})(3 \text{ in.} - 1.3953 \text{ in.})}{(0.75 \text{ in.} \times 2.5 \text{ in.})(0.3547 \text{ in.})(3 \text{ in.})} \qquad \sigma_B = -3.22 \text{ ksi} \blacktriangleleft$$



PROBLEM 4.163

The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.

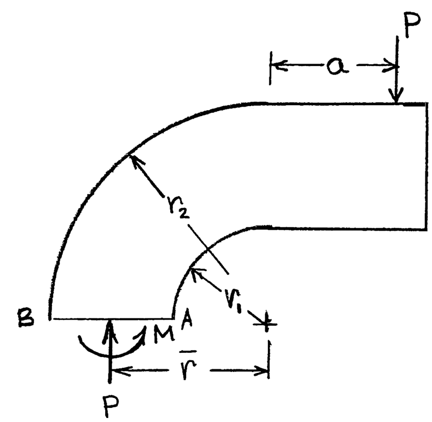
SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}$$



Also $\quad e = \bar{r} - R$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1} = -K\frac{P}{A}$$

with $\quad y_A = R - r_1$

Thus, $\quad K = 1 + \frac{(a+\bar{r})(R-r_1)}{er_1}$

Data: $\quad h = 25 \text{ mm}, \quad r_1 = 20 \text{ mm}, \quad r_2 = 45 \text{ mm}, \quad \bar{r} = 32.5 \text{ mm}$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

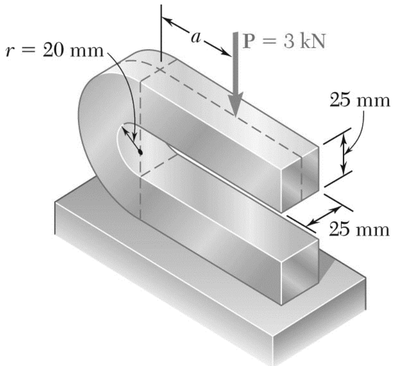
$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2 \quad R - r_1 = 10.8288 \text{ mm}$$

$$P = 3 \times 10^3 \text{ N} \cdot \text{m}, \quad \sigma_A = -150 \times 10^6 \text{ Pa}$$

$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)er_1}{R - r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$$a = 93.37 - 32.5 \qquad\qquad a = 60.9 \text{ mm} \blacktriangleleft$$



PROBLEM 4.164

The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.

SOLUTION

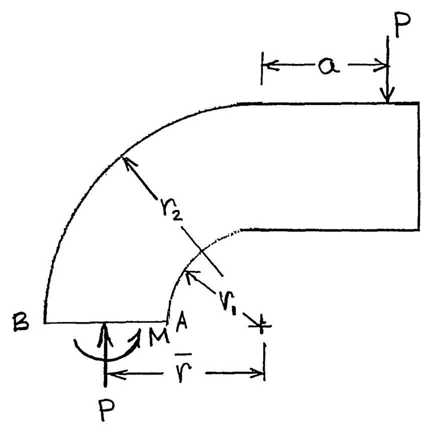
Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}.$$

Also $e = \bar{r} - R$



The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Aer_1} = -K\frac{P}{A}$$

with $\quad y_A = R - r_1$

Thus, $\quad K = 1 + \frac{(a + \bar{r})(R - r_1)}{er_1}$

Data: $\quad h = 25$ mm, $\quad r_1 = 20$ mm, $\quad r_2 = 45$ mm, $\quad \bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

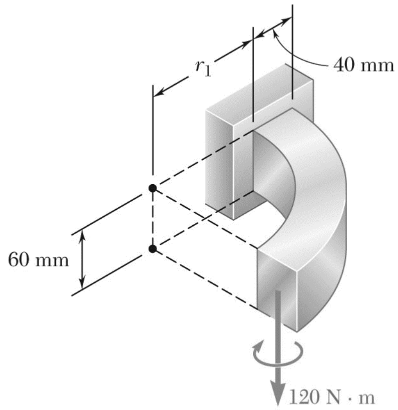
$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{m}^2$$

$$a = 60 \text{ mm}, \quad a + \bar{r} = 92.5 \text{ mm}, \quad R - r_1 = 10.8288 \text{ mm}$$

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968 \quad P = 30 \times 10^3 \text{ N}$$

$$\sigma_A = \frac{KP}{A} = -\frac{(30.968)(3 \times 10^3)}{625 \times 10^{-6}} = -148.6 \times 10^6 \text{ Pa}$$

$\sigma_A = -148.6$ MPa ◀



PROBLEM 4.165

The curved bar shown has a cross section of 40×60 mm and an inner radius $r_1 = 15$ mm. For the loading shown, determine the largest tensile and compressive stresses.

SOLUTION

$$h = 40 \text{ mm}, \quad r_1 = 15 \text{ mm}, \quad r_2 = 55\text{mm}$$

$$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6}\text{m}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{55}{40}} = 30.786 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35 \text{ mm}$$

$$e = \bar{r} - R = 4.214 \text{ mm} \qquad \sigma = -\frac{My}{Aer}$$

At $r = 15$mm, $\quad y = 30.786 - 15 = 15.756$ mm

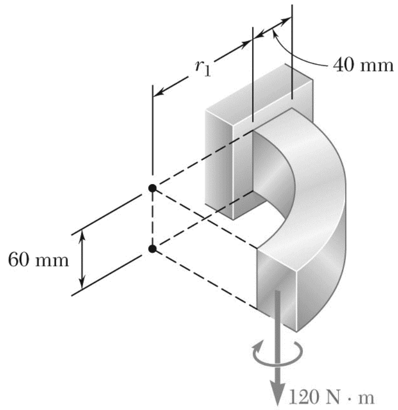
$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^{-6} \text{ Pa}$$

$\sigma = -12.49$MPa ◀

At $r = 55$mm, $\quad y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^{6} \text{ Pa}$$

$\sigma = 5.22$ MPa ◀



PROBLEM 4.166

For the curved bar and loading shown, determine the percent error introduced in the computation of the maximum stress by assuming that the bar is straight. Consider the case when (*a*) $r_1 = 20$ mm, (*b*) $r_1 = 200$ mm, (*c*) $r_1 = 2$ m.

SOLUTION

$$h = 40\text{ mm},\ A = (60)(40) = 2400\text{ mm}^2 = 2400\times10^{-6}\text{ m}^2,\quad M = 120\text{ N}\cdot\text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(40)^3 = 0.32\times10^6\text{ mm}^4 = 0.32\times10^{-6}\text{ mm}^4,\quad c = \frac{1}{2}h = 20\text{ mm}$$

Assuming that the bar is straight

$$\sigma_s = -\frac{Mc}{I} = -\frac{(120)(20\times10^{-8})}{(0.32\times10^{-6})} = 7.5\times10^6\text{Pa} = 7.5\text{ MPa}$$

(*a*) $\quad r_1 = 20$ mm $\quad r_2 = 60$ mm

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{40}{\ln\frac{60}{20}} = 36.4096\text{ mm}\qquad r_1 - R = -16.4096\text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40\text{ mm}\qquad e = \bar{r} - R = 3.5904\text{ mm}$$

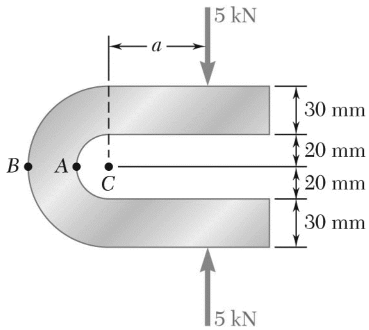
$$\sigma_a = \frac{M(r_1 - R)}{Aer} = \frac{(120)(-16.4096\times10^{-3})}{(2400\times10^{-6})(3.5904\times10^{-3})(20\times10^{-3})} = -11.426\times10^6\text{ Pa} = -11.426\text{MPa}$$

$$\%\text{ error} = \frac{-11.426 - (-7.5)}{-11.426}\times100\% = -34.4\% \quad ◀$$

For parts (*b*) and (*c*), we get the values in the table below:

	r_1, mm	r_2, mm	R, mm	$\bar{r}$, mm	e, mm	σ, MPa	% error
(*a*)	20	60	36.4096	40	3.5904	–11.426	–34.4 %
(*b*)	200	240	219.3926	220	0.6074	–7.982	6.0 % ◀
(*c*)	2000	2040	2019.9340	2020	0.0660	–7.546	0.6 % ◀

PROBLEM 4.167

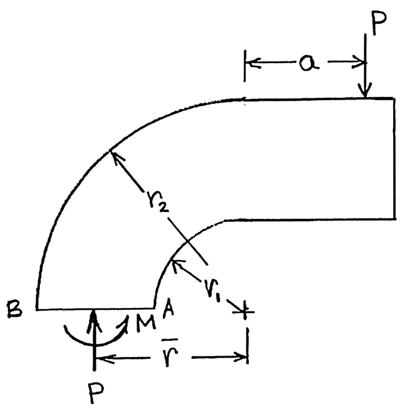


The curved bar shown has a cross section of 30×30 mm. Knowing that $a = 60$ mm, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Reduce the internal forces transmitted across section *AB* to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}.$$

Also $\quad e = \bar{r} - R$

The maximum compressive stress occurs at point *A*. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \qquad \text{with} \qquad y_A = R - r_1$$

Thus, $\qquad K = 1 + \dfrac{(a+\bar{r})(R - r_1)}{er_1}$

Data: $\qquad h = 30 \text{ mm}, \quad r_1 = 20 \text{ mm}, \quad r_2 = 50 \text{ mm}, \quad \bar{r} = 35 \text{ mm}$

$$R = \frac{30}{\ln \frac{50}{20}} = 32.7407 \text{ mm}, \quad e = 35 - 32.7407 = 2.2593 \text{ mm}$$

$$b = 30 \text{ mm}, \quad A = bh = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{m}^2$$

$$a = 60 \text{ mm}, \quad a + \bar{r} = 95 \text{ mm}, \quad R - r_1 = 12.7407 \text{ mm}$$

$$K = 1 + \frac{(95)(12.7407)}{(2.2593)(20)} = 27.786$$

$$P = 5 \times 10^3 \text{ N}$$

PROBLEM 4.167 *(Continued)*

(*a*) $$\sigma_A = -\frac{KP}{A} = -\frac{(27.786)(5\times 10^3)}{900\times 10^{-6}} = -154.4\times 10^6 \text{ Pa}$$

$\sigma_A = -154.4$ MPa ◀

(*b*) At point *B*, $y_B = r_2 - R = 50 - 32.7407 = 17.2953$ mm

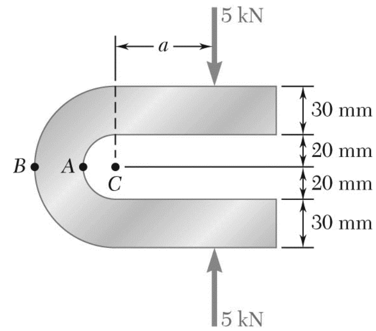
$$\sigma_B = -\frac{P}{A} + \frac{My_B}{Aer_2} = \frac{P}{A}\left[-1 + \frac{(a+\bar{r})y_B}{er_2}\right]$$

$$= \frac{K'P}{A} \quad \text{where} \quad K' = \frac{(a+\bar{r})y_B}{er_2} - 1$$

$$K' = \frac{(95)(17.2953)}{(2.2593)(50)} - 1 = 13.545$$

$$\sigma_B = \frac{(13.545)(5\times 10^3)}{900\times 10^{-6}} = 75.2\times 10^6 \text{ Pa}$$

$\sigma_B = 75.2$ MPa ◀



PROBLEM 4.168

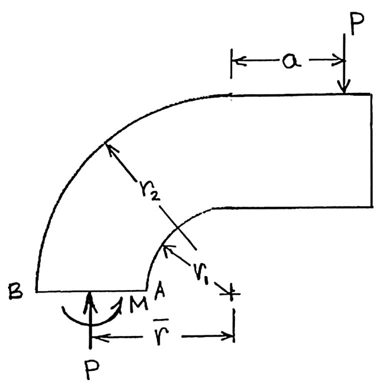
The curved bar shown has a cross section of 30×30 mm. Knowing that the allowable compressive stress is 175 MPa, determine the largest allowable distance a.

SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at $R = \dfrac{h}{\ln \dfrac{r_2}{r_1}}.$



Also $e = \bar{r} - R$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \quad \text{with} \quad y_A = R - r_1$$

Thus, $$K = 1 + \frac{(a + \bar{r})(R - r_1)}{er_1} \qquad (1)$$

Data: $h = 30 \text{ mm}, \quad r_1 = 20 \text{ mm}, \quad r_2 = 50 \text{ mm}, \quad \bar{r} = 35 \text{ mm}, \quad R = \dfrac{30}{\ln \frac{50}{20}} = 32.7407 \text{ mm}$

$$e = 35 - 32.7407 = 2.2593 \text{ mm}, \quad b = 30 \text{ mm}, \quad R - r_1 = 12.7407 \text{ mm}, \quad a = ?$$

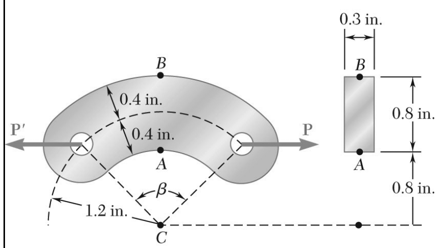
$$\sigma_A = -175 \text{ MPa} = -175 \times 10^6 \text{ Pa}, \quad P = 5 \text{ kN} = 5 \times 10^3 \text{ N}$$

$$K = -\frac{A\sigma_A}{P} = -\frac{(900 \times 10^{-6})(-175 \times 10^6)}{5 \times 10^3} = 31.5$$

Solving (1) for $a + \bar{r}$, $\quad a + \bar{r} = \dfrac{(K - 1)er_1}{R - r_1}$

$$a + \bar{r} = \frac{(30.5)(2.2593)(20)}{12.7407} = 108.17 \text{ mm}$$

$$a = 108.17 \text{ mm} - 35 \text{ mm} \qquad a = 73.2 \text{ mm} \blacktriangleleft$$



PROBLEM 4.169

Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 12 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

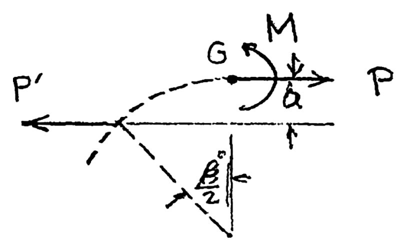
SOLUTION

Reduce section force to a force-couple system at G; the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$.

For the rectangular section, the neutral axis for bending couple only lies at



$$R = \frac{h}{\ln\frac{r_2}{r_1}}.$$

Also $e = \bar{r} - R$

At point A the tensile stress is $\sigma_A = \dfrac{P}{A} - \dfrac{My_A}{Aer_1} = \dfrac{P}{A} + \dfrac{Pay_A}{Aer_1} = \dfrac{P}{A}\left(1 + \dfrac{ay_A}{er_1}\right) = K\dfrac{P}{A}$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 1.2$ in., $r_1 = 0.8$ in., $r_2 = 1.6$ in., $h = 0.8$ in., $b = 0.3$ in.

$A = (0.3)(0.8) = 0.24\ \text{in}^2$ $\qquad R = \dfrac{0.8}{\ln\frac{1.6}{0.8}} = 1.154156$ in.

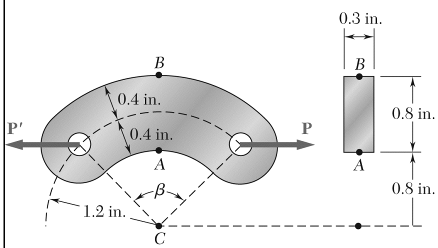
$e = 1.2 - 1.154156 = 0.045844$ in., $\qquad y_A = 1.154156 - 0.8 = 0.35416$ in.

$a = 1.2(1 - \cos 45°) = 0.35147$ in.

$$K = 1 + \frac{(0.35147)(0.35416)}{(0.045844)(0.8)} = 4.3940$$

$$P = \frac{(0.24)(12)}{4.3940} = 0.65544 \text{ kips}$$

$P = 655$ lb ◀



PROBLEM 4.170

Solve Prob. 4.169, assuming that $\beta = 60°$.

PROBLEM 4.169 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 12 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

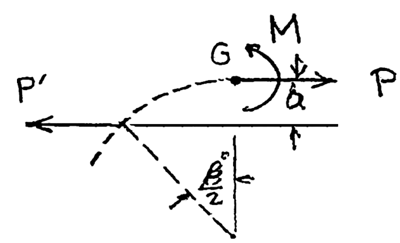
SOLUTION

Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$.

For the rectangular section, the neutral axis for bending couple only lies at



$$R = \frac{h}{\ln\frac{r_2}{r_1}}.$$

Also $e = \bar{r} - R$

At point A, the tensile stress is $$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $$K = 1 + \frac{ay_A}{er_1} \qquad \text{and} \qquad y_A = R - r_1$$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 1.2$ in., $r_1 = 0.8$ in., $r_2 = 1.6$ in., $h = 0.8$ in., $b = 0.3$ in.

$$A = (0.3)(0.8) = 0.24 \text{ in}^2 \qquad R = \frac{0.8}{\ln\frac{1.6}{0.8}} = 1.154156 \text{ in.}$$

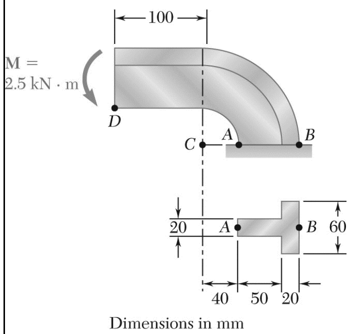
$$e = 1.2 - 1.154156 = 0.045844 \text{ in.} \qquad y_A = 1.154156 - 0.8 = 0.35416 \text{ in.}$$

$$a = (1.2)(1 - \cos 30°) = 0.160770 \text{ in.}$$

$$K = 1 + \frac{(0.160770)(0.35416)}{(0.045844)(0.8)} = 2.5525$$

$$P = \frac{(0.24)(12)}{2.5525} = 1.128 \text{ kips}$$

$P = 1128$ lb ◀

PROBLEM 4.171

A machine component has a T-shaped cross section that is orientated as shown. Knowing that $M = 2500\text{ N}\cdot\text{m}$, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Properties of the cross section.

$$R = \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} \qquad \bar{r} = \frac{\sum A_i \bar{r}_i}{\sum A_i}$$

r, mm	Part	b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}_i$, mm	$A_i \bar{r}_i$, mm^3
40	1	20	50	1000	16.2186	65	65,000
90	2	60	20	1200	12.0402	100	120,000
110	Σ			2200	28.2588		185,000

$$R = \frac{2200}{28.2588} = 77.852\text{ mm}, \qquad \bar{r} = \frac{185000}{2200} = 84.091\text{ mm}, \qquad e = \bar{r} - R = 6.239\text{ mm}$$

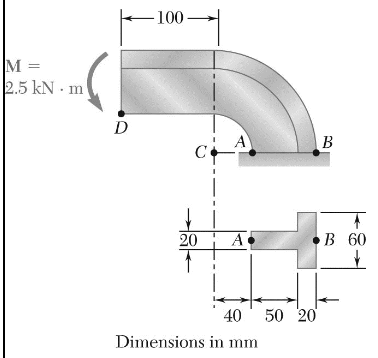
Stresses.

(*a*) Point *A*: $r = r_A = 40\text{ mm}$

$$\sigma_A = \frac{M(r-R)}{Aer} = \frac{(2.5\text{ kN}\cdot\text{m})(0.040\text{ m} - 0.0778517\text{ m})}{(2.2\times 10^{-3}\text{ m}^2)(6.2392\times 10^{-3}\text{ m})(0.040\text{ m})} \qquad \sigma_A = -172.4\text{ MPa} \blacktriangleleft$$

(*b*) Point *B*: $r = r_B = 110\text{ mm}$

$$\sigma_B = \frac{M(r-R)}{Aer} = \frac{(2.5\text{ kN}\cdot\text{m})(0.110\text{ m} - 0.0778517\text{ m})}{(2.2\times 10^{-3}\text{ m}^2)(6.2392\times 10^{-3}\text{ m})(0.110\text{ m})} \qquad \sigma_B = 53.2\text{ MPa} \blacktriangleleft$$



PROBLEM 4.172

Assuming that the couple shown is replaced by a vertical 10-kN force attached at point D and acting downward, determine the stress at (a) point A, (b) point B.

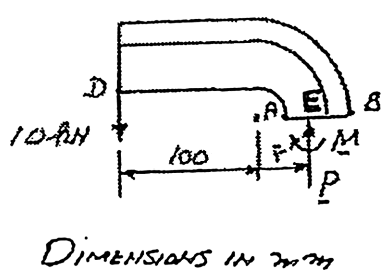
SOLUTION

Properties of the cross section. $R = \dfrac{\sum A}{\sum \int \frac{1}{r} dA} = \dfrac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \dfrac{\sum A_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} \qquad \bar{r} = \dfrac{\sum A_i \bar{r}_i}{\sum A_i}$

r, mm
40
90
110

Part	b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}_i$, mm	$A_i \bar{r}_i$, mm^3
1	20	50	1000	16.2186	65	65,000
2	60	20	1200	12.0402	100	120,000
Σ			2200	28.2588		185,000

$$R = \frac{2200}{28.2588} = 77.852 \text{ mm}, \qquad \bar{r} = \frac{185000}{2200} = 84.091 \text{ mm}, \qquad e = \bar{r} - R = 6.239 \text{ mm}$$



Force-couple system at the centroid E.

$P = 10$ kN

$M = (10 \text{ kN})(100 \text{ mm} + 84.0909 \text{ mm}) = 1840.9 \text{ N} \cdot \text{m}$

PROBLEM 4.172 *(Continued)*

Stresses.

(*a*) Point A: $r = r_A = 40$ mm

$$\sigma_A = -\frac{P}{A} + \frac{M(r-R)}{Aer} = -\frac{10\text{ kN}}{2.2\times 10^{-3}\text{ m}^2} + \frac{(1840.9\text{ N}\cdot\text{m})(0.040\text{ m} - 0.0778517\text{ m})}{(2.2\times 10^{-3}\text{ m}^2)(6.2392\times 10^{-3}\text{ m})(0.040\text{ m})}$$

$$= -4.545\text{ MPa} - 126.913\text{ MPa}$$

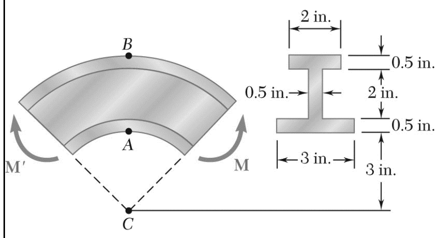
$\sigma_A = -131.5$ MPa ◀

(*b*) Point B: $r = r_B = 110$ mm

$$\sigma_B = -\frac{P}{A} + \frac{M(r-R)}{Aer} = -\frac{10\text{ kN}}{2.2\times 10^{-3}\text{ m}^2} + \frac{(1840.9\text{ N}\cdot\text{m})(0.110\text{ m} - 0.0778517\text{ m})}{(2.2\times 10^{-3}\text{ m}^2)(6.2392\times 10^{-3}\text{ m})(0.110\text{ m})}$$

$$= -4.545\text{ MPa} + 39.196\text{ MPa}$$

$\sigma_B = 34.7$ MPa ◀



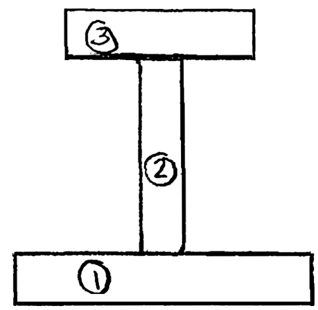
PROBLEM 4.173

Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\, dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$

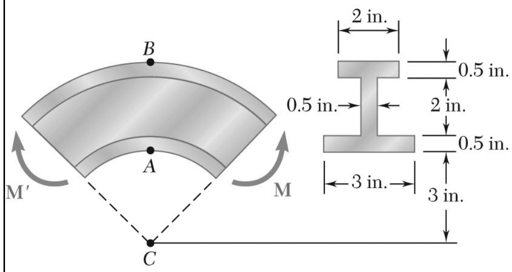


r	Part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.}$$

$e = 0.263$ in. ◀



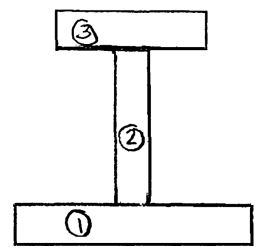
PROBLEM 4.174

Three plates are welded together to form the curved beam shown. For $M = 8$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\,dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	Part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3							
3.5	①	3	0.5	1.5	0.462452	3.25	4.875
5.5	②	0.5	2	1.0	0.225993	4.5	4.5
6	③	2	0.5	1.0	0.174023	5.75	5.75
	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \qquad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.} \qquad M = -8 \text{ kip} \cdot \text{in}$$

(*a*) $y_A = R - r_1 = 4.05812 - 3 = 1.05812$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)}$$

$\sigma_A = 3.06$ ksi ◀

(*b*) $y_B = R - r_2 = 4.05812 - 6 = -1.94188$ in.

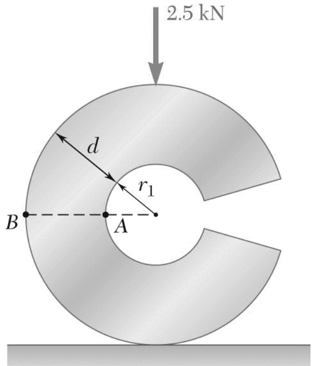
$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)}$$

$\sigma_B = -2.81$ ksi ◀

(*c*) $y_C = R - \bar{r} = -e$

$$\sigma_C = -\frac{My_C}{Ae\bar{r}} = -\frac{Me}{Ae\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-8}{(3.5)(4.32143)}$$

$\sigma_C = 0.529$ ksi ◀



PROBLEM 4.175

The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$c = \frac{1}{2}d = 16 \text{ mm}, \quad r_1 = 20 \text{ mm}, \quad r_2 = r_1 + d = 52 \text{ mm}$$

$$\bar{r} = r_1 + c = 36 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[36 + \sqrt{36^2 - 16^2}\right]$$

$$= 34.1245 \text{ mm}$$

$$e = \bar{r} - R = 1.8755 \text{ mm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6}\text{m}^2$$

$$P = 2.5 \times 10^3\text{N} \qquad M = P\bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90 \text{ N} \cdot \text{m}$$

(*a*) Point *A*: $y_A = R - r_1 = 34.1245 - 20 = 14.125$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -45.2 \times 10^6 \text{ Pa}$$

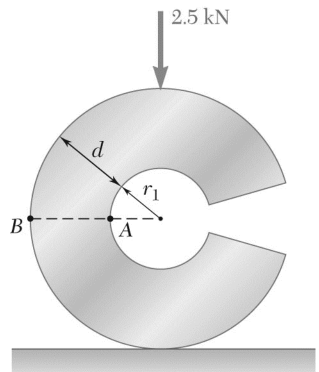
$\sigma_A = -45.2$ MPa ◀

(*b*) Point *B*: $y_B = R - r_2 = 34.1245 - 52 = -17.8755$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})}$$

$$= 17.40 \times 10^6 \text{ Pa}$$

$\sigma_B = 17.40$ MPa ◀



PROBLEM 4.176

The split ring shown has an inner radius $r_1 = 16$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$c = \frac{1}{2}d = 16 \text{ mm}, \quad r_1 = 16 \text{ mm}, \quad r_2 = r_1 + d = 48 \text{ mm}$$

$$\bar{r} = r_1 + c = 32 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[32 + \sqrt{32^2 - 16^2}\right]$$

$$= 29.8564 \text{ mm}$$

$$e = \bar{r} - R = 2.1436 \text{ mm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{m}^2$$

$$P = 2.5 \times 10^3 \text{N} \qquad M = P\bar{r} = (2.5 \times 10^3)(32 \times 10^{-3}) = 80 \text{ N} \cdot \text{m}$$

(*a*) Point *A*: $y_A = R - r_1 = 29.8564 - 16 = 13.8564$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(13.8564 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(16 \times 10^{-3})}$$

$$= -43.3 \times 10^6 \text{Pa}$$

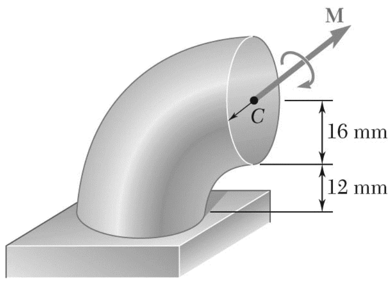
$\sigma_A = -43.3$ MPa ◀

(*b*) Point *B*: $y_B = R - r_2 = 29.8564 - 48 = -18.1436$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^6}{804.25 \times 10^{-6}} - \frac{(80)(-18.1436 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(48 \times 10^{-3})}$$

$$= 14.43 \times 10^6 \text{Pa}$$

$\sigma_B = 14.43$ MPa ◀



PROBLEM 4.177

The curved bar shown has a circular cross section of 32-mm diameter. Determine the largest couple **M** that can be applied to the bar about a horizontal axis if the maximum stress is not to exceed 60 MPa.

SOLUTION

$$c = 16 \text{ mm} \qquad \bar{r} = 12 + 16 = 28 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right]$$

$$= \frac{1}{2}\left[28 + \sqrt{28^2 - 16^2}\right] = 25.4891 \text{ mm}$$

$$e = \bar{r} - R = 28 - 25.4891 = 2.5109 \text{ mm}$$

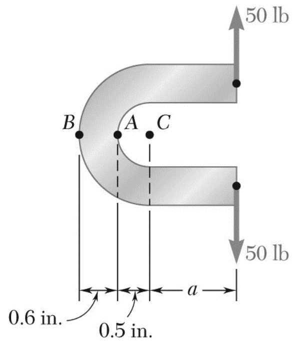
$\sigma_{\max}$ occurs at A, which lies at the inner radius.

It is given by $|\sigma_{\max}| = \left|\dfrac{My_A}{Aer_1}\right|$ from which $\qquad M = \dfrac{Aer_1|\sigma_{\max}|}{y_A}.$

Also, $\qquad A = \pi c^2 = \pi(16)^2 = 804.25 \text{ mm}^2$

Data: $\qquad y_A = R - r_1 = 25.4891 - 12 = 13.4891 \text{ mm}$

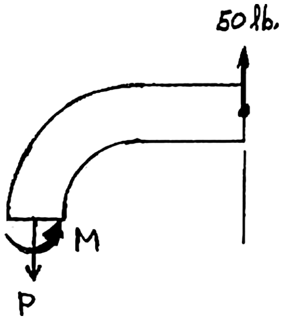
$$M = \frac{(804.25 \times 10^{-6})(2.5109 \times 10^{-3})(12 \times 10^{-3})(60 \times 10^{6})}{13.4891 \times 10^{-3}} \qquad M = 107.8 \text{ N} \cdot \text{m} \blacktriangleleft$$



PROBLEM 4.178

The bar shown has a circular cross section of 0.6 in. diameter. Knowing that $a = 1.2$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



$$c = \frac{1}{2}d = 0.3 \text{ in.} \qquad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in.}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$$

$$= 0.77081 \text{ in.}$$

$$e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$$

$$P = 50 \text{ lb}$$

$$M = -P(a + \bar{r}) = -50(1.2 + 0.8) = -100 \text{ lb} \cdot \text{in}$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$y_B = R - r_2 = 0.77081 - 1.1 = -0.32919 \text{ in.}$$

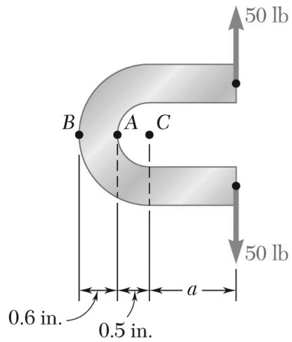
(*a*) $$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{50}{0.28274} - \frac{(-100)(0.27081)}{(0.28274)(0.02919)(0.5)} = 6.74 \times 10^3 \text{ psi}$$

$\sigma_A = 6.74$ ksi ◀

(*b*) $$\sigma_B = \frac{P}{A} - \frac{My_B}{Aer_2} = \frac{50}{0.28274} - \frac{(-100)(-0.32919)}{(0.28274)(0.02919)(1.1)} = -3.45 \times 10^3 \text{ psi}$$

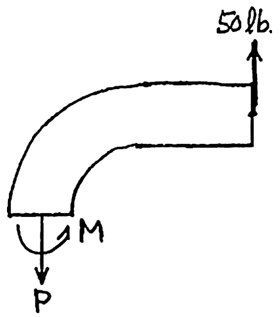
$\sigma_B = -3.45$ ksi ◀

PROBLEM 4.179



The bar shown has a circular cross section of 0.6 in. diameter. Knowing that the allowable stress is 8 ksi, determine the largest permissible distance a from the line of action of the 50-lb forces to the plane containing the center of curvature of the bar.

SOLUTION



$$c = \frac{1}{2}d = 0.3 \text{ in.}, \qquad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in.}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$$

$$= 0.77081 \text{ in.} \qquad e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$$

$$M = -P(a + \bar{r})$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{P(a + \bar{r})y_A}{Aer_1} = \frac{P}{A}\left[1 + \frac{(a + \bar{r})y_A}{er_1}\right]$$

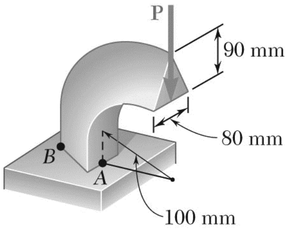
$$= \frac{KP}{A} \qquad \text{where} \qquad K = 1 + \frac{(a + \bar{r})y_A}{er_1}$$

$$K = \frac{\sigma_A A}{P} = \frac{(8 \times 10^3)(0.28274)}{50} = 45.238$$

$$a + \bar{r} = \frac{(K - 1)er_1}{y_A} = \frac{(44.238)(0.02919)(0.5)}{0.27081} = 2.384 \text{ in.}$$

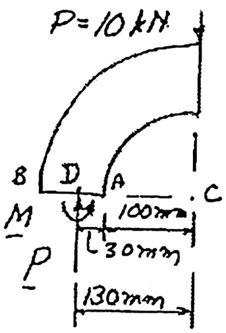
$$a = 2.384 - 0.8 \qquad\qquad a = 1.584 \text{ in.} \blacktriangleleft$$

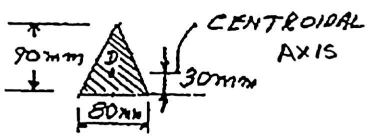
PROBLEM 4.180



Knowing that $P = 10$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION





Locate the centroid *D* of the cross section.

$$\bar{r} = 100 \text{ mm} + \frac{90 \text{ mm}}{3} = 130 \text{ mm}$$

<u>Force-couple system at *D*.</u>

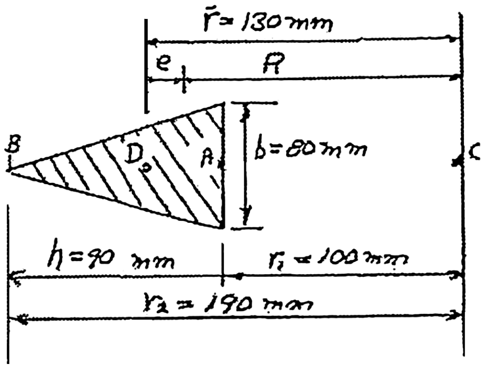
$$P = 10 \text{ kN}$$

$$M = P\bar{r} = (10 \text{ kN})(130 \text{ mm}) = 1300 \text{ N} \cdot \text{m}$$

<u>Triangular cross section.</u>

$$A = \frac{1}{2}bh = \frac{1}{2}(90 \text{ mm})(80 \text{ mm})$$

$$= 3600 \text{ mm}^2 = 3600 \times 10^{-6} \text{ m}^2$$



$$R = \frac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1} = \frac{\frac{1}{2}(90)}{\frac{190}{90}\ln\frac{190}{100} - 1} = \frac{45 \text{ mm}}{0.355025}$$

$$R = 126.752 \text{ mm}$$

$$e = \bar{r} - R = 130 \text{ mm} - 126.752 \text{ mm} = 3.248 \text{ mm}$$

(*a*) <u>Point *A*:</u> $r_A = 100 \text{ mm} = 0.100 \text{ m}$

$$\sigma_A = -\frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = -\frac{10 \text{ kN}}{3600 \times 10^{-6} \text{m}^2} + \frac{(1300 \text{ N} \cdot \text{m})(0.100 \text{ m} - 0.126752 \text{ m})}{(3600 \times 10^{-6} \text{ m}^2)(3248 \times 10^{-3} \text{ m})(0.100 \text{ m})}$$

$$= -2.778 \text{ MPa} - 29.743 \text{ MPa}$$

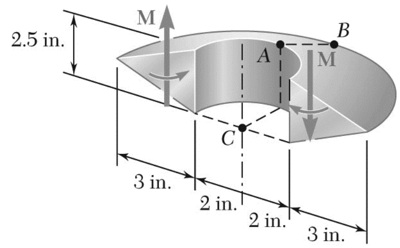
$$\sigma_A = -32.5 \text{ MPa} \blacktriangleleft$$

(*b*) <u>Point *B*:</u> $r_B = 190 \text{ mm} = 0.190 \text{ m}$

$$\sigma_B = -\frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = -\frac{10 \text{ kN}}{3600 \times 10^{-6} \text{ m}^2} + \frac{(1300 \text{ N} \cdot \text{m})(0.190 \text{ m} - 0.126752 \text{ m})}{(3600 \times 10^{-6} \text{ m}^2)(3.248 \times 10^{-3} \text{ m})(0.190 \text{ m})}$$

$$= -2.778 \text{ MPa} + 37.01 \text{ MPa}$$

$$\sigma_B = +34.2 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.181

Knowing that $M = 5$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 1 = 3.00000 \text{ in.}$$

$$b_1 = 2.5 \text{ in.}, \quad r_1 = 2 \text{ in.}, \quad b_2 = 0, \quad r_2 = 5 \text{ in.}$$

Use formula for trapezoid with $b_2 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5 + 0)}{[(2.5)(5) - (0)(2)]\ln\frac{5}{2} - (3)(2.5 - 0)} = 2.84548 \text{ in.}$$

$$e = \bar{r} - R = 0.15452 \text{ in.} \qquad M = 5 \text{ kip} \cdot \text{in}$$

(*a*) $y_A = R - r_1 = 0.84548$ in.

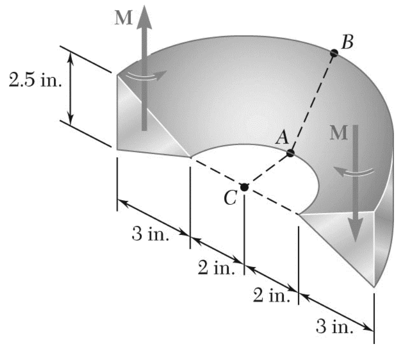
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)}$$

$\sigma_A = -3.65$ ksi ◀

(*b*) $y_B = R - r_2 = -2.15452$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)}$$

$\sigma_B = 3.72$ ksi ◀



PROBLEM 4.182

Knowing that $M = 5\,\text{kip}\cdot\text{in},$ determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = \frac{1}{2}(2.5)(3) = 3.75\ \text{in}^2$$

$$\bar{r} = 2 + 2 = 4.00000\ \text{in.}$$

$$b_1 = 0, \quad r_1 = 2\ \text{in.}, \quad b_2 = 2.5\ \text{in.}, \quad r_2 = 5\ \text{in.}$$

Use formula for trapezoid with $b_1 = 0.$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(0 + 2.5)}{[(0)(5) - (2.5)(2)]\ln\frac{5}{2} - (3)(0 - 2.5)} = 3.85466\ \text{in.}$$

$$e = \bar{r} - R = 0.14534\ \text{in.} \qquad M = 5\ \text{kip}\cdot\text{in}$$

(*a*) $y_A = R - r_1 = 1.85466$ in.

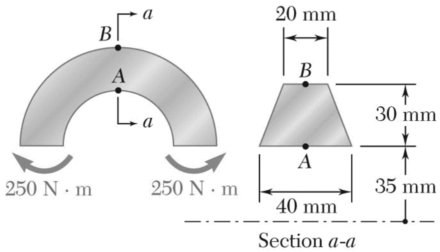
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)}$$

$\sigma_A = -8.51$ ksi ◀

(*b*) $y_B = R - r_2 = -1.14534$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)}$$

$\sigma_B = 2.10$ ksi ◀

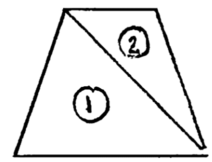


PROBLEM 4.183

For the curved beam and loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	600	45	27×10^3
②	300	55	16.5×10^3
Σ	900		43.5×10^3

$$\bar{r} = \frac{43.5\times10^3}{900} = 48.333 \text{ mm}$$

$$R = \frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_2-b_1)}$$

$$= \frac{(0.5)(30)^2(40+20)}{[(40)(65)-(20)(35)]\ln\frac{65}{35} - (30)(40-20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm} \qquad M = -250 \text{ N}\cdot\text{m}$$

(*a*) $y_A = R - r_1 = 11.8608$ mm

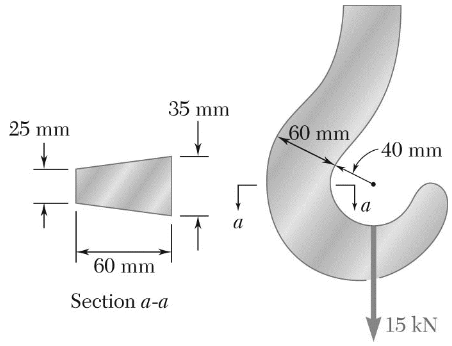
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608\times10^{-3})}{(900\times10^{-6})(1.4725\times10^{-3})(35\times10^{-3})} = 63.9\times10^6 \text{Pa}$$

$\sigma_A = 63.9$ MPa ◀

(*b*) $y_B = R - r_2 = -18.1392$ mm

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392\times10^{-3})}{(900\times10^{-6})(1.4725\times10^{-3})(65\times10^{-3})} = -52.6\times10^6 \text{Pa}$$

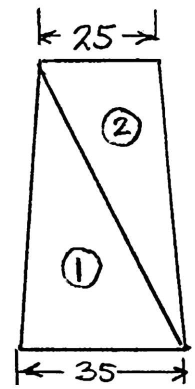
$\sigma_B = -52.6$ MPa ◀



PROBLEM 4.184

For the crane hook shown, determine the largest tensile stress in section *a-a*.

SOLUTION



Locate centroid.

	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103\times10^3}{1800} = 63.333 \text{ mm}$$

Force-couple system at centroid: $P = 15\times10^3\,\text{N}$

$$M = -P\bar{r} = -(15\times10^3)(68.333\times10^{-3}) = -1.025\times10^3\,\text{N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_1-b_2)}$$

$$= \frac{(0.5)(60)^2(35+25)}{[(35)(100)-(25)(40)]\ln\frac{100}{40} - (60)(35+25)} = 63.878 \text{ mm}$$

$$e = \bar{r} - R = 4.452 \text{ mm}$$

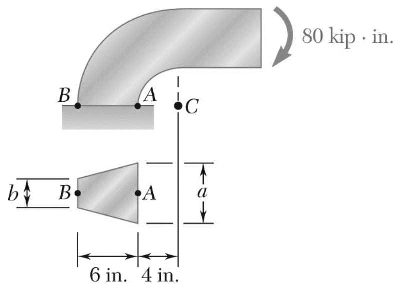
Maximum tensile stress occurs at point A.

$$y_A = R - r_1 = 23.878 \text{ mm}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1}$$

$$= \frac{15\times10^3}{1800\times10^{-6}} - \frac{-(1.025\times10^3)(23.878\times10^{-3})}{(1800\times10^{-6})(4.452\times10^{-3})(40\times10^{-3})}$$

$$= 84.7\times10^6\,\text{Pa} \qquad \sigma_A = 84.7 \text{ MPa} \blacktriangleleft$$



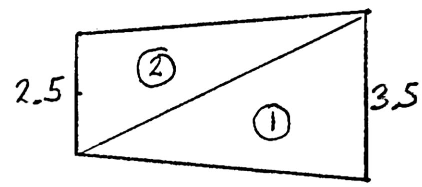
PROBLEM 4.185

Knowing that the machine component shown has a trapezoidal cross section with $a = 3.5$ in. and $b = 2.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.

	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	10.5	6	63
②	7.5	8	60
Σ	18		123



$$\bar{r} = \frac{123}{18} = 6.8333 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(3.5 + 2.5)}{[(3.5)(10) - (2.5)(4)]\ln\frac{10}{4} - (6)(3.5 - 2.5)} = 6.3878 \text{ in.}$$

$$e = \bar{r} - R = 0.4452 \text{ in.} \qquad M = 80 \text{ kip} \cdot \text{in}$$

(*a*) $y_A = R - r_1 = 6.3878 - 4 = 2.3878$ in.

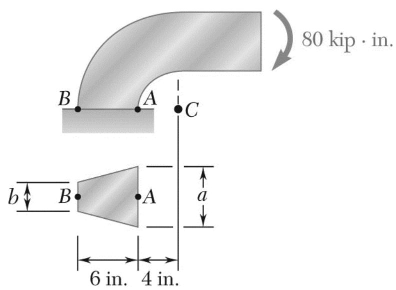
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.3878)}{(18)(0.4452)(4)}$$

$\sigma_A = -5.96$ ksi ◀

(*b*) $y_B = R - r_2 = 6.3878 - 10 = -3.6122$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.6122)}{(18)(0.4452)(10)}$$

$\sigma_B = 3.61$ ksi ◀



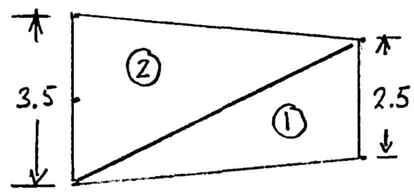
PROBLEM 4.186

Knowing that the machine component shown has a trapezoidal cross section with $a = 2.5$ in. and $b = 3.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.

	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	7.5	6	45
②	10.5	8	84
Σ	18		129



$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(2.5 + 3.5)}{[(2.5)(10) - (3.5)(4)]\ln\frac{10}{4} - (6)(2.5 - 3.5)} = 6.7168 \text{ in.}$$

$$e = \bar{r} - R = 0.4499 \text{ in.} \qquad M = 80 \text{ kip} \cdot \text{in}$$

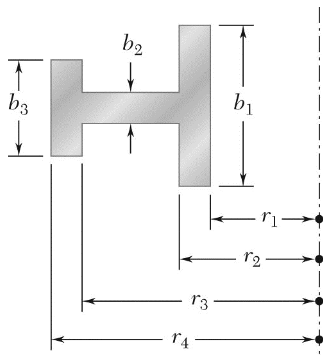
(*a*) $y_A = R - r_1 = 2.7168$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.7168)}{(18)(0.4499)(4)} \qquad \sigma_A = -6.71 \text{ ksi} \blacktriangleleft$$

(*b*) $y_B = R - r_2 = -3.2832$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.2832)}{(18)(0.4499)(10)} \qquad \sigma_B = 3.24 \text{ ksi} \blacktriangleleft$$

PROBLEM 4.187



Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

where A is the total area of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\, dA} = \frac{A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$= \frac{A}{\Sigma \ln\left(\frac{r_{i+1}}{r_i}\right)^{b_i}} = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]} \quad \blacktriangleleft$$

Note that for each rectangle, $\int \frac{1}{r} d\,A = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$

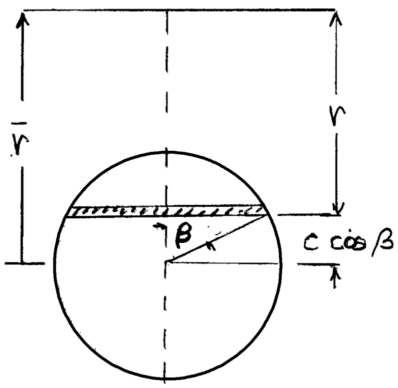
$$= b_i \int_{r_2}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$$

PROBLEM 4.188

Using Eq. (4.66), derive the expression for R given in Fig. 4.73 for a circular cross section.

SOLUTION

Use polar coordinate β as shown. Let w be the width as a function of β



$$w = 2c\sin\beta$$
$$r = \bar{r} - c\cos\beta$$
$$dr = c\sin\beta\, d\beta$$
$$dA = w\, dr = 2c^2\sin^2\beta\, d\beta$$

$$\int \frac{dA}{r} = \int_0^{\pi} \frac{2c^2\sin\beta}{\bar{r} - c\cos\beta}\, d\beta$$

$$\int \frac{dA}{r} = \int_0^{\pi} \frac{c^2(1-\cos^2\beta)}{\bar{r} - c\cos\beta}\, d\beta$$

$$= 2\int_0^{\pi} \frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2 - c^2)}{\bar{r} - c\cos\beta}\, d\beta$$

$$= 2\int_0^{\pi} (\bar{r} + c\cos\beta)\, d\beta - 2(\bar{r}^2 - c^2)\int_0^{\pi} \frac{dr}{\bar{r} - c\cos\beta}$$

$$= 2\bar{r}\beta\Big|_0^{\pi} + 2c\sin\beta\Big|_0^{\pi}$$

$$-2(\bar{r}^2 - c^2)\frac{2}{\sqrt{\bar{r}^2 - c^2}}\tan^{-1}\frac{\sqrt{\bar{r}^2 - c^2}\,\tan\left(\frac{1}{2}\beta\right)}{\bar{r} + c}\Bigg|_0^{\pi}$$

$$= 2\bar{r}(\pi - 0) + 2c(0-0) - 4\sqrt{\bar{r}^2 - c^2}\cdot\left(\frac{\pi}{2} - 0\right)$$

$$= 2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}} = \frac{1}{2}\frac{c^2}{\bar{r} - \sqrt{r^2 - c^2}} \times \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$$

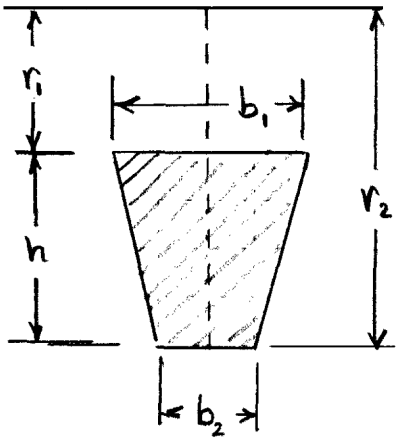
$$= \frac{1}{2}\frac{c^2\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2}\frac{c^2\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)}{c^2} = \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right) \blacktriangleleft$$

PROBLEM 4.189

Using Eq. (4.66), derive the expression for R given in Fig. 4.73 for a trapezoidal cross section.

SOLUTION

The section width w varies linearly with r.



$$w = c_0 + c_1 r$$

$$w = b_1 \text{ at } r = r_1 \text{ and } w = b_2 \quad \text{at} \quad r = r_2$$

$$b_1 = c_0 + c_1 r_1$$

$$b_2 = c_0 + c_1 r_2$$

$$b_1 - b_2 = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1)c_0 = hc_0$$

$$c_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$

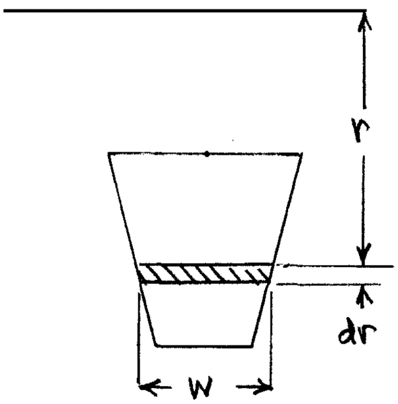
$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r} dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1(r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2)$$



$$A = \frac{1}{2}(b_1 + b_2)h$$

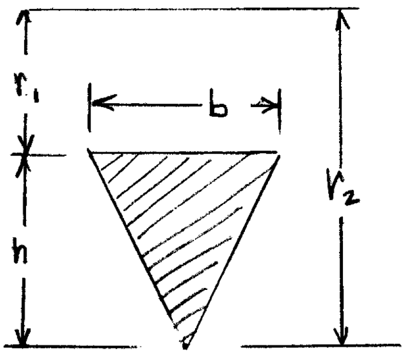
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h(b_1 - b_2)} \quad ◀$$

PROBLEM 4.190

Using Equation (4.66), derive the expression for R given in Fig. 4.73 for a triangular cross section.

SOLUTION

The section width w varies linearly with r.



$$w = c_0 + c_1 r$$

$$w = b \text{ at } r = r_1 \quad \text{and} \quad w = 0 \text{ at } r = r_2$$

$$b = c_0 + c_1 r_1$$

$$0 = c_0 + c_1 r_2$$

$$b = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b}{h} \quad \text{and} \quad c_0 = -c_1 r_2 = \frac{br_2}{h}$$

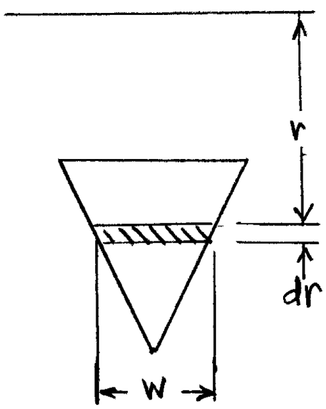
$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\,dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r}\,dr$$

$$= c_0 \ln r\Big|_{r_1}^{r_2} + c_1 r\Big|_{r_1}^{r_2}$$

$$= c_0 \ln\frac{r_2}{r_1} + c_1(r_2 - r_1)$$

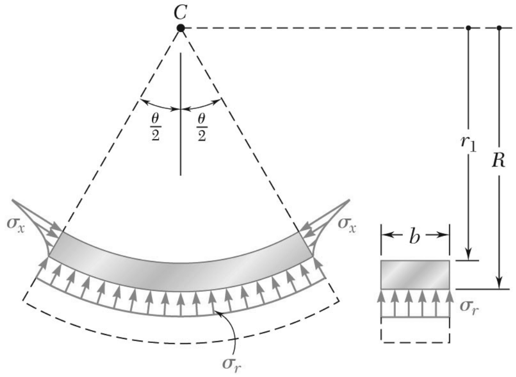
$$= \frac{br_2}{h}\ln\frac{r_2}{r_1} - \frac{b}{h}\,h$$

$$= \frac{b\,r_2}{h}\ln\frac{r_2}{r_1} - b = b\left(\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1\right)$$



$$A = \frac{1}{2}b\,h$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2}bh}{b\left(\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1\right)} = \frac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1} \quad \blacktriangleleft$$



PROBLEM 4.191

For a curved bar of rectangular cross section subjected to a bending couple **M**, show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compute the value of σ_r for the curved bar of Examples 4.10 and 4.11. (*Hint:* consider the free-body diagram of the portion of the beam located above the neutral surface.)

SOLUTION

At radial distance r, $\quad \sigma_r = \frac{M(r-R)}{Aer} = \frac{M}{Ae} - \frac{MR}{Aer}$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_r dA = \int_{r_1}^{R} \sigma_r b dr$$

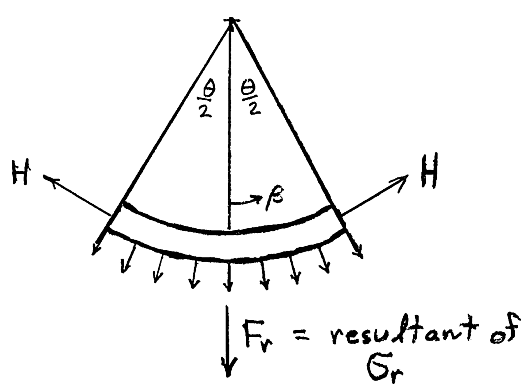
$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{R}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_n: $\quad F_r = \int \sigma_r \cos\beta \, dA$

$$= \int_{-\theta/2}^{\theta/2} \sigma_r \cos\beta (bR d\beta) = \sigma_r bR \int_{-\theta/2}^{\theta/2} \cos\beta \, d\beta$$

$$= \sigma_r bR \ \sin\beta \Big|_{-\theta/2}^{\theta/2} = 2\sigma_r bR \sin\frac{\theta}{2}$$



For equilibrium: $\quad F_r - 2H \sin\frac{\theta}{2} = 0$

$$2\sigma_r bR \sin\frac{\theta}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)\sin\frac{\theta}{2} = 0$$

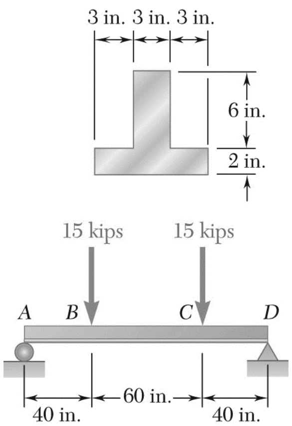
$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right) \quad ◀$$

Using results of Examples 4.10 and 4.11 as data,

$M = 8$ kip · in, $\quad A = 3.75$ in^2, $\quad R = 5.9686$ in., $\quad e = 0.0314$ in., $\quad r_1 = 5.25$ in.

$$\sigma_r = \frac{8}{(3.75)(0.0314)}\left[1 - \frac{5.25}{5.9686} - \ln\frac{5.9686}{5.25}\right]$$

$\sigma_r = -0.54$ ksi ◀

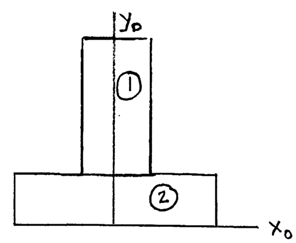


PROBLEM 4.192

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

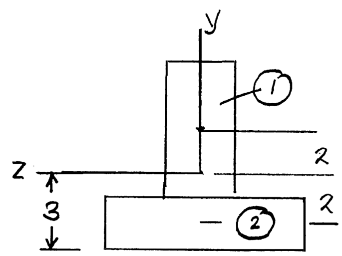
SOLUTION

	A	$\bar{y}_0$	$A\bar{y}_0$
①	18	5	90
②	18	1	18
Σ	36		108



$$\bar{Y}_0 = \frac{108}{36} = 3 \text{ in.}$$

Neutral axis lies 3 in. above the base.

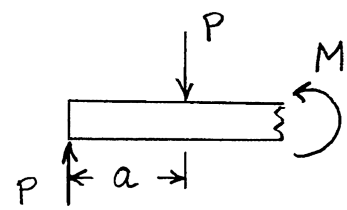


$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$y_{top} = 5$ in. $\quad y_{bot} = -3$ in.



$$M - Pa = 0$$

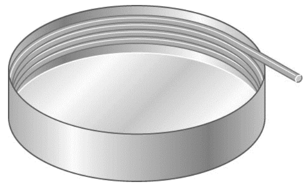
$$M = Pa = (15)(40) = 600 \text{ kip} \cdot \text{in}$$

$$\sigma_{top} = -\frac{M\,y_{top}}{I} = -\frac{(600)(5)}{204}$$

$\sigma_{top} = -14.71$ ksi (compression) ◀

$$\sigma_{bot} = -\frac{M\,y_{bot}}{I} = -\frac{(600)(-3)}{204}$$

$\sigma_{bot} = 8.82$ ksi (tension) ◀



PROBLEM 4.193

Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a coiled rod, (*b*) the corresponding bending moment in the rod. Use $E = 200$ GPa.

SOLUTION

Let $D =$ inside diameter of the drum.

$d =$ diameter of rod, $c = \frac{1}{2}d$,

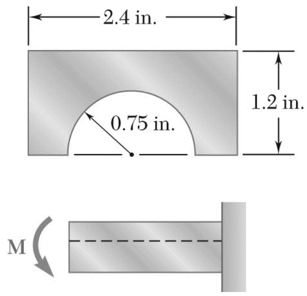
$\rho =$ radius of curvature of centerline of rods when bent.

$$\rho = \frac{1}{2}D - \frac{1}{2}d = \frac{1}{2}(1.25) - \frac{1}{2}(6\times10^{-3}) = 0.622 \text{ m}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.003)^4 = 63.617\times10^{-12}\,\text{m}^4$$

(*a*) $\sigma_{\max} = \dfrac{Ec}{\rho} = \dfrac{(200\times10^9)(0.003)}{0.622} = 965\times10^6\,\text{Pa}$ $\qquad\sigma = 965$ MPa ◀

(*b*) $M = \dfrac{EI}{\rho} = \dfrac{(200\times10^9)(63.617\times10^{-12})}{0.622} = 20.5\ \text{N}\cdot\text{m}$ $\qquad M = 20.5\ \text{N}\cdot\text{m}$ ◀



PROBLEM 4.194

Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION

① = rectangle ② = semi-circular cutout

$$A_1 = (2.4)(1.2) = 2.88 \text{ in}^2$$

$$A_2 = \frac{\pi}{2}(0.75)^2 = 0.8836 \text{ in}^2$$

$$A = 2.88 - 0.8836 = 1.9964 \text{ in}^2$$

$$\bar{y}_1 = 0.6 \text{ in.}$$

$$\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.75)}{3\pi} = 0.3183 \text{ in.}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(2.88)(0.6) - (0.8836)(0.3183)}{1.9964} = 0.7247 \text{ in.}$$

Neutral axis lies 0.7247 in. above the bottom.

Moment of inertia about the base:

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(2.4)(1.2)^3 - \frac{\pi}{8}(0.75)^4 = 1.25815 \text{ in}^4$$

Centroidal moment of inertia:

$$\bar{I} = I_b - A\bar{Y}^2 = 1.25815 - (1.9964)(0.7247)^2 = 0.2097 \text{ in}^4$$

$$y_{\text{top}} = 1.2 - 0.7247 = 0.4753 \text{ in.},$$

$$y_{\text{bot}} = -0.7247 \text{ in.}$$

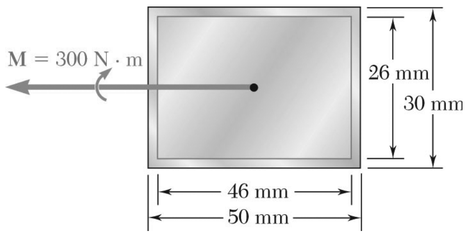
$$|\sigma| = \left|\frac{M\,y}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $M = \dfrac{(12)(0.2097)}{0.4753} = 5.29 \text{ kip}\cdot\text{in}$

Bottom: (compression) $M = \dfrac{(16)(0.2097)}{0.7247} = 4.63 \text{ kip}\cdot\text{in}$

Choose the smaller value. $M = 4.63 \text{ kip}\cdot\text{in}$ ◀

PROBLEM 4.195



In order to increase corrosion resistance, a 2-mm-thick cladding of aluminum has been added to a steel bar as shown. The modulus of elasticity is 200 GPa for steel and 70 GPa for aluminum. For a bending moment of 300 N · m, determine (*a*) the maximum stress in the steel, (*b*) the maximum stress in the aluminum, (*c*) the radius of curvature of the bar.

SOLUTION

Use aluminum as the reference material.

$$n = 1 \text{ in aluminum.} \qquad n = \frac{E_s}{E_a} = \frac{200}{70} = 2.857 \text{ in steel.}$$

Cross section geometry:

$$\text{Steel: } A_s = (46\,\text{mm})(26\,\text{mm}) = 1196\text{ mm}^2 \qquad I_s = \frac{1}{12}(46\,\text{mm})(26\,\text{mm})^3 = 67{,}375\text{ mm}^4$$

$$\text{Aluminum: } A_a = (50\,\text{mm})(30\,\text{mm}) - 1196\,\text{mm}^2 = 304\text{ mm}^2$$

$$I_a = \frac{1}{12}(50\,\text{mm})(30\,\text{mm}^3) - 67{,}375\,\text{mm}^4 = 45{,}125\,\text{mm}^4$$

<u>Transformed section.</u>

$$I = n_a I_a + n_s I_s = (1)(45{,}125) + (2.857)(67{,}375) = 237{,}615\text{ mm}^4 = 237.615\times 10^{-9}\text{ m}^4$$

<u>Bending moment.</u> $M = 300\,\text{N}\cdot\text{m}$

(*a*) Maximum stress in steel: $n_s = 2.857 \qquad y_s = 13\,\text{mm} = 0.013\,\text{m}$

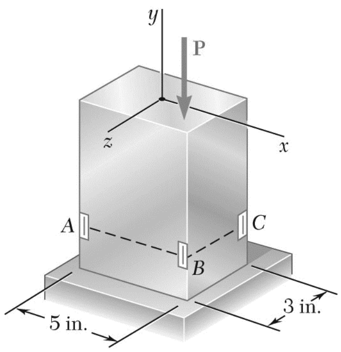
$$\sigma_s = \frac{n_s M y_s}{I} = \frac{(2.857)(300)(0.013)}{237.615\times 10^{-9}} = 46.9\times 10^6\text{ Pa} \qquad \sigma_s = 46.9\,\text{MPa} \blacktriangleleft$$

(*b*) Maximum stress in aluminum: $n_a = 1, \qquad y_a = 15\text{ mm} = 0.015\text{ m}$

$$\sigma_a = \frac{n_a M y_a}{I} = \frac{(1)(300)(0.015)}{237.615\times 10^{-9}} = 18.94\times 10^6\text{ Pa} \qquad \sigma_a = 18.94\text{ MPa} \blacktriangleleft$$

(*c*) Radius of curvative: $\rho = \dfrac{EI}{M} \qquad \rho = \dfrac{(70\times 10^9)(237.615\times 10^{-9})}{300} \qquad \rho = 55.4\text{ m} \blacktriangleleft$

PROBLEM 4.196



A single vertical force **P** is applied to a short steel post as shown. Gages located at *A, B,* and *C* indicate the following strains:

$$\in_A = -500\mu \qquad \in_B = -1000\mu \qquad \in_C = -200\mu$$

Knowing that $E = 29\times 10^6$ psi, determine (*a*) the magnitude of **P**, (*b*) the line of action of **P**, (*c*) the corresponding strain at the hidden edge of the post, where $x = -2.5$ in. and $z = -1.5$ in.

SOLUTION

$$I_x = \frac{1}{12}(5)(3)^3 = 11.25\text{ in}^4 \qquad I_z = \frac{1}{12}(3)(5)^3 = 31.25\text{ in}^4 \qquad A = (5)(3) = 15\text{ in}^2$$

$$M_x = Pz \qquad M_z = -Px$$

$$x_A = -2.5\text{ in.},\quad x_B = 2.5\text{ in.},\quad x_C = 2.5\text{ in.},\quad x_D = -2.5\text{ in.}$$

$$z_A = 1.5\text{ in.},\quad z_B = 1.5\text{ in.},\quad z_C = -1.5\text{ in.},\quad z_D = -1.5\text{ in.}$$

$$\sigma_A = E\varepsilon_A = (29\times 10^6)(-500\times 10^{-6}) = -14{,}500\text{ psi} = -14.5\text{ ksi}$$

$$\sigma_B = E\varepsilon_B = (29\times 10^6)(-1000\times 10^{-6}) = -29{,}000\text{ psi} = -29\text{ ksi}$$

$$\sigma_C = E\varepsilon_C = (29\times 10^6)(-200\times 10^{-6}) = -5800\text{ psi} = -5.8\text{ ksi}$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -0.06667\text{P} - 0.13333M_x - 0.08M_z \qquad (1)$$

$$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -0.06667\text{P} - 0.13333M_x + 0.08M_z \qquad (2)$$

$$\sigma_C = -\frac{P}{A} - \frac{M_x z_C}{I_x} + \frac{M_z x_C}{I_z} = -0.06667\text{P} + 0.13333M_x + 0.08M_z \qquad (3)$$

Substituting the values for σ_A, σ_B, and σ_C into (1), (2), and (3) and solving the simultaneous equations gives

$$M_x = 87\text{ kip}\cdot\text{in},\quad M_z = -90.625\text{ kip}\cdot\text{in},$$ (*a*) $P = 152.25$ kips ◀

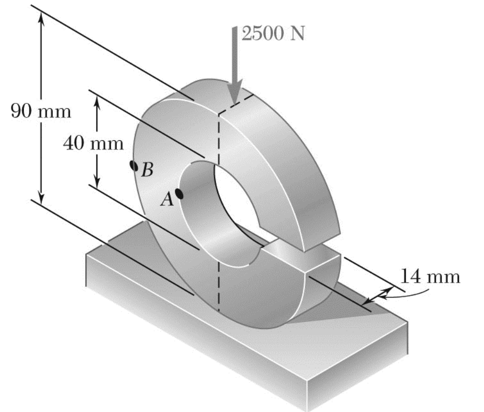
$$x = -\frac{M_z}{P} = -\frac{-90.625}{152.25}$$ (*b*) $x = 0.595$ in. ◀

$$z = \frac{M_x}{P} = \frac{87}{152.25}$$ $z = 0.571$ in. ◀

$$\sigma_D = -\frac{P}{A} - \frac{M_x z_D}{I_x} + \frac{M_z x_D}{I_z} = -0.06667\text{P} + 0.13333M_x - 0.08M_z$$

$$= -(0.06667)(152.25) + (0.13333)(87) - (0.08)(-90.625) = 8.70\text{ ksi}$$

(*c*) Strain at hidden edge: $\varepsilon = \dfrac{\sigma_D}{E} = \dfrac{8.70\times 10^3}{29\times 10^6}$ $\varepsilon = 300\,u$ ◀



PROBLEM 4.197

For the split ring shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

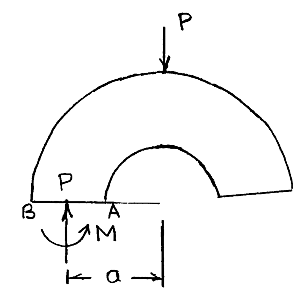
SOLUTION

$$r_1 = \frac{1}{2}40 = 20 \text{ mm}, \qquad r_2 = \frac{1}{2}(90) = 45 \text{ mm} \qquad h = r_2 - r_1 = 25 \text{ mm}$$

$$A = (14)(25) = 350 \text{ mm}^2 \qquad R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 32.5 \text{ mm} \qquad e = \bar{r} - R = 1.6712 \text{ mm}$$

Reduce the internal forces transmitted across section *AB* to a force-couple system at the centroid of the cross section. The bending couple is



$$M = Pa = P\bar{r} = (2500)(32.5\times10^{-3}) = 81.25 \text{ N}\cdot\text{m}$$

(*a*) Point *A*: $r_A = 20$ mm $\quad y_A = 30.8288 - 20 = 10.8288$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{AeR} = -\frac{2500}{350\times10^{-6}} - \frac{(81.25)(10.8288\times10^{-3})}{(350\times10^{-6})(1.6712\times10^{-3})(20\times10^{-3})}$$

$$= -82.4\times10^6 \text{ Pa}$$

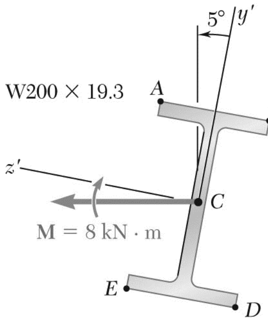
$\sigma_A = -82.4$ MPa ◀

(*b*) Point *B*: $r_B = 45$ mm $\quad y_B = 30.8288 - 45 = -14.1712$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_B} = -\frac{2500}{350\times10^{-6}} - \frac{(81.25)(-14.1712\times10^{-3})}{(350\times10^{-6})(1.6712\times10^{-3})(45\times10^{-3})}$$

$$= 36.6\times10^6 \text{ Pa}$$

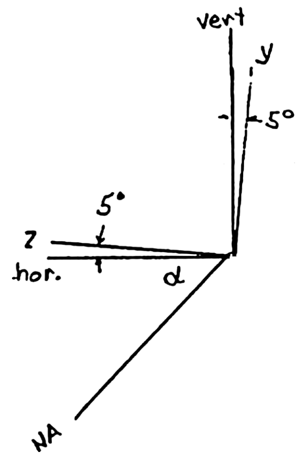
$\sigma_B = 36.6$ MPa ◀



PROBLEM 4.198

A couple **M** of moment 8 kN · m acting in a vertical plane is applied to a W200 × 19.3 rolled-steel beam as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum stress in the beam.

SOLUTION



For $W200 \times 19.3$ rolled steel sector,

$$I_{z'} = 16.6 \times 10^6 \text{ mm}^4 = 16.6 \times 10^{-6} \text{ m}^4$$

$$I_{y'} = 1.15 \times 10^6 \text{ mm}^4 = 1.15 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = -y_E = \frac{203}{2} = 101.5 \text{ mm}$$

$$z_A = -z_B = -z_D = z_E = \frac{102}{2} = 51 \text{ mm}$$

$$M_z = (8 \times 10^3)\cos 5° = 7.9696 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_y = -(8 \times 10^3)\sin 5° = -0.6972 \times 10^3 \text{ N} \cdot \text{m}$$

(*a*) $$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{16.6 \times 10^{-6}}{1.15 \times 10^{-6}}\tan(-5°) = -1.2629$$

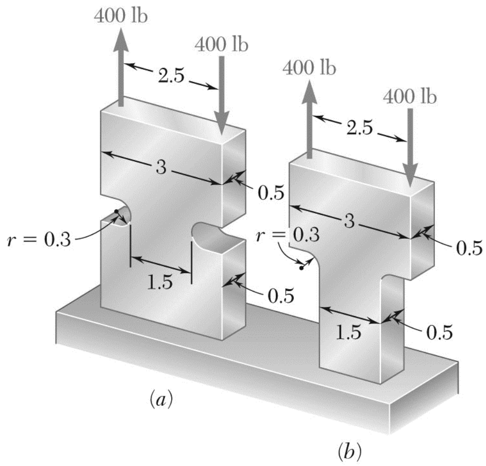
$$\varphi = -51.6°$$

$$\alpha = 51.6° - 5° \qquad \alpha = 46.6° \blacktriangleleft$$

(*b*) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(7.9696 \times 10^3)(-101.5 \times 10^{-3})}{16.6 \times 10^{-6}} + \frac{(-0.6972 \times 10^3)(-51 \times 10^{-3})}{1.15 \times 10^{-6}}$$

$$= 79.6 \times 10^6 \text{ Pa} \qquad \sigma_D = 79.6 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.199

Determine the maximum stress in each of the two machine elements shown.

SOLUTION

For each case, $M = (400)(2.5) = 1000 \text{ lb} \cdot \text{in}$

At the minimum section,

$$I = \frac{1}{12}(0.5)(1.5)^3 = 0.140625 \text{ in}^4$$

$$c = 0.75 \text{ in.}$$

(*a*) $D/d = 3/1.5 = 2$

$r/d = 0.3/1.5 = 0.2$

From Fig 4.32, $K = 1.75$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.75)(1000)(0.75)}{0.140625} = 9.33 \times 10^3 \text{ psi}$$

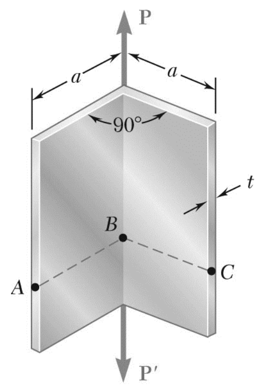
$\sigma_{\max} = 9.33 \text{ ksi}$ ◀

(*b*) $D/d = 3/1.5 = 2$ $r/d = 0.3/1.5 = 0.2$

From Fig. 4.31, $K = 1.50$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.50)(1000)(0.75)}{0.140625} = 8.00 \times 10^3 \text{ psi}$$

$\sigma_{\max} = 8.00 \text{ ksi}$ ◀

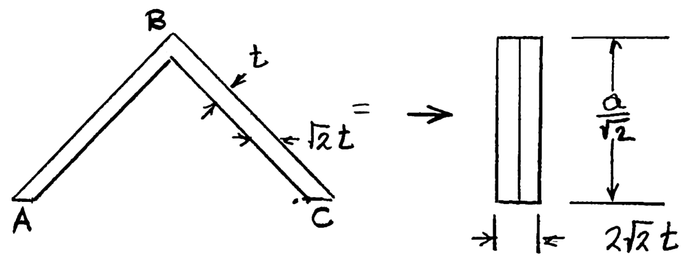


PROBLEM 4.200

The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (a) at A, (b) at B, (c) at C.

SOLUTION

Moment of inertia about centroid:



$$I = \frac{1}{12}\left(2\sqrt{2}t\right)\left(\frac{a}{\sqrt{2}}\right)^3$$

$$= \frac{1}{12}ta^3$$

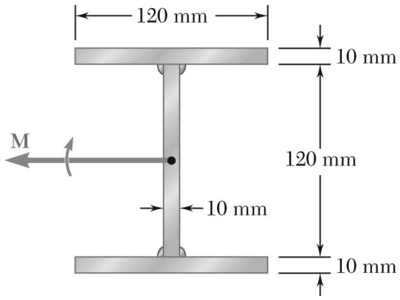
Area: $\quad A = \left(2\sqrt{2}t\right)\left(\frac{a}{\sqrt{2}}\right) = 2at, \quad c = \frac{a}{2\sqrt{2}}$

(a) $\quad \sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3}$ $\qquad \sigma_A = -\frac{P}{2at}$ ◀

(b) $\quad \sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3}$ $\qquad \sigma_B = -\frac{2P}{at}$ ◀

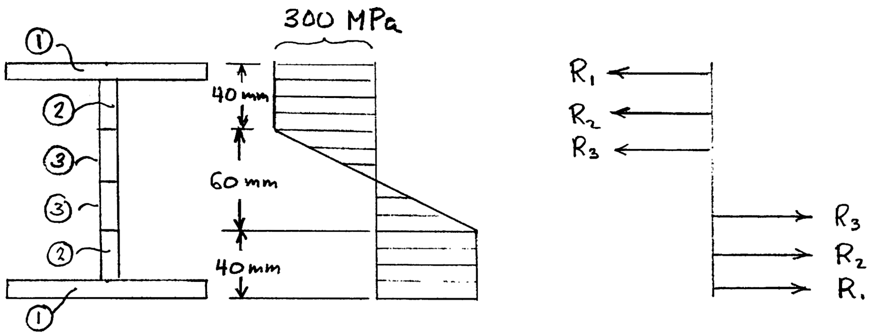
(c) $\quad \sigma_C = \sigma_A$ $\qquad \sigma_C = -\frac{P}{2at}$ ◀

PROBLEM 4.201



Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.

SOLUTION



$$A_1 = (120)(10) = 1200 \text{ mm}^2$$

$$R_1 = \sigma_Y A_1 = (300\times10^6)(1200\times10^{-6}) = 360\times10^3 \text{ N}$$

$$A_2 = (30)(10) = 300 \text{ mm}^2$$

$$R_2 = \sigma_Y A_2 = (300\times10^6)(300\times10^{-6}) = 90\times10^3 \text{ N}$$

$$A_3 = (30)(10) = 300 \text{ mm}^2$$

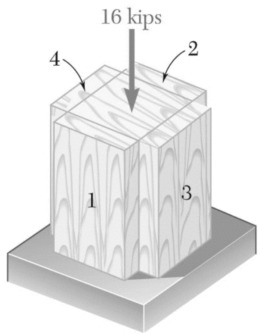
$$R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^6)(300\times10^{-6}) = 45\times10^3 \text{ N}$$

$$y_1 = 65 \text{ mm} = 65\times10^{-3} \text{ m} \qquad y_2 = 45 \text{ mm} = 45\times10^{-3} \text{ m} \qquad y_3 = 20 \text{ mm} = 20\times10^{-3} \text{ m}$$

(*a*) $M = 2(R_1 y_1 + R_2 y_2 + R_3 y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$

$= 56.7\times10^3 \text{ N}\cdot\text{m}$ $\qquad$ $M = 56.7 \text{ kN}\cdot\text{m}$ ◀

(*b*) $\dfrac{y_Y}{\rho} = \dfrac{\sigma_Y}{E} \qquad \rho = \dfrac{E y_Y}{\sigma_Y} = \dfrac{(200\times10^9)(30\times10^{-3})}{300\times10^6}$ $\qquad$ $\rho = 20$ m ◀

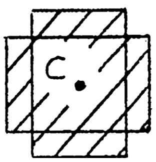


PROBLEM 4.202

A short column is made by nailing four 1×4-in. planks to a 4×4-in. timber. Determine the largest compressive stress created in the column by a 16-kip load applied as shown in the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) planks 1 and 2 are removed, (*d*) planks 1, 2, and 3 are removed, (*e*) all planks are removed.

SOLUTION

(*a*) Centric loading: $M = 0 \qquad \sigma = -\dfrac{P}{A}$

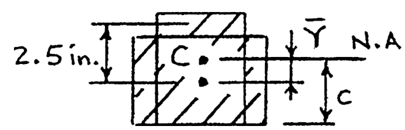


$$A = (4)(4) + (4)(1)(4) = 32 \text{ in}^2$$

$$\sigma = -\frac{16 \times 10^3}{32}$$

$\sigma = -500$ psi ◀

(*b*) Eccentric loading: $M = Pe \qquad \sigma = -\dfrac{P}{A} - \dfrac{Pec}{I}$



$$A = (4)(4) + (3)(1)(3) = 28 \text{ in}^2 \qquad e = \bar{y}$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{A} = \frac{(1)(4)(2.5)}{28} = 0.35714 \text{ in.}$$

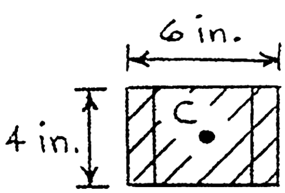
$$I = \Sigma(\bar{I} + Ad^2) = \frac{1}{12}(6)(4)^3 + (6)(4)(0.35714)^2$$

$$+\frac{1}{12}(4)(1)^3 + (4)(1)(2.14286)^2 = 53.762 \text{ in}^4$$

$$\sigma = -\frac{16\times10^3}{28} - \frac{(16\times10^3)(0.35714)(2.35714)}{53.762}$$

$\sigma = -822$ psi ◀

(*c*) Centric loading: $M = 0 \qquad \sigma = -\dfrac{P}{A}$



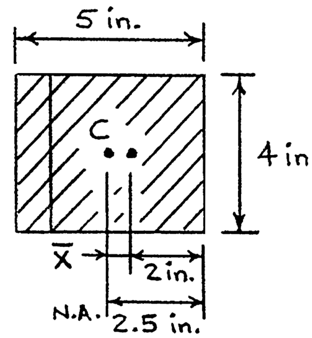
$$A = (6)(4) = 24 \text{ in}^2$$

$$\sigma = -\frac{16\times10^3}{24}$$

$\sigma = -667$ psi ◀

PROBLEM 4.202 *(Continued)*

(*d*) Eccentric loading: $M = Pe \qquad \sigma = -\dfrac{P}{A} - \dfrac{Pec}{I}$



$A = (4)(1) = (1)(4)(1) = 20 \text{ in}^2 \qquad e = \bar{x}$

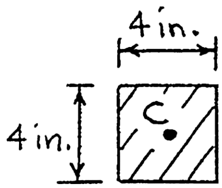
$\bar{x} = 2.5 - 2 = 0.5 \text{ in.}$

$I = \dfrac{1}{12}(4)(5)^3 = 41.667 \text{ in}^4$

$$\sigma = -\frac{16\times10^3}{20} - \frac{(16\times10^3)(0.5)(2.5)}{41.667}$$

$\sigma = -1280 \text{ psi}$ ◀

(*e*) Centric loading: $M = 0 \qquad \sigma = -\dfrac{P}{A}$

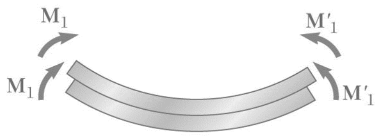


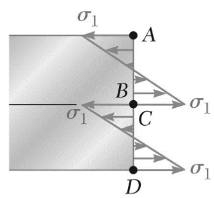
$A = (4)(4) = 16 \text{ in}^2$

$$\sigma = -\frac{16\times10^3}{16}$$

$\sigma = -1000 \text{ psi}$ ◀

PROBLEM 4.203





Two thin strips of the same material and same cross section are bent by couples of the same magnitude and glued together. After the two surfaces of contact have been securely bonded, the couples are removed. Denoting by σ_1 the maximum stress and by ρ_1 the radius of curvature of each strip while the couples were applied, determine (*a*) the final stresses at points *A, B, C,* and *D,* (*b*) the final radius of curvature.

SOLUTION

Let b = width and t = thickness of one strip.

Loading one strip, $M = M_1$

$$I_1 = \frac{1}{12}bt^3, \quad c = \frac{1}{2}t$$

$$\sigma_1 = \frac{M_1 c}{I} = \frac{\sigma M_1}{bt^2}$$

$$\frac{1}{\rho_1} = \frac{M_1}{EI_1} = \frac{12 M_1}{Et^3}$$

After M_1 is applied to each of the strips, the stresses are those given in the sketch above. They are

$$\sigma_A = -\sigma_1, \quad \sigma_B = \sigma_1, \quad \sigma_C = -\sigma_1, \quad \sigma_D = \sigma_1$$

The total bending couple is $2M_1$.

After the strips are glued together, this couple is removed.

$$M' = 2M_1, \quad I' = \frac{1}{12}b(2t)^3 = \frac{2}{3}bt^3 \quad c = t$$

The stresses removed are

$$\sigma' = -\frac{M'y}{I} = -\frac{2M_1 y}{\frac{2}{3}bt^3} = -\frac{3M_1 y}{bt^2}$$

$$\sigma'_A = -\frac{3M_1}{bt^2} = -\frac{1}{2}\sigma_1, \quad \sigma'_B = \sigma'_C = 0, \quad \sigma'_D = \frac{3M_1}{bt^2} = \frac{1}{2}\sigma_1$$

PROBLEM 4.203 *(Continued)*

(*a*) Final stresses: $\sigma_A = -\sigma_1 - (-\frac{1}{2}\sigma_1)$ $\quad \sigma_A = -\frac{1}{2}\sigma_1$ ◀

$\sigma_B = \sigma_1$ ◀

$\sigma_C = -\sigma_1$ ◀

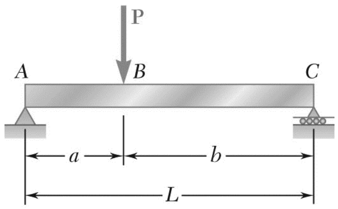
$\sigma_D = \sigma_1 - \frac{1}{2}\sigma_1$ $\quad \sigma_D = \frac{1}{2}\sigma_1$ ◀

$$\frac{1}{\rho'} = \frac{M'}{EI'} = \frac{2M_1}{E\,\frac{2}{3}bt^3} = \frac{3M_1}{Et^3} = \frac{1}{4}\frac{1}{\rho'}$$

(*b*) Final radius: $\frac{1}{\rho} = \frac{1}{\rho_1} - \frac{1}{\rho'} = \frac{1}{\rho_1} - \frac{1}{4}\frac{1}{\rho_1} = \frac{3}{4}\frac{1}{\rho_1}$

$\rho = \frac{4}{3}\rho_1$ ◀

CHAPTER 5



PROBLEM 5.1

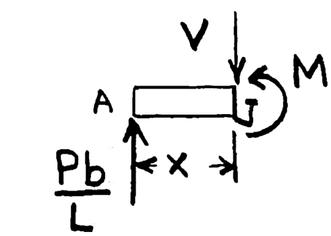
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Reactions:

$$\circlearrowleft + \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

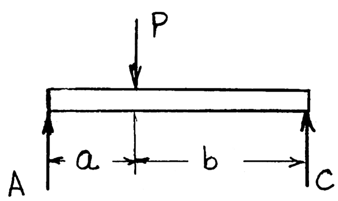
$$+\circlearrowright \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$



From A to B: $\quad 0 < x < a$

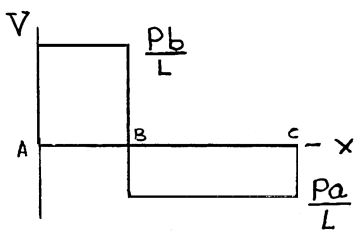
$$+\uparrow \Sigma F_y = 0: \quad \frac{Pb}{L} - V = 0$$

$$V = \frac{Pb}{L} \blacktriangleleft$$

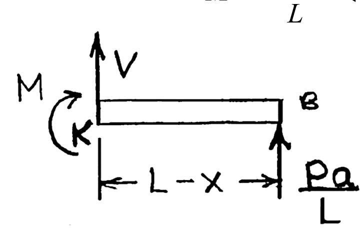


$$+\circlearrowright \Sigma M_J = 0: \quad M - \frac{Pb}{L}x = 0$$

$$M = \frac{Pbx}{L} \blacktriangleleft$$

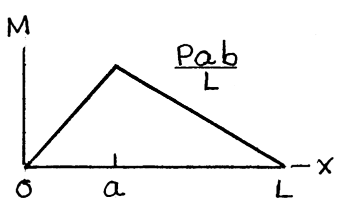


From B to C: $\quad a < x < L$



$$+\uparrow \Sigma F_y = 0: \quad V + \frac{Pa}{L} = 0$$

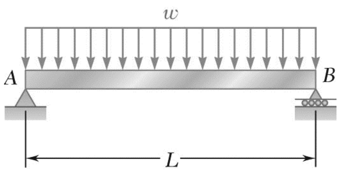
$$V = -\frac{Pa}{L} \blacktriangleleft$$



$$+\circlearrowright \Sigma M_K = 0: \quad -M + \frac{Pa}{L}(L - x) = 0$$

$$M = \frac{Pa(L - x)}{L} \blacktriangleleft$$

At section B: $$M = \frac{Pab}{L^2} \blacktriangleleft$$

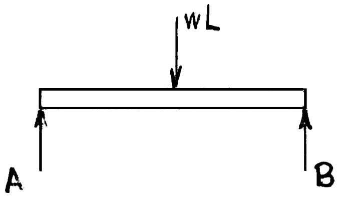


PROBLEM 5.2

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Reactions:



$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

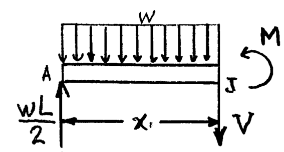
$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

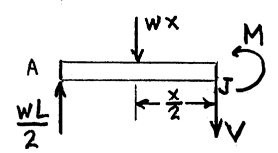
Free body diagram for determining reactions.

Over whole beam, $\quad 0 < x < L$

Place section at x.

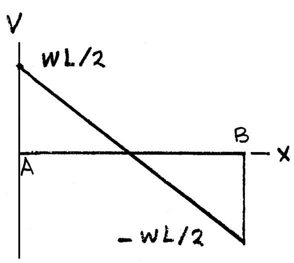
Replace distributed load by equivalent concentrated load.

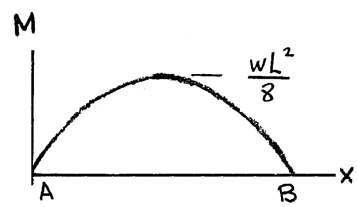




$$+\uparrow \Sigma F_y = 0: \quad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$





$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

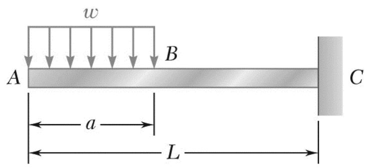
$$M = \frac{w}{2}(Lx - x^2)$$

$$M = \frac{w}{2}x(L - x) \blacktriangleleft$$

Maximum bending moment occurs at $x = \frac{L}{2}$.

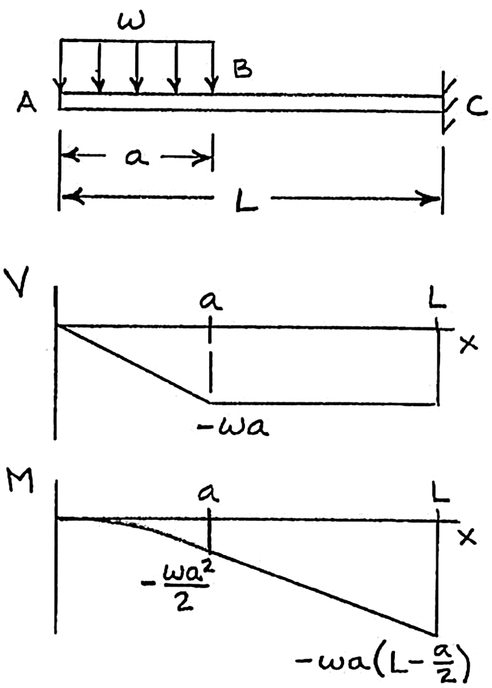
$$M_{\max} = \frac{wL^2}{8} \blacktriangleleft$$

PROBLEM 5.3

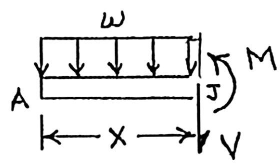


For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



From A to B $(0 < x < a)$:



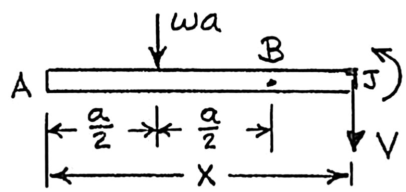
$+\uparrow \Sigma F_y = 0: \ -wx - V = 0$

$V = -wx$ ◀

$+\circlearrowleft \Sigma M_J = 0: \ (wx)\frac{x}{2} + M = 0$

$M = -\frac{wx^2}{2}$ ◀

From B to C $(a < x < L)$:

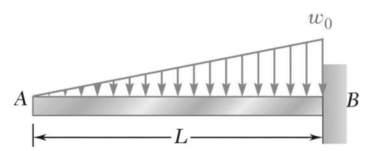


$+\uparrow \Sigma F_y = 0: \ -wa - V = 0$

$V = -wa$ ◀

$+\circlearrowleft \Sigma M_J = 0: \ (wa)\left(x - \frac{a}{2}\right) + M = 0$

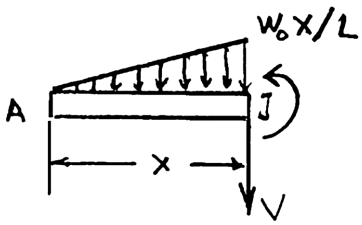
$M = -wa\left(x - \frac{a}{2}\right)$ ◀

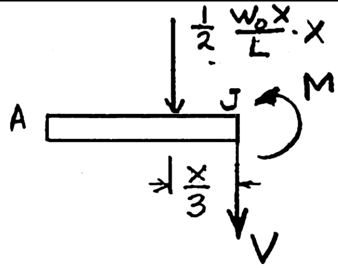


PROBLEM 5.4

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

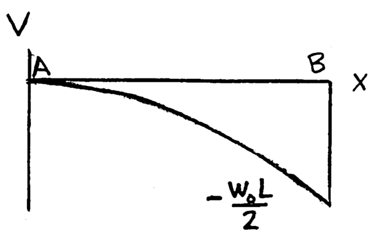
SOLUTION





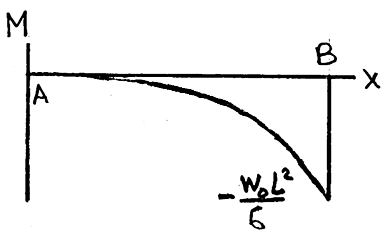
$$+\uparrow \Sigma F_y = 0: \quad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$$

$$V = -\frac{w_0 x^2}{2L} \blacktriangleleft$$



$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{1}{2}\frac{w_0 x}{L}\cdot x \cdot \frac{x}{3} + M = 0$$

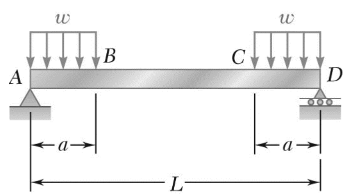
$$M = -\frac{w_0 x^3}{6L} \blacktriangleleft$$



At $x = L$,

$$V = -\frac{w_0 L}{2} \qquad |V|_{max} = \frac{w_0 L}{2} \blacktriangleleft$$

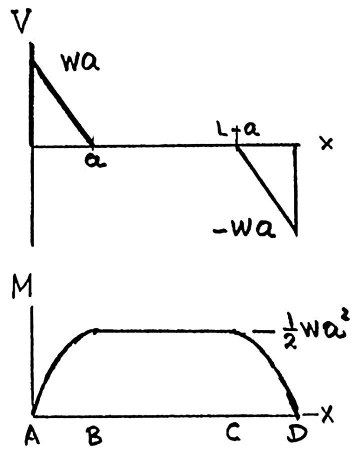
$$M = -\frac{w_0 L^2}{6} \qquad |M|_{max} = \frac{w_0 L^2}{6} \blacktriangleleft$$



PROBLEM 5.5

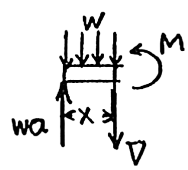
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

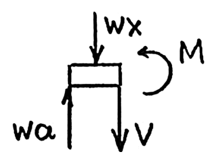
SOLUTION



Reactions: $A = D = wa$

From A to B: $0 < x < a$



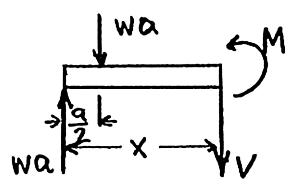


$+\uparrow \sum F_y = 0: \quad wa - wx - V = 0$

$$V = w(a - x) \blacktriangleleft$$

$+\circlearrowleft \sum M_J = 0: \quad -wax + (wx)\dfrac{x}{2} + M = 0$

$$M = w\left(ax - \frac{x^2}{2}\right) \blacktriangleleft$$

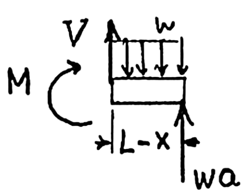


From B to C: $a < x < L - a$

$\sum F_y = 0: \quad wa - wa - V = 0$

$$V = 0 \blacktriangleleft$$

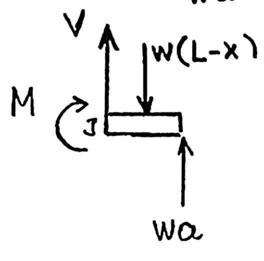
$+\circlearrowleft \sum M_J = 0: \quad -wax + wa\left(x - \dfrac{a}{2}\right) + M = 0 \qquad M = \dfrac{1}{2}wa^2 \blacktriangleleft$



From C to D: $L - a < x < L$

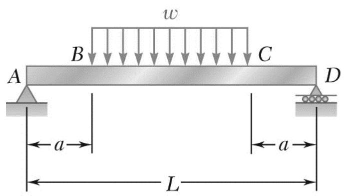
$+\uparrow \sum F_y = 0: \quad V - w(L - x) + wa = 0$

$$V = w(L - x - a) \blacktriangleleft$$



$+\circlearrowleft \sum M_J = 0: \quad -M - w(L - x)\left(\dfrac{L - x}{2}\right) + wa(L - x) = 0$

$$M = wa[(L - x) - \frac{1}{2}(L - x)^2] \blacktriangleleft$$



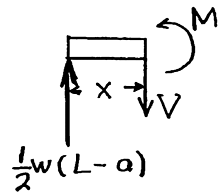
PROBLEM 5.6

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Calculate reactions after replacing distributed load by an equivalent concentrated load.

Reactions are

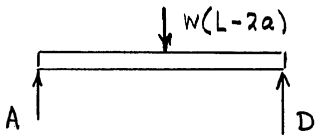


$$A = D = \frac{1}{2}w(L-2a)$$

From A to B: $\quad 0 < x < a$

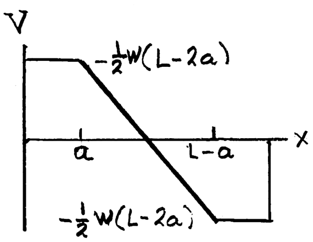
$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{2}w(L-2a) - V = 0$$

$$V = \frac{1}{2}w(L-2a) \blacktriangleleft$$



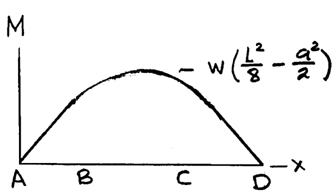
$$+\circlearrowleft \Sigma M = 0: \quad -\frac{1}{2}w(L-2a) + M = 0$$

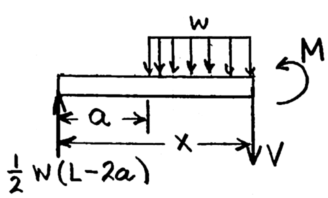
$$M = \frac{1}{2}w(L-2a)x \blacktriangleleft$$

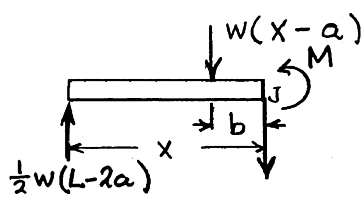


From B to C: $\quad a < x < L-a$

$$b = \frac{x-a}{2}$$







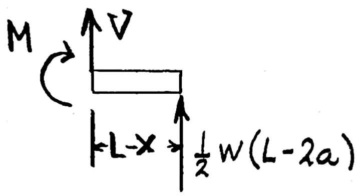
Place section cut at x. Replace distributed load by equivalent concentrated load.

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{2}w(L-2a) - w(x-a) - V = 0 \qquad V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$

$$+\circlearrowleft M_J = 0: \quad -\frac{1}{2}w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M = 0$$

$$M = \frac{1}{2}w[(L-2a)x - (x-a)^2] \blacktriangleleft$$

PROBLEM 5.6 *(Continued)*



From C to D: $L - a < x < L$

$$+\uparrow \Sigma F_y = 0: \quad V + \frac{1}{2}w(L - 2a) = 0$$

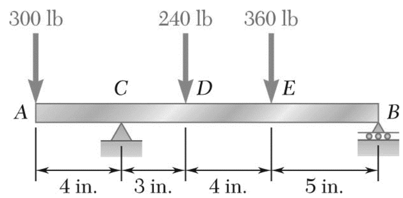
$$V = -\frac{w}{2}(L - 2a) \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -M + \frac{1}{2}w(L - 2a)(L - x) = 0$$

$$M = \frac{1}{2}w(L - 2a)(L - x) \blacktriangleleft$$

At $x = \dfrac{L}{2}$,

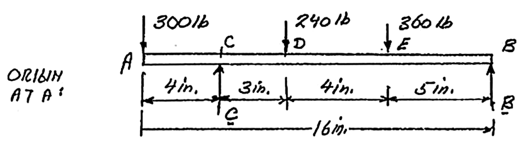
$$M_{\max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right) \blacktriangleleft$$



PROBLEM 5.7

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

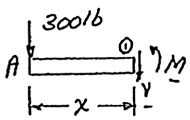
SOLUTION



Reactions:

$$+\circlearrowleft \Sigma M_C = 0: \quad (300)(4) - (240)(3) - (360)(7) + 12B = 0 \qquad \underline{\mathbf{B}} = 170 \text{ lb} \uparrow$$

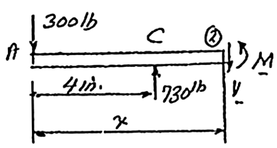
$$+\uparrow \Sigma F_y = 0: \quad -300 + C - 240 - 360 + 170 = 0 \qquad \underline{\mathbf{C}} = 730 \text{ lb} \uparrow$$



From *A* to *C*:

$$+\uparrow \Sigma F_y = 0: \quad -300 - V = 0 \qquad V = -300 \text{ lb}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad (300)(x) + M = 0 \qquad M = -300x$$

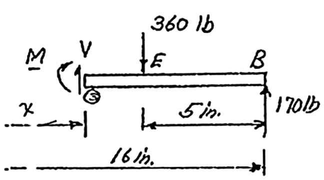


From *C* to *D*:

$$+\uparrow \Sigma F_y = 0: \quad -300 + 730 - V = 0 \qquad V = +430 \text{ lb}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad (300)x - (730)(x - 4) + M = 0$$

$$M = -2920 + 430x$$

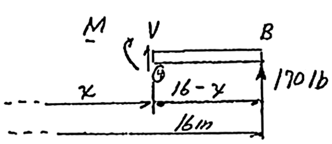


From *D* to *E*:

$$+\uparrow \Sigma F_y = 0: \quad V - 360 + 170 = 0 \qquad V = +190 \text{ lb}$$

$$+\circlearrowleft \Sigma M_3 = 0: \quad (170)(16 - x) - (360)(11 - x) - M = 0$$

$$M = -1240 + 190x$$

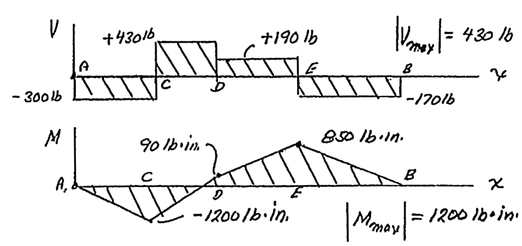


From *E* to *B*:

$$+\uparrow \Sigma F_y = 0: \quad V + 170 = 0 \qquad V = -170 \text{lb}$$

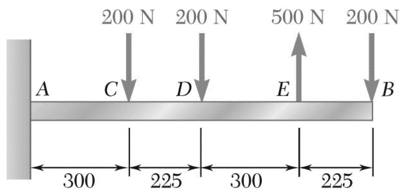
$$+\circlearrowleft \Sigma M_4 = 0: \quad (170)(16 - x) - M = 0$$

$$M = 2720 - 170x$$



(*a*) $|V|_{\max} = 430 \text{ lb}$ ◀

(*b*) $|M|_{\max} = 1200 \text{ lb} \cdot \text{in}$ ◀

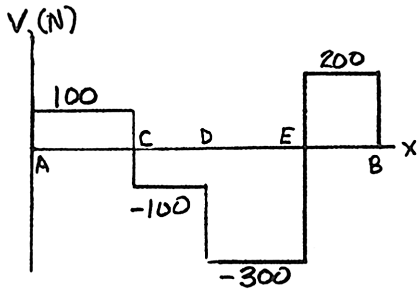


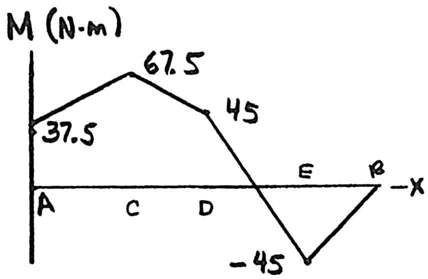
Dimensions in mm

PROBLEM 5.8

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

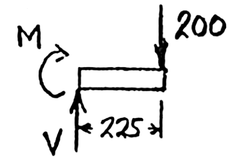
SOLUTION





At B, $V = 200\text{N}, \quad M = 0$

At E^+,

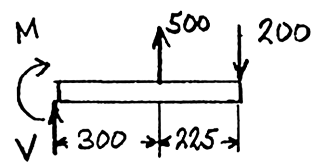


$+\uparrow \Sigma F_y = 0: \quad V - 200 = 0 \quad V = 200\text{N}$

$+\circlearrowleft \Sigma M_E = 0: \quad -M - (0.225)(200) = 0$

$M = -45\ \text{N} \cdot \text{m}$

At D^+,



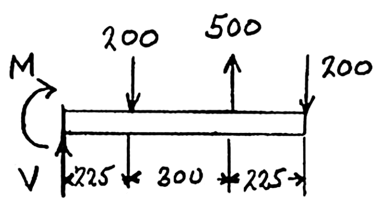
$+\uparrow \Sigma F_y = 0: \quad V + 500 - 200 = 0$

(*a*) $V = -300\ \text{N}$ ◀

$+\circlearrowleft \Sigma M_D = 0: \quad -M + (0.3)(500) - (0.525)(200) = 0$

$M = 45\ \text{N} \cdot \text{m}$

At C^+,



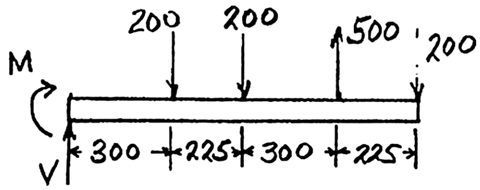
$+\uparrow \Sigma F_y = 0: \quad V - 200 + 500 - 200 = 0 \quad V = -100\text{N}$

$+\circlearrowleft \Sigma M_C = 0: \quad -M - (0.225)(200) + (0.525)(500) - (0.75)(200) = 0$

(*b*) $M = 67.5\ \text{N} \cdot \text{m}$ ◀

PROBLEM 5.8 *(Continued)*

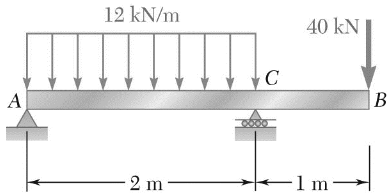
At A,



$$+\uparrow \sum F_y = 0: \quad V - 200 - 200 + 500 - 200 = 0 \qquad V = 100 \text{ N}$$

$$+\circlearrowleft \sum M_A = 0: \quad -M - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) = 0$$

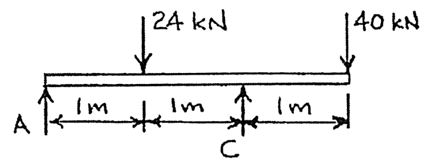
$$M = 37.5 \text{ N} \cdot \text{m}$$

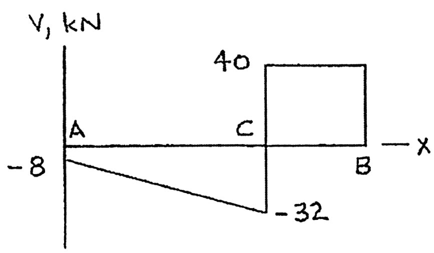


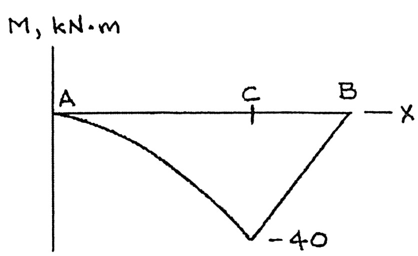
PROBLEM 5.9

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION







Reactions:

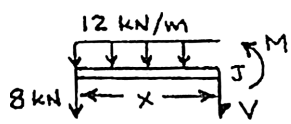
$$+\circlearrowleft \Sigma M_C = 0: \quad -2A + (1)(24) - (1)(40) = 0$$

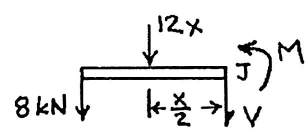
$$\boldsymbol{A} = -8\,\text{kN} = 8\,\text{kN} \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 2C - (1)(24) - (3)(40) = 0$$

$$\boldsymbol{C} = 72\,\text{kN} = 72\,\text{kN} \uparrow$$

<u>*A* to *C*.</u> $\quad 0 < x < 2\text{m}$



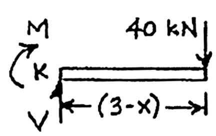


$$+\uparrow \Sigma F_y = 0: \quad -8 - 12x - V = 0 \quad V = (-8 - 12x)\,\text{kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -8x - (12x)\left(\frac{x}{2}\right) - M = 0$$

$$M = (-8x - 6x^2)\,\text{kN} \cdot \text{m}$$

<u>*C* to *B*.</u> $\quad 2\text{m} < x < 3\text{m}$



$$+\uparrow \Sigma F_y = 0: \quad V - 40 = 0$$

$$V = 40\,\text{kN}$$

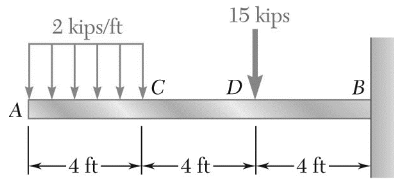
$$+\circlearrowleft \Sigma M_K = 0: \quad -M - (3 - x)(40) = 0$$

$$M = (40x - 120)\,\text{kN} \cdot \text{m}$$

From the diagrams, $\quad$ (*a*) $|V|_{max} = 40.0\,\text{kN}$ ◀

(*b*) $|M|_{max} = 40.0\,\text{kN} \cdot \text{m}$ ◀

PROBLEM 5.10

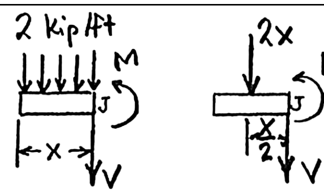


Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

A to *C*:

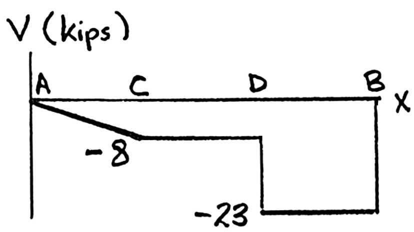
$0 < x < 4$ ft



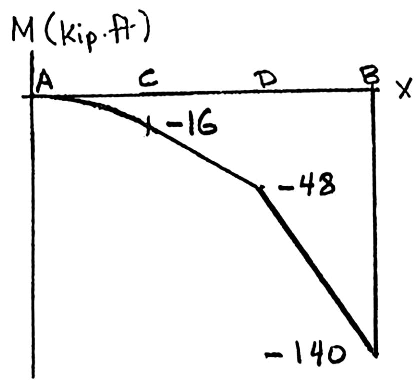
$+\uparrow \Sigma F_y = 0: \quad -V - 2x = 0 \qquad V = -2x$ kips

$+\circlearrowleft \Sigma M_J = 0: \quad M + (2x)\left(\dfrac{x}{2}\right) = 0$

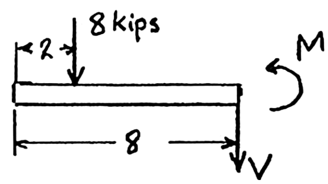
$M = -x$ kip · ft



At *C*, $\quad V = -8$ kips $\quad M = -16$ kip · ft



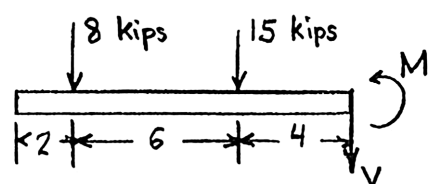
At D^-,



$+\uparrow \Sigma F_y = 0: \quad -8 - V = 0 \qquad V = -8$ kips

$+\circlearrowleft \Sigma M_D = 0: \quad (6)(8) - M = 0 \qquad M = -48$ kip · ft

At B^-,



$+\uparrow \Sigma F_y = 0: \quad -8 - 15 - V = 0$

$V = -23$ kips

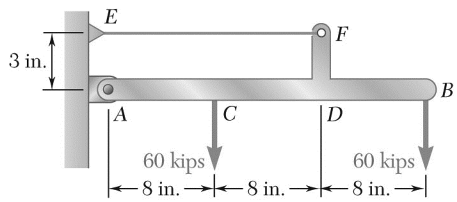
$+\circlearrowleft \Sigma M_B = 0: \quad -(10)(8) - (4)(15) - M = 0$

$M = -140$ kip · ft

From the diagrams:

(*a*) $|V|_{max} = 23.0$ kips ◀

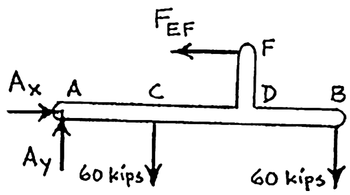
(*b*) $|M|_{max} = 140.0$ kip · ft ◀



PROBLEM 5.11

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



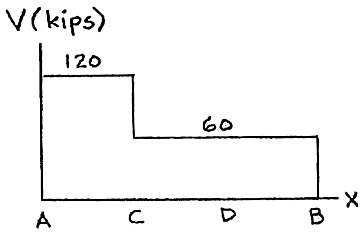
Reactions:

$$+\circlearrowleft \Sigma M_A = 0: \quad 3F_{EF} - (8)(60) - (24)(60) = 0$$

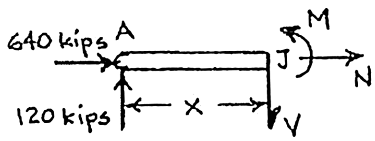
$$F_{EF} = 640 \text{ kips}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 640 = 0 \quad A_x = 640 \text{ kips} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 60 - 60 = 0 \quad A_y = 120 \text{ kips} \uparrow$$



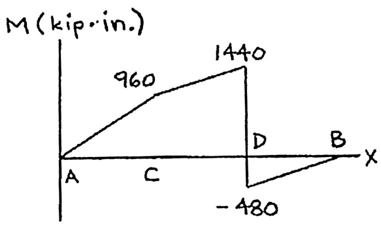
From A to C: $\quad (0 < x < 8 \text{ in.})$



$$+\uparrow \Sigma F_y = 0:$$

$$120 - V = 0$$

$$V = 120 \text{ kips}$$



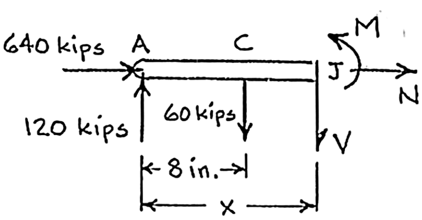
$$+\circlearrowleft \Sigma M_J = 0: \quad M - 120x = 0 \quad M = 120x \text{ kip} \cdot \text{in}$$

From C to D: $\quad (8 \text{ in.} < x < 16 \text{ in.})$

$$+\uparrow \Sigma F_Y = 0: \quad 120 - 60 - V = 0 \quad V = 60 \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - 120x + 60(x - 8) = 0$$

$$M = (60x + 480) \text{ kips} \cdot \text{in}$$

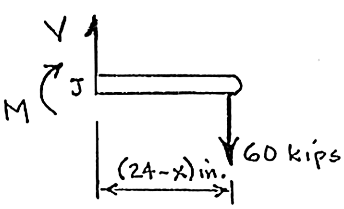


From D to B: $\quad (16 \text{ in.} < x < 24 \text{ in.})$

$$+\uparrow \Sigma F_y = 0: \quad V - 60 = 0 \quad V = 60 \text{ kips}$$

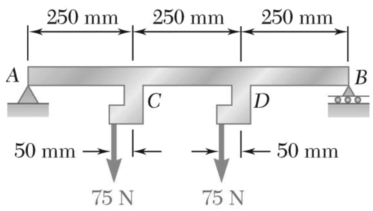
$$+\circlearrowleft \Sigma M_J = 0: \quad -M - 60(24 - x) = 0$$

$$M = (60x - 1440) \text{ kip} \cdot \text{in}$$



(*a*) $|V|_{max} = 120.0 \text{ kips}$ ◀

(*b*) $|M|_{max} = 1440 \text{ kip} \cdot \text{in} = 120.0 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 5.12

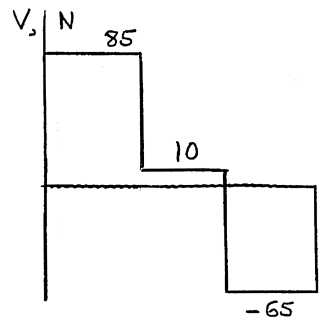
Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Reaction at A: $+\circlearrowleft M_B = 0$: $-0.750 R_A + (0.550)(75) + (0.300)(75) = 0$

$\boldsymbol{R}_A = 85\,\text{N}\uparrow$

Also, $\boldsymbol{R}_B = 65\,\text{N}\uparrow$

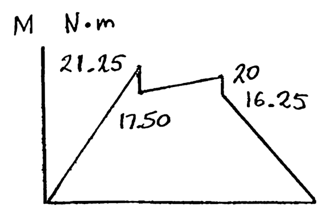


A to C: $V = 85\text{ N}$

C to D: $V = 10\text{ N}$

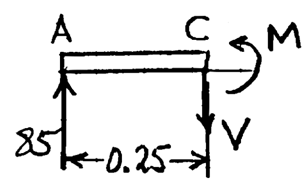
D to B: $V = -65\text{ N}$

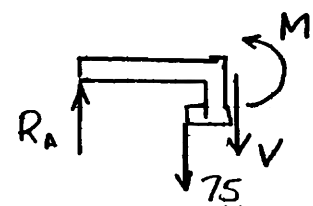
At A and B, $M = 0$



Just to the left of C,

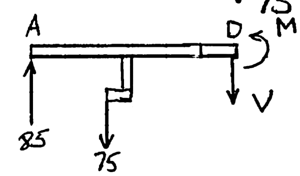
$$+\circlearrowleft \Sigma M_C = 0: \ -(0.25)(85) + M = 0$$
$$M = 21.25\text{ N}\cdot\text{m}$$





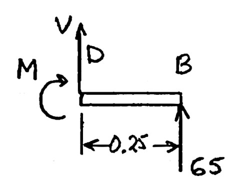
Just to the right of C,

$$+\circlearrowleft \Sigma M_C = 0: \quad -(0.25)(85) + (0.050)(75) + M = 0$$
$$M = 17.50\text{ N}\cdot\text{m}$$



Just to the left of D,

$$+\circlearrowleft \Sigma M_D = 0: \quad -(0.50)(85) + (0.300)(75) + M = 0$$
$$M = 20\text{ N}\cdot\text{m}$$

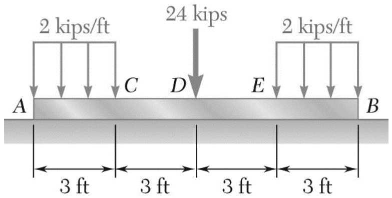


Just to the right of D,

$$+\circlearrowleft \Sigma M_D = 0: \quad -M + (0.25)(65) = 0$$
$$M = 16.25\text{ kN}$$

(*a*) $|V|_{\max} = 85.0\text{ N}$ ◀

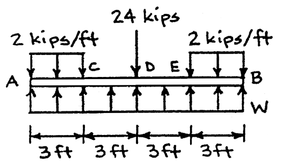
(*b*) $|M|_{\max} = 21.25\text{ N}\cdot\text{m}$ ◀

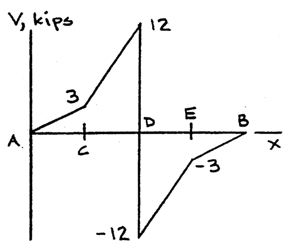


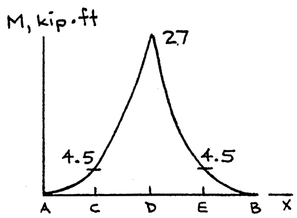
PROBLEM 5.13

Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



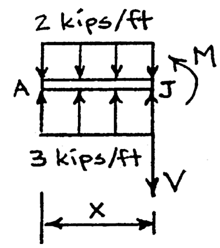




Over the whole beam,

$$+\uparrow \Sigma F_y = 0: \quad 12w - (3)(2) - 24 - (3)(2) = 0 \qquad w = 3 \text{ kips/ft}$$

A to *C*: $(0 \le x < 3 \text{ ft})$



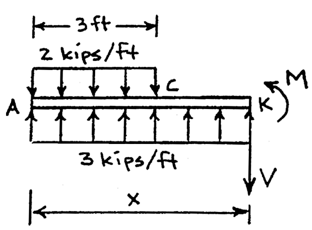
$$+\uparrow \Sigma F_y = 0: \quad 3x - 2x - V = 0 \qquad V = (x) \text{ kips}$$

$$+\Sigma M_J = 0: \quad -(3x)\frac{x}{2} + (2x)\frac{x}{2} + M = 0 \qquad M = (0.5x^2) \text{ kip} \cdot \text{ft}$$

At *C*, $x = 3$ ft

$$V = 3 \text{ kips}, \quad M = 4.5 \text{ kip} \cdot \text{ft}$$

C to *D*: $(3 \text{ ft} \le x < 6 \text{ ft})$



$$+\uparrow \Sigma F_y = 0: \quad 3x - (2)(3) - V = 0 \qquad V = (3x - 6) \text{ kips}$$

$$+\circlearrowleft \Sigma MK = 0: \quad -(3x)\left(\frac{x}{2}\right) + (2)(3)\left(x - \frac{3}{2}\right) + M = 0$$

$$M = (1.5x^2 - 6x + 9) \text{ kip} \cdot \text{ft}$$

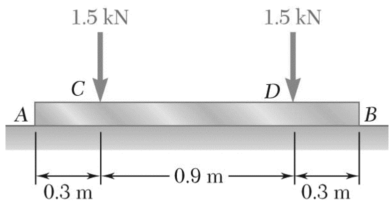
At D^-, $x = 6$ ft

$$V = 12 \text{ kips}, \qquad M = 27 \text{ kip} \cdot \text{ft}$$

D to *B*: Use symmetry to evaluate.

(*a*) $|V|_{max} = 12.00 \text{ kips}$ ◀

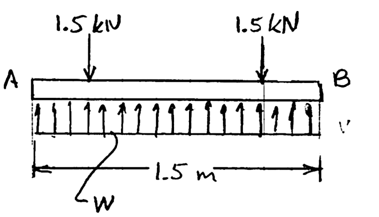
(*b*) $|M|_{max} = 27.0 \text{ kip} \cdot \text{ft}$ ◀

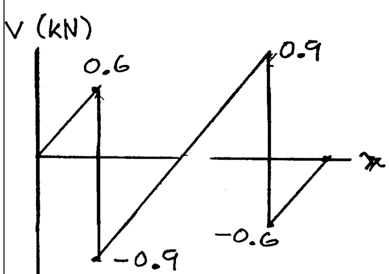


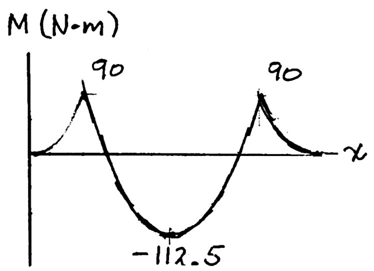
PROBLEM 5.14

Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

SOLUTION

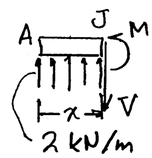






Over the whole beam,

$$+\uparrow \Sigma F_y = 0:\quad 1.5w - 1.5 - 1.5 = 0 \qquad\qquad w = 2\text{ kN/m}$$



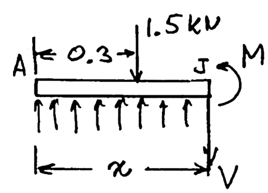
A to C: $\quad 0 \le x < 0.3$ m

$$+\uparrow \Sigma F_y = 0:\quad 2x - V = 0 \qquad\qquad V = (2x)\text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0:\quad -(2x)\left(\frac{x}{2}\right) + M = 0 \qquad\qquad M = (x^2)\text{ kN}\cdot\text{m}$$

At C^-, $\quad x = 0.3$ m

$$V = 0.6\text{ kN},\quad M = 0.090\text{ kN}\cdot\text{m}$$
$$= 90\text{ N}\cdot\text{m}$$



C to D: $\quad 0.3\text{ m} < x < 1.2$ m

$$+\uparrow \Sigma F_y = 0:\quad 2x - 1.5 - V = 0 \qquad\qquad V = (2x - 1.5)\text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0:\quad -(2x)\left(\frac{x}{2}\right) + (1.5)(x - 0.3) + M = 0$$

$$M = (x^2 - 1.5x + 0.45)\text{ kN}\cdot\text{m}$$

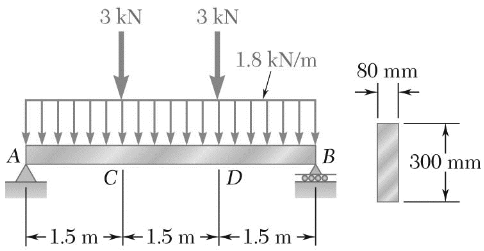
At the center of the beam: $x = 0.75$ m

$$V = 0 \qquad M = -0.1125\text{ kN}\cdot\text{m}$$
$$= -112.5\text{ N}\cdot\text{m}$$

At C^+, $\quad x = 0.3$ m, $\quad V = -0.9$ kN

(a) Maximum $|V| = 0.9\text{ kN} = 900\text{ N}$ ◀

(b) Maximum $|M| = 112.5\text{ N}\cdot\text{m}$ ◀



PROBLEM 5.15

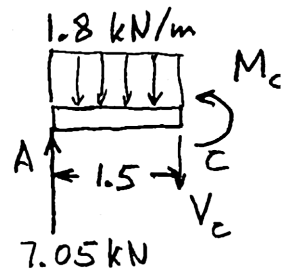
For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

Reaction at A:

$$+\circlearrowleft M_B = 0: \quad -4.5A + (3.0)(3) + (1.5)(3) + (1.8)(4.5)(2.25) = 0 \qquad \mathbf{A} = 7.05 \text{ kN} \uparrow$$

Use AC as free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (7.05)(1.5) + (1.8)(1.5)(0.75) = 0$$

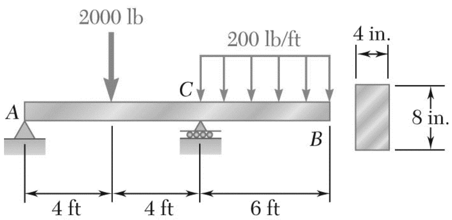
$$M_C = 8.55 \text{ kN} \cdot \text{m} = 8.55 \times 10^3 \text{ N} \cdot \text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(80)(300)^3 = 180 \times 10^6 \text{ mm}^4$$

$$= 180 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}(300) = 150 \text{ mm} = 0.150 \text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(8.55 \times 10^3)(0.150)}{180 \times 10^{-6}} = 7.125 \times 10^6 \text{ Pa} \qquad \sigma = 7.13 \text{ MPa} \blacktriangleleft$$

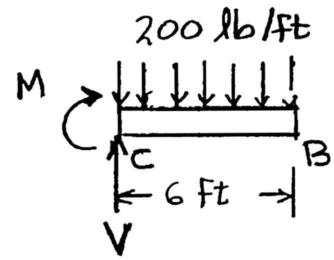


PROBLEM 5.16

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

Use *CB* as free body.



$$+\circlearrowleft M_C = 0: \quad -M - (200)(6)\left(\frac{6}{2}\right) = 0$$

$$M = -3600 \text{ lb}\cdot\text{ft}$$

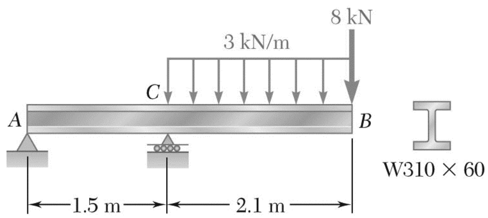
$$= -43.2\times 10^3 \text{ lb}\cdot\text{in}$$

For rectangular section,

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(4)(8)^3 = 170.667 \text{ in}^3$$

$$c = \frac{1}{2}h = 4 \text{ in.}$$

$$\sigma = \frac{|M|c}{I} = \frac{(43.2\times 10^3)(4)}{170.667} = 1.0125\times 10^3 \text{ psi} \qquad \sigma = 1.013 \text{ ksi} \blacktriangleleft$$



PROBLEM 5.17

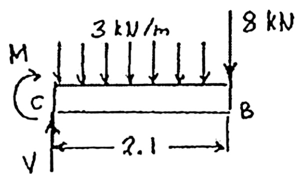
For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

Use portion CB as free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -M + (3)(2.1)(1.05) + (8)(2.1) = 0$$

$$M = 23.415 \text{ kN} \cdot \text{m} = 23.415 \times 10^3 \text{ N} \cdot \text{m}$$

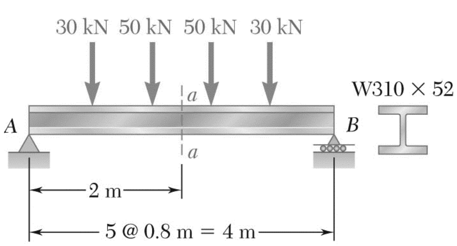


For W310 × 60 : $S = 844 \times 10^3 \text{ mm}^3$

$$= 844 \times 10^{-6} \text{ m}^3$$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{23.415 \times 10^3}{844 \times 10^{-6}} = 27.7 \times 10^6 \text{ Pa}$

$\sigma = 27.7 \text{ MPa}$ ◀



PROBLEM 5.18

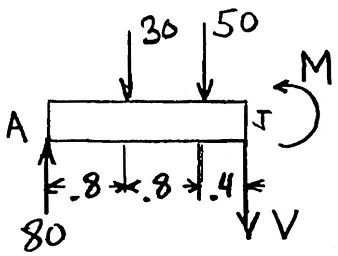
For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

SOLUTION

Reactions: By symmetry, $A = B$

$+\uparrow \sum F_y = 0: \quad \mathbf{A} = \mathbf{B} = 80 \text{ kN}\uparrow$

Using left half of beam as free body,



$+\circlearrowleft \sum M_J = 0:$

$$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$$

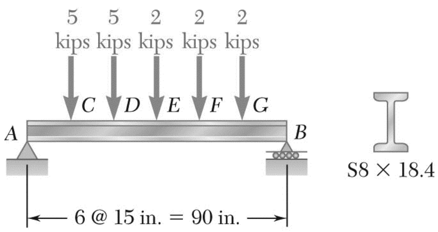
$$M = 104 \text{ kN} \cdot \text{m} = 104 \times 10^3 \text{ N} \cdot \text{m}$$

For $\quad W310 \times 52: \quad S = 747 \times 10^3 \text{mm}^3$

$$= 747 \times 10^{-6} \text{m}^3$$

Normal stress: $\quad \sigma = \dfrac{M}{S} = \dfrac{104 \times 10^3}{747 \times 10^{-6}} = 139.2 \times 10^6 \text{ Pa}$

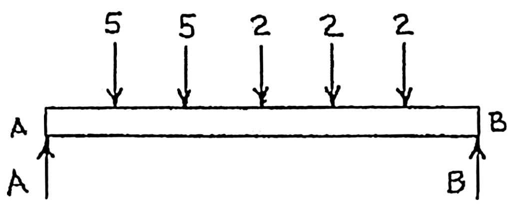
$$\sigma = 139.2 \text{ MPa} \blacktriangleleft$$



PROBLEM 5.19

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION



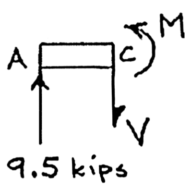
Use entire beam as free body.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$-90A + (75)(5) + (60)(5) + (45)(2) + (30)(2) + (15)(2) = 0$$

$$A = 9.5 \text{ kips} \uparrow$$

Use portion AC as free body.



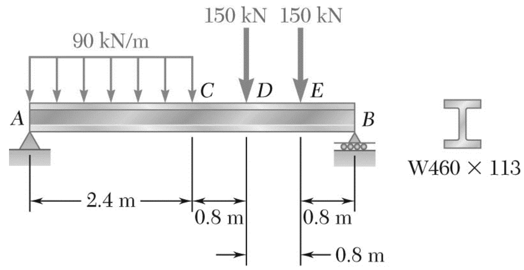
$$+\circlearrowleft \Sigma M_C = 0: \; M - (15)(9.5) = 0$$

$$M = 142.5 \text{ kip} \cdot \text{in}$$

For $S8 \times 18.4$, $\quad S = 14.4 \text{ in}^3$

Normal stress: $\qquad \sigma = \dfrac{M}{S} = \dfrac{142.5}{14.4}$

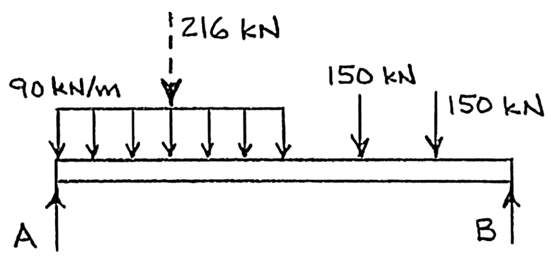
$$\sigma = 9.90 \text{ ksi} \blacktriangleleft$$



PROBLEM 5.20

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

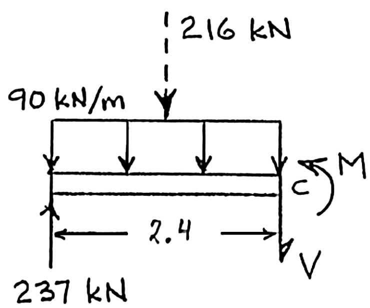


Use entire beam as free body.

$+\circlearrowleft \Sigma M_B = 0:$

$-4.8A + (3.6)(216) + (1.6)(150) + (0.8)(150) = 0$

$\boldsymbol{A} = 237 \text{ kN} \uparrow$



Use portion AC as free body.

$+\circlearrowleft \Sigma M_C = 0:$

$M - (2.4)(237) + (1.2)(216) = 0$

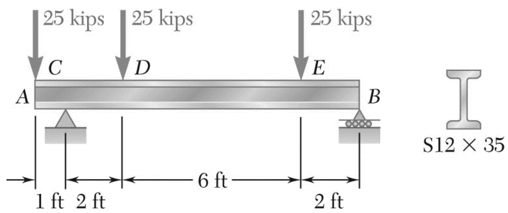
$M = 309.6 \text{ kN} \cdot \text{m}$

For W460 × 113, $S = 2390 \times 10^6 \text{mm}^3$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{309.6 \times 10^3 \text{N} \cdot \text{m}}{2390 \times 10^{-6} \text{m}^3}$$
$$= 129.5 \times 10^6 \text{ Pa}$$

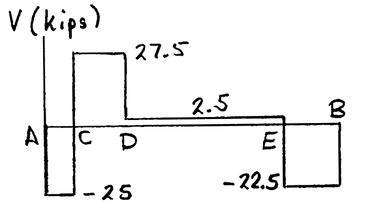
$\sigma = 129.5 \text{ MPa}$ ◀

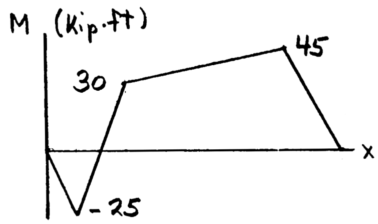


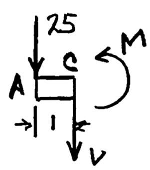
PROBLEM 5.21

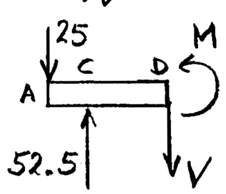
Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

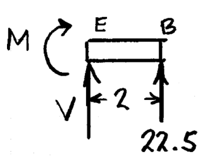
SOLUTION











$+\circlearrowleft \Sigma M_B = 0:$

$(11)(25) - 10C + (8)(25) + (2)(25) = 0 \quad \mathbf{C} = 52.5 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_C = 0:$

$(1)(25) - (2)(25) - (8)(25) + 10B = 0 \quad \mathbf{B} = 22.5 \text{ kips}$

Shear:

A to C^-: $\quad V = -25$ kips

C^+ to D^-: $\quad V = 27.5$ kips

D^+ to E^-: $\quad V = 2.5$ kips

E^+ to B: $\quad V = -22.5$ kips

Bending moments:

At C, $\quad +\circlearrowleft \Sigma M_C = 0: (1)(25) + M = 0$

$$M = -25 \text{ kip} \cdot \text{ft}$$

At D, $\quad +\circlearrowleft \Sigma M_D = 0: (3)(25) - (2)(52.5) + M = 0$

$$M = 30 \text{ kip} \cdot \text{ft}$$

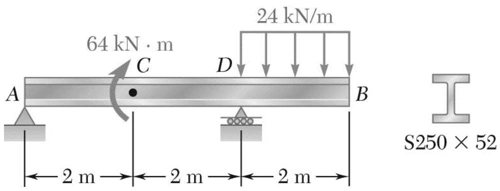
At E, $\quad +\circlearrowleft \Sigma M_E = 0: \quad -M + (2)(22.5) = 0 \quad M = 45 \text{ kip} \cdot \text{ft}$

$\max|M| = 45 \text{ kip} \cdot \text{ft} = 540 \text{ kip} \cdot \text{in}$

For $S12 \times 35$ rolled steel section: $\quad S = 38.1 \text{ in}^3$

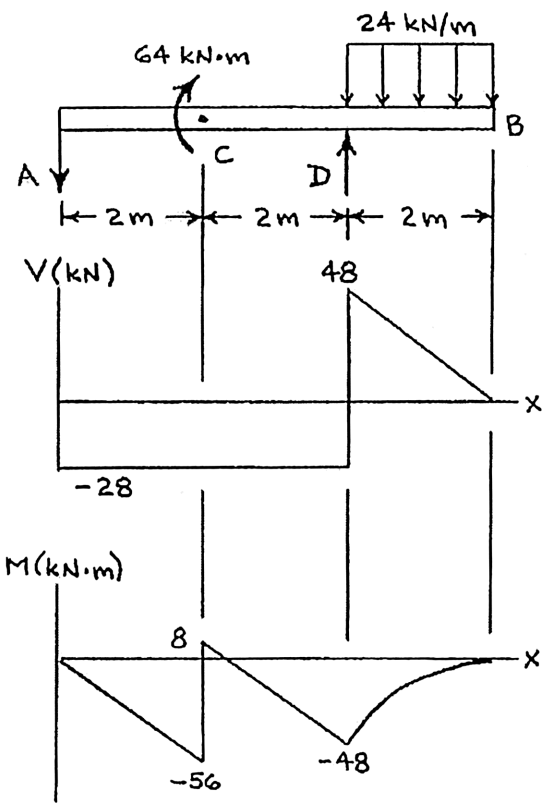
Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{540}{38.1} = 14.17 \text{ ksi} \qquad \sigma = 14.17 \text{ ksi}$ ◀

PROBLEM 5.22



Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

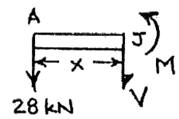


Reactions:

$$+\circlearrowleft \Sigma M_D = 0: \quad 4A - 64 - (24)(2)(1) = 0 \quad \boldsymbol{A} = 28 \text{ kN}\downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad -28 + D - (24)(2) = 0 \quad \boldsymbol{D} = 76 \text{ kN}\uparrow$$

A to C: $\quad 0 < x < 2\text{m}$



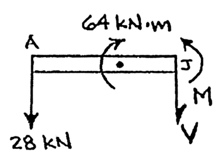
$$+\uparrow \Sigma F_y = 0: \quad -V - 28 = 0$$

$$V = -28 \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + 28x = 0$$

$$M = (-28x) \text{ kN} \cdot \text{m}$$

C to D: $\quad 2\text{m} < x < 4\text{m}$



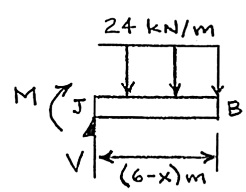
$$+\uparrow \Sigma F_y = 0: \quad -V - 28 = 0$$

$$V = -28 \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + 28x - 64 = 0$$

$$M = (-28x + 64) \text{ kN} \cdot \text{m}$$

D to B: $\quad 4\text{m} < x < 6\text{m}$



$$+\uparrow \Sigma F_y = 0:$$

$$V - 24(6 - x) = 0$$

$$V = (-24x + 144) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0:$$

$$-M - 24(6 - x)\left(\frac{6 - x}{2}\right) = 0$$

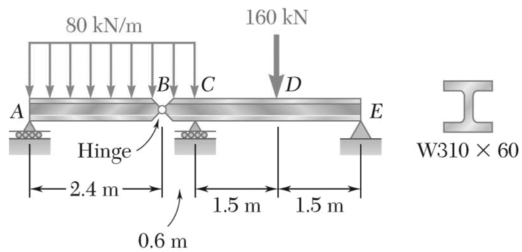
$$M = -12(6 - x)^2 \text{ kN} \cdot \text{m}$$

$$\max|M| = 56 \text{ kN} \cdot \text{m} = 56 \times 10^3 \text{ N} \cdot \text{m}$$

For $S250 \times 52$ section, $\quad S = 482 \times 10^3 \text{ mm}^3$

$$\text{Normal Stress: } \sigma = \frac{|M|}{S} = \frac{56 \times 10^3 \text{N} \cdot \text{m}}{482 \times 10^{-6} \text{m}^3} = 116.2 \times 10^6 \text{ Pa}$$

$$\sigma = 116.2 \text{ MPa} \blacktriangleleft$$

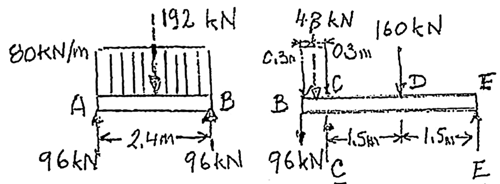


PROBLEM 5.23

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

Statics: Consider portion AB and BE separately.



Portion BE:

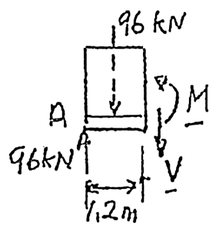
$+\circlearrowleft \sum M_E = 0:$

$(96)(3.6) + (48)(3.3) - C(3) + (160)(1.5) = 0$

$\boldsymbol{C} = 248\,\text{kN} \uparrow$

$\boldsymbol{E} = 56\,\text{kN} \uparrow$

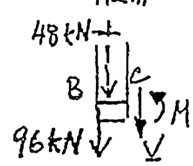
$$M_A = M_B = M_E = 0$$



At midpoint of AB:

$\sum F_y = 0: \quad V = 0$

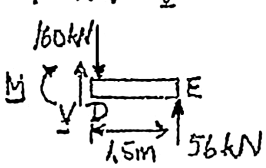
$\sum M = 0: \quad M = (96)(1.2) - (96)(0.6) = 57.6\ \text{kN} \cdot \text{m}$



Just to the left of C:

$\sum F_y = 0: \quad V = -96 - 48 = -144\ \text{kN}$

$\sum M_C = 0: \quad M = -(96)(0.6) - (48)(0.3) = -72\ \text{kN}$

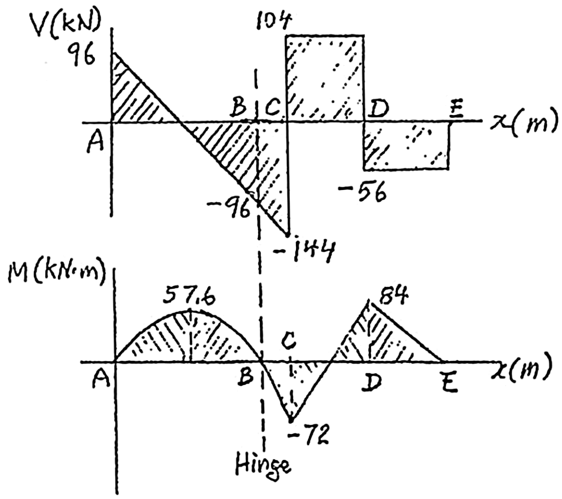


Just to the left of D:

$\sum F_y = 0: \quad V = 160 - 56 = +104\ \text{kN}$

$\sum M_D = 0: \quad M = (56)(1.5) = +84\ \text{kN} \cdot \text{m}$

PROBLEM 5.23 *(Continued)*



From the diagram:

$$|M|_{\max} = 84\ \text{kN}\cdot\text{m} = 84\times 10^3\ \text{N}\cdot\text{m}$$

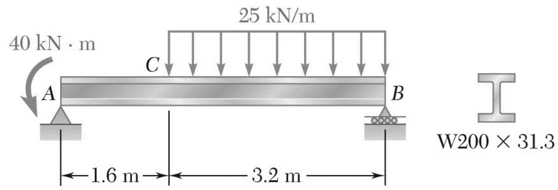
For W310 × 60 rolled steel shape,

$$S_x = 844\times 10^3\,\text{mm}^3$$
$$= 844\times 10^{-6}\,\text{m}^3$$

Stress: $\sigma_m = \dfrac{|M|_{\max}}{S}$

$$\sigma_m = \frac{84\times 10^3}{844\times 10^{-6}} = 99.5\times 10^6\ \text{Pa}$$

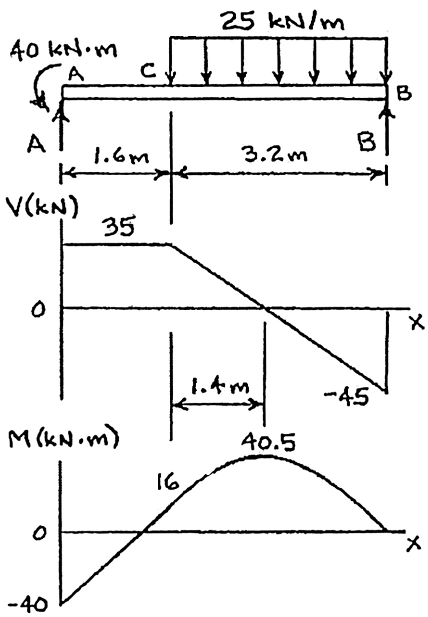
$\sigma_m = 99.5$ MPa ◀



PROBLEM 5.24

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

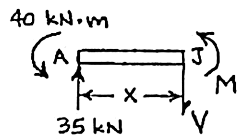


Reaction at A:

$$+\circlearrowleft \sum M_B = 0: \quad -4.8A + 40 + (25)(3.2)(1.6) = 0$$

$$\mathbf{A} = 35 \text{ kN} \uparrow$$

A to C: $\qquad 0 < x < 1.6 \text{ m}$



$$+\uparrow \sum F_y = 0: \quad 35 - V = 0 \quad V = 35 \text{ kN}$$

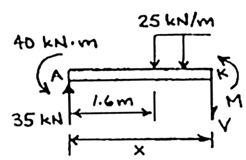
$$+\circlearrowleft \sum M_J = 0: \quad M + 40 - 35x = 0$$

$$M = (30x - 40) \text{ kN} \cdot \text{m}$$

C to B: $\qquad 1.6 \text{ m} < x < 4.8 \text{ m}$

$$+\uparrow \sum F_y = 0: \; 35 - 25(x - 1.6) - V = 0$$

$$V = (-25x + 75) \text{ kN}$$

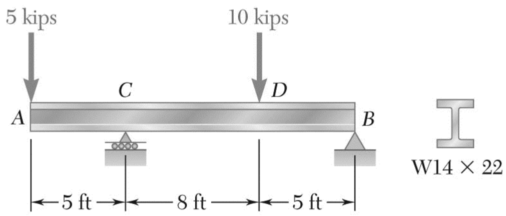


$$+\circlearrowleft \sum M_K = 0: \quad M + 40 - 35x + (25)(x - 1.6)\left(\frac{x - 1.6}{2}\right) = 0$$

$$M = (-12.5x^2 + 75x - 72) \text{ kN} \cdot \text{m}$$

Normal stress: $\qquad$ For W200 × 31.3, $S = 298 \times 10^3 \text{ mm}^3$

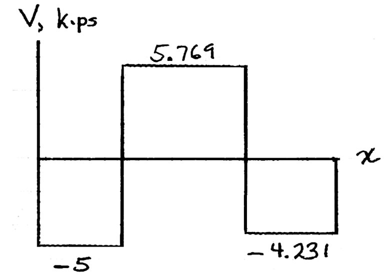
$$\sigma = \frac{|M|}{S} = \frac{40.5 \times 10^3 \text{ N} \cdot \text{m}}{298 \times 10^{-6} \text{ m}^3} = 135.9 \times 10^6 \text{ Pa} \qquad \sigma = 135.9 \text{ MPa} \blacktriangleleft$$

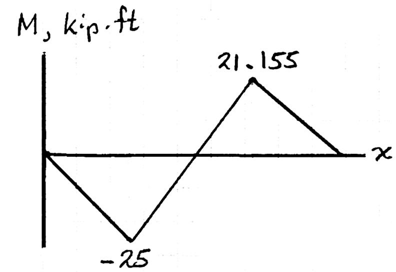


PROBLEM 5.25

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION





Reaction at C: $+\circlearrowleft \Sigma M_B = 0$: $(18)(5) - 13C + (5)(10) = 0$

$C = 10.769$ kips

Reaction at B: $+\circlearrowleft M_C = 0$: $(5)(5) - (8)(10) + 13B = 0$

$B = 4.231$ kips

Shear diagram:

A to C^-: $V = -5$ kips

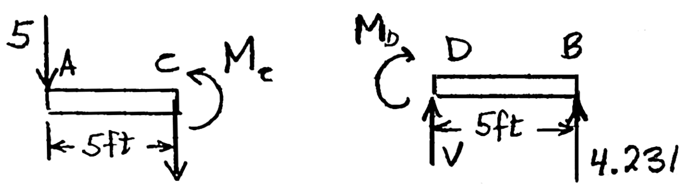
C^+ to D^-: $V = -5 + 10.769 = 5.769$ kips

D^+ to B: $V = 5.769 - 10 = -4.231$ kips

At A and B, $M = 0$

At C, $+\circlearrowleft M_C = 0$: $(5)(5) + M_C = 0$

$M_C = -25$ kip · ft



At D, $+\circlearrowleft \Sigma M_D = 0$: $-M_D + (5)(4.231)$

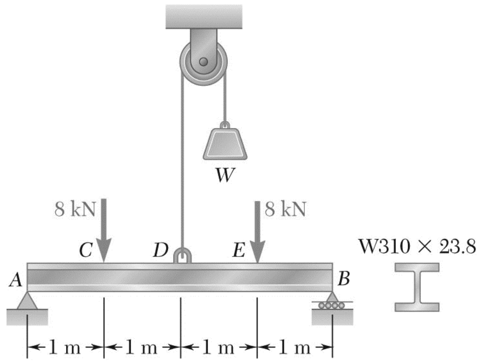
$M_D = 21.155$ kip · ft

$|V|_{max} = 5.77$ kips ◀

$|M|_{max}$ occurs at C. $|M|_{max} = 25$ kip · ft $= 300$ kip · in ◀

For W14 × 22 rolled steel section, $S = 29.0\ \text{in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{300}{29.0}$ $\sigma = 10.34$ ksi ◀



PROBLEM 5.26

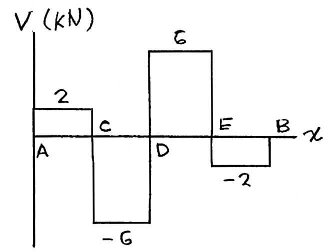
Knowing that $W = 12\,\text{kN}$, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.

SOLUTION

By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad A - 8 + 12 - 8 + B = 0$$

$$A = B = 2\text{ kN}$$

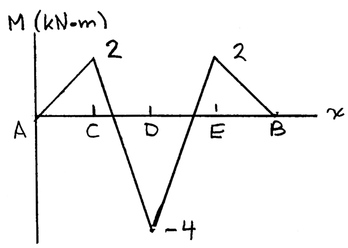


Shear: A to C^-: $V = 2$ kN ◀

C^+ to D^-: $V = -6$ kN ◀

D^+ to E^-: $V = 6$ kN ◀

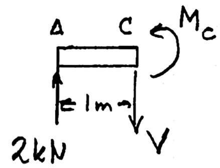
E^+ to B: $V = -2$ kN ◀

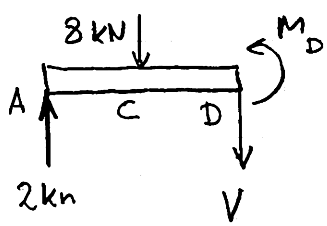


Bending moment:

At C, $\quad +\circlearrowleft \Sigma M_C = 0: \quad M_C - (1)(2) = 0$

$M_C = 2\text{ kN}\cdot\text{m}$ ◀





At D, $\quad +\Sigma M_D = 0: \quad M_D - (2)(2) + (8)(1) = 0$

$M_D - 4\text{ kN}\cdot\text{m}$ ◀

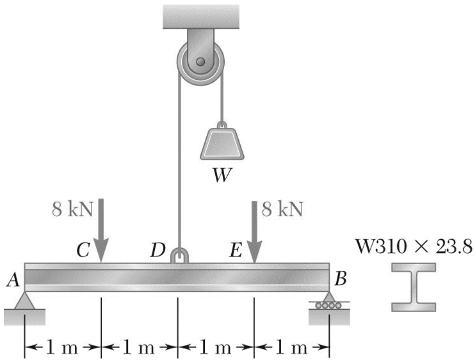
By symmetry, $\quad M = 2\text{ kN}\cdot\text{m}$ at E. $\quad M_E = 2\text{ kN}\cdot\text{m}$ ◀

$\max|M| = 4\text{ kN}\cdot\text{m}$ occurs at E.

For W310 × 23.8, $\quad S_x = 280\times10^3\text{mm}^3 = 280\times10^{-6}\text{m}^3$

Normal stress: $\quad \sigma_{\max} = \dfrac{|M|_{\max}}{S_x} = \dfrac{4\times10^3}{280\times10^{-6}}$

$= 14.29\times10^6$ Pa $\quad \sigma_{\max} = 14.29$ MPa ◀

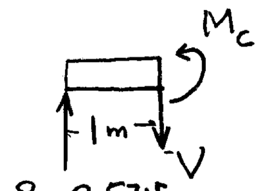


PROBLEM 5.27

Determine (*a*) the magnitude of the counterweight W for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

By symmetry, $A = B$



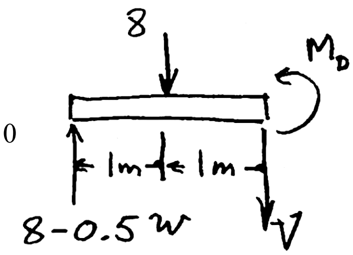
$$+\uparrow \Sigma F_y = 0: \quad A - 8 + W - 8 + B = 0$$

$$A = B = 8 - 0.5W$$

Bending moment at C: $\quad +\circlearrowleft \Sigma M_C = 0: \quad -(8 - 0.5W)(1) + M_C = 0$

$$M_C = (8 - 0.5W) \text{ kN} \cdot \text{m}$$

Bending moment at D:



$$+\circlearrowleft \Sigma M_D = 0: \; -(8 - 0.5W)(2) + (8)(1) + M_D = 0$$

$$M_D = (8 - W) \text{ kN} \cdot \text{m}$$

Equate: $\quad -M_D = M_C \qquad W - 8 = 8 - 0.5W$

$W = 10.67$ kN ◀

(*a*) $W = 10.6667$ kN

$M_C = -2.6667 \text{ kN} \cdot \text{m}$

$M_D = 2.6667 \text{ kN} \cdot \text{m} = 2.6667.10^3 \text{ N} \cdot \text{m}$

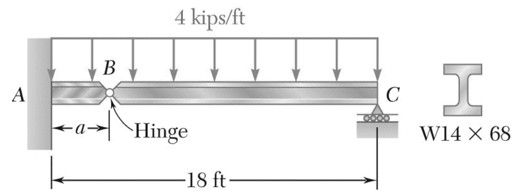
$|M|_{max} = 2.6667 \text{ kN} \cdot \text{m}$

For W310 × 23.8 rolled steel shape,

$$S_x = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{m}^3$$

(*b*) $\sigma_{max} = \dfrac{|M|_{max}}{S_x} = \dfrac{2.6667 \times 10^3}{280 \times 10^{-6}} = 9.52 \times 10^6 \text{ Pa}$

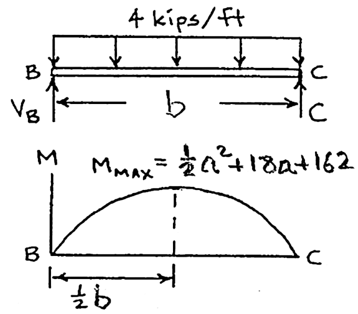
$\sigma_{max} = 9.52$ MPa ◀

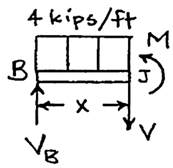


PROBLEM 5.28

Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION





For W14 × 68, $S_x = 103\text{ in}^3$

Let $b = (18 - a)$ ft

Segment *BC*:

By symmetry, $V_B = C$

$$+\uparrow \Sigma F_y = 0: \; V_B + C - 4b = 0$$

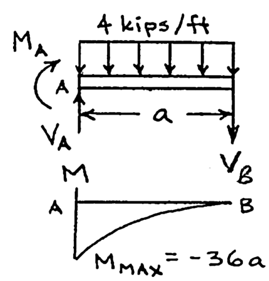
$$V_B = 2b$$

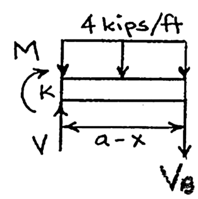
$$+\circlearrowleft \Sigma M_J = 0: \; -V_B x + (4x)\left(\frac{x}{2}\right) - M = 0$$

$$M = V_B x - 2x^2 = 2bx - 2x^2 \quad \text{lb} \cdot \text{ft}$$

$$\frac{dM}{dx} = 2b - x_m = 0 \qquad x_m = \frac{1}{2}b$$

$$M_{\max} = b^2 - \frac{1}{2}b^2 = \frac{1}{2}b^2$$





Segment *AB*:

$$+\circlearrowleft \Sigma M_K = 0: \; -4(a-x)\frac{(a-x)}{2}$$

$$-V_B(a-x) - M = 0$$

$$M = -2(a-x)^2 + 2b(a-x)$$

$|M_{\max}|$ occurs at $x = 0$.

$$|M_{\max}| = -2a^2 - 2ab = -2a^2 - 2a(18 - a) = 36a$$

(*a*) Equate the two values of $|M_{\max}|$: $\quad 36a = \frac{1}{2}b^2 = \frac{1}{2}(18-a)^2 = 162 - 18a + \frac{1}{2}a^2$

$$\frac{1}{2}a^2 - 54a + 162 = 0 \qquad a = 54 \pm \sqrt{(54)^2 - (4)\left(\tfrac{1}{2}\right)(162)}$$

$$a = 54 \pm 50.9118 = 3.0883 \text{ ft}$$

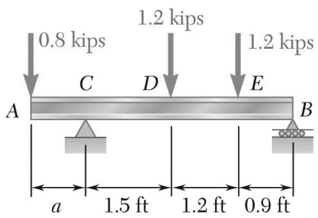
$a = 3.09$ ft ◀

(*b*) $|M|_{\max} = 36a = 111.179 \text{ kip} \cdot \text{ft} = 1334.15 \text{ kip} \cdot \text{in}$

$$\sigma = \frac{|M|_{\max}}{S_x} = \frac{1334.15}{103} = 12.95 \text{ kips/in}^2$$

$\sigma_m = 12.95$ ksi ◀

PROBLEM 5.29



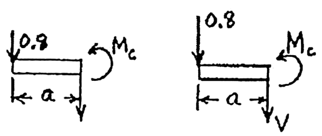
S3 × 5.7

Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

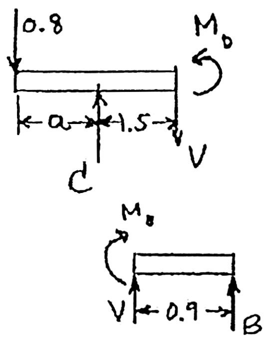
$$+\circlearrowleft \sum M_C = 0 : 0.8a - (1.5)(1.2) - (2.7)(1.2) + (3.6)B = 0 \qquad \mathbf{B} = 1.4 - 0.22222a \uparrow$$

$$+\circlearrowleft \sum M_B = 0 : (0.8)(3.6 + a) - 3.6C + (2.1)(1.2) + (0.9)(1.2) = 0 \qquad \mathbf{C} = 1.8 + 0.22222a \uparrow$$



Bending moment at C: $+\circlearrowleft \sum M_C = 0 : M_C + (0.8)(a) = 0$

$$M_C = -0.8a$$



Bending moment at D: $\sum M_D = 0 :$

$$M_D + (0.8)(a + 1.5) - 1.5C = 0$$

$$M_D = 1.5 - 0.46667a$$

Bending moment at E: $+\circlearrowleft \sum M_E = 0 : -M_E + 0.9B = 0$

$$M_E = 1.26 - 0.2a$$

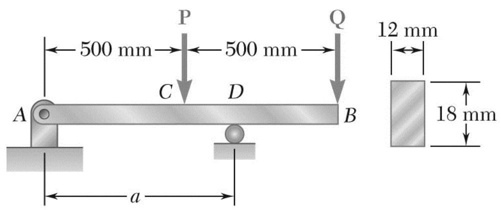
Assume $-M_C = M_E$: $\quad 0.8a = 1.26 - 0.2a \qquad a = 1.26 \text{ ft}$ ◀

$M_C = -1.008 \text{ kip} \cdot \text{ft} \qquad M_E = 1.008 \text{ kip} \cdot \text{ft} \qquad M_D = 0.912 \text{ kip} \cdot \text{ft}$

Note that $|M_D| < 1.008 \text{ kip} \cdot \text{ft} \qquad \max|M| = 1.008 \text{ kip} \cdot \text{ft} = 12.096 \text{ kip} \cdot \text{in}$

For rolled steel section $S3 \times 5.7$: $\quad S = 1.67 \text{ in}^3$

Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{12.096}{1.67} \qquad \sigma = 7.24 \text{ ksi}$ ◀

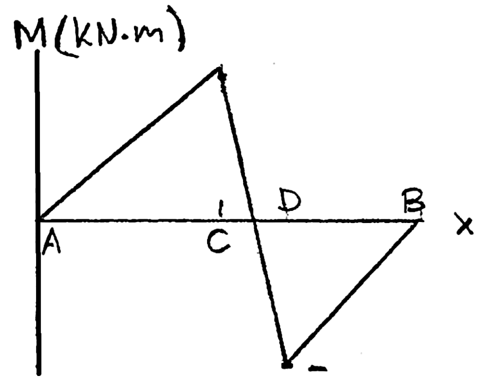


PROBLEM 5.30

Knowing that $P = Q = 480$ N, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480 \text{ N} \qquad Q = 480 \text{ N}$$

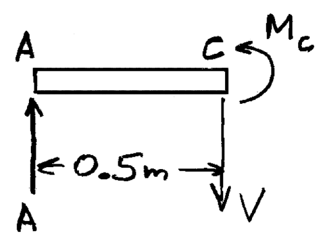


Reaction at A: $\quad +\circlearrowleft \Sigma M_D = 0: \quad -Aa + 480(a - 0.5) - 480(1 - a) = 0$

$$A = \left(960 - \frac{720}{a}\right) \text{N}$$

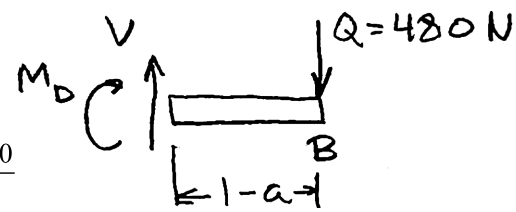
Bending moment at C: $\quad +\circlearrowleft \Sigma M_C = 0: \quad -0.5A + M_C = 0$

$$M_C = 0.5A = \left(480 - \frac{360}{a}\right) \text{N} \cdot \text{m}$$



Bending moment at D: $\quad +\circlearrowleft \Sigma M_D = 0: \quad -M_D - 480(1 - a) = 0$

$$M_D = -480(1 - a) \text{ N} \cdot \text{m}$$



(*a*) Equate: $\quad -M_D = M_C \qquad 480(1 - a) = 480 - \dfrac{360}{a}$

$$a = 0.86603 \text{ m}$$

$a = 866$ mm ◀

$$A = 128.62 \text{ N} \qquad M_C = 64.31 \text{ N} \cdot \text{m} \qquad M_D = -64.31 \text{ N} \cdot \text{m}$$

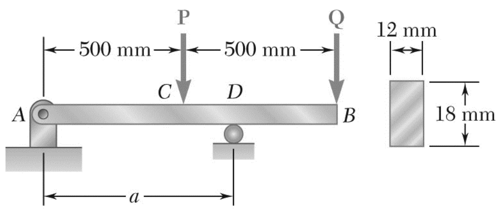
(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(13)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{64.31}{6.48 \times 10^{-9}} = 99.2 \times 10^6 \text{ Pa}$$

$\sigma_{\max} = 99.2$ MPa ◀

PROBLEM 5.31

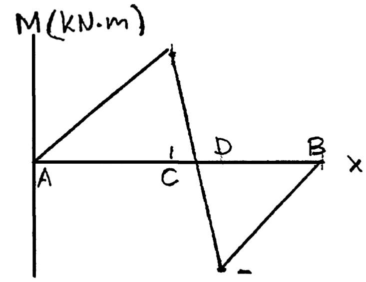


Solve Prob. 5.30, assuming that $P = 480$ N and $Q = 320$ N.

PROBLEM 5.30 Knowing that $P = Q = 480$ N, determine (*a*) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480 \text{ N} \qquad Q = 320 \text{ N}$$

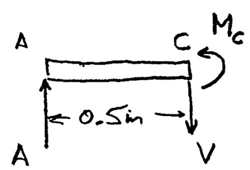


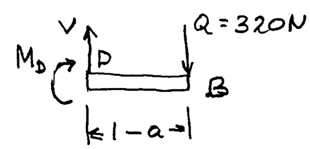
Reaction at A: $+\circlearrowleft \Sigma M_D = 0$: $Aa + 480(a - 0.5) - 320(1 - a) = 0$

$$A = \left(800 - \frac{560}{a}\right) \text{N}$$

Bending moment at C: $+\circlearrowleft \Sigma M_C = 0$: $-0.5A + M_C = 0$

$$M_C = 0.5A = \left(400 - \frac{280}{a}\right) \text{N} \cdot \text{m}$$





Bending moment at D: $+\circlearrowleft \Sigma M_D = 0$: $-M_D - 320(1 - a) = 0$

$$M_D = (-320 + 320a) \text{ N} \cdot \text{m}$$

(*a*) Equate: $-M_D = M_C \qquad 320 - 320a = 400 - \dfrac{280}{a}$

$$320a^2 + 80a - 280 = 0 \qquad a = 0.81873 \text{ m}, -1.06873 \text{ m}$$

Reject negative root. $a = 819$ mm ◀

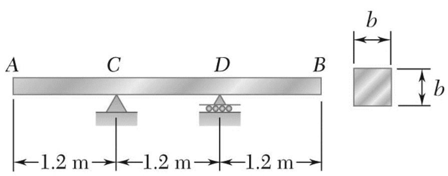
$A = 116.014$ N $\qquad M_C = 58.007$ N · m $\qquad M_D = -58.006$ N · m

(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(18)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{ m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{58.0065}{648 \times 10^{-9}} = 89.5 \times 10^6 \text{ Pa}$$

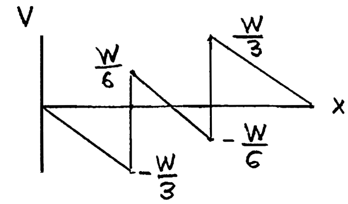
$\sigma_{\max} = 89.5$ MPa ◀

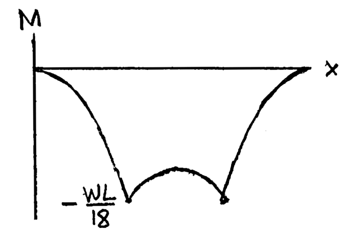


PROBLEM 5.32

A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860\ \text{kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (a) 10 MPa, (b) 50 MPa.

SOLUTION



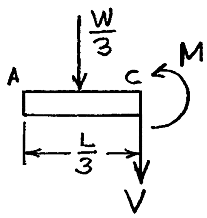


Weight density: $\gamma = \rho g$

Let L = total length of beam.

$$W = AL\rho g = b^2 L \rho g$$

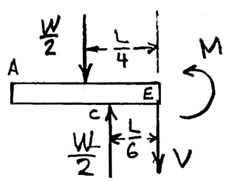
Reactions at C and D: $\quad C = D = \dfrac{W}{2}$



Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$



Bending moment at center of beam:

$$+\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0 \qquad M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2 L^2 \rho g}{18}$$

For a square section, $\quad S = \dfrac{1}{6} b^3$

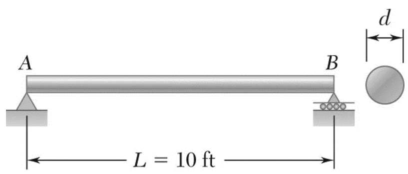
Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{b^2 L^2 \rho g/18}{b^3/6} = \dfrac{L^2 \rho g}{3b}$

Solve for b: $\quad b = \dfrac{L^2 \rho g}{3\sigma}$

Data: $\quad L = 3.6\ \text{m} \quad \rho = 7860\ \text{kg/m}^3 \quad g = 9.81\ \text{m/s}^2 \quad (a)\ \sigma = 10 \times 10^6\ \text{Pa} \quad (b)\ \sigma = 50 \times 10^6\ \text{Pa}$

(a) $\quad b = \dfrac{(3.6)^2(7860)(9.81)}{(3)(10 \times 10^6)} = 33.3 \times 10^{-3}\ \text{m}$ $\qquad b = 33.3\ \text{mm}$ ◀

(b) $\quad b = \dfrac{(3.6)^2(7860)(9.81)}{(3)(50 \times 10^6)} = 6.66 \times 10^{-3}\ \text{m}$ $\qquad b = 6.66\ \text{mm}$ ◀



PROBLEM 5.33

A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490$ lb/ft^3, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 4 ksi.

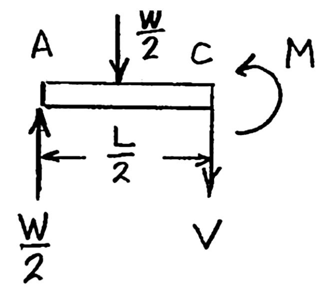
SOLUTION

Let W = total weight.

$$W = AL\gamma = \frac{\pi}{4} d^2 L\gamma$$

Reaction at A:

$$A = \frac{1}{2}W$$



Bending moment at center of beam:

$$+\circlearrowleft \Sigma M_C = 0: \ -\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section, $\left(c = \frac{1}{2}d\right)$

$$I = \frac{\pi}{4}c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4}c^3 = \frac{\pi}{32}d^3$$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32}d^2L^2\gamma}{\frac{\pi}{32}d^3} = \frac{L^2\gamma}{d}$$

Solving for d, $\qquad d = \dfrac{L^2\gamma}{\sigma}$

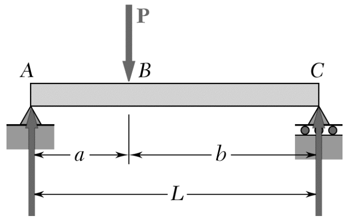
Data: $\qquad L = 10 \text{ ft} = (12)(10) = 120 \text{ in.}$

$$\gamma = 490 \text{ lb/ft}^3 = \frac{490}{12^3} = 0.28356 \text{ lb/in}^3$$

$$\sigma = 4 \text{ ksi} = 4000 \text{ lb/in}^2$$

$$d = \frac{(120)^2(0.28356)}{4000}$$

$d = 1.021$ in. ◀

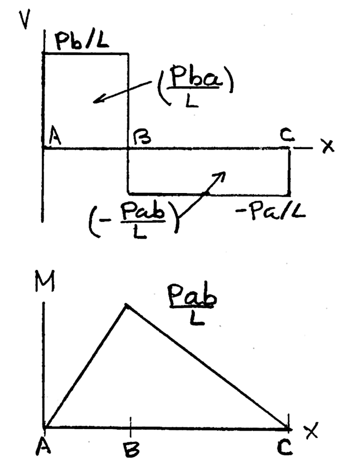


PROBLEM 5.34

Using the method of Sec. 5.3, solve Prob. 5.1*a*.

PROBLEM 5.1 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$

At A^+: $\quad V = A = \frac{Pb}{L} \qquad M = 0$

A to B^-: $\quad 0 < x < a$

$$w = 0 \qquad \int_0^x w\,dx = 0$$

$$V - V_A = 0 \qquad\qquad V = \frac{Pb}{L} \blacktriangleleft$$

$$M_B - M_A = \int_0^a V dx = \int_0^a \frac{Pb}{L} dx = \frac{Pba}{L} \qquad\qquad M_B = \frac{Pba}{L} \blacktriangleleft$$

At B^+: $\quad V = A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$

B^+ to C: $\quad a < x < L$

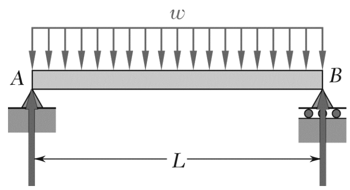
$$w = 0 \qquad \int_a^x w\,dx = 0$$

$$V_C - V_B = 0 \qquad\qquad V = -\frac{Pa}{L} \blacktriangleleft$$

$$M_C - M_B = \int_a^L V dx = -\frac{Pa}{L}(L - a) = -\frac{Pab}{L}$$

$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

$$|M|_{max} = \frac{Pab}{L} \blacktriangleleft$$



PROBLEM 5.35

Using the method of Sec. 5.3, solve Prob. 5.2*a*.

PROBLEM 5.2 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

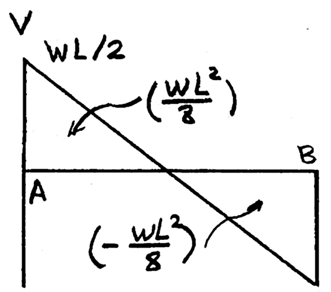
$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = A - wx \qquad V = \frac{wL}{2} - wx \blacktriangleleft$$

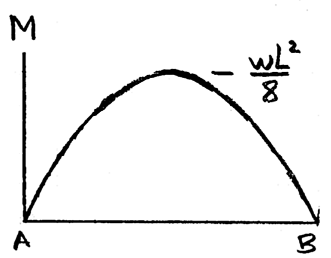


$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right) dx$$

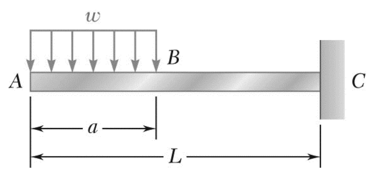
$$= \frac{wLx}{2} - \frac{wx^2}{2}$$

$$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} \qquad M = \frac{w}{2}(Lx - x^2) \blacktriangleleft$$



Maximum M occurs at $x = \frac{1}{2}$, where

$$V = \frac{dM}{dx} = 0 \qquad |M|_{\max} = \frac{wL^2}{8} \blacktriangleleft$$

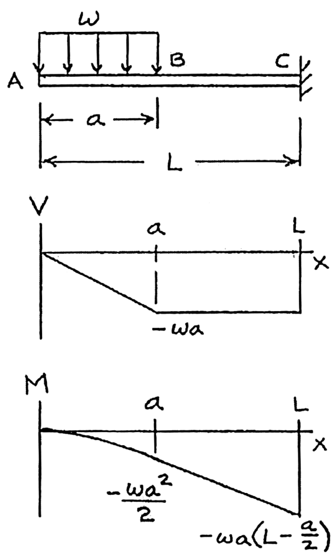


PROBLEM 5.36

Using the method of Sec. 5.3, solve Prob. 5.3*a*.

PROBLEM 5.3 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



Over AB: $V_A = 0 \quad M_A = 0$

$$V = -\int_0^x w\,dx = -wx$$

$$\frac{dM}{dx} = V = -wx$$

$$M = \int_0^x V\,dx = -\frac{wx^2}{2}\bigg|_0^x \qquad M = -\frac{wx^2}{2} \blacktriangleleft$$

At B: $\quad x = a \qquad V_B = -wa \qquad M_B = -\frac{wa^2}{2} \blacktriangleleft$

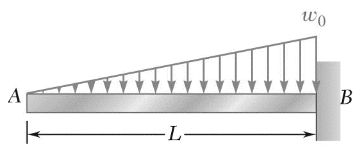
Over BC: $\quad w = 0$

$$\frac{dV}{dx} = 0 \qquad V = \text{constant} = V_B \qquad V = -wa \blacktriangleleft$$

$$\frac{dM}{dx} = V = -wa$$

$$M - M_B = \int_a^x V\,dx = -wax\Big|_a^x = -wa(x-a)$$

$$M = -wa(x-a) - \frac{wa^2}{2} \qquad M = -wa\left(x - \frac{a}{2}\right) \blacktriangleleft$$

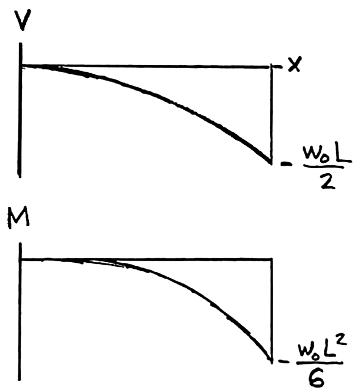


PROBLEM 5.37

Using the method of Sec. 5.3, solve Prob. 5.4*a*.

PROBLEM 5.4 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



$$w = w_0 \frac{x}{L}$$

$$V_A = 0, \qquad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{W_0 x}{L}$$

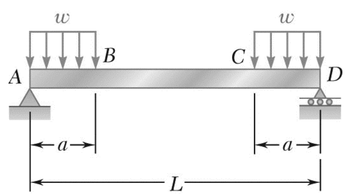
$$V - V_A = -\int_0^x \frac{w_0 x}{L} = -\frac{w_0 x^2}{2L}$$

$$V = -\frac{w_0 x^2}{2L} \blacktriangleleft$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V\,dx = -\int_0^x \frac{w_0 x^2}{2L} dx$$

$$M = -\frac{w_0 x^3}{6L} \blacktriangleleft$$



PROBLEM 5.38

Using the method of Sec. 5.3, Solve Prob. 5.5a.

PROBLEM 5.5 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

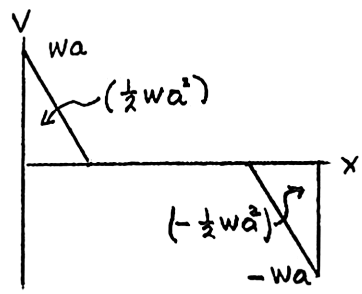
Reactions: $A = D = wa$

A to B: $0 < x < a \qquad w = w$

$V_A = A = wa, \qquad M_A = 0$

$V - V_A = -\int_0^x w\,dx = -wx$

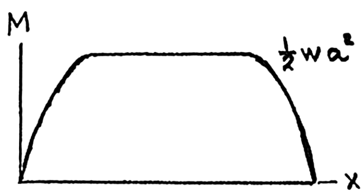
$$V = w(a - x) \blacktriangleleft$$



$\dfrac{dM}{dx} = V = wa - wx$

$M - M_A = \int_0^x V dx = \int_0^x (wa - wx)dx$

$$M = wax - \frac{1}{2}wx^2 \blacktriangleleft$$



$V_B = 0 \qquad M_B = \dfrac{1}{2}wa^2$

B to C: $a < x < L - a$ $\qquad V = 0 \blacktriangleleft$

$\dfrac{dM}{dx} = V = 0$

$M - M_B = \int_a^x V\,dx = 0$

$M = M_B \qquad M = \dfrac{1}{2}wa^2 \blacktriangleleft$

PROBLEM 5.38 *(Continued)*

C to D: $$V - V_C = -\int_{L-a}^{x} w\,dx = -w[x - (L - a)]$$

$$V = -w[x - (L - a)] \blacktriangleleft$$

$$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} -[wx - (L - a)]dx$$

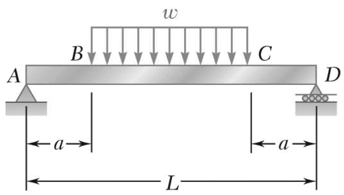
$$= -w\left[\frac{x^2}{2} - (L - a)x\right]\Bigg|_{L-a}^{x}$$

$$= -w\left[\frac{x^2}{2} - (L - a)x - \frac{(L - a)^2}{2} + (L - a)^2\right]$$

$$= -w\left[\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right]$$

$$M = \frac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right] \blacktriangleleft$$

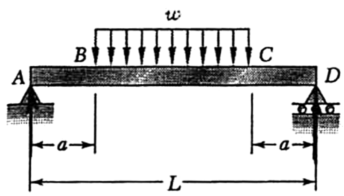
PROBLEM 5.39

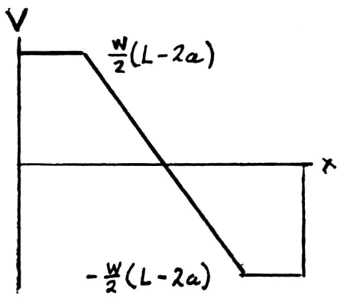


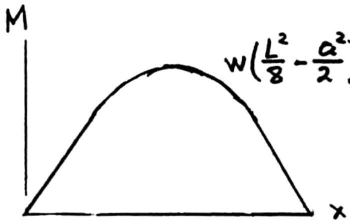
Using the method of Sec. 5.3, solve Prob. 5.6*a*.

PROBLEM 5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION







Reactions. $\quad A = D = \dfrac{1}{2}w(L-2a)$

At A. $\quad V_A = A = \dfrac{1}{2}w(L-2a), \quad M_A = 0$

A to B. $\quad 0 < x < a \qquad w = 0$

$$V_B - V_A = -\int_0^a w\,dx = 0$$

$$V_B = V_A = \frac{1}{2}w(L-2a)$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{1}{2}w(L-2a)dx$$

$$M_B = \frac{1}{2}w(L-2a)a$$

B to C. $\quad a < x < L-a \qquad w = w$

$$V - V_B = -\int_a^x w\,dx = -w(x-a)$$

$$V = \frac{1}{2}w(L-2a) - w(x-a) = \frac{1}{2}w(L-2x)$$

$$\frac{dM}{dx} = V = \frac{1}{2}w(L-2x)$$

$$M - M_B = \int_a^x V\,dx = \frac{1}{2}w(Lx - x^2)\Big|_a^x$$

$$= \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2}w(L-2a)a + \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$= \frac{1}{2}w(Lx - x^2 - a^2)$$

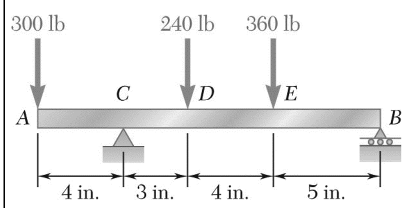
PROBLEM 5.39 *(Continued)*

At C. $\quad x = L - a \quad V_C = -\frac{1}{2}w(L - 2a) \quad M_C = \frac{1}{2}(L - 2a)a$

C to D. $\quad V = V_C = -\frac{1}{2}w(L - 2a)$

$M_D = 0$

$$At\ x = \frac{L}{2}, \qquad M_{\max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right) \blacktriangleleft$$

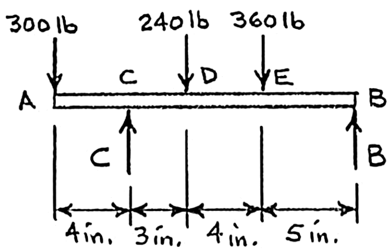


PROBLEM 5.40

Using the method of Sec. 5.3 solve Prob. 5.7.

PROBLEM 5.7 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



Reaction at C:

$$+\circlearrowleft \Sigma M_B = 0: \quad (16)(300) - 12C + (9)(240) + (5)(360) = 0$$

$$\boldsymbol{C} = 730\,\text{lb} \uparrow$$

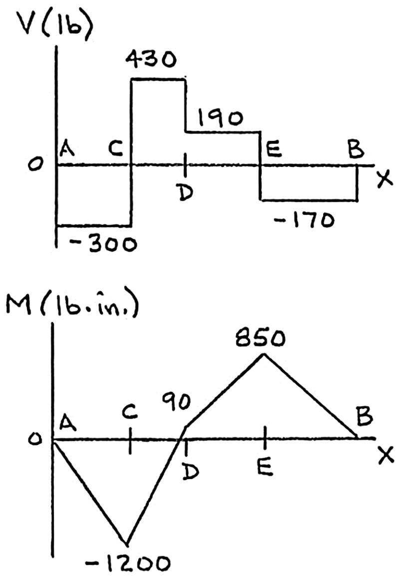
Shear diagram:

A to C: $V = -300$ lb

C to D: $V = -300 + 730 = 430$ lb

D to E: $V = 430 - 240 = 190$ lb

E to B: $V = 190 - 360 = -170$ lb



Areas of shear diagram:

A to C: $A_{AC} = (-300)(4) = -1200\ \text{lb}\cdot\text{in}$

C to D: $A_{CD} = (430)(3) = 1290\ \text{lb}\cdot\text{in}$

D to E: $A_{DE} = (190)(4) = 760\ \text{lb}\cdot\text{in}$

E to B: $A_{EB} = (-170)(5) = -850\ \text{lb}\cdot\text{in}$

Bending moments:

$M_A = 0$

$M_C = 0 - 1200 = -1200\ \text{lb}\cdot\text{in}$

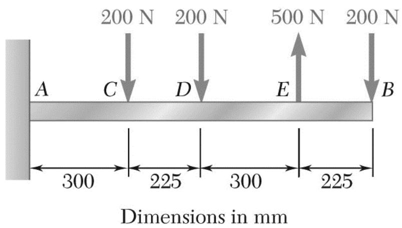
$M_D = -1200 + 1290 = 90\ \text{lb}\cdot\text{in}$

$M_E = 90 + 760 = 850\ \text{lb}\cdot\text{in}$

$M_B = 850 - 850 = 0$

(*a*) Maximum $|V| = 430$ lb ◀

(*b*) Maximum $|M| = 1200\ \text{lb}\cdot\text{in}$ ◀

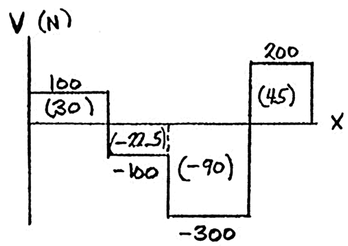


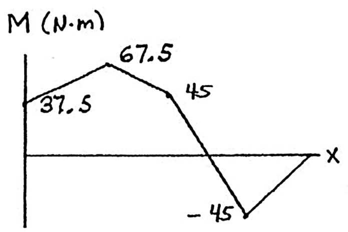
PROBLEM 5.41

Using the method of Sec. 5.3, Solve Prob. 5.8

PROBLEM 5.8 Draw the shear and bending-moment diagram for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION





$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) = 0$$

$$M_A = 37.5\,\text{N}\cdot\text{m}$$

$$+\uparrow \Sigma F_y = 0: \quad V_A - 200 - 200 + 500 - 200 = 0$$

$$V_A = 100\,\text{N}$$

Shear:

A to *C*: $V = 100\,\text{N}$

C to *D*: $V = 100 - 200 = -100\,\text{N}$

D to *E*: $V = -100 - 200 = -300\,\text{N}$

E to *B*: $V = -300 + 500 = 200\,\text{N}$

Areas under shear diagram:

A to *C*: $\int V dx = (100)(0.3) = 30\,\text{N}\cdot\text{m}$

C to *D*: $\int V dx = (-100)(0.225) = -22.5\,\text{N}\cdot\text{m}$

D to *E*: $\int V dx = (-300)(0.3) = -90\,\text{N}\cdot\text{m}$

E to *B*: $\int V dx = (200)(0.225) = 45\,\text{N}\cdot\text{m}$

Bending moments:

$$M_A = 37.5\,\text{N}\cdot\text{m}$$

$$M_C = M_A + \int_A^C V\,dx = 37.5 + 30 = 67.5\,\text{N}\cdot\text{m}$$

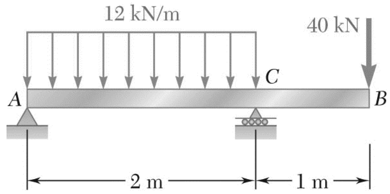
$$M_D = M_C + \int_C^D V\,dx = 67.5 - 22.5 = 45\,\text{N}\cdot\text{m}$$

$$M_E = M_D + \int_D^E V\,dx = 45 - 90 = -45\,\text{N}\cdot\text{m}$$

$$M_B = M_E + \int_E^D V\,dx = -45 + 45 = 0$$

(*a*) Maximum $|V| = 300\,\text{N}$ ◀

(*b*) Maximum $|M| = 67.5\,\text{N}\cdot\text{m}$ ◀

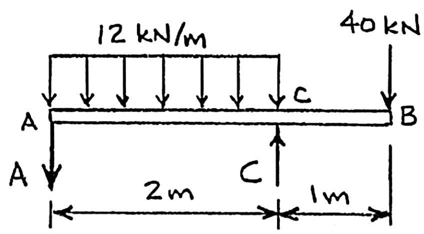


PROBLEM 5.42

Using the method of Sec. 5.3, Solve Prob. 5.9

PROBLEM 5.9 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



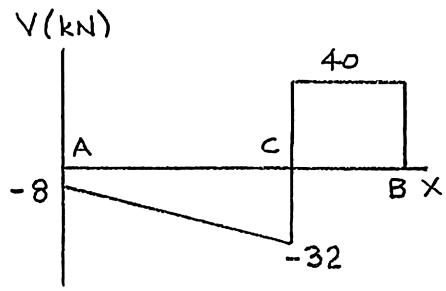
Reactions:

$$+\circlearrowleft \Sigma M_C = 0:\ 2A + (12)(2)(1) - (40)(1) = 0$$

$$\boldsymbol{A} = 8\,\text{kN} \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0:\ 2C - (12)(2)(1) - (40)(3) = 0$$

$$\boldsymbol{C} = 72\,\text{kN} \uparrow$$



Shear diagram: $V_A = -8$ kN

A to *C*: $0 < x < 2\,\text{m} \quad w = 12\text{kN/m}$

$$V_C - V_A = -\int_0^2 w\,dx = -\int_0^2 12\,dx = -24\ \text{kN}$$

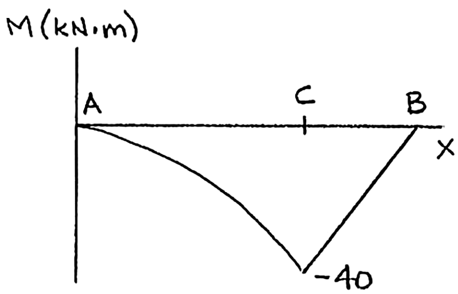
$$V_C = -24 - 8 = -32\ \text{kN}$$

C to *B*: $V_B = -32 + 72 = 40$ kN

Areas of shear diagram:

A to *C*: $\int V dx = \frac{1}{2}(-8-32)(2) = -40\ \text{kN}\cdot\text{m}$

C to *B*: $\int V dx = (1)(40) = 40\ \text{kN}\cdot\text{m}$



Bending moments:

$M_A = 0$

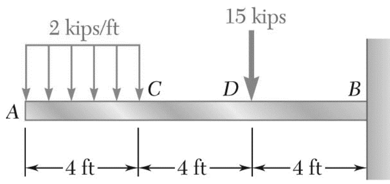
$M_C = M_A + \int V dx = 0 - 40 = -40\ \text{kN}\cdot\text{m}$

$M_B = M_C + \int V dx = -40 + 40 = 0$

(*a*) Maximum $|V| = 40.0$ kN ◀

(*b*) Maximum $|M| = 40.0$ kN · m ◀

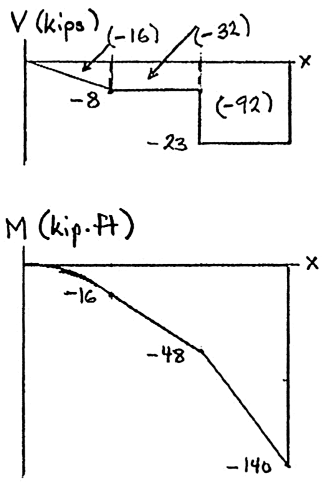
PROBLEM 5.43



Using the method of Sec. 5.3, solve Prob. 5.10

PROBLEM 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



Shear:

$V_A = 0$

$V_B = V_A - \int_A^B w dx = 0 - (4)(2) = -8 \text{ kips}$

C to D: $V = -8 \text{ kips}$

D to B: $V = -8 - 15 = -23 \text{ kips}$

Areas under shear diagram:

A to C: $\int V dx = \left(\frac{1}{2}\right)(4)(-8) = -16 \text{ kip} \cdot \text{ft}$

C to D: $\int V dx = (4)(-8) = -32 \text{ kip} \cdot \text{ft}$

D to B: $\int V dx = (4)(-23) = -92 \text{ kip} \cdot \text{ft}$

Bending moments:

$M_A = 0$

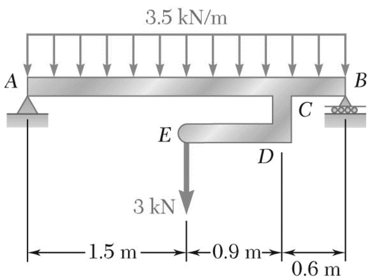
$M_C = M_A + \int V dx = 0 - 16 = -16 \text{ kip} \cdot \text{ft}$

$M_D = M_C + \int V dx = -16 - 32 = -48 \text{ kip} \cdot \text{ft}$

$M_B = M_D + \int V dx = -48 - 92 = -140 \text{ kip} \cdot \text{ft}$

(*a*) Maximum $|V| = 23 \text{ kips}$ ◀

(*b*) Maximum $|M| = 140 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 5.44

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

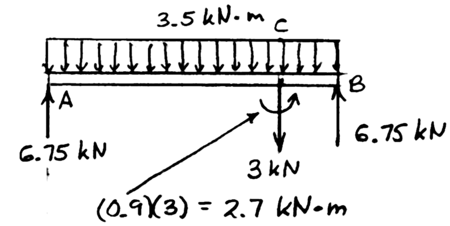
Reaction at *A*:

$$+\circlearrowleft \Sigma M_B = 0: \quad -3.0A + (1.5)(3.0)(3.5) + (1.5)(3) = 0$$

$$\boldsymbol{A} = 6.75 \text{ kN} \uparrow$$

Reaction at *B*: $\boldsymbol{B} = 6.75 \text{ kN} \uparrow$

Beam *ACB* and loading: (See sketch.)

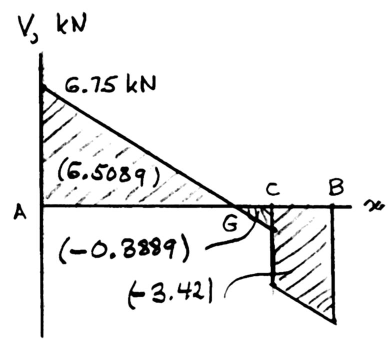


Areas of load diagram:

A to *C*: $(2.4)(3.5) = 8.4 \text{ kN}$

C to *B*: $(0.6)(3.5) = 2.1 \text{ kN}$

Shear diagram:



$$V_A = 6.75 \text{ kN}$$

$$V_{C^-} = 6.75 - 8.4 = -1.65 \text{ kN}$$

$$V_{C^+} = -1.65 - 3 = -4.65 \text{ kN}$$

$$V_B = -4.65 - 2.1 = -6.75 \text{ kN}$$

Over *A* to *C*, $V = 6.75 - 3.5x$

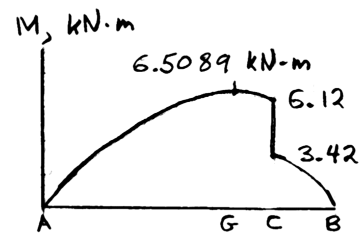
At *G*, $V = 6.75 - 3.5x_G = 0 \quad x_G = 1.9286 \text{ m}$

Areas of shear diagram:

A to *G*: $\frac{1}{2}(1.9286)(6.75) = 6.5089 \text{ kN} \cdot \text{m}$

G to *C*: $\frac{1}{2}(0.4714)(-1.65) = -0.3889 \text{ kN} \cdot \text{m}$

C to *B*: $\frac{1}{2}(0.6)(-4.65 - 6.75) = -3.42 \text{ kN} \cdot \text{m}$



PROBLEM 5.44 *(Continued)*

Bending moments:

$$M_A = 0$$

$$M_G = 0 + 6.5089 = 6.5089 \text{ kN} \cdot \text{m}$$

$$M_{C^-} = 6.5089 - 0.3889 = 6.12 \text{ kN} \cdot \text{m}$$

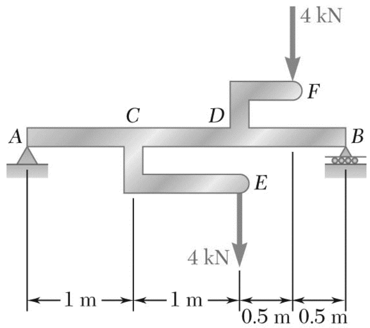
$$M_{C^+} = 6.12 - 2.7 = 3.42 \text{ kN} \cdot \text{m}$$

$$M_B = 3.42 - 3.42 = 0$$

(*a*) $|V|_{\max} = 6.75 \text{ kN}$ ◀

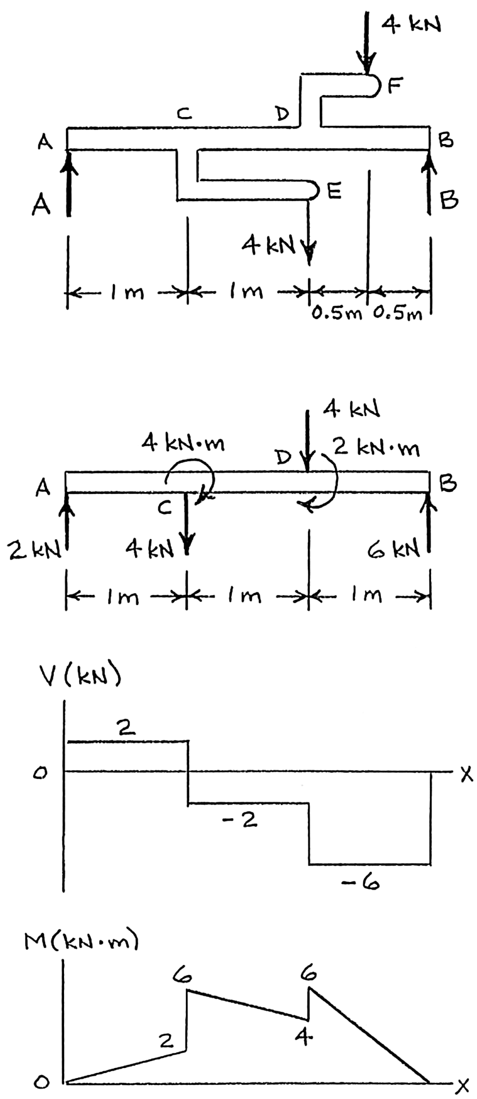
(*b*) $|M|_{\max} = 6.51 \text{ kN} \cdot \text{m}$ ◀

PROBLEM 5.45



Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



$+\circlearrowleft \Sigma M_B = 0$:

$$-3A + (1)(4) + (0.5)(4) = 0$$

$$\boldsymbol{A} = 2 \text{ kN} \uparrow$$

$+\circlearrowleft \Sigma M_A = 0$: $3B - (2)(4) - (2.5)(4) = 0$

$$\boldsymbol{B} = 6 \text{ kN} \uparrow$$

Shear diagram:

A to *C*: $V = 2 \text{ kN}$

C to *D*: $V = 2 - 4 = -2 \text{ kN}$

D to *B*: $V = -2 - 4 = -6 \text{ kN}$

Areas of shear diagram:

A to *C*: $\int V dx = (1)(2) = 2 \text{ kN} \cdot \text{m}$

C to *D*: $\int V dx = (1)(-2) = -2 \text{ kN} \cdot \text{m}$

D to *E*: $\int V dx = (1)(-6) = -6 \text{ kN} \cdot \text{m}$

Bending moments:

$M_A = 0$

$M_{C^-} = 0 + 2 = 2 \text{ kN} \cdot \text{m}$

$M_{C^+} = 2 + 4 = 6 \text{ kN} \cdot \text{m}$

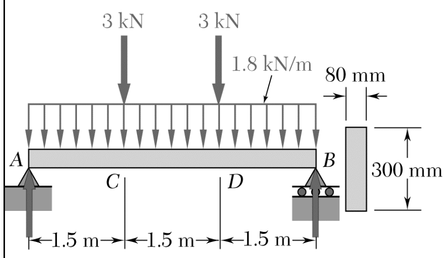
$M_{D^-} = 6 - 2 = 4 \text{ kN} \cdot \text{m}$

$M_{D^+} = 4 + 2 = 6 \text{kN} \cdot \text{m}$

$M_B = 6 - 6 = 0$

(*a*) $|V|_{\text{max}} = 6.00 \text{ kN}$ ◀

(*b*) $|M|_{\text{max}} = 6.00 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 5.46

Using the method of Sec. 5.3, solve Prob. 5.15

PROBLEM 5.15 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

By symmetry, $A = B$.

$$+\uparrow \Sigma F_y = 0: \quad A + B - 3 - 3 - (4.5)(1.8) = 0$$

$$A = B = 7.05 \text{ kN}$$

Shear diagram: $V_A = 7.05$ kN

A to C^-: $w = 1.8$ kN/m

At C^-, $V = 7.05 - (1.8)(1.5) = 4.35$ kN

At C^+, $V = 4.35 - 3 = 1.35$ kN

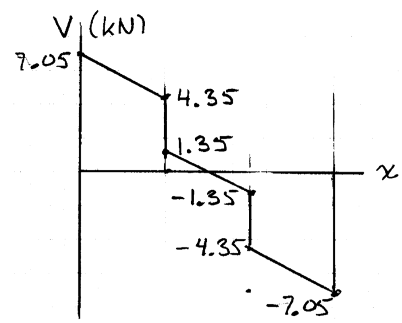
C^+ to D^-: $w = 1.8$ kN/m

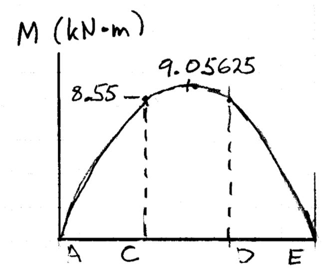
At D^-, $V = 1.35 - (1.5)(1.8) = -1.35$ kN

At D^+, $V = -1.35 - 3 = -4.35$ kN

D^+ to *B*: $w = 1.8$ kN

At *B*, $V = -4.35 - (1.5)(1.8) = -7.05$ kN





Draw the shear diagram:

$V = 0$ at point *E*, the midpoint of *CD*.

Areas of the shear diagram:

A to *C*: $\frac{1}{2}(7.05 + 4.35)(1.5) = 8.55 \text{ kN} \cdot \text{m}$

C to *E*: $\frac{1}{2}(1.35)(0.75) = 0.50625 \text{ kN} \cdot \text{m}$

E to *D*: $\frac{1}{2}(-1.35)(0.75) = -0.50625 \text{ kN} \cdot \text{m}$

D to *B*: $\frac{1}{2}(-4.35 - 7.05)(1.5) = -8.55 \text{ kN} \cdot \text{m}$

PROBLEM 5.46 *(Continued)*

Bending moments:

$$M_A = 0$$
$$M_C = 0 + 8.55 = 8.55 \text{ kN} \cdot \text{m}$$
$$M_E = 8.55 + 0.50625 = 9.05625 \text{ kN} \cdot \text{m}$$
$$M_D = 9.05625 - 0.50625 = 8.55 \text{ kN} \cdot \text{m}$$
$$M_B = 8.55 - 8.55 = 0$$
$$M_C = 8.55 \times 10^3 \text{ N} \cdot \text{m}$$

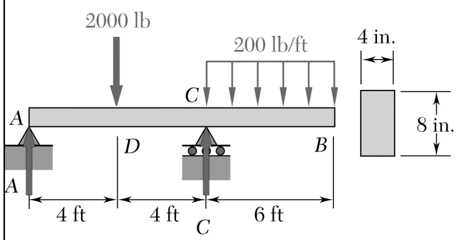
For a rectangular section,

$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(80)(300)^2$$
$$= 1.2 \times 10^6 \text{ mm}^3 = 1.2 \times 10^{-3} \text{ m}^3$$

Maximum normal stress at C:

$$\sigma = \frac{M_C}{S} = \frac{8.55 \times 10^3}{1.2 \times 10^{-3}}$$
$$= 7.125 \times 10^6 \text{ Pa}$$

$\sigma = 7.13 \text{ MPa}$ ◀



PROBLEM 5.47

Using the method of Sec. 5.3, solve Prob. 5.16.

PROBLEM 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -8A + (4)(2000) - (3)(6)(200) = 0$$

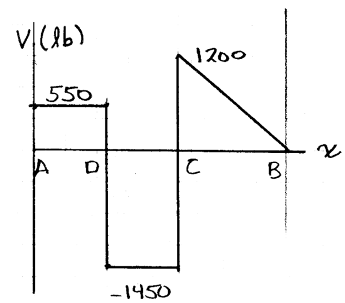
$$A = 550 \text{ lb} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 8C - (4)(2000) - (11)(6)(200) = 0$$

$$C = 2650 \text{ lb} \uparrow$$

Check: $+\uparrow \Sigma F_y = 550 - 2000 + 2650 - (6)(200) = 0$

Shear diagram:



A to D: $V = 550$ lb

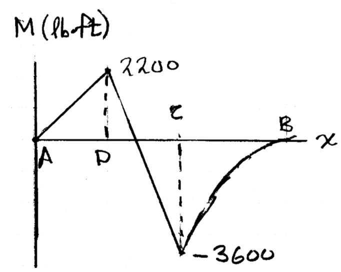
D to C^-: $V = 550 - 2000 = -1450$ lb

At C^+, $V = -1450 + 2650 = 1200$ lb

C^+ to B: $w = 200$ lb/ft

$$\int_8^{14} 200\, dx = 1200 \text{ lb}$$

At B, $V = 0$ as expected.



Areas of shear diagram:

A to D: $A_{AD} = (550)(4) = 2200 \text{ lb} \cdot \text{ft}$

D to C: $A_{DC} = (-1450)(4) = -5800 \text{ lb} \cdot \text{ft}$

C to B: $A_{CB} = \frac{1}{2}(1200)(6) = 3600 \text{ lb} \cdot \text{ft}$

Bending moments: $M_A = 0$

$$M_D = 0 + 2200 = 2200 \text{ lb} \cdot \text{ft}$$

$$M_C = 2200 + (-5800) = -3600 \text{ lb} \cdot \text{ft}$$

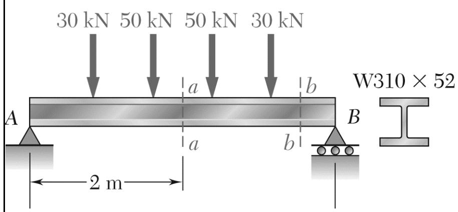
$$M_B = -3600 + 3600 = 0 \text{ as expected}$$

$$|M|_{\max} = 3600 \text{ lb} \cdot \text{ft} = 43200 \text{ lb} \cdot \text{in}$$

For a rectangular section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(4)(8)^2 = 42.667 \text{ in}^3$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{43200}{42.667} = 1012.5 \text{ psi}$$

$\sigma_{\max} = 1013$ psi ◀

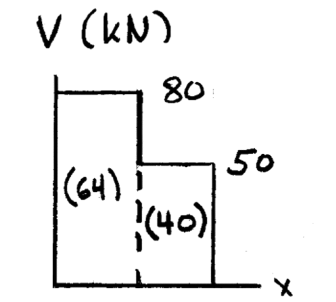


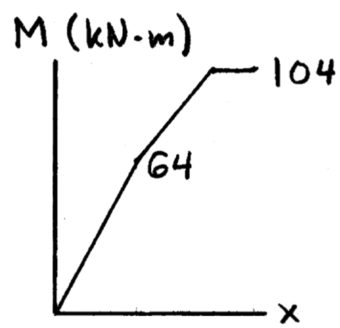
PROBLEM 5.48

Using the method of Sec. 5.3, solve Prob. 5.18.

PROBLEM 5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

SOLUTION





Reactions: By symmetry, $A = B$.

$+\!\uparrow \sum F_y = 0: \quad \boldsymbol{A} = \boldsymbol{B} = 80 \text{ kN} \uparrow$

Shear diagram:

A to C: $V = 80 \text{ kN}$

C to D: $V = 80 - 30 = 50 \text{ kN}$

D to E: $V = 50 - 50 = 0$

Areas of shear diagram:

A to C: $\int V dx = (80)(0.8) = 64 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = (50)(0.8) = 40 \text{ kN} \cdot \text{m}$

D to E: $\int V dx = 0$

Bending moments:

$M_A = 0$

$M_C = 0 + 64 = 64 \text{ kN} \cdot \text{m}$

$M_D = 64 + 40 = 104 \text{ kN} \cdot \text{m}$

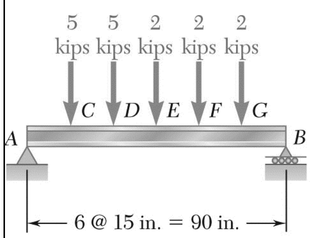
$M_E = 104 + 0 = 104 \text{ kN} \cdot \text{m}$

$$|M|_{\max} = 104 \text{ kN} \cdot \text{m} = 104 \times 10^3 \text{ N} \cdot \text{m}$$

For W310 × 52, $S = 747 \times 10^3 \text{ mm}^3 = 747 \times 10^{-6} \text{ m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{104 \times 10^3}{747 \times 10^{-6}} = 139.2 \times 10^6 \text{ Pa}$$

$$\sigma = 139.2 \text{ MPa} \blacktriangleleft$$

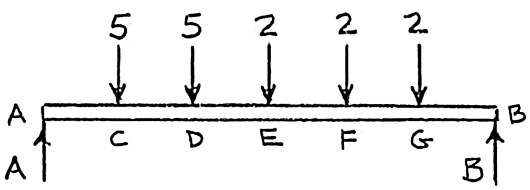


PROBLEM 5.49

Using the method of Sec. 5.3, solve Prob.5.19.

PROBLEM 5.19 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION



Use entire beam as free body.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$-90A + (75)(5) + (60)(5) + (45)(2) + (30)(2) + (15)(2) = 0$$

$$\boldsymbol{A} = 9.5 \text{ kips} \uparrow$$

Shear A to C: $V = 9.5 \text{ kips}$

Area under shear curve A to C: $\int V dx = (15)(9.5) = 142.5 \text{ kip} \cdot \text{in}$

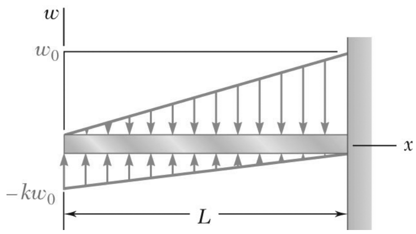
$$M_A = 0$$

$$M_C = 0 + 142.5 = 142.5 \text{ kip} \cdot \text{in}$$

For $\text{S}8 \times 18.4$, $S = 14.4 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{142.5}{14.4}$ $\sigma = 9.90 \text{ ksi}$ ◀

PROBLEM 5.50



For the beam and loading shown, determine the equations of the shear and bending-moment curves, and the maximum absolute value of the bending moment in the beam, knowing that (*a*) $k = 1$, (*b*) $k = 0.5$.

SOLUTION

$$w = \frac{w_0 x}{L} - \frac{kw_0(L-x)}{L} = (1+k)\frac{w_0 x}{L} - kw.$$

$$\frac{dV}{dx} = -w = kw_0 - (1+k)\frac{w_0 x}{L}$$

$$V = kw_0 x - (1+k)\frac{w_0 x^2}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \quad C_1 = 0$$

$$\frac{dM}{dx} = V = kw_0 x - (1+k)\frac{w_0 x^2}{2L}$$

$$M = \frac{kw_0 x^2}{2} - (1+k)\frac{w_0 x^3}{6L} + C_2$$

$$M = 0 \quad \text{at} \quad x = 0 \quad C_2 = 0$$

$$M = \frac{kw_0 x^2}{2} - \frac{(1+k)w_0 x^3}{6L}$$

(*a*) $\underline{k = 1}$.

$$V = w_0 x - \frac{w_0 x^2}{L} \blacktriangleleft$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L} \blacktriangleleft$$

Maximum M occurs at $x = L$.

$$|M|_{\max} = \frac{w_0 L^2}{6} \blacktriangleleft$$

(*b*) $\underline{k = \frac{1}{2}}$.

$$V = \frac{w_0 x}{2} - \frac{3w_0 x^2}{4L} \blacktriangleleft$$

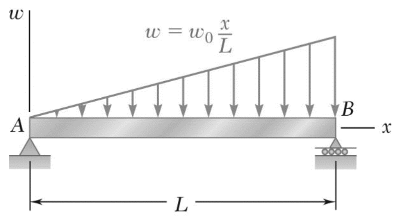
$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L} \blacktriangleleft$$

$$V = 0 \quad \text{at} \quad x = \frac{2}{3}L$$

At $x = \frac{2}{3}L$, $\quad M = \frac{w_0\left(\frac{2}{3}L\right)^2}{4} - \frac{w_0\left(\frac{2}{3}L\right)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704\ w_0 L^2$

At $x = L$, $\quad M = 0$

$$|M|_{\max} = \frac{w_0 L^2}{27} \blacktriangleleft$$



PROBLEM 5.51

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \frac{x}{L}$$

$$V = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + C_1 x + C_2$$

$$M = 0 \quad \text{at} \quad x = 0 \qquad C_2 = 0$$

$$M = 0 \quad \text{at} \quad x = L \qquad 0 = -\frac{1}{6} w_0 L^2 + C_1 L \qquad C_1 = \frac{1}{6} w_0 L$$

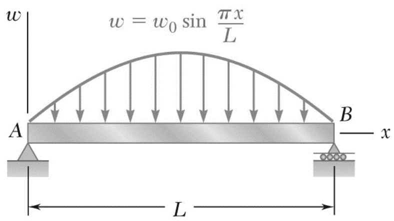
(*a*) $V = -\frac{1}{2} w_0 \frac{x^2}{L} + \frac{1}{6} w_0 L^2$ $\qquad V = \frac{1}{6} w_0 (L^2 - 3x^2)/L$ ◀

$M = -\frac{1}{6} w_0 \frac{x^3}{L} + \frac{1}{6} w_0 Lx$ $\qquad M = \frac{1}{6} w_0 (Lx - x^3/L)$ ◀

(*b*) $M_{\max}$ occurs when $\frac{dM}{dx} = V = 0.\quad L^2 - 3x_m^2 = 0$

$$x_m = \frac{L}{\sqrt{3}} \qquad M_{\max} = \frac{1}{6} w_0 \left(\frac{L^2}{\sqrt{3}} - \frac{L^2}{3\sqrt{3}} \right)$$

$M_{\max} = 0.0642 w_0 L^2$ ◀



PROBLEM 5.52

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0 \qquad C_2 = 0$$

$$M = 0 \text{ at } x = L \qquad 0 = 0 + C_1 L + 0$$

$$C_1 = 0$$

(*a*)

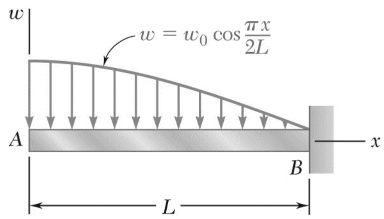
$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} \blacktriangleleft$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} \blacktriangleleft$$

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

(*b*) $M_{\max} = \dfrac{w_0 L^2}{\pi^2} \sin \dfrac{\pi}{2}$

$$M_{\max} = \frac{w_0 L^2}{\pi^2} \blacktriangleleft$$



PROBLEM 5.53

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\frac{2Lw_0}{\pi} \sin \frac{\pi x}{2L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{4L^2 w_0}{\pi^2} \cos \frac{\pi x}{2L} + C_1 x + C_2$$

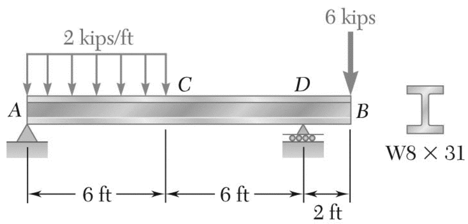
$V = 0$ at $x = 0$. Hence, $C_1 = 0$.

$M = 0$ at $x = 0$. Hence, $C_2 = -\dfrac{4L^2 w_0}{\pi^2}$.

(*a*) $V = -(2Lw_0 / \pi)\sin(\pi x / 2L)$ ◀

$M = -(4L^2 w_0/\pi^2)[1 - \cos(\pi x/2L)]$ ◀

(*b*) $|M|_{\max} = 4 w_0 L^2/\pi^2$ ◀

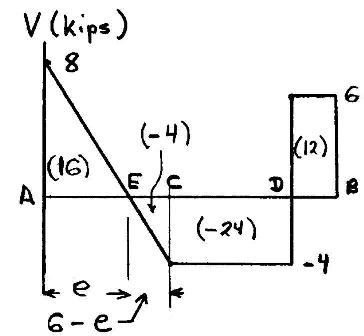


PROBLEM 5.54

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft M_D = 0: \; -12A + (9)(6)(2) - (2)(6) = 0$$
$$A = 8 \text{ kips}$$
$$+\circlearrowleft M_A = 0: \; -(3)(6) + 12D - (14)(6) = 0$$
$$D = 10 \text{ kips}$$



Shear: $V_A = 8$ kips

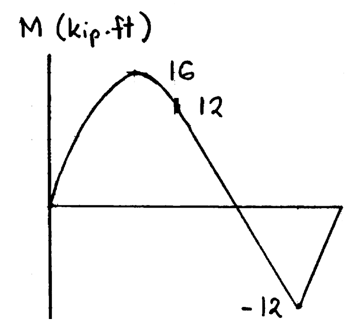
$V_C = 8 - (6)(2) = -4$ kips

C to D: $V = -4$ kips

D to B: $V = -4 + 10 = 6$ kips

Locate point E where $V = 0$.

$$\frac{e}{8} = \frac{6-e}{4} \qquad 12e = 48$$
$$e = 4 \text{ ft} \qquad 6 - e = 2 \text{ ft}$$



Areas of the shear diagram:

A to E: $\int V dx = \left(\frac{1}{2}\right)(4)(8) = 16 \text{ kip} \cdot \text{ft}$

E to C: $\int V dx = \left(\frac{1}{2}\right)(2)(-4) = -4 \text{ kip} \cdot \text{ft}$

C to D: $\int V dx = (6)(-4) = -24 \text{ kip} \cdot \text{ft}$

D to B: $\int V dx = (2)(6) = 12 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

$M_E = 0 + 16 = 16 \text{ kip} \cdot \text{ft}$

$M_C = 16 - 4 = 12 \text{ kip} \cdot \text{ft}$

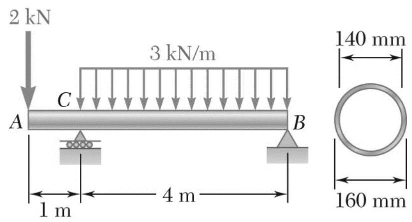
$M_D = 12 - 24 = -12 \text{ kip} \cdot \text{ft}$

$M_B = -12 + 12 = 0$

Maximum $|M| = 16 \text{ kip} \cdot \text{ft} = 192 \text{ kip} \cdot \text{in}$

For W8× 31 rolled steel section, $S = 27.5 \text{ in}^3$

Normal stress: $\sigma = \frac{|M|}{S} = \frac{192}{27.5}$ $\qquad \sigma = 6.98$ ksi ◀



PROBLEM 5.55

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0 : (2)(1) - (3)(4)(2) + 4B = 0$$
$$B = 5.5 \text{ kN}$$

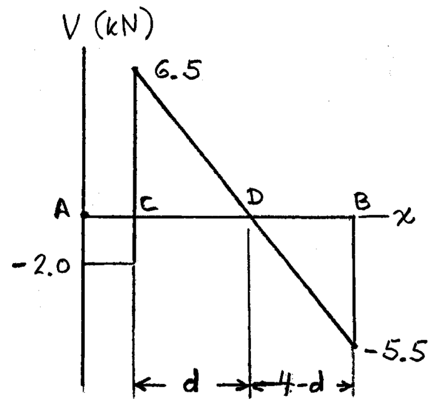
$$+\circlearrowleft \Sigma M_B = 0 : (5)(2) + (3)(4)(2) - 4C = 0$$
$$C = 8.5 \text{ kN}$$

Shear:

A to *C*: $V = -2 \text{ kN}$

C^+ : $V = -2 + 8.5 = 6.5 \text{ kN}$

B: $V = 6.5 - (3)(4) = -5.5 \text{ kN}$



Locate point *D* where $V = 0$.

$$\frac{d}{6.5} = \frac{4-d}{5.5} \qquad 12d = 26$$

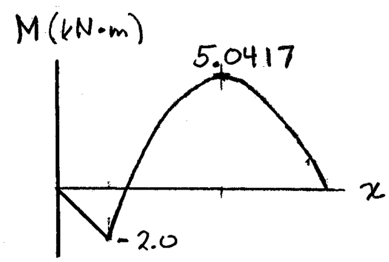
$$d = 2.1667\,\text{m} \qquad 4 - d = 3.8333\text{ m}$$

Areas of the shear diagram:

A to *C*: $\int V dx = (-2.0)(1) = -2.0 \text{ kN}\cdot\text{m}$

C to *D*: $\int V dx = \frac{1}{2}(2.16667)(6.5) = 7.0417 \text{ kN}\cdot\text{m}$

D to *B*: $\int V dx = \frac{1}{2}(3.83333)(-5.5) = -5.0417 \text{ kN}\cdot\text{m}$



Bending moments: $M_A = 0$

$$M_C = 0 - 2.0 = -2.0 \text{ kN}\cdot\text{m}$$

$$M_D = -2.0 + 7.0417 = 5.0417 \text{ kN}\cdot\text{m}$$

$$M_B = 5.0417 - 5.0417 = 0$$

Maximum $|M| = 5.0417 \text{ kN}\cdot\text{m} = 5.0417\times 10^3 \text{N}\cdot\text{m}$

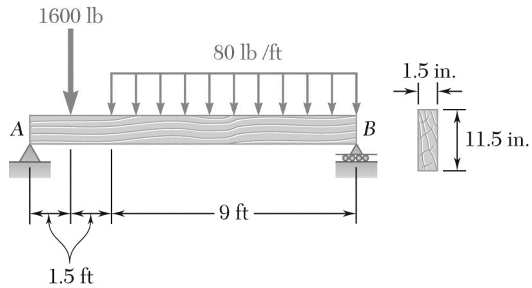
PROBLEM 5.55 *(Continued)*

For pipe: $c_o = \frac{1}{2}d_o = \frac{1}{2}(160) = 80 \text{ mm}, \quad c_i = \frac{1}{2}d_i = \frac{1}{2}(140) = 70 \text{ mm}$

$$I = \frac{\pi}{4}\left(c_o^4 - c_i^4\right) = \frac{\pi}{4}\left[(80)^4 - (70)^4\right] = 13.3125 \times 10^6 \text{ mm}^4$$

$$S = \frac{I}{c_o} = \frac{13.3125 \times 10^6}{80} = 166.406 \times 10^3 \text{ mm}^3 = 166.406 \times 10^{-6} \text{ m}^3$$

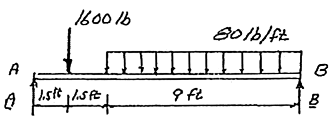
Normal stress: $\sigma = \frac{M}{S} = \frac{5.0417 \times 10^3}{166.406 \times 10^{-6}} = 30.3 \times 10^6 \text{ Pa}$ $\sigma = 30.3 \text{ MPa}$ ◀



PROBLEM 5.56

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

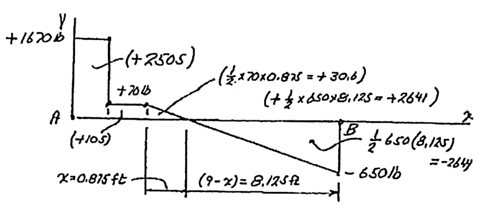


$$+\circlearrowleft \Sigma M_A = 0 : (1600 \text{ lb})(1.5 \text{ ft}) + \left[(80 \text{ lb/ft})(9 \text{ ft})\right](7.5 \text{ ft}) - 12B = 0$$

$$B = +650 \text{ lb} \qquad \underline{\boldsymbol{B}} = 650 \text{ lb} \uparrow$$

$$+\uparrow \Sigma F_y = 0 : A - 1600 \text{ lb} - \left[(80 \text{ lb/ft})(9 \text{ ft})\right] + 650 \text{ lb} = 0$$

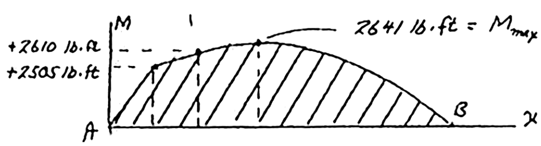
$$A = +1670 \text{ lb} \qquad \underline{\boldsymbol{A}} = 1670 \text{ lb} \uparrow$$



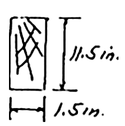
$$\frac{x}{70 \text{ lb}} = \frac{9-x}{650 \text{ lb}} \qquad x = 0.875 \text{ ft} \qquad 2641 \text{ lb} \cdot \text{ft} = M_{\max}$$

$$c = \frac{1}{2}(11.5 \text{ in.}) = 5.75 \text{ in.}$$

$$I = \frac{1}{12}(1.5 \text{ in.})(11.5 \text{ in.})^3 = 190.1 \text{ in}^4$$

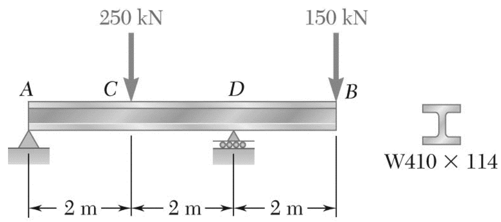


◀



$$M_{\max} = 2641 \text{ lb} \cdot \text{ft} = 31{,}690 \text{ lb} \cdot \text{in}$$

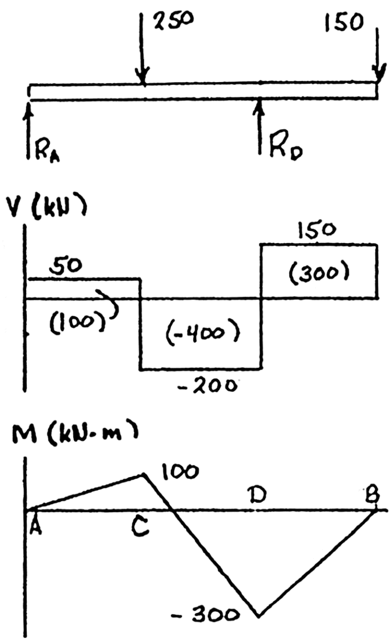
$$\sigma_m = \frac{M_{\max} c}{I} = \frac{(31{,}690 \text{ lb} \cdot \text{in})(5.75 \text{ in.})}{190.1 \text{ in}^4} \qquad \sigma_m = 959 \text{ psi} ◀$$



PROBLEM 5.57

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION



$$w = 0$$

$$+\circlearrowleft \Sigma M_D = 0:$$

$$-4R_A + (2)(250) - (2)(150) = 0$$

$$\mathbf{R}_A = 50 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0:$$

$$4R_D - (2)(250) - (6)(150) = 0$$

$$\mathbf{R}_D = 350 \text{ kN} \uparrow$$

Shear: $V_A = 50$ kN

A to C: $V = 50$ kN

C to D: $V = 50 - 250 = -200$ kN

D to B: $V = -200 + 350 = 150$ kN

Areas of shear diagram:

A to C: $\int V dx = (50)(2) = 100 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = (-200)(2) = -400 \text{ kN} \cdot \text{m}$

D to B: $\int V dx = (150)(2) = 300 \text{ kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$$M_C = M_A + \int V dx = 0 + 100 = 100 \text{ kN} \cdot \text{m}$$

$$M_D = M_C + \int V dx = 100 - 400 = -300 \text{ kN} \cdot \text{m}$$

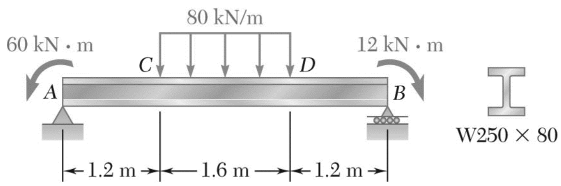
$$M_B = M_D + \int V dx = -300 + 300 = 0$$

Maximum $|M| = 300 \text{ kN} \cdot \text{m} = 300 \times 10^3 \text{ N} \cdot \text{m}$

For $W410 \times 114$ rolled steel section, $S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa}$$

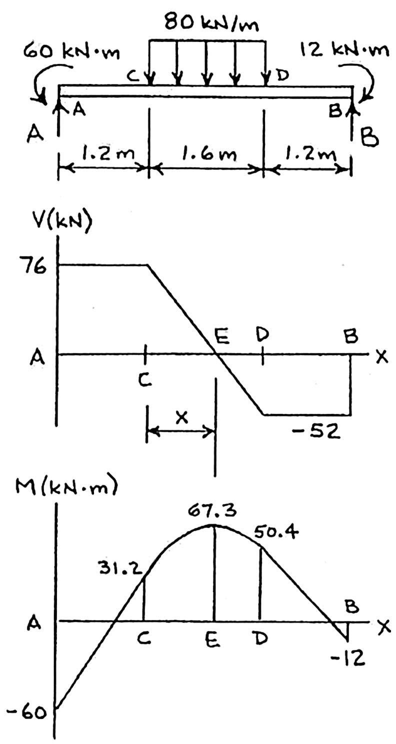
$$\sigma_m = 136.4 \text{ MPa} \blacktriangleleft$$



PROBLEM 5.58

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION



Reaction:

$$+\circlearrowleft \Sigma M_B = 0 :- 4A + 60 + (80)(1.6)(2) - 12 = 0$$

$$\mathbf{A} = 76\,\text{kN} \uparrow$$

Shear: $V_A = 76\,\text{kN}$

A to C: $V = 76\text{ kN}$ ◀

$V_D = 76 - (80)(1.6) = -52\text{ kN}$

D to C: $V = -52\text{ kN}$

Locate point where $V = 0$:

$V(x) = -80x + 76 = 0 \qquad x = 0.95\text{ m}$

Areas of shear diagram:

A to C: $\int V dx = (1.2)(76) = 91.2\text{ kN} \cdot \text{m}$

C to E: $\int V dx = \dfrac{1}{2}(0.95)(76) = 36.1\text{ kN} \cdot \text{m}$

E to D: $\int V dx = \dfrac{1}{2}(0.65)(-52) = -16.9\text{ kN} \cdot \text{m}$

D to B: $\int V dx = (1.2)(-52) = -62.4\text{ kN} \cdot \text{m}$

Bending moments: $M_A = -60\text{ kN} \cdot \text{m}$

$M_C = -60 + 91.2 = 31.2\text{ kN} \cdot \text{m}$

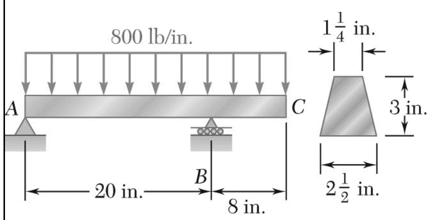
$M_E = 31.2 + 36.1 = 67.3\text{ kN} \cdot \text{m}$ ◀

$M_D = 67.3 - 16.9 = 50.4\text{ kN} \cdot \text{m}$

$M_B = 50.4 - 62.4 = -12\text{ kN} \cdot \text{m}$

For W250 × 80, $S = 983 \times 10^3\text{mm}^3$

Normal stress: $\sigma_{\max} = \dfrac{|M|}{S} = \dfrac{67.3 \times 10^3\text{ N} \cdot \text{m}}{983 \times 10^{-6}\text{ m}^3} = 68.5 \times 10^6\text{ Pa} \qquad \sigma_m = 68.5\text{ MPa}$ ◀



PROBLEM 5.59

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

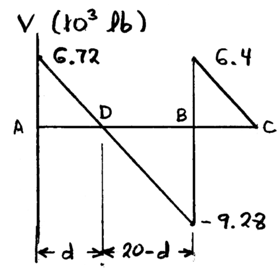
$$+\circlearrowleft \Sigma M_B = 0: \quad -20A + (6)(28)(800) = 0$$

$$A = 6.72 \times 10^3 \text{ lb}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 20B - (14)(28)(800) = 0$$

$$B = 15.68 \times 10^3 \text{ lb}$$

Shear: $\qquad V_A = 6.72 \times 10^3 \text{ lb}$



B^-: $\qquad V_{B^-} = 6.72 \times 10^3 - (20)(800) = -9.28 \times 10^3 \text{ lb}$

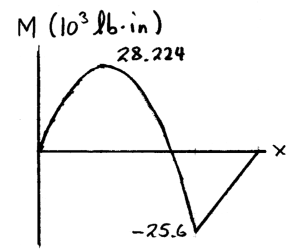
B^+: $\qquad V_{B^+} = -9.28 \times 10^3 + (15.68 \times 10^3) = 6.4 \times 10^3 \text{ lb}$

C: $\qquad V_C = 6.4 \times 10^3 - (8)(800) = 0$

Locate point D where $\quad V = 0$.

$$\frac{d}{6.72} = \frac{20-d}{9.28} \qquad 16d = 134.4$$

$$d = 8.4 \text{ in.} \qquad 20 - d = 11.6 \text{ in.}$$



Areas of the shear diagram:

A to D: $\qquad \int V dx = \left(\frac{1}{2}\right)(8.4)(6.72 \times 10^3) = 28.224 \times 10^3 \text{ lb} \cdot \text{in}$

D to B: $\qquad \int V dx = \left(\frac{1}{2}\right)(11.6)(-9.28 \times 10^3) = -53.824 \times 10^3 \text{ lb} \cdot \text{in}$

B to C: $\qquad \int V dx = \left(\frac{1}{2}\right)(8)(6.4 \times 10^3) = 25.6 \times 10^3 \text{ lb} \cdot \text{in}$

Bending moments:

$$M_A = 0$$

$$M_D = 0 + 28.224 \times 10^3 = 28.224 \times 10^3 \text{ lb} \cdot \text{in}$$

$$M_B = 28.224 \times 10^3 - 53.824 \times 10^3 = -25.6 \times 10^3 \text{ lb} \cdot \text{in}$$

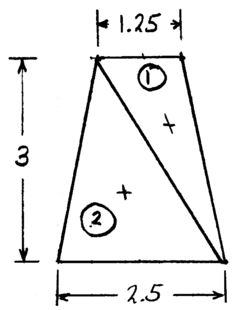
$$M_C = -25.6 \times 10^3 + 25.6 \times 10^3 = 0$$

Maximum $|M| = 28.224 \times 10^3 \text{ lb} \cdot \text{in}$

PROBLEM 5.59 *(Continued)*

Locate centroid of cross section. See table below.

$$\overline{Y} = \frac{7.5}{5.625} = 1.3333 \text{ in. from bottom.}$$



For each triangle, $\overline{I} = \dfrac{1}{36}bh^3$

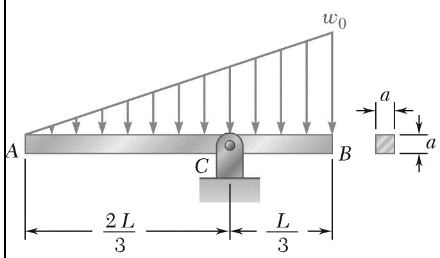
Moment of inertia:

$$I = \Sigma\overline{I} + \Sigma A d^2$$
$$= 1.25 + 2.8125 = 4.0625 \text{ in}^4$$

Normal stress: $\sigma = \dfrac{Mc}{I} = \dfrac{(28.224\times 10^3)(1.6667)}{4.0625} = 11.58\times 10^3 \text{ psi}$ $\quad\sigma = 11.58$ ksi ◀

Part	A, in^2	$\overline{y}$, in.	$A\overline{y}$, in^3	d, in.	Ad^2 in^4	$\overline{I}$ in^4
①	1.875	2	3.75	0.6667	0.8333	0.9375
②	3.75	1	3.75	0.3333	0.4167	1.875
Σ	5.625		7.5		1.25	2.8125

PROBLEM 5.60



Beam AB, of length L and square cross section of side a, is supported by a pivot at C and loaded as shown. (a) Check that the beam is in equilibrium. (b) Show that the maximum stress due to bending occurs at C and is equal to $w_0L^2/(1.5a)^3$.

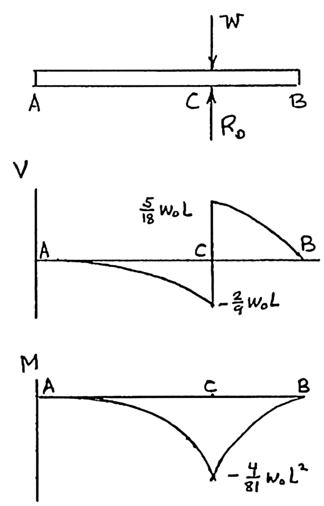
SOLUTION

(a) Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

For the triangular distribution, the centroid lies at $x = \dfrac{2L}{3}$. $\quad W = \dfrac{1}{2}w_0L$

(a) $+\uparrow \sum F_y = 0 : R_D - W = 0 \qquad R_D = \dfrac{1}{2}w_0L \qquad +\circlearrowleft \sum M_C = 0 : 0 = 0$ equilibrium ◀

$$V = 0,\ M = 0,\ \text{at } x = 0$$



$$0 < x < \frac{2L}{3}, \quad \frac{dV}{dx} = -w = -\frac{w_0x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0x^2}{2L} + C_1 = -\frac{w_0x^2}{2L}$$

$$M = -\frac{w_0x^3}{6L} + C_2 = -\frac{w_0x^3}{6L}$$

Just to the left of C, $\quad V = -\dfrac{w_0(2L/3)^2}{2L} = -\dfrac{2}{9}w_0L$

Just to the right of C, $\quad V = -\dfrac{2}{9}w_0L + R_D = \dfrac{5}{18}w_0L$

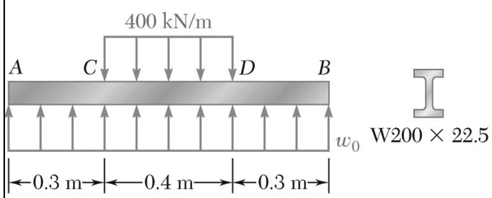
Note sign change. Maximum $|M|$ occurs at C. $\quad M_C = -\dfrac{w_0(2L/3)^3}{6L} = -\dfrac{4}{81}w_0L^2$

Maximum $|M| = \dfrac{4}{81}w_0L^2$

For square cross section, $\quad I = \dfrac{1}{12}a^4 \qquad c = \dfrac{1}{2}a$

(b) $\quad \sigma_m = \dfrac{|M|_{\max}c}{I} = \dfrac{4}{81}\dfrac{w_0L^2 6}{a^3} = \dfrac{8}{27}\dfrac{w_0L^2}{a^3} = \left(\dfrac{2}{3}\right)^3\dfrac{w_0L^2}{a^3}$

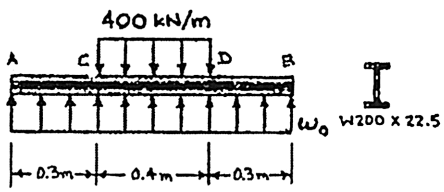
$$\sigma_m = \frac{w_0L^2}{(1.5a)^3} \ ◀$$



PROBLEM 5.61

Knowing that beam AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION

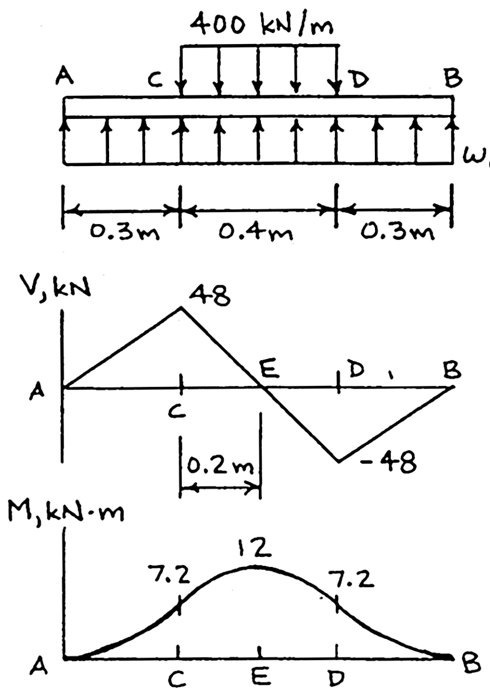


$$+\uparrow \sum F_y = 0: \ (1)(w_0) - (0.4)(400) = 0$$

$$w_0 = 160 \text{ kN/m}$$

Shear diagram: $V_A = 0$

$$V_C = 0 + (0.3)(160) = 48 \text{ kN}$$
$$V_D = 48 - (0.3)(400) + (0.3)(160) = -48 \text{ kN}$$
$$V_B = -48 + (0.3)(160) = 0$$



Locate point E where $V = 0$.

By symmetry, E is the midpoint of CD.

Areas of shear diagram:

A to C: $\frac{1}{2}(0.3)(48) = 7.2 \text{ kN} \cdot \text{m}$

C to E: $\frac{1}{2}(0.2)(48) = 4.8 \text{ kN} \cdot \text{m}$

E to D: $\frac{1}{2}(0.2)(-48) = -4.8 \text{ kN} \cdot \text{m}$

D to B: $\frac{1}{2}(0.3)(-48) = -7.2 \text{ kN} \cdot \text{m}$

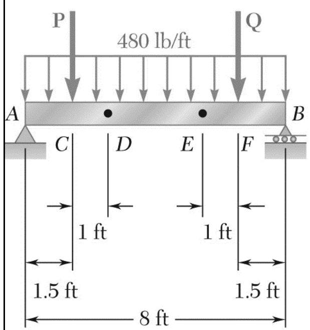
Bending moments: $M_A = 0$

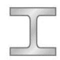
$$M_C = 0 + 7.2 = 7.2 \text{ kN}$$
$$M_E = 7.2 + 4.8 = 12.0 \text{ kN}$$
$$M_D = 12.0 - 4.8 = 7.2 \text{ kN}$$
$$M_B = 7.2 - 7.2 = 0$$

$$|M|_{\max} = 12.0 \text{ kN} \cdot \text{m} = 12.0 \times 10^3 \text{ N} \cdot \text{m}$$

For W200 × 22.5 rolled steel shape, $S_x = 193 \times 10^3 \text{ mm}^3 = 193 \times 10^{-6} \text{ m}^3$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{12.0 \times 10^3}{193 \times 10^{-6}} = 62.2 \times 10^6 \text{ Pa}$ $\qquad \sigma = 62.2 \text{ MPa}$ ◀

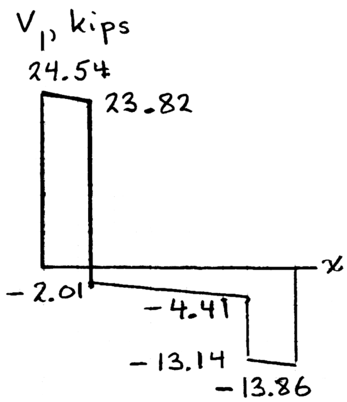



W8 × 31

PROBLEM 5.62

The beam *AB* supports a uniformly distributed load of 480 lb/ft and two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the lower flange is +14.85 ksi at *D* and +10.65 ksi at *E*. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending that occurs in the beam.

SOLUTION



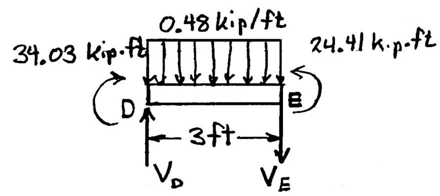
(*a*) For W8 × 31 rolled steel section, $S = 27.5\ \text{in}^3$

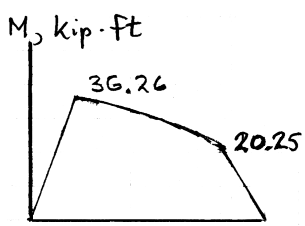
$$M = S\sigma$$

At *D*, $M_D = (27.5)(14.85) = 408.375\ \text{kip} \cdot \text{in}$

At *E*, $M_E = (27.5)(10.65) = 292.875\ \text{kip} \cdot \text{in}$

$$M_D = 34.03\ \text{kip} \cdot \text{ft} \quad M_E = 24.41\ \text{kip} \cdot \text{ft}$$





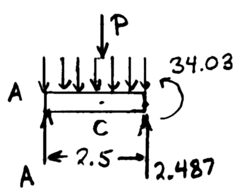
Use free body *DE*:

$$+\circlearrowleft \Sigma M_E = 0 : -34.03 + 24.41 + (1.5)(3)(0.48) - 3V_D = 0$$

$$V_D = -2.487\ \text{kips}$$

$$+\circlearrowleft M_D = 0 : -34.03 + 24.41 - (1.5)(3)(0.48) - 3V_E = 0$$

$$V_E = -3.927\ \text{kips}$$



Use free body *ACD*:

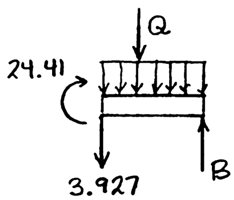
$$+\circlearrowleft \Sigma M_A = 0 : -1.5P - (1.25)(2.5)(0.48) + (2.5)(2.487) + 34.03 = 0$$

$$\boldsymbol{P} = 25.83\ \text{kips} \downarrow$$

$$+\uparrow \Sigma F_y = 0 : A - (2.5)(0.48) + 2.487 - 25.83 = 0$$

$$\boldsymbol{A} = 24.54\ \text{kips} \uparrow$$

PROBLEM 5.62 *(Continued)*



Use free body *EFB*:

$$+\circlearrowleft \sum M_B = 0 : 1.5Q + (1.25)(2.5)(0.48) + (2.5)(3.927) - 24.41 = 0$$

$$Q = 8.728 \text{ kips}$$

$$+\uparrow \sum F_y = 0 : B - 3.927 - (2.5)(0.48) - 8.7 = 0$$

$$B = 13.855 \text{ kips}$$

Areas of load diagram:

A to C: $(1.5)(0.48) = 0.72 \text{ kip} \cdot \text{ft}$

C to F: $(5)(0.48) = 2.4 \text{ kip} \cdot \text{ft}$

F to B: $(1.5)(0.48) = 0.72 \text{ kip} \cdot \text{ft}$

Shear diagram: $V_A = 24.54 \text{ kips}$

$$V_C^- = 24.54 - 0.72 = 23.82 \text{ kips}$$

$$V_C^+ = 23.82 - 25.83 = -2.01 \text{ kips}$$

$$V_F^- = -2.01 - 2.4 = 4.41 \text{ kips}$$

$$V_F^+ = -4.41 - 8.728 = -13.14 \text{ kips}$$

$$V_B = -13.14 - 0.72 = -13.86 \text{ kips}$$

Areas of shear diagram:

A to C: $\frac{1}{2}(1.5)(24.52 + 23.82) = 36.23 \text{ kip} \cdot \text{ft}$

C to F: $\frac{1}{2}(5)(-2.01 - 4.41) = -16.05 \text{ kip} \cdot \text{ft}$

F to B: $\frac{1}{2}(1.5)(-13.14 - 13.86) = 20.25 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

$$M_C = 0 + 36.26 = 36.26 \text{ kip} \cdot \text{ft}$$

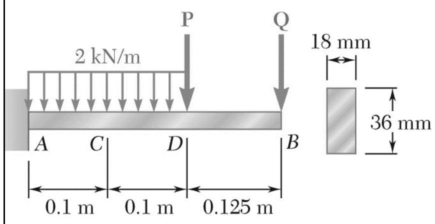
$$M_F = 36.26 - 16.05 = 20.21 \text{ kip} \cdot \text{ft}$$

$$M_B = 20.21 - 20.25 \approx 0$$

Maximum $|M|$ occurs at C: $|M|_{\text{max}} = 36.26 \text{ kip} \cdot \text{ft} = 435.1 \text{ kip} \cdot \text{in}$

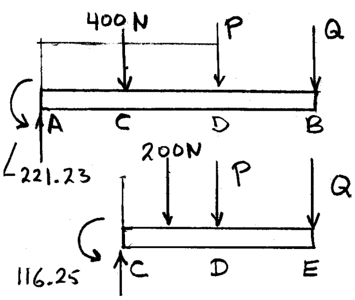
(*b*) Maximum stress: $\sigma = \dfrac{|M|_{\text{max}}}{S} = \dfrac{435.1}{27.5}$ $\quad \sigma = 15.82 \text{ ksi}$ ◀

PROBLEM 5.63*



Beam AB supports a uniformly distributed load of $2\,\text{kN/m}$ and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending in bottom edge of the beam is -56.9 MPa at A and -29.9 MPa at C. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads **P** and **Q**.

SOLUTION



$$I = \frac{1}{12}(18)(36)^3 = 69.984\times 10^3\,\text{mm}^4$$

$$c = \frac{1}{2}d = 18\text{ mm}$$

$$S = \frac{I}{c} = 3.888\times 10^3\,\text{mm}^3 = 3.888\times 10^{-6}\,\text{m}^3$$

At A, $\quad M_A = S\sigma_A = (3.888\times 10^{-6})(-56.9) = -221.25\text{ N}\cdot\text{m}$

At C, $\quad M_C = S\sigma_C = (3.888\times 10^{-6})(-29.9) = -116.25\text{ N}\cdot\text{m}$

$$+\circlearrowleft \Sigma M_A = 0:\quad 221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \qquad (1)$$

$$+\circlearrowleft \Sigma M_C = 0:\quad 116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

$$0.1P + 0.225Q = 106.25 \qquad (2)$$

Solving (1) and (2) simultaneously, $\qquad P = 500\text{ N}$ ◀

$Q = 250\text{ N}$ ◀

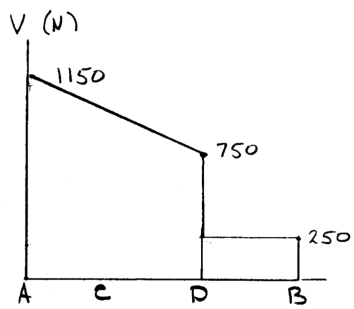
Reproduction force at A: — Reaction force at A: $\quad R_A - 400 - 500 - 250 = 0 \quad R_A = 1150\text{ N}\cdot\text{m}$

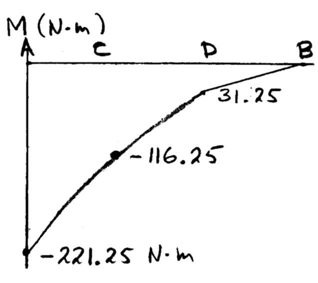
$V_A = 1150\text{ N} \quad V_D = 250$

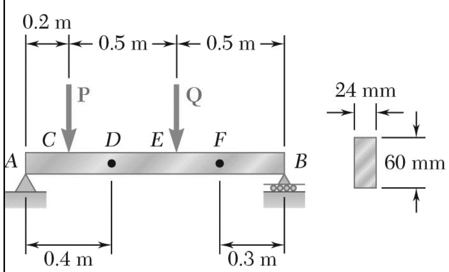
$M_A = -221.25\text{ N}\cdot\text{m} \quad M_C = -116.25\text{ N}\cdot\text{m} \quad M_D = -31.25\text{ N}\cdot\text{m}$

$|V|_{\max} = 1150\text{ N}$ ◀

$|M|_{\max} = 221\text{ N}\cdot\text{m}$ ◀



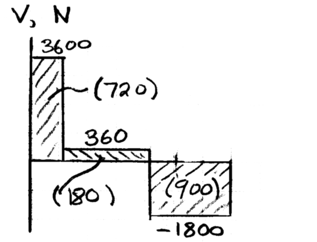


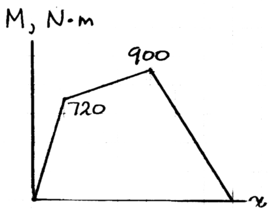


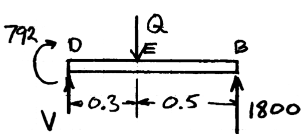
PROBLEM 5.64*

The beam AB supports two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the beam is $+55\,\text{MPa}$ at D and $+37.5\,\text{MPa}$ at F. (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending that occurs in the beam.

SOLUTION





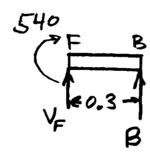


(a) $\quad I = \dfrac{1}{12}(24)(60)^3 = 432\times10^3\,\text{mm}^4 \qquad c = 30\text{ mm}$

$$S = \frac{I}{c} = 14.4\times10^3\,\text{mm}^3 = 14.4\times10^{-6}\,\text{m}^3 \qquad M = S\sigma$$

At D, $\quad M_D = (14.4\times10^{-6})(55\times10^6) = 792\text{ N}\cdot\text{m}$

At F, $\quad M_F = (14.4\times10^{-6})(37.5\times10^6) = 540\text{ N}\cdot\text{m}$



Using free body FB, $\quad +\circlearrowleft\Sigma M_F = 0: \ -540 + 0.3B = 0$

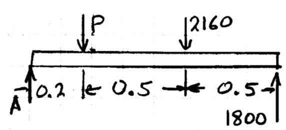
$$B = \frac{540}{0.3} = 1800\text{ N}$$

Using free body $DEFB$, $\quad +\circlearrowleft\Sigma M_D = 0: \ -792 - 3Q + (0.8)(1800) = 0$

$$Q = 2160\text{ N}$$

Using entire beam, $\quad +\circlearrowleft\Sigma M_A = 0: \ -0.2P - (0.7)(2160) + (1.2)(1800) = 0$

$$P = 3240\text{ N}$$



$+\uparrow\Sigma F_y = 0: \ A - 3240 - 2160 + 1800 = 0$

$$A = 3600\text{ N}$$

Shear diagram and its areas:

A to C^-: $\quad V = 3600\text{ N} \qquad A_{AC} = (0.2)(3600) = 720\text{ N}\cdot\text{m}$

C^+ to E^-: $\quad V = 3600 - 3240 = 360\text{ N} \qquad A_{CE} = (0.5)(360) = 180\text{ N}\cdot\text{m}$

E^+ to B: $\quad V = 360 - 2160 = -1800\text{ N} \qquad A_{EB} = (0.5)(-1800) = -900\text{ N}\cdot\text{m}$

Bending moments:

$M_A = 0$

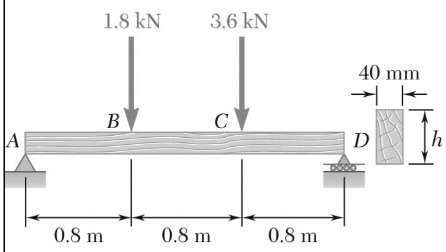
$M_C = 0 + 720 = 720\text{ N}\cdot\text{m}$

$M_E = 720 + 180 = 900\text{ N}\cdot\text{m} \qquad |M|_{\max} = 900\text{ N}\cdot\text{m}$

$M_B = 900 - 900 = 0$

(b) $\quad$ Normal stress. $\quad \sigma_{\max} = \dfrac{|M|_{\max}}{S} = \dfrac{900}{14.4\times10^{-6}} = 62.5\times10^6\,\text{Pa}$

$$\sigma_{\max} = 62.5\text{ MPa} \ \blacktriangleleft$$



PROBLEM 5.65

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

Reactions:

$$+\circlearrowleft \Sigma M_D = 0: \quad -2.4A + (1.6)(1.8) + (0.8)(3.6) = 0 \qquad A = 2.4 \text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0 \qquad D = 3 \text{ kN}$$

Construct shear and bending moment diagrams:

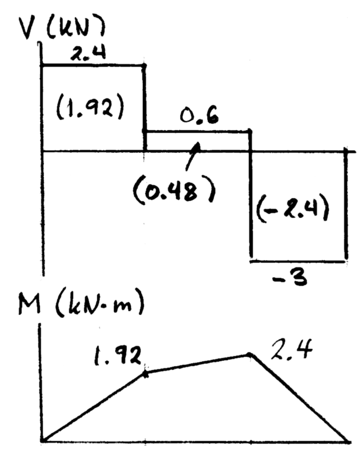
$$|M|_{\max} = 2.4 \text{ kN}\cdot\text{m} = 2.4\times10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12\times10^6 \text{ Pa}$$

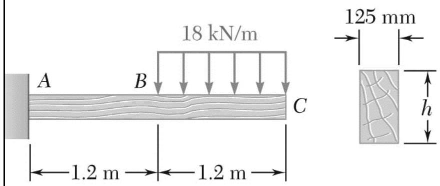
$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{2.4\times10^3}{12\times10^6} = 200\times10^{-6} \text{ m}^3 = 200\times10^3 \text{ mm}^3$$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(40)h^2 = 200\times10^3$$

$$h^2 = \frac{(6)(200\times10^3)}{40} = 30\times10^3 \text{ mm}^2$$



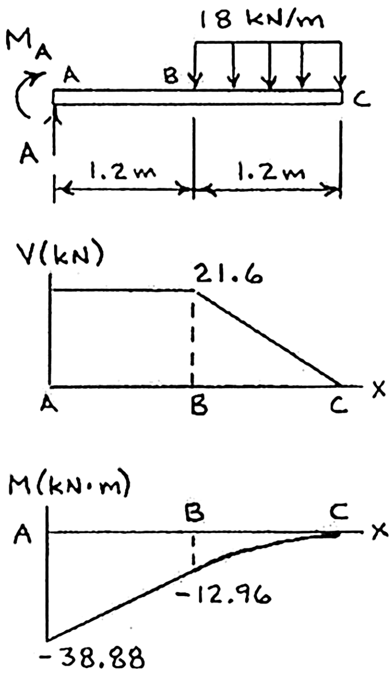
$h = 173.2$ mm ◀



PROBLEM 5.66

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION



Reactions: $+\uparrow \sum F_y = 0: \quad A - (1.2)(18) = 0$

$$\boldsymbol{A} = 21.6 \text{ kN} \uparrow$$

$$+\circlearrowleft \sum M_A = 0: \quad -M_A - (1.8)(1.2)(18) = 0$$

$$M_A = -38.88 \text{ kN}\cdot\text{m}$$

Shear diagram: $\quad V_A = V_B = 21.6 \text{ kN}$

$$V_C = 21.6 - (1.2)(18) = 0$$

Areas of shear diagram:

$$A \text{ to } B: \ (1.2)(21.6) = 25.92 \text{ kN}\cdot\text{m}$$

$$B \text{ to } C: \frac{1}{2}(1.2)(21.6) = 12.96 \text{ kN}\cdot\text{m}$$

Bending moments: $\quad M_A = -38.88 \text{ kN}\cdot\text{m}$

$$M_B = -38.88 + 25.92 = -12.96 \text{ kN}\cdot\text{m}$$

$$M_C = -12.96 + 12.96 = 0$$

$$|M|_{\max} = 38.88 \text{ kN}\cdot\text{m} = 38.8 \times 10^3 \text{ N}\cdot\text{m}$$

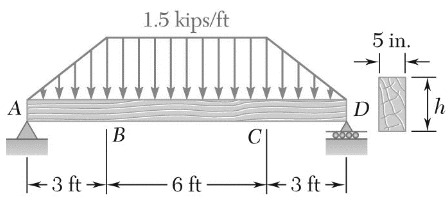
$$\sigma_{\max} = \frac{|M|_{\max}}{S}$$

$$S = \frac{|M|_{\max}}{\sigma_{\max}} = \frac{38.8 \times 10^3 \text{ N}\cdot\text{m}}{12 \times 10^6 \text{ Pa}} = 3240 \times 10^{-6} \text{ m}^3 = 3240 \times 10^3 \text{ mm}^3$$

For a rectangular section, $\quad S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(3240 \times 10^3)}{125}} = 394 \text{ mm}$$

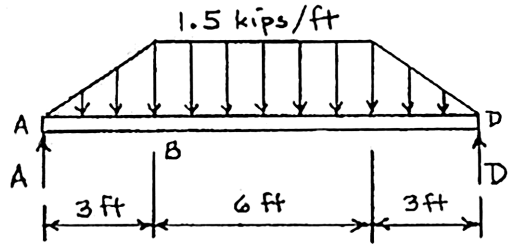
$$h = 394 \text{ mm} \blacktriangleleft$$

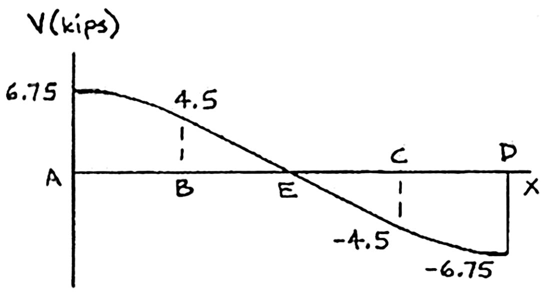


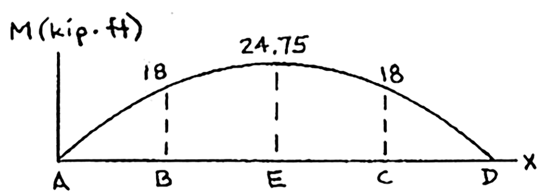
PROBLEM 5.67

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION







Reactions: By symmetry, $A = D$.

$$+\uparrow \Sigma F_y = 0: \; A - \frac{1}{2}(3)(1.5) - (6)(1.5) - \frac{1}{2}(3)(1.5) - D = 0$$

$$A = D = 6.75 \text{ kips} \uparrow$$

Shear diagram: $V_A = 6.75$ kips

$$V_B = 6.75 - \frac{1}{2}(3)(1.5) = 4.5 \text{ kips}$$

$$V_C = 4.5 - (6)(1.5) = -4.5 \text{ kips}$$

$$V_D = -4.5 - \frac{1}{2}(3)(1.5) = -6.75 \text{ kips}$$

Locate point E where $V = 0$:

By symmetry, E is the midpoint of BC.

Areas of the shear diagram:

$$A \text{ to } B: \; (3)(4.5) + \frac{2}{3}(3)(2.25) = 18 \text{ kip} \cdot \text{ft}$$

$$B \text{ to } E: \; \frac{1}{2}(3)(4.5) = 6.75 \text{ kip} \cdot \text{ft}$$

$$E \text{ to } C: \; \frac{1}{2}(3)(-4.5) = -6.75 \text{ kip} \cdot \text{ft}$$

C to D: By antisymmetry, -18 kip · ft

PROBLEM 5.67 *(Continued)*

Bending moments: $M_A = 0$

$$M_B = 0 + 18 = 18 \text{ kip} \cdot \text{ft}$$

$$M_E = 18 + 6.75 = 24.75 \text{ kip} \cdot \text{ft}$$

$$M_C = 24.75 - 6.75 = 18 \text{ kip} \cdot \text{ft}$$

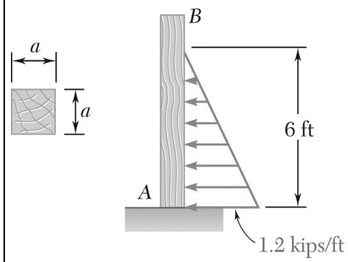
$$M_D = 18 - 18 = 0$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} \qquad S = \frac{|M|_{\max}}{\sigma_{\max}} = \frac{(24.75 \text{ kip} \cdot \text{ft})(12 \text{ in/ft})}{1.750 \text{ ksi}} = 169.714 \text{ in}^3$$

For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(169.714)}{5}} = 14.27 \text{ in.}$$

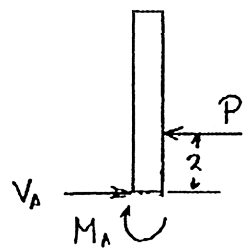
$h = 14.27$ in. ◀



PROBLEM 5.68

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION



Equivalent concentrated load:

$$P = \left(\frac{1}{2}\right)(6)(1.2) = 3.6 \text{ kips}$$

Bending moment at A:

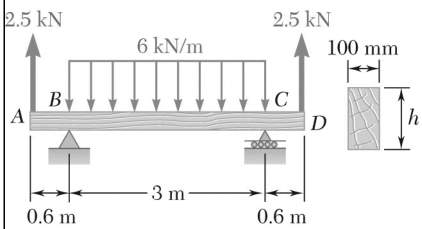
$$M_A = (2)(3.6) = 7.2 \text{ kip} \cdot \text{ft} = 86.4 \text{ kip} \cdot \text{in}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3$$

For a square section, $\qquad S = \dfrac{1}{6}a^3$

$$a = \sqrt[3]{6S}$$

$$a_{\min} = \sqrt[3]{(6)(49.37)} \qquad\qquad a_{\min} = 6.67 \text{ in.} \blacktriangleleft$$



PROBLEM 5.69

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

By symmetry, $B = C$

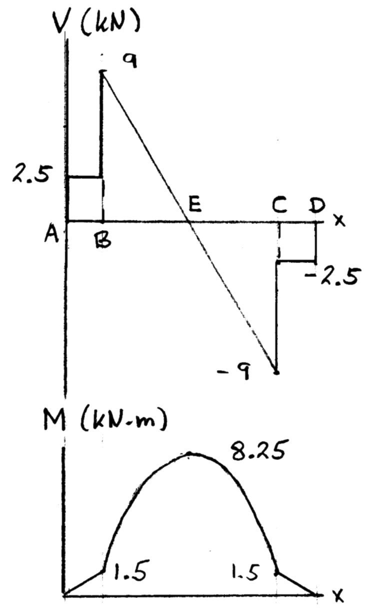
$$+\uparrow \Sigma F_y = 0: \quad B + C + 2.5 + 2.5 - (3)(6) = 0 \qquad B = C = 6.5 \text{ kN}$$

Shear:

A to B: $V = 2.5 \text{ kN}$

$$V_{B^+} = 2.5 + 6.5 = 9 \text{ kN}$$
$$V_{C^-} = 9 - (3)(6) = -9 \text{ kN}$$

C to D: $V = -9 + 6.5 = -2.5 \text{ kN}$



Areas of the shear diagram:

A to B: $\int V dx = (0.6)(2.5) = 1.5 \text{ kN} \cdot \text{m}$

B to E: $\int V dx = \left(\frac{1}{2}\right)(1.5)(9) = 6.75 \text{ kN} \cdot \text{m}$

E to C: $\int V dx = -6.75 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = -1.5 \text{ kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 + 1.5 = 1.5 \text{ kN} \cdot \text{m}$$
$$M_E = 1.5 + 6.75 = 8.25 \text{ kN} \cdot \text{m}$$
$$M_C = 8.25 - 6.75 = 1.5 \text{ kN} \cdot \text{m}$$
$$M_D = 1.5 - 1.5 = 0$$

Maximum $|M| = 8.25 \text{ kN} \cdot \text{m} = 8.25 \times 10^3 \text{ N} \cdot \text{m}$

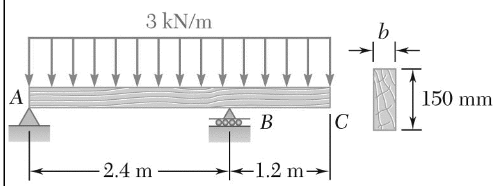
$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$$

For a rectangular section, $S = \frac{1}{6} b h^2$

$$687.5 \times 10^3 = \left(\frac{1}{6}\right)(100) h^2$$

$$h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2 \qquad h = 203 \text{ mm} \blacktriangleleft$$



PROBLEM 5.70

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

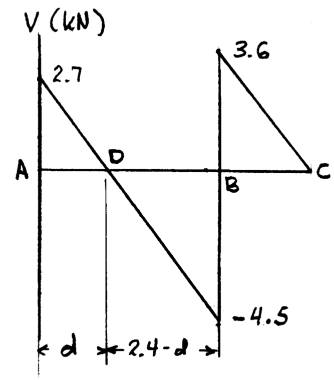
SOLUTION

$$+\circlearrowleft M_B = 0: \quad -2.4A + (0.6)(3.6)(3) = 0 \qquad A = 2.7 \text{ kN}$$

$$+\circlearrowleft M_A = 0: \quad -(1.8)(3.6)(3) + 2.4B = 0 \qquad B = 8.1 \text{ kN}$$

Shear:

$$V_A = 2.7 \text{ kN}$$
$$V_{B^-} = 2.7 - (2.4)(3) = -4.5 \text{ kN}$$
$$V_{B^+} = -4.5 + 8.1 = 3.6 \text{ kN}$$
$$V_C = 3.6 - (1.2)(3) = 0$$



Locate point D where $V = 0$. $\quad \dfrac{d}{2.7} = \dfrac{2.4-d}{4.5} \qquad 7.2d = 6.48$

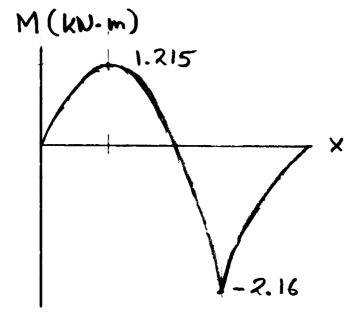
$$d = 0.9 \text{ m} \qquad 2.4 - d = 1.5 \text{ m}$$

Areas of the shear diagram:

A to D: $\quad \int V dx = \left(\dfrac{1}{2}\right)(0.9)(2.7) = 1.215 \text{ kN}\cdot\text{m}$

D to B: $\quad \int V dx = \left(\dfrac{1}{2}\right)(1.5)(-4.5) = -3.375 \text{ kN}\cdot\text{m}$

B to C: $\quad \int V dx = \left(\dfrac{1}{2}\right)(1.2)(3.6) = 2.16 \text{ kN}\cdot\text{m}$



Bending moments:

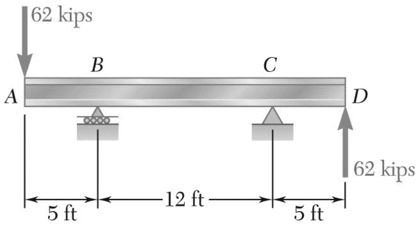
$$M_A = 0$$
$$M_D = 0 + 1.215 = 1.215 \text{ kN}\cdot\text{m}$$
$$M_B = 1.215 - 3.375 = -2.16 \text{ kN}\cdot\text{m}$$
$$M_C = -2.16 + 2.16 = 0$$

Maximum $|M| = 2.16 \text{ kN}\cdot\text{m} = 2.16\times10^3 \text{ N}\cdot\text{m}$ $\qquad \sigma_{\text{all}} = 12 \text{ MPa} = 12\times10^6 \text{ Pa}$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{2.16\times10^3}{12\times10^6} = 180\times10^{-6} \text{ m}^3 = 180\times10^3 \text{ mm}^3$$

For rectangular section, $\quad S = \dfrac{1}{6}bh^2 = \dfrac{1}{6}b(150)^2 = 180\times10^3$

$$b = \frac{(6)(180\times10^3)}{150^2} \qquad b = 48.0 \text{ mm} \blacktriangleleft$$



PROBLEM 5.71

Knowing that the allowable stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0: \ (17)(62) - 12B + (5)(62) = 0$ $\qquad$ $\mathbf{B} = 113.667 \text{ kips} \uparrow$

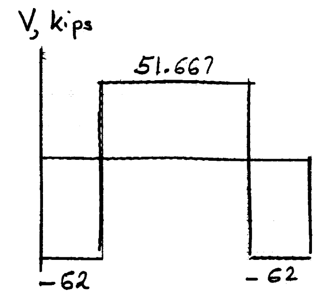
$+\circlearrowleft \Sigma M_B = 0: \ (5)(62) + 12C + (17)(62) = 0$ $\qquad$ $C = -113.667 \text{ kips}$ or $\mathbf{C} = 113.667 \text{ kips} \downarrow$

Shear diagram:

A to B^-: $\quad V = -62 \text{ kips}$

B^+ to C^-: $\quad V = -62 + 113.667 = 51.667 \text{ kips}$

C^+ to D: $\quad V = 51.667 - 113.667 = -62 \text{ kips.}$



Areas of shear diagram:

A to B: $\quad (5)(-62) = -310 \text{ kip} \cdot \text{ft}$

B to C: $\quad (12)(51.667) = 620 \text{ kip} \cdot \text{ft}$

C to D: $\quad (5)(-62) = -310 \text{ kip} \cdot \text{ft}$

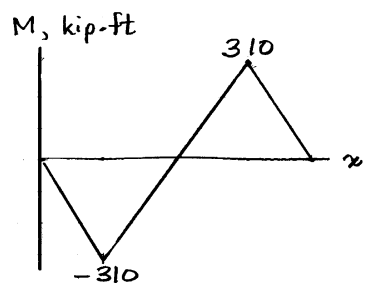
Bending moments: $\quad M_A = 0$

$M_B = 0 - 310 = -310 \text{ kip} \cdot \text{ft}$

$M_C = -310 + 620 = 310 \text{ kip} \cdot \text{ft}$

$M_D = 310 - 310 = 0$

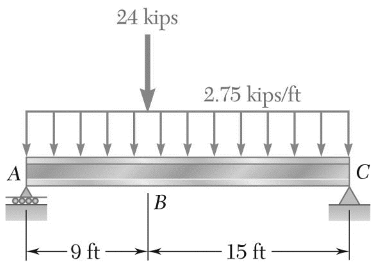
$|M|_{\max} = 310 \text{ kip} \cdot \text{ft} = 3.72 \times 10^3 \text{ kip} \cdot \text{in}$



Required $S_{\min}$: $\quad S_{\min} = \dfrac{|M|_{\max}}{\sigma_{\text{all}}} = \dfrac{3.72 \times 10^3}{24} = 155 \text{ in}^3$

Shape	$S(\text{in}^3)$
$W27 \times 84$	213
$W21 \times 101$	227
$W18 \times 106$	204
$W14 \times 232$	232

Use $W27 \times 84$ ◀



PROBLEM 5.72

Knowing that the allowable stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

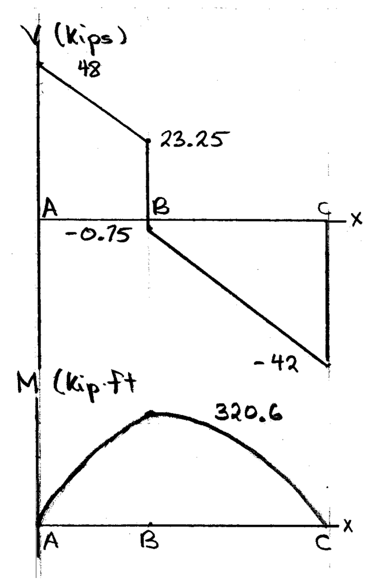
SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -24A + (12)(24)(2.75) + (15)(24) = 0 \qquad A = 48 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 24C - (12)(24)(2.75) - (9)(24) = 0 \qquad C = 42 \text{ kips}$$

Shear:

$$V_A = 48$$
$$V_{B^-} = 48 - (9)(2.75) = 23.25 \text{ kips}$$
$$V_{B^+} = 23.25 - 24 = -0.75 \text{ kips}$$
$$V_C = -0.75 - (15)(2.75) = -42 \text{ kips}$$



Areas of the shear diagram:

A to B: $$\int V dx = \left(\frac{1}{2}\right)(9)(48 + 23.25) = 320.6 \text{ kip} \cdot \text{ft}$$

B to C: $$\int V dx = \left(\frac{1}{2}\right)(15)(-0.75 - 42) = -320.6 \text{ kip} \cdot \text{ft}$$

Bending moments:

$$M_A = 0$$
$$M_B = 0 + 320.6 = 320.6 \text{ kip} \cdot \text{ft}$$
$$M_C = 320.6 - 320.6 = 0$$

Maximum $|M| = 320.6 \text{ kip} \cdot \text{ft} = 3848 \text{ kip} \cdot \text{in}$

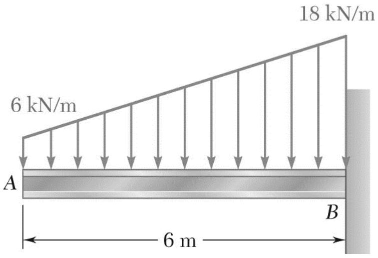
$$\sigma_{\text{all}} = 24 \text{ ksi}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3848}{24} = 160.3 \text{ in}^3$$

Shape	S, (in^3)
W30×99	269
W27×84	213 ←
W24×104	258
W21×101	227
W18×106	204

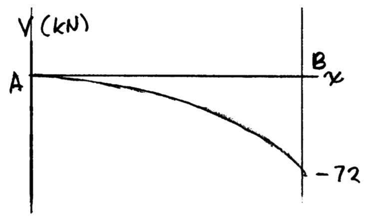
Lightest wide flange beam: W27×84 @ 84 lb/ft ◀

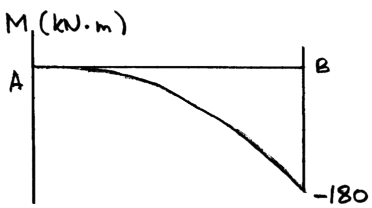
PROBLEM 5.73



Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION





$$w = \left(6 + \frac{18-6}{6}x\right) = (6 + 2x)\ \text{kN/m}$$

$$\frac{dV}{dx} = -w = -6 - 2x$$

$$V = -6x - x^2 + C_1$$

$$V = 0 \quad \text{at} \quad x = 0, \quad C_1 = 0$$

$$\frac{dM}{dx} = V = -6x - x^2$$

$$M = -3x^2 - \frac{1}{3}x^3 + C_2$$

$$M = 0 \quad \text{at} \quad x = 0, \quad C_2 = 0$$

$$M = -3x^2 - \frac{1}{3}x^3$$

$|M|_{\max}$ occurs at $x = 6$ m.

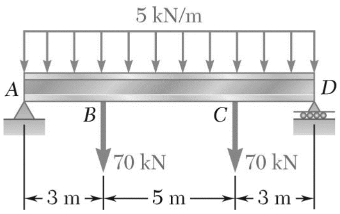
$$|M|_{\max} = \left|-(3)(6)^2 - \left(\frac{1}{3}\right)(6)^3\right| = 80\ \text{kN}\cdot\text{m} = 180\times10^3\ \text{N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 160\ \text{MPa} = 160\times10^6\ \text{Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{180\times10^3}{160\times10^6} = 1.125\times10^{-3}\ \text{m}^3 = 1125\times10^3\ \text{mm}^3$$

Shape	S, $(10^3\,\text{mm}^3)$
$W530\times66$	1340 ←
$W460\times74$	1460
$W410\times85$	1510
$W360\times79$	1270
$W310\times107$	1600
$W250\times101$	1240

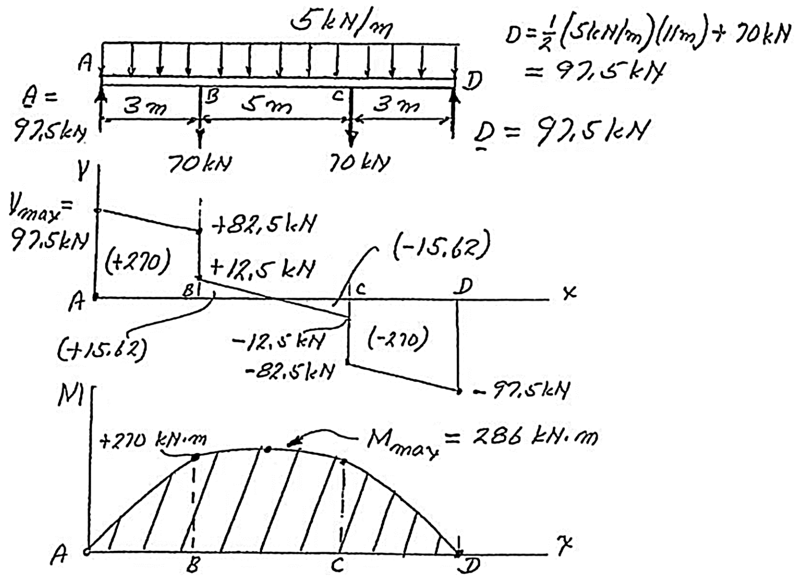
Lightest acceptable wide flange beam: $W530\times66$ ◀



PROBLEM 5.74

Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION



Section modulus

$\sigma_{\text{all}} = 160$ Mpa

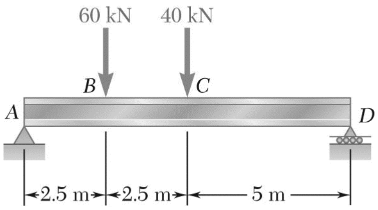
$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{286\ \text{kN}\cdot\text{m}}{160\ \text{MPa}} = 1787 \times 10^{-6}\ \text{m}^3$$

$$= 1787 \times 10^3\ \text{mm}^3$$

Shape	S, $(10^3\,\text{mm}^3)$
W610 × 101	2520
W530 × 92	2080 ← ◀
W460 × 113	2390
W410 × 114	2200
W360 × 122	2020
W310 × 143	2150

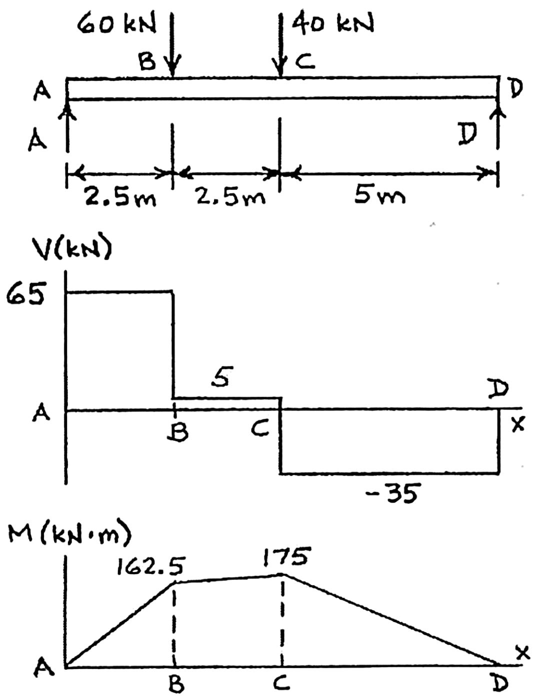
Use W530 × 92 ◀

PROBLEM 5.75



Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

SOLUTION



Reaction: $+\circlearrowleft \sum M_D = 0 : -10A + (7.5)(60) + (5)(40) = 0$

$\mathbf{A} = 65\,\text{kN} \uparrow$

Shear diagram:

A to B: $V = 65$ kN

B to C: $V = 65 - 60 = 5$ kN

C to D: $V = 5 - 40 = -35$ kN

Areas of shear diagram:

A to B: $(2.5)(65) = 162.5\ \text{kN}\cdot\text{m}$

B to C: $(2.5)(5) = 12.5\ \text{kN}\cdot\text{m}$

C to D: $(5)(-35) = -175\ \text{kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$M_B = 0 + 162.5 = 162.5\ \text{kN}\cdot\text{m}$

$M_C = 162.5 + 12.5 = 175\ \text{kN}\cdot\text{m}$

$M_D = 175 - 175 = 0$

$|M|_{\text{max}} = 175\ \text{kN}\cdot\text{m} = 175\times10^3\ \text{N}\cdot\text{m}$

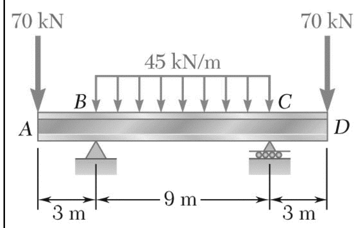
$\sigma_{\text{all}} = 160\ \text{MPa} = 160\times10^6\ \text{Pa}$

Shape	S_x, (10^3mm^3)
$S610\times119$	2870
$S510\times98.2$	1950
$S460\times81.4$	1460 ←

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{175\times10^3}{160\times10^6} = 1093.75\times10^{-6}\ \text{m}^3$$

$$= 1093.75\times10^3\ \text{mm}^3$$

Lightest S-section: $S460\times81.4$ ◀



PROBLEM 5.76

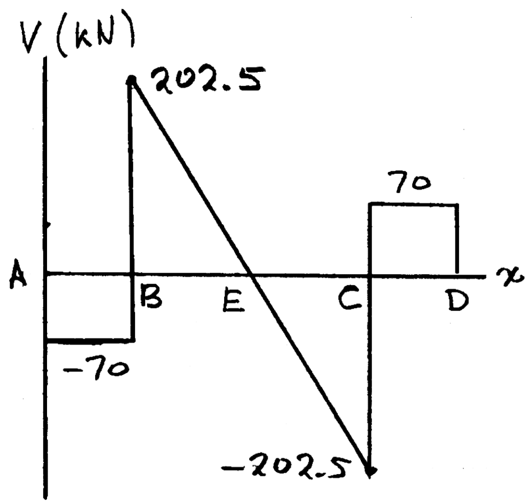
Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

SOLUTION

Reactions: By symmetry, $B = C$.

$$+\uparrow \sum F_y = 0: -70 + B - (9)(45) + C - 70 = 0$$

$$\boldsymbol{B} = \boldsymbol{C} = 272.5 \text{ kN} \uparrow$$



Shear: $V_A = -70$ kN

$V_{B^-} = -70 + 0 = -70$ kN

$V_{B^+} = -70 + 272.5 = 202.5$ kN

$V_{C^-} = 202.5 - (9)(45) = -202.5$ kN

$V_{C^+} = -202.5 + 272.5 = 70$ kN

$V_D = 70$ kN

Draw shear diagram. Locate point E where $V = 0$.

E is the midpoint of BC.

Areas of the shear diagram:

A to B: $\int V dx = (3)(-70) = -210 \text{ kN} \cdot \text{m}$

B to E: $\int V dx = \frac{1}{2}(4.5)(202.5) = 455.625 \text{ kN} \cdot \text{m}$

E to C: $\int V dx = \frac{1}{2}(4.5)(-202.5) = -455.625 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = (3)(70) = 210 \text{ kN} \cdot \text{m}$

PROBLEM 5.76 *(Continued)*

Bending moments: $M_A = 0$

$M_B = 0 - 210 = -210.5 \text{ kN} \cdot \text{m}$

$M_E = -210 + 455.625 = 245.625 \text{ kN}$

$M_C = 245.625 - 455.625 = -210 \text{ kN}$

$M_D = -210 + 210 = 0$

$|M|_{\max} = 245.625 \text{kN} \cdot \text{m} = 245.625 \times 10^3 \text{ N} \cdot \text{m}$

$\sigma_{\text{all}} = 160 \text{MPa} = 160 \times 10^6 \text{ Pa}$

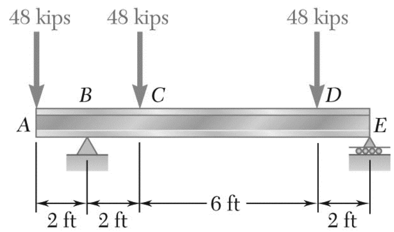
$$\sigma = \frac{|M|}{S}$$

$$S = \frac{|M|}{\sigma} = \frac{245.625 \times 10^3}{160 \times 10^6} = 1.5352 \times 10^{-3} \text{ m}^3$$

$$= 1535.2 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{mm}^3)$
$\text{S}610 \times 119$	2870
$\text{S}510 \times 98.2$	1950 $\leftarrow$
$\text{S}460 \times 104$	1690

Lightest S-shape $\text{S}510 \times 98.2$ ◀



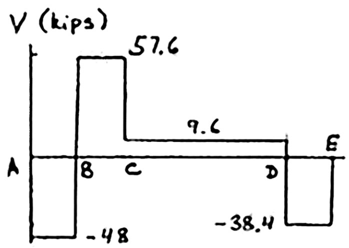
PROBLEM 5.77

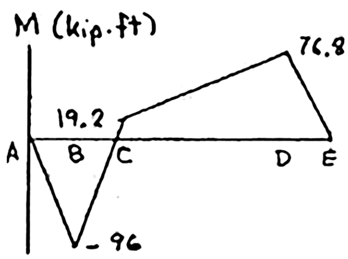
Knowing that the allowable stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0 : (12)(48) - 10B + (8)(48) + (2)(48) = 0 \qquad \mathbf{B} = 105.6 \text{ kips}\uparrow$$

$$+\circlearrowleft \Sigma M_B = 0 : (2)(48) - (2)(48) - (8)(48) + 10E = 0 \qquad \mathbf{E} = 38.4 \text{ kips}\uparrow$$





Shear: A to B: $V = -48$ kips

B to C: $V = -48 + 105.6 = 57.6$ kips

C to D: $V = 57.6 - 48 = 9.6$ kips

D to E: $V = 9.6 - 48 = -38.4$ kips

Areas: A to B: $(2)(-48) = -96 \text{ kip}\cdot\text{ft}$

B to C: $(2)(57.6) = 115.2 \text{ kip}\cdot\text{ft}$

C to D: $(6)(9.6) = 57.6 \text{ kip}\cdot\text{ft}$

D to E: $(2)(-38.4) = 76.8 \text{ kip}\cdot\text{ft}$

Bending moments: $M_A = 0$

$M_B = 0 - 96 = -96 \text{ kip}\cdot\text{ft}$

$M_C = -96 + 115.2 = 19.2 \text{ kip}\cdot\text{ft}$

$M_D = 19.2 + 57.2 = 76.8 \text{ kip}\cdot\text{ft}$

$M_E = 76.8 - 76.8 = 0$

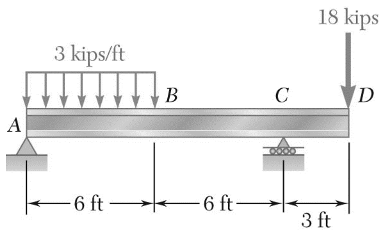
Maximum $|M| = 96 \text{ kip}\cdot\text{ft} = 1152 \text{ kip}\cdot\text{in}$

$\sigma_{\text{all}} = 24$ ksi

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1152}{24} = 48 \text{ in}^3$$

Shape	$S\,(\text{in}^3)$
$S15 \times 42.9$	59.4
$S12 \times 50$	50.6

Lightest S-shaped beam: $S15 \times 42.9$ ◀



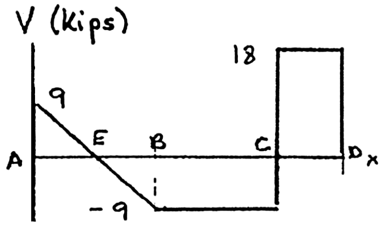
PROBLEM 5.78

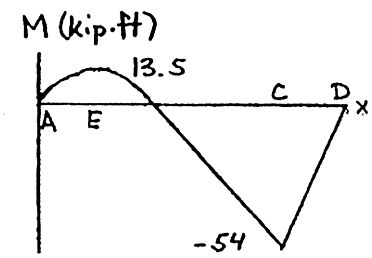
Knowing that the allowable stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \sum M_C = 0: -12A + (9)(6)(3) - (3)(18) = 0 \quad A = 9 \text{ kips}$$

$$+\circlearrowleft \sum M_A = 0: 12C - (3)(6)(3) - (15)(18) = 0 \quad C = 27 \text{ kips}$$





Shear: $V_A = 9$ kips

B to C: $V = 9 - (6)(3) = -9$ kips

C to D: $V = -9 + 27 = 18$ kips

Areas:

A to E: $(0.5)(3)(9) = 13.5 \text{ kip} \cdot \text{ft}$

E to B: $(0.5)(3)(-9) = -13.5 \text{ kip} \cdot \text{ft}$

B to C: $(6)(-9) = -54 \text{ kip} \cdot \text{ft}$

C to D: $(3)(18) = 54 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

$M_E = 0 + 13.5 = 13.5 \text{ kip} \cdot \text{ft}$

$M_B = 13.5 - 13.5 = 0$

$M_C = 0 + 54 = 54 \text{ kip} \cdot \text{ft}$

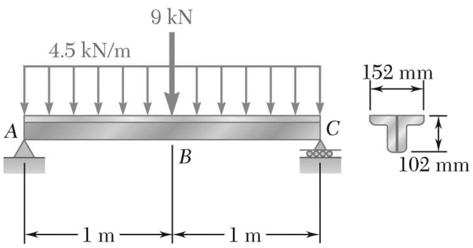
$M_D = 54 - 54 = 0$

Maximum $|M| = 54 \text{ kip} \cdot \text{ft} = 648 \text{ kip} \cdot \text{in} \quad \sigma_{\text{all}} = 24 \text{ ksi}$

$$S_{\min} = \frac{648}{24} = 27 \text{ in}^3$$

Shape	$S(\text{in}^3)$
$\text{S}12 \times 31.8$	36.2
$\text{S}10 \times 35$	29.4

Lightest S-shaped beam: $\text{S}12 \times 31.8$ ◀



PROBLEM 5.79

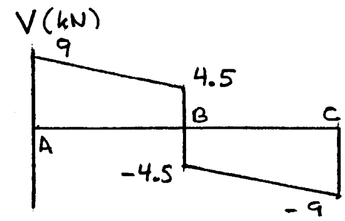
Two $L102 \times 76$ rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine the maximum angle of thickness that can be used.

SOLUTION

Reactions: By symmetry, $A = C$

$$+\uparrow \Sigma F_y = 0: \quad A - (2)(4.5) - 9 + C = 0$$

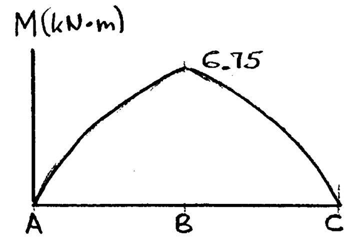
$$\boldsymbol{A} = \boldsymbol{C} = 9 \text{ kN}\uparrow$$



Shear:

$$V_A = 9 \text{ kN}$$
$$V_{B^-} = 9 - (1)(4.5) = 4.5 \text{ kN}$$
$$V_{B^+} = 4.5 - 9 = -4.5 \text{ kN}$$
$$V_C = -4.5 - (1)(4.5) = -9 \text{ kN}$$

Areas of shear diagram:



A to B: $$\int V dx = \frac{1}{2}(1)(9 + 4.5) = 6.75 \text{ kN}\cdot\text{m}$$

B to C: $$\int V dx = \frac{1}{2}(1)(-9 - 4.5) = -6.75 \text{ kN}\cdot\text{m}$$

Bending moments:

$$M_A = 0$$
$$M_B = 0 + 6.75 = 6.75 \text{ kN}\cdot\text{m}$$
$$M_C = 6.75 - 6.75 = 0$$

Maximum $|M| = 6.75 \text{ kN}\cdot\text{m} = 6.75 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma_{\text{all}} = 140 \text{ MPa} = 140 \times 10^6 \text{ Pa}$$

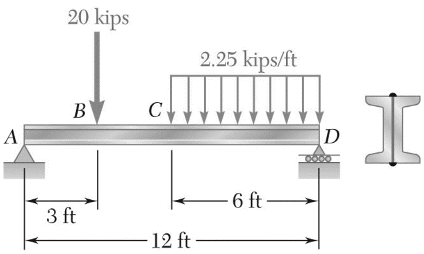
For the section of two angles, $$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{6.75 \times 10^3}{140 \times 10^6} = 48.21 \times 10^{-6} \text{ m}^3$$
$$= 48.21 \times 10^3 \text{ mm}^3$$

For each angle, $$S_{\min} = \frac{1}{2}(48.21) = 24.105 \times 10^3 \text{ mm}^3$$

Shape	$S\,(10^3 \text{ mm}^3)$
$L102 \times 76 \times 12.7$	31.1 ←
$L102 \times 76 \times 9.5$	24.0
$L102 \times 76 \times 6.4$	16.6

Lightest angle is $L102 \times 76 \times 12.7$ ◀

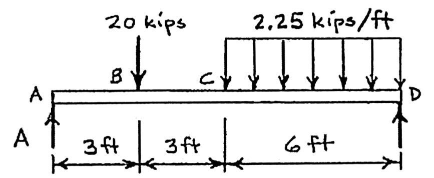
$t_{\min} = 12.7$ mm ◀

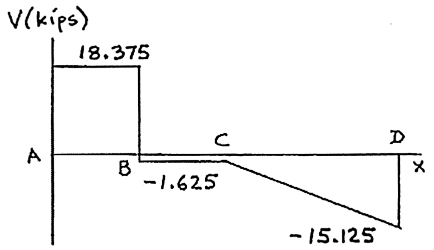


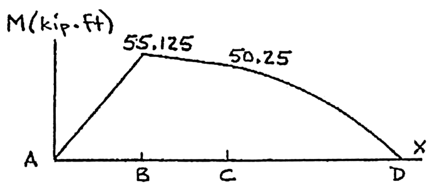
PROBLEM 5.80

Two rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 30 ksi, determine the most economical channels that can be used.

SOLUTION







Shape	$S(\text{in})^3$
$C10 \times 15.3$	13.5
$C9 \times 15$	11.3 ←
$C8 \times 18.7$	11.0

Reaction: $+\circlearrowleft \Sigma M_D = 0: \quad -12A + 9(20) + (6)(2.25)(3) = 0$

$$\boldsymbol{A} = 18.375 \text{ kips} \uparrow$$

Shear diagram:

A to B: $V = 18.375$ kips

B to C: $V = 18.375 - 20 = -1.625$ kips

$$V_D = -1.625 - (6)(2.25) = -15.125 \text{ kips}$$

Areas of shear diagram:

A to B: $(3)(18.375) = 55.125 \text{ kip} \cdot \text{ft}$

B to C: $(3)(-1.625) = -4.875 \text{ kip} \cdot \text{ft}$

C to D: $0.5(6)(-1.625 - 15.125) = -50.25 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

$M_B = 0 + 55.125 = 55.125 \text{ kip} \cdot \text{ft}$

$M_C = 55.125 - 4.875 = 50.25 \text{ kip} \cdot \text{ft}$

$M_D = 50.25 - 50.25 = 0$

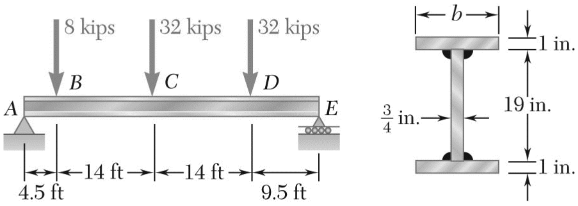
$|M|_{max} = 55.125 \text{ kip} \cdot \text{ft} = 661.5 \text{ kip} \cdot \text{in}$

$\sigma_{all} = 30$ ksi

For double channel, $S_{min} = \dfrac{|M|}{\sigma_{all}} = \dfrac{661.5}{30} = 22.05 \text{ in}^3$

For single channel, $S_{min} = 0.5(22.05) = 11.025 \text{ in}^3$

Lightest channel section: $C9 \times 15$ ◀



PROBLEM 5.81

Three steel plates are welded together to form the beam shown. Knowing that the allowable normal stress for the steel used is 22 ksi, determine the minimum flange width b that can be used.

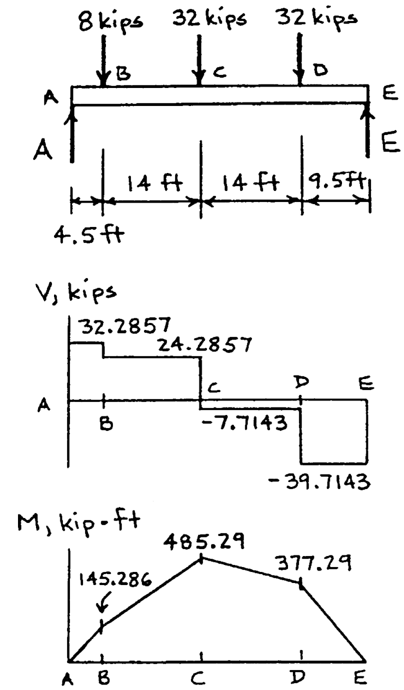
SOLUTION

Reactions: $+\circlearrowleft \Sigma M_E = 0: \quad -42A + (37.5)(8) + (23.5)(32) - (9.5)(32) = 0$

$\boldsymbol{A} = 32.2857 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_A = 0: \quad 42E - (4.5)(8) - (18.5)(32) - (32.5)(32) = 0$

$\boldsymbol{E} = 39.7143 \text{ kips} \uparrow$



Shear:

A to B: 32.2857 kips

B to C: $32.2857 - 8 = 24.2857$ kips

C to D: $24.2857 - 32 = -7.7143$ kips

D to E: $-7.7143 - 32 = -39.7143$ kips

Areas:

A to B: $(4.5)(32.2857) = 145.286 \text{ kip} \cdot \text{ft}$

B to C: $(14)(24.2857) = 340 \text{ kip} \cdot \text{ft}$

C to D: $(14)(-7.7143) = -108 \text{ kip} \cdot \text{ft}$

D to E: $(9.5)(-39.7143) = -377.286 \text{ kip} \cdot \text{ft}$

Bending moments:

$M_A = 0$

$M_B = 0 + 145.286 = 145.286 \text{ kip} \cdot \text{ft}$

$M_C = 145.286 + 340 = 485.29 \text{ kip} \cdot \text{ft}$

$M_D = 485.29 - 108 = 377.29 \text{ kip} \cdot \text{ft}$

$M_E = 377.29 - 377.286 = 0$

Maximum $|M| = 485.29 \text{ kip} \cdot \text{ft} = 5.2834 \times 10^3 \text{ kip} \cdot \text{in} \qquad \sigma_{\text{all}} = 22 \text{ ksi}$

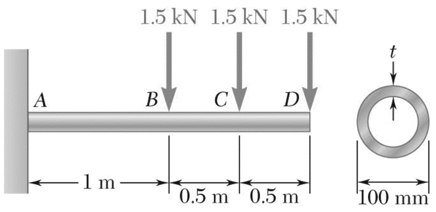
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{5.2834 \times 10^3}{22} = 264.70 \text{ in}^3$$

$$I = \frac{1}{12}\left(\frac{3}{4}\right)(19)^3 + 2\left[\frac{1}{12}(b)(1)^3 + (b)(1)(10)^2\right] = 428.69 + 200.17b$$

$$c = 9.5 + 1 = 10.5 \text{ in.}$$

$$S_{\min} = \frac{I}{c} = 40.828 + 19.063b = 264.70$$

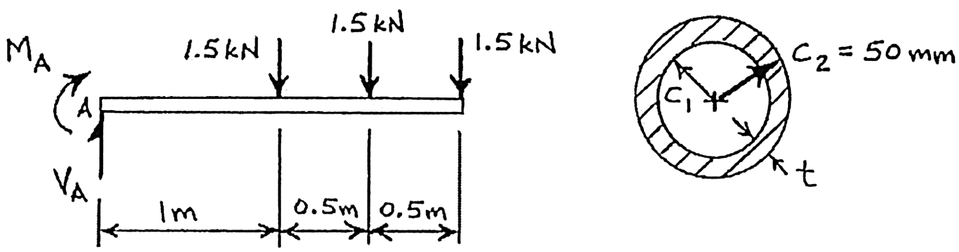
$b = 11.74$ in. ◀



PROBLEM 5.82

A steel pipe of 100-mm diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from 6 mm to 24 mm in 3-mm increments, and that the allowable normal stress for the steel used is 150 MPa, determine the minimum wall thickness *t* that can be used.

SOLUTION



$$+\circlearrowleft\ \Sigma M_A = 0: \quad -M_A - (1)(1.5) - (1.5)(1.5) - (2)(1.5) = 0 \qquad M_A = -6.75\ \text{kN}\cdot\text{m}$$

$$|M|_{\max} = |M_A| = 6.75\ \text{kN}\cdot\text{m}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{6.75\times 10^3\ \text{N}\cdot\text{m}}{150\times 10^6\ \text{Pa}} = 45\times 10^{-6}\ \text{m}^3 = 45\times 10^3\ \text{mm}^3$$

$$S_{\min} = \frac{I_{\min}}{c_2} \qquad I_{\min} = c_2 S_{\min} = (50)(45\times 10^3) = 2.25\times 10^6\ \text{mm}^4$$

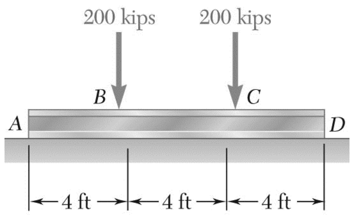
$$I_{\min} = \frac{\pi}{4}\left(c_2^4 - c_{1\max}^4\right)$$

$$c_{1\max}^4 = c_2^4 - \frac{4}{\pi} I_{\min} = (50)^4 - \frac{4}{\pi}(2.25\times 10^6) = 3.3852\times 10^6\ \text{mm}^4$$

$$c_{1\max} = 42.894\ \text{mm}$$

$$t_{\min} = c_2 - c_{1\max} = 50 - 42.894 = 7.106\ \text{mm}$$

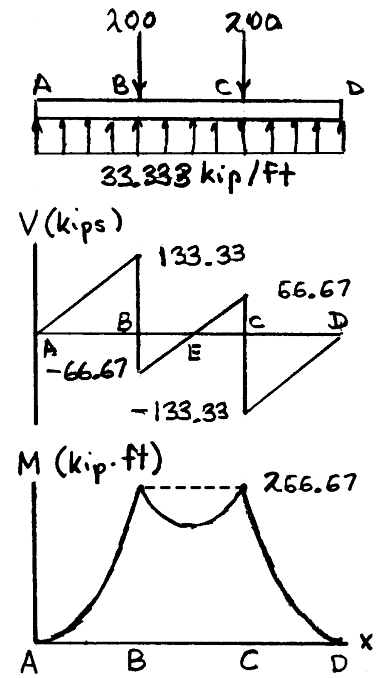
$$t = 9\ \text{mm} \blacktriangleleft$$



PROBLEM 5.83

Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION



Distributed reaction: $$q = \frac{400}{12} = 33.333 \text{ kip/ft}$$

Shear:

$$V_A = 0$$
$$V_{B^-} = 0 + (4)(33.333) = 133.33 \text{ kips}$$
$$V_{B^+} = 133.33 - 200 = -66.67 \text{ kips}$$
$$V_{C^-} = -66.67 + 4(33.333) = 66.67 \text{ kips}$$
$$V_{C^+} = 66.67 - 200 = -133.33 \text{ kips}$$
$$V_D = -133.33 + (4)(33.333) = 0 \text{ kips}$$

Areas:

A to B: $$\left(\frac{1}{2}\right)(4)(133.33) = 266.67 \text{ kip} \cdot \text{ft}$$

B to E: $$\left(\frac{1}{2}\right)(2)(-66.67) = -66.67 \text{ kip} \cdot \text{ft}$$

E to C: $$\left(\frac{1}{2}\right)(2)(66.67) = 66.67 \text{ kip} \cdot \text{ft}$$

C to D: $$\left(\frac{1}{2}\right)(4)(-133.33) = -266.67 \text{ kip} \cdot \text{ft}$$

Bending moments:

$$M_A = 0$$
$$M_B = 0 + 266.67 = 266.67 \text{ kip} \cdot \text{ft}$$
$$M_E = 266.67 - 66.67 = 200 \text{ kip} \cdot \text{ft}$$
$$M_C = 200 + 66.67 = 266.67 \text{ kip} \cdot \text{ft}$$
$$M_D = 266.67 - 266.67 = 0$$

Maximum $|M| = 266.67 \text{ kip} \cdot \text{ft} = 3200 \text{ kip} \cdot \text{in.}$

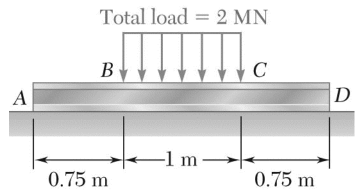
$$\sigma_{\text{all}} = 24 \text{ ksi}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3200}{24} = 133.3 \text{ in}^3$$

PROBLEM 5.83 *(Continued)*

Shape	$S(\text{in}^3)$	
$W 27 \times 84$	213	
$W 24 \times 68$	154	$\leftarrow$
$W 21 \times 101$	227	
$W 18 \times 76$	146	
$W 16 \times 77$	134	
$W 14 \times 145$	232	

Lightest W-shaped section: $W 24 \times 68$ ◀



PROBLEM 5.84

Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

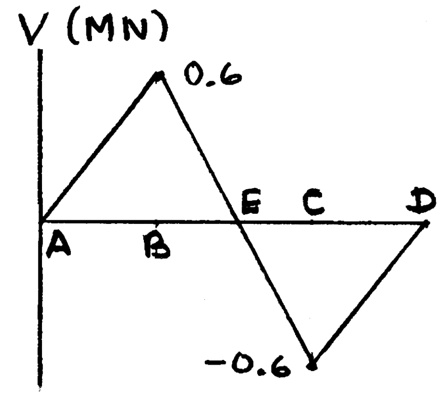
Downward distributed load: $w = \dfrac{2}{1.0} = 2 \text{ MN/m}$

Upward distributed reaction: $q = \dfrac{2}{2.5} = 0.8 \text{ MN/m}$

Net distributed load over BC: 1.2 MN/m

Shear:

$$V_A = 0$$
$$V_B = 0 + (0.75)(0.8) = 0.6 \text{ MN}$$
$$V_C = 0.6 - (1.0)(1.2) = -0.6 \text{ MN}$$
$$V_D = -0.6 + (0.75)(0.8) = 0$$



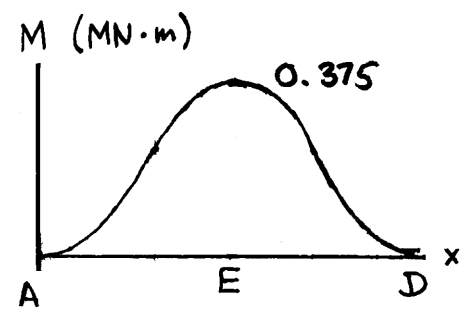
Areas:

A to B: $\left(\dfrac{1}{2}\right)(0.75)(0.6) = 0.225 \text{ MN} \cdot \text{m}$

B to E: $\left(\dfrac{1}{2}\right)(0.5)(0.6) = 0.150 \text{ MN} \cdot \text{m}$

E to C: $\left(\dfrac{1}{2}\right)(0.5)(-0.6) = -0.150 \text{ MN} \cdot \text{m}$

C to D: $\left(\dfrac{1}{2}\right)(0.75)(-0.6) = -0.225 \text{ MN} \cdot \text{m}$



Bending moments:

$$M_A = 0$$
$$M_B = 0 + 0.225 = 0.225 \text{ MN} \cdot \text{m}$$
$$M_E = 0.225 + 0.150 = 0.375 \text{ MN} \cdot \text{m}$$
$$M_C = 0.375 - 0.150 = 0.225 \text{ MN} \cdot \text{m}$$
$$M_D = 0.225 - 0.225 = 0$$

Maximum $|M| = 0.375 \text{ MN} \cdot \text{m} = 375 \times 10^3 \text{ N} \cdot \text{m}$

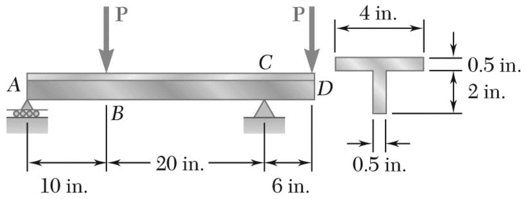
$$\sigma_{\text{all}} = 170 \text{ MPa} = 170 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3} \text{ m}^3 = 2206 \times 10^3 \text{ mm}^3$$

PROBLEM 5.84 *(Continued)*

Shape	$S\,(10^3\ \text{mm}^3)$	
$W\,690 \times 125$	3510	
$W\,610 \times 101$	2530	$\leftarrow$
$W\,530 \times 150$	3720	
$W\,460 \times 113$	2400	

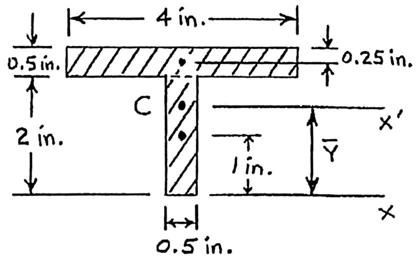
Lightest wide flange section: $W\,610 \times 101$ ◀



PROBLEM 5.85

Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +6 ksi in tension and −18 ksi in compression.

SOLUTION



$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{(1)(0.5)(2) + (2.25)(4)(0.5)}{(0.5 \times 2) + (4 \times 0.5)}$$

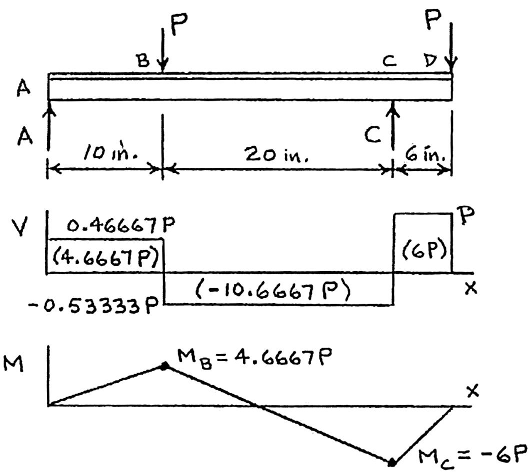
$$= 1.83333 \text{ in.}$$

$$I_{x'} = \Sigma\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \left[\frac{1}{12}(0.5)(2)^3 + (0.5)(2)(1.83333 - 1)^2\right]$$

$$+\left[\frac{1}{12}(4)(0.5)^3 + (4)(0.5)(2.25 - 1.83333)^2\right]$$

$$= 1.41667 \text{ in}^4$$



$$+\circlearrowleft \Sigma M_C = 0: \quad -30A + (20)P - (6)P = 0$$

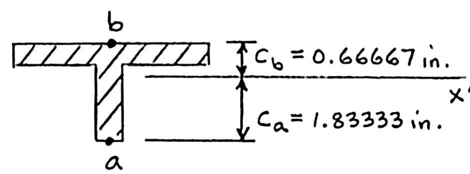
$$\boldsymbol{A} = 0.46667P \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 0.46667P - 2P + \boldsymbol{C} = 0$$

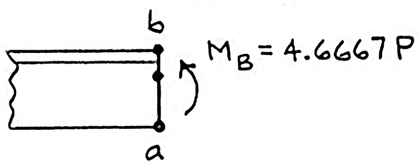
$$\boldsymbol{C} = 1.53333P \uparrow$$

$$\sigma_{\text{all}} = +6 \text{ ksi}$$

$$\sigma_{\text{all}} = -18 \text{ ksi}$$



At section B: For $\sigma_b = 18$ ksi (compression):



$$\sigma_b = M_B \frac{c_b}{I_{x'}}$$

$$18 = (4.6667P)\frac{0.66667}{1.41667} \qquad P = 8.20 \text{ kips}$$

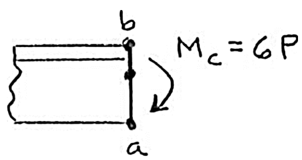
PROBLEM 5.85 *(Continued)*

For $\sigma_a = 6\text{ ksi}$ (tension),

$$\sigma_a = M_B \frac{c_a}{I_{x'}}$$

$$6 = (4.6667P)\frac{1.83333}{1.41667} \qquad P = 0.994\text{ kips}$$

At section C:



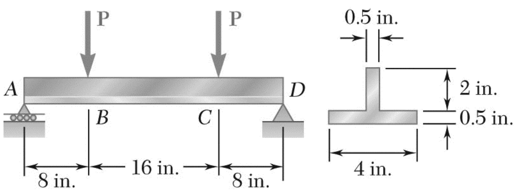
For $\sigma_b = 6\text{ ksi}$ (tension),

$$\sigma_b = M_C \frac{c_b}{I_{x'}} \qquad 6 = (6P)\frac{0.66667}{1.41667} \qquad P = 2.13\text{ kips}$$

For $\sigma_a = 18\text{ ksi}$ (Compression),

$$\sigma_a = M_C \frac{c_a}{I_x'} \qquad 18 = (6P)\frac{1.83333}{1.41667} \qquad P = 2.32\text{ kips}$$

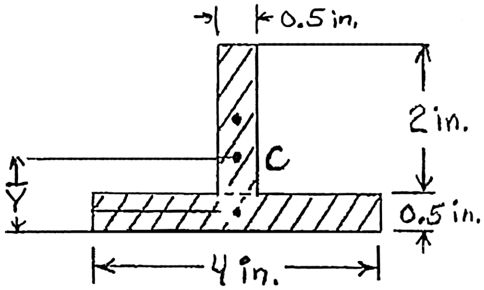
Choose smallest value of P: $P = 0.994\text{ kips}$ ◀



PROBLEM 5.86

Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +6 ksi in tension and −18 ksi in compression.

SOLUTION



$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{(1.5)(0.5)(2) + (0.25)(4)(0.5)}{(0.5)(2) + (4)(0.5)}$$

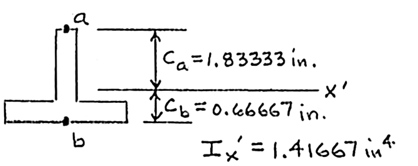
$$= 0.66667 \text{ in.}$$

$$I = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(0.5)(2)^3 + (0.5)(2)(1.5 - 0.66667)^2$$

$$+ \frac{1}{12}(4)(0.5)^3 + (4)(0.5)(0.66667 - 0.25)^2$$

$$= 1.41667 \text{ in}^4$$

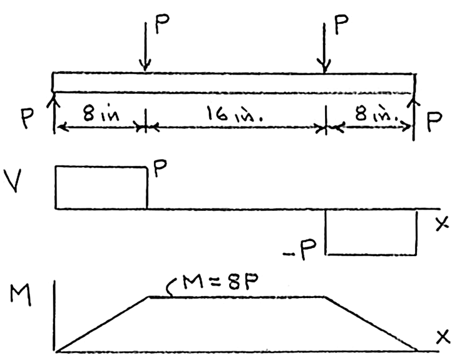


$$\sigma_{\text{all}} = +6 \text{ ksi}, \quad -18 \text{ ksi}$$

For $\sigma_a = 18 \text{ ksi}$ (compression),

$$\sigma_a = M_{\max}\frac{c_a}{I_{x'}}$$

$$18 = (8P)\frac{1.83333}{1.41667} \qquad P = 1.739 \text{ kips}$$

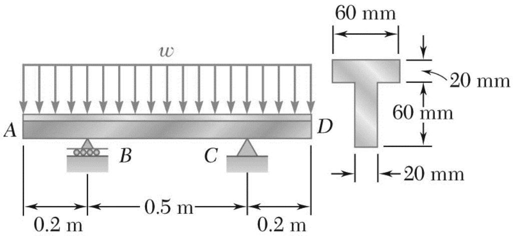


For $\sigma_b = 6 \text{ ksi}$ (tension),

$$\sigma_b = M_{\max}\frac{c_b}{I_{x'}}$$

$$6 = (8P)\frac{0.66667}{1.41667} \qquad P = 1.594 \text{ kips}$$

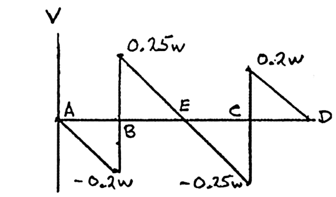
Choose smallest value of P: $P = 1.594$ kips ◀

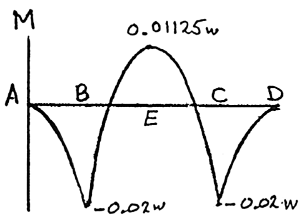


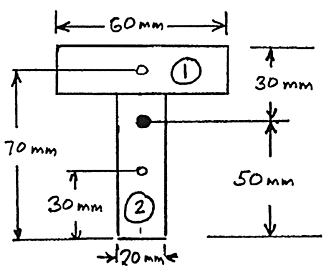
PROBLEM 5.87

Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and −130 MPa in compression.

SOLUTION







Reactions. By symmetry, $B = C$

$+\uparrow \sum F_y = 0 : B + C - 0.9w = 0$

$$B = C = 0.45w \uparrow$$

Shear: $V_A = 0$

$$V_{B^-} = 0 - 0.2w = -0.2w$$
$$V_{B^+} = -0.2w + 0.45w = 0.25w$$
$$V_{C^-} = 0.25w - 0.5w = -0.25w$$
$$V_{C^+} = -0.25w + 0.45w = 0.2w$$
$$V_D = 0.2w - 0.2w = 0$$

Areas: A to B. $\frac{1}{2}(0.2)(-0.2w) = -0.02w$

B to E $\frac{1}{2}(0.25)(0.25w) = 0.03125w$

Bending moments: $M_A = 0$
$$M_B = 0 - 0.02w = -0.02w$$
$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia:

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}(10^3\ \text{mm}^3)$	d, mm.	$Ad^2(10^3\ \text{mm}^4)$	$\bar{I}(10^3\ \text{mm}^4)$
①	1200	70	84	20	480	40
②	1200	30	36	20	480	360
Σ	2400		120		960	400

$$\bar{Y} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 1360 \times 10^3 \text{ mm}^4$$

PROBLEM 5.87 *(Continued)*

Top: $I/y = (1360 \times 10^3)/30 = 45.333 \times 10^3 \text{ mm}^3 = 45.333 \times 10^{-6} \text{ m}^3$

Bottom: $I/y = (1360 \times 10^3)/(-50) = -27.2 \times 10^3 \text{ mm}^3 = -27.2 \times 10^{-6} \text{ m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02\,w = -(80 \times 10^6)(45.333 \times 10^{-6})$ $w = 181.3 \times 10^3$ N/m

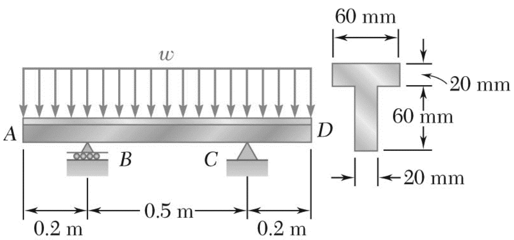
Compression at B and C: $-0.02\,w = -(-130 \times 10^6)(27.2 \times 10^{-6})$ $w = 176.8 \times 10^3$ N/m

Tension at E: $0.01125\,w = -(80 \times 10^6)(27.2 \times 10^{-6})$ $w = 193.4 \times 10^3$ N/m

Compression at E: $0.01125\,w = -(-130 \times 10)(45.333 \times 10^{-6})$ $w = 523.8 \times 10^3$ N/m

The smallest allowable load controls: $w = 176.8 \times 10^3$ N/m

$w = 176.8$ kN/m ◀

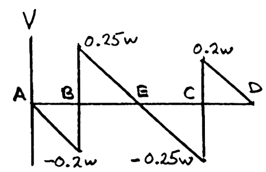


PROBLEM 5.88

Solve Prob. 5.87, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at B and C.

PROBLEM 5.87 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and –130 MPa in compression.

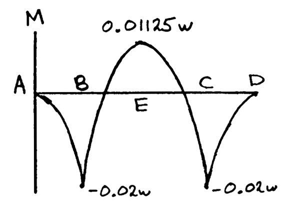
SOLUTION



Reactions: By symmetry, $B = C$

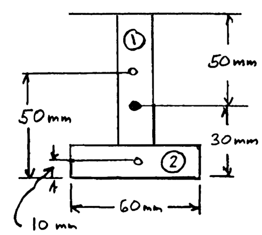
$$+\uparrow \Sigma F_y = 0:\ B + C - 0.9w = 0$$

$$\boldsymbol{B} = \boldsymbol{C} = 0.45w \uparrow$$



Shear:

$$V_A = 0$$
$$V_{B^-} = 0 - 0.2w = -0.2w$$
$$V_{B^+} = -0.2w + 0.45w = 0.25w$$
$$V_{C^-} = 0.25w - 0.5w = -0.25w$$
$$V_{C^+} = -0.25w + 0.45w = 0.2w$$
$$V_D = 0.2w - 0.2w = 0$$



Areas:

A to B: $\frac{1}{2}(0.2)(-0.2w) = -0.02w$

B to E: $\frac{1}{2}(0.25)(0.25w) = 0.03125w$

Bending moments:

$$M_A = 0$$
$$M_B = 0 - 0.02w = -0.02w$$
$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia:

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, (10^3 mm^3)	d, mm	Ad^2(10^3 mm^4)	$\bar{I}$, (10^3 mm^4)
①	1200	50	60	20	480	360
②	1200	10	12	20	480	40
Σ	2400		72		960	400

$$\bar{Y} = \frac{72 \times 10^3}{2400} = 30\,\text{mm} \qquad I = \Sigma Ad^2 + \Sigma \bar{I} = 1360 \times 10^3\ \text{mm}^3$$

PROBLEM 5.88 *(Continued)*

Top: $I/y = (1360\times10^3)/(50) = 27.2\times10^3 \text{ mm}^3 = 27.2\times10^{-6} \text{ m}^3$

Bottom: $I/y = (1360\times10^3)/(-30) = -45.333\times10^8 \text{ mm}^3 = -45.333\times10^{-6} \text{ m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02\,w = -(80\times10^6)(27.2\times10^{-6})$ $\quad w = 108.8\times10^3 \text{ N/m}$

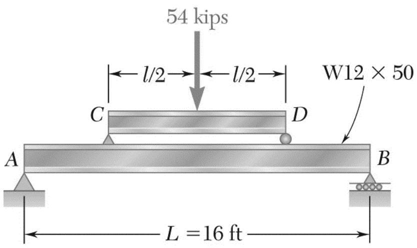
Compression at B and C: $-0.02\,w = -(-130\times10^6)(-45.333\times10^{-6})$ $\quad w = 294.7\times10^3 \text{ N/m}$

Tension at E: $0.01125\,w = -(80\times10^6)(-45.333\times10^{-6})$ $\quad w = 322.4\times10^3 \text{ N/m}$

Compression at E: $0.01125\,w = -(-130\times10^6)(27.2\times10^{-6})$ $\quad w = 314.3\times10^3 \text{ N/m}$

The smallest allowable load controls: $\quad w = 108.8\times10^3 \text{ N/m}$

$w = 108.8 \text{ kN/m}$ ◀

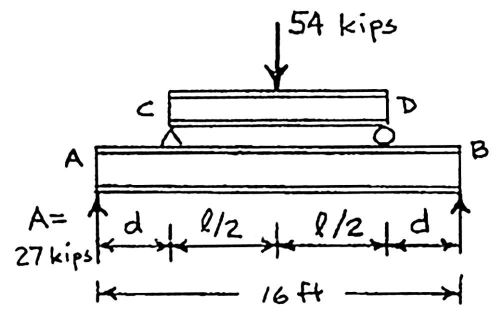


PROBLEM 5.89

A 54-kip is load is to be supported at the center of the 16-ft span shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine (*a*) the smallest allowable length l of beam *CD* if the $W12 \times 50$ beam *AB* is not to be overstressed, (*b*) the most economical W shape that can be used for beam *CD*. Neglect the weight of both beams.

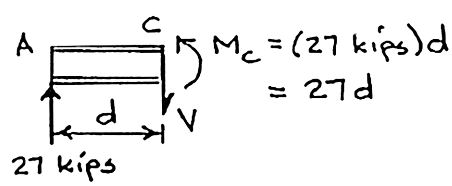
SOLUTION

(*a*)



$$d = 8\,\text{ft} - \frac{l}{2} \qquad l = 16\,\text{ft} - 2d \tag{1}$$

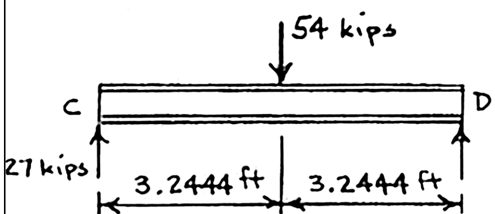
Beam *AB* (Portion *AC*):

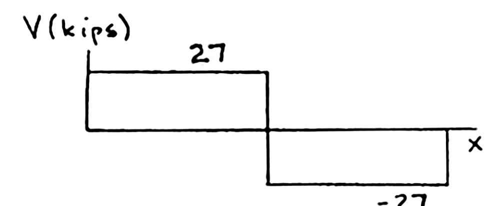


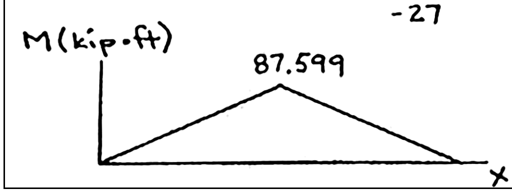
For $W12 \times 50$, $S_x = 64.2\text{ in}^3$ $\sigma_{\text{all}} = 24\text{ ksi}$

$$M_{\text{all}} = \sigma_{\text{all}} S_x = (24)(64.2) = 1540.8\text{ kip}\cdot\text{in} = 128.4\text{ kip}\cdot\text{ft}$$
$$M_C = 27d = 128.4\text{ kip}\cdot\text{ft} \qquad d = 4.7556\text{ ft}$$

Using (1), $l = 16 - 2d = 16 - 2(4.7556) = 6.4888\text{ ft}$ $\qquad l = 6.49\text{ ft}$ ◀







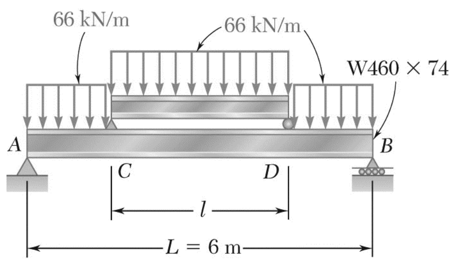
(*b*)

Beam *CD*: $l = 6.4888\text{ ft}$ $\sigma_{\text{all}} = 24\text{ ksi}$

$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{(87.599 \times 12)\text{ kip}\cdot\text{in}}{24\text{ ksi}}$$
$$= 43.800\text{ in}^3$$

Shape	$S(\text{in}^3)$	
$W18 \times 35$	57.6	
$W16 \times 31$	47.2	←
$W14 \times 38$	54.6	
$W12 \times 35$	45.6	
$W10 \times 45$	49.1	

$W16 \times 31$. ◀



PROBLEM 5.90

A uniformly distributed load of 66 kN/m is to be supported over the 6-m span shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine (*a*) the smallest allowable length l of beam CD if the $W460\times74$ beam AB is not to be overstressed, (*b*) the most economical W shape that can be used for beam CD. Neglect the weight of both beams.

SOLUTION

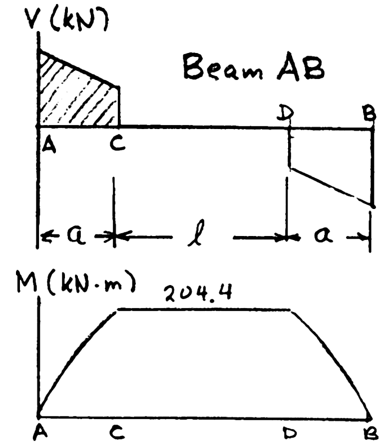
For $W460\times74$,

$$S = 1460\times10^3\ \text{mm}^3 = 1460\times10^{-6}\ \text{m}^3$$

$$\sigma_{\text{all}} = 140\,\text{MPa} = 140\times10^6\ \text{Pa}$$

$$M_{\text{all}} = S\sigma_{\text{all}} = (1460\times10^{-6})(140\times10^6)$$

$$= 204.4\times10^3\ \text{N}\cdot\text{m} = 204.4\ \text{kN}\cdot\text{m}$$



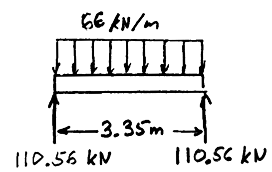
Reactions: By symmetry, $A = B, \quad C = D$

$$+\uparrow\Sigma F_y = 0:\ A + B - (6)(66) = 0$$

$$A = B = 198\ \text{kN} = 198\times10^3\ \text{N}$$

$$+\Sigma F_y = 0:\ C + D - 66l = 0$$

$$C = D = (33l)\ \text{kN} \qquad (1)$$



Shear and bending moment in beam AB:

$$0 < x < a, \qquad V = 198 - 66x\ \text{kN}$$

$$M = 198x - 33x^2\ \text{kN}\cdot\text{m}$$

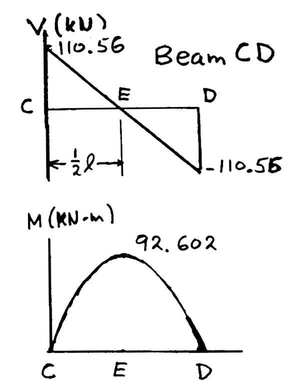
At C, $x = a$. $\quad M = M_{\max}$

$$M = 198a - 33a^2\ \text{kN}\cdot\text{m}$$

Set $M = M_{\text{all}}$. $\quad 198a - 33a^2 = 204.4$

$$33a^2 - 198a + 204.4 = 0$$

$$a = \cancel{4.6751\ \text{m}}, \qquad 1.32487\ \text{m}$$



(*a*) By geometry, $\quad l = 6 - 2a = 3.35\ \text{m}$ $\qquad l = 3.35\ \text{m}$ ◀

From (1), $\quad C = D = 110.56\ \text{kN}$

Draw shear and bending moment diagrams for beam CD. $V = 0$ at point E, the midpoint of CD.

PROBLEM 5.90 *(Continued)*

Area from A to E: $\int V dx = \frac{1}{2}(110.560)\left(\frac{1}{2}l\right) = 92.602 \text{ kN} \cdot \text{m}$

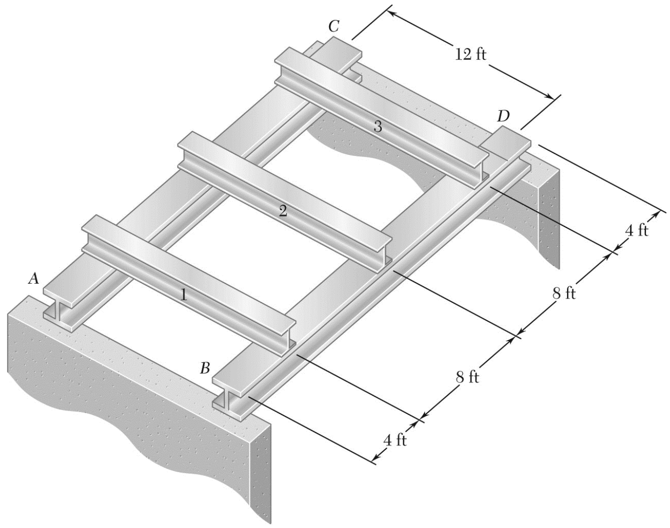
$$M_E = 92.602 \text{ kN} \cdot \text{m} = 92.602 \times 10^3 \text{ N} \cdot \text{m}$$

$$S_{\text{min}} = \frac{M_E}{\sigma_{\text{all}}} = \frac{92.602 \times 10^3}{140 \times 10^6} = 661.44 \times 10^{-6} \text{ m}^3$$

$$= 661.44 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$	
W410 × 46.1	774	
W360 × 44	693	←
W310 × 52	748	
W250 × 58	693	
W200 × 71	709	

(*b*) Use W360 × 44. ◀



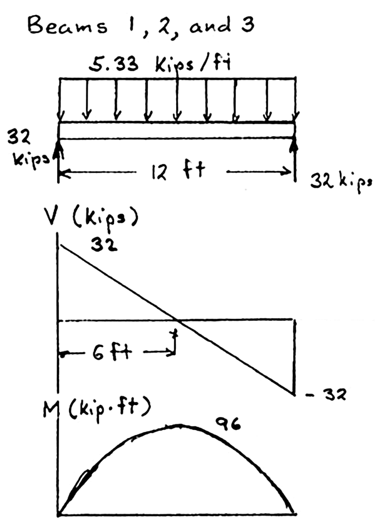
PROBLEM 5.91

Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders *AC* and *BD*. Knowing that the allowable normal stress for the steel used is 24 ksi, select (*a*) the most economical S shape for the three beams, (*b*) the most economical W shape for the two girders.

SOLUTION

For beams 1, 2, and 3

$$\text{Maximum } M = \left(\frac{1}{2}\right)(6)(32) = 96 \text{ kip} \cdot \text{ft} = 1152 \text{ kip} \cdot \text{in}$$

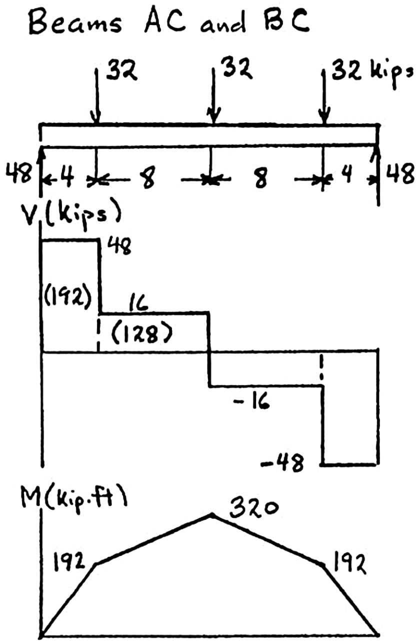


$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1152}{24} = 48 \text{ in}^3$$

Shape	$S(\text{in}^3)$
$S15 \times 42.9$	59.4
$S12 \times 50$	50.6

(*a*) Use $S15 \times 42.9$. ◀

PROBLEM 5.91 *(Continued)*



For beams AC and BC

Areas under shear digram:

$(4)(48) = 192 \text{ kip} \cdot \text{ft}$

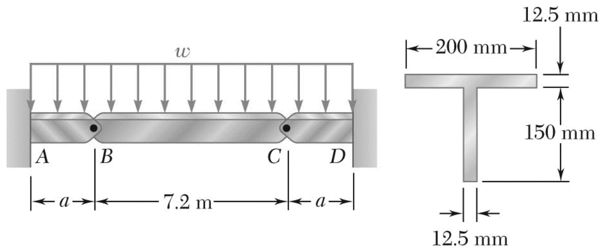
$(8)(16) = 128 \text{ kip} \cdot \text{ft}$

Maximum $M = 192 + 128 = 320 \text{ kip} \cdot \text{ft} = 3840 \text{ kip} \cdot \text{in}$

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3840}{24} = 160 \text{ in}^3$$

Shape	$S(\text{in}^3)$
$W30 \times 99$	269
$W27 \times 84$	213
$W24 \times 104$	258
$W21 \times 101$	227
$W18 \times 106$	204

(*b*) Use $W27 \times 84$. ◀



PROBLEM 5.92

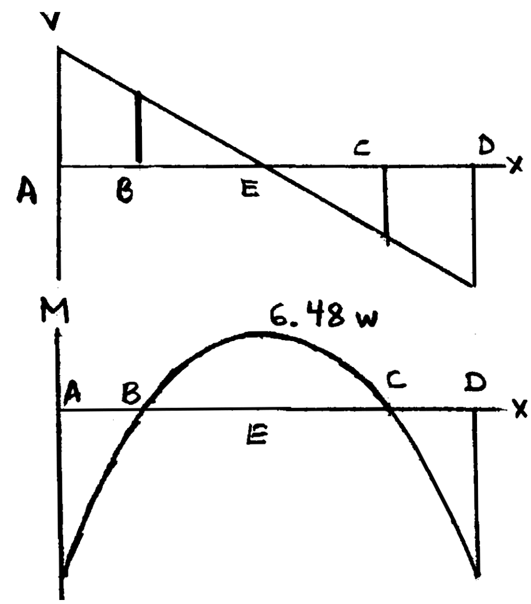
Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of *w* if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.

SOLUTION

$$M_B = M_C = 0$$

$$V_B = -V_C = \left(\frac{1}{2}\right)(7.2)w = 3.6w$$

Area *B* to *E* of shear diagram:



$$\left(\frac{1}{2}\right)(3.6)(3.6\,w) = 6.48\,w$$

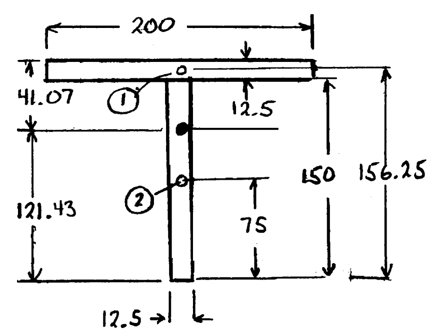
$$M_E = 0 + 6.48\,w = 6.48\,w$$

Centroid and moment of inertia:

Part	$A\,(\text{mm}^2)$	$\bar{y}\,(\text{mm})$	$A\bar{y}\,(\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531250}{4375} = 121.43 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6 \text{ mm}^4$$



Location	$y\,(\text{mm})$	$I/y\,(10^3\text{ mm}^3)$ ← also (10^{-6} m^3)
Top	41.07	258.6
Bottom	−121.43	−87.47

PROBLEM 5.92 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E: $-(110\times10^{6})(-87.47\times10^{-6}) = 9.622\times10^{3}\ \text{N}\cdot\text{m}$

Compression at E: $-(-150\times10^{-6})(258.6\times10^{-6}) = 38.8\times10^{3}\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times10^{6})(258.6\times10^{-6}) = -28.45\times10^{3}\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times10^{6})(-87.47\times10^{-6}) = -13.121\times10^{3}\ \text{N}\cdot\text{m}$

(a) Allowable load w: $6.48\,w = 9.622\times10^{3}$ $w = 1.485\times10^{3}\ \text{N/m}$ $w = 1.485\ \text{kN/m}$ ◀

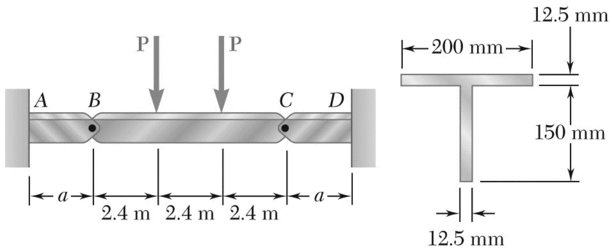
Shear at A: $V_A = (a+3.6)\,w$

Area A to B of shear diagram: $\frac{1}{2}a(V_A+V_B) = \frac{1}{2}a(a+7.2)\,w$

Bending moment at A (also D): $M_A = -\frac{1}{2}a(a+7.2)\,w$

$$-\frac{1}{2}a(a+7.2)(4.485\times10^{3}) = -13.121\times10^{3}$$

(b) Distance a: $\frac{1}{2}a^2 + 3.6a - 8.837 = 0$ $a = 1.935\ \text{m}$ ◀



PROBLEM 5.93

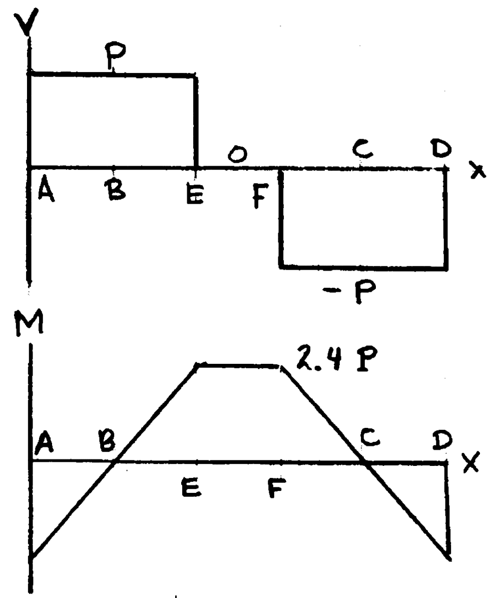
Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of **P** if beam BC is not to be overstressed, (*b*) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.

SOLUTION

$$M_B = M_C = 0$$
$$V_B = -V_C = P$$

Area B to E of shear diagram: 2.4 P

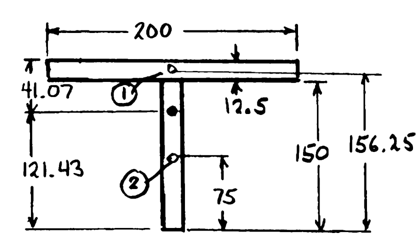
$$M_E = 0 + 2.4\text{ P} = 2.4\text{ P} = M_F$$



Centroid and moment of inertia:

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531250}{4375} = 121.43\text{ mm}$$
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6\text{ mm}^4$$



Location	$y(\text{mm})$	$I/y(10^3\text{ mm}^3)$
Top	41.07	258.6
Bottom	−121.43	−87.47

← also (10^{-6} m^3)

PROBLEM 5.93 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E and F: $-(110\times 10^6)(-87.47\times 10^{-6}) = 9.622\times 10^3\ \text{N}\cdot\text{m}$

Compression at E and F: $-(-150\times 10^6)(258.6\times 10^{-6}) = 38.8\times 10^3\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times 10^6)(258.6\times 10^{-6}) = -28.45\times 10^3\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times 10^6)(-87.47\times 10^{-6}) = -13.121\times 10^3\ \text{N}\cdot\text{m}$

(*a*) Allowable load P: $2.4\ \text{P} = 9.622\times 10^3$ $P = 4.01\times 10^3\ \text{N}$ $P = 4.01\ \text{kN}$ ◀

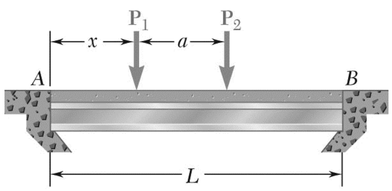
Shear at A: $V_A = P$

Area A to B of shear diagram: $aV_A = aP$

Bending moment at A: $M_A = -aP = -4.01\times 10^3 a$

(*b*) Distance a: $-4.01\times 10^3 a = -13.121\times 10^3$ $a = 3.27\ \text{m}$ ◀

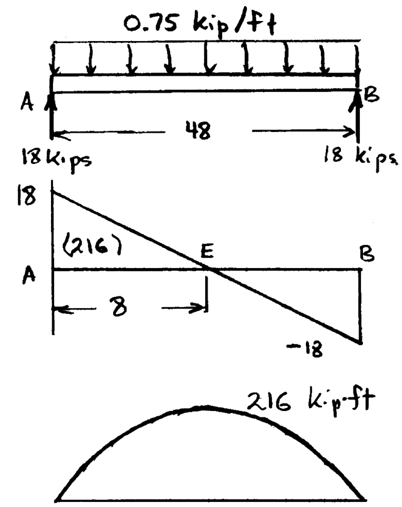
PROBLEM 5.94*



A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength of $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

SOLUTION

$$L = 48 \text{ ft} \quad a = 14 \text{ ft} \quad P_1 = 24 \text{ kips}$$
$$P_2 = 6 \text{ kips} \quad W = 0.75 \text{ kip/ft}$$



Dead load: $R_A = R_B = \left(\frac{1}{2}\right)(48)(0.75) = 18$ kips

Area A to E of shear diagram:

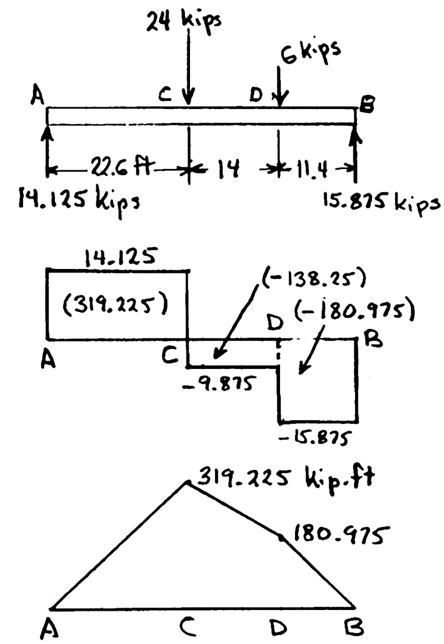
$$\left(\frac{1}{2}\right)(8)(18) = 216 \text{ kip}\cdot\text{ft}$$

$$M_{\text{max}} = 216 \text{ kip}\cdot\text{ft} = 2592 \text{ kip}\cdot\text{in} \text{ at point } E.$$

Live load: $u = \dfrac{aP_2}{2(P_1 + P_2)} = \dfrac{(14)(6)}{(2)(30)} = 1.4$ ft

$$x = \frac{L}{2} - u = 24 - 1.4 = 22.6 \text{ ft}$$
$$x + a = 22.6 + 14 = 36.6 \text{ ft}$$
$$L - x - a = 48 - 36.6 = 11.4 \text{ ft}$$
$$+\circlearrowleft \Sigma M_B = 0: -48\,R_A + (25.4)(24) + (11.4)(6) = 0$$
$$R_A = 14.125 \text{ kips}$$

PROBLEM 5.94* *(Continued)*



Shear:

A to C: $V = 14.125$ kips

C to D: $V = 14.125 - 24 = -9.875$ kips

D to B: $V = -15.875$ kips

Area:

A to C: $(22.6)(14.125) = 319.225 \text{ kip} \cdot \text{ft}$

Bending moment: $M_C = 319.225 \text{ kip} \cdot \text{ft} = 3831 \text{ kip} \cdot \text{in}$

Design:

$$\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S_{\min}$$

$$S_{\min} = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U}$$

$$= \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)}$$

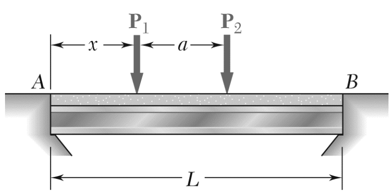
$$= 184.2 \text{ in}^3$$

Shape	$S(\text{in}^3)$	
W30×99	269	
W27×84	213	←
W24×104	258	
W21×101	227	
W18×106	204	

Use W27×84. ◀

PROBLEM 5.95

Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.94, determine how much heavier a truck could safely cross the bridge designed in that problem.



PROBLEM 5.94* A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength of $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

SOLUTION

$$L = 48 \text{ ft} \qquad a = 14 \text{ ft} \quad P_1 = 24 \text{ kips}$$
$$P_2 = 6 \text{ kips} \quad W = 0.75 \text{ kip/ft}$$

See solution to Prob. 5.94 for calculation of the following:

$$M_D = 2592 \text{ kip} \cdot \text{in} \qquad M_L = 3831 \text{ kip} \cdot \text{in}$$

For rolled steel section W27 × 84, $S = 213 \text{ in}^3$

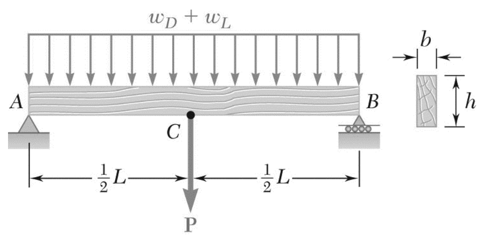
Allowable live load moment M_L^*:

$$\gamma_D M_D + \gamma_L M_L^* = \varphi M_U = \varphi \sigma_U S$$
$$M_L^* = \frac{\varphi \sigma_U S - \gamma_D M_D}{\gamma_L}$$
$$= \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75}$$
$$= 4721 \text{ kip} \cdot \text{in}$$

Ratio:
$$\frac{M_L^*}{M_L} = \frac{4721}{3831} = 1.232 = 1 + 0.232$$

Increase 23.2%. ◀

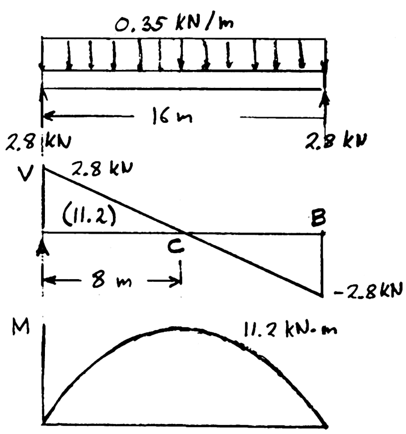
PROBLEM 5.96



A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION

$$L = 16 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m}$$
$$w_L = 600 \text{ N/m} = 0.6 \text{ kN/m}, \quad P = 6 \text{ kN}$$



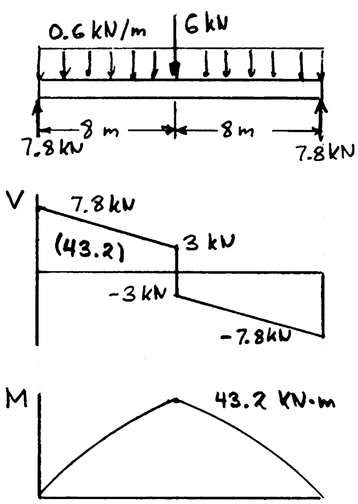
Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8$ kN

Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(2.8) = 11.2 \text{ kN}\cdot\text{m}$

Bending moment at C: $11.2 \text{ kN}\cdot\text{m} = 11.2\times10^3 \text{ N}\cdot\text{m}$

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8$ kN

Shear at C^-: $V = 7.8 - (8)(0.6) = 3$ kN



Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(7.8 + 3) = 43.2 \text{ kN}\cdot\text{m}$

Bending moment at C: $43.2 \text{ kN}\cdot\text{m} = 43.2\times10^3 \text{ N}\cdot\text{m}$

Design: $\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi\sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi\sigma_U} = \frac{(1.2)(11.2\times10^3) + (1.6)(43.2\times10^3)}{(0.9)(50\times10^6)}$$
$$= 1.8347\times10^{-3} \text{ m}^3 = 1.8347\times10^6 \text{ mm}^3$$

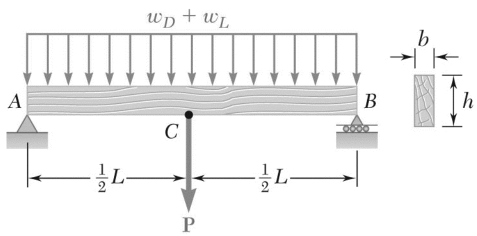
For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347\times10^6)}{75}}$$

$h = 383$ mm ◀

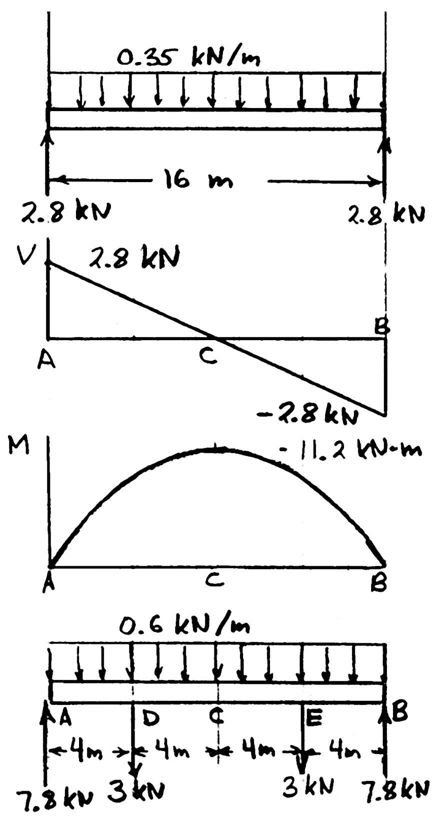
PROBLEM 5.97*

Solve Prob. 5.96, assuming that the 6-kN concentrated load **P** applied to each beam is replaced by 3-kN concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ applied at a distance of 4 m from each end of the beams.



PROBLEM 5.96* A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION



$$L = 16 \text{ m}, \quad a = 4 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m}$$
$$w_L = 600 \text{ N/m} = 0.6 \text{ kN/m}, \quad P = 3 \text{ kN}$$

Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8 \text{ kN}$

Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(2.8) = 11.2 \text{ kN}\cdot\text{m}$

Bending moment at C: $11.2 \text{ kN}\cdot\text{m} = 11.2\times 10^3 \text{ N}\cdot\text{m}$

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8 \text{ kN}$

Shear at D^-: $7.8 - (4)(0.6) = 5.4 \text{ kN}$

Shear at D^+: $5.4 - 3 = 2.4 \text{ kN}$

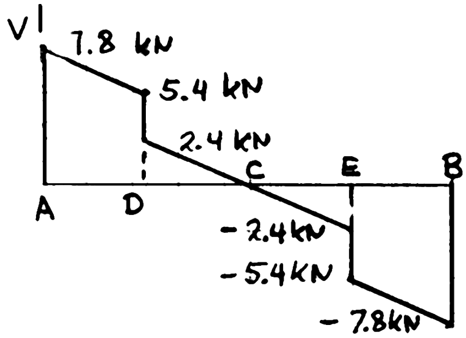
Area A to D:

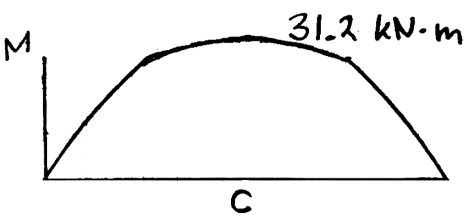
$\left(\frac{1}{2}\right)(4)(7.8 + 5.4) = 26.4 \text{ kN}\cdot\text{m}$

Area D to C: $\left(\frac{1}{2}\right)(4)(2.4) = 4.8 \text{ kN}\cdot\text{m}$

Bending moment at C: $26.4 + 4.8 = 31.2 \text{ kN}\cdot\text{m}$
$$= 31.2\times 10^3 \text{ N}\cdot\text{m}$$

PROBLEM 5.97* *(Continued)*





Design: $\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U} = \frac{(1.2)(11.2\times 10^3) + (1.6)(31.2\times 10^3)}{(0.9)(50\times 10^6)}$$

$$= 1.408\times 10^{-3}\,\text{m}^3 = 1.408\times 10^6\,\text{mm}^3$$

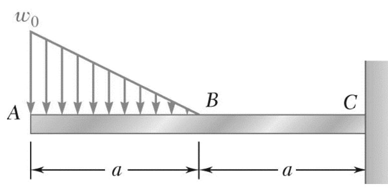
For a rectangular section,

$$S = \frac{1}{6}bh^2$$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408\times 10^6)}{75}}$$

$h = 336$ mm ◀

PROBLEM 5.98



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a}\langle x-a\rangle^1$$
$$= -\frac{dV}{dx}$$

(*a*) $$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$ ◀

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a}\langle x-a\rangle^3$$ ◀

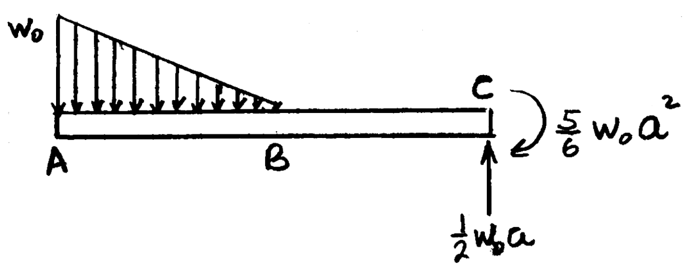
At point C, $x = 2a$

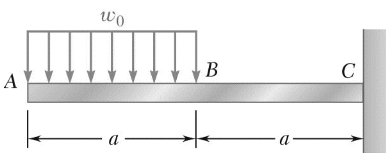
(*b*) $$M_C = -\frac{w_0(2a)^2}{2} + \frac{w_0(2a)^3}{6a} - \frac{w_0 a^3}{6a}$$

$$M_C = -\frac{5}{6}w_0 a^2$$ ◀

Check: $+\circlearrowleft \Sigma M_C = 0$: $\left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\frac{5}{6}w_0 a^2$$





PROBLEM 5.99

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - w_0\langle x-a\rangle^0$$
$$= -\frac{dV}{dx}$$

(*a*) $V = -w_0 x + w_0\langle x-a\rangle^1 = \dfrac{dM}{dx}$ ◀

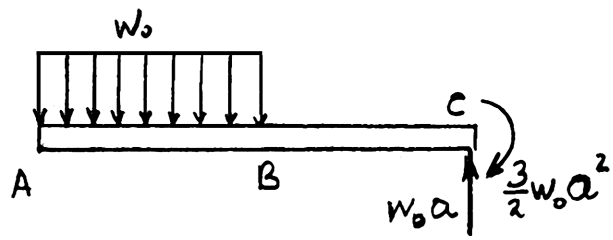
$$M = -\frac{1}{2}w_0x^2 + \frac{1}{2}w_0\langle x-a\rangle^2$$ ◀

At point C, $x = 2a$

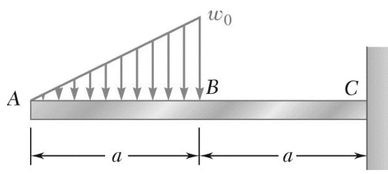
(*b*) $M_C = -\dfrac{1}{2}w_0(2a)^2 + \dfrac{1}{2}w_0a^2$ $M_C = -\dfrac{3}{2}w_0a^2$ ◀

Check: $+\circlearrowleft \Sigma M_C = 0$: $\left(\dfrac{3a}{2}\right)(w_0a) + M_C = 0$

$$M_C = -\frac{3}{2}w_0a^2$$



PROBLEM 5.100



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = \frac{w_0 x}{a} - w_0 \langle x-a \rangle^0 - \frac{w_0}{a}\langle x-a \rangle^1$$
$$= -\frac{dV}{dx}$$

(*a*) $$V = -\frac{w_0 x^2}{2a} + w_0 \langle x-a \rangle^1 + \frac{w_0}{2a}\langle x-a \rangle^2 = \frac{dM}{dx}$$ ◀

$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2}\langle x-a \rangle^2 + \frac{w_0}{6a}\langle x-a \rangle^3$$ ◀

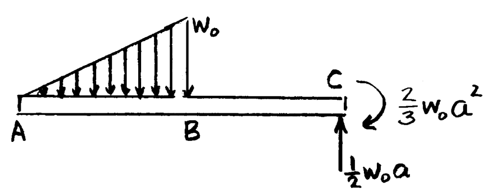
At point C, $x = 2a$

(*b*) $$M_C = -\frac{w_0 (2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a}$$

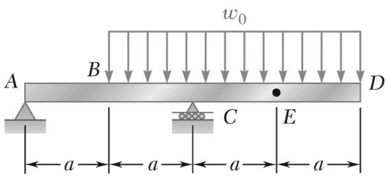
$$M_C = -\frac{2}{3} w_0 a^2$$ ◀

Check: $+\circlearrowleft \Sigma M_C = 0$: $\left(\frac{4a}{3}\right)\left(\frac{1}{2} w_0 a\right) + M_C = 0$

$$M_C = -\frac{2}{3} w_0 a^2$$



PROBLEM 5.101



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -2aA - \left(\frac{a}{2}\right) + (3aw_0) = 0 \qquad A = -\frac{3}{4} w_0 a$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 2aC - \left(\frac{5a}{2}\right) + (3aw_0) = 0 \qquad C = \frac{15}{4} w_0 a$$

$$w = w_0 \langle x - a \rangle^0 = -\frac{dV}{dx}$$

(*a*) $$V = -w_0 \langle x - a \rangle^1 - \frac{3}{4} w_0 a + \frac{15}{4} w_0 a \langle x - 2a \rangle^0 = \frac{dM}{dx}$$ ◀

$$M = -\frac{1}{2} w_0 \langle x - a \rangle^2 - \frac{3}{4} w_0 a x + \frac{15}{4} w_0 a \langle x - 2a \rangle^1 + 0$$ ◀

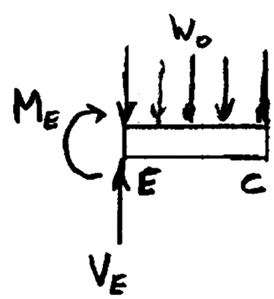
At point E, $\quad x = 3a$

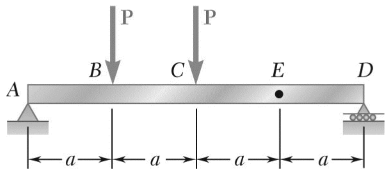
(*b*) $$M_E = -\frac{1}{2} w_0 (2a)^2 - \frac{3}{4} w_0 a(3a) + \frac{15}{4} w_0 a(a)$$

$$M_E = -\frac{1}{2} w_0 a^2$$ ◀

Check: $\quad +\circlearrowleft \Sigma M_E = 0: \quad -M_E - \frac{a}{2}(w_0 a) = 0$

$$M_E = -\frac{1}{2} w_0 a^2$$





PROBLEM 5.102

(a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of the beam to the right of E.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -4aA + 3aP + 2aP = 0 \qquad A = 1.25P$$

(a) $V = 1.25P - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$ ◀

$$M = 1.25Px - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1$$ ◀

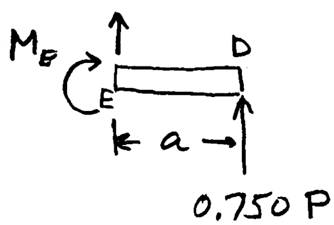
(b) At point E, $x = 3a$

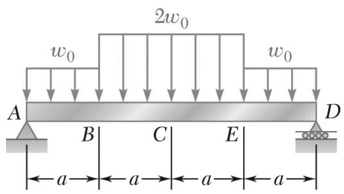
$$M_E = 1.25P(3a) - P(2a) - P(a) = 0.750\ Pa$$ ◀

Reaction: $+\uparrow \Sigma F_y = 0: \quad A - P - P + D = 0 \qquad \boldsymbol{D} = 0.750P \uparrow$

$$+\circlearrowleft \Sigma M_E = 0: \quad -M_E + 0.750\ Pa = 0$$

$$M_E = 0.750\ Pa$$

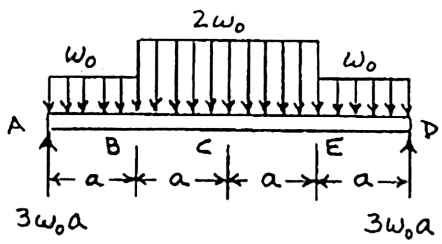




PROBLEM 5.103

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of the beam to the right of E.

SOLUTION



$$w = w_0 + w_0\langle x-a\rangle^0 - w_0\langle x-3a\rangle^0$$

(*a*) $\quad V = 3w_0a - \int w\,dx$

$$= 3w_0a - w_0x - w_0\langle x-a\rangle^1 + w_0\langle x-3a\rangle^1 \quad \blacktriangleleft$$

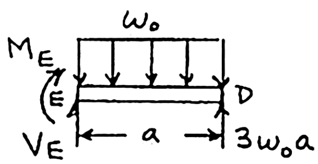
$$M = \int V\,dx = 3w_0ax - w_0x^2/2 - w_0\langle x-a\rangle^2/2$$

$$+w_0\langle x-3a\rangle^2/2 \quad \blacktriangleleft$$

(*b*) At point E, $\quad x = 3a$

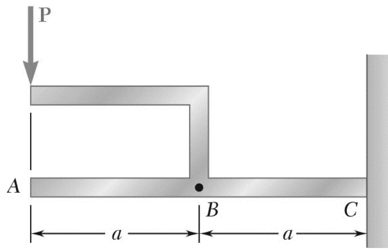
$$M_E = 3w_0a(3a) - w_0(3a)^2/2 - w_0(2a)^2/2$$

$$M_E = 5w_0a^2/2 \quad \blacktriangleleft$$



$$+\circlearrowleft \sum M_E = 0: \quad 3w_0a(a) - (w_0a)\left(\tfrac{a}{2}\right) - M_E = 0$$

$$M_E = 5w_0a^2/2 \text{ (checks)}$$



PROBLEM 5.104

(a) Using singularity functions, write the equations for the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for M to determine the bending moment just to the right of point B.

SOLUTION

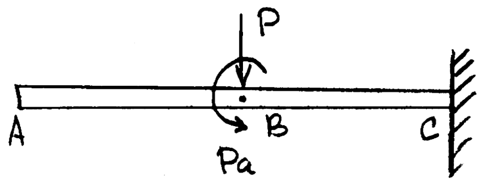
(a) $V = -P\langle x-a\rangle^0$ ◀

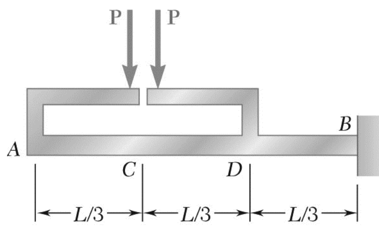
$$\frac{dM}{dx} = -P\langle x-a\rangle^0$$

$$M = -P\langle x-a\rangle^1 - Pa\langle x-a\rangle^0 \quad ◀$$

Just to the right of B, $x = a^1$.

(b) $M = -\ 0 - Pa$ $\qquad M = -Pa$ ◀

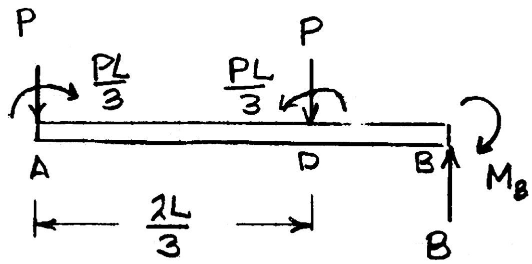




PROBLEM 5.105

(a) Using singularity functions, write the equations for the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for M to determine the bending moment just to the right of point D.

SOLUTION



(a) $V = -P - P\left\langle x - \frac{2L}{3}\right\rangle^0 = \frac{dM}{dx}$ ◀

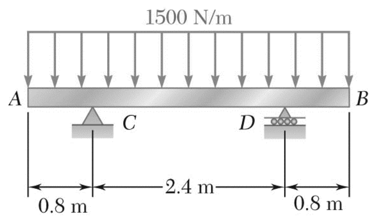
$$M = -Px + \frac{PL}{3} - P\left\langle x - \frac{2L}{3}\right\rangle^1 - \frac{PL}{3}\left\langle x - \frac{2L}{3}\right\rangle^0$$ ◀

Just to the right of D, $x = \frac{2L}{3}$.

(b) $M_D^+ = -P\left(\frac{2L}{3}\right) + \frac{PL}{3} - P(0) - \frac{PL}{3}$

$$M_D^+ = -\frac{4PL}{3}$$ ◀

PROBLEM 5.106



(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$w = 1.5 \text{ kN/m}$$

By statics,

$$\boldsymbol{C} = \boldsymbol{D} = 3 \text{ kN} \uparrow$$

(*a*) $V = -1.5x + 3\langle x - 0.8\rangle^0 + 3\langle x - 3.2\rangle^0 \text{ kN}$ ◀

$$M = -0.75x^2 + 3\langle x - 0.8\rangle^1 + 3\langle x - 3.2\rangle^1 \text{ kN} \cdot \text{m} \blacktriangleleft$$

Locate point E where $V = 0$. Assume $x_C < x_E < x_D$

$$0 = -1.5x_E + 3(x_E - 0.8) + 0 \qquad x_E = 2.0 \text{ m}$$

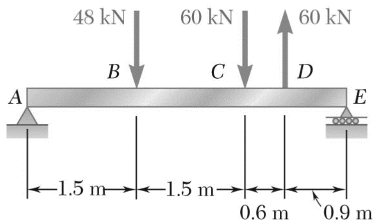
$$M_C = -(0.75)(0.8)^2 + 0 + 0 = -0.480 \text{ kN} \cdot \text{m}$$

$$M_E = -(0.75)(2.0)^2 + (3)(1.2) + 0 = 0.600 \text{ kN} \cdot \text{m}$$

$$M_D = -(0.75)(3.2)^2 + (3)(2.4) + 0 = -0.480 \text{ kN} \cdot \text{m}$$

(*b*) $|M|_{\max} = 0.600 \text{ kN} \cdot \text{m}$

$$|M|_{\max} = 600 \text{ N} \cdot \text{m} \blacktriangleleft$$



PROBLEM 5.107

(a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad -4.5R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

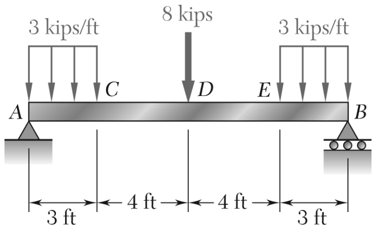
$$R_A = 40 \text{ kN}$$

(a) $V = 40 - 48\langle x-1.5\rangle^0 - 60\langle x-3.0\rangle^0 + 60\langle x-3.6\rangle^0 \text{ kN}$ ◀

$M = 40x - 48\langle x-1.5\rangle^1 - 60\langle x-3.0\rangle^1 + 60\langle x-3.6\rangle^1 \text{ kN}\cdot\text{m}$ ◀

Pt.	x(m)	M(kN·m)
A	0	0
B	1.5	$(40)(1.5) = 60 \text{ kN}\cdot\text{m}$
C	3.0	$(40)(3.0) - (48)(1.5) = 48 \text{ kN}\cdot\text{m}$
D	3.6	$(40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2 \text{ kN}\cdot\text{m}$
E	4.5	$(40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$

(b) $M_{\max} = 60 \text{ kN}\cdot\text{m}$ ◀



PROBLEM 5.108

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -14A + (12.5)(3)(3) + (7)(8) + (1.5)(3)(3) = 0$$

$$\mathbf{A} = 13 \text{ kips} \uparrow$$

$$w = 3 - 3\langle x-3\rangle^0 + 3\langle x-11\rangle^0 = -\frac{dV}{dx}$$

(*a*) $V = 13 - 3x + 3\langle x-3\rangle^1 - 8\langle x-7\rangle^0 - 3\langle x-11\rangle^1$ kips ◀

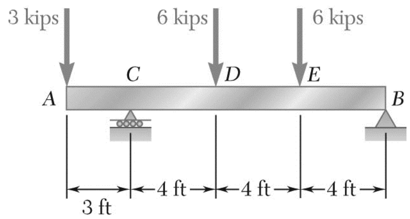
$M = 13x - 1.5x^2 + 1.5\langle x-3\rangle^2 - 8\langle x-7\rangle^1 - 1.5\langle x-11\rangle^2$ kip · ft ◀

$$V_C = 13 - (3)(3) = 4 \text{ kips}$$
$$V_{D^-} = 13 - (3)(7) + (3)(4) = 4 \text{ kips}$$
$$V_{D^+} = 13 - (3)(7) + (3)(4) - 8 = -4 \text{ kips}$$
$$V_E = 13 - (3)(11) + (3)(8) - 8 = -4 \text{ kips}$$
$$V_B = 13 - (3)(14) + (3)(11) - 8 - (3)(3) = -13 \text{ kips}$$

(*b*) Note that V changes sign at D.

$$|M|_{\max} = M_D = (13)(7) - (1.5)(7)^2 + (1.5)(4)^2 - 0 - 0$$

$|M|_{\max} = 41.5$ kip · ft ◀



PROBLEM 5.109

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \ (15)(3) - 12C + (8)(6)C + (4)(6) = 0$$

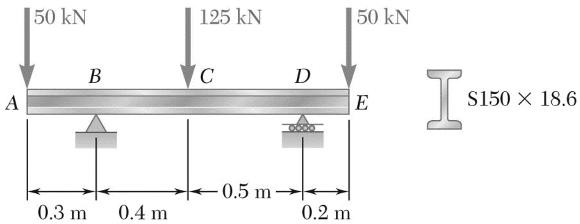
$$\mathbf{C} = 9.75 \text{ kips} \uparrow$$

(*a*) $V = -3 + 9.75\langle x-3\rangle^0 - 6\langle x-7\rangle^0 - 6\langle x-11\rangle^0$ kips ◀

$M = -3x + 9.75\langle x-3\rangle^1 - 6\langle x-7\rangle^1 - 6\langle x-11\rangle^1$ kip · ft ◀

Pt.	x(ft)	M(kip · ft)	
A	0	0	
C	3	$-(3)(3) = -9$	
D	7	$-(3)(7) + (9.75)(4) = 18$	
E	11	$-(3)(11) + (9.75)(8) - (6)(4) = 21$	← maximum
B	15	$-(3)(15) + (9.75)(12) - (6)(8) - (6)(4) = 0$	

(*b*) $|M|_{max} = 21.0$ kip · ft ◀



PROBLEM 5.110

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0:$$

$$(1.2)(50) - 0.9B + (0.5)(125) - (0.2)(50) = 0$$

$$\boldsymbol{B} = 125 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0:$$

$$(0.3)(50) - (0.4)(125) + 0.9D - (1.1)(50) = 0$$

$$\boldsymbol{D} = 100 \text{ kN} \uparrow$$

(*a*) $V = -50 + 125\langle x - 0.3\rangle^0 - 125\langle x - 0.7\rangle^0 + 100\langle x - 1.2\rangle^0 \text{ kN}$ ◀

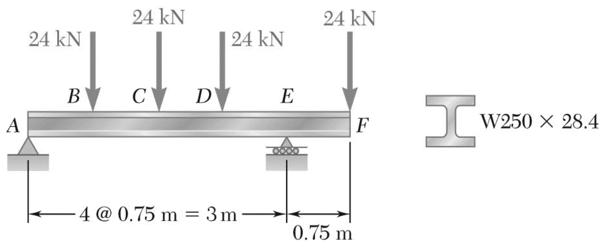
$M = -50x + 125\langle x - 0.3\rangle^1 - 125\langle x - 0.7\rangle^1 + 100\langle x - 1.2\rangle^1 \text{ kN}\cdot\text{m}$ ◀

Point	x(m)	M(kN·m)
B	0.3	$-(50)(0.3) + 0 - 0 + 0 = -15 \text{ kN}\cdot\text{m}$
C	0.7	$-(50)(0.7) + (125)(0.4) - 0 + 0 = 15 \text{ kN}\cdot\text{m}$
D	1.2	$-(50)(1.2) + (125)(0.9) - (125)(0.5) + 0 = -10 \text{ kN}\cdot\text{m}$
E	1.4	$-(50)(1.4) + (125)(1.1) - (125)(0.7) + (100)(0.2) = 0$ (checks)

Maximum $|M| = 15 \text{ kN}\cdot\text{m} = 15\times 10^3 \text{ N}\cdot\text{m}$

For S150×18.6 rolled steel section, $S = 120\times 10^3 \text{ mm}^3 = 120\times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{15\times 10^3}{120\times 10^{-6}} = 125\times 10^6 \text{ Pa}$ $\qquad \sigma = 125.0 \text{ MPa}$ ◀



PROBLEM 5.111

(a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad -3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$$
$$R_A = 30 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$$
$$R_E = 66 \text{ kips}$$

(a) $V = 30 - 24\langle x - 0.75\rangle^0 - 24\langle x - 1.5\rangle^0 - 24\langle x - 2.25\rangle^0 + 66\langle x - 3\rangle^0$ kN ◀

$M = 30x - 24\langle x - 0.75\rangle^1 - 24\langle x - 1.5\rangle^1 - 24\langle x - 2.25\rangle^1 + 66\langle x - 3\rangle^1$ kN · m ◀

Point	x(m)	M(kN · m)
B	0.75	$(30)(0.75) = 22.5$ kN · m
C	1.5	$(30)(1.5) - (24)(0.75) = 27$ kN · m
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5$ kN · m
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18$ kN · m
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$ ✓

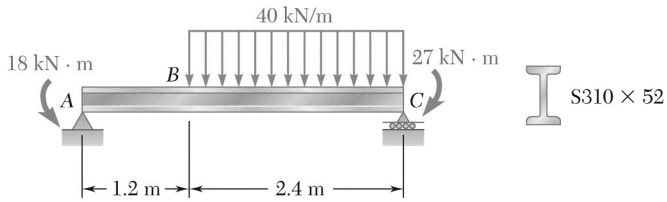
Maximum $|M| = 27$ kN · m $= 27 \times 30^3$ N · m

For rolled steel section W250×28.4, $S = 308 \times 10^3 \text{ mm}^3 = 308 \times 10^{-6} \text{ m}^3$

(b) Normal stress:

$$\sigma = \frac{|M|}{S} = \frac{27 \times 10^3}{308 \times 10^{-6}} = 87.7 \times 10^6 \text{ Pa}$$

$\sigma = 87.7$ MPa ◀



PROBLEM 5.112

(a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.

SOLUTION

$+\circlearrowleft M_c = 0: \quad 18 - 3.6A + (1.2)(2.4)(40) - 27 = 0$

$\quad \boldsymbol{A} = 29.5 \text{ kN} \uparrow$

$V = 29.5 - 40\langle x - 1.2\rangle^1 \text{ kN}$

<u>Point D</u>. $\quad V = 0 \qquad 29.5 - 40(x_D - 1.2) = 0$

$$x_D = 1.9375 \text{ m}$$

$M = -18 + 29.5x - 20\langle x - 1.2\rangle^2 \text{ kN} \cdot \text{m}$

$M_A = -18 \text{ kN} \cdot \text{m}$

$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN} \cdot \text{m}$

$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN} \cdot \text{m}$

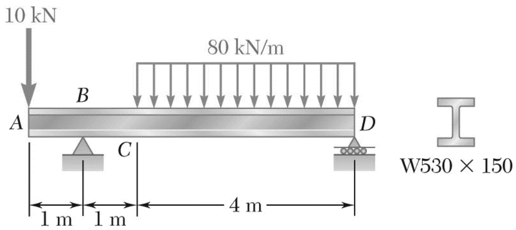
(a) Maximum $\qquad |M| = 28.278 \text{ kN} \cdot \text{m at } x = 1.9375 \text{ m}$ ◀

For $S310 \times 52$ rolled steel section, $\qquad S = 624 \times 10^3 \text{ mm}^3$

$$= 624 \times 10^{-6} \text{ m}^3$$

(b) Normal stress: $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{28.278 \times 10^3}{624 \times 10^{-6}} = 45.3 \times 10^6 \text{ Pa}$

$\sigma = 45.3 \text{ MPa}$ ◀



PROBLEM 5.113

(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft M_D = 0: \quad (6)(10) - 5R_B + (2)(4)(80) = 0$$

$$R_B = 140 \text{ kN}$$

$$w = 80\langle x-2\rangle^0 \text{ kN/m} = -dV/dx$$

$$V = -10 + 140\langle x-1\rangle^0 - 80\langle x-2\rangle^1 \text{ kN}$$

A to *B*: $\quad V = -10 \text{ kN}$

B to *C*: $\quad V = -10 + 140 = 130 \text{ kN}$

D: $\quad (x = 6) \quad V = -10 + 140 - 80(4) = -190 \text{ kN}$

V changes sign at B and at point E $(x = x_E)$ between C and D.

$$V = 0 = -10 + 140\langle x_E - 1\rangle^0 - 80\langle x_E - 2\rangle^1$$

$$= -10 + 140 - 80(x_E - 2) \qquad x_E = 3.625 \text{ m}$$

$$M = -10x + 140\langle x-1\rangle^1 - 40\langle x-2\rangle^2 \text{ kN}\cdot\text{m}$$

At pt. *B*, $\quad x = 1 \quad M_B = -(10)(1) = -10 \text{ kN}\cdot\text{m}$

At pt. *E*, $\quad x = 3.625$

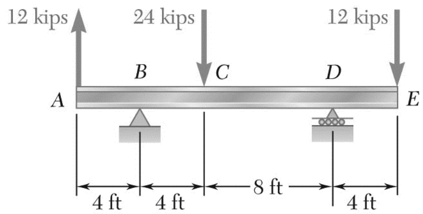
$$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6 \text{ kN}\cdot\text{m}$$

(*a*) $\quad |M|_{\max} = 225.6 \text{ kN}\cdot\text{m at } x = 3.625 \text{ m}$ ◀

For W530 × 150, $\quad S = 3720 \times 10^3 \text{ mm}^3 = 3720 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6 \text{ Pa}$

$\sigma = 60.6 \text{ MPa}$ ◀



PROBLEM 5.114

A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -(16)(12) - 12B + (8)(24) - (4)(12) = 0$$

$B = -4$ kips $\quad \mathbf{B} = 4$ kips $\downarrow$

$$+\circlearrowleft \Sigma M_B = 0: \quad -(4)(12) - (4)(24) + 12D - (16)(12) = 0$$

$\mathbf{D} = 28$ kips $\uparrow$

Check: $\quad +\uparrow \Sigma F_y = 12 - 4 - 24 + 28 - 12 = 0 \ \checkmark$

$$V = 12 - 4\langle x-4\rangle^0 - 24\langle x-8\rangle^0 + 28\langle x-16\rangle^0$$
$$M = 12x - 4\langle x-4\rangle^1 - 24\langle x-8\rangle^1 + 28\langle x-16\rangle^1$$

At A, $\quad x = 0, \quad M = 0$

At B, $\quad x = 4$ ft, $\quad M = (12)(4) = 48$ kip·ft

At C, $\quad x = 8$ ft, $\quad M = (12)(8) - (4)(4) = 80$ kip·ft

At D, $\quad x = 16$ ft, $\quad M = (12)(16) - (4)(12) - (24)(8) = -48$ kip·ft

At E, $\quad x = 20$ ft, $\quad M = (12)(20) - (4)(16) - (24)(12) + (28)(4) = 0 \quad$ (checks)

(*a*) $\quad |M|_{max} = 80$ kip·ft $\quad$ at $\quad C$. ◀

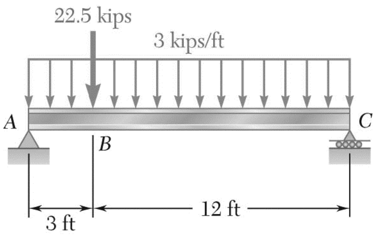
(*b*) $\quad |M|_{max} = 960$ kip·in $\qquad \sigma_{all} = 24$ ksi

$$\sigma = \frac{|M|}{S} \qquad S_{min} = \frac{|M|_{max}}{\sigma} = \frac{960}{24} = 40 \text{ in}^3$$

Shape	S_x (in^3)
W18×35	57.6
W16×31	47.2
W14×30	42.0 ←
W12×35	45.6
W10×39	42.1
W8×48	43.5

Lightest wide flange shape: W14×30 ◀

PROBLEM 5.115



A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -15R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$\boldsymbol{R}_A = 40.5 \text{ kips} \uparrow$$

$$w = 3 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 40.5 - 3x - 22.5\langle x-3\rangle^0 \text{ kips}$$

$$M = 40.5x - 1.5x^2 - 22.5\langle x-3\rangle^1 \text{ kip}\cdot\text{ft}$$

(*a*) Location of point D where $V = 0$. Assume $3 \text{ ft} < x_D < 12 \text{ ft}$.

$$0 = 40.5 - 3x_D - 22.5 \qquad x_D = 6 \text{ ft}$$

At point D, $(x = 6 \text{ ft})$.

$$M = (40.5)(6) - (1.5)(6)^2 - (22.5)(3)$$
$$= 121.5 \text{ kip}\cdot\text{ft} = 1458 \text{ kip}\cdot\text{in}$$

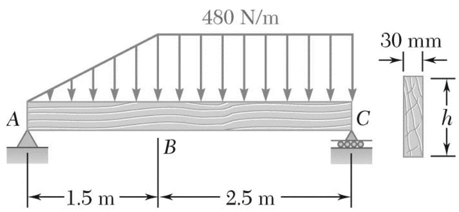
Maximum $|M|$: $\quad |M|_{\max} = 121.5 \text{ kip}\cdot\text{ft} \quad \text{at} \quad x = 6.00 \text{ ft}$ ◀

$$S_{\min} = \frac{M}{\sigma_{\text{all}}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

(*b*)

Shape	$S\,(\text{in}^3)$	
W21×44	81.6	
W18×50	88.9	
W16×40	64.7	←
W14×43	62.6	
W12×50	64.2	
W10×68	75.7	

Wide-flange shape: W16×40 ◀



PROBLEM 5.116

A timber beam is being designed with supports and loads as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$$480 \text{ N/m} = 0.48 \text{ kN/m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -4R_A + (3)\left(\frac{1}{2}\right)(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$$

$$\mathbf{R}_A = 0.645 \text{ kN} \uparrow$$

$$w = \frac{0.48}{1.5}x - \frac{0.48}{1.5}\langle x-1.5\rangle^1 = 0.32x - 0.32\langle x-1.5\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16x^2 + 0.16\langle x-1.5\rangle^2 \text{ kN}$$

$$M = 0.645x - 0.05333x^3 + 0.05333\langle x-1.5\rangle^3 \text{ kN}\cdot\text{m}$$

(*a*) Locate point D where $V = 0$.

Assume $1.5 \text{ m} < x_D < 4 \text{ m}$.

$$0 = 0.645 - 0.16x_D^2 + 0.16(x_D - 1.5)^2$$

$$= 0.645 - \cancel{0.16x_D^2} + \cancel{0.16x_D^2} - 0.48x_D + 0.36$$

$$x_D = 2.09375 \text{ m} \blacktriangleleft$$

At point D, $\quad M_D = (0.645)(2.09375) - (0.05333)(2.09375)^3 + (0.05333)(0.59375)^3$

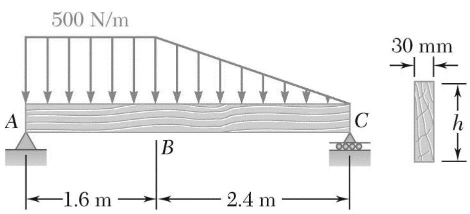
$$M_D = 0.87211 \text{ kN}\cdot\text{m} \blacktriangleleft$$

$$S_{\text{min}} = \frac{M_D}{\sigma_{\text{all}}} = \frac{0.87211\times10^3}{12\times10^6} = 72.6758\times10^{-6} \text{ m}^3 = 72.6758\times10^3 \text{ mm}^3$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}}$

$$h_{\text{min}} = \sqrt{\frac{(6)(72.6758\times10^3)}{30}} = 120.56 \text{ mm}$$

(*b*) At next larger 10-mm increment, $\quad h = 130 \text{ mm} \blacktriangleleft$



PROBLEM 5.117

A timber beam is being designed with supports and loads as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth *h* varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$$500 \text{ N/m} = 0.5 \text{ kN/m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -4R_A + (3.2)(1.6)(0.5) + (1.6)\left(\frac{1}{2}\right)(2.4)(0.5) = 0 \qquad R_A = 0.880 \text{ kN} \uparrow$$

$$w = 0.5 - \frac{0.5}{2.4}\langle x - 1.6\rangle^1 = 0.5 - 0.20833\langle x - 1.6\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.880 - 0.5x + 0.104167\langle x - 1.6\rangle^2 \text{ kN}$$

$$V_A = 0.880 \text{ kN}$$

$$\left.\begin{aligned} V_B &= 0.880 - (0.5)(1.6) = 0.080 \text{ kN} \\ V_C &= 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520 \text{ kN} \end{aligned}\right\} \text{Sign change}$$

Locate point *D* (between *B* and *C*) where $V = 0$. $\quad 0 = 0.880 - 0.5x_D + 0.104167\,(x_D - 1.6)^2$

$$0.104167\,x_D^2 - 0.83333\,x_D + 1.14667 = 0$$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)} = \cancel{6.2342}, \quad 1.7658 \text{ m} \blacktriangleleft$$

$$M = 0.880x - 0.25x^2 + 0.347222\langle x - 1.6\rangle^3 \text{ kN}\cdot\text{m}$$

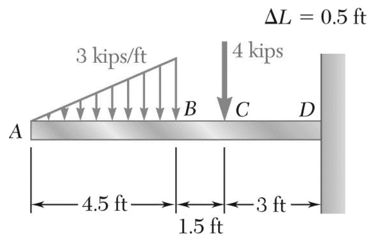
$$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.34722)(0.1658)^3 = 0.776 \text{ kN}\cdot\text{m}$$

(*a*) $$M_{\max} = 0.776 \text{ kN}\cdot\text{m} \quad \text{at} \quad x = 1.7658 \text{ m} \blacktriangleleft$$

$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{0.776\times10^3}{12\times10^6} = 64.66\times10^{-6} \text{ m}^3 = 64.66\times10^3 \text{ mm}^3$$

For a rectangular cross section, $S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}} \qquad h_{\min} = \sqrt{\frac{(6)(64.66\times10^3)}{30}} = 113.7 \text{ mm}$

(*b*) At next higher 10-mm increment, $\qquad h = 120 \text{ mm} \blacktriangleleft$



PROBLEM 5.118

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

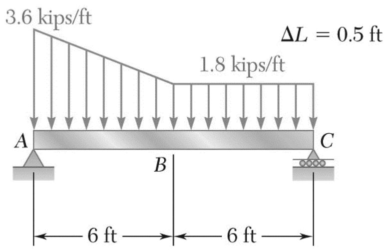
$$w = \frac{3}{4.5}x - 3\langle x-4.5\rangle^0 - \frac{3}{4.5}\langle x-4.5\rangle^1$$

$$= \frac{2}{3}x - 3\langle x-4.5\rangle^0 - \frac{2}{3}\langle x-4.5\rangle^1 = -\frac{dV}{dx}$$

$$V = -\frac{1}{3}x^2 + 3\langle x-4.5\rangle^1 + \frac{1}{3}\langle x-4.5\rangle^2 - 4\langle x-6\rangle^0$$

$$M = -\frac{1}{9}x^3 + \frac{3}{2}\langle x-4.5\rangle^2 + \frac{1}{9}\langle x-4.5\rangle^3 - 4\langle x-6\rangle^1$$

x	V	M
ft	kips	kip · ft
0.0	0.00	0.00
0.5	−0.08	−0.01
1.0	−0.33	−0.11
1.5	−0.75	−0.38
2.0	−1.33	−0.89
2.5	−2.08	−1.74
3.0	−3.00	−3.00
3.5	−4.08	−4.76
4.0	−5.33	−7.11
4.5	−6.75	−10.13
5.0	−6.75	−13.50
5.5	−6.75	−16.88
6.0	−10.75	−20.25
6.5	−10.75	−25.63
7.0	−10.75	−31.00
7.5	−10.75	−36.38
8.0	−10.75	−41.75
8.5	−10.75	−47.13
9.0	−10.75	−52.50



PROBLEM 5.119

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -12R_A + (6)(12)(1.8) + (10)\left(\frac{1}{2}\right)(6)(1.8) = 0$$

$$R_A = 15.3 \text{ kips}$$

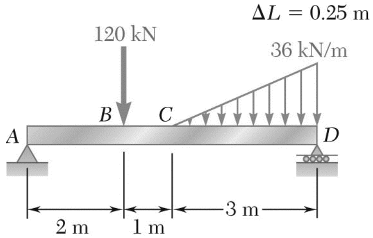
$$w = 3.6 - \frac{1.8}{6}x + \frac{1.8}{6}\langle x-6\rangle^1$$
$$= 3.6 - 0.3x + 0.3\langle x-6\rangle^1$$

$$V = 15.3 - 3.6x + 0.15x^2 - 0.15\langle x-6\rangle^2 \text{ kips} \blacktriangleleft$$

$$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05\langle x-6\rangle^3 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

x	V	M
ft	kips	kip · ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	−0.90	37.8
6.5	−1.80	37.1
7.0	−2.70	36.0
7.5	−3.60	34.4
8.0	−4.50	32.4
8.5	−5.40	29.9
9.0	−6.30	27.0

x	V	M
ft	kips	kip · ft
9.5	−7.20	23.6
10.0	−8.10	19.8
10.5	−9.00	15.5
11.0	−9.90	10.8
11.5	−10.80	5.6
12.0	−11.70	0.0



PROBLEM 5.120

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -6R_A + (4)(120) + (1)\left(\frac{1}{2}\right)(3)(36) = 0$$

$$R_A = 89 \text{ kN}$$

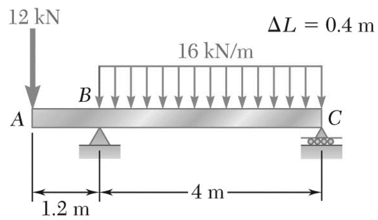
$$w = \frac{36}{3}\langle x-3\rangle^1 = 12\langle x-3\rangle^1$$

$$V = 89 - 120\langle x-2\rangle^0 - 6\langle x-3\rangle^2 \text{ kN} \blacktriangleleft$$

$$M = 89x - 120\langle x-2\rangle^1 - 2\langle x-3\rangle^3 \text{ kN}\cdot\text{m} \blacktriangleleft$$

x	V	M
m	kN	kN · m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	–31.0	178.0
2.3	–31.0	170.3
2.5	–31.0	162.5
2.8	–31.0	154.8
3.0	–31.0	147.0
3.3	–31.4	139.2
3.5	–32.5	131.3
3.8	–34.4	122.9
4.0	–37.0	114.0
4.3	–40.4	104.3
4.5	–44.5	93.8
4.8	–49.4	82.0

x	V	M
m	kN	kN · m
5.0	–55.0	69.0
5.3	–61.4	54.5
5.5	–68.5	38.3
5.8	–76.4	20.2
6.0	–85.0	–0.0



PROBLEM 5.121

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad (5.2)(12) - 4B + (2)(4)(16) = 0$$

$$\boldsymbol{B} = 47.6 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (1.2)(12) - (2)(4)(16) + 4C = 0$$

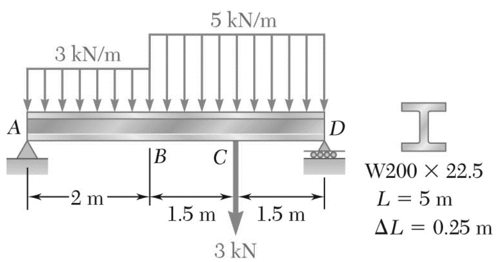
$$\boldsymbol{C} = 28.4 \text{ kN} \uparrow$$

$$w = 16\langle x - 1.2\rangle^0 = -\frac{dV}{dx}$$

$$V = -16\langle x-1.2\rangle^1 - 12 + 47.6\langle x-1.2\rangle^0 \blacktriangleleft$$

$$M = -8\langle x-1.2\rangle^2 - 12x + 47.6\langle x-1.2\rangle^1 \blacktriangleleft$$

x	V	M
m	kN	kN · m
0.0	−12.0	0.00
0.4	−12.0	−4.80
0.8	−12.0	−9.60
1.2	35.6	−14.40
1.6	29.2	−1.44
2.0	22.8	8.96
2.4	16.4	16.80
2.8	10.0	22.08
3.2	3.6	24.80
3.6	−2.8	24.96
4.0	−9.2	22.56
4.4	−15.6	17.60
4.8	−22.0	10.08
5.2	−28.4	−0.00



PROBLEM 5.122

For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x-axis at end A of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0:$$

$$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$$

$$R_A = 10.2 \text{ kN}$$

$$w = 3 + 2\langle x-2\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

(*a*) $$V = 10.2 - 3x - 2\langle x-2\rangle^1 - 3\langle x-3.5\rangle^0 \text{ kN}$$ ◀

(*b*) $$M = 10.2x - 1.5x^2 - \langle x-2\rangle^2 - 3\langle x-3.5\rangle^1 \text{ kN}\cdot\text{m}$$ ◀

For rolled steel section W200 × 22.5,

$$S = 193\times10^3 \text{ mm}^3 = 193\times10^{-6} \text{ m}^3$$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{16.164\times10^3}{193\times10^{-6}} = 83.8\times10^6 \text{ Pa}$$

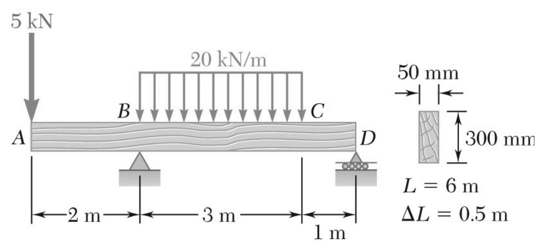
$$\sigma = 83.8 \text{ MPa}$$ ◀

x	V	M	σ
m	kN	kN · m	MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.73	24.5
0.75	7.95	6.81	35.3
1.00	7.20	8.70	45.1
1.25	6.45	10.41	53.9
1.50	5.70	11.93	61.8
1.75	4.95	13.26	68.7
2.00	4.20	14.40	74.6
2.25	2.95	15.29	79.2
2.50	1.70	15.88	82.3
2.75	0.45	16.14	83.6

PROBLEM 5.122 *(Continued)*

x	V	M	σ	
m	kN	kN · m	MPa	
3.00	−0.80	16.10	83.4	
3.25	−2.05	15.74	81.6	
3.50	−6.30	15.08	78.1	
3.75	−7.55	13.34	69.1	
4.00	−8.80	11.30	58.5	
4.25	−10.05	8.94	46.3	
4.50	−11.30	6.28	32.5	
4.75	−12.55	3.29	17.1	
5.00	−13.80	0.00	0.0	
2.83	0.05	16.164	83.8	
2.84	0.00	16.164	83.8	←
2.85	−0.05	16.164	83.8	

PROBLEM 5.123



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the *x*-axis at end *A* of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -4R_B + (6)(5) + (2.5)(3)(20) = 0 \qquad R_B = 45 \text{ kN}$$

$$w = 20\langle x-2\rangle^0 - 20\langle x-5\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = -5 + 45\langle x-2\rangle^0 - 20\langle x-2\rangle^1 + 20\langle x-5\rangle^1 \text{ kN} \quad \blacktriangleleft$$

$$M = -5x + 45\langle x-2\rangle^1 - 10\langle x-2\rangle^2 + 10\langle x-5\rangle^2 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

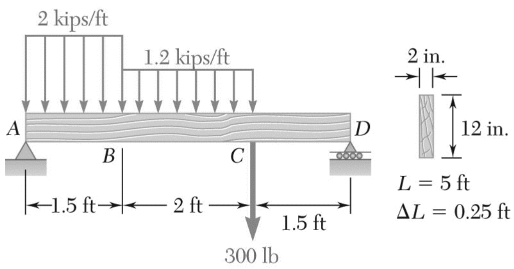
(*a*)

x	*V*	*M*	stress	
m	kN	kN · m	MPa	
0.00	–5	0.00	0.0	
0.50	–5	–2.50	–3.3	
1.00	–5	–5.00	–6.7	
1.50	–5	–7.50	–10.0	
2.00	40	–10.00	–13.3	
2.50	30	7.50	10.0	
3.00	20	20.00	26.7	
3.50	10	27.50	36.7	
4.00	0	30.00	40.0	←
4.50	–10	27.50	36.7	
5.00	–20	20.00	26.7	
5.50	–20	10.00	13.3	
6.00	–20	0.00	0.0	

(*b*) Maximum $|M| = 30 \text{ kN}\cdot\text{m}$ at $x = 4.0$ m

For rectangular cross section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(50)(300)^2 = 750\times10^3 \text{ mm}^3 = 750\times10^{-6} \text{ m}^3$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{30\times10^3}{750\times10^{-6}} = 40\times10^6 \text{ Pa} \qquad \sigma_{\max} = 40.0 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 5.124

For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$300 \text{ lb} = 0.3 \text{ kips}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$$

$$R_A = 3.84 \text{ kips}$$

$$w = 2 - 0.8\langle x - 1.5\rangle^0 - 1.2\langle x - 3.5\rangle^0 \text{ kip/ft}$$

$$V = 3.84 - 2x + 0.8\langle x - 1.5\rangle^1 + 1.2\langle x - 3.5\rangle^1 - 0.3\langle x - 3.5\rangle^0 \text{ kips} \quad ◀$$

$$M = 3.84x - x^2 + 0.4\langle x - 1.5\rangle^2 + 0.6\langle x - 3.5\rangle^2 - 0.3\langle x - 3.5\rangle^1 \text{ kip}\cdot\text{ft} \quad ◀$$

x	V	M	stress
ft	kips	kip · ft	ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	−0.06	3.80	0.951
2.50	−0.36	3.75	0.937
2.75	−0.66	3.62	0.906
3.00	−0.96	3.42	0.855
3.25	−1.26	3.14	0.786
3.50	−1.86	2.79	0.697
3.75	−1.86	2.32	0.581

PROBLEM 5.124 *(Continued)*

x	V	M	stress
ft	kips	kip · ft	ksi
4.00	–1.86	1.86	0.465
4.25	–1.86	1.39	0.349
4.50	–1.86	0.93	0.232
4.75	–1.86	0.46	0.116
5.00	–1.86	–0.00	–.000
2.10	0.12	3.80	0.949
2.20	0.00	3.80	0.951 ←
2.30	–0.12	3.80	0.949

Maximum $|M| = 3.804\text{ kip}\cdot\text{ft} = 45.648\text{ kip}\cdot\text{in}$ at $x = 2.20$ ft

Rectangular section:

$$2\text{ in.}\times 12\text{ in.}$$

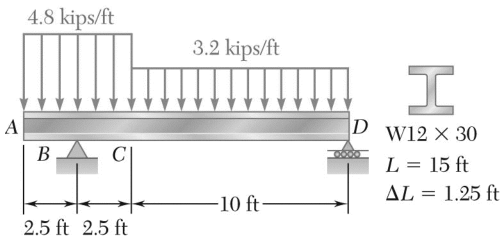
$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(2)(12)^2$$

$$= 48\text{ in}^3$$

$$\sigma = \frac{M}{S} = \frac{45.648}{48}$$

$\sigma = 0.951$ ksi ◀

PROBLEM 5.125



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -12.5 R_B + (12.5)(5.0)(4.8) + (5)(10)(3.2) = 0$$

$$R_B = 36.8 \text{ kips}$$

$$w = 4.8 - 1.6\langle x-5\rangle^0 \text{ kips/ft}$$

$$V = -4.8x + 36.8\langle x-2.5\rangle^0 + 1.6\langle x-5\rangle^1 \text{ kips} \quad ◀$$

$$M = -2.4x^2 + 36.8\langle x-2.5\rangle^1 + 0.8\langle x-5\rangle^2 \text{ kip}\cdot\text{ft} \quad ◀$$

x	V	M	stress
ft	kips	kip · ft	ksi
0.00	0.00	0.00	0.00
1.25	–6.0	–3.75	–1.17
2.50	24.8	–15.00	–4.66
3.75	18.8	12.25	3.81
5.00	12.8	32.00	9.95
6.25	8.8	45.50	14.15
7.50	4.8	54.00	16.79
8.75	0.8	57.50	17.88
10.00	–3.2	56.00	17.41
11.25	–7.2	49.50	15.39
12.50	–11.2	38.00	11.81
13.75	–15.2	21.50	6.68
15.00	–19.2	0.00	0.00
8.90	0.32	57.58	17.90
9.00	–0.00	57.60	17.91 ←
9.10	–0.32	57.58	17.90

PROBLEM 5.125 *(Continued)*

Maximum $|M| = 57.6 \text{ kip}\cdot\text{ft} = 691.2 \text{ kip}\cdot\text{in}$ at $x = 9.0 \text{ ft}$

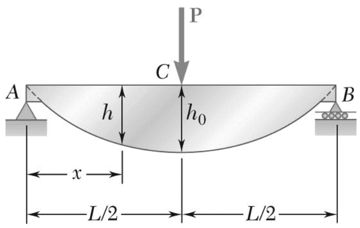
For rolled steel section $W12\times30$,

$$S = 38.6 \text{ in}^3$$

Maximum normal stress:

$$\sigma = \frac{M}{S} = \frac{691.2}{38.6}$$

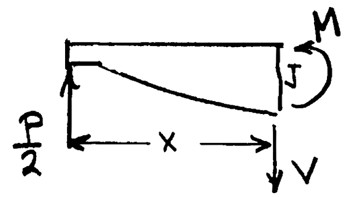
$\sigma = 17.91 \text{ ksi}$ ◀



PROBLEM 5.126

The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{\text{all}} = 72$ MPa.

SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{P}{2}\uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px} {2} \qquad \left(0 < x < \frac{L}{2}\right)$$

$$S = \frac{M}{\sigma_{\text{all}}} = \frac{Px}{2\sigma_{\text{all}}}$$

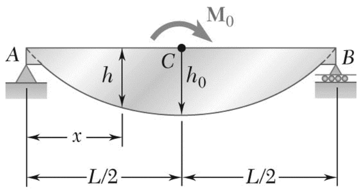
For a rectangular cross section, $\quad S = \dfrac{1}{6}bh^2$

Equating, $\quad \dfrac{1}{6}bh^2 = \dfrac{Px}{2\sigma_{\text{all}}} \qquad h = \sqrt{\dfrac{3Px}{\sigma_{\text{all}}b}}$

(a) At $x = \dfrac{L}{2}$, $\quad h = h_0 = \sqrt{\dfrac{3PL}{2\sigma_{\text{all}}b}} \qquad h = h_0\sqrt{\dfrac{2x}{L}}, \quad 0 < x < \dfrac{L}{2}$ ◀

For $x > \dfrac{L}{2}$, replace x by $L - x$.

(b) Solving for P, $\quad P = \dfrac{2\sigma_{\text{all}}bh_0^2}{3L} = \dfrac{(2)(72\times10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60\times10^3 \text{ N} \qquad P = 60 \text{ kN}$ ◀

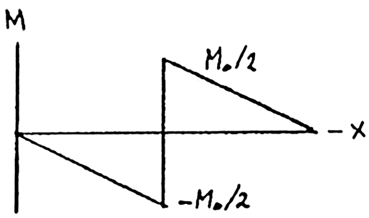


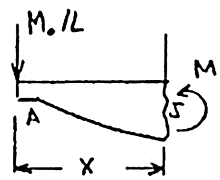
PROBLEM 5.127

The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{\text{all}} = 72$ MPa.

SOLUTION

$$\boldsymbol{A} = M_0/L \downarrow \qquad \boldsymbol{B} = M_0/L \uparrow$$





$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{M_0}{L}x + M = 0$$

$$M = -\frac{M_0 x}{L} \qquad \left(0 < x < \frac{L}{2}\right)$$

For $x > \dfrac{L}{2}$, $$M = \frac{M_0(L-x)}{L} \qquad \left(\frac{L}{2} < x < L\right)$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{M_0 x}{\sigma_{\text{all}} L} \qquad \text{for} \qquad \left(0 < x \frac{L}{2}\right)$$

For $x > \dfrac{L}{2}$, replace x by $L - x$.

For a rectangular cross section, $$S = \frac{1}{6}bh^2$$

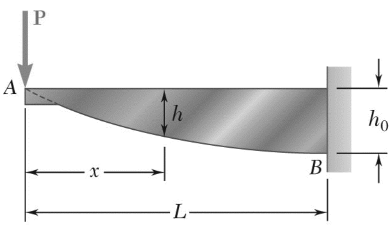
Equating, $$\frac{1}{6}bh^2 = \frac{M_0 x}{\sigma_{\text{all}} L} \qquad h = \sqrt{\frac{6M_0 x}{\sigma_{\text{all}} bL}}$$

(a) At $x = \dfrac{L}{2}$, $$h = h_0 = \sqrt{\frac{3M_0}{\sigma_{\text{all}} b}}$$

$$h = h_0\sqrt{2x/L} \blacktriangleleft$$

(b) Solving for M_0, $$M_0 = \frac{\sigma_{\text{all}} b h_0^2}{3} = \frac{(72 \times 10^6)(0.025)(0.200)^2}{3} = 24 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_0 = 24 \text{ kN} \cdot \text{m} \blacktriangleleft$$



PROBLEM 5.128

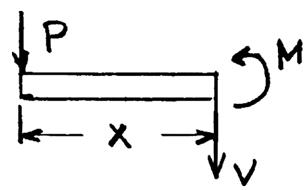
The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{\text{all}} = 24$ ksi.

SOLUTION

$$V = -P$$

$$M = -Px \qquad |M| = Px$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{P}{\sigma_{\text{all}}}x$$



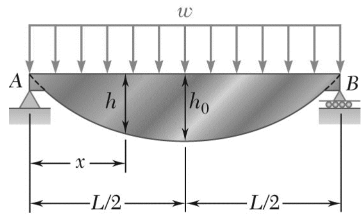
For a rectangular cross section, $$S = \frac{1}{6}bh^2$$

Equating, $$\frac{1}{6}bh^2 = \frac{Px}{\sigma_{\text{all}}} \qquad h = \left(\frac{6Px}{\sigma_{\text{all}}b}\right)^{1/2} \tag{1}$$

At $x = L$, $$h = h_0 = \left\{\frac{6PL}{\sigma_{\text{all}}b}\right\}^{1/2} \tag{2}$$

(a) Divide Eq. (1) by Eq. (2) and solve for h. $h = h_0(x/L)^{1/2}$ ◀

(b) Solving for P, $$P = \frac{\sigma_{\text{all}}bh_0^2}{6L} = \frac{(24)(1.25)(12)^2}{(6)(36)}$$ $P = 20.0$ kips ◀



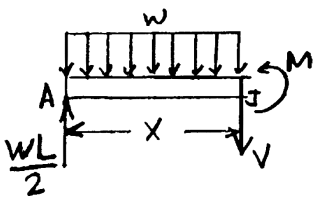
PROBLEM 5.129

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{\text{all}} = 24$ ksi.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0 \qquad R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_J = 0:$$



$$\frac{wL}{2}x - wx\frac{x}{2} + M = 0 \qquad M = \frac{w}{2}x(L-x)$$

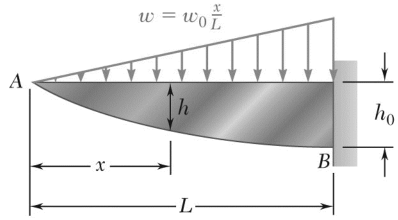
$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{wx(L-x)}{2\sigma_{\text{all}}}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{wx(L-x)}{2\sigma_{\text{all}}}$ $\qquad h = \left\{\frac{3wx(L-x)}{\sigma_{\text{all}}b}\right\}^{1/2}$

(a) At $x = \frac{L}{2}$, $\qquad h = h_0 = \left\{\frac{3wL^2}{4\sigma_{\text{all}}b}\right\}^{1/2}$ $\qquad h = h_0\left[\frac{x}{L}\left(1 - \frac{x}{L}\right)\right]^{1/2}$ ◀

(b) Solving for w, $\qquad w = \frac{4\sigma_{\text{all}}bh_0^2}{3L^2} = \frac{(4)(24)(1.25)(12)^2}{(3)(36)^2}$ $\qquad w = 4.44$ kip/in ◀



PROBLEM 5.130

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{\text{all}} = 200$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \qquad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{w_0 x^3}{6L\sigma_{\text{all}}}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{w_0 x^3}{6L\sigma_{\text{all}}} \qquad h = \sqrt{\frac{w_0 x^3}{\sigma_{\text{all}} bL}}$

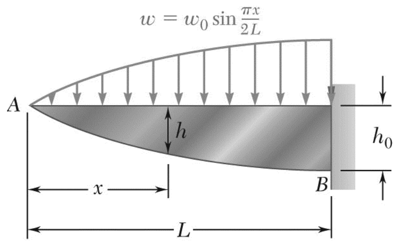
At $x = L$, $h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{\text{all}} b}}$

(*a*) $h = h_0 \left(\frac{x}{L}\right)^{3/2}$ ◀

Data: $L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$w_0 = 300 \text{ kN/m} = 300 \times 10^3 \text{ N/m}, \quad \sigma_{\text{all}} = 200 \text{ MPa} = 200 \times 10^6 \text{ Pa}$

(*b*) $h_0 = \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 167.7 \times 10^{-3} \text{ m}$ $\qquad h_0 = 167.7 \text{ mm}$ ◀



PROBLEM 5.131

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{\text{all}} = 200$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{2L}$$

$$V = \frac{2w_0 L}{\pi} \cos \frac{\pi x}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \rightarrow C_1 = \frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi}\left(1 - \cos \frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right) \qquad |M| = \frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{L}\right)$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{2w_0 L}{\pi \sigma_{\text{all}}}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

For a rectangular cross section, $\quad S = \frac{1}{6} b h^2$

Equating, $\quad \frac{1}{6} b h^2 = \frac{2w_0 L}{\pi \sigma_{\text{all}}}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$

$$h = \left\{ \frac{12 w_0 L}{\pi \sigma_{\text{all}} b}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)\right\}^{1/2}$$

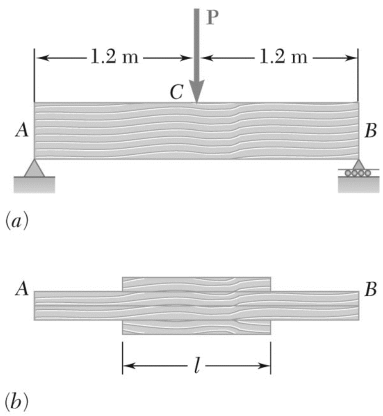
At $x = L$, $\quad h = h_0 = \left\{ \frac{12 w_0 L^2}{\pi \sigma_{\text{all}} b}\left(1 - \frac{2}{\pi}\right)\right\}^{1/2} = 1.178 \sqrt{\frac{w_0 L^2}{\sigma_{\text{all}} b}}$

(a) $\quad h = h_0 \left[\left(\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L}\right) \Big/ \left(1 - \frac{2}{\pi}\right)\right]^{1/2} \qquad h = 1.659\, h_0 \left[\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L}\right]^{1/2}$ ◀

Data: $\quad L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$$w_0 = 300 \text{ kN/m} = 300 \times 10^3 \text{ N/m}, \quad \sigma_{\text{all}} = 200 \text{ MPa} = 200 \times 10^6 \text{ Pa}$$

(b) $\quad h_0 = 1.178 \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 197.6 \times 10^{-3} \text{ m} \qquad h_0 = 197.6 \text{ mm}$ ◀



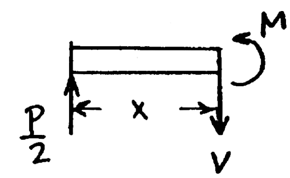
PROBLEM 5.132

A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50×50-mm cross section. Determine the length l of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$

$$0 < x < \frac{1}{2}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \quad \text{or} \quad M = \frac{M_{\max} x}{1.2}$$

Bending moment diagram is two straight lines.

At C, $\qquad S_C = \frac{1}{6} b h_C^2 \qquad M_C = M_{\max}$

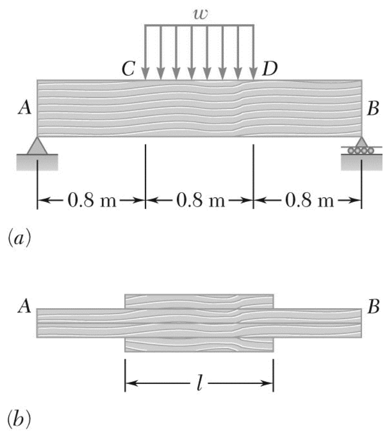
Let D be the point where the thickness changes.

At D, $\qquad S_D = \frac{1}{6} b h_D^2 \qquad M_D = \frac{M_{\max} x_D}{1.2}$

$$\frac{S_D}{S_C} = \frac{h_D^2}{h_C^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_C} = \frac{x_D}{1.2} \qquad x_D = 0.3 \text{ m}$$

$$\frac{l}{2} = 1.2 - x_D = 0.9$$

$l = 1.800$ m ◀



PROBLEM 5.133

A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part a of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part b of the figure, four pieces of the same timber as the original beam and of 50×50-mm cross section. Determine the length l of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{0.8\text{ N}}{2} = 0.4\,w$$

Shear:

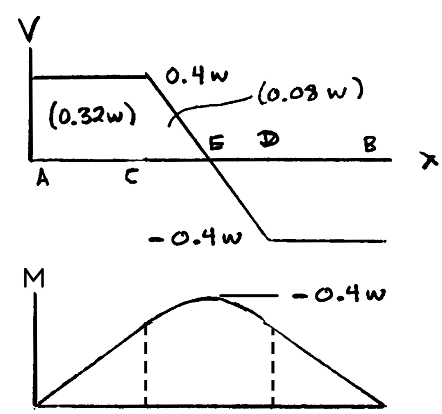
A to C: $\quad V = 0.4\,w$

D to B: $\quad V = -0.4\,w$

Areas:

A to C: $\quad (0.8)(0.4)\,w = 0.32\,w$

C to E: $\quad \left(\frac{1}{2}\right)(0.4)(0.4)\,w = 0.08\,w$



Bending moments:

At C, $\quad M_C = 0.40\,w$

A to C: $\quad M = 0.40\,wx$

At C, $\quad S_C = \frac{1}{6}bh_C^2 \qquad M_C = M_{\max} = 0.40\,w$

Let F be the point were the thickness changes.

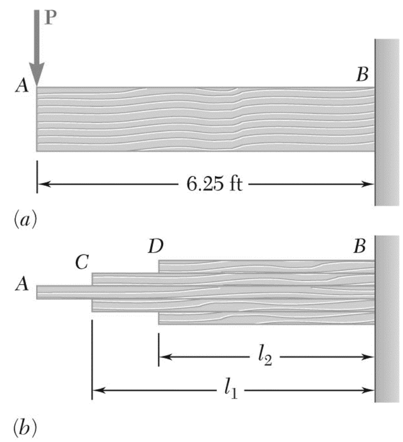
At F, $\quad S_F = \frac{1}{6}bh_F^2 \qquad M_F = 0.40\,wx_F$

$$\frac{S_F}{S_C} = \frac{h_F^2}{h_C^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_C} = \frac{0.40\,wx_F}{0.40\,w}$$

$$x_F = 0.25\text{ m} \qquad \frac{l}{2} = 1.2 - x_F = 0.95\text{ m}$$

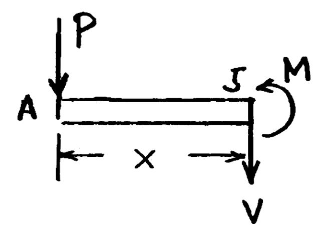
$l = 1.900$ m ◀

PROBLEM 5.134



A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2×2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



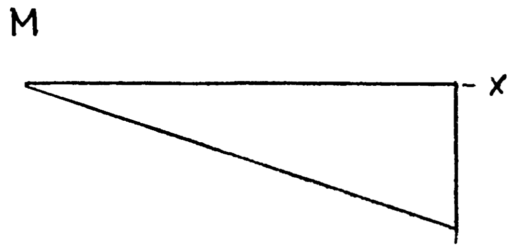
$$+\circlearrowleft \Sigma M_J = 0: \quad Px + M = 0 \qquad M = -Px$$

$$|M| = Px$$

At B, $\quad |M|_B = M_{\max}$

At C, $\quad |M|_C = M_{\max} x_C/6.25$

At D, $\quad |M|_D = M_{\max} x_D/6.25$



$$S_B = \frac{1}{6}bh^2 = \frac{1}{6}\cdot b(5b)^2 = \frac{25}{6}b^3$$

A to C: $\quad S_C = \frac{1}{6}\cdot b(b)^2 = \frac{1}{6}b^3$

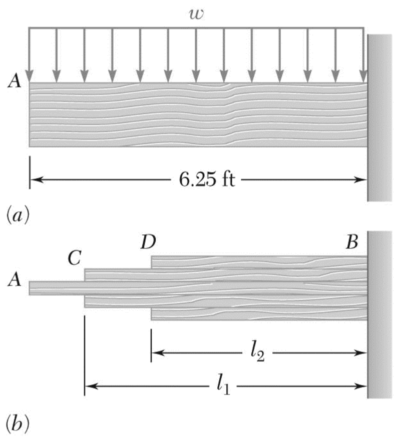
C to D: $\quad S_D = \frac{1}{6}b\,(3b)^2 = \frac{9}{6}b^3$

$$\frac{|M|_C}{|M|_B} = \frac{x_C}{6.25} = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$$

$$l_1 = 6.25 - 0.25 \qquad l_1 = 6.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \frac{x_D}{6.25} = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$$

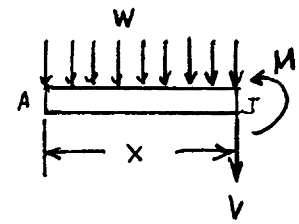
$$l_2 = 6.25 - 2.25 \qquad l_2 = 4.00 \text{ ft} \blacktriangleleft$$



PROBLEM 5.135

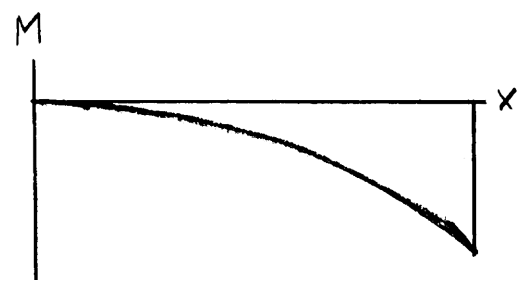
A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2×2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



$$+\circlearrowleft \Sigma M_J = 0: \quad wx\frac{x}{2} + M = 0$$

$$M = -\frac{wx^2}{2} \qquad |M| = \frac{wx^2}{2}$$



At B, $\quad |M|_B = |M|_{\max}$

At C, $\quad |M|_C = |M|_{\max}\ (x_C/6.25)^2$

At D, $\quad |M|_D = |M|_{\max}\ (x_D/6.25)^2$

At B, $\quad S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to C: $\quad S_C = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

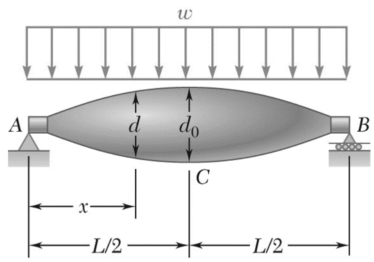
C to D: $\quad S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

$$\frac{|M|_C}{|M|_B} = \left(\frac{x_C}{6.25}\right)^2 = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$$

$$l_1 = 6.25 - 1.25 \text{ ft} \qquad l_1 = 5.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{6.25\sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$$

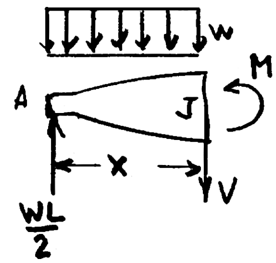
$$l_2 = 6.25 - 3.75 \text{ ft} \qquad l_2 = 2.50 \text{ ft} \blacktriangleleft$$



PROBLEM 5.136

A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

SOLUTION



$$R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_J = 0: \; -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

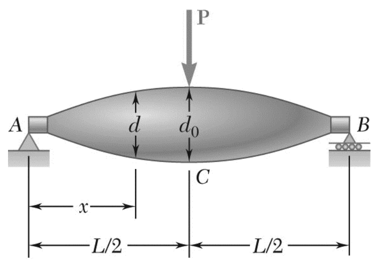
$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{wx(L-x)}{2\sigma_{\text{all}}}$$

For a solid circular cross section, $c = \frac{d}{2} \quad I = \frac{\pi}{4}c^3 \quad S = \frac{I}{c} = \frac{\pi d^3}{32}$

Equating, $\frac{\pi d^3}{32} = \frac{wx(L-x)}{2\sigma_{\text{all}}} \qquad d = \left\{\frac{16\,wx(L-x)}{\pi\sigma_{\text{all}}}\right\}^{1/3}$

At $x = \frac{L}{2}$, $d = d_0 = \left\{\frac{4wL^2}{\pi\sigma_{\text{all}}}\right\}^{1/3}$ $\qquad d = d_0\left\{4\frac{x}{L}\left(1-\frac{x}{L}\right)\right\}^{1/3}$ ◀

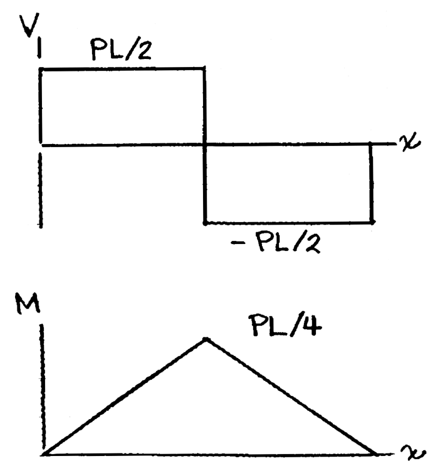


PROBLEM 5.137

A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

SOLUTION

Draw shear and bending moment diagrams.



$$0 \le x \le \frac{L}{2}, \qquad M = \frac{Px}{2}$$

$$\frac{L}{2} \le x \le L, \qquad M = \frac{P(L-x)}{2}$$

For a solid circular section, $c = \frac{1}{2}d$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4 \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3$$

For constant strength design, $\sigma = \text{constant}$. $\qquad S = \dfrac{M}{\sigma}$

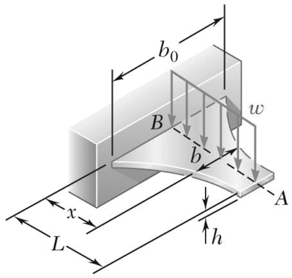
For $\quad 0 \le x \le \frac{L}{2}, \quad \frac{\pi}{32}d^3 = \frac{Px}{2} \qquad$ (1a)

For $\quad \frac{L}{2} \le x \le L, \quad \frac{\pi}{32}d^3 = \frac{P(L-x)}{2} \qquad$ (1b)

At point C, $\quad \frac{\pi}{32}d_0^3 = \frac{PL}{4} \qquad$ (2)

Dividing Eq. (1a) by Eq. (2), $\quad 0 \le x \le \frac{L}{2}, \quad \frac{d^3}{d_0^3} = \frac{2x}{L} \qquad d = d_0(2x/L)^{1/3}$ ◀

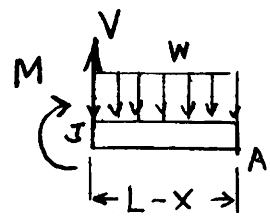
Dividing Eq. (1b) by Eq. (2), $\quad \frac{L}{2} \le x \le L, \quad \frac{d^3}{d_0^3} = \frac{2(L-x)}{L} \qquad d = d_0[2(L-x)/L]^{1/3}$ ◀



PROBLEM 5.138

A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support the distributed load w along its centerline AB. (a) Knowing that the beam is to be of constant strength, express b in terms of x, L, and b_0. (b) Determine the maximum allowable value of w if $L = 15$ in., $b_0 = 8$ in., $h = 0.75$ in., and $\sigma_{\text{all}} = 24$ ksi.

SOLUTION



$$+\circlearrowleft \Sigma M_J = 0: \quad -M - w(L-x)\frac{L-x}{2} = 0$$

$$M = -\frac{w(L-x)^2}{2} \qquad |M| = \frac{w(L-x)^2}{2}$$

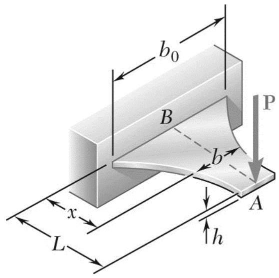
$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{w(L-x)^2}{2\sigma_{\text{all}}}$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2$

$$\frac{1}{6}bh^2 = \frac{w(L-x)^2}{2\sigma_{\text{all}}} \qquad b = \frac{3w(L-x)^2}{\sigma_{\text{all}}h^2}$$

(a) At $\quad x = 0, \quad b = b_0 = \dfrac{3wL^2}{\sigma_{\text{all}}h^2}$ $\qquad b = b_0\left(1 - \dfrac{x}{L}\right)^2$ ◀

(b) Solving for w, $\quad w = \dfrac{\sigma_{\text{all}}b_0h^2}{3L^2} = \dfrac{(24)(8)(0.75)^2}{(3)(15)^2} = 0.160$ kip/in $\qquad w = 160.0$ lb/in ◀



PROBLEM 5.139

A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support the concentrated load **P** at point A. (a) Knowing that the beam is to be of constant strength, express b in terms of x, L, and b_0. (b) Determine the smallest allowable value of h if $L = 300$ mm, $b_0 = 375$ mm, $P = 14.4$ kN, and $\sigma_{\text{all}} = 160$ MPa.

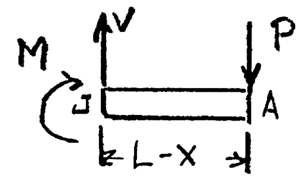
SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0$$

$$M = -P(L-x)$$

$$|M| = P(L-x)$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{P(L-x)}{\sigma_{\text{all}}}$$



For a rectangular cross section, $\quad S = \frac{1}{6}bh^2$

Equating, $\quad \frac{1}{6}bh^2 = \frac{P(L-x)}{\sigma_{\text{all}}} \qquad b = \frac{6P(L-x)}{\sigma_{\text{all}}h^2}$

(a) At $x = 0$, $\quad b = b_0 = \frac{6PL}{\sigma_{\text{all}}h^2}$ $\qquad b = b_0\left(1 - \frac{x}{L}\right)$ ◀

Solving for h, $\quad h = \sqrt{\frac{6PL}{\sigma_{\text{all}}b_0}}$

Data: $\quad L = 300 \text{ mm} = 0.300 \text{ m}, \quad b_0 = 375 \text{ mm} = 0.375 \text{ m}$

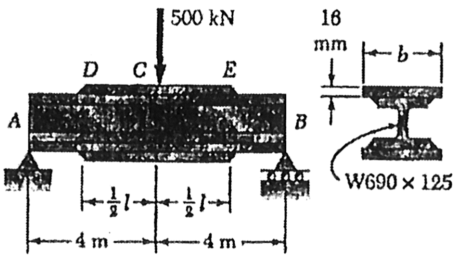
$P = 14.4 \text{ kN} = 14.4 \times 10^3 \text{ N} \cdot \text{m}, \quad \sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

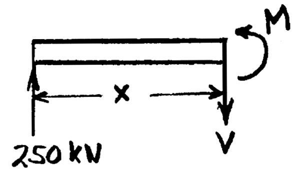
(b) $\quad h = \sqrt{\frac{(6)(14.4\times 10^3)(0.300)}{(160\times 10^6)(0.375)}} = 20.8 \times 10^{-3} \text{ m}$ $\qquad h = 20.8$ mm ◀

PROBLEM 5.140

Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4$ m and $b = 285$ mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION





$$A = B = 250 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -250x + M = 0$$

$$M = 250x \text{ kN} \cdot \text{m}$$

At center of beam, $\quad x = 4 \text{ m} \quad M_C = (250)(4) = 1000 \text{ kN} \cdot \text{m}$

At *D*, $\quad x = \frac{1}{2}(8 - l) = \frac{1}{2}(8 - 4) = 2 \text{ m} \qquad M_0 = 500 \text{ kN} \cdot \text{m}$

At center of beam, $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 1190 \times 10^6 + 2\left\{(285)(16)\left(\frac{678}{2} + \frac{16}{2}\right)^2 + \frac{1}{12}(285)(16)^3\right\}$$

$$= 2288 \times 10^6 \text{ mm}^4$$

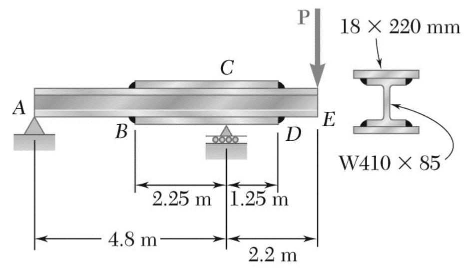
$$c = \frac{678}{2} + 16 = 355 \text{ mm} \qquad S = \frac{I}{c} = 6445 \times 10^3 \text{mm}^3$$

$$= 6445 \times 10^{-6} \text{m}^3$$

(*a*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6 \text{Pa} \qquad \sigma = 155.2 \text{ MPa} \blacktriangleleft$

At *D*, $\quad S = 3490 \times 10^3 \text{mm}^3 = 3510 \times 10^{-6} \text{m}^3$

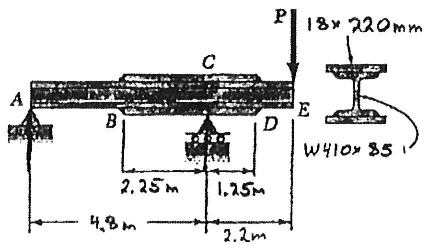
(*b*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{500 \times 10^3}{3490 \times 10^{-6}} = 143.3 \times 10^6 \text{ Pa} \qquad \sigma = 143.3 \text{ MPa} \blacktriangleleft$



PROBLEM 5.141

Knowing that $\sigma_{\text{all}} = 150$ MPa, determine the largest concentrated load **P** that can be applied at end E of the beam shown.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad -4.8A - 2.2P = 0$$

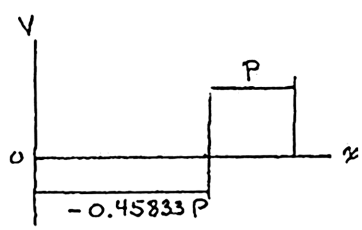
$$A = -0.45833P \qquad \boldsymbol{A} = 0.45833P \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 4.8D - 7.0P = 0$$

$$\boldsymbol{D} = 1.45833P \uparrow$$

Shear: A to C: $V = -0.45833P$

C to E: $V = P$



Bending moments: $M_A = 0$

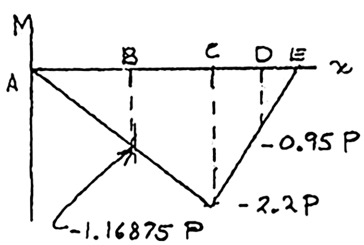
$$M_C = 0 + (4.8)(-0.45833P) = -2.2P$$

$$M_E = -2.2P + 2.2P = 0$$

$$M_B = \left(\frac{4.8 - 2.25}{48}\right)(-2.2P) = -1.16875P$$

$$M_D = \left(\frac{2.2 - 1.25}{2.2}\right)(-2.2P) = -0.95P$$

$$|M_D| < |M_B|$$



For W410 × 85, $S = 1510 \times 10^3 \text{mm}^3 = 1510 \times 10^{-6} \text{m}^3$

Allowable value of P based on strength at B. $\sigma = \dfrac{|M_B|}{S}$

$$150 \times 10^6 = \frac{1.16875P}{1510 \times 10^{-6}} \qquad P = 193.8 \times 10^3 \text{N}$$

PROBLEM 5.141 *(Continued)*

Section properties over portion *BCD*:

$$\text{W410} \times 85: \quad d = 417 \text{ mm}, \quad \frac{1}{2}d = 208.5 \text{ mm}, \quad I_x = 316 \times 10^6 \text{mm}^4$$

Plate: $$A = (18)(220) = 3960 \text{ mm}^2 \qquad d = 208.5 + \left(\frac{1}{2}\right)(18) = 217.5 \text{ mm}$$

$$\bar{I} = \frac{1}{12}(220)(18)^3 = 106.92 \times 10^3 \text{ mm}^4 \qquad Ad^2 = 187.333 \times 10^6 \text{mm}^4$$

$$I_x = \bar{I} + Ad^2 = 187.440 \times 10^6 \text{ mm}^4$$

For section, $$I = 316 \times 10^6 + (2)(187.440 \times 10^6) = 690.88 \times 10^6 \text{ mm}^4$$

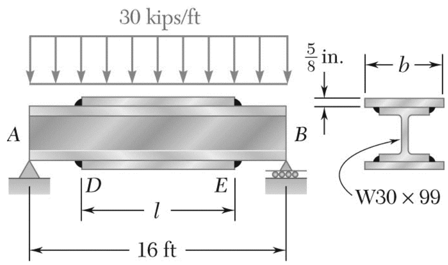
$$c = 208.5 + 18 = 226.5 \text{ mm}$$

$$S = \frac{I}{c} = \frac{690.88 \times 10^6}{226.5} = 3050.2 \times 10^3 \text{mm}^3 = 3050.2 \times 10^{-6} \text{ m}^3$$

Allowable load based on strength at *C*: $$\sigma = \frac{|M_C|}{S}$$

$$150 \times 10^6 = \frac{2.2P}{3050.2 \times 10^{-6}} \qquad P = 208.0 \times 10^3 \text{ N}$$

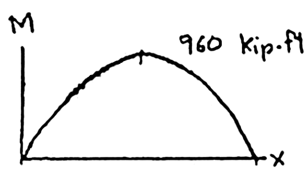
The smaller allowable load controls. $P = 193.8 \times 10^3 \text{N}$ $P = 193.8 \text{ kN}$ ◀



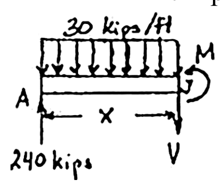
PROBLEM 5.142

Two cover plates, each $\frac{5}{8}$ in. thick, are welded to a W30×99 beam as shown. Knowing that $l = 9$ ft and $b = 12$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION



$\boldsymbol{A} = \boldsymbol{B} = 240 \text{ kips} \uparrow$



$$+\circlearrowleft \Sigma M_J = 0: \; -240x + 30x\frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip} \cdot \text{ft}$$

At center of beam, $\quad x = 8 \text{ ft}$

$$M_C = 960 \text{ kip} \cdot \text{ft} = 11{,}520 \text{ kip} \cdot \text{in}$$

At point *D*, $\quad x = \frac{1}{2}(16 - 9) = 3.5 \text{ ft}$

$$M_D = 656.25 \text{ kip} \cdot \text{ft} = 7875 \text{ kip} \cdot \text{in}$$

At center of beam, $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

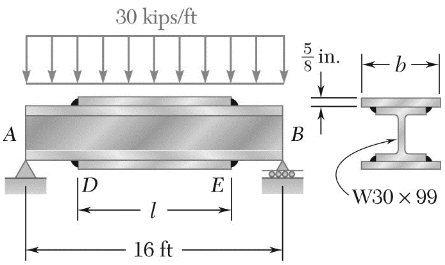
$$I = 3990 + 2\left\{(12)(0.625)\left(\frac{29.7}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(12)(0.625)^3\right\} = 7439 \text{ in}^4$$

$$c = \frac{29.7}{2} + 0.625 = 15.475 \text{ in.}$$

(*a*) Normal stress: $\quad \sigma = \frac{Mc}{I} = \frac{(11{,}520)(15.475)}{7439} \qquad \sigma = 24.0 \text{ ksi}$ ◀

At point *D*, $\quad S = 269 \text{ in}^3$

(*b*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{7875}{269} \qquad \sigma = 29.3 \text{ ksi}$ ◀

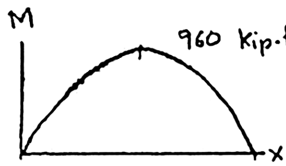


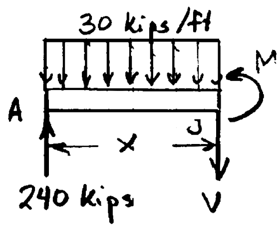
PROBLEM 5.143

Two cover plates, each $\frac{5}{8}$ in. thick, are welded to a W30×99 beam as shown. Knowing that $\sigma_{\text{all}} = 22$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 240 \text{ kips} \uparrow$$





$$+\circlearrowleft \Sigma M_J = 0: \quad -240x + 30x\frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip} \cdot \text{ft}$$

For W30 × 99 rolled steel section, $S = 269 \text{ in}^3$

Allowable bending moment:

$$M_{\text{all}} = \sigma_{\text{all}} S = (22)(269) = 5918 \text{ kip} \cdot \text{in} = 493.167 \text{ kip} \cdot \text{ft}$$

To locate points *D* and *E*, set $M = M_{\text{all}}$.

$$240x - 15x^2 = 493.167 \qquad 15x^2 - 240x + 493.167 = 0$$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft}, \quad 13.58 \text{ ft}$$

(*a*) $\quad l = x_E - x_D = 13.58 - 2.42 \qquad l = 11.16 \text{ ft}$ ◀

Center of beam: $\quad M = 960 \text{ kip} \cdot \text{ft} = 11520 \text{ kip} \cdot \text{in}$

$$S = \frac{M}{\sigma_{\text{all}}} = \frac{11520}{22} = 523.64 \text{ in}^3 \qquad c = \frac{29.7}{2} + 0.625 = 15.475 \text{ in.}$$

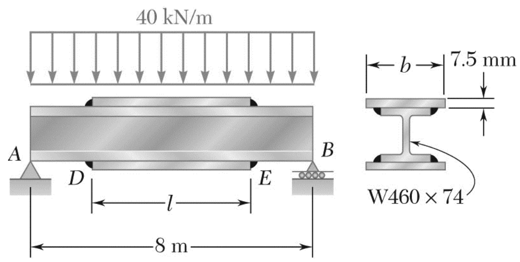
Required moment of inertia: $\quad I = Sc = 8103.3 \text{ in}^4$

But $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$8103.3 = 3990 + 2\left\{(b)(0.625)\left(\frac{29.7}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(b)(0.625)^3\right\}$$

$$= 3990 + 287.42b$$

(*b*) $\quad b = 14.31 \text{ in.}$ ◀

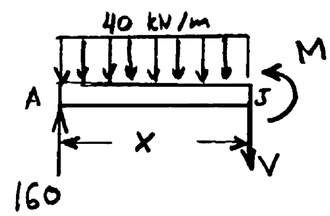


PROBLEM 5.144

Two cover plates, each 7.5 mm thick, are welded to a $W460\times 74$ beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 160 \text{ kN}\uparrow$$



$$+\circlearrowleft \Sigma M_J = 0:\quad -160x + (40x)\frac{x}{2} + M = 0$$

$$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$$

At center of beam, $\quad x = 4\text{m} \qquad M_C = 320 \text{ kN}\cdot\text{m}$

At *D*, $\quad x = \frac{1}{2}(8 - l) = 1.5 \text{ m} \qquad M_D = 195 \text{ kN}\cdot\text{m}$

At center of beam, $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 333\times 10^6 + 2\left\{(200)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(200)(7.5)^3\right\}$$

$$= 494.8\times 10^6 \text{mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$$

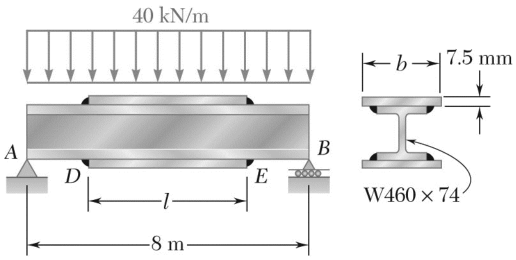
$$S = \frac{I}{c} = 2097\times 10^3 \text{mm}^3 = 2097\times 10^{-6} \text{m}^3$$

(*a*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{320\times 10^3}{2097\times 10^{-6}} = 152.6\times 10^6 \text{Pa}$

$\sigma = 152.6$ MPa ◀

At *D*, $\quad S = 1460\times 10^3 \text{mm}^3 = 1460\times 10^{-6} \text{m}^3$

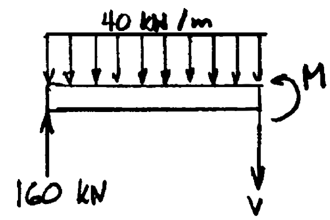
(*b*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{195\times 10^3}{1460\times 10^{-6}} = 133.6\times 10^6 \text{Pa} \qquad \sigma = 133.6$ MPa ◀



PROBLEM 5.145

Two cover plates, each 7.5 mm thick, are welded to a W460×74 beam as shown. Knowing that $\sigma_{\text{all}} = 150$ MPa for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

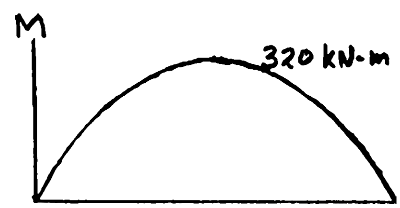
SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = 160 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -160x + (40x)\left(\frac{x}{2}\right) + M = 0$$

$$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$$



For W460×74 rolled steel beam,

$$S = 1460\times10^3 \text{ mm}^3 = 1460\times10^{-6} \text{ m}^3$$

Allowable bending moment:

$$M_{\text{all}} = \sigma_{\text{all}}\, S = (150\times10^6)(1460\times10^{-6})$$
$$= 219\times10^3 \text{ N}\cdot\text{m} = 219 \text{ kN}\cdot\text{m}$$

To locate points *D* and *E*, set $\quad M = M_{\text{all}}$

$$160x - 20x^2 = 219 \qquad 20x^2 - 160x + 219 = 0$$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} \qquad x = 1.753 \text{ m and } x = 6.247 \text{ m}$$

(*a*) $\quad x_D = 1.753$ ft $\quad x_E = 6.247$ ft $\qquad l = x_E - x_D = 4.49$ m ◀

At center of beam, $\quad M = 320 \text{ kN}\cdot\text{m} = 320\times10^3 \text{N}\cdot\text{m} \quad c = \dfrac{457}{2} + 7.5 = 236 \text{ mm}^4$

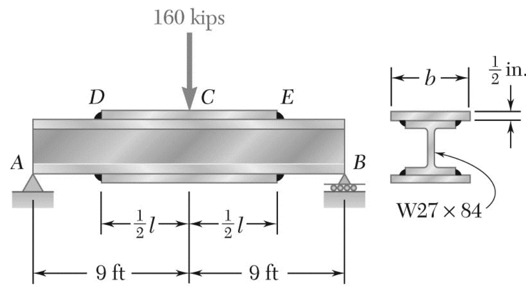
$$S = \frac{M}{\sigma_{\text{all}}} = \frac{320\times10^3}{150\times10^6} = 2133\times10^{-6} \text{ m}^3 = 2133\times10^3 \text{ mm}^3$$

Required moment of inertia: $\quad I = Sc = 503.4\times10^6 \text{ mm}^4$

But $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$503.4\times10^6 = 333\times10^6 + 2\left\{(b)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(b)(7.5)^3\right\}$$

(*b*) $\qquad = 333\times10^6 + 809.2\times10^3 b \qquad b = 211$ mm ◀

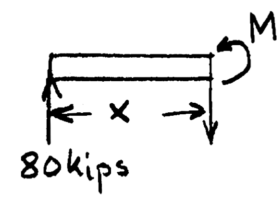


PROBLEM 5.146

Two cover plates, each $\frac{1}{2}$ in. thick, are welded to a W27×84 beam as shown. Knowing that $l = 10$ ft and $b = 10.5$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 80 \text{ kips} \uparrow$$



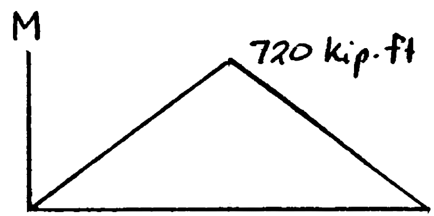
$$+\circlearrowleft \Sigma M_J = 0: \quad -80x + M = 0$$

$$M = 80x \text{ kip} \cdot \text{ft}$$

At *C*, $\quad x = 9 \text{ ft} \quad M_C = 720 \text{ kip} \cdot \text{ft} = 8640 \text{ kip} \cdot \text{in}$

At *D*, $\quad x = 9 - 5 = 4 \text{ ft}$

$$M_D = (80)(4) = 320 \text{ kip} \cdot \text{ft} = 3840 \text{ kip} \cdot \text{in}$$



At center of beam, $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$I = 2850 + 2\left\{(10.5)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(10.5)(0.500)^3\right\}$$

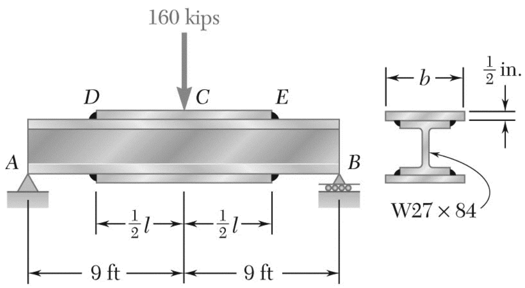
$$= 4794 \text{ in}^3$$

$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in.}$$

(*a*) Normal stress: $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(8640)(13.855)}{4794} \qquad \sigma = 25.0 \text{ ksi} \blacktriangleleft$

At point *D*, $\quad S = 213 \text{ in}^3$

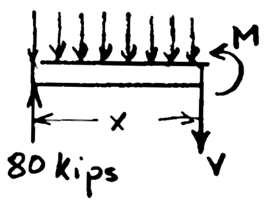
(*b*) Normal stress: $\quad \sigma = \dfrac{M}{S} = \dfrac{3840}{213} \qquad \sigma = 18.03 \text{ ksi} \blacktriangleleft$



PROBLEM 5.147

Two cover plates, each $\frac{1}{2}$ in. thick, are welded to a W27×84 beam as shown. Knowing that $\sigma_{\text{all}} = 24$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

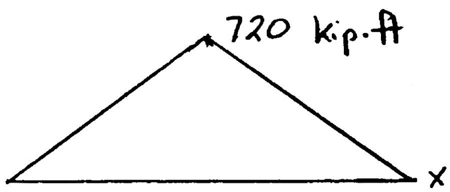
SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = 80 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -80x + M = 0$$

$$M = 80x \text{ kip} \cdot \text{ft}$$



At D, $\quad S = 213 \text{ in}^3$

Allowable bending moment:

$$M_{\text{all}} = \sigma_{\text{all}} S = (24)(213) = 5112 \text{ kip} \cdot \text{in}$$

$$= 426 \text{ kip} \cdot \text{ft}$$

Set $M_D = M_{\text{all}}$. $\quad 80x_D = 426 \quad x_D = 5.325$ ft

(*a*) $\quad l = 18 - 2x_D \quad$ $l = 7.35$ ft ◀

At center of beam, $\quad M = (80)(9) = 720 \text{ kip} \cdot \text{ft} = 8640 \text{ kip} \cdot \text{in}$

$$S = \frac{M}{\sigma_{\text{all}}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$c = \frac{26.7}{2} + 0.500 = 13.85 \text{ in.}$$

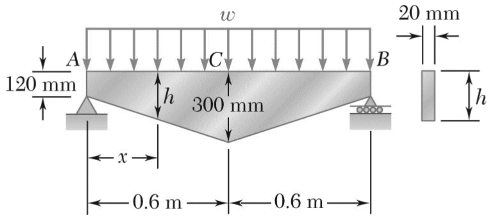
Required moment of inertia: $\quad I = Sc = 4986 \text{ in}^4$

But $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$4986 = 2850 + 2\left\{(b)(0.500)\left(\frac{26.7}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(b)(0.500)^3\right\}$$

$$= 2850 + 184.981b$$

(*b*) $\quad b = 11.55$ in. ◀

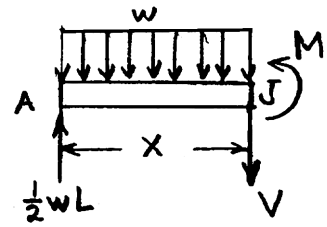


PROBLEM 5.148

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{\text{all}} = 140$ MPa.

SOLUTION

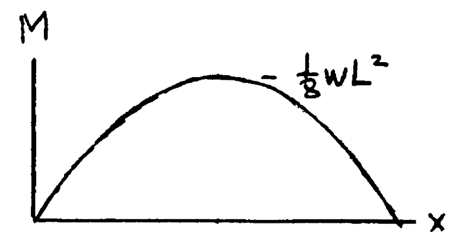
$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2} wL \uparrow \qquad L = 1.2 \text{ m}$$



$$+\circlearrowleft \Sigma M_J = 0: \; -\frac{1}{2} wL + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



For the tapered beam, $\quad h = a + kx$

$$a = 120 \text{ mm}$$

$$k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For rectangular cross section, $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress: $\quad \sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0.$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a + kL)x}{(a + kx)^3}\right\} = 0$$

PROBLEM 5.148 *(Continued)*

(*a*) $x_m = \dfrac{aL}{2a + kL} = \dfrac{(120)(1.2)}{(2)(120) + (300)(1.2)}$ $\qquad x_m = 0.24 \text{ m}$ ◀

$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$

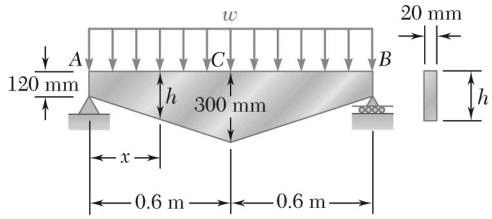
$S_m = \dfrac{1}{6}bh_m^2 = \dfrac{1}{6}(20)(192)^2 = 122.88\times10^3 \text{ mm}^3 = 122.88\times10^{-6} \text{ m}^3$

Allowable value of M_m: $M_m = S_m\sigma_{\text{all}} = (122.88\times10^{-6})(140\times10^6)$

$= 17.2032\times10^3 \text{ N}\cdot\text{m}$

(*b*) Allowable value of *w*: $w = \dfrac{2M_m}{x_m(L - x_m)} = \dfrac{(2)(17.2032\times10^3)}{(0.24)(0.96)}$

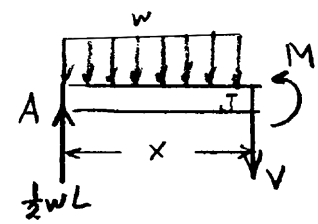
$= 149.3\times10^3 \text{ N/m}$ $\qquad w = 149.3 \text{ kN/m}$ ◀



PROBLEM 5.149

For the tapered beam shown, knowing that w = 160 kN/m, determine (a) the transverse section in which the maximum normal stress occurs, (b) the corresponding value of the normal stress.

SOLUTION

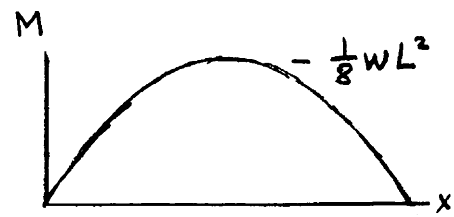


$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}\ wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



where w = 160 kN/m and L = 1.2 m.

For the tapered beam, $h = a + kx$

$$a = 120 \text{ mm}$$

$$k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For a rectangular cross section, $S = \dfrac{1}{6}bh^2 = \dfrac{1}{6}b(a + kx)^2$

Bending stress: $\sigma = \dfrac{M}{S} = \dfrac{3w}{b}\dfrac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress, set $\dfrac{d\sigma}{dx} = 0.$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - 2ax + kLx}{(a + kx)^3}\right\} = 0$$

PROBLEM 5.149 *(Continued)*

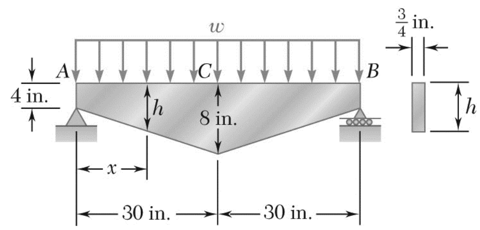
(*a*) $x_m = \dfrac{aL}{2a+kL} = \dfrac{(120)(1.2)}{(2)(120)+(300)(1.2)}$ $\qquad x_m = 0.240 \text{ m}$ ◀

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

$$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88\times 10^3 \text{ mm}^3 = 122.88\times 10^{-6} \text{ m}^3$$

$$M_m = \frac{w}{2}x_m(L-x_m) = \frac{160\times 10^3}{2}(0.24)(0.96) = 18.432\times 10^3 \text{ N}\cdot\text{m}$$

(*b*) Maximum bending stress: $\sigma_m = \dfrac{M_m}{S_m} = \dfrac{18.432\times 10^3}{122.88\times 10^{-6}} = 150\times 10^6 \text{ Pa}$ $\qquad \sigma_m = 150.0 \text{ MPa}$ ◀

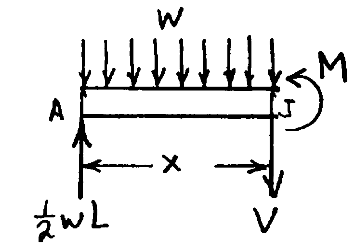


PROBLEM 5.150

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{\text{all}} = 24$ ksi.

SOLUTION

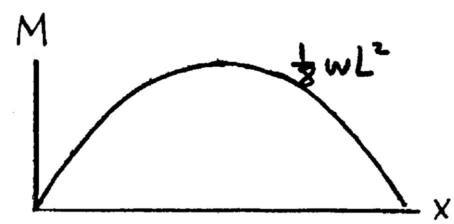
$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow \quad L = 60 \text{ in.}$$



$$+\circlearrowleft \Sigma M_J = 0: \ -\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



For the tapered beam, $\quad h = a + kx$

$$a = 4 \text{ in.} \quad k = \frac{8-4}{30} = \frac{2}{15} \text{ in./in.}$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\quad \sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0.$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx - x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx - x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a + kL)x}{(a+kx)^3}\right\} = 0$$

PROBLEM 5.150 *(Continued)*

(*a*)

$$x_m = \frac{aL}{2a+kL} = \frac{(4)(60)}{(2)(4)+\left(\frac{2}{15}\right)(6.0)}$$

$x_m = 15$ in. ◀

$$h_m = a + kx_m = 4 + \left(\frac{2}{15}\right)(15) = 6.00 \text{ in.}$$

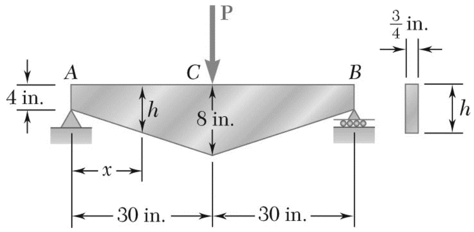
$$S_m = \frac{1}{6}bh_m^3 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(6.00)^2 = 4.50 \text{ in}^3$$

Allowable value of M_m: $\quad M_m = S_m\,\sigma_{\text{all}} = (4.50)(24) = 180.0 \text{ kip}\cdot\text{in}$

(*b*) Allowable value of *w*:

$$w = \frac{2M_m}{x_m(L-x_m)} = \frac{(2)(108.0)}{(15)(45)} = 0.320 \text{ kip/in}$$

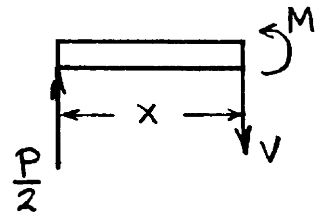
$w = 320$ lb/in ◀



PROBLEM 5.151

For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs, (b) the largest concentrated load **P** that can be applied, knowing that $\sigma_{\text{all}} = 24$ ksi.

SOLUTION



$$\mathbf{R}_A = \mathbf{R}_B = \frac{P}{2}\uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \quad \left(0 < x < \frac{L}{2}\right)$$

For a tapered beam, $\quad h = a + kx$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\quad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0.$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\frac{(a+kx)^2 - x - 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\frac{a-kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Data: $\quad a = 4$ in., $\quad k = \frac{8-4}{30} = 0.13333$ in/in

(a) $\quad x_m = \frac{4}{0.13333} = 30$ in. $\qquad x_m = 30.0$ in. ◀

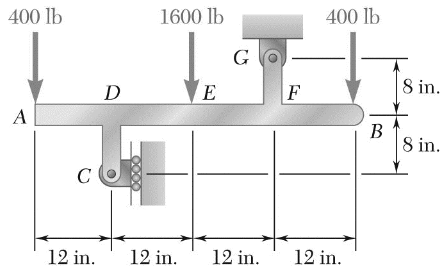
$$h_m = a + kx_m = 8 \text{ in.}$$

$$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$$

$$M_m = \sigma_{\text{all}}S_m = (24)(8) = 192 \text{ kip}\cdot\text{in}$$

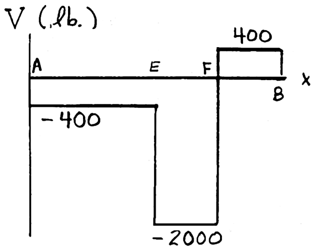
(b) $\quad P = \frac{2M_m}{x_m} = \frac{(2)(192)}{30} = 12.8$ kips $\qquad P = 12.80$ kips ◀

PROBLEM 5.152



Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

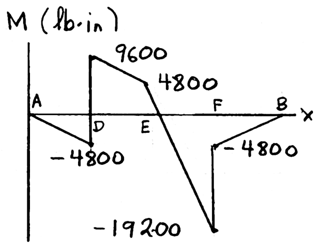


$$+\circlearrowleft \Sigma M_G = 0: \; -16C + (36)(400) + (12)(1600)$$

$$-(12)(400) = 0 \qquad C = 1800 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \; -C + G_x = 0 \qquad G_x = 1800 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \; -400 - 1600 + G_y - 400 = 0 \qquad G_y = 2400 \text{ lb}$$

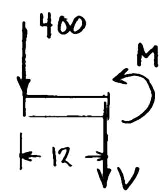


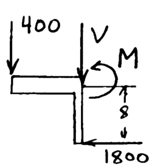
A to *E*: $V = -400$ lb

E to *F*: $V = -2000$ lb

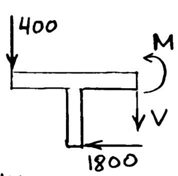
F to *B*: $V = 400$ lb

At *A* and *B*, $M = 0$





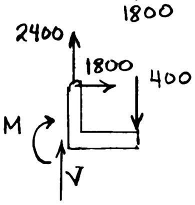
At D^-, $+\circlearrowleft \Sigma M_D = 0: \; (12)(400) + M = 0 \qquad M = -4800 \text{ lb} \cdot \text{in}$



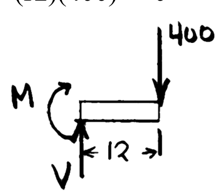
At D^+, $+\circlearrowleft \Sigma M_D = 0: \; (12)(400) - (8)(1800) + M = 0 \qquad M = 9600 \text{ lb} \cdot \text{in}$

At E, $+\circlearrowleft \Sigma M_E = 0: \; (24)(400) - (8)(1800) + M = 0 \qquad M = 4800 \text{ lb} \cdot \text{in}$

At F^-, $+\circlearrowleft \Sigma M_F = 0: \; -M - (8)(1800) - (12)(400) = 0 \qquad M = -19200 \text{ lb} \cdot \text{in}$

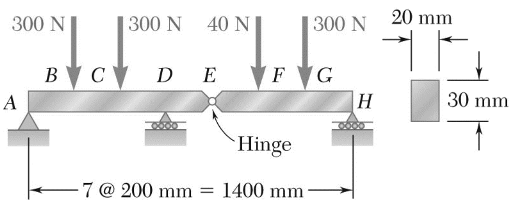


At F^+, $+\circlearrowleft \Sigma M_F = 0: \; -M - (12)(400) = 0 \qquad M = -4800 \text{ lb} \cdot \text{in}$



(*a*) Maximum $|V| = 2000$ lb ◀

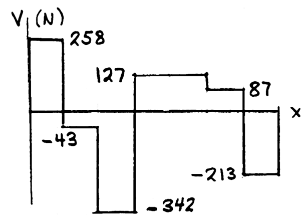
(*b*) Maximum $|M| = 19200$ lb · in ◀

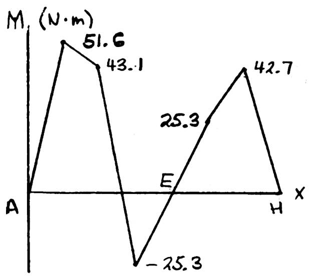


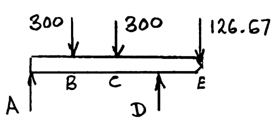
PROBLEM 5.153

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION



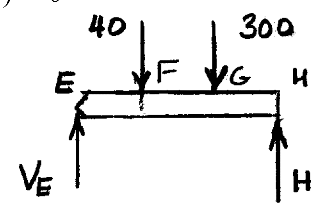




Free body *EFGH*. Note that $M_E = 0$ due to hinge.

$$+\circlearrowleft \Sigma M_E = 0: \quad 0.6\,H - (0.2)(40) - (0.40)(300) = 0$$

$$H = 213.33 \text{ N}$$



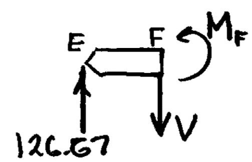
$$+\uparrow \Sigma F_y = 0: \quad V_E - 40 - 300 + 213.33 = 0$$

$$V_E = 126 \cdot 67 \text{ N}$$

Shear:

E to F:	$V = 126.67 \text{ N} \cdot \text{m}$
F to G:	$V = 86.67 \text{ N} \cdot \text{m}$
G to H:	$V = -213.33 \text{ N} \cdot \text{m}$

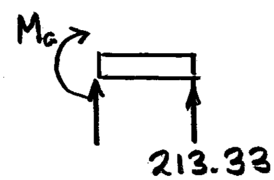
Bending moment at F:



$$+\circlearrowleft \Sigma M_F = 0: \quad M_F - (0.2)(126.67) = 0$$

$$M_F = 25.33 \text{ N} \cdot \text{m}$$

Bending moment at G:



$$+\circlearrowleft \Sigma M_G = 0: \quad -M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N} \cdot \text{m}$$

Free body *ABCDE*.

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.63) = 0$$

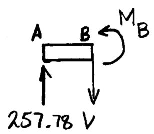
$$A = 257.78 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6D = 0$$

$$D = 468.89 \text{ N}$$

PROBLEM 5.153 *(Continued)*

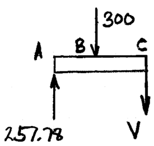
Bending moment at *B*.



$$+\circlearrowleft \Sigma M_B = 0: \quad -(0.2)(257.78) + M_B = 0$$

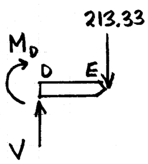
$$M_B = 51.56 \text{ N} \cdot \text{m}$$

Bending moment at *C*.



$$+\circlearrowleft \Sigma M_C = 0: \quad -(0.4)(257.78) + (0.2)(300) + M_C = 0$$

$$M_C = 43.11 \text{ N} \cdot \text{m}$$



Bending moment at *D*.

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (0.2)(213.33) = 0$$

$$M_D = -25.33 \text{ N} \cdot \text{m}$$

$$\max|M| = 51.56 \text{ N} \cdot \text{m}$$

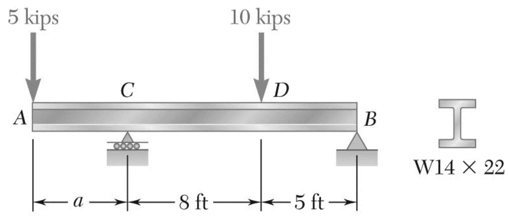
$$S = \frac{1}{6}bh^2 = \frac{1}{6}(20)(30)^2$$

$$= 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$$

Normal stress.

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa}$$

$$\sigma = 17.19 \text{ MPa} \blacktriangleleft$$



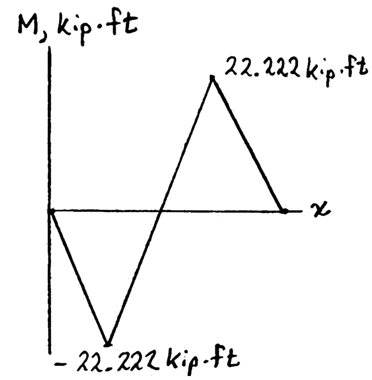
PROBLEM 5.154

Determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

Reaction at *B*: $+\circlearrowleft \Sigma M_C = 0: \quad 5a - (8)(10) + 13R_B = 0$

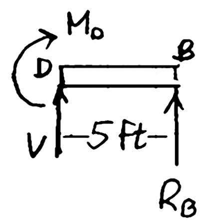
$$R_B = \frac{1}{18}(80 - 5a)$$



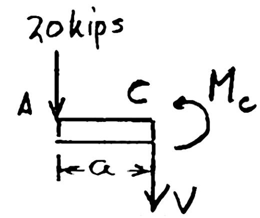
Bending moment at *D*:

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D + 5R_B = 0$$

$$M_D = 5R_B = \frac{5}{13}(80 - 5a)$$



Bending moment at *C*:



$$+\circlearrowleft M_C = 0 \quad 5a + M_C = 0$$

$$M_C = -5a$$

Equate: $-M_C = M_D$

$$5a = \frac{5}{13}(80 - 5a)$$

$$a = 4.4444 \text{ ft}$$

(*a*) $a = 4.44$ ft ◀

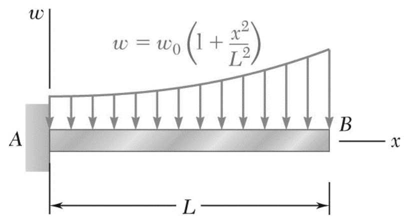
Then $-M_C = M_D = (5)(4.4444) = 22.222 \text{ kip} \cdot \text{ft}$

$$|M|_{\max} = 22.222 \text{ kip} \cdot \text{ft} = 266.67 \text{ kip} \cdot \text{in}$$

For W14 × 22 rolled steel section, $S = 29.0 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{266.67}{29.0} = 9.20 \text{ ksi}$

(*b*) 9.20 ksi ◀



PROBLEM 5.155

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

(*a*)

$$\frac{dV}{dx} = -w = -w_0\left(1 + \frac{x^2}{L^2}\right)$$

$$V = -w_0\left(x + \frac{x^3}{3L^2}\right) + C_1$$

$$V = 0 \quad \text{at} \quad x = L.$$

$$0 = -w_0\left(L + \frac{1}{3}L\right) + C_1 \quad C_1 = \frac{4}{3}w_0 L$$

$$\frac{dM}{dx} = V = w_0\left(\frac{4}{3}L - x - \frac{1}{3}\frac{x^3}{L^2}\right) \quad \blacktriangleleft$$

$$M = w_0\left(\frac{4}{3}Lx - \frac{1}{2}x^2 - \frac{1}{12}\frac{x^4}{L^2}\right) + C_2$$

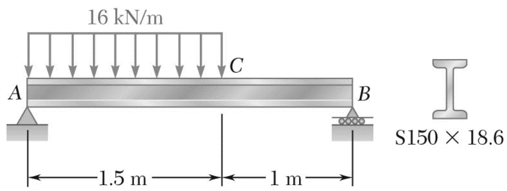
$$M = 0 \quad \text{at} \quad x = L. \quad w_0\left(\frac{4}{3}L^2 - \frac{1}{2}L^2 - \frac{1}{12}L^2\right) + C_2 = 0$$

$$C_2 = -\frac{3}{4}w_0 L^2$$

$$M = w_0\left(\frac{4}{3}Lx - \frac{1}{2}x^2 - \frac{1}{12}\frac{x^4}{L^2} - \frac{3}{4}L^2\right) \quad \blacktriangleleft$$

(*b*) $|M|_{\max}$ occurs at $x = 0$.

$$|M|_{\max} = \frac{3}{4}w_0 L^2 \quad \blacktriangleleft$$



PROBLEM 5.156

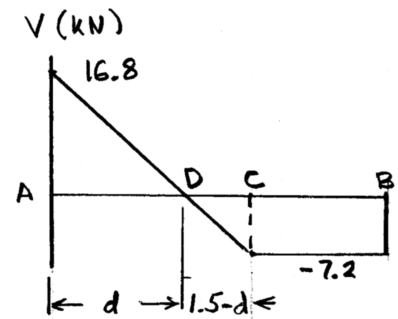
Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -2.5A + (1.75)(1.5)(16) = 0$$
$$A = 16.8 \text{ kN}$$
$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.75) + (1.5)(16) + 2.5B = 0$$
$$B = 7.2 \text{ kN}$$

Shear diagram:

$$V_A = 16.8 \text{ kN}$$
$$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$$
$$V_B = -7.2 \text{ kN}$$



Locate point D where $V = 0$.

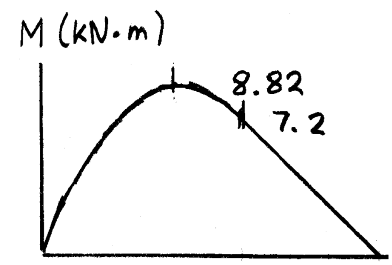
$$\frac{d}{16.8} = \frac{1.5 - d}{7.2} \qquad 24d = 25.2$$
$$d = 1.05 \text{ m} \quad 1.5 - d = 0.45 \text{ m}$$

Areas of the shear diagram:

A to D: $\int V dx = \left(\frac{1}{2}\right)(1.05)(16.8) = 8.82 \text{ kN}\cdot\text{m}$

D to C: $\int V dx = \left(\frac{1}{2}\right)(0.45)(-7.2) = -1.62 \text{ kN}\cdot\text{m}$

C to B: $\int V dx = (1)(-7.2) = -7.2 \text{ kN}\cdot\text{m}$



Bending moments:

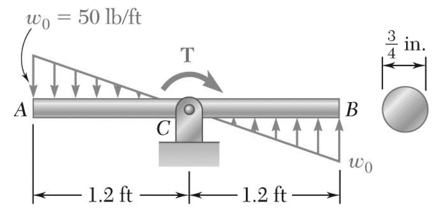
$$M_A = 0$$
$$M_D = 0 + 8.82 = 8.82 \text{ kN}\cdot\text{m}$$
$$M_C = 8.82 - 1.62 = 7.2 \text{ kN}\cdot\text{m}$$
$$M_B = 7.2 - 7.2 = 0$$

Maximum $|M| = 8.82 \text{ kN}\cdot\text{m} = 8.82 \times 10^3 \text{ N}\cdot\text{m}$

For S150×18.6 rolled steel section, $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa}$$

$\sigma = 73.5$ MPa ◀



PROBLEM 5.157

Knowing that beam AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION

A to C: $\quad 0 < x < 1.2$ ft

$$w = 50\left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$$

$$\frac{dV}{dx} = -w = 41.667x - 50$$

$$V = V_A + \int_0^x (41.667x - 50)dx$$

$$= 0 + 20.833x^2 - 50x = \frac{dM}{dx}$$

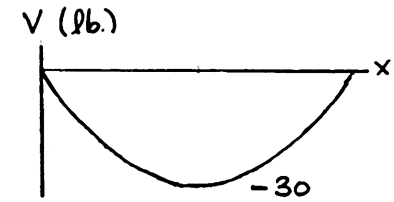
$$M = M_A + \int_0^x V\,dx$$

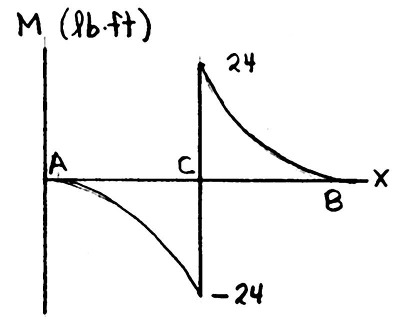
$$= 0 + \int_0^x (20.833x^2 - 50x)dx$$

$$= 6.944x^3 - 25x^2$$

At $\quad x = 1.2$ ft, $\quad V = -30$ lb

$M = -24$ lb $\cdot$ in





C to B: Use symmetry conditions.

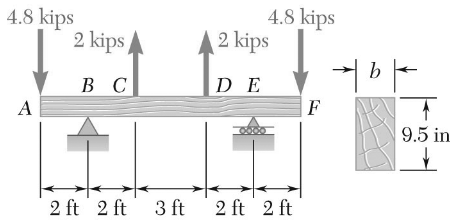
Maximum $|M| = 24$ lb $\cdot$ ft $= 288$ lb $\cdot$ in

Cross section: $\quad c = \frac{d}{2} = \left(\frac{1}{2}\right)(0.75) = 0.375$ in.

$$I = \frac{\pi}{4}c^4 = \left(\frac{\pi}{4}\right)(0.375) = 15.532\times 10^{-3}\,\text{in}^4$$

Normal stress: $\quad \sigma = \frac{|M|c}{I} = \frac{(2.88)(0.375)}{15.532\times 10^{-3}} = 6.95\times 10^3\,\text{psi}$ $\qquad \sigma = 6.95$ ksi ◄

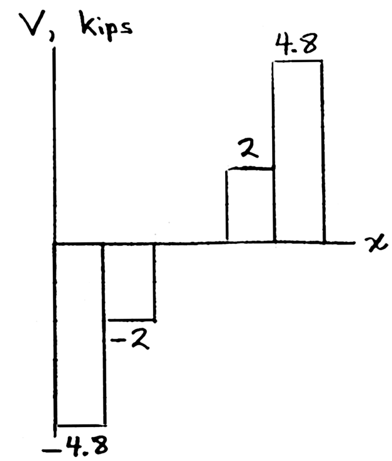
PROBLEM 5.158



For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION

For equilibrium, $B = E = 2.8$ kips



Shear diagram:

A to B^-: $V = -4.8$ kips

B^+ to C^-: $V = -4.8 + 2.8 = -2$ kips

C^+ to D^-: $V = -2 + 2 = 0$

D^+ to E^-: $V = 0 + 2 = 2$ kips

E^+ to F: $V = 2 + 2.8 = 4.8$ kips

Areas of shear diagram:

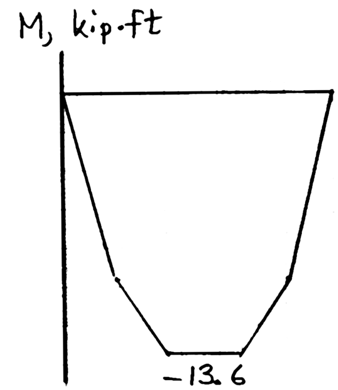
A to B: $(2)(-4.8) = -9.6 \text{ kip}\cdot\text{ft}$

B to C: $(2)(-2) = -4 \text{ kip}\cdot\text{ft}$

C to D: $(3)(0) = 0$

D to E: $(2)(2) = 4 \text{ kip}\cdot\text{ft}$

E to F: $(2)(4.8) = 9.6 \text{ kip}\cdot\text{ft}$



Bending moments:

$M_A = 0$

$M_B = 0 - 9.6 = -9.6 \text{ kip}\cdot\text{ft}$

$M_C = -9.6 - 4 = -13.6 \text{ kip}\cdot\text{ft}$

$M_D = -13.6 + 0 = -13.6 \text{ kip}\cdot\text{ft}$

$M_E = -13.6 + 4 = -9.6 \text{ kip}\cdot\text{ft}$

$M_F = -9.6 + 9.6 = 0$

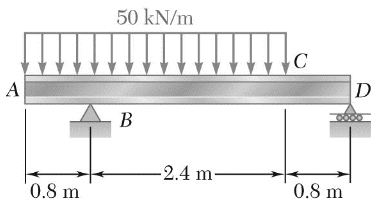
$$|M|_{\max} = 13.6 \text{ kip}\cdot\text{ft} = 162.3 \text{ kip}\cdot\text{in} = 162.3\times 10^3 \text{ lb}\cdot\text{in}$$

Required value for S:

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{162.3\times 10^3}{1750} = 93.257 \text{ in}^3$$

For a rectangular section, $I = \frac{1}{12}bh^3, \quad c = \frac{1}{2}h \qquad S = \frac{I}{c} = \frac{bh^2}{6} = \frac{(b)(9.5)^2}{6} = 15.0417b$

Equating the two expressions for S, $15.0417\, b = 93.257$ $\qquad b = 6.20$ in. ◀



PROBLEM 5.159

Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

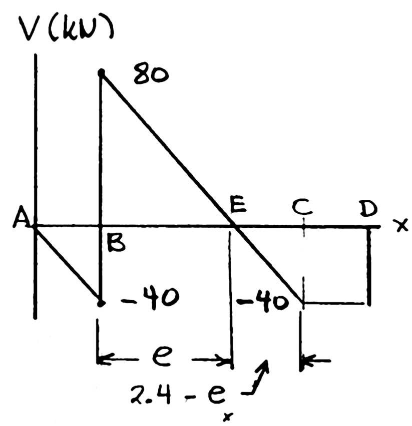
SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -3.2B + (24)(3.2)(50) = 0 \qquad B = 120 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 3.2D - (0.8)(3.2)(50) = 0 \qquad D = 40 \text{ kN}$$

Shear:

$$V_A = 0$$
$$V_{B^-} = 0 - (0.8)(50) = -40 \text{ kN}$$
$$V_{B^+} = -40 + 120 = 80 \text{ kN}$$
$$V_C = 80 - (2.4)(50) = -40 \text{ kN}$$
$$V_D = -40 + 0 = -40 \text{ kN}$$



Locate point E where $V = 0$.

$$\frac{e}{80} = \frac{2.4 - e}{40} \qquad 120e = 192$$

$$e = 1.6 \text{ m} \qquad 2.4 - e = 0.8 \text{ m}$$

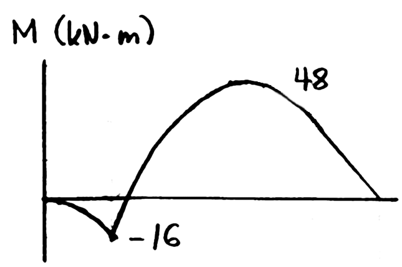
Areas:

A to B: $\int V dx = \left(\frac{1}{2}\right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m}$

B to E: $\int V dx = \left(\frac{1}{2}\right)(1.6)(80) = 64 \text{ kN} \cdot \text{m}$

E to C: $\int V dx = \left(\frac{1}{2}\right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = (0.8)(-40) = -32 \text{ kN} \cdot \text{m}$



Bending moments:

$$M_A = 0$$
$$M_B = 0 - 16 = -16 \text{ kN} \cdot \text{m}$$
$$M_E = -16 + 64 = 48 \text{ kN} \cdot \text{m}$$
$$M_C = 48 - 16 = 32 \text{ kN} \cdot \text{m}$$
$$M_D = 32 - 32 = 0$$

PROBLEM 5.159 *(Continued)*

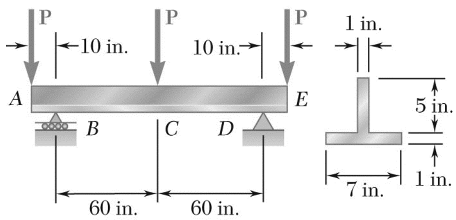
Maximum $|M| = 48 \text{ kN} \cdot \text{m} = 48 \times 10^3 \text{ N} \cdot \text{m}$

$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$
$W\,310 \times 32.7$	415
$W\,250 \times 28.4$	308 $\leftarrow$
$W\,200 \times 35.9$	342

Lightest wide flange beam: $W\,250 \times 28.4$ @ 28.4 kg/m ◀

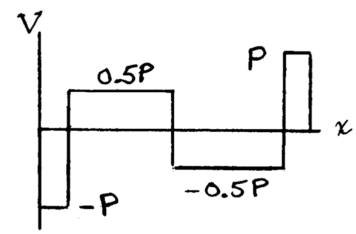


PROBLEM 5.160

Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +8 ksi in tension and −18 ksi in compression.

SOLUTION

Reactions: $\boldsymbol{B} = \boldsymbol{D} = 1.5\text{ P}\uparrow$



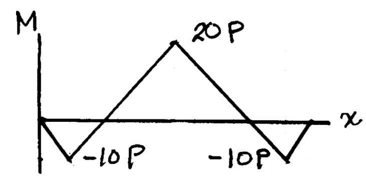
Shear diagram:

A to B^-: $V = -P$

B^+ to C^-: $V = -P + 1.5\text{ P} = 0.5\text{ P}$

C^+ to D^-: $V = 0.5\text{ P} - P = -0.5\text{ P}$

D^+ to E: $V = -0.5\text{ P} + 1.5\text{ P} = P$



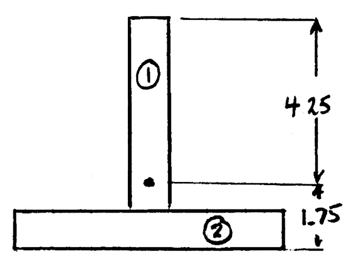
Areas:

A to B: $(10)(-P) = -10\text{ P}$

B to C: $(60)(0.5\text{ P}) = 30\text{ P}$

C to D: $(60)(-0.5\text{ P}) = -30\text{ P}$

D to E: $(10)(P) = 10\text{ P}$



Bending moments:

$$M_A = 0$$
$$M_B = 0 - 10\text{ P} = -10\text{ P}$$
$$M_C = -10\text{ P} + 30\text{ P} = 20\text{ P}$$
$$M_D = 20\text{ P} - 30\text{ P} = -10\text{ P}$$
$$M_E = -10\text{ P} + 10\text{ P} = 0$$

Largest positive bending moment: 20 P

Largest negative bending moment: −10 P

Centroid and moment of inertia:

Part	A, in^2	$\bar{y}_0$, in.	$A\bar{y}_0$, in^3	d, in.	Ad^2, in^4	$\bar{I}$, in^4
①	5	3.5	17.5	1.75	15.3125	10.417
②	7	0.5	3.5	1.25	10.9375	0.583
Σ	12		21		26.25	11.000

$$\bar{Y} = \frac{21}{12} = 1.75\text{ in.} \qquad I = \Sigma Ad^2 + \Sigma I = 37.25\text{ in}^4$$

Top: $y = 4.25$ in. Bottom: $y = -1.75$ in. $\sigma = -\dfrac{My}{I}$

PROBLEM 5.160 *(Continued)*

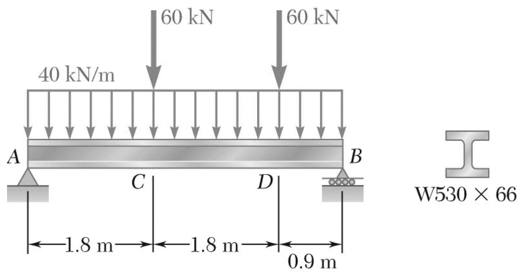
Top, tension: $8 = -\dfrac{(-10\text{ P})(4.25)}{37.25}$ $P = 7.01\text{ kips}$

Top, comp.: $-18 = -\dfrac{(20\text{ P})(4.25)}{37.25}$ $P = 7.89\text{ kips}$

Bottom. tension: $8 = -\dfrac{(20\text{ P})(-1.75)}{37.25}$ $P = 8.51\text{ kips}$

Bottom. comp.: $-18 = -\dfrac{(-10\text{ P})(-1.75)}{37.25}$ $P = 38.3\text{ kips}$

Smallest value of P is the allowable value. $P = 7.01\text{ kips}$ ◀



PROBLEM 5.161

(a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -4.5A + (2.25)(4.5)(40) + (2.7)(60) + (0.9)(60) = 0$$

$$\boldsymbol{A} = 138 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(2.25)(4.5)(40) - (1.8)(60) - (3.6)(60) + 4.5B = 0$$

$$\boldsymbol{B} = 162 \text{ kN} \uparrow$$

$$w = 40 \text{ kN/m} = \frac{dV}{dx}$$

$$V = -40x + 138 - 60\langle x - 1.8\rangle^0 - 60\langle x - 3.6\rangle^0 = \frac{dM}{dx}$$

$$M = -20x^2 - 138x - 60\langle x - 1.8\rangle^1 - 60\langle x - 3.6\rangle^1$$

$$V_C^+ = -(40)(1.8) + 138 - 60 = 6 \text{ kN}$$

$$V_D^- = -(40)(3.6) + 138 - 60 = -66 \text{ kN}$$

Locate point E where $V = 0$. It lies between C and D.

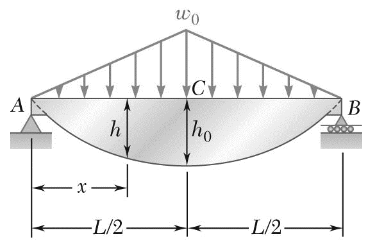
$$V_E = -40x_E + 138 - 60 + 0 = 0 \qquad x_E = 1.95 \text{ m}$$

$$M_E = -(20)(1.95)^2 + (138)(1.95) - (60)(1.95 - 1.8) = 184 \text{ kN} \cdot \text{m}$$

(a) $|M|_{\text{max}} = 184 \text{ kN} \cdot \text{m} = 184 \times 10^3 \text{ N} \cdot \text{m}$ at $x = 1.950$ m ◀

For W530×66 rolled steel section, $S = 1340 \times 10^3 \text{ mm}^3 = 1340 \times 10^{-6} \text{ m}^3$

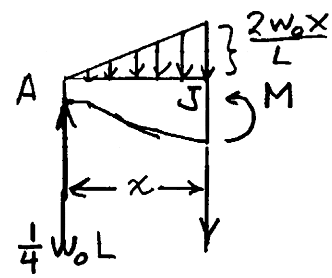
(b) Normal stress: $\sigma = \dfrac{|M|_{\text{max}}}{S} = \dfrac{184 \times 10^3}{1340 \times 10^{-6}} = 137.3 \times 10^6$ Pa $\qquad \sigma = 137.3$ MPa ◀



PROBLEM 5.162

The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{\text{all}} = 72$ MPa.

SOLUTION



By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad A - \frac{1}{2}w_0L + B = 0 \qquad \boldsymbol{A} = \boldsymbol{B} = \frac{1}{4}w_0L \uparrow$$

For $0 \le x \le \dfrac{L}{2}$, $\quad w = \dfrac{2w_0x}{L} \qquad \dfrac{dV}{dx} = -w = -\dfrac{2w_0x}{L} \qquad V = C_1 - \dfrac{w_0x^2}{L}$

At $x = 0$, $\quad V = \dfrac{1}{4}w_0L: \qquad C_1 = \dfrac{1}{4}w_0L$

$$\frac{dM}{dx} = V = \frac{1}{4}w_0L - \frac{w_0x^2}{L} \qquad M = C_2 + \frac{1}{4}w_0Lx - \frac{1}{3}\frac{w_0x^3}{L}$$

At $x = 0$, $\quad M = 0: \qquad C_2 = 0$

$$M = \frac{1}{2}\frac{w_0}{L}(3L^2 - 4x^3)$$

(a) At $x = \dfrac{L}{2}$, $\quad M = M_C = \dfrac{1}{12}\dfrac{w_0}{L}\left[3L^2\left(\dfrac{L}{2}\right) - 4\left(\dfrac{L}{2}\right)^3\right] = \dfrac{1}{12}w_0L^2$

For constant strength, $\quad S = \dfrac{M}{\sigma_{\text{all}}}, \qquad S_0 = \dfrac{M_0}{\sigma_{\text{all}}} = \dfrac{M_C}{\sigma_{\text{all}}} \qquad \dfrac{S}{S_0} = \dfrac{M}{M_0} = \dfrac{1}{L^3}(3L^2x - 4x^3)$

For a rectangular section, $\quad S = \dfrac{1}{6}bh^2 \qquad S_0 = \dfrac{1}{6}bh_0^2 \qquad \dfrac{S}{S_0} = \left(\dfrac{h}{h_0}\right)^2$

$$h = h_0\sqrt{\frac{3L^2x - 4x^3}{L^3}} \blacktriangleleft$$

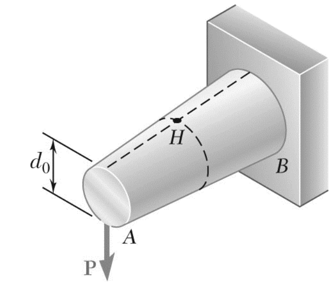
(b) Data: $\quad L = 800$ mm $\qquad h_0 = 200$ mm $\qquad b = 25$ mm $\qquad \sigma_{\text{all}} = 72$ MPa

$$S_0 = \frac{1}{6}bh_0^2 = \frac{1}{6}(25)(200)^2 = 166.667\times 10^3\,\text{mm}^3 = 166.667\times 10^{-6}\,\text{m}^3$$

$$M_C = \sigma_{\text{all}}S_0 = (72\times 10^6)(166.667\times 10^{-6}) = 12\times 10^3\,\text{N}\cdot\text{m}$$

$$w_0 = \frac{12M_C}{L^2} = \frac{(12)(12\times 10^3)}{(0.800)^2} = 225\times 10^3\,\text{N/m}$$

$$w_0 = 225\ \text{kN/m} \blacktriangleleft$$



PROBLEM 5.163

A transverse force **P** is applied as shown at end A of the conical taper AB. Denoting by d_0 the diameter of the taper at A, show that the maximum normal stress occurs at point H, which is contained in a transverse section of diameter $d = 1.5d_0$.

SOLUTION

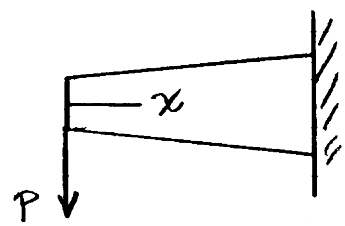
$$V = -P = \frac{dM}{dx} \qquad M = -Px$$

Let $$d = d_0 + kx$$

For a solid circular section, $$I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^3$$

$$c = \frac{d}{2} \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3 = \frac{\pi}{32}(d_0 + kx)^3$$

$$\frac{dS}{dx} = \frac{3\pi}{32}(d_0 + kx)^2 k = \frac{3\pi}{32}d^2 k$$



Stress: $$\sigma = \frac{|M|}{S} = \frac{Px}{S}$$

At H, $$\frac{d\sigma}{dx} = \frac{1}{S^2}\left(PS - Px_H\frac{dS}{dx}\right) = 0$$

$$S - x_H\frac{dS}{dx} = \frac{\pi}{32}d^3 - x_H\frac{3\pi}{32}d^2 k$$

$$kx_H = \frac{1}{3}d = \frac{1}{3}(d_0 + k_H x_H) \qquad kx_H = \frac{1}{2}d_0$$

$$d = d_0 + \frac{1}{2}d_0 = \frac{3}{2}d_0$$

$d = 1.5d_0$ ◀