تقدم لجنة

ملخص لمادة:

معالجة إشارات و أنظمة

جزيل الشكر للطالب:

حمزة اسماعيل

Signals and Systems :-

* Improper Integrals and Infinite Series :-

1. Improper Integrals :-

$$\int_0^\infty f(t)\,dt = \lim_{X\to\infty} \int_0^X f(t)\,dt .$$ → Convergent / divergent.

$$\int_{-\infty}^\infty f(t)\,dt = \lim_{X\to\infty} \int_{-X}^X f(t)\,dt .$$

2. Infinite series :-

$$\sum_{n=0}^\infty X[n] = \lim_{N\to\infty} \sum_{n=0}^N X[n]$$ → Convergent / divergent.

$$\sum_{-\infty}^\infty X[n] = \lim_{N\to\infty} \sum_{-N}^N X[n]$$

* Type of signals :-

1. Continuous time signal. (CT).
   $X(t)$ → Continuous time signal.
2. discrete time signal (DT).
   $X[n]$ → discrete time signal.
   n: Integer.

* Signals :-

Signal : function of one or more independent Variable.

$f(t)$, $X[n]$ → Signals.

t, n → Independent Variable.

f, X → dependent Variable.

$f(t)$ → Continuous time Signal.

$X[n]$ → Discrete time Signal.

the defference between (CT) and (DT) is that (DT) takes only integer numbers

* Signals :-

1. Real Valued Signal.
   → Continuous time signal
   $f(t) = a(t)$ → real Value.
   → discrete time signal
   $X[n] = a[n]$ → real value.
2. Complex Valued Signal :-
   → Polar Coordinate. $X = a + jb$.
   magnitude $|X| = \sqrt{a^2 + b^2}$.
   Phase $\theta = \tan^{-1}\left(\frac{b}{a}\right)$.
   → Cartesian coordinate :-
   $X = a + jb$.
   a → real Part b → Imaginary Part.

$X(t) = a(t) + j\,b(t)$ ⇒ Complex signal

Complex Signal, real signal, Imaginary signal.

* Signal Power and Energy :-

1. Physical Power, energy :-
   $\text{Power} = \frac{d\,\text{energy}}{dt}$. $\text{Energy} = \int_{t_1}^{t_2} \text{Power}\,dt$
2. Signal Power, energy :-

→ $\text{average} = \int f(t)\,dt$.

→ $\text{average}_{CT} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} X(t)\,dt$.

→ $\text{average}_{DT} = \frac{1}{n_2 - n_1 + 1} \sum_{n_1}^{n_2} X[n]$.

Continuous → Integration ($\int$).

Discrete → Series (Σ).

* Signals and Systems:-

* Total Energy:-

1. total energy for CT $x[t_1 \cdot t_2]$.

$$E_{total} = \int_{t_1}^{t_2} |x(t)|^2 \, dt \geq 0$$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

2. total energy for DT $x[n_1 \cdot n_2]$

$$E_{total} = \sum_{n=n_1}^{n_2} |x[n]|^2 .$$

$$E_\infty = \sum_{-\infty}^{\infty} |x[n]|^2 = \lim_{N \to \infty} \sum_{-N}^{N} |x[n]|^2$$

* Average Power:-

1. average power for CT $x[t_1 \cdot t_2]$

$$P_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt .$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt .$$

2. average power for DT $x[n_1 \cdot n_2]$

$$P_{avg} = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2 .$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2 .$$

* linear transformation of the independent Variable:-

Linear transformation (CT) signal

$x(t) \to x(\alpha t + \beta)$.

Linear transformation (DT) signal.

$x[n] \to x[\alpha n + \beta]$.

$\alpha, \beta \to$ constant

(real value not complex).

* Time shifting and Time scaling:-

1. β: represent time shifting.

$\alpha = 1 . \beta > 0 \to$ Left (time advance).

$\alpha = 1 . \beta < 0 \to$ right (time delay).

2. α: represent time scaling/reversal:-

$\beta = 0 . \alpha = -1 \to$ reverse about y-axis.

$|\alpha| > 1 \to$ Compression الرسمة تنضغط

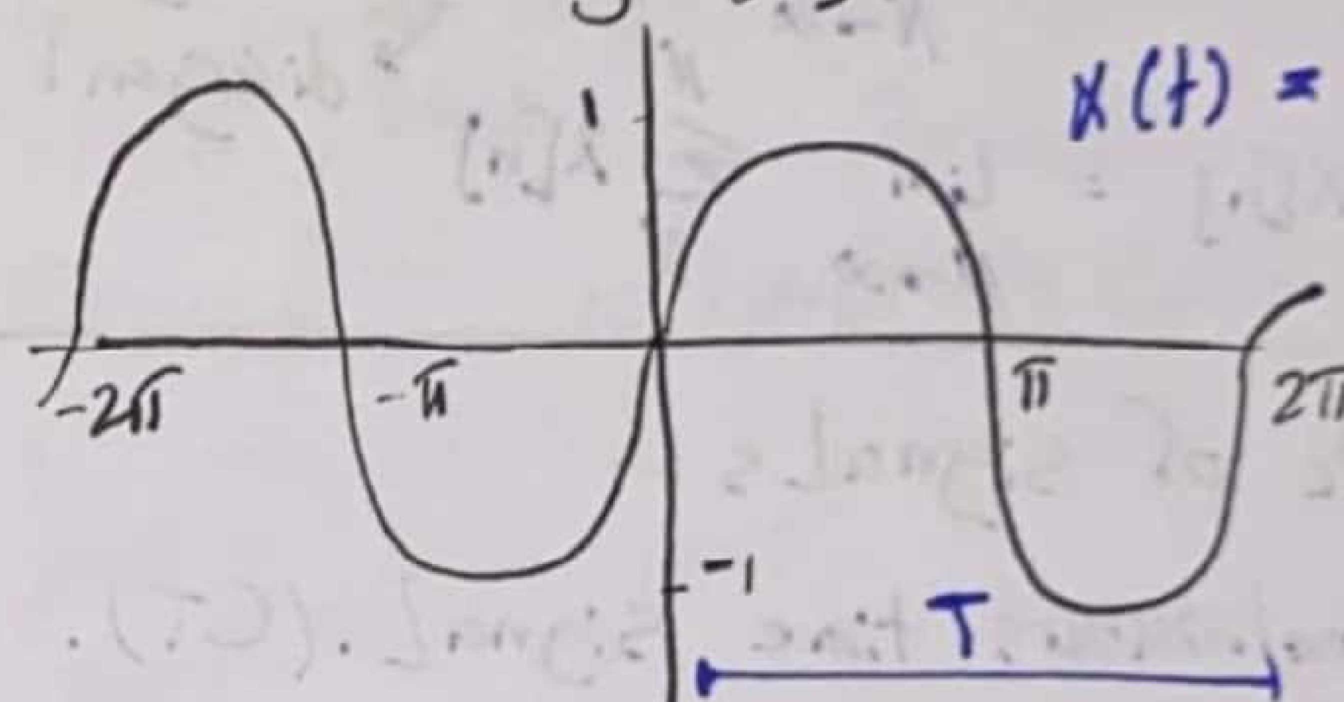
$|\alpha| < 1 \to$ streching الرسمة تتمدد

→ Compression and streching

(بيصير بالعرض. على محور ال X وليس Y).

* Periodic signals:-

$x(t) = \sin(t)$



T:- Period . $T = 2\pi$

f:- frequency $f = \frac{1}{T} = \frac{1}{2\pi}$ (H)

W:- angular frequency $w = 2\pi f = \frac{2\pi}{T}$

$\sin(t) = \sin(t + 2\pi) = \sin(t + mT)$

m: is an integer.

the fundamental period → (T_0), (N_0)

. (Period) اصغر قيمة موجبة لل

* formal Definitions of Periodic signals:-

1. CT $x(t)$ is Periodic if:

$x(t) = x(t + mT)$ اذا كان (T) رقم حقيقي يحقق المعادلة لكل قيم (t).

2. DT $x[n]$ is Periodic if:

$x[n] = x[n + mN]$ اذا كان (N) رقم صحيح يحقق المعادلة.

$n, m, N \to$ Integers.

Signals and Systems :-

The fundamental Period (T_0) :-

$x(t) = \cos(w_0 t)$.

$x(t) = x(t+2\pi)$

$\cos(w_0 t) = \cos(w_0(t+2\pi))$.

$\cos(w_0 t) = \cos(w_0 t + nT w_0)$.

$T_0 w_0 = 2\pi \Rightarrow \boxed{T_0 = \frac{2\pi}{|w_0|}}$

$w_0 = \frac{2\pi}{T_0}$: w_0 : fundamental frequency.

Odd and even Signals :-

- A signal is even if :-
  1. CT case :- $x(t) = x(-t)$
  2. DT case :- $x[n] = x[-n]$.
- A signal is odd if :-
  1. CT case :- $x(t) = -x(-t)$.
  2. DT case :- $x[n] = -x[-n]$.

Euler Identity :-

$e^{J\theta} = \cos\theta + J\sin\theta$.

real Part, imaginary Part.

Any signal ⇒ Can express it by 2 sub segments → odd فردي + even زوجي

CT case : $x(t) = Od\{x(t)\} + Ev\{x(t)\}$.

$od\{x(t)\} = \frac{x(t) - x(-t)}{2}$.

$Ev\{x(t)\} = \frac{x(t) + x(t)}{2}$

$od\{x(t)\} + Ev\{x(t)\} = x(t)$.

Basic CT Signals (exponential and sinusoidal signals).

general complex exponential signal

$\boxed{x(t) = Ce^{at}}$, C, a → complex number

$C = |C|e^{J\theta}$ $a = r + Jw_0$.

$x(t) = |C|e^{J\theta}e^{(r+Jw_0)t}$

$x(t) = |C|e^{rt}e^{J(\theta+w_0 t)}$

$x(t) = |C|e^{rt}(\cos(\theta+w_0 t) + J\sin(\theta+w_0 t))$

amplitude. real Part   imaginary Part.

$x(t) = Ce^{at}$. ~~C, a → Real number~~.

$C=1$. a → doesn't have real Part.

$x(t) = e^{Jw_0 t}$ → Periodic.

$x(t) = e^{Jw_0 t} = \cos(w_0 t) + J\sin(w_0 t)$.

$x(t) = x(t+T)$.

$e^{Jw_0 t} \overset{?}{=} e^{Jw_0 t}e^{Jw_0 T}$.

$e^{Jw_0 T} = 1$. $e^0 = 1$

∴ $w_0 T = 0, 2\pi, 4\pi, \dots$

$\boxed{T_0 = \frac{2\pi}{|w_0|}}$

Basic DT signals (exponential and sinusoidal signals) :-

general complex exponential signal

$x[n] = C\alpha^n$. $x[n] = Ce^{Bn}$.

C, α → complex number.

$C = |C|e^{J\theta}$. $\alpha = |\alpha|e^{Jw_0}$

$x[n] = C\alpha^n$.

$x[n] = |C|e^{J\theta}|\alpha|^n e^{Jw_0 n}$

$x[n] = |C||\alpha|^n e^{J(w_0 n+\theta)}$.

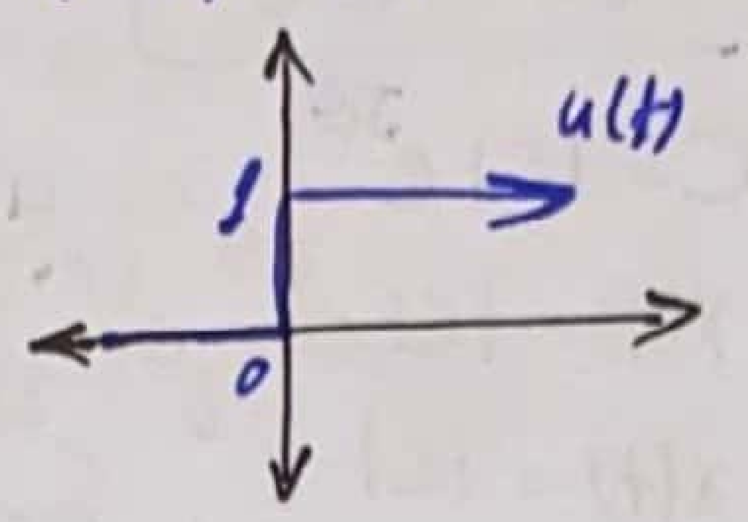
$x[n] = |C||\alpha|^n(\cos(w_0 n+\theta) + J\sin(w_0 n+\theta))$

* Signals and Systems :-

* Unit step function and dirac delta function (Impulse) :-

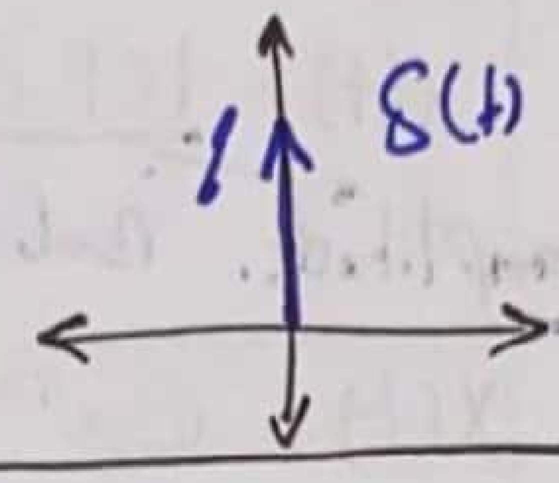
1. Unit step :-

$$u(t) = \begin{cases} 1 , & t \geq 0 \\ 0 , & t > 0 \end{cases}$$



2. Impulse function $\delta(t)$

$$\delta(t) = \begin{cases} 1 , & t = 0 \\ 0 , & \text{Otherwize.} \end{cases}$$



* Relationship between $\delta(t)$, $u(t)$:-

1. for Continuous signals :-

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau \ , \quad \delta(\tau) = \frac{d}{dt}[u(t)].$$

2. for discrete signals :-

$$u[n] = \sum_{-\infty}^{n} \delta[m] .$$

* Properties of unit step and Unit Impulse

1. Unit step :-

- $u(t)^2 = u(t)$
- $u(Kt - t_0) = u(t - \frac{t_0}{K})$
- $u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau .$

2. Unit Impulse :-

- $\delta(t) = \frac{d}{dt}[u(t)]$
- $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$, $\int_{-\infty}^{t} \delta(t)\, dt = u(t)$
- $f(t)\, \delta(t - t_0) = f(t_0)\, \delta(t - t_0) .$
- $\int_{-\infty}^{\infty} f(t)\, \delta(t - t_0)\, dt = f(t_0) .$
- $\delta(at + b) = \frac{1}{|a|}\, \delta(t + \frac{b}{a}) .$
- $\delta[n] = u[n] - u[n-1] .$

* Properties of Systems :-

1. Memory less system :- لا ذاكرة

Output at time (t) depends on the Input at the same time .

2. Invertable system :- Inverse الـ

اذا قدرنا نعرف الـ (input) بمعرفتنا للـ (output) .

3. Causal system :- النظام منطقي

الـ (output) يعتمد ع زمن الـ input نفسه او على زمن قبله .

4. stability system :- نظام مستقر

Bounded input Bounded output.

5. Time Invariance (TI) system :-

Output doesn't depend on the input signal is applied.

الـ output ما بتغير مع مرور الزمن .

6. linearity system :- نظام خطي

$$a x_1 + b x_2 = a y_1 + b y_2$$

* Convolution for LTI systems :-

1. Convolution sum (DT LTI system) :-

sifting Properties :-

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$$

$$x[n] \longrightarrow \boxed{T_{LTI}} \longrightarrow y[n] .$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = x[n] * h[n]$$

$\delta[n] \rightarrow$ Impulse signal.

$h[n] \rightarrow$ Impulse response.

مهم

$$* \int_{-\infty}^{\infty} f(t)\, \delta(t - t_0)\, dt = f(t_0) .$$

$$\sum_{-\infty}^{\infty} f[n]\, \delta[n - n_0] = f[n_0] .$$

Signals and Systems:-

* Convolution for LTI systems:-

2. Convolution Integrals (CT LTI system)

Sifting Properties:-

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$$x[t] \longrightarrow \boxed{T_{LTI}} \longrightarrow y[t].$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau.$$

* Properties of LTI systems:-

1. Convolution is commutative تبديلية

$$x(t) * h(t) = h(t) * x(t)$$

$$x[n] * h[n] = h[n] * x[n]$$

$$* \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

$$* \sum_{-\infty}^{\infty} x[k]\,h[n-k] = \sum_{-\infty}^{\infty} h[k]\,x[n-k]$$

2. Convolution is distributive:- توزيعية

$$* \; x(t) * (h_1(t) \pm h_2(t)) = x(t) * h_1 \pm x(t) * h_2$$

$$* \; x[n] * (h_1[n] \pm h_2[n]) = x[n] * h_1 \pm x[n] * h_2$$

3. Convolution is associative:-

$$* \; x(t) * h_1(t) * h_2(t) = (x * h_1) * h_2$$

$$= x * (h_1 * h_2) = (x * h_2) * h_1.$$

4. System with and without memory:-

for DT LTI system:-

$$y[n] = \sum_{-\infty}^{\infty} x[k]\,h[n-k] = x[n] * h[n].$$

System memoryless ⇒ $y[n] = 0$ every where unless the point $[n]$.

$h[n-k] = 0$ for $n \neq k$.

$h[n-k] = \text{constant}$ $n = k$.

⇒ $\boxed{h[n] = C\,\delta[n]}$. for DT LTI

⇒ $\boxed{h(t) = C\,\delta(t)}$ for CT LTI

* Properties of LTI systems:-

5- Invertibility of LTI systems:-

$$x \longrightarrow \overset{LTI}{\boxed{h}} \longrightarrow \boxed{h_2} \longrightarrow y = x.$$

$$x \longrightarrow \boxed{h * h_1} \longrightarrow y = x \qquad x * (h * h_1) = x$$

Conclusion $h * h_2 = \delta$.

* for CT: $h(t) * h_i(t) = \delta(t)$
* for DT: $h[n] * h_i[n] = \delta[n]$

شغل بالك
عشان تطلع
الانفرس

6- Causality of LTI systems:-

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$$* \boxed{y[n] = \sum_{k=-\infty}^{n} x[k]\,h[n-k]} + \sum_{k=n+1}^{\infty} \cancel{x[k]\,h[n-k]} \rightarrow 0.$$

$n \geq k \rightarrow n-k \geq 0$ $\qquad$ $n < k \rightarrow n-k < 0$

or:-

$$* \; y[n] = \sum_{-\infty}^{\infty} h[k]\,x[n-k] = \boxed{\sum_{0}^{\infty} h[k]\,x[n-k]}.$$

for CT LTI system.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau.$$

$$* \boxed{y(t) = \int_{-\infty}^{t} x(\tau)\,h(t-\tau)\,d\tau} + \int_{t}^{\infty} \cancel{x(\tau)\,h(t-\tau)} \rightarrow 0 \; d\tau.$$

$t \geq \tau \rightarrow t-\tau \geq 0$ $\qquad$ $t < \tau \rightarrow t-\tau < 0$

or:-

$$* \; y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau = \boxed{\int_{0}^{\infty} h(\tau)\,x(t-\tau)\,d\tau}$$

7- Stability of LTI system:-

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{-\infty}^{\infty} h[k]\,x[n-k]$$

$$BIBO \Rightarrow |x[n]| < B_1, \; |y[n]| < B_2$$

$$|y[n]| = \left|\sum_{k=-\infty}^{\infty} h[k]\,x[n-k]\right| \leq \sum_{k=-\infty}^{\infty} |h[k]\,x[n-k]|$$

$$|y[n]| = \sum_{k=-\infty}^{\infty} |h[k]|\,|x[n-k]| = B_1 \sum_{-\infty}^{\infty} |h[k]|$$

$$, \; \sum_{-\infty}^{\infty} |h[k]| < \infty$$

$$|y[n]| < B_2$$

$|y[n]| < \infty$ ⇒ stable system.

• $h[n]$ is absolutely summable.

- for CT LTI system:-

$$|y(t)| = B \int_{-\infty}^{\infty} |h(\tau)|\,d\tau < \infty$$

• $h(t)$ is absolutely integrable.

Signals and Systems :-

* Properties of LTI system :-

8- unit step response of LTI system $s(t)$, $s[n]$ and relation to $h(t)$, $h[n]$:-

$\delta(t) \rightarrow \boxed{LTI} \rightarrow h(t)$. Impulse response

$u(t) \rightarrow \boxed{LTI} \rightarrow s(t)$. step response

$y(t) = x(t) * h(t)$.

$s(t) = u(t) * h(t)$.

$$s(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau.$$

$$s(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau.$$

$$\boxed{s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau \qquad h(t) = \frac{d[s(t)]}{dt}}$$

$\int$ → Integrals, $\frac{d}{dt}$ → direvative

for DT LTI system :-

$s[n] = u[n] * h[n]$

$$s[n] = \sum_{k=-\infty}^{\infty} u[k]\, h[n-k]$$

$$s[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$$

$$\boxed{s[n] = \sum_{-\infty}^{n} h[k]. \qquad s[n] = s[n-1] + h[n] \qquad h[n] = s[n] - s[n-1]}$$

→ difference

* Fourier series representation of Periodic signals :-

* Eigen function and Eigen Value :-

$f(t) \rightarrow \boxed{I} \rightarrow A f(t)$

$f(t) \rightarrow$ Eigen function.

$A \rightarrow$ Eigen Value.

- for a complex exponential (e^{st}) continu

$e^{st} \rightarrow \boxed{LTI} \rightarrow H(s)\, e^{st}$.

s: complex number

$e^{st} \rightarrow$ Eigen function . $H(s) \rightarrow$ eigen Value

- for a DT of complex exponential :-

$z^{n} \rightarrow \boxed{LTI} \rightarrow H(z)\, z^{n}$.

Z: complex number

$z^{n} \rightarrow$ Eigen function . $H(z) \rightarrow$ Eigen Value.

- Continuous Time signal :-

$$\boxed{H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau.}$$

- Discrete Time signal :-

$$\boxed{H(z) = \sum_{k=-\infty}^{\infty} h[k]\, z^{-k}}$$

$x(t) \longrightarrow y(t)$

$e^{s_1 t} \longrightarrow H(s)\, e^{s_1 t}$

$a_1 e^{s_1 t} \longrightarrow a_1 H(s_1)\, e^{s_1 t}$.

$a_1 e^{s_1 t} + a_2 e^{s_2 t} \longrightarrow a_1 H(s_1) e^{s_1 t} + a_2 H(s_2) e^{s_2 t}$.

CT LTI & Eigen function لما بدخل أكثر من linear Combination وبتسمي

$$\sum_{k} a_k e^{s_k t} \xrightarrow[LTI]{CT} \sum a_k H(s_k) e^{s_k t}$$

for DT :-

$$\sum_{k} a_k z_k^{n} \xrightarrow[LTI]{DT} \sum_{k} a_k H(z_k) z_k^{n}.$$

Signals and Systems:-

* fourier series (FS) of continuous time Periodic signals (CTFS):-

* fourier series (FS):-

$x(t) \rightarrow$ CT and Periodic signal

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t} \qquad w_0 = \frac{2\pi}{T}$$

if $x(t)$ is real value $\Rightarrow x(t) = x^*(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t} = x^*(t).$$

$$= \sum_{-\infty}^{\infty} a_k^* e^{-jkw_0 t} = \sum_{k=-\infty}^{\infty} a_{-k}^* e^{jkw_0 t}$$

$$\Rightarrow \boxed{a_k = a_{-k}^*} \Leftrightarrow \boxed{a_k^* = a_{-k}.}$$

* fourier series Coefficients (a_k):-

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t} \qquad w_0 = \frac{2\pi}{T}.$$

$$\boxed{a_k = \frac{1}{T} \int_0^T x(t)\, e^{-jkw_0 t}\, dt}$$

a_k: fourier series coefficients.

a_k: fourier series analysis equations.

$$a_0 = \frac{1}{T} \int_T x(t)\, dt.$$

a_0: constant (DC) compound of $x(t)$

* fourier series (FS):-

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t} = a_0 + \sum_{k=1}^{\infty} 2|a_k| \cos(kw_0 t + \theta)$$

$$a_k = \frac{1}{T} \int_T x(t)\, e^{-jkw_0 t} \rightarrow \text{complex}.$$

$$a_k = |a_k| \angle \theta = |a_k|\, e^{j\theta} = \alpha + j\beta.$$

$$\boxed{x(t) = a_0 + \sum^{\infty} 2|a_k| \cos(kw_0 t + \theta_k)}$$

$|a_k| = \sqrt{\alpha^2 + \beta^2}$ Amplitude spectram

$\theta_k = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$ phase spectram.

* Eular formala:-

- $e^{j\theta} = \cos\theta + j\sin\theta.$
- $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
- $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

* Properties of continuous time of Fourier seiries :-

let we have aperiodic signals $x(t), y(t)$ and have the same fundamental Period (T)

1. Linearity :-

$x(t) \xrightarrow{FS} a_k$, $y(t) \xrightarrow{FS} b_k$

$$\underbrace{A x(t) + B y(t)}_{\text{linear combination.}} \xrightarrow{FS} A a_k + B b_k.$$

2. Time shifting :-

$x(t) \xrightarrow{FS} a_k$

$x(t - t_0) \longrightarrow a_k\, e^{-jkw_0 t_0}$

3. Time reversal :-

$x(t) \xrightarrow{FS} a_k.$

$x(-t) \longrightarrow a_{-k}$

4- Time scaling :-

$x(t) \xrightarrow{FS} a_k$

$x(at) \longrightarrow a_k.$

5. Multiplication Property :-

$x(t) \xrightarrow{FS} a_k$, $y(t) \xrightarrow{FS} b_k.$

$$x(t)\, y(t) \xrightarrow{FS} \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

$$\sum_{-\infty}^{\infty} a_l b_{k-l} = a_k * b_k.$$

6. Conjugate and conjugate symmetry

$x(t) \xrightarrow{FS} a_k.$, $x^*(t) \xrightarrow{FS} a_{-k}$

if $x(t)$ is real.

$x(t) = x^*(t) \qquad a_k = a_{-k}^*$

$|a_k| = |a_{-k}|$, $\measuredangle a_k = -\measuredangle a_{-k}.$

Signals and Systems:-

* Parseval's Relation for CT Periodic signals :-

- Parseval's Relation:-

$$\sum |a_k|^2 = \frac{1}{T}\int_T |x(t)|^2\, dt.$$

$$x(t) = \sum a_k e^{jk\omega_0 t}$$

$$\frac{1}{T}\int_T | a_k e^{jk\omega_0 t}|^2 dt$$

$$= \frac{1}{T}\int_T |a_k|^2 \, |\cancel{e^{jk\omega_0 t}}| \, dt . = \sum_{-\infty}^{\infty} |a_k|^2$$

* Fourier series (FS) of a Discrete Time (DT) Periodic signals ($x[n]$):-

DT harmonically related complex exponential ($\phi_k[n]$) :

$$\phi_k[n] = e^{jk\omega_0 n} \qquad \omega_0 = \frac{2\pi}{N}$$

$$\phi_k[n] = \phi_{k+N}[n] = \phi_{k+mN}[n] \rightarrow \text{Periodic}$$

DTFS- Synthesis equation

$$x[n] = \sum_{k=k_0}^{k_0+N-1} a_k \, \phi_k[n] = \sum_{\langle N \rangle} a_k \, \phi_k[n]$$

$$= \sum_{\langle N \rangle} a_k e^{jk\omega_0 n} \qquad \omega_0 = \frac{2\pi}{N}$$

$$a_k = \frac{1}{N}\sum_{n\in\langle N \rangle} x[n] \, e^{-jk\omega_0 n}$$

a_k :- fourier series coefficients

a_k :- DTFS - Analysis equation.

* CTFS and DTFS :-

CTFS ⇒ have an convergence issues and Gibbs Phenomenon.

DTFS ⇒ has no convergence issues and no Gibbs Phenomenon.

* Properties of DT of FS :-

1. linearity :-

$$x[n] \xrightarrow{FS} a_k \quad , \quad y[n] \rightarrow b_k.$$

$$A\,x[n] + B\,y[n] \xrightarrow{FS} A a_k + B b_k.$$

2- Time shifting :-

$$x[n-n_0] \xrightarrow{FS} a_k \, e^{-jk\omega_0 n}$$

3- Conjugation :-

$$x^*[n] \xrightarrow{FS} a^*_{-k}$$

4- Time reversal :-

$$x[n] \xrightarrow{FS} a_{-k}.$$

5- Multiplication :-

$$x[n]\,y[n] \xrightarrow{FS} \sum_{l=\langle N \rangle} a_l \, b_{k-l}$$

6- Parseval's Relation :-

$$\sum_{k=\langle N \rangle} |a_k|^2 = \frac{1}{N}\sum_{k=\langle N \rangle} |x[n]|^2$$

* frequency response of LTI system:-

1- CT LTI system.

Eigen function $e^{st} \rightarrow \boxed{LTI} \rightarrow H(s)\, e^{st}$ — Eigen Value

$$H(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt = \mathcal{L}[h(t)].$$

$$s = j\omega \Rightarrow H(j\omega) \rightarrow \text{frequency Response}$$

$$e^{j\omega t} \rightarrow \boxed{LTI} \rightarrow H(j\omega)\, e^{j\omega t}.$$

2- DT LTI system.

Eigen function $z^n \rightarrow \boxed{LTI} \rightarrow H(z)\, z^n$ — Eigen Value.

$$H(z) = \sum_{-\infty}^{\infty} h[n]\, z^{-n}$$

$$z = e^{j\omega} \Rightarrow H(e^{j\omega}) \rightarrow \text{frequency response}$$

$$e^{j\omega n} \rightarrow \boxed{LTI} \rightarrow H(e^{j\omega})\, e^{j\omega n}$$

* Signals and Systems :-

* frequency response of LTI system:-

1- CT LTI System :-

$e^{Jwt} \rightarrow \boxed{LTI} \rightarrow H(Jw)\, e^{Jwt}$

$H(Jw) = \int_{-\infty}^{\infty} h(t)\, e^{-Jwt}\, dt$.

$x(t) \rightarrow \boxed{LTI} \rightarrow y(t) = x(t) * h(t)$

$\sum a_k e^{JKw_0t} \rightarrow \boxed{LTI} \rightarrow \sum a_k H(Jw)\, e^{JKw_0t}$

$x(t) = \sum_{-\infty}^{\infty} a_k e^{JKw_0t}$.

$y(t) = \sum_{-\infty}^{\infty} \underline{a_k H(JKw_0)} e^{JKw_0t} = \sum_{-\infty}^{\infty} \underline{b_k} e^{JKw_0t}$

2- DT LTI system :-

$e^{Jwn} \rightarrow \boxed{LTI} \rightarrow H(e^{Jw})\, e^{Jwn}$

$H(e^{Jw}) = \sum_{-\infty}^{\infty} h[n]\, e^{-Jwn}$.

$x[n] \rightarrow \boxed{LTI} \rightarrow y[n] = x[n] * h[n]$

$\sum a_k e^{JKw_0n} \rightarrow \boxed{LTI} \rightarrow \sum a_k H(e^{Jw})\, e^{JKw_0n}$

$x[n] = \sum_{-\infty}^{\infty} a_k e^{JKw_0n}$.

$y[n] = \sum_{-\infty}^{\infty} a_k H(e^{Jw})\, e^{JKw_0n} = \sum_{-\infty} b_k e^{JKw_0n}$

* The Continuous -time fourier transform:-

* fourier Transform (CTFT) :-

$x(t) \rightarrow$ CT and APeriodic or Periodic

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(Jw)\, e^{Jwt}\, dw$

synthesis equation for inverse fourier Transform

$X(Jw) = \int_{-\infty}^{\infty} x(t)\, e^{-Jwt}\, dt$.

analysis equation fourier transform

synthesis equ + analysis equ

$\Rightarrow$ fourier Transform Pair

* fourier Transform for Periodic Signal :-

$x(t) \rightarrow$ CT and Periodic signal.

$X(Jw) = \sum_{K=-\infty}^{\infty} 2\pi\, a_k\, \delta(w - Kw_0)$.

$x(t) = \sum_{K=-\infty}^{\infty} a_k\, e^{JKw_0t}$.

* Properties of the fourier Transform :-

$x(t)\,,\; y(t) \rightarrow$ CT and A Periodic.

$x(t) \xleftrightarrow{FT} X(Jw)\;,\; y(t) \xleftrightarrow{FT} Y(Jw)$.

1- linearity :-

$a\,x(t) + b\,y(t) \xleftrightarrow{FT} a\,X(Jw) + b\,Y(Jw)$

2- Time shifting :-

$x(t - t_0) \xleftrightarrow{FT} X(Jw)\, e^{-Jwt_0}$.

3- frequency shifting :-

$e^{Jw_0t}\, x(t) \xleftrightarrow{FT} X(J(w - w_0))$.

4- Conjugation :-

$x^*(t) \xleftrightarrow{FT} X^*(-Jw)$.

5- Time Reversal :-

$x(-t) \xleftrightarrow{FT} X(-Jw)$.

6- Time scaling :-

$x(at) \xleftrightarrow{FT} \frac{1}{|a|} X\left(\frac{Jw}{a}\right)$

7- Convolution :-

$x(t) * y(t) \xleftrightarrow{FT} X(Jw)\, Y(Jw)$.

8- Multiplication :-

$x(t) \cdot y(t) \xleftrightarrow{FT} \frac{1}{2\pi} X(Jw) * Y(Jw)$

9- differentiation in time :-

$\frac{d}{dt}[x(t)] \xleftrightarrow{FT} Jw\, X(Jw)$.

10- Integration :-

$\int_{-\infty}^{t} x(t)\, dt \xleftrightarrow{FT} \frac{1}{Jw} X(Jw) + \pi X(0)\, \delta(w)$.

Signals and Systems :-

Parseval's Relation for Aperiodic Sig :-

- Parsevals Relation :-

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 dw$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) e^{jwt} dw .$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw) e^{jwt}|^2 dw = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 dw$$

Basic Fourier Transform Pairs :-

$$\text{Signal} \xrightleftharpoons[IFT]{FT} \text{fourier Transform}$$

$$\sum a_k e^{jkw_0t} \xleftrightarrow{FT} 2\pi \sum a_k \delta(w - kw_0)$$

$$e^{jkw_0t} \xleftrightarrow{FT} 2\pi \delta(w - w_0).$$

$$\cos w_0t \xleftrightarrow{FT} \pi[\delta(w-w_0) + \delta(w+w_0)]$$

$$\sin w_0t \xleftrightarrow{FT} \frac{\pi}{j}[\delta(w-w_0) - \delta(w+w_0)]$$

$$x(t) = 1 \xleftrightarrow{FT} 2\pi \delta(w)$$

$$\delta(t) \xleftrightarrow{FT} 1$$

$$u(t) \xleftrightarrow{FT} \frac{1}{jw} + \pi \delta(w)$$

$$\delta(t - t_0) \xleftrightarrow{FT} e^{-jwt_0}.$$

$$x(t) = \begin{cases} 1 \\ 0 \end{cases} \xleftrightarrow{FT} \sum \frac{2 \sin k w_0 T_1}{k} \delta(w - kw_0)$$

$$\frac{\sin Wt}{\pi t} \xleftrightarrow{FT} X(jw) = \begin{cases} 1 \\ 0 \end{cases}$$

$$e^{-at} u(t) \xleftrightarrow{FT} \frac{1}{a + jw}$$

$$t e^{-at} u(t) \xleftrightarrow{FT} \frac{1}{(a + jw)^2}$$

$$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \xleftrightarrow{FT} \frac{1}{(a + jw)^n}$$

Sinc function :-

$$\text{Sinc}\,\theta = \frac{\sin \pi\theta}{\pi\theta}$$

The Discrete time fourier Transform

Fourier Transform (DTFT) :-

$x[n] \rightarrow$ DT and Aperiodic or Periodic

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw \rightarrow \underline{\underline{DT}}$$

Synthesis equation Fourier Transform

$$X(e^{jw}) = \sum_{k=-\infty}^{\infty} x[n] e^{-jwn} \rightarrow \underline{CT}.$$

analysis equation fourier Transform

Synthesis equ + analysis equ.

$\Rightarrow$ fourier Transform Pairs.

Properties of the discrete time Fourier Transform :-

$$x[n] \xleftrightarrow{FT} X(e^{jw}), \quad y[n] \xleftrightarrow{FT} Y(e^{jw})$$

1. Linearity :-

$$a x[n] + b y[n] \longleftrightarrow a X(e^{jw}) + b Y(e^{jw}).$$

2. Time shifting :-

$$x[n - n_0] \longleftrightarrow X(e^{jw}) e^{-jwn_0}.$$

3. frequency shifting :-

$$e^{jw_0n} x[n] \longleftrightarrow X(e^{j(w-w_0)}).$$

4. Conjugation :-

$$x^*[n] \longleftrightarrow X^*(e^{-jw})$$

5. Time Reversal :-

$$x[-n] \longleftrightarrow X(e^{-jw}).$$

6. Convolution :-

$$x[n] * y[n] \longleftrightarrow X(e^{jw}) \cdot Y(e^{jw}).$$

7. Multiplication :-

$$x[n] \cdot y[n] \longleftrightarrow \frac{1}{2\pi} X(e^{jw}) * Y(e^{jw}).$$

8. Differencing :-

$$x[n] - x[n-1] \longleftrightarrow (1 - e^{-jw}) X(e^{jw}).$$

9. Accumulation

$$\sum x[k] \longleftrightarrow \left(\frac{1}{1 - e^{-jw}}\right) X(e^{jw})$$

* Signals and Systems:-

* fourier Transform for Periodic Signal :-

$X[n] \rightarrow$ DT and Periodic signal.

$$X[n] = \sum_{k=<N>} a_k e^{jk w_0 n}$$

$$X(e^{jw}) = \sum_{-\infty}^{\infty} 2\pi a_k \delta(w - k w_0)$$

---

* Properties of the fourier Transform :-

Real . odd and even CT and DT

$$\text{Signal} \underset{IFT}{\overset{FT}{\rightleftarrows}} \text{fourier Transform}$$

$x(t)$ real, even $\longleftrightarrow$ $X(jw)$ real, even

$x(t)$ real, odd $\longleftrightarrow$ $X(jw)$ odd Purely imaginary

$x[n]$ real, even $\longleftrightarrow$ $X(e^{jw})$ real, even

$x[n]$ real, odd $\longleftrightarrow$ $X(e^{jw})$ odd Purely imaginary.

Parsevals Relation

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 dw$$

Parsevals Relation:-

$$\sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{jw})|^2 dw$$