EICoM
ELECTRICAL COMPUTER MECHATRONICS

ديناميكا واهتزازات

م.ايات الجراح

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chapter (2) :- Kinematics of particles:

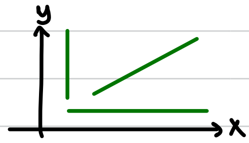
→ static: deals with bodies at rest/equilibrium
→ Dynamics: // // // in motion

Kinematics — s, v, a (without force)

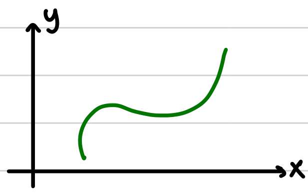
Kinetics — s, v, a (with force)

→ particles: point or a body of zero dimensions.
→ Rigid body. distances between points before and after motion

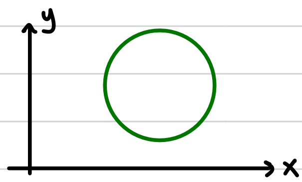
→ motion: $\boxed{1}$ Rectilinear motion: it refers to a straight line



$\boxed{2}$ curvilinear motion: it's a motion on a curved path.



$\boxed{3}$ Rotational Motion:



2.2 Rectilinear Motion

$\Delta s = s' - s$ = displacement

s = distance

$v = \dfrac{\Delta s}{\Delta t}$ = velocity

$a = \dfrac{\Delta v}{\Delta t}$

* Distance (s): Scalar quantity
  total length of path over which the particle travels.

* Displacement ($\vec{s}$, Δs): Vector quantity
  change in particle's position

  +ve → move to the right
  -ve → move to the left

* Speed: +ve scalar quantity

  $$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{\text{total distance } (s)}{\text{total time } (t)} \quad (m/s)$$

* Velocity: Vector quantity

  $$\text{Velocity} = \frac{\text{displacement}}{\text{time}} \quad (m/s)$$

  can be +ve or -ve.

Basic equations
(with non-constant acceleration)

$\boxed{1}$ $V = \frac{ds}{dt}$

$\boxed{2}$ $a = \frac{dv}{dt}$

$\boxed{3}$ $a = \frac{d^2s}{dt^2}$

$\boxed{4}$ $a\,ds = v\,dv$

with constant acceleration

$\boxed{5}$ $v = v_0 + a_c t$

$\boxed{6}$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$\boxed{7}$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

← can't be used if the acceleration is not constant.

SAMPLE PROBLEM 2/1

The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 24t + 6$, where s is measured in meters from a convenient origin and t is in seconds. Determine (*a*) the time required for the particle to reach a velocity of 72 m/s from its initial condition at $t = 0$, (*b*) the acceleration of the particle when $v = 30$ m/s, and (*c*) the net displacement of the particle during the interval from $t = 1$ s to $t = 4$ s.

a) $V = \frac{ds}{dt} = 6t^2 - 24 = 72$

$6t^2 = 96$

$\boxed{t = 4\,s}$ (we don't take the negative 4 because time can't be negative).

b) $a = \frac{dv}{dt} = 12t$ → changes with time (can't use it with equations $\boxed{5}$-$\boxed{7}$)

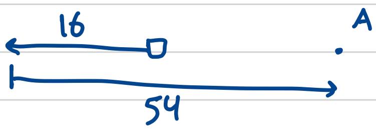
$$30 = 6t^2 - 24 \Rightarrow \boxed{t = 3s}$$

$$a = 12(3) = 36 \ m/s^2$$

c) $S\big|_{t=4} = 2(4)^3 - 24(4) + 6 = 38$

$$S_0\big|_{t=1} = 2(1)^3 - 24(1) + 6 = -16$$

$$\Delta s = S - S_0 = 38 - -16 = 54 \ m$$



2/8 A particle moves along a straight line with a velocity in millimeters per second given by $v = 400 - 16t^2$, where t is in seconds. Calculate the net displacement Δs and total distance D traveled during the first 6 seconds of motion.

$$V = 400 - 16t^2 \quad (mm/s)$$

from $\boxed{1}$

$$V = \frac{ds}{dt} \qquad ds = v\,dt$$

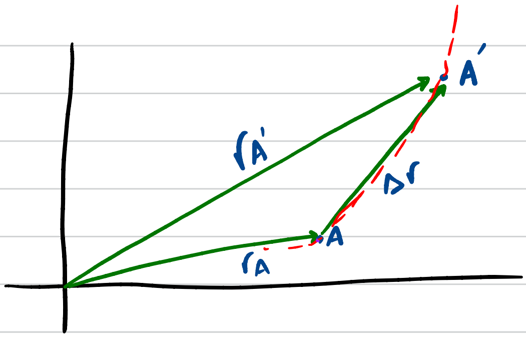
$$\int_{S_0=0}^{S} ds = \int_0^6 (400 - 16t^2)\,dt$$

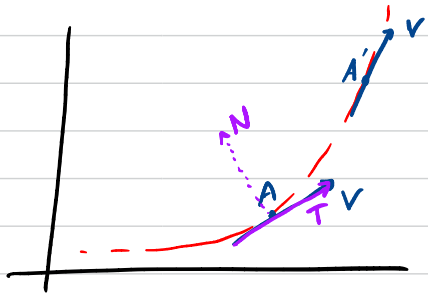
$$S - S_0 = 400t - \frac{16t^3}{3}\Big]_0^6$$

$$\boxed{\begin{aligned} S &= 1248 \ mm \\ S &= 1.248 \ m \end{aligned}}$$

2.3 plane curvilinear Motion

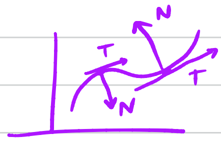
(it is motion along a curved path in a single line).





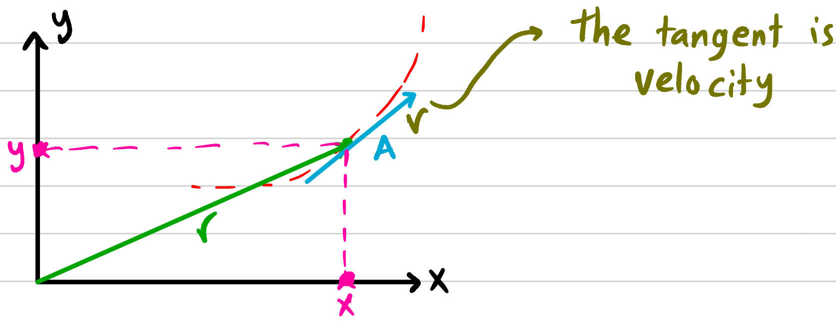
→ coordinate systems:

1. Rectangular coordinate (x - y)
2. Normal - Tangential coordinate (n - t) *



x 3. polar coordinate (r-θ)

2.4 Rectangular coordinate (x - y)



Vectors

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$$

$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

SAMPLE PROBLEM 2/5

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that $x = 0$ when $t = 0$. Plot the path of the particle and determine its velocity and acceleration when the position $y = 0$ is reached.

at $y = 0 \Longrightarrow V = ?$ magnitude and direction since it's velocity.

$a = ?$

$y = 100 - 4t^2$

$0 = 100 - 4t^2$

$\boxed{t = 5s}$

$V_y = -8t$, at 5 sec $= -40\ m/s$

$V_x = 50 - 16t$

$V_x\ (t = 5) = 50 - 16(5)$

$= -30\ m/s$

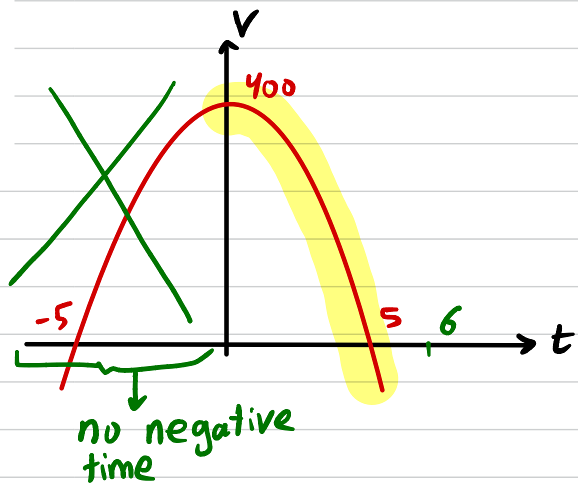
$V = V_x\ \hat{i} + V_y\ \hat{j}$

$= -30\ \hat{i} - 40\ \hat{j}$

$|V| = \sqrt{(-30)^2 + (-40)^2}$

problems 2/2

2/8 A particle moves along a straight line with a velocity in millimeters per second given by $v = 400 - 16t^2$, where t is in seconds. Calculate the net displacement Δs and total distance D traveled during the first 6 seconds of motion.



$v = 400 - 16t^2 \ (mm/s)$

$t = 0 - 6$ seconds

$$v = \frac{ds}{dt} = 400 - 16t^2$$

$$\int_{s_0}^{s} ds = \int_0^6 400 - 16t^2 \, dt$$

$$s - s_0 = 400t - \frac{16t^3}{3}\Big]_0^6$$

$$\Delta s = 1248 \text{ mm}$$
$$= 1.248 \text{ m}$$

$$\text{Distance} = |\Delta s_1| + |\Delta s_2|$$

$$= \int_0^5 400 - 16t^2 \, dt + \int_5^6 400 - 16t^2 \, dt$$

$$= 400t - \frac{16}{3}t^3\Big]_0^5 + 400t - \frac{16t^3}{3}\Big]_5^6$$

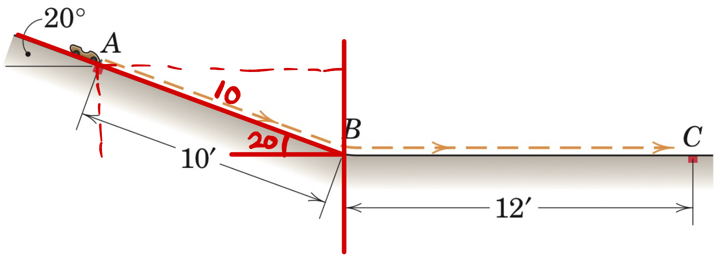
$$= |1333.33| + |1248 - 1333.33|$$

absolute value because we want total distance not displacement.

$$= 1333.33 + 85.33$$

$$\text{Distance} = 1418.66 \text{ mm}$$
$$= 1.418 \text{ m}$$

2/14 In the pinewood-derby event shown, the car is released from rest at the starting position A and then rolls down the incline and on to the finish line C. If the constant acceleration down the incline is 9 ft/sec^2 and the speed from B to C is essentially constant, determine the time duration t_{AC} for the race. The effects of the small transition area at B can be neglected.

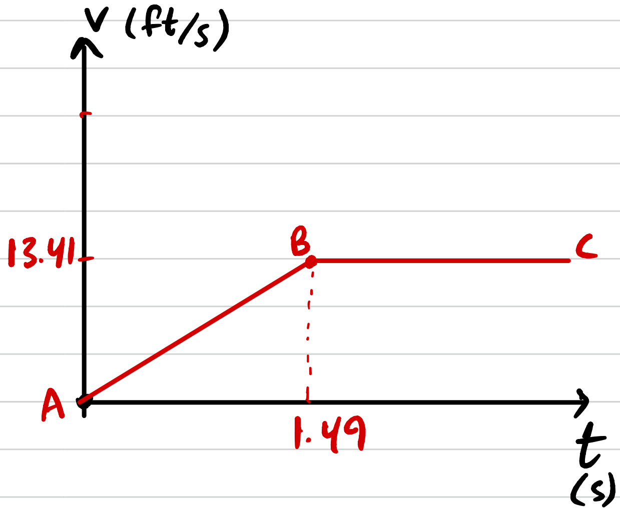


Problem 2/14

$t_{AC} = ?$

$x = 10\cos 20 = 9.4$

$y = 10 \sin 20 = 3.42$



from $\boxed{7}$

$$S = \cancel{S_0}^{\,0} + V_0 t + \tfrac{1}{2} a t^2$$

from A→B $\quad 10 = (0 * t) + \tfrac{1}{2} * 9 * t^2 \quad$ → released from rest $\therefore V_0 = 0$

$10 = 4.5\,t^2 \rightarrow t^2 = \frac{10}{4.5}$

$t_B = 1.49\,s$ (time to reach point B from A).

from $\boxed{5}$ $\quad V = V_0 + a t$

$V = 0 + (9 * 1.49)$

$V = 13.41$ ft/s (velocity when the car reaches point B).

velocity from B to c is constant $\therefore V_B = V_c$

$\therefore$ acceleration $= 0$

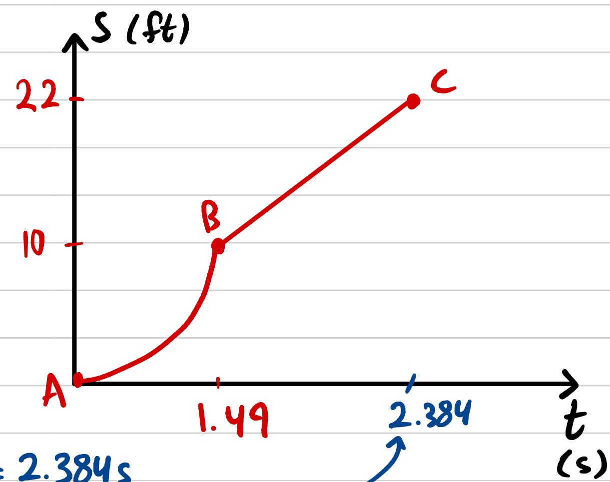
because velocity is constant

slope from B to c = V

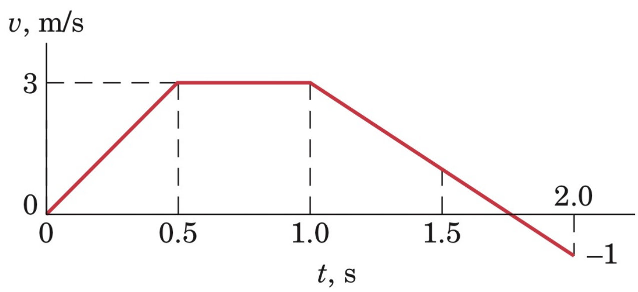
$V_B = V_c = \frac{\Delta s}{\Delta t} \Rightarrow 13.41 = \frac{12}{\Delta t}$

$\Delta t = 0.894\,s \qquad \therefore t_c - t_B = 0.894$

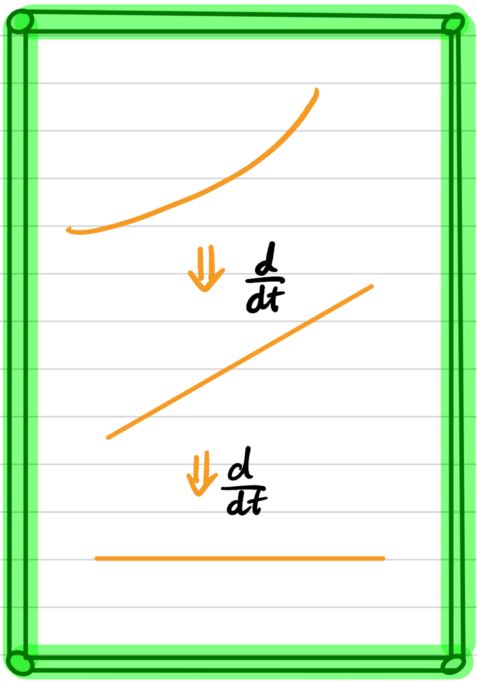
$t_c = 0.894 + 1.49 = 2.384\,s$

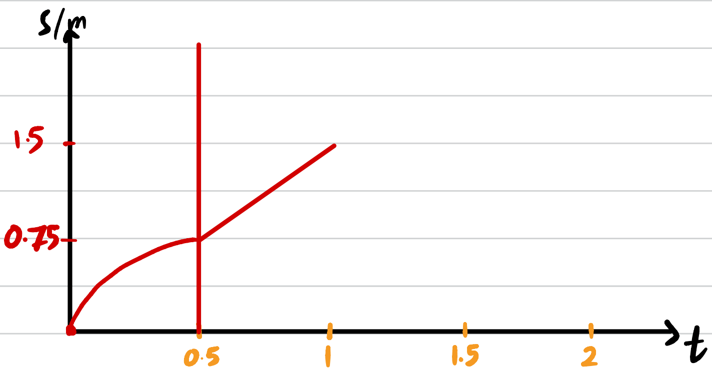


2/29 A particle starts from rest at $x = -2$ m and moves along the x-axis with the velocity history shown. Plot the corresponding acceleration and the displacement histories for the 2 seconds. Find the time t when the particle crosses the origin.



Problem 2/29

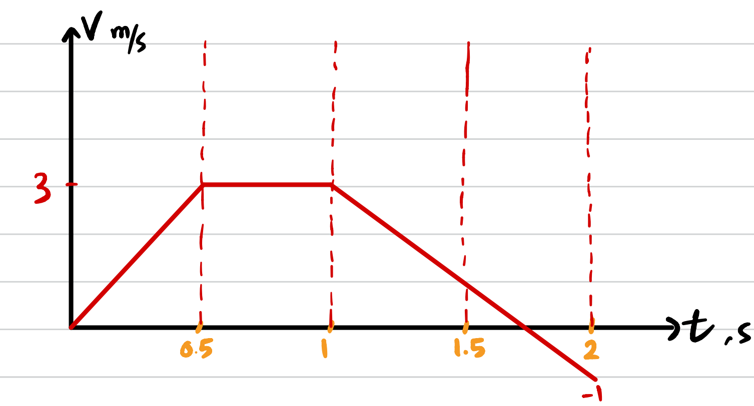


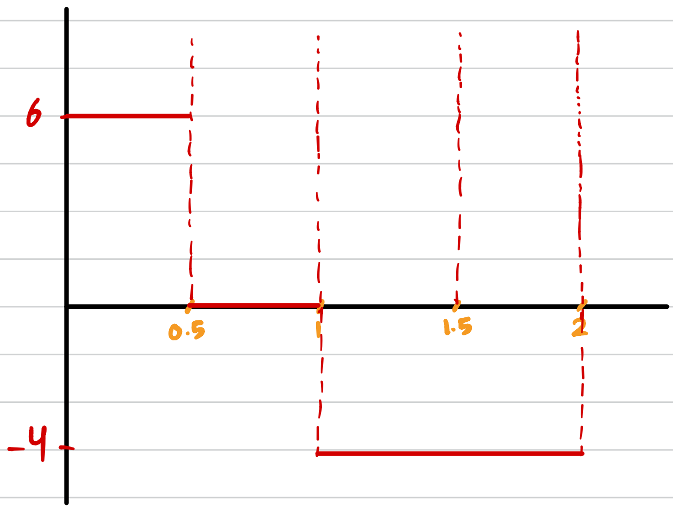


area gives displacement

$\Delta s = \frac{1}{2} \times 0.5 \times 3 = 0.75$

$s_f = 0.75 + s_0 = 0.75 - 2$
$= -1.25$





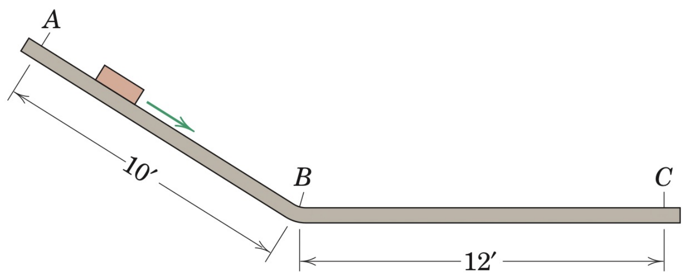
for $t \leq 0.5$

$a = \frac{dv}{dt} = \frac{3}{0.5} = 6$

for $1 < t \leq 2$

$a = \frac{3 - -1}{1 - 2} = -4$

2/35 Packages enter the 10-ft chute at A with a speed of 4 ft/sec and have a $0.3g$ acceleration from A to B. If the packages come to rest at C, calculate the constant acceleration a of the packages from B to C. Also find the time required for the packages to go from A to C.



Problem 2/35

$$a = 0.3g = 0.3 * 0.98\ m/s^2 = 2.94\ m/s^2$$
$$= 9.66\ ft/s^2$$

from $\boxed{7}$ $\quad S_{AB} = V_0 * t_{AB} + \frac{1}{2} a_{AB} * t_{AB}^2$

$$10 = 4 * t_{AB} + \frac{1}{2}(9.66)(t_{AB}^2)$$

$$t_{AB} = 1.083\ s$$

from $\boxed{5}$ $\quad V_B = V_A + a_{AB}\, t_{AB}$

$$= 4 + 9.66 * 1.083$$
$$= 14.462$$

from $\boxed{6}$ $\quad V_C^2 = V_B^2 + 2\, a_{BC} * S_{BC}$

$$0 = 14.462^2 + 2 * a_{BC} * 12$$

comes at rest

$$\boxed{a_{BC} = -8.714\ ft/s^2.}$$

$$V_C = V_B + a_{BC} * t_{BC}$$
$$0 = 14.462 + (-8.714)\, t_{BC}$$

$$t_{BC} = 1.66\ s$$

$$t_{total} = t_{AB} + t_{BC}$$
$$= 1.083 + 1.66$$
$$\boxed{t_{total} = 2.743\ s}$$

→ Projectile Motion: (Motion under gravity)

$a_x = 0$ → no wind

$V_x = V_{x,0}$

$x = x_0 + V_{x,0}\, t$ $\boxed{7}$

if $a_x \neq 0$

$x = x_0 + V_{x,0}\, t + \frac{1}{2} a_x t^2$

$V_{x,0}$ is always constant

x direction

$a_y = -g$ if ↑ +y ↓ -y

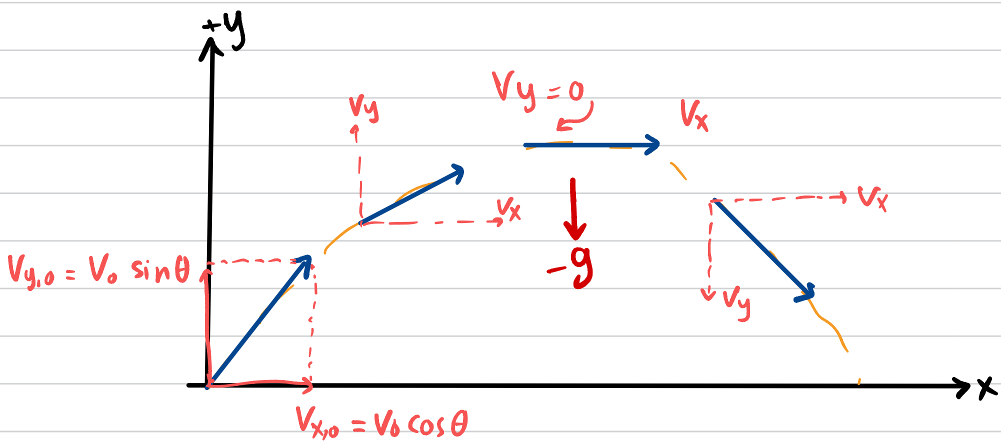
$V_y = V_{y,0} - g t$

$y = y_0 + V_{y,0}\, t - \frac{1}{2} g t^2$ $\boxed{7}$

$V_y^2 = V_{y,0}^2 - 2g(y - y_0)$

V_{y_0} is always changing

y direction



problems 2/4

2/63 The x-coordinate of a particle in curvilinear motion is given by $x = 2t^3 - 3t$, where x is in feet and t is in seconds. The y-component of acceleration in feet per second squared is given by $a_y = 4t$. If the particle has y-components $y = 0$ and $\dot{y} = 4$ ft/sec when $t = 0$, find the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ when $t = 2$ sec. Sketch the path for the first 2 seconds of motion, and show the velocity and acceleration vectors for $t = 2$ sec.

$$\boxed{\begin{aligned} x &= 2t^3 - 3t \\ \dot{x} &= 6t^2 - 3 \\ \ddot{x} &= 12t = a_x \end{aligned}}$$

$\frac{d}{dt}$ $\frac{d}{dt}$

@ $t=0 \rightarrow \dot{y} = 4$ ft/s & $y = 0$

@ $t=2 \rightarrow |V| = ?$ $|a| = ?$

$$\ddot{y} = 4t = a_y$$

$$* \quad \dot{y} = \int a_y \, dt = \int 4t \, dt = 2t^2 + c$$

$\dot{y}|_{t=0} = 4$ ft/s $\quad \therefore \dot{y} = 2(0)^2 + c = 4$

$\therefore c = 4$

$$\therefore \boxed{\dot{y} = 2t^2 + 4}$$

$$|V| = \sqrt{V_x^2 + V_y^2} = \sqrt{(6(t^2)-3)^2 + (2(t^2)+4)^2}$$

$$= \sqrt{(6(2)^2-3)^2 + (2(2)^2+4)^2} \quad \leftarrow \text{at } t=2$$

$$|V| = 24.2 \text{ ft/s}$$

$$|a| = \sqrt{a_x^2 + a_y^2} = \sqrt{(12t)^2 + (4t)^2}$$

$$|a| = \sqrt{24^2 + 8^2} \quad \leftarrow \text{at } t=2$$

$$|a| = 25.3 \text{ m/s}^2$$

2/64 The y-coordinate of a particle in curvilinear motion is given by $y = 4t^3 - 3t$, where y is in inches and t is in seconds. Also, the particle has an acceleration in the x-direction given by $a_x = 12t$ in./sec^2. If the velocity of the particle in the x-direction is 4 in./sec when $t = 0$, calculate the magnitudes of the velocity **v** and acceleration **a** of the particle when $t = 1$ sec. Construct **v** and **a** in your solution.

$y = 4t^3 - 3t$
$\dot{y} = 12t^2 - 3$
$\ddot{y} = 24t$

$@\ t = 0 \longrightarrow \dot{x} = 4\ m/s$

$@\ t = 1 \longrightarrow |v|\ ,\ |a| = ?$

$\ddot{x} = 12t$

$\dot{x} = \int \ddot{x}\, dt = \int 12t\, dt = 6t^2 + C$

$\dot{x}\big|_{t=0} = 6(0)^2 + C = 4$

$\therefore C = 4$

$\therefore \boxed{\dot{x} = 6t^2 + 4}$

$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{(6(t)^2+4)^2 + (12(t)^2 - 3)^2}$

$= \sqrt{10^2 + 9^2}$ $\swarrow @\ t=1$

$|v| = 13.45\ m/s$

$|a| = \sqrt{a_x^2 + a_y^2} = \sqrt{(12t)^2 + (24t)^2}$

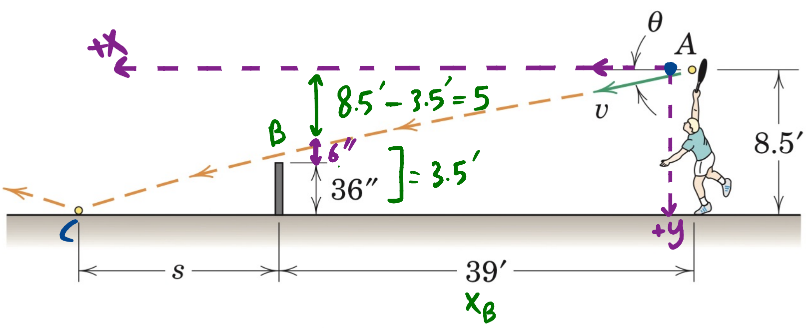
$@\ t = 1$

$= \sqrt{12^2 + 24^2}$

$|a| = 26.8\ m/s^2$

2/77 If the tennis player serves the ball horizontally ($\theta = 0$), calculate its velocity v if the center of the ball clears the 36-in. net by 6 in. Also find the distance s from the net to the point where the ball hits the court surface. Neglect air resistance and the effect of ball spin.

36 in = 3 ft
6 in = 0.5 ft
$g = 9.81 \, m/s^2 = 32.2 \, ft/s^2$



Problem 2/77

1

no wind ∴ $a_x = 0$

$x_B = x_{B_0} + V_{x_0} t_B$ $\qquad x_{B_0} = 0$

$x_B = V_{x_0} t_B$

$V_{x_0} = \frac{x_B}{t_B} = \frac{39}{t_B}$ ... ①

$y_B = y_{B_0} + V_{y_{B,0}} t_B + \frac{1}{2} g t_B^2$ ... ② $\qquad y_{B_0} = 0$

$V_{y_{B,0}} = 0$ (served horizontally).

$y_B = \frac{1}{2} g t^2$

$5 = \frac{1}{2}(32.2) t_B^2$

$\therefore \boxed{t_B = 0.557 \, s}$

∴ from ①

$V_{x_B} = \frac{39}{t_B} = \frac{39}{0.557} = \boxed{70 \, ft/s = V_{x_B}}$

2

$y_c = V_{y_0} t + \frac{1}{2} g t_c^2$

$8.5 = \frac{1}{2} g t^2 \Rightarrow \therefore \boxed{t_c = 0.727 \, s}$

$x_c = V_{x_0} t_c$ $\qquad x_c = 39 + s$

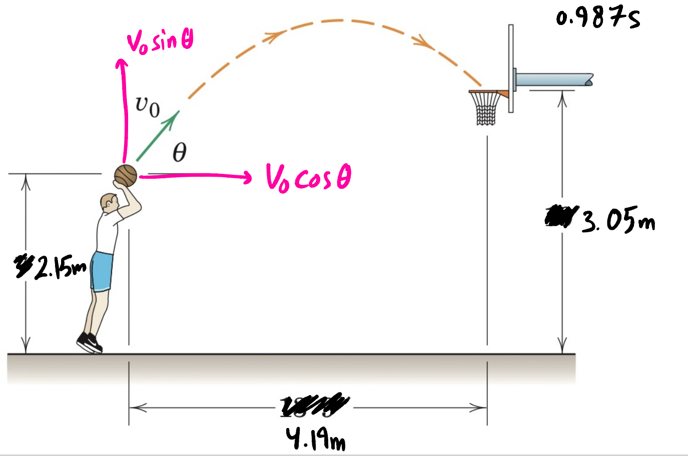
$39 + s = 70 * 0.727$

$\boxed{s = 11.89 \, ft}$

Since there is no acceleration on the x-axis means that velocity is constant. ($V_{A_x} = V_{B_x} = V_{C_x}$)

2/78 The basketball player likes to release his foul shots with an initial speed $v_0 = 7.15\ m/s$ What value(s) of the initial angle θ will cause the ball to pass through the center of the rim? Neglect clearance considerations as the ball passes over the front portion of the rim.

if the flying time is 0.987s



$$y_f = y_0 + V_0 t - \frac{1}{2} g t^2$$

$$3.05 = 2.15 + (7.15 \sin\theta)(0.987) - \frac{1}{2}(9.81)(0.987)^2$$

$$\boxed{\theta = 53.57^\circ} \quad \#$$

OR:

$$X = X_0 + V_{x,0}\, t$$

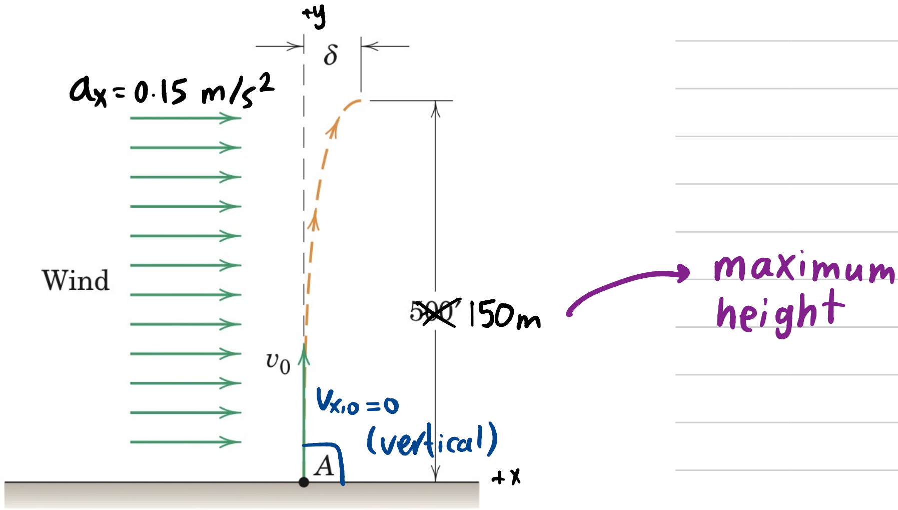
$$4.14 = 0 + V_0 \cos\theta\, t$$

$$4.14 = 0 + 7.15 \cos\theta (0.987)$$

$$\cos\theta = \frac{4.14}{7.15 * 0.987}$$

$$\theta = \cos^{-1}\left(\frac{4.14}{7.15 * 0.987}\right) = 54^\circ \approx 53.57^\circ$$

2/87 A fireworks shell is launched vertically from point A with speed sufficient to reach a maximum altitude of 150 m ~~500 ft~~. A steady horizontal wind causes a constant horizontal acceleration of 0.5 ft/sec^2, but does not affect the vertical motion. Determine the deviation δ at the top of the trajectory caused by the wind.



Problem 2/87

$$V_y^2 = V_{y,0}^2 - 2g(y - y_0)$$

at max height $V_y = 0$

$$0 = V_{y,0}^2 - 2(9.81)(150)$$

$$\boxed{V_{y,0} = 54.25\ m/s}$$

$$V_y = V_{y_0} - g\,t_B$$

$$0 = V_{y_0} - 9.81\,t_B$$

$$\boxed{t_B = 5.53s}$$

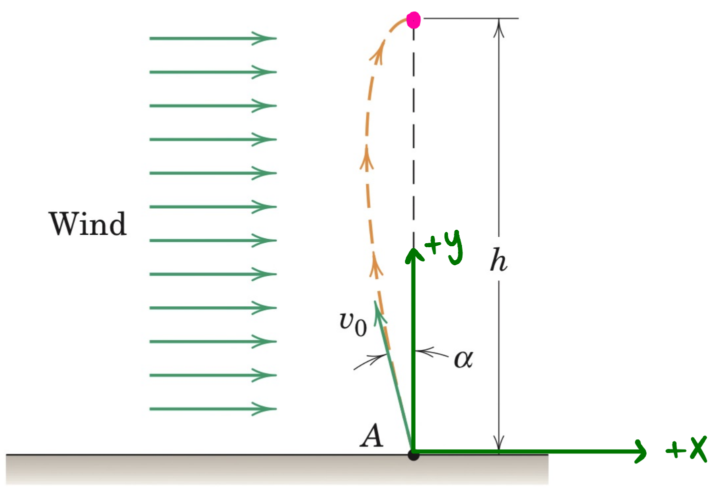
$$x_B = x_A + \cancel{V_{xA}}^{\,0}\,t_B + \tfrac{1}{2}a\,t_B^2$$

initially, velocity $= 0$

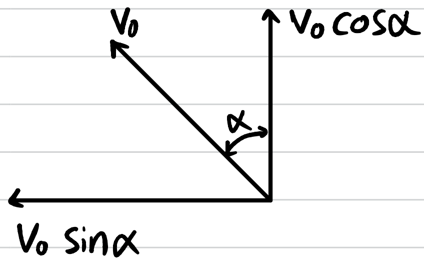
$$x_B - x_A = \tfrac{1}{2}(0.15)(5.53)^2$$

$$\boxed{\delta = 2.3m}$$

2/88 Consider the fireworks shell of the previous problem. What angle α compensates for the wind in that the shell peaks directly over the launch point A? All other information remains as stated in the previous problem, including the fact that the initial launch velocity $\mathbf{v}_0$ if vertical would result in a maximum altitude of 500 ft. What is the maximum height h possible in this problem?



Problem 2/88



* constant horizontal acceleration = 0.15 m/s²

$$2gy = V_0^2 \cos^2\alpha$$

$V_y = 0$ (max height)

$$y = \frac{V_0^2}{19.62}$$

$V_0 = 54.25$ m/s from previous question.

$y = 150$ m which is the maximum height.

* The shell would reach its maximum height only when fired vertically up and at maximum height vertical component of the velocity is zero.

$$V_y = V_{y_0} - gt$$

$V_y = 0$ (@ max height)
$V_{y_0} = V_0 \cos\alpha$

$$0 = V_0 \cos\alpha - 9.81\,t$$
$$V_0 \cos\alpha = 9.81\,t \quad \text{--- (1)}$$

$$X = X_0 + V_{X_0} t + \tfrac{1}{2} a_x t^2$$
$$0 = 0 - V_0 \sin\alpha\, t + \tfrac{1}{2} a_x t^2$$

* X and $X_0 = 0$ because we want to launch the shell up (only change in y)

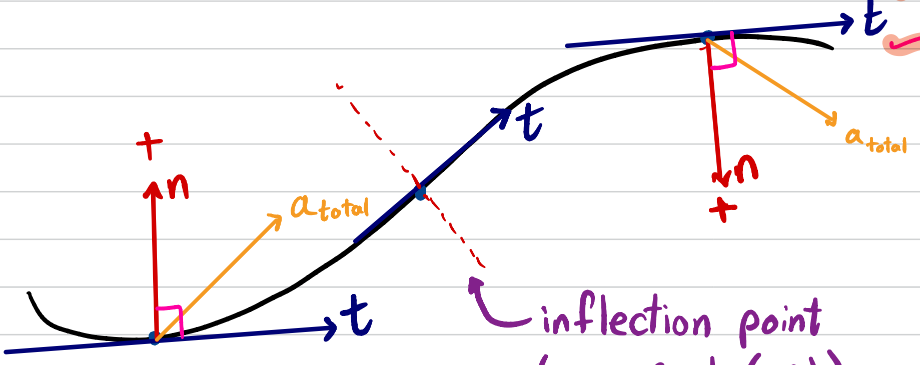
$$V_0 \sin\alpha\, t = (0.5)(0.15)\, t^2$$
$$V_0 \sin\alpha = 0.075\, t \quad \text{---- (2)}$$

Divide (1) by (2) to find the angle

$$\frac{V_0 \cos\alpha}{V_0 \sin\alpha} = \frac{9.81\,t}{0.075\,t} \Rightarrow \alpha = 0.44^\circ$$

2/5 Normal – Tangintial coordinates.

* n-t axes are perpendicular.



tangent to the path

+ve t-axis in direction of increasing displacement.

inflection point (no normal (=0)).

* Normal vector is always in the direction of the center of the curve.

What's the difference between x-y and n-t coordinates?
x-y coordinates are fixed, while n-t coordinates move along the path w/ the particle.

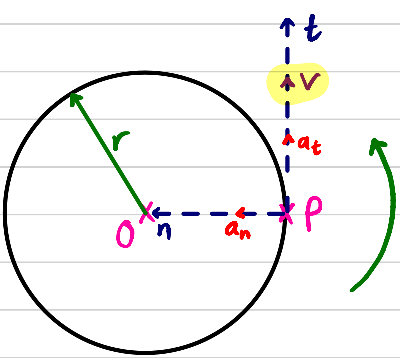
$$\vec{a} = \vec{a}_n + \vec{a}_t \Rightarrow a = \sqrt{a_n^2 + a_t^2}$$

$a_n = \frac{v^2}{\rho}$ ≡ normal acceleration

ρ ≡ Radius of curveture (from center of mass of an object to the center of the curve).

$a_t = \dot{v} = \ddot{s}$

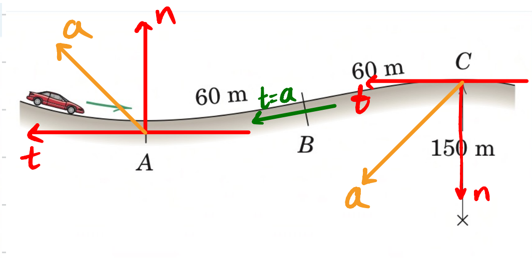
* circular motion (special case of curvelinear motion).



where $\dot{\theta}$: angular speed.
$\ddot{\theta}$: angular acceleration.

SAMPLE PROBLEM 2/7

To anticipate the dip and hump in the road, the driver of a car applies her brakes to produce a uniform deceleration. Her speed is 100 km/h at the bottom A of the dip and 50 km/h at the top C of the hump, which is 120 m along the road from A. If the passengers experience a total acceleration of 3 m/s^2 at A and if the radius of curvature of the hump at C is 150 m, calculate (a) the radius of curvature ρ at A, (b) the acceleration at the inflection point B, and (c) the total acceleration at C.



a is always between n and t vectors, t is to the left because it's deceleration.

$$V_A = 100\,Km/h = \frac{100 \times 10^3}{3600} = 27.8\ m/s$$

$$V_c = 50\ Km/h = \frac{50 \times 10^3}{3600} = 13.89\ m/s$$

$a_A = 3m/s^2$ ← total acceleration

$\rho_c = 150m$

$\boxed{1}$

$$(a_n)_A = \frac{V_A^2}{\rho_A} = \frac{27.8^2}{\rho_A}$$

$$V_c^2 = V_A^2 + 2(a_t)_A(S_c - S_A)$$

$$(13.89)^2 = (27.8)^2 + 2\,a_t\,(120)$$

$$* \ a_t = -2.41\ m/s^2$$

$$(a)_A^2 = (a_n)_A^2 + (a_t)_A^2$$

$$(3)_A^2 = (a_n)_A^2 + ((a_t)_A^2)$$

$$(3)^2 = (a_n)^2 + (-2.41)^2$$

$$\sqrt{9 - (-2.41)^2} = (a_n)_A$$

$$(a_n)_A = 1.785\ m/s^2$$

$\boxed{2}$ $a_B = ?$

Constant acceleration

$$(a_t)_B = -2.41\ m/s^2 = (a_t)_A$$

$$(a_n)_B = \frac{V_B^2}{\rho_B} = \frac{V_B^2}{\infty} = 0$$

$$\therefore\ a_B = -2.41\ m/s^2$$

$\boxed{3}$ $(a)_c$

$$(a_t)_c = -2.41\ m/s^2$$

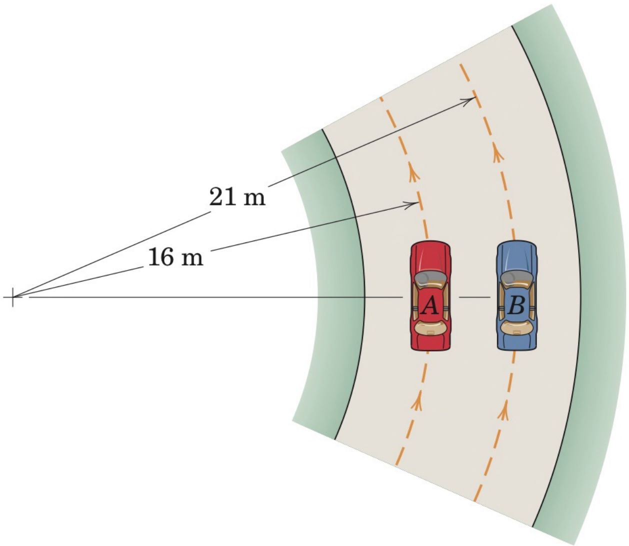
$$(a_n)_c = \frac{V_c^2}{\rho_c} = \frac{13.89^2}{150} = 1.286\ m/s^2$$

$$(a)_c = \sqrt{(-2.41)^2 + (1.286)^2}$$

$$= 2.73\ m/s^2$$

problems 2/5

2/97 Determine the maximum speed for each car if the normal acceleration is limited to $0.88g$. The roadway is unbanked and level.



$$a_n = 0.88\,g = 0.88\,(9.81\ m/s^2) = 8.633\ m/s^2$$

$$a_n = \frac{v^2}{r} \Rightarrow v^2 = r\,a_n$$

* for car A :-

$$v^2 = (16)(8.633)$$

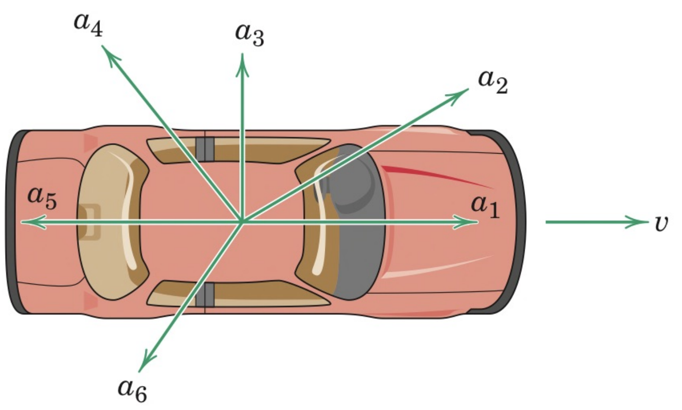
$$\boxed{v = 11.75\ m/s}$$

* for car B :-

$$v^2 = (21)(8.633)$$

$$\boxed{v = 13.46\ m/s}$$

2/99 Six acceleration vectors are shown for the car whose velocity vector is directed forward. For each acceleration vector describe in words the instantaneous motion of the car.



Vector a_1 :- it is pointed in the same forward direction as the velocity vector. Therefore, the speed of the car will increase and there is no change in path curvature of the car.

Vector a_2 :- it is pointed towards the left in the forward direction with the velocity vector forward, Therefore, the speed of the car will increase and the car will turn to the left.

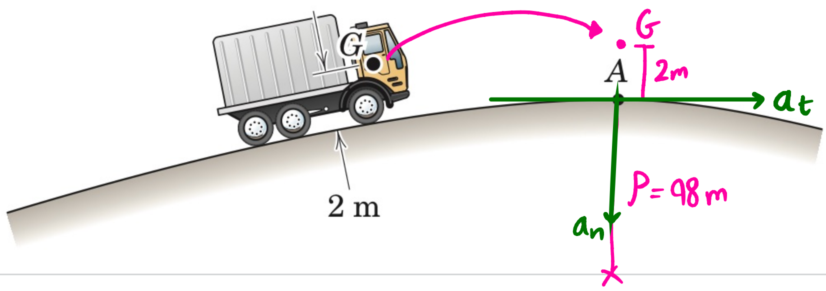
Vector a_3 :- The speed of the car will be stationary and the car will turn to the left.

Vector a_4 :- speed of the car will decrease, the car will turn to the left.

Vector a_5 :- speed of the car will decrease, no change in path curvature.

Vector a_6 :- speed of the car will decrease, car will turn to the right

2/100 The driver of the truck has an acceleration of 0.4g as the truck passes over the top A of the hump in the road at constant speed. The radius of curvature of the road at the top of the hump is 98 m, and the center of mass G of the driver (considered a particle) is 2 m above the road. Calculate the speed v of the truck.



$$a^2 = a_n^2 + a_t^2$$

* at constant speed the tangential acceleration $(a_t) = 0$

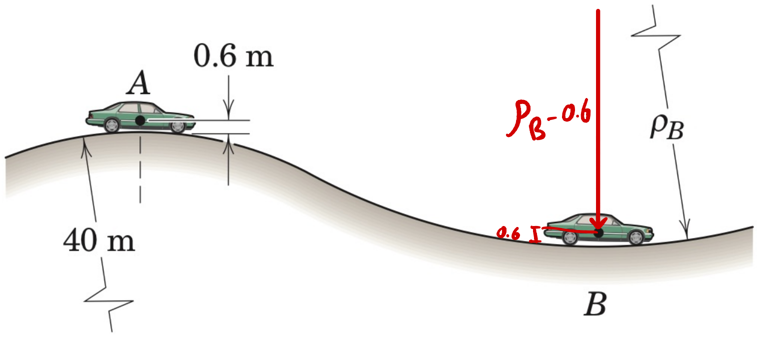
$$a = a_n = 0.4\,g = 0.4\,(9.81\ m/s^2) = 3.924\ m/s^2$$

$$a_n = \frac{V^2}{\rho}$$

$$V = \sqrt{a_n\,\rho} = \sqrt{(3.924)(98+2)} = 19.81\ m/s$$

2/107 The speed of a car increases uniformly with time from 50 km/h at A to 100 km/h at B during 10 seconds. The radius of curvature of the hump at A is 40 m. If the magnitude of the total acceleration of the mass center of the car is the same at B as at A, compute the radius of curvature ρ_B of the dip in the road at B. The mass center of the car is 0.6 m from the road.

* velocity increases uniformly = acceleration is constant



$$50\ Km/h * \frac{1000}{3600} = 13.89\ m/s \quad A$$

$$100\ Km/h * \frac{1000}{3600} = 27.78\ m/s \quad B$$

$$a_A = a_B$$

Using kinematics equation for constant acceleration between point A and B :-

$$V_B = V_A + a_t \cdot t$$
$$27.78 = 13.89 + a_t * 10$$
$$\boxed{a_t = 1.39\ m/s^2}$$

$$\rho_A = 40 + 0.6 = 40.6\ m$$

$$a_A^2 = a_n^2 + a_t^2 = \frac{V_A^2}{\rho_A} + a_t^2$$

$$a_A^2 = \left(\frac{(13.89)^2}{40.6}\right)^2 + (1.39)^2$$

$$\boxed{a_A = 4.95\ m/s^2}$$

Since $a_A = a_B$

$$a^2 = a_{nB}^2 + a_t^2$$

$$(4.95)^2 = a_{nB}^2 + (1.39)^2$$

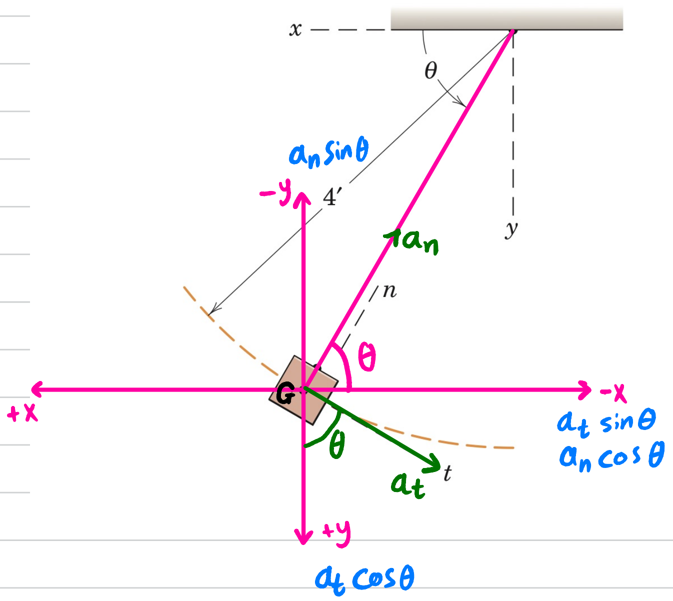
$$a_{nB} = 4.75\ m/s^2$$

$$a_{nB} = \frac{V_B^2}{\rho_B - 0.6}$$

$$\rho_B - 0.6 = \frac{(27.78)^2}{(4.75)}$$

$$\boxed{\rho_B = 163.1\ m}$$ #

2/112 Write the vector expression for the acceleration **a** of the mass center G of the simple pendulum in both n-t and x-y coordinates for the instant when $\theta = 60°$ if $\dot{\theta} = 2$ rad/sec and $\ddot{\theta} = 4.025$ rad/sec^2.



n-t coordinate

$$\vec{a} = \vec{a_n} + \vec{a_t}$$

$$= \frac{V^2}{\rho}\,n + \dot{V}\,t \qquad = r\dot{\theta}^2\,n + r\ddot{\theta}\,t$$

$$= 4(4)\,n + 4(4.025)\,t$$

$$\therefore a = 16\,n + 16.1\,t$$

x-y coordinate :-

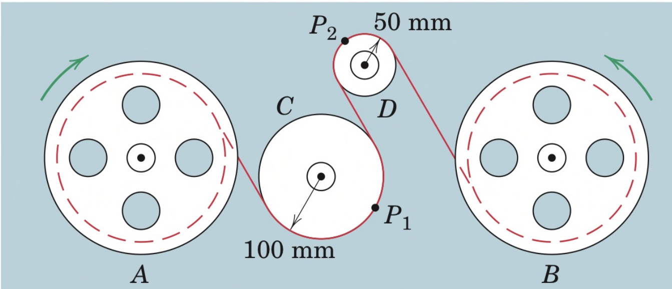
$$a_x = -a_t \sin\theta - a_n \cos\theta = -16.1 \sin(60) - 16 \cos(60) = -21.94$$

$$a_y = a_t \cos\theta - a_n \sin\theta = 16 \cos(60) - 16.1 \sin(60) = -5.94$$

$$\therefore a = -21.94\,\hat{i} - 5.94\,\hat{j}$$

2/114 Magnetic tape is being transferred from reel A to reel B and passes around idler pulleys C and D. At a certain instant, point P_1 on the tape is in contact with pulley C and point P_2 is in contact with pulley D. If the normal component of acceleration of P_1 is 40 m/s^2 and the tangential component of acceleration of P_2 is 30 m/s^2 at this instant, compute the corresponding speed v of the tape, the magnitude of the total acceleration of P_1, and the magnitude of the total acceleration of P_2.

* Since the pullys are connected by a single tape then the tangential acceleration at different points is the same.



$$a_{P_1}^2 = a_{nP_1}^2 + a_{tP_1}^2 = (40)^2 + a_{tP_1}^2$$

$$a_{P_2}^2 = a_{nP_2}^2 + a_{tP_2}^2 = a_{nP_2}^2 + (30)^2$$

$$a_n = \frac{v^2}{\rho} \Rightarrow 40 = \frac{v^2}{0.1} \Rightarrow \boxed{v = 2 \text{ m/s}}$$

* $a_{P_1} = \sqrt{(40)^2 + (30)^2} = 50 \text{ m/s}^2$

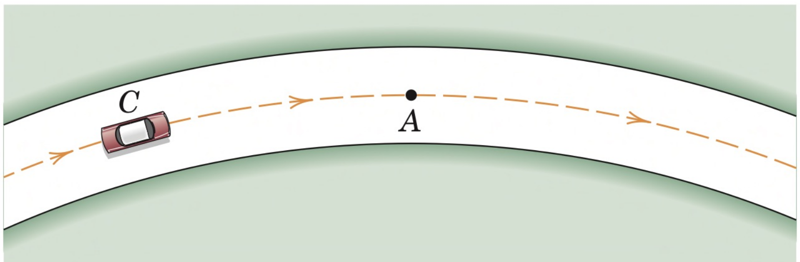
* $a_{P_2} = \sqrt{\left(\frac{2^2}{0.05}\right)^2 + (30)^2} = 85.44 \text{ m/s}^2$

2/115 The car C increases its speed at the constant rate of 1.5 m/s^2 as it rounds the curve shown. If the magnitude of the total acceleration of the car is 2.5 m/s^2 at the point A where the radius of curvature is 200 m, compute the speed v of the car at this point.

$a_t = 1.5 \text{ m/s}^2$

$a = 2.5 \text{ m/s}^2$

$\rho_A = 200 \text{ m}$



$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{2.5^2 - 1.5^2} = 2 = \frac{v^2}{\rho} \Rightarrow 2 = \frac{v^2}{200} \Rightarrow v = 20 \text{ m/s}$$

$m \rightarrow \frac{1}{1000} \text{ Km}$

$s \rightarrow \frac{1}{3600} \text{ h}$

$$v = 3.6 \times 20 = \boxed{72 \text{ Km/h}}$$

2/118 A particle moving in the x-y plane has a position vector given by $\mathbf{r} = \frac{3}{2}t^2\mathbf{i} + \frac{2}{3}t^3\mathbf{j}$, where $\mathbf{r}$ is in inches and t is in seconds. Calculate the radius of curvature ρ of the path for the position of the particle when $t = 2$ sec. Sketch the velocity $\mathbf{v}$ and the curvature of the path for this particular instant.

$$a_n = \frac{V^2}{\rho} \ ?$$

$$r = \tfrac{3}{2}t^2\,\hat{i} + \tfrac{2}{3}t^3\,\hat{j}$$

$$V = 3t\,\hat{i} + 2t^2\,\hat{j} \qquad @\, t=2 \rightarrow V = 6\hat{i} + 8\hat{j}$$

$$a = 3\hat{i} + 4t\,\hat{j} \qquad @\, t=2 \rightarrow a = 3\hat{i} + 8\hat{j}$$

$$|V| = \sqrt{6^2 + 8^2} = 10 \text{ m/s}$$

$$|a| = \sqrt{3^2 + 8^2} = 8.54 \text{ m/s}^2$$

unit vector in the direction of the tangintial acceleration which is also in the diriction of the velocity.

$$a_t = a \cdot \hat{e}_t = a \cdot \frac{V}{|V|}$$

$$= (3\hat{i} + 8\hat{j}) \cdot \frac{6\hat{i} + 8\hat{j}}{10}$$

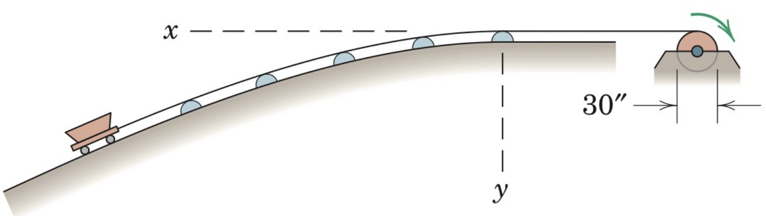
$$= 1.8 + 6.4 = 8.2 \text{ in/s}^2$$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{(8.54)^2 - (8.2)^2} = 2.4 \text{ in/s}^2$$

$$a_n = \frac{V^2}{\rho} \Rightarrow 2.4 = \frac{10^2}{\rho} \Rightarrow \boxed{\rho = 41.7 \text{ in}} \ \#$$

2/125 The mine skip is being hauled to the surface over the curved track by the cable wound around the 30-in. drum, which turns at the constant clockwise speed of 120 rev/min. The shape of the track is designed so that $y = x^2/40$, where x and y are in feet. Calculate the magnitude of the total acceleration of the skip as it reaches a level of 2 ft below the top. Neglect the dimensions of the skip compared with those of the path. Recall that the radius of curvature is given by

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$



Diameter = 30 in = 2.5 feet

$\dot{\theta}$ = 120 rpm

Tangential velocity :-

$V = r\dot{\theta}$

$V = (2.5)(120)\frac{(\pi)}{60} = 15.7$ m/s

$y = \frac{x^2}{40} \Rightarrow x = \sqrt{40y}$

y ≡ position of the skip when it reaches 2 ft below the top = 2 ft

$x = \sqrt{40(2)} = 8.94$ ft

$\frac{dy}{dx} = \frac{x}{20}$ $\frac{d^2y}{dx^2} = \frac{1}{20}$

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = 20\left[1 + \left(\frac{x}{20}\right)^2\right]^{\frac{3}{2}}$

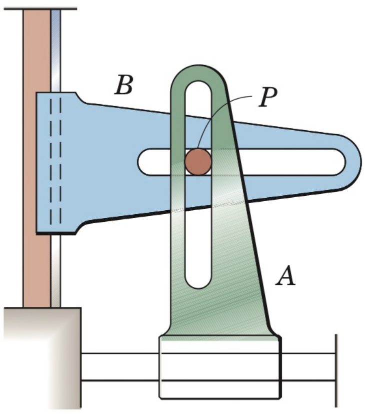
x ≡ position of the carriage = 8.94 ft

$\rho = 26.28$ ft

$a = \frac{v^2}{\rho} = \frac{(15.7)^2}{26.28}$

$a = 9.38$ ft/s²

▶**2/129** The pin P is constrained to move in the slotted guides which move at right angles to one another. At the instant represented, A has a velocity to the right of 0.2 m/s which is decreasing at the rate of 0.75 m/s each second. At the same time, B is moving down with a velocity of 0.15 m/s which is decreasing at the rate of 0.5 m/s each second. For this instant determine the radius of curvature ρ of the path followed by P. Is it possible to also determine the time rate of change of ρ?



A: 0.2 m/s (+x direction)

$\vec{V}_A = 0.2\hat{i}$

B: 0.15 m/s (−y direction)

$\vec{V}_B = -0.15\hat{j}$

$\vec{V} = \vec{V}_A + \vec{V}_B$
$= 0.2\hat{i} - 0.15\hat{j}$

magnitude of velocity at point p

$V = \sqrt{(0.2)^2 + (-0.15)^2}$

$V = 0.25$ m/s

$\vec{a}_A = 0.75\hat{i}$ m/s² $\qquad \vec{a}_B = 0.5\hat{j}$ m/s²

$\vec{a} = \vec{a}_A + \vec{a}_B = 0.75\hat{i} + 0.5\hat{j}$

$|\vec{a}| = \sqrt{(0.75)^2 + (0.5)^2} = 0.9$ m/s²

For the direction of the acceleration:-

Unit vector in direction of v ↗ $\quad \vec{n} = \dfrac{\vec{V}}{|\vec{V}|} = \dfrac{0.2\hat{i} - 0.15\hat{j}}{0.25} = 0.8\hat{i} - 0.6\hat{j}$

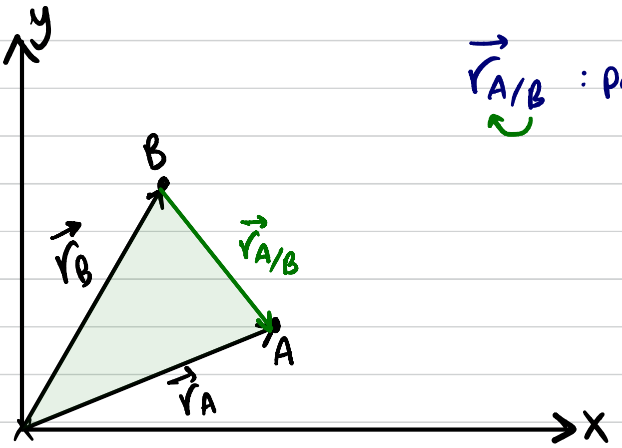
$\vec{a}_t = a \cdot \vec{n}$
$= -0.9(0.8\hat{i} - 0.6\hat{j}) = -0.72\hat{i} + 0.54\hat{j}$

$\vec{a} = \vec{a}_n + \vec{a}_t$
$a_n = \vec{a} - \vec{a}_t$

2/8 Relative Motion

Absolute motion:

Relative motion:

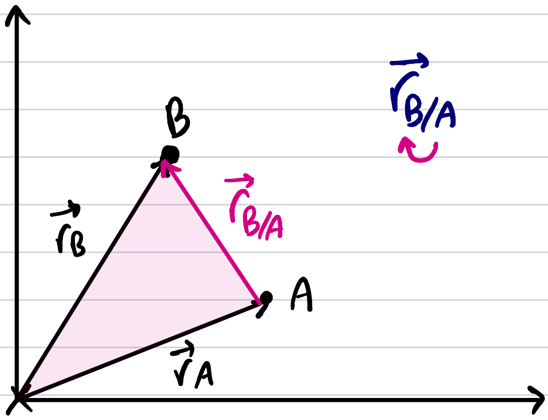


$\vec{r}_{A/B}$: position of A relative to B.

$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

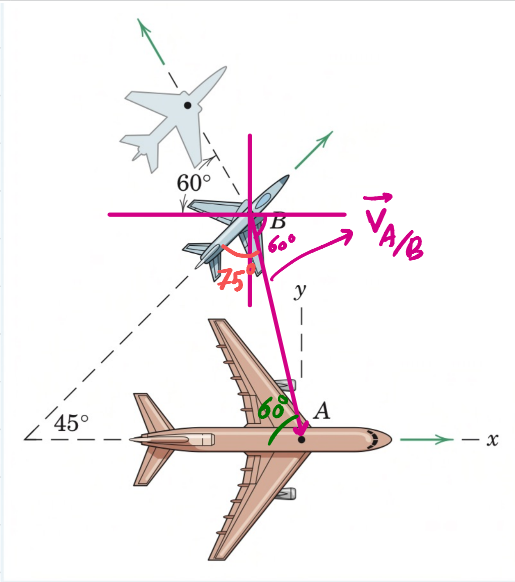


$\vec{r}_{B/A}$

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

SAMPLE PROBLEM 2/13

Passengers in the jet transport A flying east at a speed of 800 km/h observe a second jet plane B that passes under the transport in horizontal flight. Although the nose of B is pointed in the 45° northeast direction, plane B appears to the passengers in A to be moving away from the transport at the 60° angle as shown. Determine the true velocity of B.



$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$$

$$V_A = \frac{800 \times 10^3}{3600} = 222.22 \text{ m/s}$$

$$222.22\,\hat{i} = V_B \cos 45^\circ\,\hat{i} + V_B \sin 45^\circ\,\hat{j} + V_{A/B} \cos 60\,\hat{i} - V_{A/B} \sin 60\,\hat{j}$$

$$222.22 = V_B \cos 45^\circ + V_{A/B} \cos 60^\circ \quad \text{— (1)}$$

$$0 = V_B \sin 45^\circ - V_{A/B} \sin 60^\circ \quad \text{— (2)}$$

$$V_B = 717 \text{ Km/h}$$

$$V_{A/B} = 586 \text{ Km/h}$$

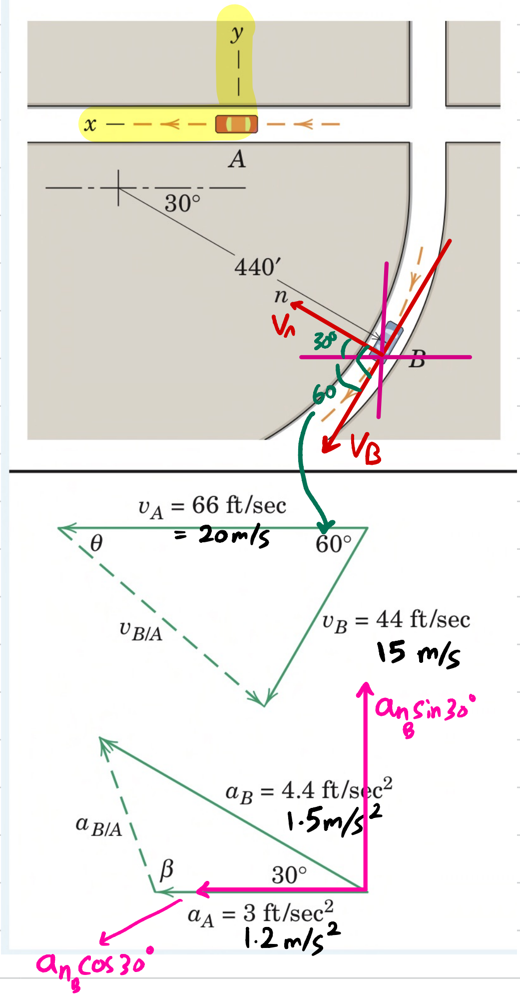
OR:-

Sin Law:-

$$\frac{800}{\sin(75^\circ)} = \frac{V_B}{\sin(60^\circ)} = \frac{V_{A/B}}{\sin 45}$$

SAMPLE PROBLEM 2/14

Car A is accelerating in the direction of its motion at the rate of $1.2\ m/s^2$. Car B is rounding a curve of 150m radius at a constant speed of 54 km/h. Determine the velocity and acceleration which car B appears to have to an observer in car A if car A has reached a speed of 72 km/h for the positions represented.



$*(a_t)_B = 0$ → because $V_B = 54$ km/h and constant

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

$$54 \cos 60° \hat{i} - 54 \sin 60° \hat{j} = 72 \hat{i} + (V_{B/A})_x \hat{i} + (V_{B/A})_y \hat{j}$$

$$54 \cos 60 = 72 + (V_{B/A})_x$$
$$-54 \sin 60 = (V_{B/A})_y$$

$$\theta = \tan^{-1}\left(\frac{(V_{B/A})_y}{(V_{B/A})_x}\right)$$

$$V_{B/A} = 18.03\ m/s$$

$$\theta = 46.1°$$

OR:-

Cosine Law:

$$V_A^2 + V_B^2 - 2 V_A V_B \cos 60° = V_{B/A}^2$$

$$20^2 + 15^2 - 2(20)(15) \cos 60 = V_{B/A}^2$$

$$V_{B/A} = 18.03\ m/s$$

*can be solved using the cosine law.

$(a_t)_B = 0$

$$a_n = \frac{V_B^2}{\rho_B} = \frac{15^2}{150} = 1.5\ m/s^2$$

(2) $\vec{a}_{B/A} = ?$ $\qquad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$$(a_n)_B \cos 30° \hat{i} + (a_n)_B \sin 30° \hat{j} = 1.2 \hat{i} + (a_{B/A})_x + (a_{B/A})_y$$

$$(a_{B/A})_x = 1.5 \cos 30° - 1.2 = 0.099\ m/s^2$$
$$(a_{B/A})_y = 1.5 \sin 30° = 0.75\ m/s^2$$

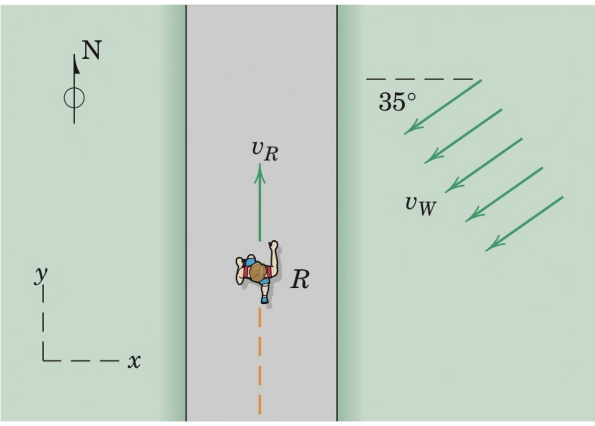
$$a_{B/A} = \sqrt{(a_{B/A})_x^2 + (a_{B/A})_y^2} = \sqrt{(0.099)^2 + (0.75)^2} = 0.757\ m/s^2$$

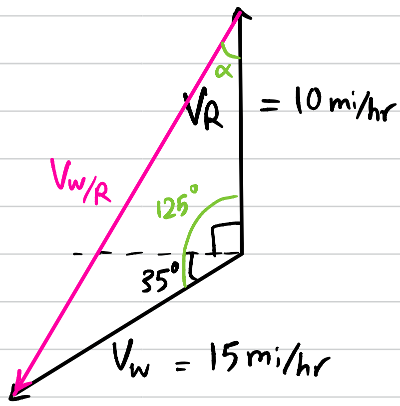
problems 2/8

$V_{W/R}$?

$$\boxed{V_W = V_R + V_{W/R}}$$

2/186 A marathon participant R is running north at a speed $v_R = 10$ mi/hr. A wind is blowing in the direction shown at a speed $v_W = 15$ mi/hr. (*a*) Determine the velocity of the wind relative to the runner. (*b*) Repeat for the case when the runner is moving directly to the south at the same speed. Express all answers both in terms of the unit vectors **i** and **j** and as magnitudes and compass directions.

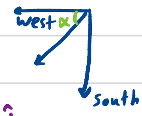




a) $V_{W/R} = \sqrt{V_W^2 + V_B^2 - 2V_W V_B \cos\theta}$ ⟵ using cosine law

$= \sqrt{(15)^2 + (10)^2 - 2(15)(10)\cos(125)} \Rightarrow V_{W/R} = 22.3$ mi/hr

finding the angle ⟶ $\dfrac{V_W}{\sin\alpha} = \dfrac{V_{W/R}}{\sin(125)} \Rightarrow \dfrac{15}{\sin\alpha} = \dfrac{22.3}{\sin(125)} \Rightarrow \alpha = 33.4°$ towards west of south

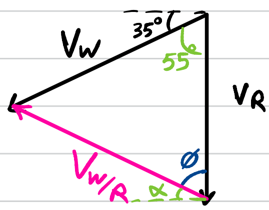


cartesian vector ⟶ $V_{W/R} = -V_{W/R}\sin\alpha\,\hat{i} - V_{W/R}\cos\alpha\,\hat{j} = -22.3\sin(33.4)\hat{i} - 22.3\cos(33.4)\hat{j}$

$V_{W/R} = -12.3\hat{i} - 18.6\hat{j}$

b) $V_{W/R} = \sqrt{V_W^2 + V_R^2 - 2V_W V_R \cos\theta}$

$= \sqrt{15^2 + 10^2 - 2(15)(10)\cos(55)} = 12.4$ mi/hr



find angle ⟶ $\dfrac{V_W}{\sin\varnothing} = \dfrac{V_{W/R}}{\sin(55)} \Rightarrow \dfrac{15}{\sin\varnothing} = \dfrac{12.4}{\sin(55)} \Rightarrow \varnothing = 82.27°$

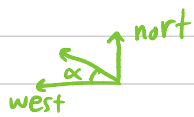
$\alpha = 90 - \varnothing$

$\alpha = 7.73°$ towards north of west

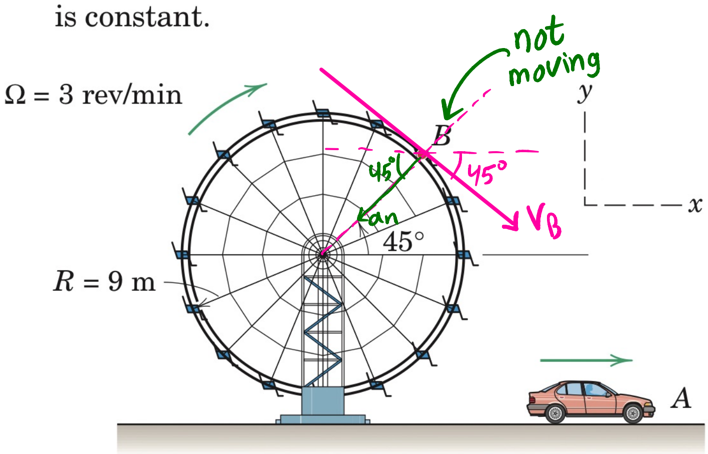
cartesian vector ⟶ $V_{W/R} = V_{W/R}\cos\alpha\,\hat{i} + V_{W/R}\sin\alpha\,\hat{j}$

$= -12.4\cos(7.73) + 12.4\sin(7.73)$

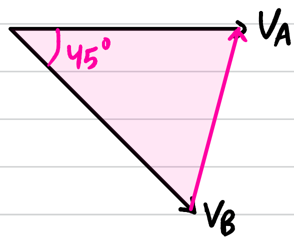
$= -12.3\hat{i} + 1.67\hat{j}$



2/188 The car A has a forward speed of 18 km/h and is accelerating at 3 m/s². Determine the velocity and acceleration of the car relative to observer B, who rides in a nonrotating chair on the Ferris wheel. The angular rate $\Omega = 3$ rev/min of the Ferris wheel is constant.



$18\,km/h = 5\,m/s = V_A$



rev → 2π
min → 60 s

angular velocity (rate Ω) is constant ∴ angular acceleration = 0.

a) $V = r\dot\theta = \dot\theta\,(r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j})$

$= \dot\theta\,(9\cos(45)\,\hat{i} + 9\sin(45)\,\hat{j})$

$= 3 * \frac{2\pi}{60}\,(9\cos(45)\,\hat{i} - 9\sin(45)\,\hat{j})$

$V_B = 2\hat{i} - 2\hat{j}$

$V_A = V_B + V_{A/B}$

$V_{A/B} = V_A - V_B = 5\hat{i} - 2\hat{i} + 2\hat{j} = 3\hat{i} + 2\hat{j}$

b) $a^2 = a_t^2 + a_n^2$ → at constant speed, $a_t = 0$

$a = a_n = \frac{V_B^2}{\rho} = r\dot\theta^2 = \left(3 * \frac{2\pi}{60}\right)^2 \left(-9\cos(45°)\,\hat{i} - 9\sin(45°)\,\hat{j}\right)$

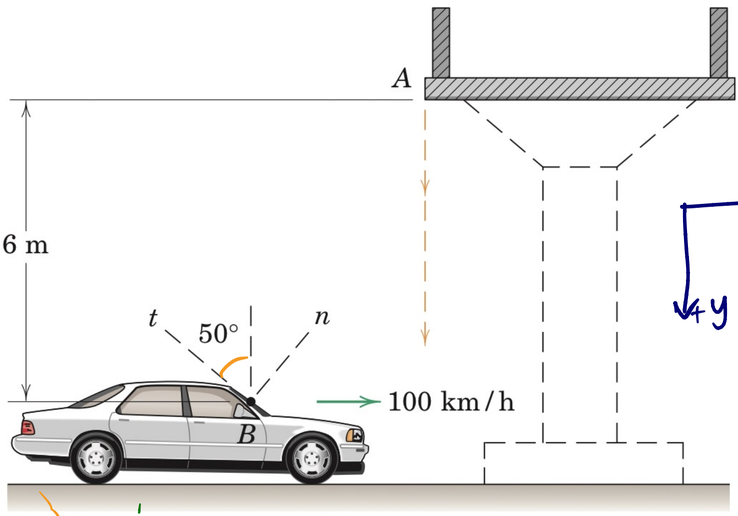
$= -0.628\,\hat{i} - 0.628\,\hat{j}$

$a_A = a_B + a_{A/B}$ $\qquad a_{A/B} = a_A - a_B$

$a_{A/B} = 3\hat{i} + 0.628\hat{i} + 0.628\hat{j}$

$a_{A/B} = 3.628\hat{i} + 0.628\hat{j}$

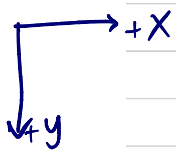
2/192 A drop of water falls with no initial speed from point A of a highway overpass. After dropping 6 m, it strikes the windshield at point B of a car which is traveling at a speed of 100 km/h on the horizontal road. If the windshield is inclined 50° from the vertical as shown, determine the angle θ relative to the normal n to the windshield at which the water drop strikes.



$$V_A^2 = V_{A_0}^2 + 2gh$$

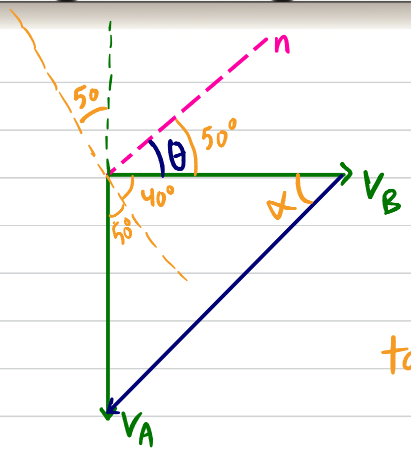
+y axis assumed to be down ↓

* no initial velocity so $V_{A_0} = 0$



$$V_A = \sqrt{2gh} = \sqrt{2(9.81)(6)}$$

$$V_A = 10.85\ m/s$$



$\theta_{A/n}$?

$$\tan\alpha = \frac{10.85}{\frac{100 \times 1000}{3600}} \Rightarrow \alpha = \tan^{-1}\left(\frac{10.85 * 3.6}{100}\right)$$

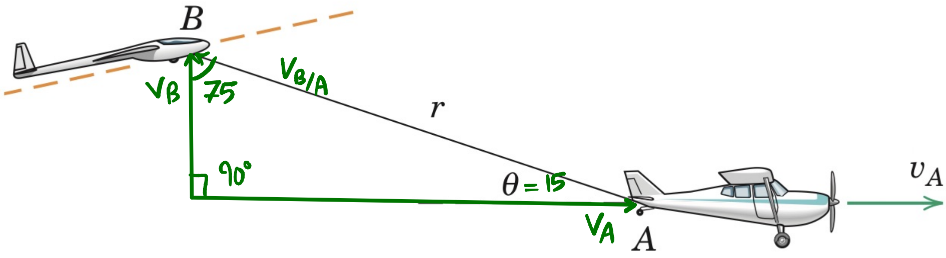
$$\alpha = 21.34$$

2/196 Airplane A is flying horizontally with a constant speed of 200 km/h and is towing the glider B, which is gaining altitude. If the tow cable has a length $r = 60$ m and θ is increasing at the constant rate of 5 degrees per second, determine the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the glider for the instant when $\theta = 15°$.

$V_A = 200 \text{ km/hr} = 55.56 \text{ m/s}$

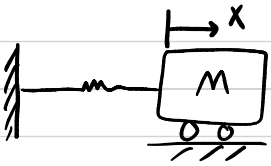
$\dot{\theta} = 5°/s = \frac{5\pi}{180} \text{ rad/s}$

$\dot{\theta} = 0.0873 \text{ rad/s}$



$\frac{V_{B/A}}{\sin 90} = \frac{55.}{\sin 75}$ $V_{B/A} =$

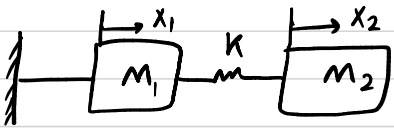
2.9 constrained Motion of connected particles.



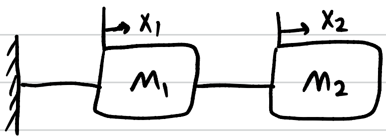
1 DOF ✓

Degree of freedoms (only 1 DoF in this course)

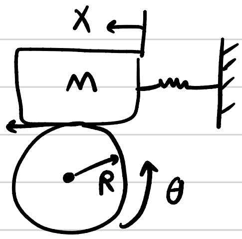
number of independant variables.



2 DoF



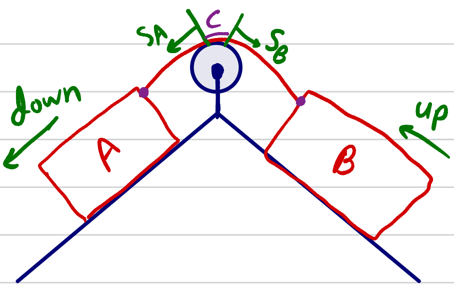
1 DoF ✓



$X = R\theta$

1 DoF

there is a relation between the two variables.



Cable length

$$\boxed{L = S_A + S_B + C}$$

derivate

$$0 = V_A + V_B + 0$$

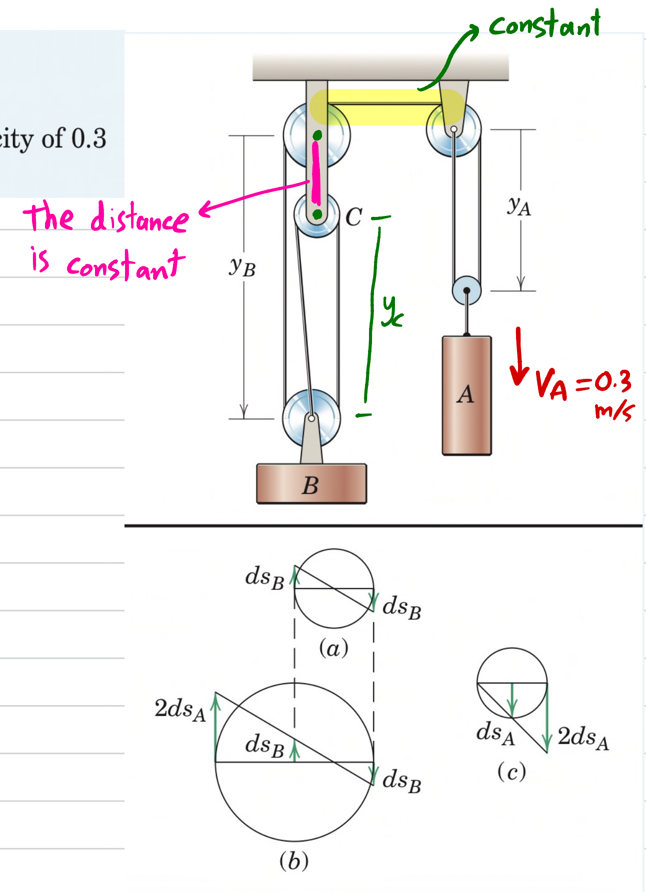
$$V_A = -V_B$$

$$a_A = -a_B$$

only for this

SAMPLE PROBLEM 2/15

In the pulley configuration shown, cylinder A has a downward velocity of 0.3 m/s. Determine the velocity of B. Solve in two ways.



$$L = 2y_A + 3y_B + \text{constants}$$

$$0 = 2\dot{y}_A + 3\dot{y}_B$$

$$2(0.3) = -3\dot{y}_B$$

$$* \quad \dot{y}_B = -0.2\ \text{m/s} \quad (\text{or } 0.2\ \text{m/s upward})$$

$$L = y_A + y_A + y_B + y_c + y_c + C$$

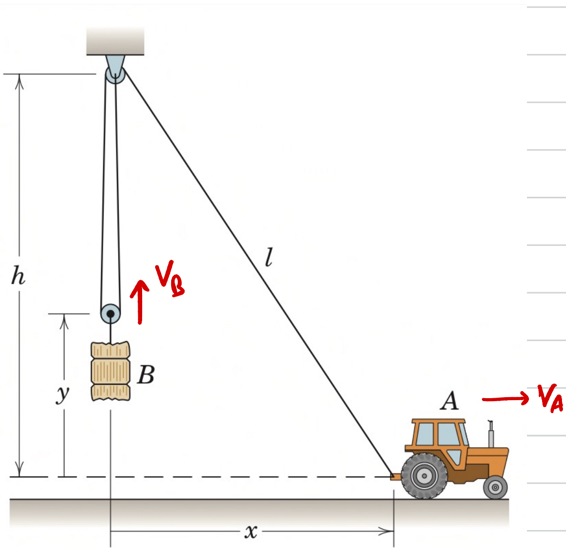
$$L = 2y_A + y_B + 2y_c + C$$

$$L = 2y_A + y_B + 2(y_B - c_1) + C$$

$$L = 2y_A + 3y_B + C$$

SAMPLE PROBLEM 2/16

The tractor A is used to hoist the bale B with the pulley arrangement shown. If A has a forward velocity v_A, determine an expression for the upward velocity v_B of the bale in terms of x.



$$L = 2(h - y_B) + l$$

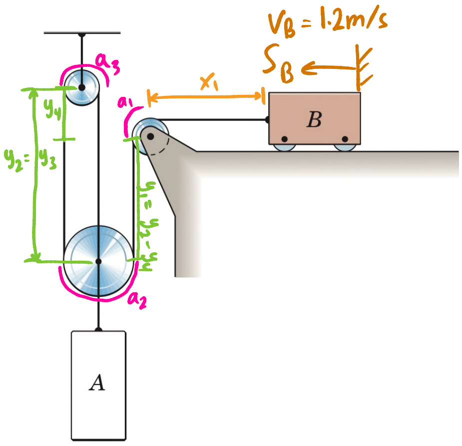
$$L = 2(h - y_B) + \sqrt{h^2 + x^2}$$

$$0 = -2V_B + \frac{2x\dot{x}}{2\sqrt{h^2 + x^2}}$$

$$= -2V_B + \frac{x V_A}{\sqrt{h^2 + x^2}} \Rightarrow V_B = \frac{x V_A}{2\sqrt{h^2 + x^2}}$$

problems 2/19

2/207 If block B has a leftward velocity of 1.2 m/s, determine the velocity of cylinder A.

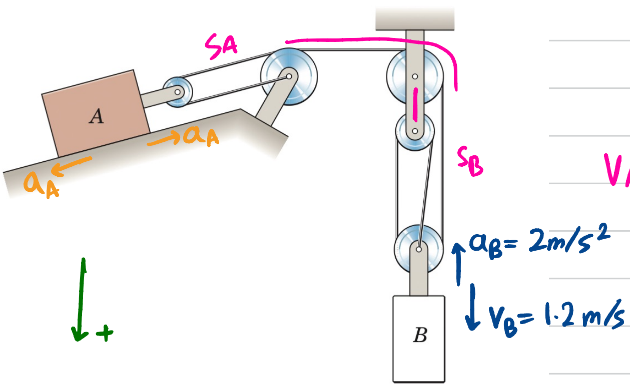


$$L = x_1 + a_1 + y_1 + a_2 + y_2 + a_3 + y_3$$
$$L = x_1 + y_1 + 2y_2 + C$$
$$L = x_1 + 3y_2 - y_4 + C$$
$$L = x_1 + 3y_2 + C$$ (assuming y_4 is constant)

$$0 = \dot{x}_1 + 3\dot{y}_2 + 0$$
$$0 = 1.2 + 3\dot{y}_2$$

$$\dot{y}_2 = -0.4 \text{ m/s}$$ (in downward direction.)

2/208 At a certain instant, the velocity of cylinder B is 1.2 m/s down and its acceleration is 2 m/s^2 up. Determine the corresponding velocity and acceleration of block A.



$$L = 2s_A + 3s_B + C$$

$$0 = 2\dot{s}_A + 3\dot{s}_B + 0$$
$$0 = 2\dot{s}_A + 3(1.2)$$

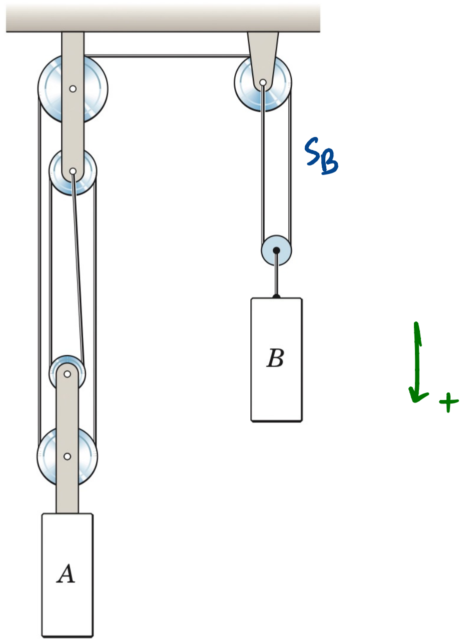
$$V_A = \dot{s}_A = -1.8 \text{ m/s}$$ (upward towards the incline)

$$0 = 2\ddot{s}_A + 3\ddot{s}_B$$

$$0 = 2\ddot{s}_A + 3(-2)$$

$$a_A = \ddot{s}_A = 3 \text{ m/s}^2$$ (downward along the incline).

2/209 Cylinder B has a downward velocity in feet per second given by $v_B = t^2/2 + t^3/6$, where t is in seconds. Calculate the acceleration of A when $t = 2$ sec.



$$L = 2s_B + 4s_A + C$$

$$0 = 2\dot{s}_B + 4\dot{s}_A$$

$$0 = 2\left(\frac{t^2}{2} + \frac{t^3}{6}\right) + 4\dot{s}_A$$

$$-4\dot{s}_A = t^2 + \frac{t^3}{3}$$

$$-4\ddot{s}_A = 2t + t^2$$

$$\ddot{s}_A \Big|_{t=2} = \frac{2(2)+(2)^2}{-4} = -2 \text{ ft/s}^2$$

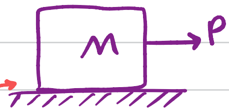
$\therefore a_A(t=2) = -2$ ft/s^2 (negative sign means in upward direction).

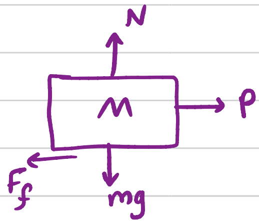
3 Kinetics of particles

Friction:-

→ Friction force :- is a tangential force generated between contacting surfaces

moving object has → Kinetic friction

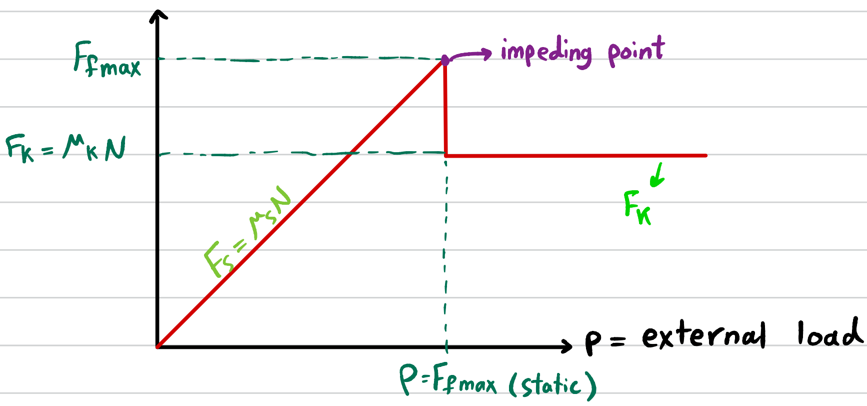




$F_s > F_k$

$\mu_s > \mu_k$

$F_f \equiv$ friction force



$F_s = \mu_s N$

$F_k = \mu_k N$

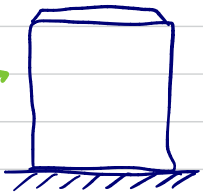
$\mu_s \equiv$ static coefficient friction

$\mu_k \equiv$ kinetic coefficient friction

$N \equiv$ normal force

Body starts moving only when $P = F_{s\,max}$

not moving



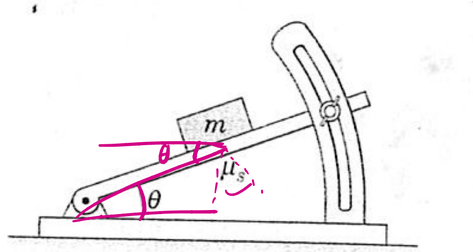
static object has static friction. (F_s)

static force > kinetic force

Sample Problem 6/1

static force because "before moving".

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.



$\swarrow^{+} \; \Sigma F_x = 0$

$mg\sin\theta - \mu_s N = 0$

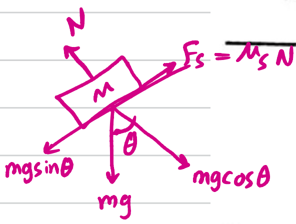
$mg\sin\theta = \mu_s N$

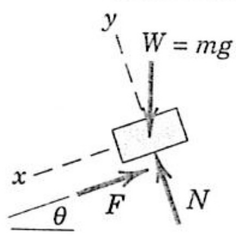
$\sin\theta = \dfrac{\mu_s N}{mg}$

$+\uparrow \; \Sigma F_y = 0$

$N - mg\cos\theta = 0$

$N = mg\cos\theta$

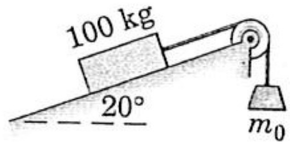




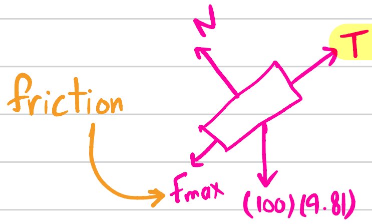
$\sin\theta = \dfrac{\mu_s \cancel{mg}\cos\theta}{\cancel{mg}} \Rightarrow \tan\theta = \mu_s \Rightarrow \theta = \tan^{-1}(\mu_s)$

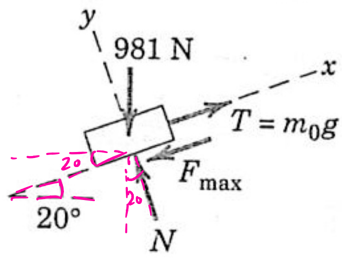
Sample Problem 6/2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.

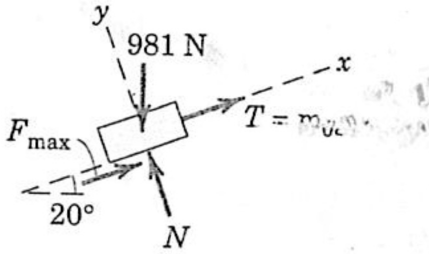


$\mu_s = 0.3$





Case I



Case II

$\Sigma F_y = 0 \qquad N - 981\cos 20° = 0 \qquad N = 922\,N$

$F_{max} = \mu_s N \qquad F_{max} = 0.3(922) = 277\,N$

$\Sigma F_x = 0 \qquad m_0(9.81) - 277 - 981\sin 20° = 0$

$\boxed{m_0 = 62.4\,kg}$ Case I

Case II

$\Sigma F_x = \quad m_0(9.81) + 277 - 981\sin 20 = 0$

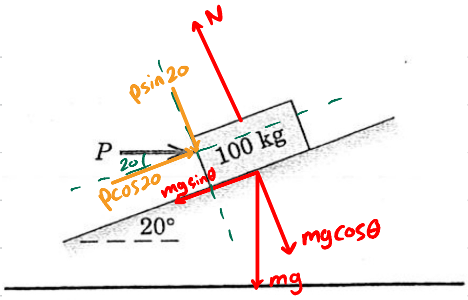
$\boxed{m_0 = 6.01\,kg}$

$\therefore$ m_0 may have any value from 6.01 kg to 62.4 kg, and the block will remain at rest.

* friction is bad in bearing, gears, ...
good in breaks, walking, ...

Sample Problem 6/3

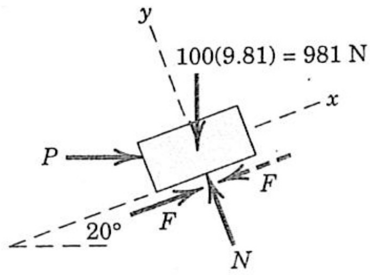
Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, $P = 500$ N and, second, $P = 100$ N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



@ $P = 500$

$\Sigma F_y = 0$

$$N - mg\cos 20 - P\sin 20 = 0$$
$$N = (100)(9.81)\cos 20° + 500 \sin 20° = 0$$
$$N = 1093 \text{ N}$$



$\Sigma F_x = 0$ (at rest)

assume that static friction is downward ↙

$$mg\sin 20 + F_s - P\cos 20° = 0$$
$$F_s = 500\cos 20° - (100)(9.81)\sin 20°$$
$$F_s = 134.3 \text{ N}$$

$$F_{s\,max} = \mu_s N = (0.2)(1093) = 218.6 \text{ N}$$

* since $F_s < F_{s\,max}$ the block isn't moving and the assumption is correct.

@ $P = 100$

$\Sigma F_y = 0$

$$N = 956 \text{ N}$$

$$F_{s\,max} = \mu_s N = (0.2)(956) = 191.2$$

$\Sigma F_x = 0$

$$F = -241.5 \text{ N}$$

The negative only indicates that the friction is in the opposite direction and we compare the magnitude with $F_{s\,max}$ $(|F_s| < F_{s\,max})$.

* since $F_s > F_{s\,max}$ that means that the block is moving so we need to use the kinetic friction.

$$F_k = \mu_k N = (0.17)(956) = 162.52$$

⇒ So the friction is 162.5 N in the upward direction ↗

★ Kinetics: relationship between force acting on a body and change its motion.

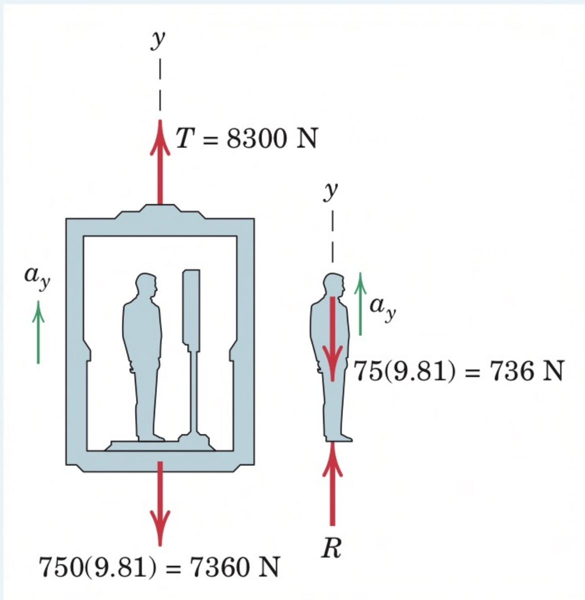
$\boxed{3.4}$ Rectilinear Motion

$$\Sigma F_x = m a_x \qquad \Sigma F_y = m a_y \qquad \Sigma F_z = m a_z$$

x

SAMPLE PROBLEM 3/1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.



$m_{man} = 75$ kg

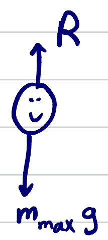
$V_o = 0$ (from rest)

$m_T = 750$ kg

V_{up}? @t=3

$\boxed{1}$ $R = ?$

Scale reading

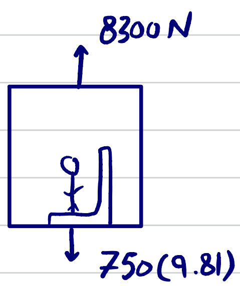


For man:-

$+\uparrow \; \Sigma F_y = m a_y$

$R - 75(9.81) = 75 a_y \Rightarrow R = 75(9.81) + 75(1.257)$

$R = 830$ N



For elevator:-

$+\uparrow \; \Sigma F_y = m a_y$

$8300 - 750(9.81) = 750 a_y$

$a_y = 1.257 \, m/s^2$

$\boxed{2}$ $V_f = \cancel{V_i}^{\,0} + a_y t$

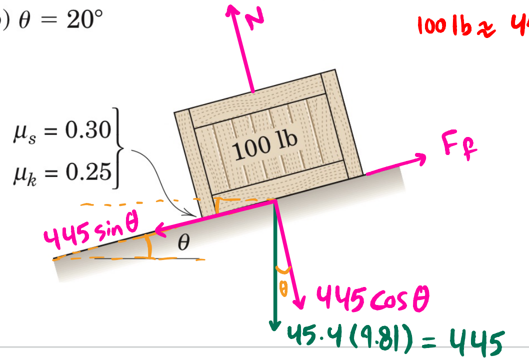
$V_f = 0 + (1.257)(3) \Rightarrow V_f = 3.77$ m/s

problems 3/4

3/3 The 100-lb crate is carefully placed with zero velocity on the incline. Describe what happens if (*a*) $\theta = 15°$ and (*b*) $\theta = 20°$

→ no acceleration

$100\,lb \approx 45.4\,Kg$



$\Sigma F_y = 0$

$N - 445\cos\theta = 0$

$N = 445\cos\theta$

$\Sigma F_x = 0$

$445\sin\theta - F_f = 0$

$445\sin\theta = F_f$

@ $\theta = 15°$

$F_f = 445\sin(15°) = 115.2\,N$

$F_{s\,max} = \mu_s N = (0.3)(445\cos\theta)$

$= (0.3)(445\cos 15°)$

$= 129$

since $F_f < F_{s\,max}$ ∴ The crate is not moving and have static friction upward.

@ $\theta = 20°$

$F_f = 445\sin(20°)$

$= 152.2\,N$

$F_{s\,max} = \mu_s N$

$= (0.3)(445\cos 20) = 125.45\,N$

⇒ since $F_f > F_{s\,max}$ ∴ The crate is moving and the static friction is upward.

$\Sigma F_x =$

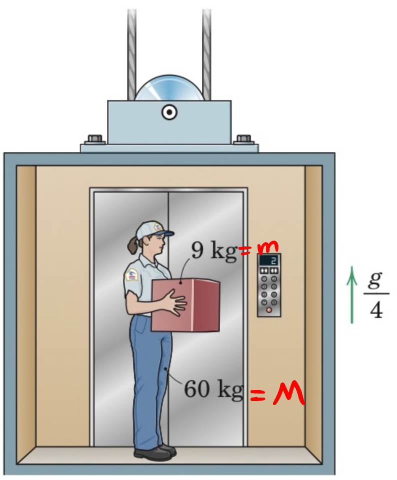
$445\sin\theta - \underbrace{F_f}_{\mu_k N} = m\,a_y$

$445\sin 20° - (0.25)(445\cos 20°) = 45.4\,a_y$

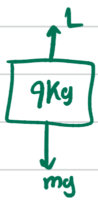
$47.6 = 45.4\,a_y$

$a_y = 1.05\,m/s^2$

3/4 A 60-kg woman holds a 9-kg package as she stands within an elevator which briefly accelerates upward at a rate of $g/4$. Determine the force R which the elevator floor exerts on her feet and the lifting force L which she exerts on the package during the acceleration interval. If the elevator support cables suddenly and completely fail, what values would R and L acquire?



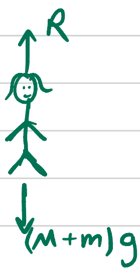
$a_y = \frac{g}{4}$



$\Sigma F_z = m a_y$

$L - 9(9.81) = 9\,\frac{(9.81)}{4}$

$L = 110.4\,N$



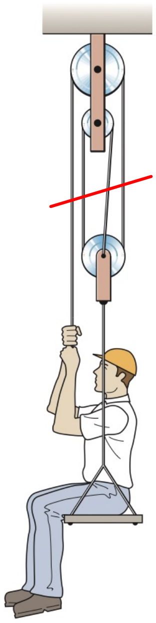
$\Sigma F_y = (M+m)\,a_y$

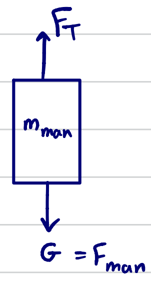
$R - (M+m)(9.81) = (M+m)\frac{(9.81)}{4}$

$R = (60+9)\frac{(9.81)}{4} + (60+9)(9.81)$

$R = 846.11\,N$

3/8 The 180-lb man in the bosun's chair exerts a pull of 50 lb on the rope for a short interval. Find his acceleration. Neglect the mass of the chair, rope, and pulleys.





$m_{man} = 81.6467$ kg
$m_{pull} = 22.68$ kg

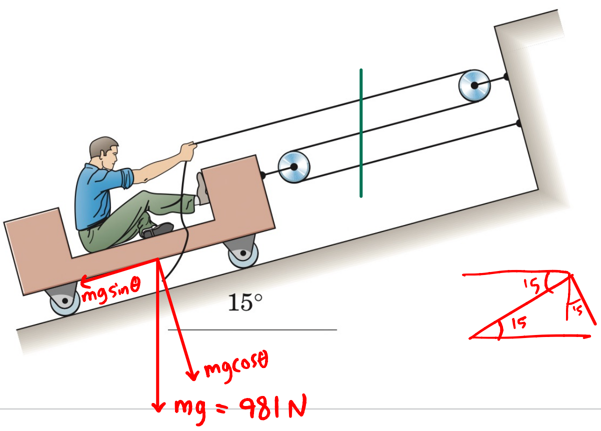
$F_{man} = m_{man} \cdot g$
$= 81.6467 * 9.81$
$= 800.95$ N

$F_T = 4 F_{pull}$
$= 4(222.5)$
$= 890$ N

$F_{pull} = m_{pull} \cdot g$
$= 22.68 * 9.81$
$= 222.5$ N

$\Sigma F_y = m_{man} a$
$F_T - G = m_{man} a$
$890 - 800.95 = 81.6467\, a$
$89.05 = 81.6467\, a$
$a = 1.2\ m/s^2$

3/9 A man pulls himself up the 15° incline by the method shown. If the combined mass of the man and cart is 100 kg, determine the acceleration of the cart if the man exerts a pull of 250 N on the rope. Neglect all friction and the mass of the rope, pulleys, and wheels.

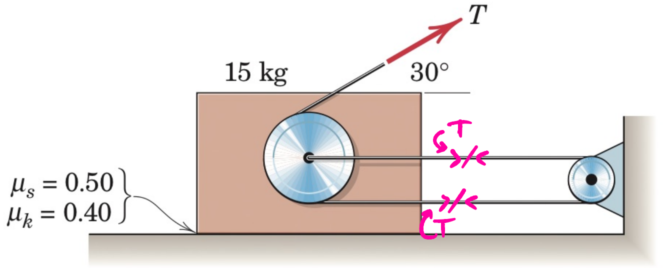


$T = 250$ N

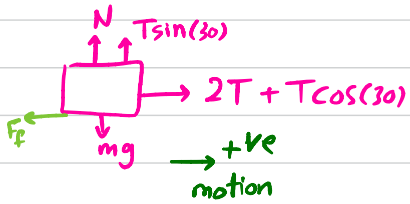
$\Sigma F_x = m a$
$3T - mg\sin\theta = m a$

$3(250) - 981\sin(15°) = 100\, a$
$a = 4.96\ m/s^2$

3/21 Determine the initial acceleration of the 15-kg block if (*a*) $T = 23$ N and (*b*) $T = 26$ N. The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.



1 $T = 23$ N



$\xrightarrow{+} \Sigma F_x = m a_x = 0$

assume that there is no motion until proven otherwise.

$2T + T\cos(30) - F_f = 0$

$F_f = 2T + T\cos(30)$
$= 2(23) + (23)\cos(30) = 65.9$ N

$F_{max} = \mu_s N = \mu_s(T\sin(30) - mg)$
$= 0.5(23\sin(30) - (15)(9.81))$

$F_{max} = 67.8$

Since $F_f < F_{max}$

it will <u>not</u> move.

2 $T = 26$N $\rightarrow a_x = ?$

X-axis:

$2T + T\cos(30) - F_f = 0$
$2(26) + 26\cos(30) - F_f = 0$
$F_f = 74.5$ N

y-axis:

$T\sin(30) + N - mg = 0$
$N = mg - T\sin(30)$
$N = 15(9.81) - 26(\sin 30)$
$= 134.15$ N

$F_{max} = N \cdot \mu_s$
$= 134.15(0.5) = 67$ N

$F_f = N \cdot \mu_k = 134.15(0.4)$
$= 53.66$ N

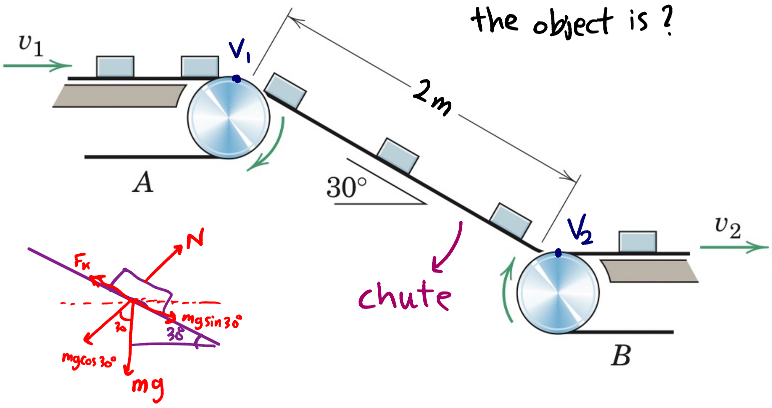
$F_{max} < F_f$
$\therefore$ it will move.

$2T + T\cos 30 - F_k = m a_x$
$2(26) + 26\cos(30) - 53.66 = 15 a_x$
$a_x = 1.39 \text{ m/s}^2$

years

3/23 Small objects are delivered to the 2m . inclined chute by a conveyor belt A which moves at a speed $v_1 =$ 0.4 m/s If the conveyor belt B has a speed $v_2 =$ 0.9 m/s and the objects are delivered to this belt with no slipping, calculate the coefficient of friction μ_k between the objects and the chute.

The acceleration of the object is ?



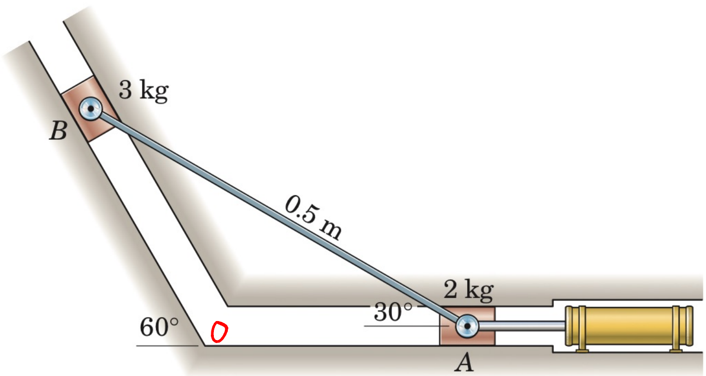
$$V_2^2 = V_1^2 + 2a(x - x_0)$$
$$V_2^2 - V_1^2 = 2a(x - x_0)$$
$$(0.9)^2 - (0.4)^2 = 2a(2)$$
$$a_x = 0.1625\ m/s^2$$

$$\Sigma F_x = m a_x$$
$$mg\sin 30 - \mu_k N = m a_x \qquad F_k = \mu_k N$$

$$\Sigma F_y = 0$$
$$N = mg\cos 30$$

$$\cancel{m}g(\sin 30 - \mu_k \cos 30) = \cancel{m}(0.1625)$$
$$\mu_k = 0.558$$

3/33 The sliders A and B are connected by a light rigid bar and move with negligible friction in the slots, both of which lie in a horizontal plane. For the position shown, the hydraulic cylinder imparts a velocity and acceleration to slider A of 0.4 m/s and 2 m/s^2, respectively, both to the right. Determine the acceleration of slider B and the force in the bar at this instant.



$$\frac{\sin 120}{x} = \boxed{\frac{\sin 30}{OA} = \frac{\sin 30}{OB}}$$

$$OA = OB$$

$$\frac{\sin 120}{0.5} = \frac{\sin 30}{OA} \qquad \therefore OA = OB = 0.288m$$

$$x^2 = OA^2 + OB^2\ 2(OA)(OB)\cos(\theta)$$
$$\frac{d(x^2)}{dt} = \frac{d}{dt}\left[OA^2 + OB^2 - 2(OA)(OB)\cos(\theta)\right]$$

$$2x\dot{x} = 2(OA)(\dot{OA}) + 2(OB)(\dot{OB}) - 2\cos(120°)\left[(OA)(\dot{OB}) + (OB)(\dot{OA})\right]$$

$$2(0.5)(0) = 2(0.288)(0.4) + 2(0.288)(\dot{OB}) - 2\cos(120°)\left[(0.288)(\dot{OB}) + (0.288)(0.4)\right]$$

$$(\dot{OB}) = V_B = -0.4\ m/s$$

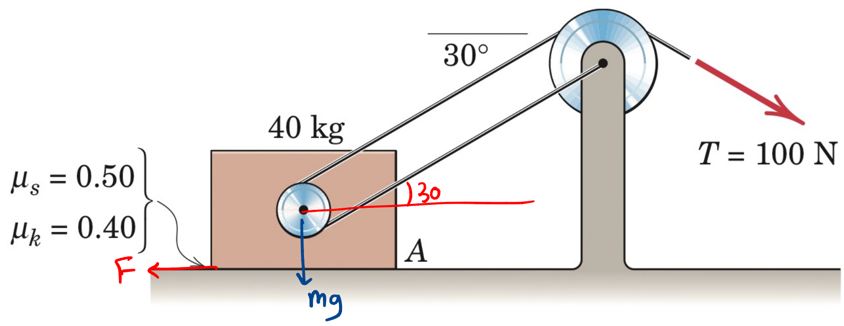
$$\frac{d}{dt}(0) = \frac{d}{dt}\left(2(OA)(\dot{OA}) + 2(OB)(\dot{OB}) - 2\cos(120°)\left[(OA)(\dot{OB}) + (OB)(\dot{OA})\right]\right)$$

$$0 = 2(OA)(\ddot{OA}) + 2(\dot{OA})^2 + 2(OB)(\ddot{OB}) + 2(\dot{OB})^2 - 2\cos(120°)\left[(OA)(\ddot{OB}) + (\dot{OB})(\dot{OA}) + (OB)(\ddot{OA}) + (\dot{OA})(\dot{OB})\right]$$

$$0 = 2(0.288)(2) + 2(0.4)^2 + 2(0.288)(\ddot{OB}) + 2(-0.4)^2 - 2\cos(120°)\left[(0.288)(\ddot{OB}) + (-0.4)(0.4) + (0.288)(2) + (0.4)(-0.4)\right]$$

$(\ddot{OB}) = a_B = -2.37\ m/s^2$ or $a_B = 2.37 m/s^2$ Downward.

3/37 Compute the acceleration of block A for the instant depicted. Neglect the masses of the pulleys.



$\Sigma F_y = 0$

$N = -2(100\sin(30°)) + (40)(9.81)$

$N = 292.4\ N$

Assume static equilibrium:

$\Sigma F_x = 0 \; ; \; -F + 200\cos(30°) = 0$

$F = 173.2\ N$

$F_{max} = \mu_s N = 0.5(292.4) = 146.2 < F$ ∴ Assumption is wrong, the Block will move.

$\Sigma F_x = ma$

$2(100\cos(30)) - \mu_k N = m a_x$

$200\cos(30) - 0.4(292.4) = 40\ a_x$

$a_x = 1.406\ m/s^2$

Chapter 3/5 : curvilinear motion

$\boxed{1}$ Rectangular coordinate

$$\Sigma F_x = ma_x \qquad a_x = \ddot{x}$$
$$\Sigma F_y = ma_y \qquad a_y = \ddot{y}$$

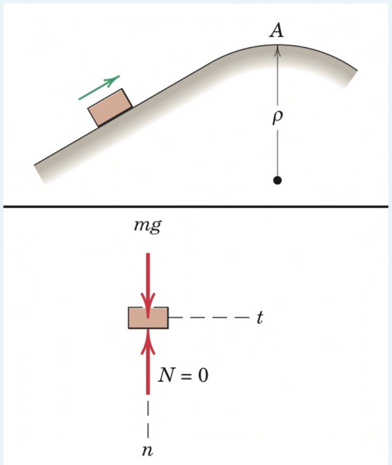
$\boxed{2}$ Normal & Tangential coordinates

$$\Sigma F_n = ma_n \qquad V = \rho\dot{\theta}$$
$$\Sigma F_t = ma_t \qquad a_t = \dot{V} = \rho\ddot{\theta}$$
$$a_n = \frac{V^2}{\rho} = \rho\dot{\theta}^2$$

positive, since it's towards the center of curvature.

SAMPLE PROBLEM 3/6

Determine the maximum speed v which the sliding block may have as it passes point A without losing contact with the surface.



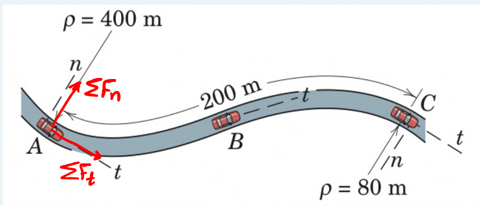
Solution. The condition for loss of contact is that the normal force N which the surface exerts on the block goes to zero. Summing forces in the normal direction gives

$[\Sigma F_n = ma_n]$ $\qquad mg = m\dfrac{v^2}{\rho} \qquad v = \sqrt{g\rho}$ *Ans.*

If the speed at A were less than $\sqrt{g\rho}$, then an upward normal force exerted by the surface on the block would exist. In order for the block to have a speed at A which is greater than $\sqrt{g\rho}$, some type of constraint, such as a second curved surface above the block, would have to be introduced to provide additional downward force.

SAMPLE PROBLEM 3/8

A 1500-kg car enters a section of curved road in the horizontal plane and slows down at a uniform rate from a speed of 100 km/h at A to a speed of 50 km/h as it passes C. The radius of curvature ρ of the road at A is 400 m and at C is 80 m. Determine the total horizontal force exerted by the road on the tires at positions A, B, and C. Point B is the inflection point where the curvature changes direction.



$V_A = 100\,\text{km/h} = 27.7\ \text{m/s}$

$V_c = 50\,\text{km/h} = 13.89\ \text{m/s}$

$$V_c^2 = V_A^2 + 2a_t(s_c - s_A)$$
$$(13.89)^2 = (27.7)^2 + 2(200)a_t$$
$$a_t = -1.43\ \text{m/s}^2$$

$$\Sigma F_n = ma_n = m\frac{V_A^2}{\rho_A} = 1500\left(\frac{27.7^2}{400}\right) = 2877\text{N}$$

$$\Sigma F_t = ma_t = 1500a_t$$
$$\Sigma F_t = -2145\ \text{N}$$

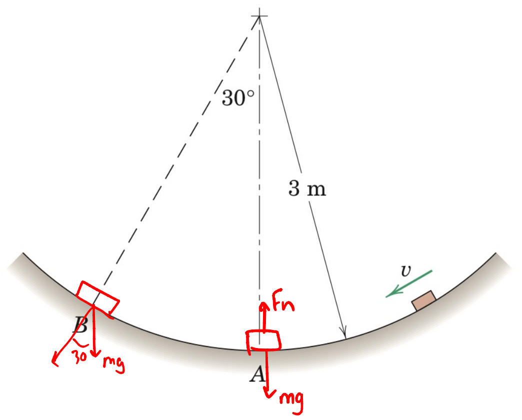
$$F_T = \sqrt{(\Sigma F_n)^2 + (\Sigma F_t)^2} = \sqrt{(2877)^2 + (2145)^2}$$

@ A : $F_T = 3588.6\,\text{N}$

continue @ B & C alone.

problems 3/5

3/47 The small 0.6-kg block slides with a small amount of friction on the circular path of radius 3 m in the vertical plane. If the speed of the block is 5 m/s as it passes point A and 4 m/s as it passes point B, determine the normal force exerted on the block by the surface at each of these two locations.



A: $\Sigma F_n = m a_n = m\left(\frac{v_A^2}{\rho}\right)$

$F_n - mg = 0.6\left(\frac{5^2}{3}\right) \Rightarrow F_n = 5 + (0.6)(9.81)$

$= 10.886\ N$

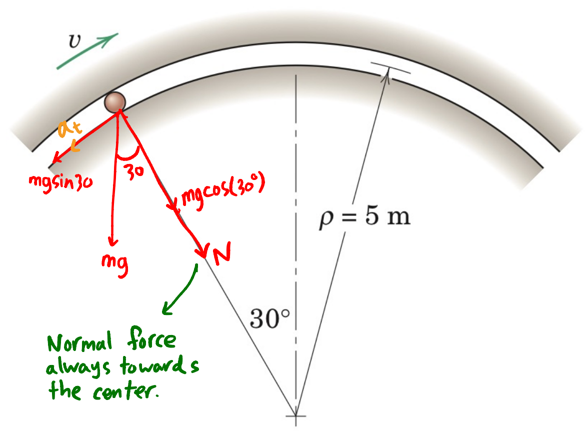
B: $\Sigma F_n = m a_n = m\left(\frac{v_B^2}{\rho}\right)$

$F_n - mg\cos(30^\circ) = 0.6\left(\frac{4^2}{3}\right)$

$= 3.2 + (0.6)(9.81)\cos(30^\circ)$

$= 8.3\ N$

3/49 The 0.1-kg particle has a speed $v = 10$ m/s as it passes the 30° position shown. The coefficient of kinetic friction between the particle and the vertical-plane track is $\mu_k = 0.20$. Determine the magnitude of the total force exerted by the track on the particle. What is the deceleration of the particle?



$\Sigma F_n = m a_n$

$N + mg\cos(30^\circ) = m\left(\frac{v^2}{\rho}\right)$

$N = 0.1\left(\frac{10^2}{5}\right) - (0.1)(9.81)\cos(30^\circ)$

$N = 1.15\ N$

$F_K = \mu_K N = (0.2)(1.15) = 0.23\ N$

Total force R exerted by the track:

$R = \sqrt{N^2 + F_K^2} = \sqrt{1.15^2 + 0.23^2} = 1.173 N$

friction + Normal forces

b)

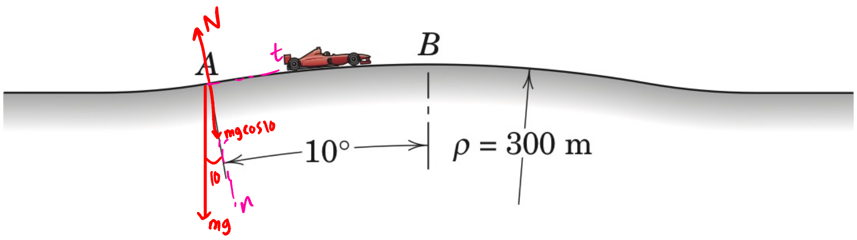
$\Sigma F_t = m a_t$

$F + mg\sin 30^\circ = m a_t$

$0.23 + (0.1)(9.81)\sin(30^\circ) = (0.1)\,a_t$

$a_t = 7.205\ m/s^2$ ← deceleration

3/57 A Formula-1 car encounters a hump which has a circular shape with smooth transitions at either end. (*a*) What speed v_B will cause the car to lose contact with the road at the topmost point B? (*b*) For a speed $v_A = 190$ km/h, what is the normal force exerted by the road on the 640-kg car as it passes point A?



Lost contact → Normal force = 0

$\Sigma F_n = ma_n$

$mg = N + m\frac{V_B^2}{\rho}$

$\cancel{m}g = \cancel{m}\frac{V_B^2}{\rho}$

$g = \frac{V_B^2}{\rho}$

$V_B = \sqrt{g\rho} = \sqrt{(9.81)(300)} = 54.25\ m/s$

$= 195.3\ Km/hr$

$\Sigma F_y = ma_y$

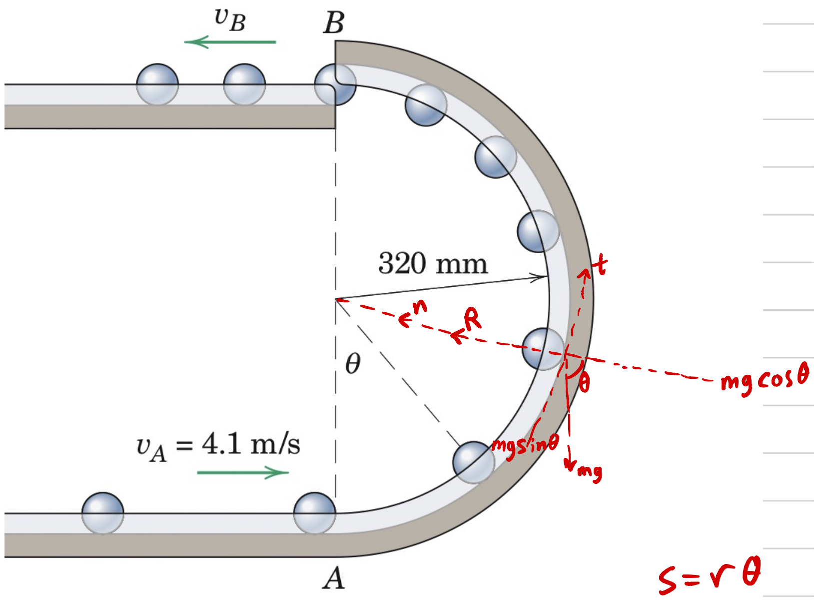
$mg\cos(10°) - N = m\left(\frac{V_A^2}{\rho}\right)$

$V_A = 160\ Km/hr = 52.7\ m/s$

$N = (640)(9.81)\cos(10°) - (640)\left(\frac{52.7^2}{300}\right) = 258.13\ N$

3/77 Small steel balls, each with a mass of 65 g, enter the semicircular trough in the vertical plane with a horizontal velocity of 4.1 m/s at A. Find the force R exerted by the trough on each ball in terms of θ and the velocity v_B of the balls at B. Friction is negligible.

$m = 65\,g$



$s = r\theta$

$ds = (r\,d\theta)$

$\Sigma F_n = m a_n$

$R - mg\cos\theta = m\dfrac{v^2}{0.32}$

$R = \dfrac{0.065}{0.32} v^2 + 0.065(9.81)\cos\theta$

$\Sigma F_t = m a_t$

$-\cancel{m}g\sin\theta = \cancel{m}a_t$

$a_t = -g\sin\theta$

$v\,dv = a_t\,ds = -g\sin\theta(r\,d\theta)$

$\int_{4.1}^{v} v\,dv = \int_{0}^{\theta} -gr\sin\theta\,d\theta$

$\dfrac{v^2}{2}\Big]_{4.1}^{v} = -gr\left(-\cos\theta\Big]_0^\theta\right)$

$\dfrac{v^2}{2} - \dfrac{4.1^2}{2} = +9.81(0.32)[\cos\theta - 1]$

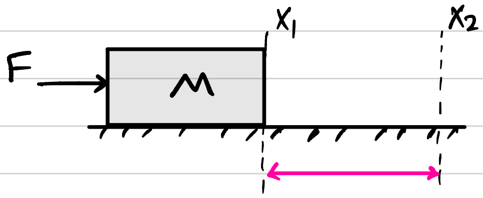
$v^2 = 10.53 + 6.28\cos\theta$

$\therefore R = 2.14 + 1.913\cos\theta$ N

for $\theta = 180°$: $v_B^2 = 10.53 + 6.28(-1)$

$v_B = 2.06$ m/s

$\boxed{3/6}$ work and energy



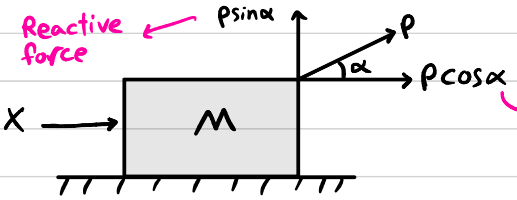
work

$$U_{1\to 2} = F(x_2 - x_1) \quad \text{in Joule} = N\cdot m$$

$\boxed{1}$ work associated by constant external force

dot product: $\hat{i}\cdot\hat{i} = 1$, $\hat{i}\cdot\hat{j} = 0$, $\hat{i}\cdot\hat{k} = 0$; $\hat{j}\cdot\hat{i} = 0$, $\hat{j}\cdot\hat{j} = 1$, $\hat{j}\cdot\hat{k} = 0$; $\hat{k}\cdot\hat{i} = 0$, $\hat{k}\cdot\hat{j} = 0$, $\hat{k}\cdot\hat{k} = 1$

Reactive force



active force

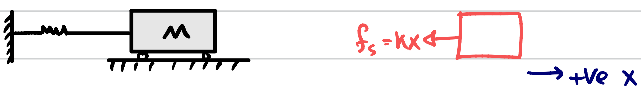
$$U_{1\to 2} = \int_{r_1}^{r_2} \vec{F}\cdot dr = \int_1^2 (P\cos\alpha\,\hat{i} + P\sin\alpha\,\hat{j})\cdot dx\,\hat{i} = \int_{x_1}^{x_2} P\cos\alpha\, dx = P\cos\alpha\,(x_2 - x_1)$$

Because in this case, it doesn't move in the $\hat{j}$ axis.

$\boxed{2}$ work associated with a spring force

+Ve $U_{1\to 2} \Rightarrow P$ with x direction.

-Ve $U_{1\to 2} \Rightarrow P$ opposite x direction.



$$U_{1\to 2} = \int_1^2 F.dr$$

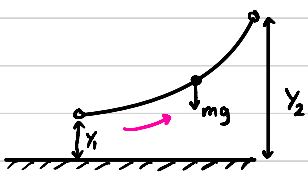
$$U_{1\to 2} = \int_1^2 -kx\,dx = -\frac{k}{2}x^2\Big]_{x_1}^{x_2} = \frac{1}{2}k(x_1^2 - x_2^2)$$

$\boxed{3}$ work associated with weight

$$U_{1\to 2} = \int_1^2 F.dr = \int_{x_1}^{x_2} -mg\,dy$$

$$= -mg(y_2 - y_1)$$

$$= mg(y_1 - y_2)$$



principle of working Energy

Kinetic energy $(T) = \frac{1}{2} m V^2$

$T_1 + U_{1\to2} = T_2$ } → work-energy equation

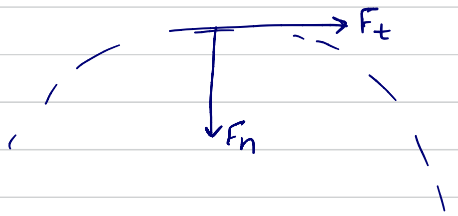
constant external force — weight — spring

$\Delta T = T_2 - T_1 = \frac{1}{2} m V_2^2 = \frac{1}{2} m V^2$

in n-t coordinates:-

F_n : does no work (reactive force).
F_t : does work (Active force).

→ work positive if F_t in the direction of the displacement.
→ // negative // // // // opposite direction of the displacement.



Power $= \frac{U_{1\to2}}{time}$: time rate at which it can do work or deliver energy.

$= \vec{F} \cdot \vec{V}$

η = efficiency $= \frac{P_{out}}{P_{in}}$

SAMPLE PROBLEM 3/11

Calculate the velocity v of the 50-kg crate when it reaches the bottom of the chute at B if it is given an initial velocity of 4 m/s down the chute at A. The coefficient of kinetic friction is 0.30.

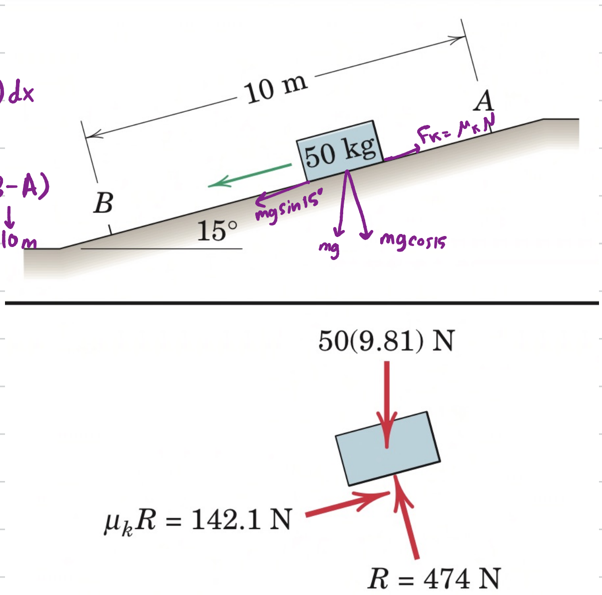
$\mu_K = 0.3$
$V_A = 4 m/s$
$V_B = ??$

$$U_{A\to B} = \int_A^B \vec{F}\,dx = \int_A^B mg\sin 15^\circ - \mu_K(mg\cos 15^\circ)\,dx$$

$$U_{A\to B} = (mg\sin 15^\circ - \mu_K\, mg\cos 15^\circ)(B-A)$$

10 m

$$T_A + U_{A\to B} = T_B$$



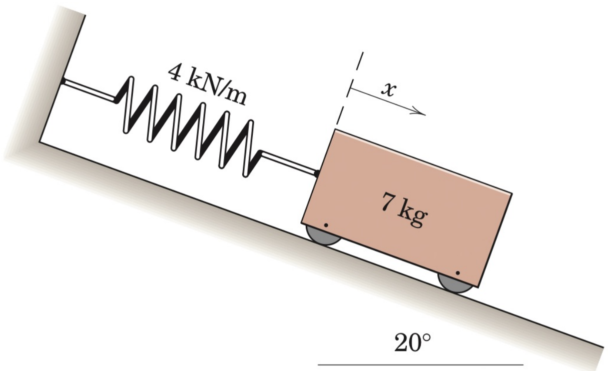
$$\frac{1}{2} m V_A^2 + (mg\sin 15^\circ - \mu_k\, mg\cos 15^\circ)(10) = \frac{1}{2} m V_B^2$$

$$\therefore V_B = 3.15 \text{ m/s}$$

problems 3/6

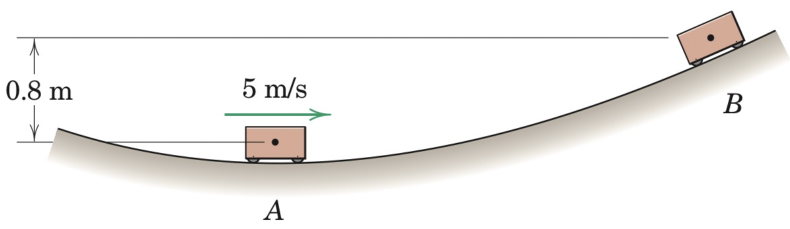
3/97 The spring is unstretched when $x = 0$. If the body moves from the initial position $x_1 = 100$ mm to the final position $x_2 = 200$ mm, (a) determine the work done by the spring on the body and (b) determine the work done on the body by its weight.

$X_1 = 100$ mm
$X_2 = 200$ mm
$K = 4$ kN/m



$$U_{1\to 2} = \frac{1}{2} K(x_1^2 - x_2^2) = \frac{1}{2}\, 4\times 10^3\,(0.1^2 - 0.2^2) = -60\text{ J}$$

3/98 The small body has a speed $v_A = 5$ m/s at point A. Neglecting friction, determine its speed v_B at point B after it has risen 0.8 m. Is knowledge of the shape of the track necessary?



$T_A = \frac{1}{2} m V_A^2 = \frac{25}{2} m$

$T_B = \frac{1}{2} m V_B^2$

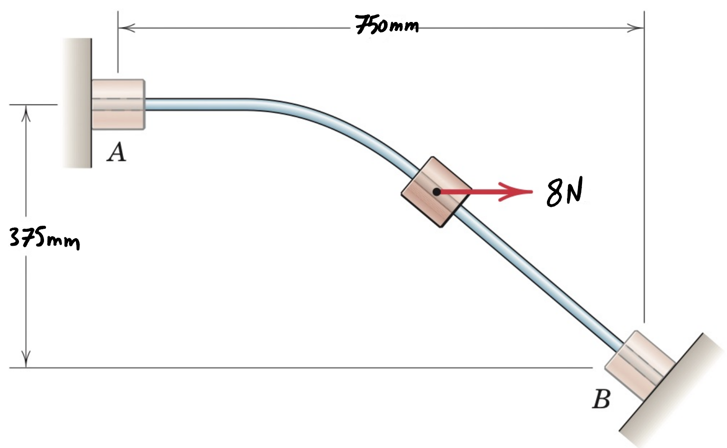
$T_1 + U_{1\to 2} = T_2$

$\frac{25}{2}\cancel{m} + \cancel{m} g(y_1 - y_2) = \frac{1}{2}\cancel{m} V_B^2$

$\frac{25}{2} + (9.81)(0-0.8) = \frac{1}{2} V_B^2$

$V_B = 3.05$ m/s

3/100 The 0.8 Kg collar slides with negligible friction on the fixed rod in the vertical plane. If the collar starts from rest at A under the action of the constant 8-N horizontal force, calculate its velocity v as it hits the stop at B.



$V_A = 0$

$T_1 + U_{1\to 2} = T_2$

$0 + mg(y_1 - y_2) + F(x_2 - x_1) = \frac{1}{2} m V_B^2$

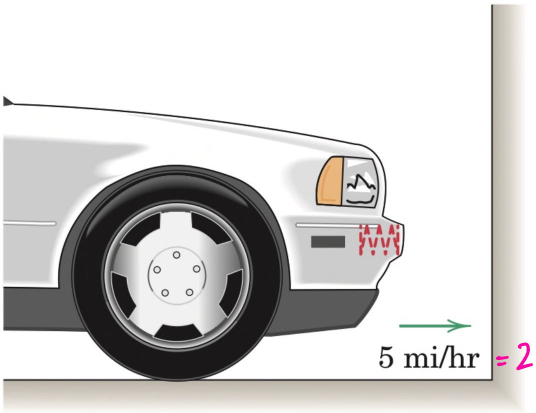
$(0.8)(9.81)(0.375) + 8(0.75) = \frac{1}{2}(0.8) V_B^2$

$V_B = 4.73$ m/s

1587.57 kg

15.24 m

3/101 In the design of a spring bumper for a 3500-lb car, it is desired to bring the car to a stop from a speed of 5 mi/hr in a distance equal to 6 in. of spring deformation. Specify the required stiffness k for each of the two springs behind the bumper. The springs are undeformed at the start of impact.



= 2.235 m/s

$$T_1 + U_{1\to 2} = T_2$$

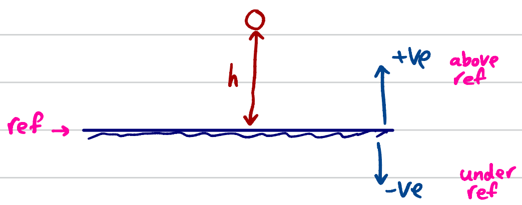
$$0 + 2 \cdot \frac{1}{2} k x^2 = \frac{1}{2} m V^2$$

$$k(15.24)^2 = \frac{1}{2}(1587.57)(2.235)^2$$

$$k = 17.07 \text{ N/m}$$

3/7 Potential Energy P.E:

1 Gravitational P.E:



$V_g = mgh$

2 Elastic P.E

$V_e = \frac{1}{2} k x^2$ (always positive)

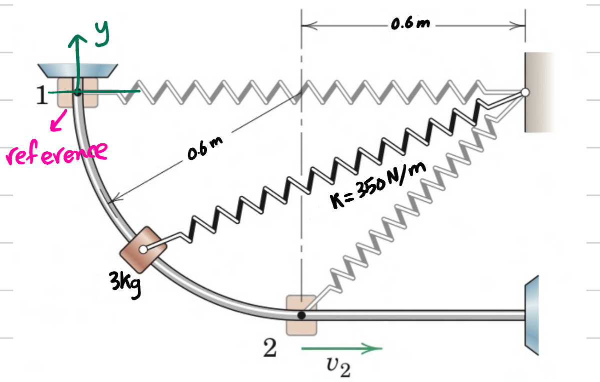
→ work - energy equation

$$T_1 + V_{e_1} + V_{g_1} + U'_{1\to2} = T_2 + V_{e_2} + V_{g_2}$$

$U'_{1\to2}$ → work done by all external forces except gravity & spring forces.

SAMPLE PROBLEM 3/16

The 6-lb slider is released from rest at position 1 and slides with negligible friction in a vertical plane along the circular rod. The attached spring has a stiffness of 2 lb/in. and has an unstretched length of 24 in. Determine the velocity of the slider as it passes position 2.



$V_1 = 0$ m/s

Unstretched length = 0.6 m

$V_2 = ??$

from rest $V_1 = 0$ no external forces under the reference

$$T_1 + V_{e_1} + V_{g_1} + U'_{1\to2} = T_2 + V_{e_2} + V_{g_2}$$

$$0 + \frac{1}{2} k x_1^2 + \cancel{mgh_1}^{0} + 0 = \frac{1}{2} m V_2^2 + \frac{1}{2} k x_2^2 - mgh_2$$

$$\frac{1}{2}(350)(0.6)^2 = \frac{1}{2}(3)(V_2)^2 + \frac{1}{2}(350)(\sqrt{2}(0.6) - 0.6)^2 - 3(9.81)(0.6)$$

$$V_2 = 6.82 \text{ m/s}$$

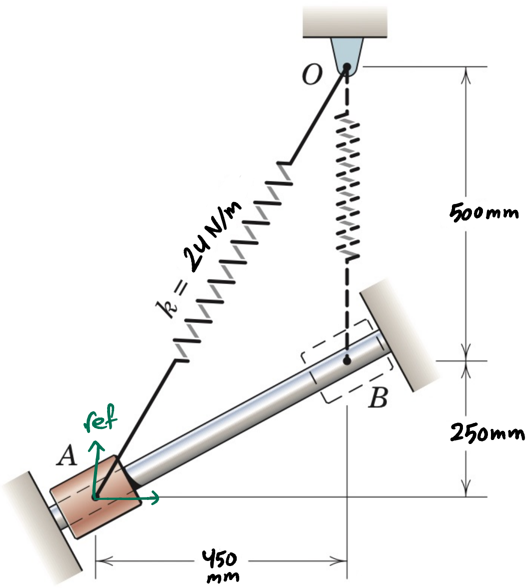
x_1 = final length - unstretched length
$= 1.2 - 0.6$
$= 0.6$

$x_2 = \sqrt{0.6^2 + 0.6^2} - 0.6$
$= \sqrt{2}(0.6) - 0.6$

problems 3/7

3/139 The 2-lb collar is released from rest at A and slides freely up the inclined rod, striking the stop at B with a velocity v. The spring of stiffness $k = 1.60$ lb/ft has an unstretched length of 15 in. Calculate v.

0.9 Kg



$V_A = 0$ $\quad V_B$?

unstretched length = 375mm

$\cancel{T_1}^{0} + Ve_1 + \cancel{Vg_1}^{0} + U_{1\to2} = T_2 + Ve_2 + Vg_2$

$$\frac{1}{2} k x_1^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x_2^2 + mgh_2$$

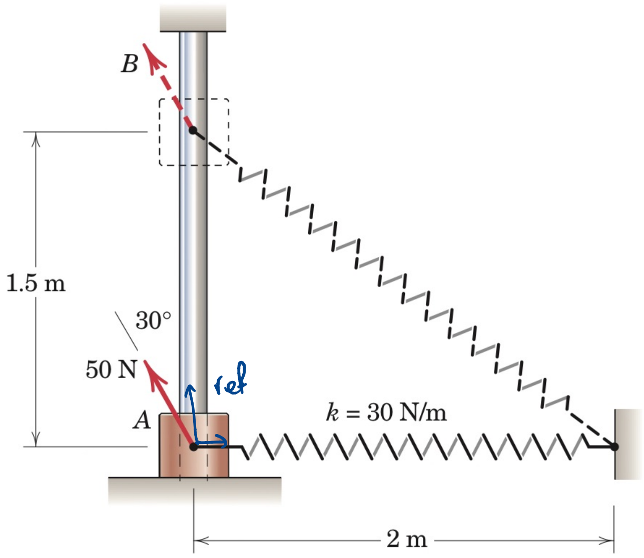
$x_1 = \sqrt{0.75^2 + 0.45^2} - 0.375 = 0.5\,m$

$x_2 = 0.5 - 0.375 = 0.125\,m$

$$\frac{1}{2}(24)(0.5)^2 = \frac{1}{2}(0.9)V^2 + \frac{1}{2}(24)(0.125)^2 + (0.9)(9.81)(0.25)$$

$V = 1.156\ m/s$

3/158 The collar has a mass of 2 kg and is attached to the light spring, which has a stiffness of 30 N/m and an unstretched length of 1.5 m. The collar is released from rest at A and slides up the smooth rod under the action of the constant 50-N force. Calculate the velocity v of the collar as it passes position B.



$$X_1 = 2 - 1.5 = 0.5$$

$$X_2 = \sqrt{2^2 + 1.5^2} - 1.5 = 1$$

$$\cancel{T_1}^{0} + V_{e_1} + \cancel{V_{g_1}}^{0} + U_{1\to2} = T_2 + V_{e_2} + V_{g_2}$$

$$\frac{1}{2}kx_1^2 + 50\cos 30^\circ (y_2 - y_1) = \frac{1}{2}mV^2 + \frac{1}{2}kx_2^2 + mgh$$

$$\frac{1}{2}(30)(0.5)^2 + 50\cos 30^\circ (1.5) = \frac{1}{2}(2)V^2 + \frac{1}{2}(30)(1)^2 + (2)(9.81)(1.5)$$

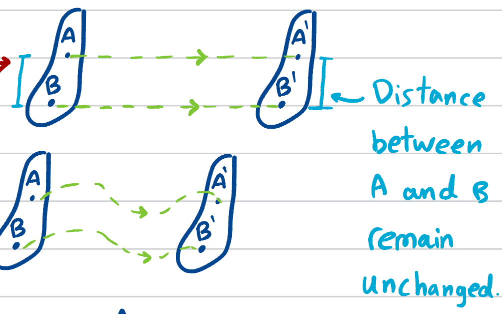
$$V = 4.93 \text{ m/s}$$

→ Chapter 5:- Kinematics of Rigid Bodies:

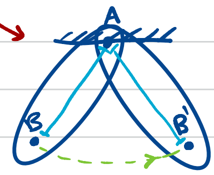
⇒ **Rigid body:** a system of particles for which distances between the particles remain unchanged.

→ plane Motion:

1. Translational
   - → Rectilinear.
   - → curvilinear.

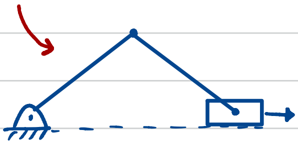


2. Rotation about fixed axis.



3. General plane Motion.

A combination of 1 and 2



$$V = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$a\,ds = v\,dv$$

→ Remember

$\boxed{5/2}$ Rotation

$$(rad/s) \leftarrow w = \frac{d\theta}{dt} = \dot{\theta} \quad \dots (1)$$

$$\alpha = \frac{dw}{dt} = \dot{w} \quad \dots (2)$$

$$\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta} \quad \dots (3)$$

$$\alpha\, d\theta = w\, dw \quad \dots (4) \longrightarrow V dv = a ds$$

at constant angulalar acceleration (α_c)

$$w = w_o + \alpha_c t \quad \dots (5)$$

$$w^2 = w_o^2 + 2\alpha_c(\theta - \theta_o) \quad \dots (6)$$

$$\theta = \theta_o + w_o t + \frac{1}{2}\alpha_c t^2 \quad \dots (7)$$

Remember:
1 rev = 2π rad

SAMPLE PROBLEM 5/1

A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. The torque produces a counterclockwise angular acceleration $\alpha = 4t$ rad/s^2, where t is the time in seconds during which the torque is applied. Determine (a) the time required for the flywheel to reduce its clockwise angular speed to 900 rev/min, (b) the time required for the flywheel to reverse its direction of rotation, and (c) the total number of revolutions, clockwise plus counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

w

*we use :
cw → -ve
ccw → +ve

$w_1 = 1800$ rev/min cw

$\alpha = 4t$ rad/s^2 ccw

$\boxed{a}$ $t = ?$ at $w_2 = 900$ rev/min cw
$= 30\pi$ rad/s cw

$w_1 = 1800\left(\frac{2\pi}{60}\right) = 60\pi$ rad/s cw

$$\alpha = \frac{dw}{dt} \longrightarrow \int_{t=0}^{t} 4t\,dt = \int_{-60\pi}^{w} dw$$

$$2t^2\Big]_0^t = w - -60\pi = w + 60\pi \qquad \boxed{2t^2 - 60\pi = w}$$

$$-30\pi = 2t^2 - 60\pi \qquad \boxed{t = 6.86 \text{ sec}}$$

$\boxed{b}$ time to reverse direction:

when the direction reverses there is a time where $(w) = 0$.

$$0 = -60\pi + t^2 \qquad \boxed{t = 9.71 \text{ sec}}$$

$\boxed{c}$

$$w = \frac{d\theta}{dt} \qquad w\,dt = d\theta$$

cw

$$\int_{t=0}^{9.71} (-60\pi + 2t^2)\,dt = \int_0^{\theta_1} d\theta$$

$$-60\pi t + \frac{2}{3}t^3\Big]_0^{9.71} = \theta_1$$

$\rightarrow$ $\boxed{-1220 \text{ rad} = \theta_1 \text{ cw}}$

ccw

$$\int_{9.71}^{14} (-60\pi + 2t^2)\,dt = \int_0^{\theta_2} d\theta$$

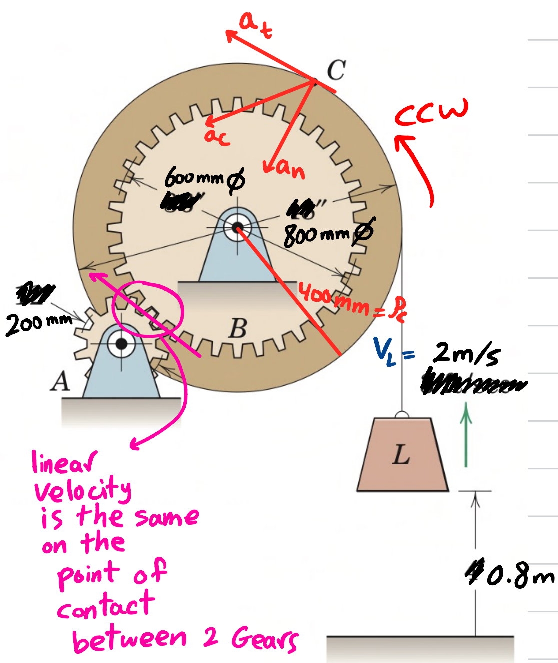
$\boxed{\theta_2 = 410 \text{ rad ccw}}$

$$\frac{\theta_1}{2\pi} + \frac{\theta_2}{2\pi} = N \longrightarrow 194.2 + 65.3 \cong 259 \text{ rev}$$

we don't take negative

SAMPLE PROBLEM 5/2

The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upward velocity of 2m/s in a vertical rise of 0.8m with constant acceleration. As the load passes this position, compute (a) the acceleration of point C on the cable in contact with the drum and (b) the angular velocity and angular acceleration of the pinion A.



a)

$$\vec{a}_c^{\,2} = (\vec{a}_n)_c^2 + (\vec{a}_t)_c^2$$

$$= \frac{V_c^2}{r_c} + (a_t)_c$$

linear velocity wich is the same for the entire cable (= 2m/s)

$$a_n = \frac{2^2}{0.4} = 10 \text{ m/s}^2$$

* constant acceleration (from the question) which means we can use equation 5,6,7.

$V_i = 0$ (lifted from rest)

final velocity

$$V_c^2 = V_i^2 + 2a_t(y - y_0)$$

$$4 = 0 + 2a_t(0.8)$$

$$a_t = 2.5 \text{ m/s}^2$$

$$a_c = \sqrt{a_n^2 + a_t^2} = \sqrt{10^2 + 2.5^2} = 10.31 \text{ m/s}^2$$

Gear Ratios :-

Angular position:

$$\theta_A r_A = \theta_B r_B$$

Angular Velocity:

$$w_A r_A = w_B r_B$$

Angular Acceleration:

$$\alpha_A r_A = \alpha_B r_B$$

b) convert linear velocity to angular velocity and angular acceleration to use in Gear ratios to find the unknown (α_A ? w_A ?)

$$V_c = r_c w_c$$

$$2 = 0.4 w_c$$

$$w_c = 5 \text{ rad/s}$$

$w_c = w_B$ (rotates together)

$$a_t = r\alpha$$

$$2.5 = 0.4\alpha_c$$

$$\alpha_c = \frac{2.5}{0.4} = 6.25 \text{ m/s}^2 = \alpha_B$$

$$w_B r_B = w_A r_A$$

$$\alpha_B r_B = \alpha_A r_A \Rightarrow \alpha_A = \frac{r_B}{r_A}\alpha_B$$

$$w_A = w_B \left(\frac{r_B}{r_A}\right) = 5\left(\frac{0.3}{0.1}\right) = 15 \text{ rad/s}$$ CW (opposite of the other gear).

Gear ratios

$$= \frac{0.3}{0.1}(6.25)$$

$$= 18.75 \text{ rad/s}^2$$ CW

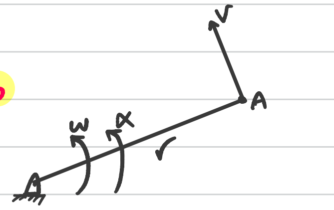
→ Alternative approach using vector notation

$$\vec{V} = \vec{\omega} \times \vec{r}_A$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}_A$$

$$\vec{a}_n = \vec{\omega} \times (\vec{\omega} \times \vec{r}_A) = \vec{\omega} \times \vec{V}$$

$\vec{\omega}$ is normal to the plane of rotation



* Use Right Hand Rule

$$a = a_t + a_n$$
$$= \alpha \times r + \omega \times (\omega \times r)$$

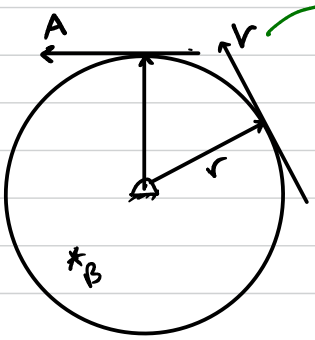
$\vec{\omega}$ is increasing → $\vec{\alpha}$ is in the direction of $\vec{\omega}$
$\vec{\omega}$ is decreasing → $\vec{\alpha}$ is in the opposite direction of $\vec{\omega}$

$\vec{\omega}$ is increasing → $\vec{a}_t$ is in the direction of $\vec{v}$
$\vec{\omega}$ is decreasing → $\vec{a}_t$ is in the opposite direction of $\vec{v}$

i, j, k

cw	ccw
$i \times j = k$	$i \times k = -j$
$j \times k = i$	$k \times j = -i$
$k \times i = j$	$j \times i = -k$

Rotation about fixed axis:



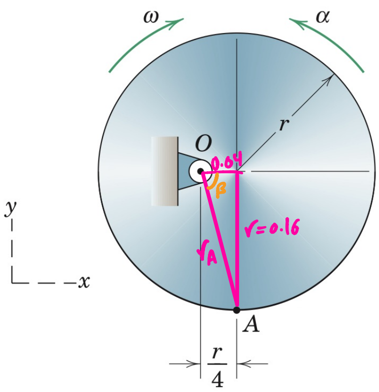
linear Velocity — angular Velocity

$$V_A = r\omega$$

$$a_t = r\alpha \qquad a_n = \frac{V^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$$

problems 5/2

5/1 X The circular disk of radius $r = 0.16$ m rotates about a fixed axis through point O with the angular properties $\omega = 2$ rad/s and $\alpha = 3$ rad/s^2 with directions as shown in the figure. Determine the instantaneous values of the velocity and acceleration of point A.



$$r_A = \sqrt{0.16^2 + 0.04^2} = 0.165 \text{ m}$$

$$\beta = \tan^{-1}\left(\frac{0.16}{0.04}\right) = 76^\circ$$

$$r_A = 0.165\cos\beta\,\hat{i} - 0.165\sin\beta\,\hat{j}$$
$$= 0.04\hat{i} - 0.16\hat{j}$$

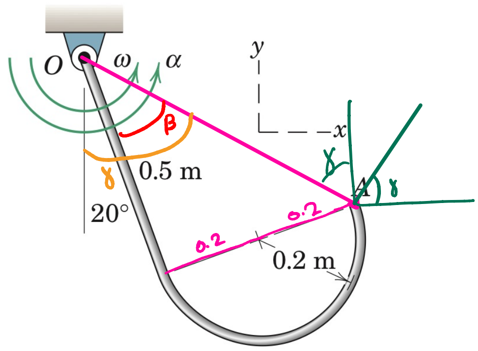
$$\vec{V}_A = \vec{\omega} \times \vec{r}_A = -2\hat{k} \times (0.04\hat{i} - 0.16\hat{j})$$
$$= -0.08\hat{j} - 0.32\hat{i} \quad m/s$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}_A = 3\hat{k} \times (0.04\hat{i} - 0.16\hat{j})$$
$$= 0.12\hat{j} + 0.48\hat{i} \quad m/s^2$$

$$\vec{a}_n = \vec{\omega} \times \vec{V}_A = -2\hat{k} \times (-0.32\hat{i} - 0.08\hat{j})$$
$$= 0.64\hat{j} - 0.16\hat{i} \quad m/s^2$$

$$\vec{a}_A = \vec{a}_t + \vec{a}_n = (0.48\hat{i} + 0.12\hat{j}) + (-0.16\hat{i} + 0.64\hat{j}) = 0.32\hat{i} + 0.78\hat{j} \quad m/s^2$$

5/3 The body is formed of slender rod and rotates about a fixed axis through point O with the indicated angular properties. If $\omega = 4$ rad/s and $\alpha = 7$ rad/s^2, determine the instantaneous velocity and acceleration of point A.



$$\vec{r}_{OA} = \sqrt{0.5^2 + 0.4^2}$$
$$= 0.64 \text{m}$$

$$\beta = \tan^{-1}\left(\frac{0.4}{0.5}\right) = 38.66^\circ$$

$$\gamma = 20^\circ + \beta$$
$$= 20^\circ + 38.66^\circ = 58.66^\circ$$

$$\vec{r}_A = 0.64\sin\gamma\,\hat{i} - 0.64\cos\gamma\,\hat{j}$$
$$= 0.55\hat{i} - 0.333\hat{j}$$

$$\vec{a}_n = \vec{\omega} \times \vec{V}_A$$
$$= 4\hat{k} \times (1.32\hat{i} + 2.2\hat{j})$$
$$= 5.28\hat{j} - 8.8\hat{i}$$

$$\vec{V}_A = \vec{\omega} \times \vec{r}_A$$
$$= 4\hat{k} \times (0.55\hat{i} - 0.33\hat{j})$$
$$= 2.2\hat{j} + 1.32\hat{i} \quad m/s$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}_A$$
$$= 7\hat{k} \times (0.55\hat{i} - 0.33\hat{j})$$
$$= 3.85\hat{j} + 2.31\hat{i} \quad m/s^2$$

$$\vec{a}_A = \vec{a}_t + \vec{a}_n$$
$$= (2.31\hat{i} + 3.85\hat{j}) + (-8.8\hat{i} + 5.28\hat{j})$$
$$= -6.49\hat{i} + 9.13\hat{j} \quad m/s^2$$

5/4 A torque applied to a flywheel causes it to accelerate uniformly from a speed of 200 rev/min to a speed of 800 rev/min in 4 seconds. Determine the number of revolutions N through which the wheel turns during this interval. (*Suggestion:* Use revolutions and minutes for units in your calculations.)

$$w_2^2 = w_1^2 + 2\alpha\Delta\theta$$

$$w_2 = w_1 + \alpha t$$

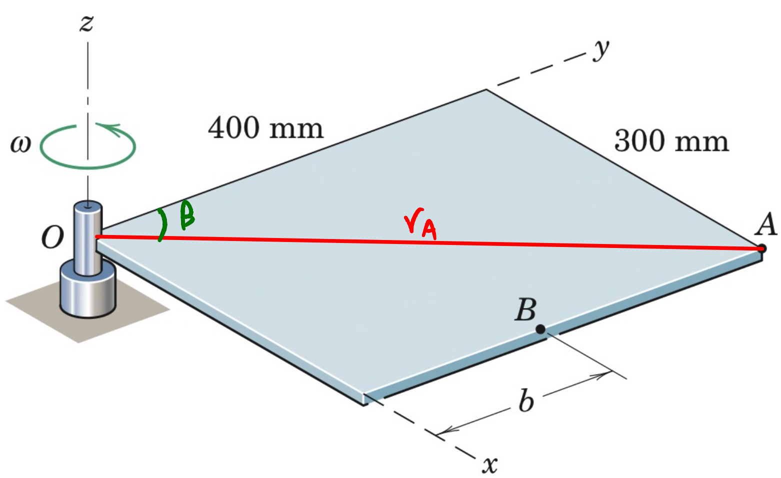
$$800\left(\frac{2\pi}{60}\right) = 200\left(\frac{2\pi}{60}\right) + \alpha(4)$$

$$\left(800\left(\frac{2\pi}{60}\right)\right)^2 = \left(200\left(\frac{2\pi}{60}\right)\right)^2 + 2(15.7)\Delta\theta$$

$$\alpha = 15.7 = 5\pi \text{ rad/s}^2$$

$$\boxed{\Delta\theta = 209.54 \text{ rad} = 33.3 \text{ rev}}$$

5/7 The rectangular plate is rotating about its corner axis through O with a constant angular velocity $\omega = 10$ rad/s. Determine the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the corner A by (a) using the scalar relations and (b) using the vector relations.



a)

$$r_A = \sqrt{0.3^2 + 0.4^2} = 0.5 \text{ m}$$

$$v_A = \omega r_A = 10(0.5) = 5 \text{ m/s}$$

$$a = a_n = \omega^2 r_A = 10^2(0.5) = 50 \text{ m/s}^2$$

($a_t = 0$ ($\alpha = 0$) constant angular velocity).

b)

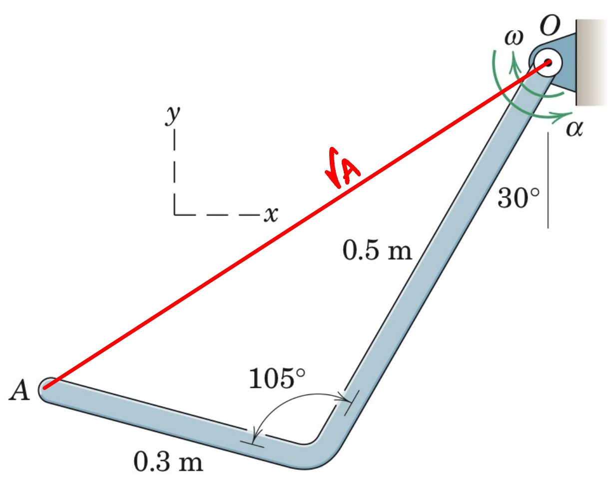
$$\beta = \tan^{-1}\left(\frac{0.3}{0.4}\right) = 36.87^\circ$$

$$r_A = 0.5 \sin\beta\, \hat{i} + 0.5 \cos\beta\, \hat{j} = 0.3\hat{i} + 0.4\hat{j}$$

$$\vec{V}_A = \vec{\omega} \times \vec{r}_A = 10\hat{k} \times (0.3\hat{i} + 0.4\hat{j}) = 3\hat{j} - 4\hat{i} \text{ m/s}$$

$$\vec{a}_A = \vec{a}_n = \vec{\omega} \times \vec{V} = 10\hat{k} \times (-4\hat{i} + 3\hat{j}) = -40\hat{j} - 30\hat{i}$$

5/10 The bent flat bar rotates about a fixed axis through point O. At the instant depicted, its angular properties are $\omega = 5$ rad/s and $\alpha = 8$ rad/s^2 with directions as indicated in the figure. Determine the instantaneous velocity and acceleration of point A.



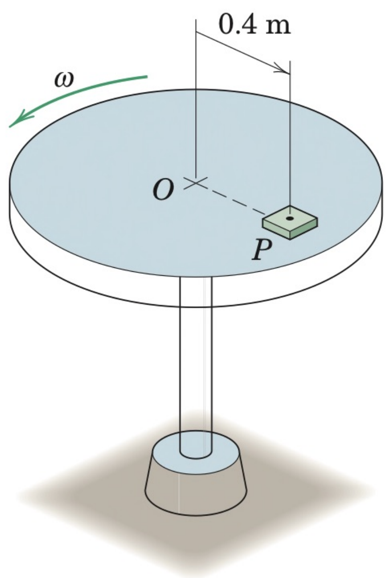
$$r_A = \sqrt{0.5^2 + 0.3^2 - 2(0.5)(0.3)\cos(105°)}$$
$$= 0.646 m$$

$$V_A = \omega r_A = 5(0.646)$$
$$= 3.28 \text{ m/s}$$

$$a_n = \omega^2 r_A = 25(0.646) = 16.15 \text{ m/s}^2$$
$$a_t = \alpha r_A = 8(0.646) = 5.168 \text{ m/s}^2$$

$$a_A = \sqrt{a_t^2 + a_n^2} = \sqrt{(5.168)^2 + (16.15)^2}$$
$$a_A = 16.95 \text{ m/s}^2$$

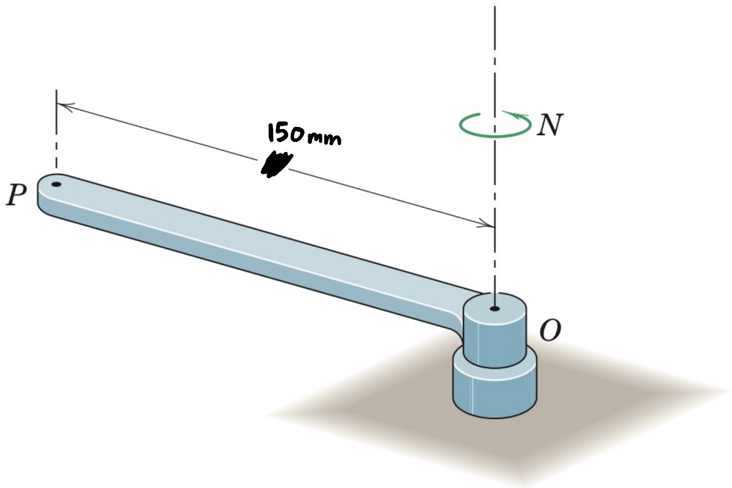
5/13 In order to test an intentionally weak adhesive, the bottom of the small 0.3-kg block is coated with adhesive and then the block is pressed onto the turntable with a known force. The turntable starts from rest at time $t = 0$ and uniformly accelerates with $\alpha = 2$ rad/s^2. If the adhesive fails at exactly $t = 3$ s, determine the ultimate shear force which the adhesive supports. What is the angular displacement of the turntable at the time of failure?



$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$= 0 + \tfrac{1}{2}(2)(3)^2$$
$$\underline{\Delta\theta = 9 \text{ rad}}$$

years

5/16 The rotating arm starts from rest and acquires a rotational speed $N = 600$ rev/min in 2 seconds with constant angular acceleration. Find the time t after starting before the acceleration vector of end P makes an angle of 45° with the arm OP.



$\omega_i = 0$ rev

$\omega_f = 600\left(\frac{2\pi}{60}\right) = 20\pi$ rad/s

$\omega_f = \omega_i + \alpha t$

$20\pi = 0 + \alpha \cdot 2$

$\alpha = 10\pi$ rad/s^2

if asked for velocity @ P:

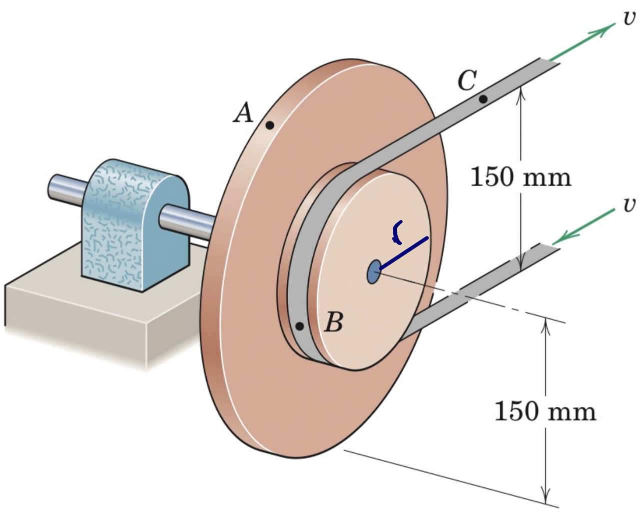
$V = r\omega = (0.15)(20\pi)$
$= 3\pi$ m/s

$\theta = \theta_0 + \omega_i t + \frac{1}{2}\alpha t^2$

45° $\frac{\pi}{4} = 0 + 0 + \frac{1}{2}(10\pi)t^2$

$t = 0.224$ s

5/17 The belt-driven pulley and attached disk are rotating with increasing angular velocity. At a certain instant the speed v of the belt is 1.5 m/s, and the total acceleration of point A is 75 m/s^2. For this instant determine (a) the angular acceleration α of the pulley and disk, (b) the total acceleration of point B, and (c) the acceleration of point C on the belt.



$r = \frac{0.15}{2}\ m = 0.075\ m$

$V_C = r\,w_c$

$1.5 = 0.075\ w_c$

$w_c = 20\ rad/s$ ← for the pully (which is the same for the disk).

$V_A = R\,w$

$= (0.15)(20)$

$= 3\ m/s$

$a_A^2 = a_t^2 + a_n^2$

$(a_A)^2 = (R\alpha)^2 + \left(\frac{V_A^2}{R}\right)^2$

$(75)^2 = (0.15)^2(\alpha)^2 + \left(\frac{(3)^2}{0.15}\right)^2$

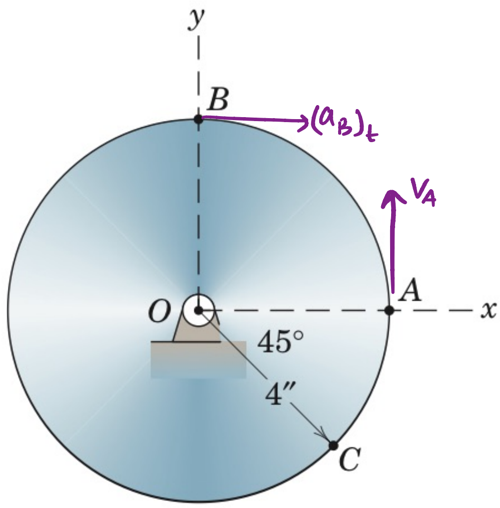
a) $\alpha = 300\ rad/s$

$a_B^2 = a_{t_B}^2 + a_{n_B}^2$

$a_B = \sqrt{(\alpha r)^2 + \left(\frac{V_B^2}{r}\right)^2}$

$= \sqrt{(300 * 0.075)^2 + \left(\frac{(1.5)^2}{0.075}\right)^2} = 37.5\ m/s^2$

5/19 The circular disk rotates about its center O. For the instant represented, the velocity of A is $\mathbf{v}_A = 8\mathbf{j}$ in./sec and the tangential acceleration of B is $(\mathbf{a}_B)_t = 6\mathbf{i}$ in./sec^2. Write the vector expressions for the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the disk. Use these results to write the vector expression for the acceleration of point C.



$$W_A = \frac{V_A}{r_A} = \frac{8}{4} = 2\hat{k} \text{ rad/s}$$

$$\alpha_b = \frac{(a_B)_t}{r_B} = \frac{6}{4} = -\frac{3}{2}\hat{k} \text{ rad/s}^2$$

$$r_c = 4\cos 45°\,\hat{i} - 4\sin 45°\,\hat{j}$$
$$= 2\sqrt{2}\,\hat{i} - 2\sqrt{2}\,\hat{j}$$

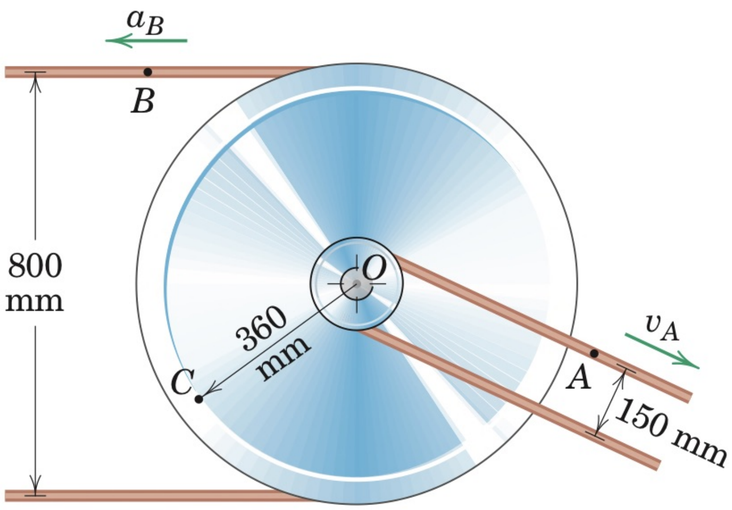
$$a_n = \vec{\omega} \times (\vec{\omega} \times \vec{r_c})$$
$$= 2\hat{k} \times (2\hat{k} \times (2\sqrt{2}\,\hat{i} - 2\sqrt{2}\,\hat{j}))$$
$$= 2\hat{k} \times (4\sqrt{2}\,\hat{j} + 4\sqrt{2}\,\hat{i})$$
$$= -8\sqrt{2}\,\hat{i} + 8\sqrt{2}\,\hat{j}$$

$$a_t = \vec{\alpha} \times \vec{r_c}$$
$$= -\frac{3}{2}\hat{k} \times (2\sqrt{2}\,\hat{i} - 2\sqrt{2}\,\hat{j})$$
$$= -3\sqrt{2}\,\hat{j} - 3\sqrt{2}\,\hat{i}$$

$$a_c = a_t + a_n = (-3\sqrt{2}\,\hat{i} - 3\sqrt{2}\,\hat{j}) + (-8\sqrt{2}\,\hat{i} + 8\sqrt{2}\,\hat{j})$$
$$a_c = -11\sqrt{2}\,\hat{i} + 5\sqrt{2}\,\hat{j} \text{ in/s}^2$$

years

5/26 The two V-belt pulleys form an integral unit and rotate about the fixed axis at O. At a certain instant, point A on the belt of the smaller pulley has a velocity $v_A = 1.5$ m/s, and point B on the belt of the larger pulley has an acceleration $a_B = 45$ m/s^2 as shown. For this instant determine the magnitude of the acceleration $\mathbf{a}_C$ of point C and sketch the vector in your solution.



$$w_A = \frac{V_A}{r_A} = \frac{1.5}{0.075} = 20 \text{ rad/s}$$

$$\alpha = \frac{a_B}{r_B} = \frac{45}{0.4} = 112.5 \text{ rad/s}^2$$

$$a_c^2 = a_n^2 + a_t^2$$

$$a_c^2 = \left(\frac{V_c^2}{r}\right)^2 + (\alpha r)^2$$

$$a_c^2 = (w^2 r)^2 + (\alpha r)^2$$

$$= ((20)^2 * (0.36))^2 + (112.5 * 0.36)^2$$

$$a_c = 149.59 \text{ m/s}^2$$

The velocity at poin B :-

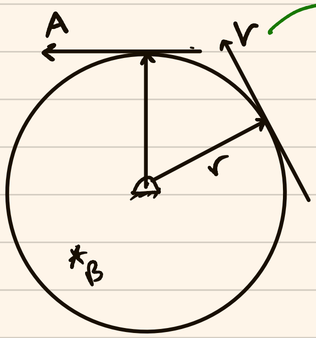
$$V_B = r_B \underbrace{w_B}_{= w_A}$$

$$V_B = (0.4)(20)$$

$$= 8 \text{ m/s}$$

5/3 Absolute Motion :-

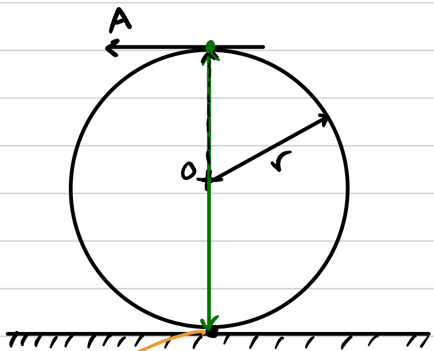
Rotation about fixed axis:



$V_A = r\omega$ (linear velocity; angular velocity)

$a_t = r\alpha$

$$a_n = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$$



$V_A = \underline{2r}\,\omega$

The distance between the point I want to calculate the velocity for and the point that has zero velocity.

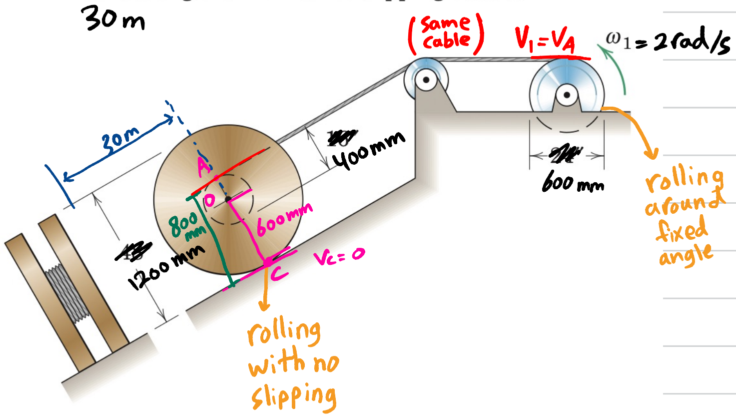
has zero velocity

Rolling without slipping

linear velocity ≠ angular velocity.

5/31 The telephone-cable reel is rolled down the incline by the cable leading from the upper drum and wrapped around the inner hub of the reel. If the upper drum is turned at the constant rate $\omega_1 = 2$ rad/sec, calculate the time required for the center of the reel to move ~~100 ft~~ along the incline. No slipping occurs.

30 m



$$V_o = \frac{S}{t}$$

$$t = \frac{S}{V_o}$$

$$\boxed{t = \frac{30}{V_o}}$$

$V_o = r_o\,\omega_o = 0.6\,\omega_o$

$\omega_1 r_1 = \omega_A r_A$

$2(0.3) = 0.8\,\omega_A$

$\omega_A = 0.75$ rad/s $= \omega_o$

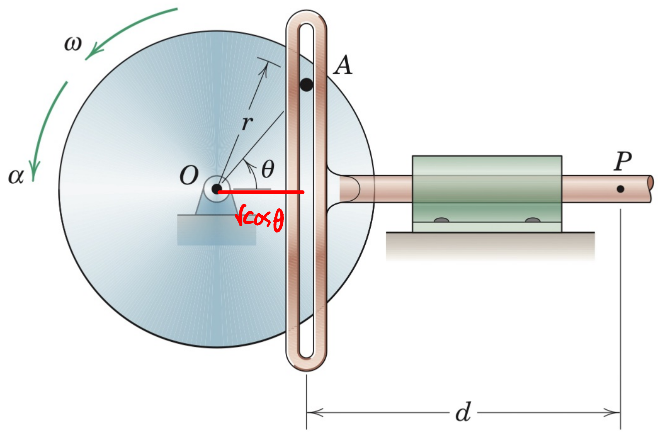
* Because it's on the same reel

$V_o = 0.6\,(0.75) = 0.45$ m/s

$t = \frac{30}{0.45} = 66.7$ seconds.

problems 5/3

5/33 The Scotch-yoke mechanism converts rotational motion of the disk to oscillatory translation of the shaft. For given values of θ, ω, α, r, and d, determine the velocity and acceleration of point P of the shaft.



$x_P = r\cos\theta + d$

differentiate with respect to time.

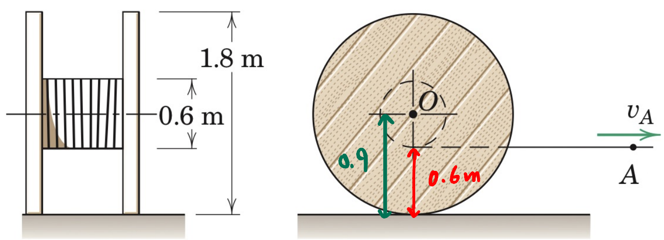
$\dot{x}_P = -r\sin\theta\,(\dot\theta)$

$v_P = -r\omega\sin\theta$

$\ddot{x}_P = \left(-r\sin\theta\,(\ddot\theta) + (\dot\theta)(-r\cos\theta(\dot\theta))\right)$

$\alpha_P = -r\alpha\sin\theta - r\omega^2\cos\theta$

5/40 The telephone-cable reel rolls without slipping on the horizontal surface. If point A on the cable has a velocity $v_A = 0.8$ m/s to the right, compute the velocity of the center O and the angular velocity ω of the reel. (Be careful not to make the mistake of assuming that the reel rolls to the left.)



V_o ?

$V_A = r\,\omega_A$

$\omega_A = \dfrac{V_A}{r_A} = \dfrac{0.8}{0.6} = 1.33\ \text{rad/s}$

angular velocity of the reel

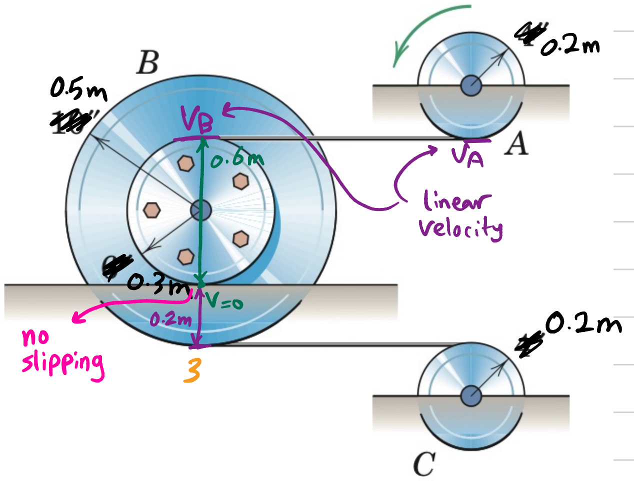
$\omega_A = \omega_o$

$V_o = r_o\,\omega_o$

$= 0.9(1.33)$

$= 1.2\ \text{m/s}$

5/47 The cable from drum A turns the double wheel B, which rolls on its hubs without slipping. Determine the angular velocity ω and angular acceleration α of drum C for the instant when the angular velocity and angular acceleration of A are 4 rad/sec and 3 rad/sec^2, respectively, both in the counterclockwise direction.



$\omega_A = 4\,rad/s$
$\alpha_A = 3\,rad/s^2$

$V_B = r_A \omega_A = 0.2(4) = 0.8\ m/s$

$V_B = r_B \omega_B \Rightarrow 0.8 = 0.6\,\omega_B$
$\Rightarrow \omega_B = \frac{0.8}{0.6} = 1.33\ rad/s$

$r_B \omega_B = r_C \omega_C$
$0.2(1.33) = 0.2(\omega_C)$

$\boxed{\omega_C = 1.33\ rad/s}$ ccw #

$a_{t_A} = a_{t_B} = 0.2 \times 3 = 0.6\ m/s^2$

$a_{t_B} = (0.6)\alpha_B$

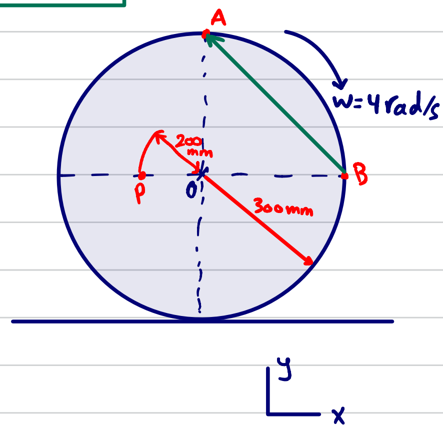
$0.6 = 0.6\,\alpha_B$

$\alpha_B = 1\ rad/s^2$

$a_{t_3} = a_{t_C} \Rightarrow 0.2 * 1 = 0.2\,\alpha_C$

$\boxed{\alpha_C = 1\ rad/s^2}$ ccw #

Question: The circular disk rolls without slipping with a clockwise angular velocity ω= 4 rad/s. For the instant represented, the vector expression for the velocity of A with respect to B is:



$V_{A/B} = \omega \times r_{A/B}$

$= -4\hat{k} \times (-0.3\hat{i} + 0.3\hat{j})$

$V_{A/B} = 1.2\hat{j} + 1.2\hat{i}\ \ m/s$

$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$
$0.3\hat{j} = 0.3\hat{i} + \vec{r}_{A/B}$
$\vec{r}_{A/B} = -0.3\hat{i} + 0.3\hat{j}$

$\boxed{5/4}$ + $\boxed{5/6}$ Relative Velocity and Relative acceleration

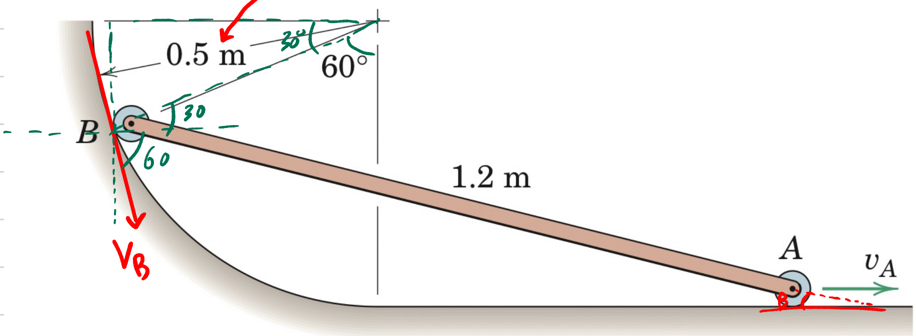
$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$$

$$\vec{V}_A = \vec{V}_B + \underbrace{w \times \vec{r}_{A/B}}_{\text{cross product}}$$

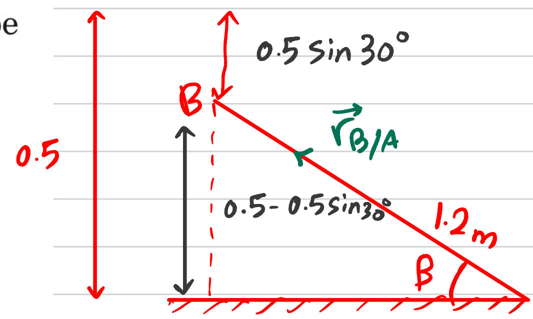
$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$$\vec{a}_A = \vec{a}_B + \underbrace{\alpha \times \vec{r}_{A/B}}_{\text{tangent acceleration}} - \underbrace{w^2 r_{A/B}}_{\text{normal acceleration}}$$

5/73 At the instant represented, the velocity of point A of the 1.2-m bar is 3 m/s to the right. Determine the speed v_B of point B and the angular velocity ω of the bar. The diameter of the small end wheels may be neglected.



$V_A = 3 m/s$



$$\sin\beta = \frac{0.5 - 0.5\sin 30^\circ}{1.2}$$

$$\Rightarrow \beta = 12.02^\circ$$

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

$$\vec{V}_B = 3\hat{i} + w \times \vec{r}_{B/A}$$

ccw out of the page (thumb)

$$V_B \cos 60^\circ \hat{i} - V_B \sin 60^\circ \hat{j} = 3\hat{i} + w \times (-1.2\cos 12.02^\circ \hat{i} + 1.2 \sin 12.02^\circ \hat{j})$$

$$V_B \cos 60^\circ \hat{i} - V_B \sin 60^\circ \hat{j} = 3\hat{i} + -1.2\, w \cos 12.02^\circ \hat{j} - 1.2 \sin 12.02^\circ \hat{i}$$

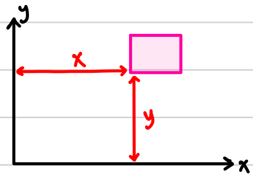
$$V_B \cos 60^\circ = 3 - 1.2 \sin 12.02^\circ w \quad \dots ①$$

$$V_B \sin 60^\circ = -1.2\, w \cos 12.02^\circ \quad \dots ②$$

$$\boxed{\begin{array}{l} V_B = 4.38 \text{ m/s} \\ w = 3.23 \text{ rad/s ccw.} \end{array}}$$

Ch6 Kinetics of Rigid bodies.

→ mass moment of inertia



$F = ma$

$M = I\alpha$

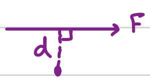
moment = F*d

→ mass moment of inertia is a measure of resistance of a body to angular acceleration.

$I_x = \int y^2 \, dA$

$I_y = \int x^2 \, dA$ → unit m^4

area moment of inertia: is a measure of the distribution of area about an axis.



$M = F d$

↳ the distance that is perpendicular to the force.

cw : −ve moment.
ccw : +ve moment.

$m = \rho V$

mass
density
Volume

$I = \int_m r^2 \, dm$

→ integral of the "second moment" about an axis of all the elements of mass dm which compose the body.

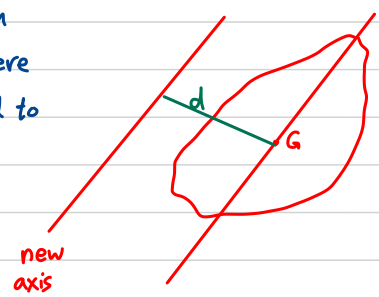
always +ve
$[kg\, m^2]$ → unit

→ parallel-axis Theorem :-

$I = I_G + md^2$

G : center of mass

d: radius of gyration: is a distance from a given axis up to the point where the entire area is assumed to be concentrated.



$I = md^2$ $\quad d = \sqrt{\frac{I}{m}}$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS

(m = mass of body shown)

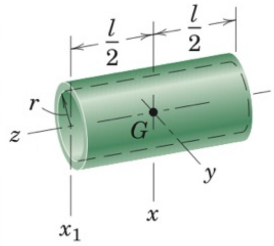
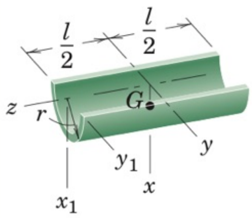
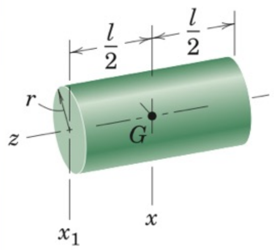
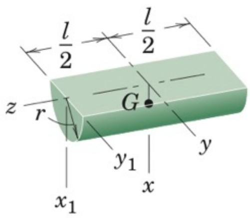
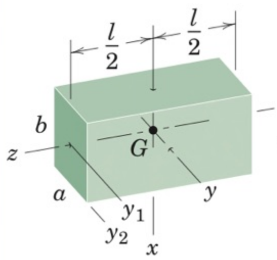
BODY		MASS CENTER	MASS MOMENTS OF INERTIA
	Circular Cylindrical Shell	—	$I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$
	Half Cylindrical Shell	$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$ $\bar{I}_{zz} = \left(1 - \frac{4}{\pi^2}\right)mr^2$
	Circular Cylinder	—	$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$
	Semicylinder	$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
	Rectangular Parallelepiped	—	$I_{xx} = \frac{1}{12}m(a^2 + l^2)$ $I_{yy} = \frac{1}{12}m(b^2 + l^2)$ $I_{zz} = \frac{1}{12}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12}mb^2 + \frac{1}{3}ml^2$ $I_{y_2y_2} = \frac{1}{3}m(b^2 + l^2)$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS ***Continued***
(m = mass of body shown)

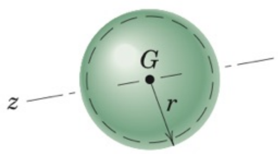
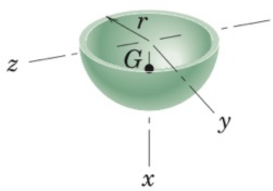
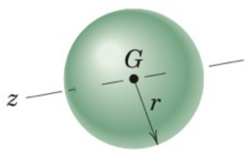
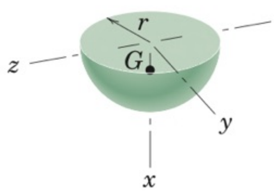
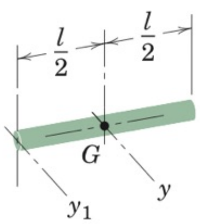
BODY		MASS CENTER	MASS MOMENTS OF INERTIA
	Spherical Shell	—	$I_{zz} = \frac{2}{3}mr^2$
	Hemispherical Shell	$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{5}{12}mr^2$
	Sphere	—	$I_{zz} = \frac{2}{5}mr^2$
	Hemisphere	$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{83}{320}mr^2$
	Uniform Slender Rod	—	$I_{yy} = \frac{1}{12}ml^2$ $I_{y_1y_1} = \frac{1}{3}ml^2$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS ***Continued***

(m = mass of body shown)

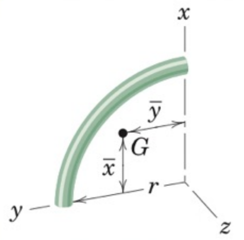
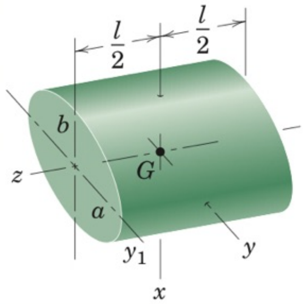
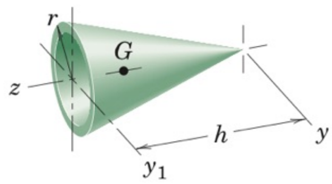
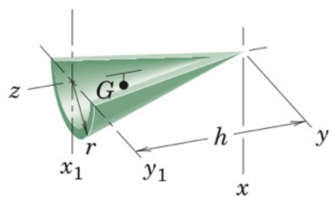
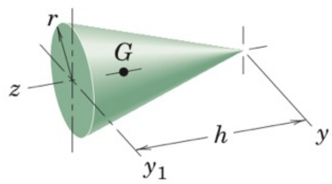
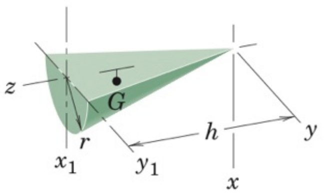
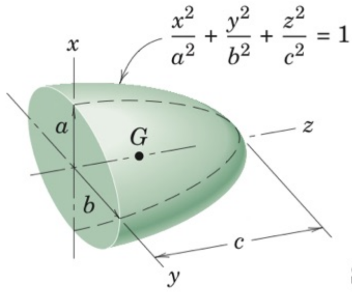
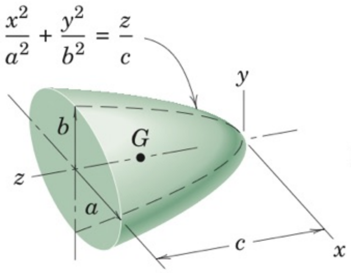
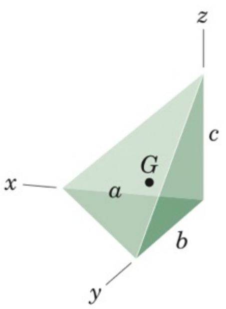
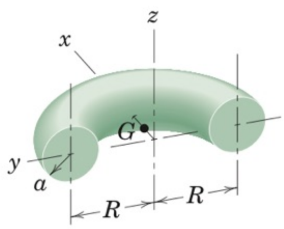
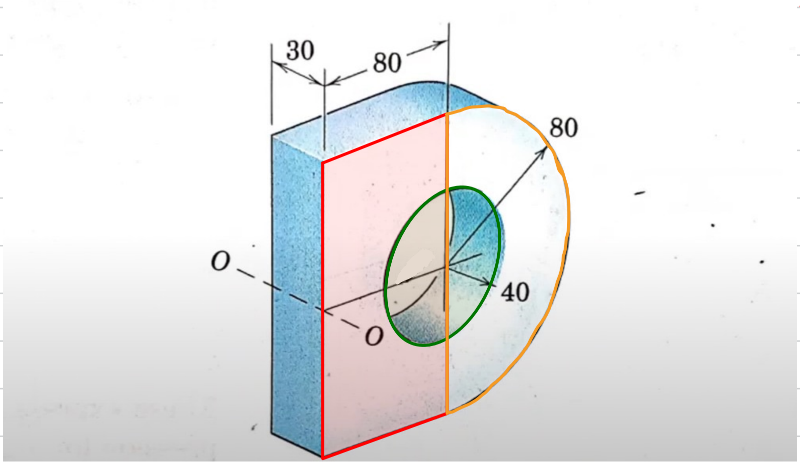
BODY		MASS CENTER	MASS MOMENTS OF INERTIA
	Quarter-Circular Rod	$\bar{x} = \bar{y}$ $= \dfrac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2}mr^2$ $I_{zz} = mr^2$
	Elliptical Cylinder	—	$I_{xx} = \frac{1}{4}ma^2 + \frac{1}{12}ml^2$ $I_{yy} = \frac{1}{4}mb^2 + \frac{1}{12}ml^2$ $I_{zz} = \frac{1}{4}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{4}mb^2 + \frac{1}{3}ml^2$
	Conical Shell	$\bar{z} = \dfrac{2h}{3}$	$I_{yy} = \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{yy} = \frac{1}{4}mr^2 + \frac{1}{18}mh^2$
	Half Conical Shell	$\bar{x} = \dfrac{4r}{3\pi}$ $\bar{z} = \dfrac{2h}{3}$	$I_{xx} = I_{yy}$ $= \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\dfrac{1}{2} - \dfrac{16}{9\pi^2}\right)mr^2$
	Right-Circular Cone	$\bar{z} = \dfrac{3h}{4}$	$I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{yy} = \frac{3}{20}mr^2 + \frac{3}{80}mh^2$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS *Continued*

(m = mass of body shown)

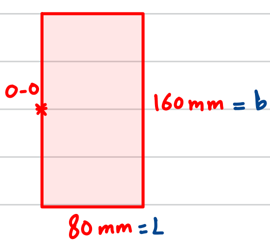
BODY	MASS CENTER	MASS MOMENTS OF INERTIA
 Half Cone	$\bar{x} = \frac{r}{\pi}$ $\bar{z} = \frac{3h}{4}$	$I_{xx} = I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{zz} = \left(\frac{3}{10} - \frac{1}{\pi^2}\right)mr^2$
 Semiellipsoid	$\bar{z} = \frac{3c}{8}$	$I_{xx} = \frac{1}{5}m(b^2 + c^2)$ $I_{yy} = \frac{1}{5}m(a^2 + c^2)$ $I_{zz} = \frac{1}{5}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{5}m(b^2 + \frac{19}{64}c^2)$ $\bar{I}_{yy} = \frac{1}{5}m(a^2 + \frac{19}{64}c^2)$
 Elliptic Paraboloid	$\bar{z} = \frac{2c}{3}$	$I_{xx} = \frac{1}{6}mb^2 + \frac{1}{2}mc^2$ $I_{yy} = \frac{1}{6}ma^2 + \frac{1}{2}mc^2$ $I_{zz} = \frac{1}{6}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{6}m(b^2 + \frac{1}{3}c^2)$ $\bar{I}_{yy} = \frac{1}{6}m(a^2 + \frac{1}{3}c^2)$
 Rectangular Tetrahedron	$\bar{x} = \frac{a}{4}$ $\bar{y} = \frac{b}{4}$ $\bar{z} = \frac{c}{4}$	$I_{xx} = \frac{1}{10}m(b^2 + c^2)$ $I_{yy} = \frac{1}{10}m(a^2 + c^2)$ $I_{zz} = \frac{1}{10}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{3}{80}m(b^2 + c^2)$ $\bar{I}_{yy} = \frac{3}{80}m(a^2 + c^2)$ $\bar{I}_{zz} = \frac{3}{80}m(a^2 + b^2)$
 Half Torus	$\bar{x} = \frac{a^2 + 4R^2}{2\pi R}$	$I_{xx} = I_{yy} = \frac{1}{2}mR^2 + \frac{5}{8}ma^2$ $I_{zz} = mR^2 + \frac{3}{4}ma^2$

B/54 The machine element is made of steel and is designed to rotate about axis O-O. Calculate its radius of gyration k_O about this axis.



→ This is a composite Body
↳ add algebraically I of parts.
↳ Holes are substracted or -ve quantities.
↳ $I = \Sigma(I_G + md^2)$

①



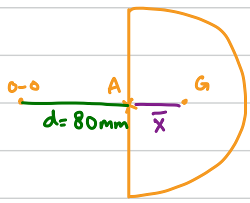
$$I_{o\text{-}o} = I_{y_1\text{-}y_1} = \frac{1}{12} m_1 b^2 + \frac{1}{3} m_1 L^2 \qquad m_1 = \rho V_1 = \rho A_1 h$$

$$= \frac{1}{12} m_1 (0.16)^2 + \frac{1}{3} m_1 (0.08)^2$$

$$= \frac{1}{12} \rho(0.08)(0.16)(0.16)^2 + \frac{1}{3}\rho(0.08)(0.16)(0.08)^2$$

$$I_{o\text{-}o_1} = 54.6 * 10^{-6} \rho \ \text{Kg.m}^2$$

②



$$I_{o\text{-}o_2} = I_G + md^2$$

$$m_2 = \rho V = \rho A_2 h = \rho \frac{\pi}{2} r^2 = \frac{\pi}{2}\rho(0.08)^2 = 0.015005\rho$$

$$G = \bar{X} = \frac{4r}{3\pi} = \frac{4(0.08)}{3\pi} = 0.034 \text{ m}$$

$$I_{z\text{-}z} = I_A = \underbrace{\frac{1}{2} m_2 r_2^2}_{} = I_G + mk^2$$

from the table
but originally comes
from the parallel axis theorem.

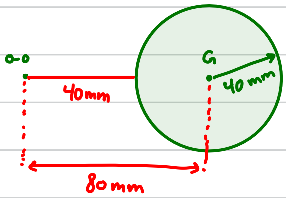
$$I_G = I_A - m_2 d^2 = \frac{1}{2}(0.015005\rho)(0.08)^2 - (0.015005\rho)(0.034)^2$$

$$= 20.6 * 10^{-6} \rho \ \text{Kg.m}^2$$

$$I_{o\text{-}o_2} = 20.6 * 10^{-6} + (0.015005\rho)(0.08 + 0.034)^2$$

$$= 151.1 * 10^{-6} \rho \ \text{Kg.m}^2$$

③



$$I_{o-o_3} = I_G + m_3 d^2$$

$$= \frac{1}{2} m_3 r_3^2 + m_3 d^2$$

$$= \frac{1}{2}\left[\pi (0.04)^2 \rho\right](0.04)^2 + (0.04)^2 \rho (0.08)^2$$

$$= 36.2 * 10^{-6} \rho \ kg.m^2$$

$$K_o = \sqrt{\frac{I_o}{m}} = \sqrt{\frac{I_{o-o_1} + I_{o-o_2} - I_{o-o_3}}{m_1 + m_2 - m_3}}$$

$$= 97.5 \ mm$$

Chapter 6 : plane kinetics of Rigid Bodies

$\Sigma F_x = m a_{G,x}$ → for curvilinear motion

$\Sigma F_y = m a_{G,y}$ $\Sigma F_n = m a_{G,n}$

$\Sigma M = I_G \alpha$ → for rotational motion $\Sigma F_t = m a_{G,t}$

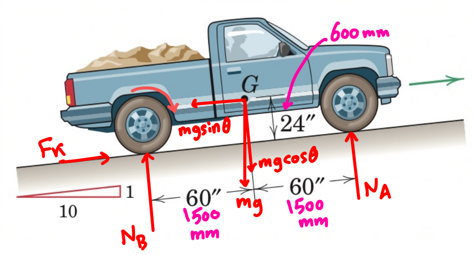
$\Sigma M_G = 0$ → for rectilinear motion $\Sigma M_G = 0$ → where $\alpha = 0$, $\omega = 0$

G, $f(t)$

* Because there is no rotation so moment is zero.

SAMPLE PROBLEM 6/1

The pickup truck weighs 3220 lb (1500 kg) and reaches a speed of 30 mi/hr (50 Km/h) from rest in a distance of 200 ft (60 m) up the 10-percent incline with constant acceleration. Calculate the normal force under each pair of wheels and the friction force under the rear driving wheels. The effective coefficient of friction between the tires and the road is known to be at least 0.8. → we can't use it.



for the truck to move uphill the wheels have to rotate cw so friction is in the opposite direction (ccw)

$\theta = \tan^{-1}\left(\frac{1}{10}\right) = 5.71°$

$V_f^2 = V_i^2 + 2a_x(s - s_0)$

$\left(\frac{50}{3.6}\right)^2 = 0 + 2a_x(60)$

$a_x = 1.608 \ m/s^2$

$\nearrow^{+} \Sigma F_x = m a_x$

$f_k - mg\sin\theta = m a_x$

$f_k - 1500(9.81)(\sin(5.71°)) = 1500(1.608)$

$f_k = 3880 \ N$

$\uparrow^{+ve} \Sigma F_y = 0$

$N_A + N_B - mg\cos 5.71° = 0$

$\circlearrowleft^{+} \Sigma M_G = I\alpha = 0$

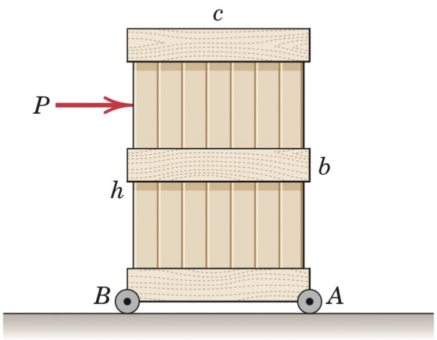
$+N_A(1.5) - N_B(1.5) + 3880(0.6) = 0$

$N_A = 6550 \ N$

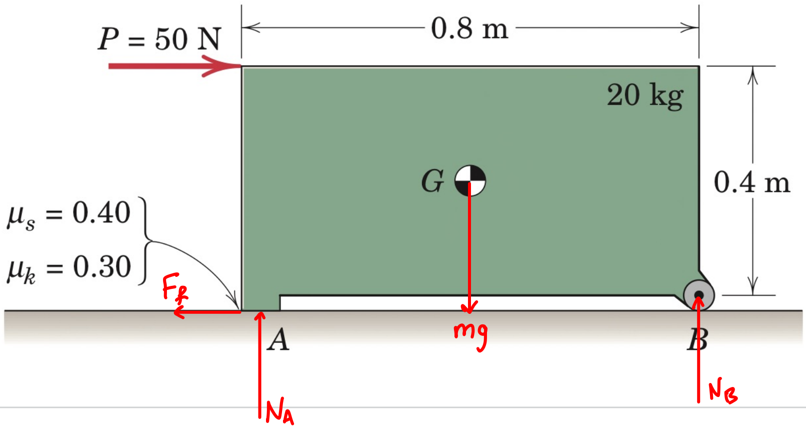
$N_B = 8100 \ N$

problems 6/3

6/7 The homogeneous create of mass m is mounted on small wheels as shown. Determine the maximum force P which can be applied without overturning the crate about (a) its lower front edge with $h = b$ and (b) its lower back edge with $h = 0$.



6/9 Determine the acceleration of the initially stationary 20-kg body when the 50-N force P is applied as shown. The small wheels at B are ideal, and the feet at A are small.



Assume no motion

$\Sigma F_x = 0$

$50 - F_f = 0$

$F_f = 50\ N$

$\Sigma F_y = 0$

$N_A + N_B - 20(9.81) = 0$

$\circlearrowleft + \Sigma M_G = 0$

$-N_A(0.4) + 0.4 N_B - 50(0.2) - F_f(0.2) = 0$

$N_A = 73.1\ N$

$N_B = 123.1\ N$

$F_{max} = \mu_s(N_A) = 0.4(73.1) = 29.2\ N$

$F_{max} < \underset{F_f}{\underline{50}} \Rightarrow$ motion occurs

$\Sigma F_x = ma_x$

$50 - \mu_k N_A = ma_x \Rightarrow 50 - 0.3 N_A = 20 a_x$

$\Sigma F_y = 0$

$N_A + N_B - mg = 0$

$\circlearrowleft + \Sigma M_G = 0$ → Because translational motion

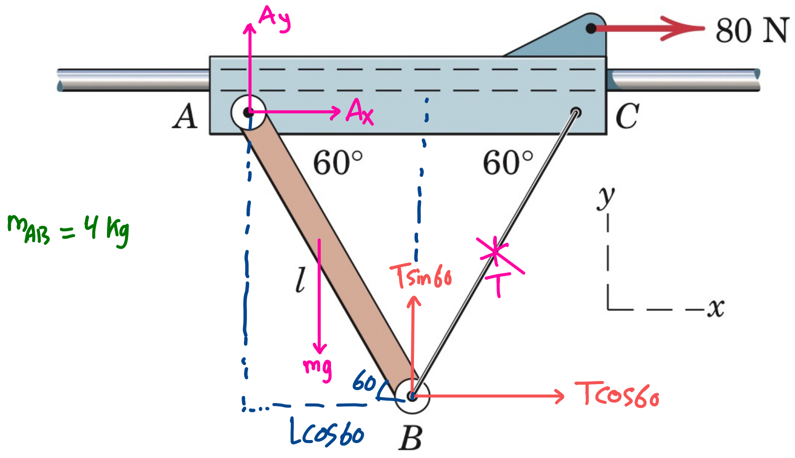
$-N_A(0.4) + 0.4(N_B) - 50(0.2) - 0.3 N_A(0.2) = 0$

$N_A = 79.6\ N$

$N_B = 116.6\ N$

$a = 1.306\ m/s^2\ (\rightarrow)$

6/13 The 6-kg frame AC and 4-kg uniform slender bar AB of length l slide with negligible friction along the fixed horizontal rod under the action of the 80-N force. Calculate the tension T in wire BC and the x- and y-components of the force exerted on the bar by the pin at A. The x-y plane is vertical.



$\overset{+}{\rightarrow} \Sigma F_x = m a_x$ → for the whole body

$80 = (6+4)\, a_x$

$a_x = 8\ m/s^2$

$\overset{+}{\rightarrow} \Sigma F_x = m a_x$ → Because the body moves

$A_x + T\cos 60^\circ = 4\,(8)$

$+\uparrow \Sigma F_y = 0$ → no motion in the y direction

$A_y - 4g + T\sin 60^\circ = 0$

$\circlearrowleft + \Sigma M_c = m a_x d$

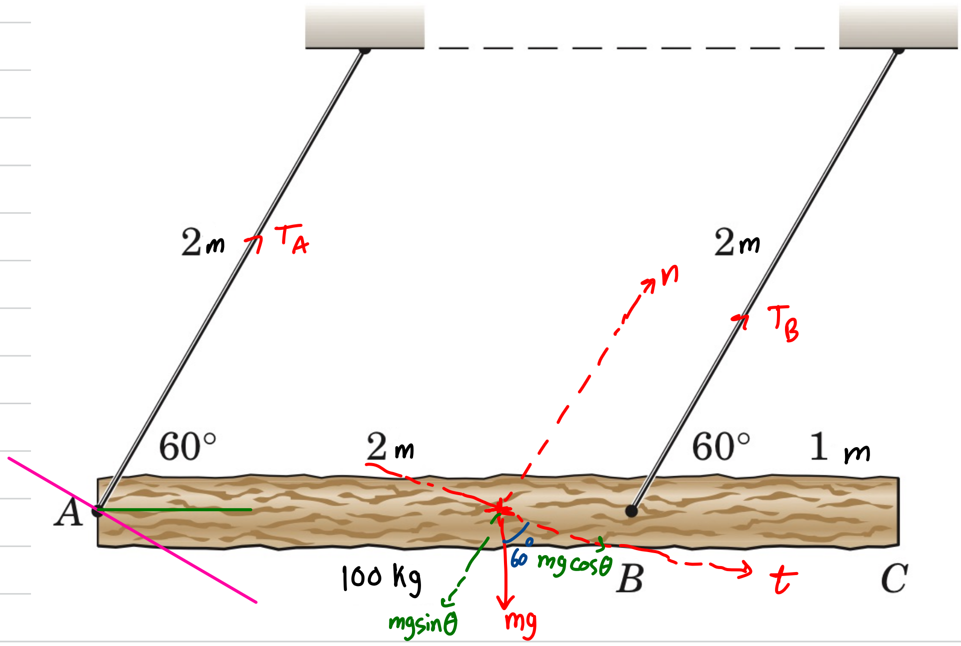
$-A_y\, 2l\cos 60 + 4g\left(\frac{3}{2} l\cos 60^\circ\right) = 4(8)\frac{l}{2}\sin 60^\circ$

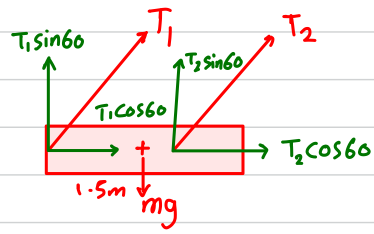
$A_x = 18.34\ N$ →

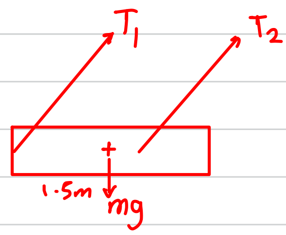
$A_y = 15.57\ N$ ↑

$T = 27.3\ N$

6/19 The uniform 100Kg [redacted] log is supported by the two cables and used as a battering ram. If the log is released from rest in the position shown, calculate the initial tension induced in each cable immediately after release and the corresponding angular acceleration α of the cables.







Since motion is in curvilinear we use n-t coordinate.
↳ always from center of mass

$\Sigma F_t = m a_t$

$\cancel{m} g\cos\theta = \cancel{m} a_t$

$g\cos\theta = a_t$

$g\cos\theta = r\alpha$

$(9.81)\cos(60) = 2\alpha$

$\alpha = 2.4525$

$\Sigma F_n = m a_n = m\frac{v^2}{r} = 0$ (from rest and immediatly)

$T_A + T_B - mg\sin60 = 0$ ①

curvilinear motion
&
around center of mass

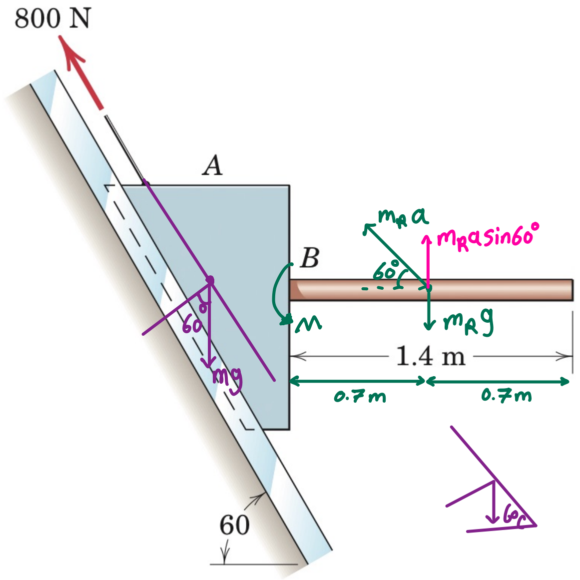
$\circlearrowleft \Sigma M_G = 0$

مسافة عامودية

$-T_A * \underbrace{1.5\sin60°}_{\text{مسافة عامودية}} + T_B(0.5)\sin60° = 0$ ②

from ① and ② → $T_A = 212\,N$ $T_B = 637\,N$

6/22 The block A and attached rod have a combined mass of 60 kg and are confined to move along the 60° guide under the action of the 800-N applied force. The uniform horizontal rod has a mass of 20 kg and is welded to the block at B. Friction in the guide is negligible. Compute the bending moment M exerted by the weld on the rod at B.



first we find the acceleration

$+\nwarrow \Sigma F_y = ma$

$800 - mg \sin(60°) = ma$

$800 - (60)(9.81)\sin(60°) = 60\,a$

$a = 4.84\ m/s^2$

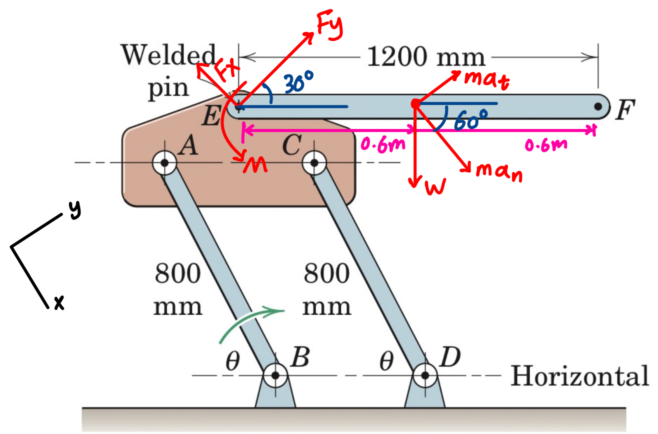
$\circlearrowleft \Sigma M_B = mad$

$M - 0.7 m_R g = 0.7\, m_R a \sin 60°$

$M = 0.7(4.84)(20)\sin 60° + 0.7(9.81)(20)$

$= 196\ N\,m$

6/23 The parallelogram linkage shown moves in the vertical plane with the uniform 8-kg bar EF attached to the plate at E by a pin which is welded both to the plate and to the bar. A torque (not shown) is applied to link AB through its lower pin to drive the links in a clockwise direction. When θ reaches 60°, the links have an angular acceleration and an angular velocity of 6 rad/s^2 and 3 rad/s, respectively. For this instant calculate the magnitudes of the force F and torque M supported by the pin at E.



$a_t = r\alpha$
$= 0.8 * 6$
$= 4.8\ m/s^2$

$a_n = r\omega^2$
$= 0.8 * 3^2$
$= 7.2\ m/s^2$

$W_{EF} = mg = 8 * 9.81 = 78.5\ N$

$\circlearrowleft \Sigma M_E = mad$

$M - 0.6 W_{EF} = 0.6\cos 60\ ma_t - 0.6 \sin 60\ ma_n$

$M = 28.7\ N.m$ ccw

$\Sigma F_x = ma_n$

$-F_x + W\sin 60° = 8 * 7.2$

$F_x = 10.4\ N$

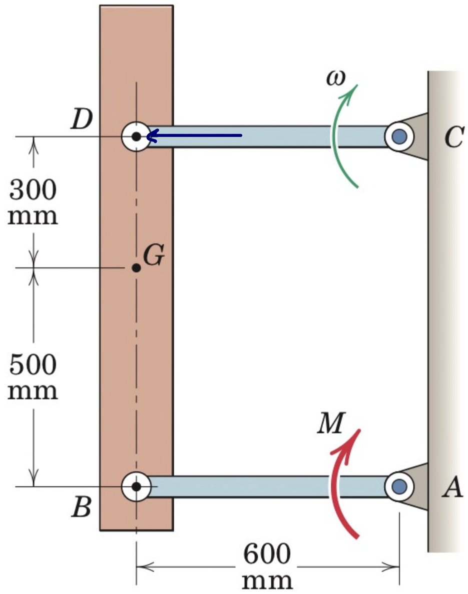
$\Sigma F_y = ma_t$

$F_y - W\cos 60° = 8 * 4.8$

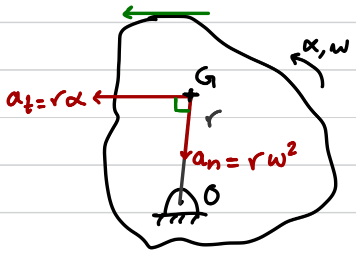
$F_y = 77.6\ N$

$F = \sqrt{F_x^2 + F_y^2} = 78.3\ N$

6/25 The 25-kg bar BD is attached to the two light links AB and CD and moves in the vertical plane. The lower link is subjected to a clockwise torque $M = 200$ N·m applied through its shaft at A. If each link has an angular velocity $\omega = 5$ rad/s as it passes the horizontal position, calculate the force which the upper link exerts on the bar at D at this instant. Also find the angular acceleration of the links at this position.



$\boxed{6/4}$ Fixed - Axis Rotation



$$\Sigma F_n = m a_n = m r \omega^2$$

$$\Sigma F_t = m a_t = m r \alpha$$

$$\circlearrowleft + \Sigma M_o = I_o \alpha$$

$\hookrightarrow$ inertia

a_n is always in the direction of the center of rotation (o), a_t is perpendicular to a_n and in the direction of rotation.

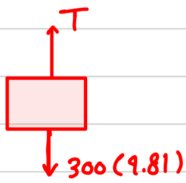
$$\circlearrowleft + M_o = m r^2 \alpha + I_G \alpha$$
$$= (m r^2 + I_G)\alpha$$
$$= \underline{\underline{I_o \alpha}}$$

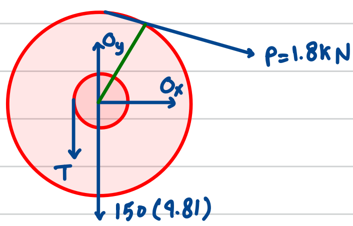
Remember : $I_o = I_G + m r^2$

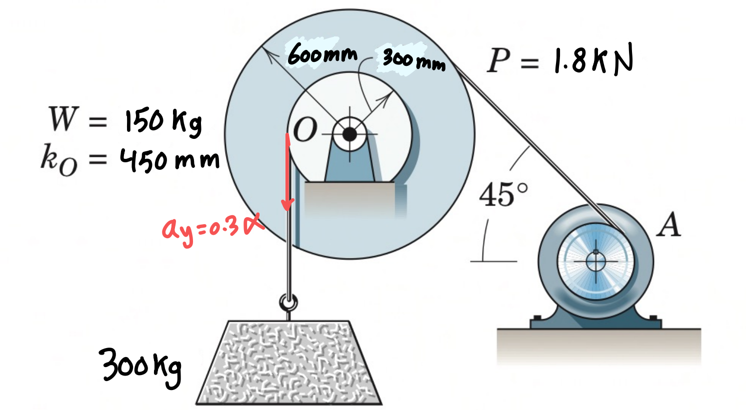
parallel axis theorm

SAMPLE PROBLEM 6/3

The concrete block weighing 644 lb is elevated by the hoisting mechanism shown, where the cables are securely wrapped around the respective drums. The drums, which are fastened together and turn as a single unit about their mass center at O, have a combined weight of 322 lb and a radius of gyration about O of 18 in. If a constant tension P of 400 lb is maintained by the power unit at A, determine the vertical acceleration of the block and the resultant force on the bearing at O.







$+\uparrow \Sigma F_y = 300 a_y \qquad T - 300(9.81) = 300(0.3\alpha) \quad —(1)$

$\circlearrowleft + \Sigma M_o = I_o \alpha \qquad T(0.3) - 1800(0.6) = I_o \alpha \quad —(2)$

$T(0.3) - 1800(0.6) = 30.4\alpha$

$I = m(K_o)^2$

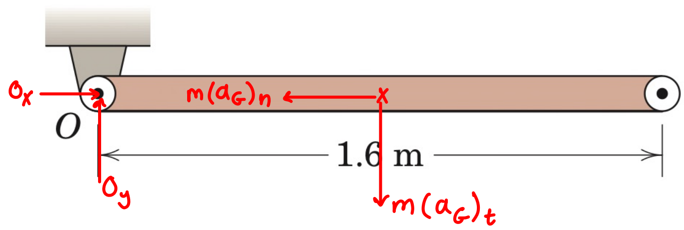
$I_o = 150 K_o^2 = 150(0.45)^2$

$I_o = 30.4 \text{ Kg.m}^2$

$T = 3250 \text{ N}$ $\qquad \alpha = 3.44 \text{ rad/s}^2$ $\qquad a_y = 1.031 \text{ m/s}^2$

problems 6/4

6/33 The uniform 20-kg slender bar is pivoted at O and swings freely in the vertical plane. If the bar is released from rest in the horizontal position, calculate the initial value of the force R exerted by the bearing on the bar an instant after release.



$$\Sigma F_n = m a_n$$

$$O_x = m r \omega^2$$

$$O_x = (20)\left(\frac{1.6}{2}\right)\underbrace{(0)^2}_{\text{from rest}}$$

$$+\downarrow \Sigma F_t = m a_t$$

$$mg - O_y = m(\alpha r)$$

$$O_y = (20)(4.81) - (20)\left(\frac{1.6}{2}\alpha\right) \quad \Rightarrow \quad O_y = (20)(9.81) - 20\left(\frac{1.6}{2}(0.1875\, O_y)\right)$$

$$O_y = 49.05\,\text{N}$$

$$\Sigma M_G = I_G \alpha$$

from sheet

$$O_y * \frac{1.6}{2} = \left[\frac{1}{12} m L^2\right]\alpha$$

$$O_y * \frac{1.6}{2} = \left[\frac{1}{12}(20)(1.6)^2\right]\alpha$$

$$\alpha = 0.1875\, O_y$$

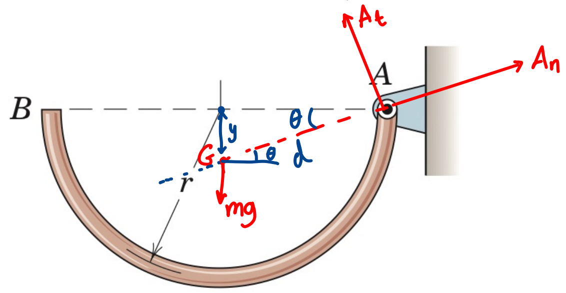
$$R = \sqrt{O_x^2 + O_y^2}$$

$$= \sqrt{0^2 + (49.05)^2}$$

$$R = 49.05\,\text{N} \quad \#$$

6/59 The uniform semicircular bar of mass m and radius r is hinged freely about a horizontal axis through A. If the bar is released from rest in the position shown, where AB is horizontal, determine the initial angular acceleration α of the bar and the expression for the force exerted on the bar by the pin at A. (Note carefully that the initial tangential acceleration of the mass center is not vertical.)

$$I_A = I_G + mr^2 = mr^2 + mr^2 = 2mr^2$$



$$+\circlearrowleft \Sigma M_A = I_A \alpha$$

$$mgr = 2mr^2 \alpha$$

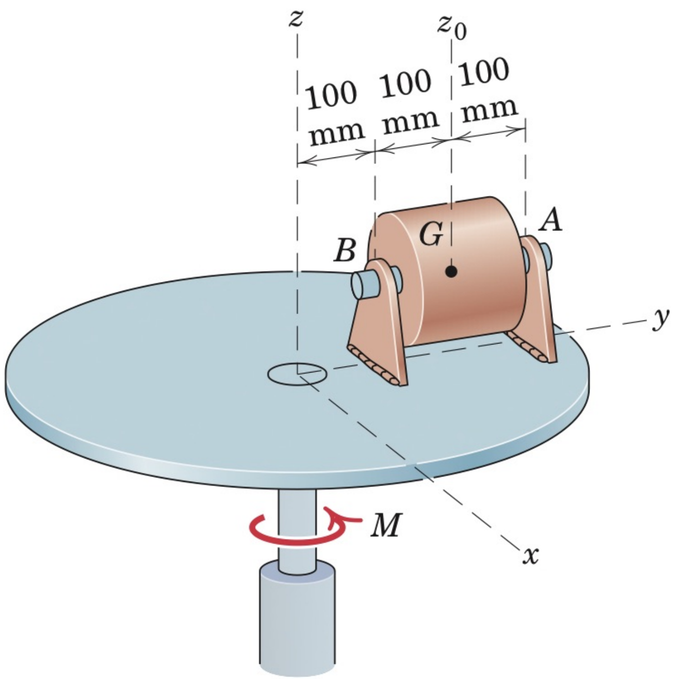
$$\alpha = \frac{g}{2r}$$

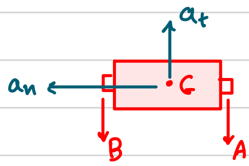
$$\Sigma F_n = ma_n = 0$$

$$A_n = mg\sin\theta$$

$$A_n = mg\,\frac{y}{d}$$

6/61 The 12-kg cylinder supported by the bearing brackets at A and B has a moment of inertia about the vertical z_0-axis through its mass center G equal to $0.080\ \text{kg}\cdot\text{m}^2$. The disk and brackets have a moment of inertia about the vertical z-axis of rotation equal to $0.60\ \text{kg}\cdot\text{m}^2$. If a torque $M = 16\ \text{N}\cdot\text{m}$ is applied to the disk through its shaft with the disk initially at rest, calculate the horizontal x-components of force supported by the bearings at A and B.





$I_z = I_b + I_G + md^2$
$= 0.6 + 0.08 + 12(0.2)^2$
$I_z = 1.16\ \text{Kg.m}^2$

$+\circlearrowleft \sum M_z = I_z \alpha$
$16 = 1.16\ \alpha$
$\alpha = 13.8\ \text{rad/s}^2$

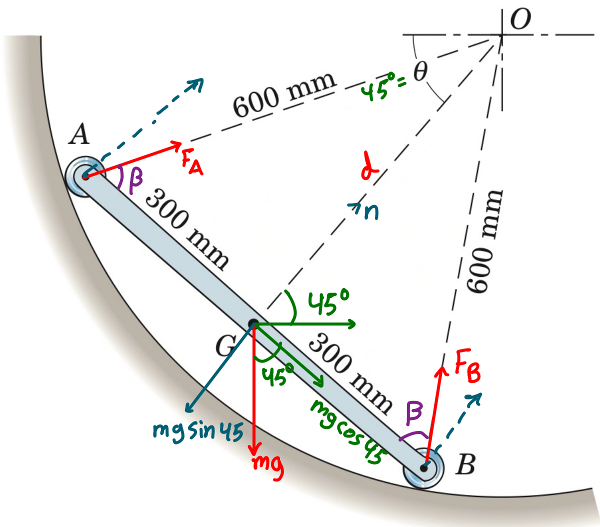
$a_t = r\alpha$
$= 0.2(13.8)$
$= 2.76\ \text{m/s}^2$

$+\downarrow \sum F_x = m a_t$
$A + B = 12(2.76)$
$A + B = 33.12$

$+\circlearrowleft \sum M_G = I_G \alpha$
$0.1A - 0.1B = 0.08 * 13.8$
$A - B = 11.04$

$A = 22.08\ \text{N}$
$B = 11.04\ \text{N}$

6/62 The 24-kg uniform slender bar AB is mounted on end rollers of negligible mass and rotates about the fixed point O as it follows the circular path in the vertical plane. The bar is released from a position which gives it an angular velocity $\omega = 2$ rad/s as it passes the position $\theta = 45°$. Calculate the forces F_A and F_B exerted by the guide on the rollers for this instant.



$$+\circlearrowright \sum M_o = I_o \alpha$$

$$I_o = I_G + md^2$$

$$I_o = \frac{mL^2}{12} + m\left(\sqrt{0.6^2 - 0.3^2}\right)^2$$

$$= \frac{24(0.6)^2}{12} + 24\sqrt{0.6^2 - 0.3^2}^2$$

$$I_o = 7.2 \text{ kg.m}^2$$

$$d\,mg\cos 45 = I_o \alpha$$

$$\left(\sqrt{0.6^2 - 0.3^2}\right)(24)(9.81)\cos 45° = 7.2\alpha$$

$$\alpha = 12.015 \text{ rad/s}^2$$

$$\cos\beta = \frac{300}{600} = \frac{1}{2}$$

$$\beta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

$$a_n = r\omega^2$$

$$= d\,\omega^2$$

$$= \left(\sqrt{0.6^2 - 0.3^2}\right)(2^2)$$

$$= 2.078 \text{ rad/s}^2$$

$$a_t = r\alpha$$

$$= \left(\sqrt{0.6^2 - 0.3^2}\right)(12.015)$$

$$= 6.24 \text{ rad/s}^2$$

$$+\nearrow \sum F_n = m a_n$$

$$F_A \sin\beta + F_B \sin\beta - mg\sin 45 = m a_n$$

$$(F_A + F_B)\sin 60° - (24)(9.81)\sin 45 = (24)(2.078)$$

$$F_A + F_B = 249.8$$

$$+\searrow \sum F_t = m a_t$$

$$F_A\cos\beta - F_B\cos\beta + mg\cos 45° = m a_t$$

$$(F_A - F_B)\cos(60°) + (24)(9.81)\cos(45°) = (24)(6.24)$$

$$F_A - F_B = -33.44$$

$$F_A = 108.18 \text{ N}$$

$$F_B = 141.62 \text{ N}$$

$\boxed{6/6}$ work and energy

$$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}I\omega^2 + U_{1\to 2} = \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$$

$$U_{1\to 2} = \underbrace{F}_{\text{constant}} \cdot d + \underbrace{M}_{\text{constant}} \cdot \theta + \Delta V_g + \Delta V_e$$

Kinetic energy $(T) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

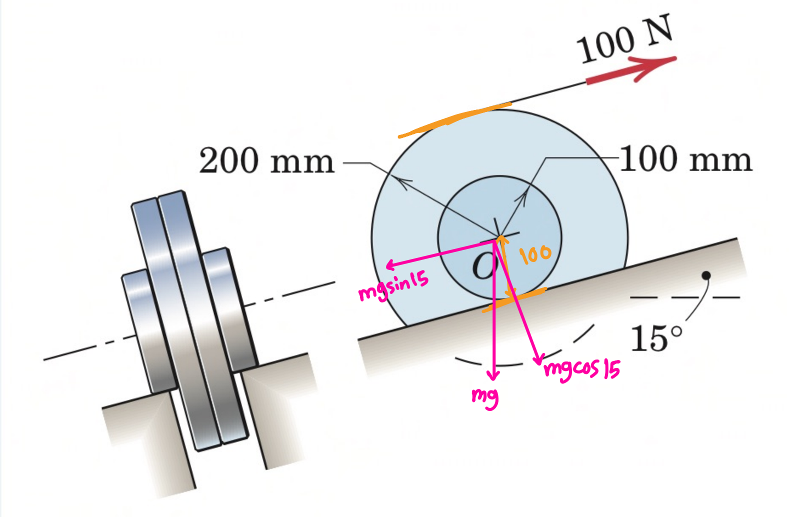
potential energy $= V_e + V_g$

$$T_1 + V_{e_1} + V_{g_1} + U'_{1\to 2} = T_2 + V_{e_2} + V_{g_2}$$

power $= F \cdot V + M\omega$

SAMPLE PROBLEM 6/9

The wheel rolls up the incline on its hubs without slipping and is pulled by the 100-N force applied to the cord wrapped around its outer rim. If the wheel starts from rest, compute its angular velocity ω after its center has moved a distance of 3 m up the incline. The wheel has a mass of 40 kg with center of mass at O and has a centroidal radius of gyration of 150 mm. Determine the power input from the 100-N force at the end of the 3-m motion interval.



$$T_1 + U_{1\to 2} = T_2$$

$$0 + U_{1\to 2} = \underbrace{\frac{1}{2}mv^2 + \frac{1}{2}I_o\omega^2}_{\text{moves linearly and rotation motion at the same time.}}$$

$$(-mg\sin 15°)(3)\underbrace{\left(\frac{300}{100}(100)(3)\right)}_{\text{to bring the force to the center, we use a ratio.}} = \frac{1}{2}m(0.1\omega)^2 + \frac{1}{2}(mk_o^2)\omega^2$$

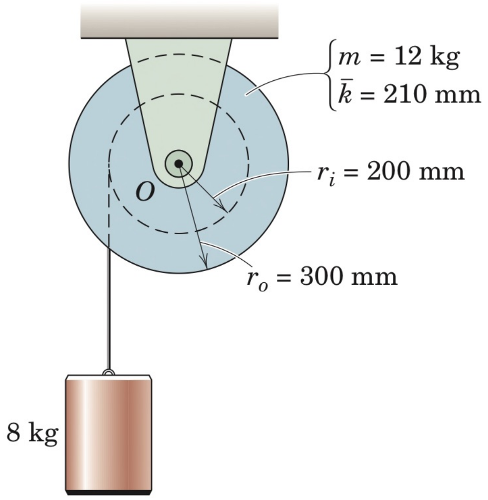
$\omega = 30.3$ rad/s

$P = F \cdot V = F \cdot r \cdot \omega$

$= (100)(0.3)(30.3) = 908$ W

Problems 6/6

6/113 The velocity of the 8-kg cylinder is 0.3 m/s at a certain instant. What is its speed v after dropping an additional 1.5 m? The mass of the grooved drum is 12 kg, its centroidal radius of gyration is $\bar{k} = 210$ mm, and the radius of its groove is $r_i = 200$ mm. The frictional moment at O is a constant 3 N·m.



$$T_1 + U_{1\to2} = T_2$$

$$T_1 = \underbrace{\tfrac{1}{2}(8)(0.3)^2}_{\text{from translation movement}} + \underbrace{\tfrac{1}{2} I \omega_1^2}_{\text{from rotational movement.}}$$

$$I = m k_o^2 = 12(0.21)^2 = 0.53\ \text{kg}\cdot\text{m}^2$$

$$V_1 = r\,\omega_1 \Rightarrow \omega_1 = \frac{V_1}{r_i} = \frac{0.3}{0.2} = 1.5\ \text{rad/s}$$

$$T_1 = \tfrac{1}{2}(8)(0.3)^2 + \tfrac{1}{2}(0.53)(1.5)^2 = 0.955\ \text{J}$$

$$T_2 = \tfrac{1}{2}(8)V_2^2 + \tfrac{1}{2}(0.53)\frac{V_2^2}{0.2^2} \qquad \omega_2 = \frac{V_2^2}{0.2^2}$$

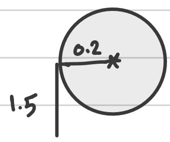
$$U_{1\to2} = mgh$$
$$= 8(9.81)(1.5) - 3\left(\frac{1.5}{0.2}\right)$$

→ positive because it's under the reference (unlike potential would be negative).

frictional Moment

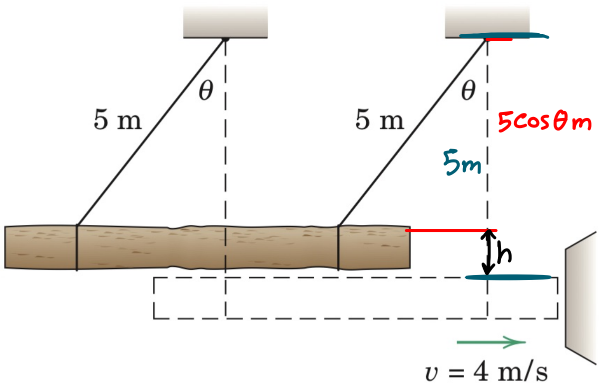
$-M\cdot\theta$

$x = r\,\theta$



$$V_2 = 3.01\ \text{m/s}$$

6/114 The log is suspended by the two parallel 5-m cables and used as a battering ram. At what angle θ should the log be released from rest in order to strike the object to be smashed with a velocity of 4 m/s?



$$T_1 + U_{1\to 2} = T_2$$

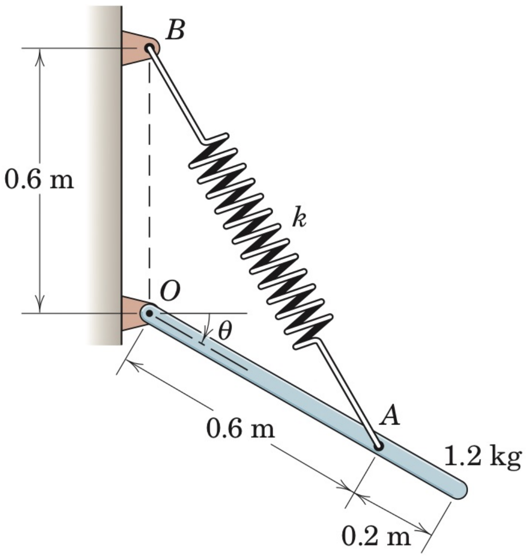
$$0 + mgh = \frac{1}{2} m v^2$$

$$\cancel{m} g (5 - 5\cos\theta) = \frac{1}{2} \cancel{m} v^2$$

$$(9.81)(5 - 5\cos\theta) = \frac{1}{2}(4^2)$$

$$\theta = 33.2^\circ$$

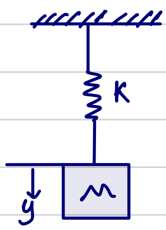
6/120 The 1.2-kg uniform slender bar rotates freely about a horizontal axis through O. The system is released from rest when it is in the horizontal position $\theta = 0$ where the spring is unstretched. If the bar is observed to momentarily stop in the position $\theta = 50°$, determine the spring constant k. For your computed value of k, what is the angular velocity of the bar when $\theta = 25°$?

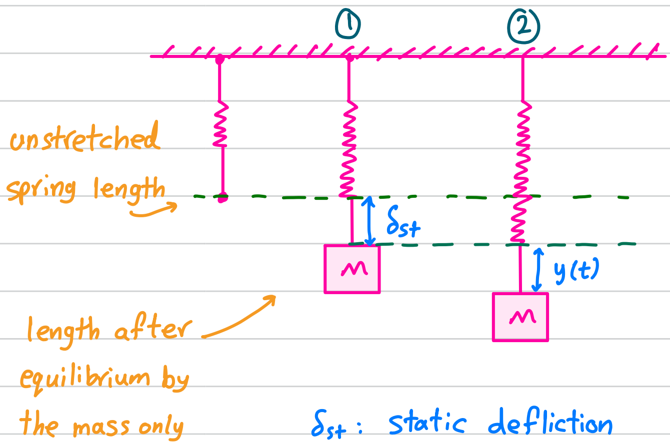


Ch8 Vibration

- Vibration
  - Free vibration
    - Damped
      - Underdamped
      - critically damped
      - overdamped
    - undamped
      - spring-mass
  - Forced vibration
    - Damped
    - undamped

1 Undamped free vibration.





$\downarrow + \Sigma F = m\ddot{y}$

$mg - F_s = m\ddot{y}$

δ_{st} : static defliction

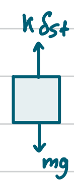
$mg - k(y+\delta_{st}) = m\ddot{y}$

$\cancel{mg} - ky - \cancel{k\delta_{st}} = m\ddot{y}$ $\quad mg = k\,\delta_{st}$

$m\ddot{y} + ky = 0$

$\ddot{y} + \boxed{\frac{k}{m}}\, y = 0$

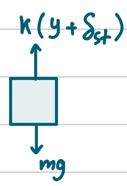
$\hookrightarrow \omega_n = \sqrt{\frac{k}{m}}$

① 

$\Sigma F = 0$ (static)

$mg - k\delta_{st} = 0$

$mg = k\,\delta_{st}$

② 

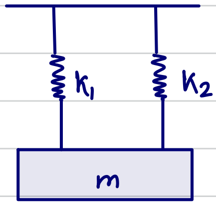
$\Sigma F = m\ddot{y}$

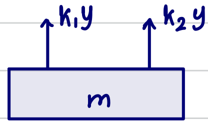
$mg - k(y+\delta_{st}) = m\ddot{y}$

The coefficient of x (or y) is ω_n^2 → Natural frequency

ω_n → rad/s

$f_n = \frac{\omega_n}{2\pi}$ → cycles /s





$-k_1 y - k_2 y = m\ddot{y}$

$m\ddot{y} + k_1 y + k_2 y = 0$

$1\ddot{y} + \frac{(k_1 + k_2)}{m} y = 0$

always must be 1.

$\omega_n = \sqrt{\frac{k_1 + k_2}{m}}$

* There is no (mg) because it got cancelled with the static deflection.

SAMPLE PROBLEM 8/1

A body weighing 25 lb is suspended from a spring of constant $k = 160$ lb/ft. At time $t = 0$, it has a downward velocity of [redacted] 0.5 m/s as it passes through the position of static equilibrium. Determine

$\dot{y}(0) = 0.5$ m/s
$t_o = 0$ s
initial conditions

(*a*) the static spring deflection δ_{st}

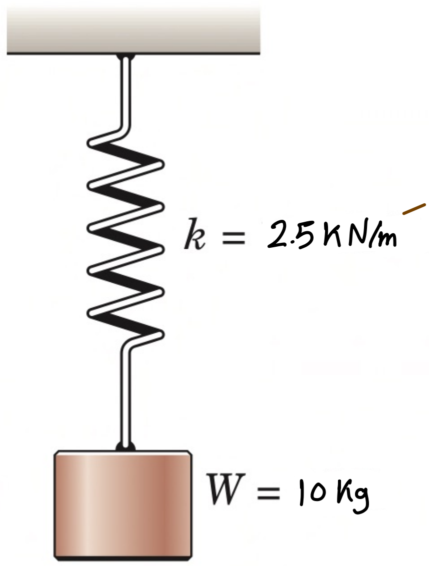
(*b*) the natural frequency of the system in both rad/sec (ω_n) and cycles/sec (f_n)

(*c*) the system period τ

(*d*) the displacement ~~x~~ y as a function of time, where ~~x~~ y is measured from the position of static equilibrium → $y(0) = 0$

(*e*) the maximum velocity v_{max} attained by the mass

(*f*) the maximum acceleration a_{max} attained by the mass.



a) δ_{st} ? $mg = K\delta_{st} \Rightarrow \delta_{st} = \frac{mg}{K} = \frac{10(9.81)}{2.5 * 10^3} = 39$ mm

b) ω_n & f_n ? $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{2.5 \times 10^3}{10}} = 15.8$ rad/s

$f_n = \frac{\omega_n}{2\pi} = 2.52$ cycles/s

c) τ ? $\tau = \frac{1}{f_n} = \frac{1}{2.52} = 0.4$ s

system period

we use initial conditions to find c and ψ

d) $y = C\sin(\omega_n t + \psi)$

$y(0) = C\sin(\psi) = 0 \Rightarrow \sin(\psi) = 0$ $\boxed{\psi = 0}$

or

① $C\sin(\psi) = 0$

② $C\omega_n\cos(\psi) = 0.5$

$\dot{y} = C\omega_n\cos(\omega_n t + \psi) \Rightarrow \dot{y}(0) = C\omega_n\cos(0) = 0.5$

$C(15.8) = 0.5$

$\frac{①}{②}$ $\frac{1}{\omega_n}\tan(\psi) = 0$

$\boxed{C = 0.0316}$

$\therefore \psi = 0$

$y(t) = 0.0316\sin(15.8t)$

e) $V = \dot{y} = 0.0316 * 15.8\cos(15.8t + 0)$

max speed is when the cos = 1

$V_{max} = 0.0316 * 15.8 = 0.57$ m/s

f) $a = \ddot{y} = -0.0316 * \omega_n^2\sin(\omega_n t) = -7.9$ m/s^2

Undamped Free Vibration System Response	must be radians $x = C \sin(\omega_n t + \psi)$
Underdamped Free Vibration System Response	$x = Ce^{-\zeta\omega_n t} \sin(\omega_d t + \psi)$
Critically damped Free Vibration System Response	$x = (A_1 + A_2 t)e^{-\omega_n t}$
Overdamped Free Vibration System Response	$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ $\lambda_1 = \omega_n(-\zeta + \sqrt{\zeta^2 - 1})$ $\lambda_2 = \omega_n(-\zeta - \sqrt{\zeta^2 - 1})$
Undamped Forced Vibration System Steady-State Amplitude	$X = \dfrac{F_0/k}{1 - (\omega/\omega_n)^2}$
Underdamped Forced Vibration System Steady-State Amplitude	$X = \dfrac{F_0/k}{\{[1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2\}^{1/2}}$ $\phi = \tan^{-1}\left[\dfrac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2}\right]$
Underdamped Forced Vibration System Response	$x = Ce^{-\zeta\omega_n t} \sin(\omega_d t + \psi) + X \sin(\omega t - \phi)$

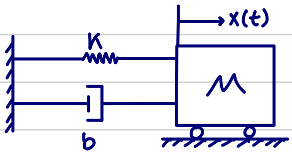
[2] Damped free vibration.

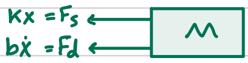
[1] $0 < \zeta < 1 \rightarrow$ underdamped

[2] $\zeta = 1 \rightarrow$ critically damped

[3] $\zeta > 1 \rightarrow$ overdamped

$\zeta = 0 \rightarrow$ undamped ([1])





$$\overset{+}{\rightarrow}\sum F = m\ddot{x}$$

$$-b\dot{x} - kx = m\ddot{x}$$

$$\frac{m}{m}\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$\frac{b}{m} \rightarrow 2\zeta\omega_n$

$$\boxed{\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0}$$

ζ = damping ratio

$$\zeta = \frac{b}{b_{cr}} = \frac{b}{2m\omega_n}$$

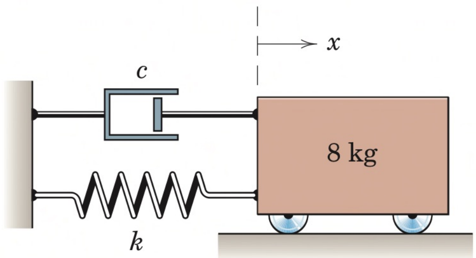
$b_{cr} \rightarrow$ critical damping

damped frequency

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

SAMPLE PROBLEM 8/2

The 8-kg body is moved 0.2 m to the right of the equilibrium position and released from rest at time $t = 0$. Determine its displacement at time $t = 2$ s. The viscous damping coefficient c is 20 N·s/m, and the spring stiffness k is 32 N/m.



$x(0) = 0.2$ m $\qquad \dot{x}(0) = 0$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \frac{20}{8}\dot{x} + \frac{32}{8}x = 0$$

$$\ddot{x} + 2.5\dot{x} + 4x = 0$$

$\omega_n^2 = 4 \Rightarrow \omega_n = 2$ rad/s

$2\zeta\omega_n = 2.5 \Rightarrow \zeta = 0.625$

$\therefore\ 0 < \zeta < 1 \rightarrow$ underdamped

$$\omega_d = 2\sqrt{1-\zeta^2} = 2\sqrt{1-(0.625)^2} = 1.561$$

$$x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$$

(1) $\quad x(0) = A\sin(\psi) = 0.2$

$$\dot{x}(t) = Ae^{-\zeta\omega_n t}\cdot\omega_d\cos(\omega_d t + \psi) - A\zeta\omega_n e^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$$

$$\dot{x}(0) = A\omega_d\cos(\psi) - A\zeta\omega_n\sin(\psi) = 0$$

$$A(1.561)\cos(\psi) = 1.25A\sin(\psi)$$

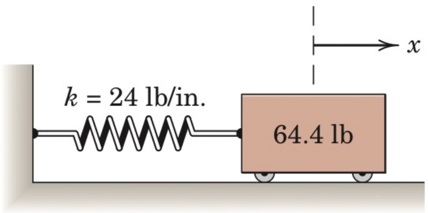
(2) $\quad A\cos(\psi) = \frac{1.25(0.2)}{1.561} = 0.16$

$\frac{(1)}{(2)} \rightarrow \psi = 51.34° * \frac{\pi}{180} = 0.896$ rad $\Rightarrow A = 0.256$ (from (1))

✓ $x(t) = 0.256e^{-1.25t}\sin(1.561t + 0.896) \rightarrow x(2) = 1.47$ mm

problems 8/2

8/2 Determine the natural frequency of the spring-mass system in both rad/sec and cycles/sec (Hz).



Problem 8/2

8/3 For the system of Prob. 8/2, determine the displacement x of the mass as a function of time if the mass is released from rest at time $t = 0$ from a position 2 in. to the right of the equilibrium position.

8/4 For the system of Prob. 8/2, determine the displacement x of the mass as a function of time if the mass is released at time $t = 0$ from a position 2 in. to the left of the equilibrium position with an initial velocity of 7 in./sec to the right. Determine the amplitude C of the motion.

$$\overset{+}{\rightarrow} \Sigma F_x = m\ddot{x}$$

$$-kx = m\ddot{x}$$

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(\frac{24\,lb}{in}\right)\left(\frac{12\,in}{ft}\right)}{\frac{64.4\,lb}{32.2\,ft/s}}} = 12\,rad/s$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{12}{2\pi} = 12 \text{ cycles/s (Hz)}$$

$$x = C\sin(\omega_n t + \psi)$$

$$\dot{x} = C\omega_n\cos(\omega_n t + \psi)$$

$$2 = C\sin(\psi) \quad ①$$

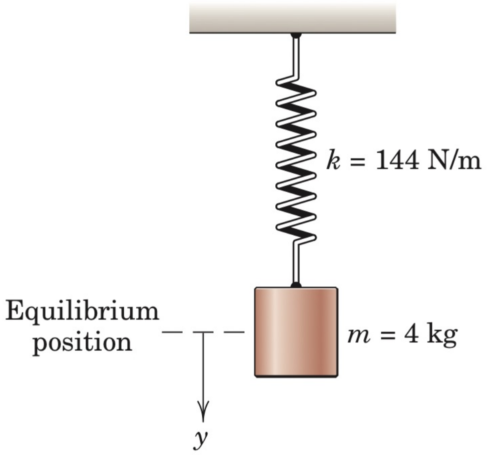
$$0 = C(12)\cos(\psi) \quad ②$$

$$\frac{②}{①} \Rightarrow \quad 0 = 12\frac{1}{\tan(\psi)}$$

8/5 For the spring-mass system shown, determine the static defection δ_{st}, the system period τ, and the maximum velocity v_{max} which result if the cylinder is displaced 0.1 m downward from its equilibrium position and released.

$$\delta_{st} K = mg$$

$$\delta_{st} = \frac{mg}{K} = \frac{4(9.81)}{144} = 0.2735\ m$$



$$+\downarrow \Sigma F_y = m\ddot{y}$$

$$-yk = m\ddot{y}$$

$$\ddot{y} + \frac{k}{m} y = 0$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{144}{4}} = 6$$

$$\tau = \frac{1}{f_n} = \frac{2\pi}{\omega_n} = 1.047\ s$$

$$y = C\sin(\omega_n t + \psi)$$

$$\dot{y} = C\omega_n \cos(\omega_n t + \psi)$$

$$0.1 = C\sin(6(0) + \psi) \Rightarrow C\sin(\psi) = 0.1$$

$C = 0.1$

$$0 = C(6)\cos(6(0) + \psi) \Rightarrow 6C\cos(\psi) = 0 \Rightarrow \psi = 90^\circ = \frac{\pi}{2}\ \text{rad}$$

$$\dot{y} = 0.1(6)\cos\left(6(t) + \frac{\pi}{2}\right)$$

max velocity when $\cos\left(6t + \frac{\pi}{2}\right) = 1$

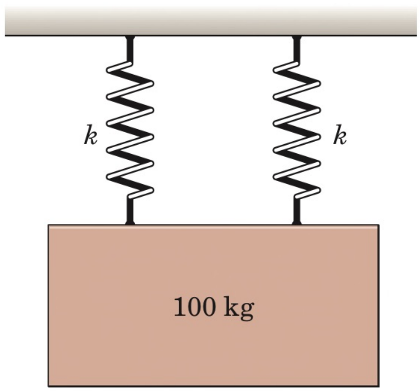
$$\therefore \dot{y}_{max} = 0.6\ m/s$$

8/8 If the 100-kg mass has a downward velocity of 0.5 m/s as it passes through its equilibrium position, calculate the magnitude a_{max} of its maximum acceleration. Each of the two springs has a stiffness k = 180 kN/m.

$$y = C\sin(\omega_n t + \psi)$$

$$\dot{y} = C\omega_n \cos(\omega_n t + \psi)$$

$$\ddot{y} = -C\omega_n^2 \sin(\omega_n t + \psi)$$



$$\downarrow \Sigma F_y = m\ddot{y}$$

$$-2ky = m\ddot{y}$$

$$\ddot{y} + \frac{2k}{m} y = 0$$

$$\omega_n = \sqrt{\frac{2(180 \times 10^3)}{100}} = 60\ rad/s$$

$V_{max} = 0.5$ when $\cos(\omega_n t + \psi) = 1$

$$\therefore 0.5 = C\omega_n \Rightarrow 0.5 = C(60)$$

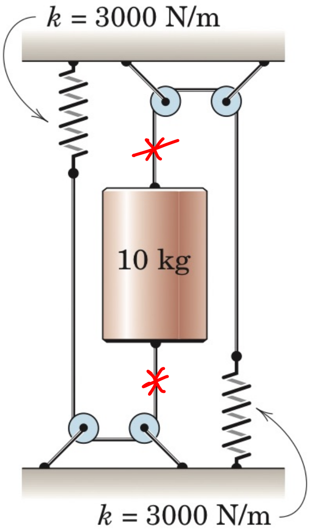
$$C = 8.33 \times 10^{-3}$$

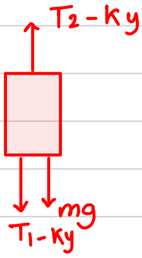
$|a_{max}|$ when $\sin(\omega_n t + \psi) = 1$

$$\therefore |a_{max}| = C\omega_n^2$$

$$|a_{max}| = 8.33 \times 10^{-3}(60)^2 = 30\ m/s^2$$

8/9 Calculate the natural frequency f_n of vertical oscillation of the spring-loaded cylinder when it is set into motion. Both springs are in tension at all times.





$$T_1 + mg = T_2$$

$$+\downarrow \sum F_y = m\ddot{y}$$

$$T_1 + mg - T_2 = m\ddot{y}$$

$$T_1 - ky + mg - T_2 - ky = m\ddot{y}$$

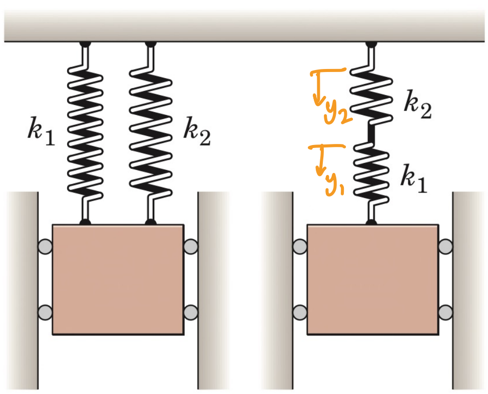
$$\cancel{T_1} - ky + \cancel{mg} - \cancel{T_1} - \cancel{mg} - ky = m\ddot{y}$$

$$\ddot{y} + \frac{2k}{m} y = 0$$

$$\omega_n = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2(3000)}{10}} = 24.5 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = 3.9 \text{ Hz}$$

8/13 Replace the springs in each of the two cases shown by a single spring of stiffness k (equivalent spring stiffness) which will cause each mass to vibrate with its original frequency.



(a) (b)

The two springs are acting as one ∴ their forces are the same but the displacement is different.

a)

$$\downarrow \sum F_y = m\ddot{y}$$

$$-k_1 y - k_2 y = m\ddot{y}$$

$$\ddot{y} + y\frac{(k_1 + k_2)}{m} = 0$$

$$\therefore k = k_1 + k_2$$

b)

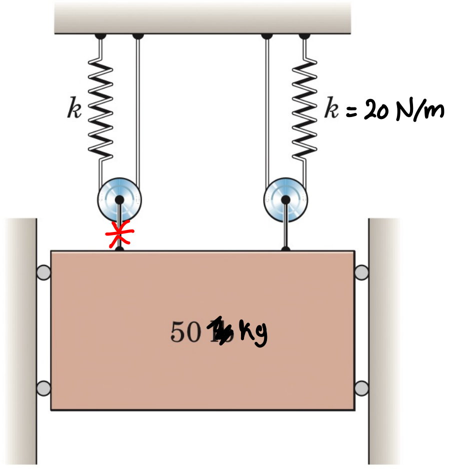
$$y = y_1 + y_2 \qquad F = ky$$

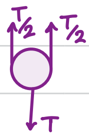
$$\left(\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2}\right) \times \frac{1}{F}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

8/21 Calculate the frequency f_n of vertical oscillation of the 50-lb block when it is set in motion. Each spring has a stiffness of 6 lb/in. Neglect the mass of the pulleys.





$T/2 = ky$

$T = 2ky$

$+\downarrow \sum F_y = m\ddot{y}$

$-2T = m\ddot{y}$

$m\ddot{y} + 2(2ky) = 0$

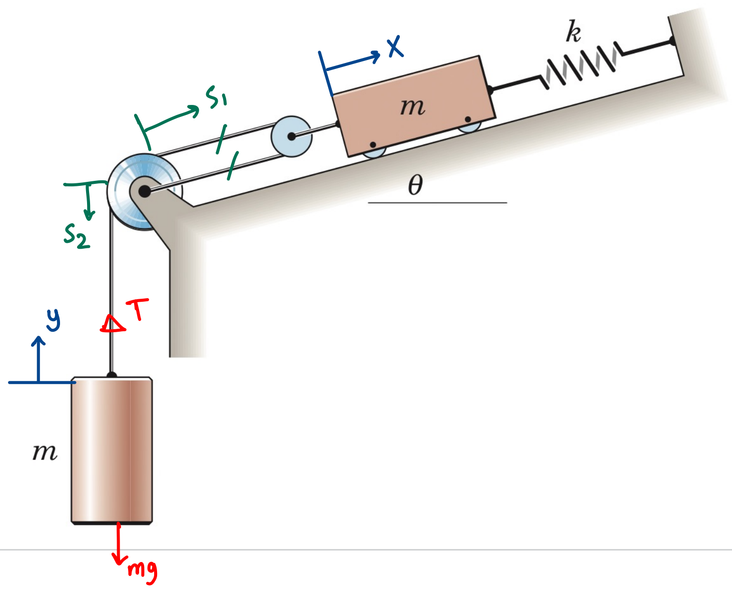
$\ddot{y} + \frac{4k}{m} y = 0$

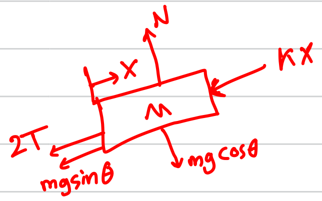
$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(20)}{50}} = 1.265 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = 0.2 \text{ Hz}$$

8/23 Calculate the natural circular frequency ω_n of the system shown in the figure. The mass and friction of the pulleys are negligible.

1DoF





$\overset{+}{\nearrow}\Sigma F = m\ddot{x}$

$-2T - kx = m\ddot{x}$

$+\downarrow \Sigma F_y = m\ddot{y}$

$-T = m\ddot{y}$

mg doesn't appear because it got cancelled with the static deflection

$L = 2S_1 + S_2 + \text{constants} \Rightarrow L = 2x + y + C$

$0 = 2v_1 + v_2 + 0$

$0 = 2\ddot{x} + \ddot{y} \Rightarrow \ddot{y} = -2\ddot{x}$

$-2(-m\ddot{y}) - kx = m\ddot{x}$

$2m\ddot{y} - kx = m\ddot{x}$

$2m(-2\ddot{x}) - kx = m\ddot{x}$

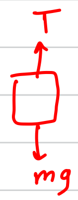
$-4m\ddot{x} - kx = m\ddot{x}$

$5m\ddot{x} + kx = 0$

$\ddot{x} + \frac{k}{5m}x = 0$

$\boxed{\omega_n = \sqrt{\frac{k}{5m}} \text{ rad/s}}$ # ✓

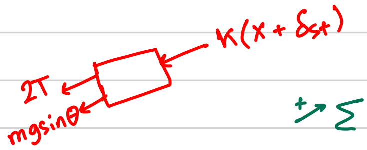
if we're asked to find δ_{st}



$+\downarrow\Sigma F = m\ddot{y} \Rightarrow mg - T = m\ddot{y}$

$T = m(g - \ddot{y})$

$T = m(g + 2\ddot{x})$



$\overset{+}{\nearrow}\Sigma F = m\ddot{x} \Rightarrow -mg\sin\theta - 2T - k(x+\delta_{st}) = m\ddot{x}$

$-mg\sin\theta - 2mg - 2m\ddot{y} - kx - k\delta_{st} = m\ddot{x}$

$-mg\sin\theta - 2mg - 4m\ddot{x} - kx - k\delta_{st} = m\ddot{x}$

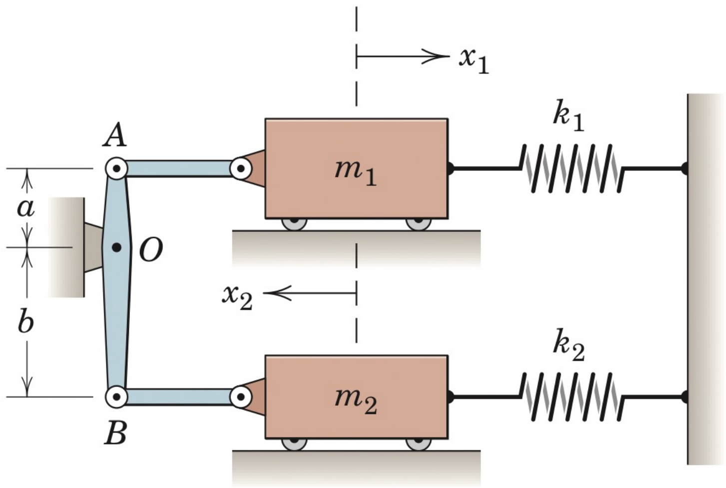
$+mg\sin\theta + 2mg + \underset{0}{\cancel{5m\ddot{x}}} + \underset{0}{\cancel{kx}} + k\delta_{st} = 0$

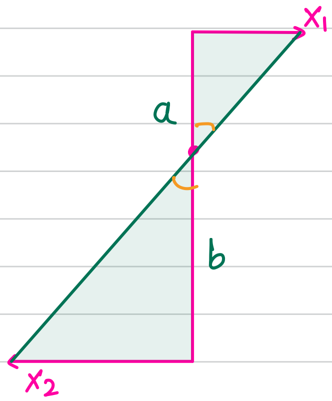
Since we're looking for static deflection, it means there is no movement.

$\ddot{x} = 0$ $\quad x = 0$ (change in position $x(t)$)

$\boxed{\delta_{st} = \frac{2mg + mg\sin\theta}{k}}$

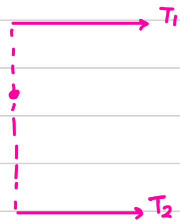
8/24 Derive the differential equation of motion for the system shown in terms of the variable x_1. The mass of the linkage is negligible. State the natural frequency ω_n' in rad/s for the case $k_1 = k_2 = k$ and $m_1 = m_2 = m$. Assume small oscillations throughout.





$$\frac{x_1}{a} = \frac{x_2}{b}$$

$$\boxed{\frac{x_1}{x_2} = \frac{a}{b}}$$



$$\Sigma F = m\ddot{x}_1$$
$$-T_1 - kx_1 = m\ddot{x}_1$$
$$T_1 = -m\ddot{x}_1 - kx_1$$

$$\Sigma F = m\ddot{x}_2$$
$$T_2 - kx_2 = m\ddot{x}_2$$
$$\frac{a}{b}T_1 - k\frac{b}{a}x_1 = m\frac{b}{a}\ddot{x}_1$$

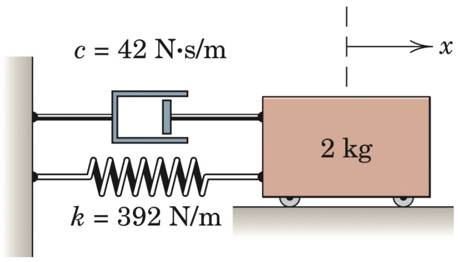
$$\circlearrowleft \Sigma M_o = 0$$
$$-T_1 a + T_2 b = 0$$
$$T_2 = \frac{a}{b}T_1$$

$$-\frac{a}{b}m\ddot{x}_1 - \frac{a}{b}kx_1 - \frac{b}{a}kx_1 = \frac{b}{a}m\ddot{x}_1$$

$$-m\ddot{x}_1\left(\frac{a^2+b^2}{ab}\right) - kx_1\left(\frac{a^2+b^2}{ab}\right) = 0$$

$$w_n = \sqrt{\frac{-k\cancel{\left(\frac{a^2+b^2}{ab}\right)}}{-m\cancel{\left(\frac{a^2+b^2}{ab}\right)}}} = \sqrt{\frac{k}{m}} \text{ rad/s}$$

8/25 Determine the value of the damping ratio ζ for the simple spring-mass-dashpot system shown.



$$\Sigma F_x = m\ddot{x}$$
$$-kx - c\dot{x} = m\ddot{x}$$
$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

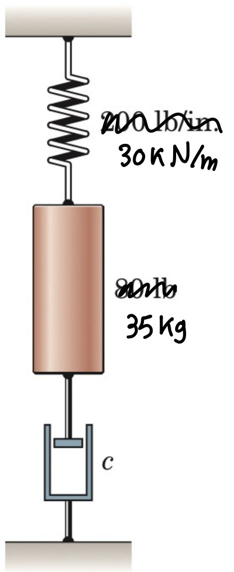
$$2\zeta w_n = \frac{c}{m}$$

$$2\zeta\sqrt{\frac{k}{m}} = \frac{c}{m} \Rightarrow 2\zeta\sqrt{\frac{392}{2}} = \frac{42}{2}$$

$$\boxed{\zeta = 0.75} < 1$$

∴ underdamped

8/29 Determine the value of the viscous damping coefficient c for which the system shown is critically damped.



$\downarrow \Sigma F_y = m\ddot{y}$

$-ky - c\dot{y} = m\ddot{y}$

$\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m}y = 0$

$2\zeta\omega_n = \frac{c}{m}$

$\omega_n = \sqrt{\frac{k}{m}}$

$= \sqrt{\frac{30 \times 10^3}{35}}$

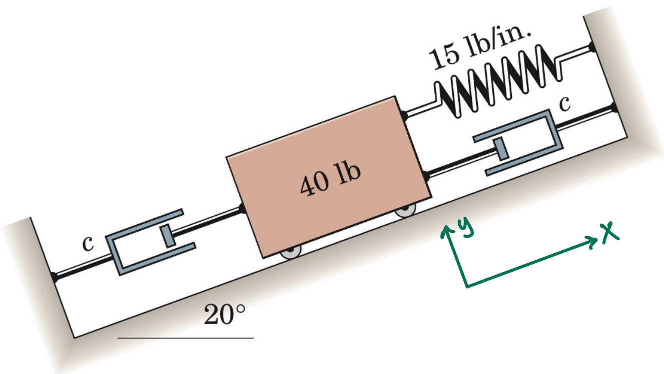
$= 29.7$ rad/s

* $\zeta = 1$ when critically damped.

$2(29.7)(35) = c$

$c = 2049.4$ N·s/m

8/34 Determine the values of the viscous damping coefficient c for which the system has a damping ratio of (a) 0.5 and (b) 1.5.



$\overset{+}{\nearrow} \Sigma F_x = m\ddot{x}$

$-kx - 2c\dot{x} = m\ddot{x}$

$\ddot{x} + \frac{2c}{m}\dot{x} + \frac{k}{m}x = 0$

$2\zeta\omega_n = \frac{2c}{m}$

$2\zeta\sqrt{\frac{k}{m}} = \frac{2c}{m}$

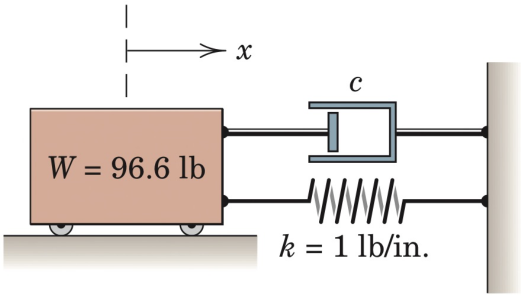
a) $2(0.5)\sqrt{\frac{15 \cdot 12}{\frac{40}{32.174}}} = \frac{2c}{\frac{40}{32.174}}$

$c = 7.48$ lb·s/ft

b) $2(1.5)\sqrt{\frac{15 \cdot 12}{\frac{40}{32.174}}} = \frac{2c}{\frac{40}{32.174}}$

$c = 22.44$ lb·s/ft

8/38 The mass of the system shown is released from rest at $x_0 = 6$ in. when $t = 0$. Determine the displacement x at $t = 0.5$ sec if (*a*) $c = 12$ lb-sec/ft and (*b*) $c = 18$ lb-sec/ft.



$$\Sigma F_x = m\ddot{x}$$
$$-c\dot{x} - kx = m\ddot{x}$$
$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

a) $2\zeta\omega_n = \frac{c}{m} \Rightarrow 2\zeta\sqrt{\frac{k}{m}} = \frac{c}{m}$

$\omega_n = 2$ rad/s

$$2\zeta\sqrt{\frac{1*12}{\frac{96.6}{32.174}}} = \frac{12}{\frac{96.6}{32.174}}$$

$\zeta = 1$ ∴ critically damped.

b) $2\zeta\omega_n = \frac{c}{m} \Rightarrow 2\zeta\sqrt{\frac{k}{m}} = \frac{c}{m}$

$\omega_n = 2$ rad/s

$$2\zeta\sqrt{\frac{1*12}{\frac{96.6}{32.174}}} = \frac{18}{\frac{96.6}{32.174}}$$

$\zeta = 1.5 > 1$
∴ overdamped.

a)

$$x = (A_1 + A_2 t)e^{-\omega_n t}$$
$$\dot{x} = -\omega_n A_1 e^{-\omega_n t} + -A_2\omega_n t + A_2 e^{-\omega_n t}$$

$6 = (A_1 + A_2(0))e^{-\omega_n(0)} \Rightarrow A_1 = 6$

$0 = -\omega_n A_1 - 0 + A_2 \Rightarrow 0 = -2(6) + A_2 \Rightarrow A_2 = 12$

$x = (6 + 12t)e^{-2t}$ @ $t = 0.5$ s $\rightarrow x = 4.41$ in

b)

$$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$
$$\dot{x} = A_1\lambda_1 e^{\lambda_1 t} + A_2\lambda_2 e^{\lambda_2 t}$$

$\zeta = 1.5$

$\lambda_1 = \omega_n(-\zeta + \sqrt{\zeta^2 - 1}) = -0.76$

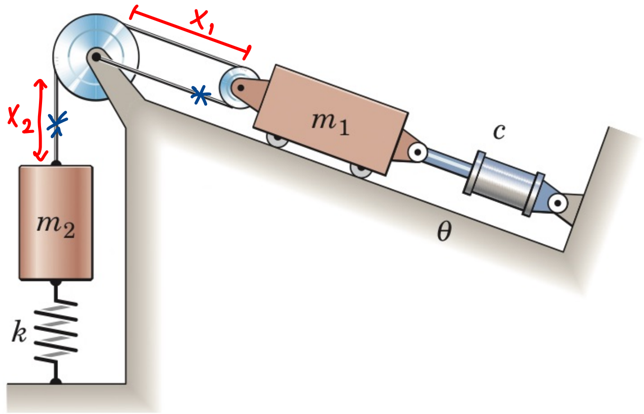
$\lambda_2 = \omega_n(-\zeta - \sqrt{\zeta^2 - 1}) = -5.24$

$6 = A_1 + A_2$

$0 = -0.76A_1 - 5.24A_2 \Rightarrow A_1 = 7.02 \quad A_2 = -1.02$

$x = (7.02)e^{-0.76t} + (-1.02)e^{-5.24t}$ @ $t = 0.5 \rightarrow x = 4.73$ in.

8/41 Determine the damping ratio ζ of the system depicted in the figure. The mass and friction of the pulleys are negligible, and the cable remains taut at all times.



$x_2 = 2x_1$

$\dot{x}_2 = 2\dot{x}_1$

$\ddot{x}_2 = 2\ddot{x}_1$

$\Sigma F_1 = m_1\ddot{x}_1$

$2T - c\dot{x}_1 = m_1\ddot{x}_1$

$m_1\ddot{x}_1 + c\dot{x}_1 - 2T = 0$

$\Sigma F_2 = m\ddot{x}_2$

$-T - kx_2 = m_2\ddot{x}_2$

$-m_2\ddot{x}_2 - kx_2 = T$

$m\ddot{x}_1 + c\dot{x}_1 - 2(-m_2\ddot{x}_2 - kx_2) = 0$

$m_1\ddot{x}_1 + c\dot{x}_1 + 2m_2\ddot{x}_2 + 2kx_2 = 0$

$m_1\dfrac{\ddot{x}_2}{2} + c\dfrac{\dot{x}_2}{2} + 2m_2\ddot{x}_2 + 2kx_2 = 0$

$m_1\ddot{x}_2 + c\dot{x}_2 + 4m_2\ddot{x}_2 + 4kx_2 = 0$

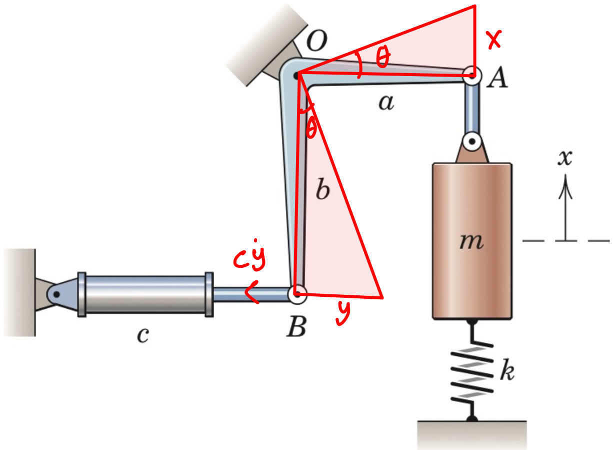
$\ddot{x}_2(m_1 + 4m_2) + c\dot{x}_2 + 4kx_2 = 0$

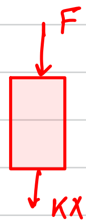
$\ddot{x}_2 + \dfrac{c}{m_1 + 4m_2}\dot{x}_2 + \dfrac{4k}{m_1 + 4m_2}x_2 = 0$

$2\zeta\omega_n = \dfrac{c}{m_1 + 4m_2}$

$\zeta = \dfrac{c}{m_1 + 4m_2} * \dfrac{1}{2\sqrt{\dfrac{4k}{m_1+4m_2}}} = \dfrac{c}{4\sqrt{\dfrac{k(m_1+4m_2)^2}{m_1+4m_2}}} = \dfrac{c}{4\sqrt{k(m_1+4m_2)}}$

8/43 Develop the equation of motion in terms of the variable x for the system shown. Determine an expression for the damping ratio ζ in terms of the given system properties. Neglect the mass of the crank AB and assume small oscillations about the equilibrium position shown.





$$c\dot{y}(b) = F(a)$$

$$F = c\dot{y}\frac{b}{a}$$

$$\theta = \frac{x}{a} = \frac{y}{b} \qquad y = \frac{bx}{a}$$