

اشارات وانظمة لهندسة الميكاترونكس

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→ improper integrals :-

$$\int_a^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$$

$$\int_{-\infty}^b f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx$$

$$\int_{-\infty}^{\infty} f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx + \lim_{t \rightarrow \infty} \int_a^t f(x) dx$$

→ infinit series :-

$$\sum_{n=0}^{\infty} x[n] = \lim_{N \rightarrow \infty} \sum_{n=0}^N x[n]$$

$$\sum_{n=-\infty}^{\infty} x[n] = \lim_{N \rightarrow \infty} \sum_{n=-N}^N x[n]$$

* **Signal**:- it is a function of one or more independent variables.
ex: $f(t) \rightarrow f$: dependent variable, t : independent variable.

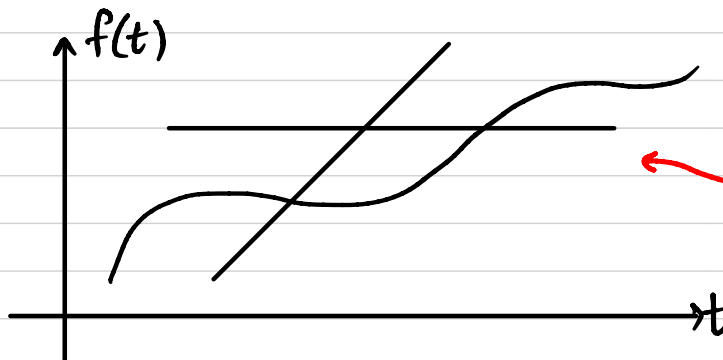
→ Two types of signals :-

□ continuous-time signals (CT)

↳ $f(t)$ where t (the time) is the independent variable.
↳ it is a signal that can take a continuum of values, (it can be 1.3456) (t must be a real number).

* examples on continuous signals :- car speed, Audio waves, room's temperature, images.

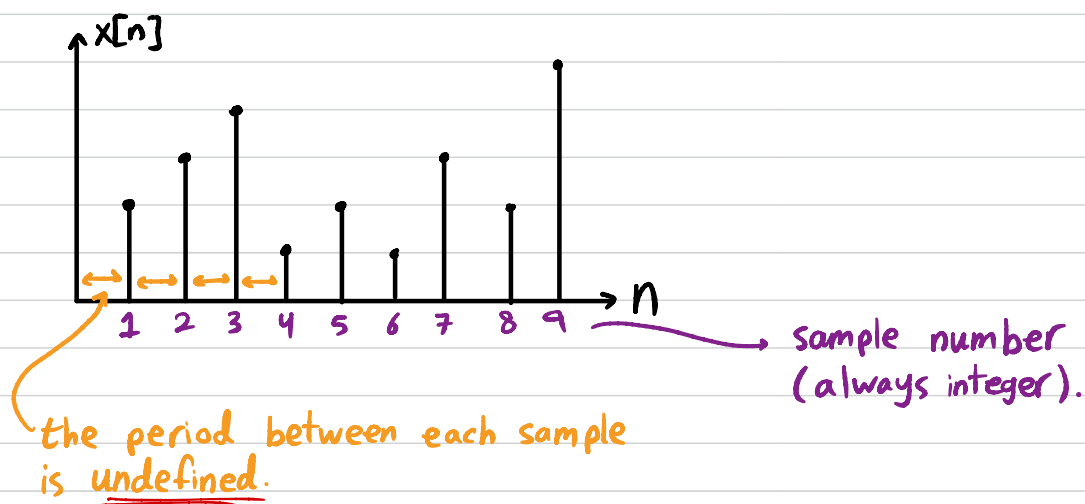
(The independent variable in images is the coordinates of the pixels).



examples of continuous graphs

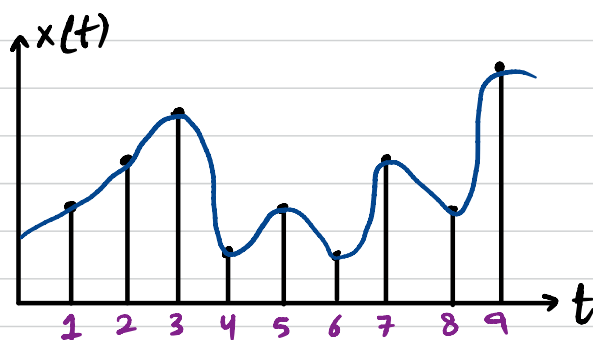
② discrete - time signal (DT)

↳ $x[n]$ where n is the number of samples (must be integer).



also, time between each sample must be the same.
(for uniformly sampled signal).

* we can get discrete from a continuous signal.



* Digital signals can only be discrete signal

however, Analog signals can be both discrete and continuous.

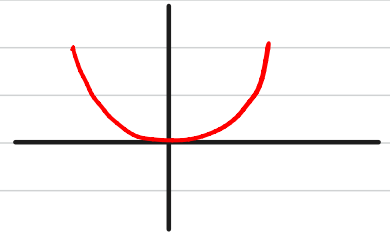
odd & even signals :-

* A signal is even if :-

$$\text{CT case: } x(-t) = x(t)$$

$$\text{DT case: } x[-n] = x[n]$$

$$\int \text{even} \neq 0$$



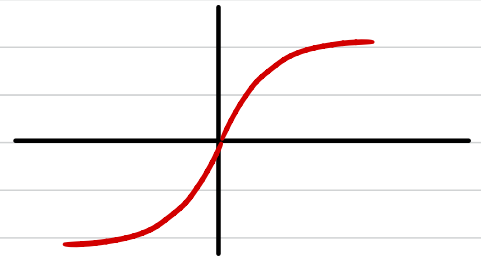
* Symmetric around the y-axis $\therefore$ even signal

* A signal is odd if :-

$$\text{CT case: } x(-t) = -x(t)$$

$$\text{DT case: } x[-n] = -x[n]$$

$$\int \text{odd} = 0$$



* A signal is neither odd nor even if :-

the signal is a combination of even and odd.

* not symmetric around the y-axis $\therefore$ odd.

Ex1:

$$x(t) = t^2 + t^4$$

$$x(-t) = (-t)^2 + (-t)^4$$

$$x(-t) = t^2 + t^4$$

$$x(t) = x(-t) \therefore \underline{\text{even}}$$

Ex3:

$$x(t) = \sin(t) + \cos(t)$$

$$x(-t) = \sin(-t) + \cos(-t)$$

$$x(-t) = \underbrace{-\sin(t)}_{\text{odd}} + \underbrace{\cos(t)}_{\text{even}}$$

$\therefore$ neither even nor odd.

Ex2:

$$x(t) = t + t^3$$

$$x(-t) = -t - t^3 = -(t + t^3)$$

$$x(-t) = -x(t) \therefore \underline{\text{odd}}$$

$$\sin(-t) = -\sin(t)$$

$$\cos(-t) = \cos(t)$$

>> Any continuous time signal can be represented as sum of odd and even components.

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{1}{2} (x(t) + x(-t)) \quad \text{even part of the signal.}$$

$$x_o(t) = \frac{1}{2} (x(t) - x(-t)) \quad \text{odd part of the signal.}$$

Ex:

$$x(t) = \cos(t) + \sin(t) + \sin(t) \cos(t)$$

$$x_e(t) = \frac{1}{2} [\cos(t) + \sin(t) + \sin(t) \cos(t) + \cos(-t) + \sin(-t) + \sin(-t) \cos(-t)]$$

$$x_e(t) = \frac{1}{2} [\cos(t) + \cancel{\sin(t)} + \cancel{\sin(t) \cos(t)} + \cos(t) - \cancel{\sin(t)} - \cancel{\sin(t) \cos(t)}]$$

$$x_e(t) = \frac{1}{2} [2 \cos(t)] = \cos(t)$$

$$x_o(t) = \frac{1}{2} [\cos(t) + \sin(t) + \sin(t) \cos(t) - \sin(-t) - \cos(-t) - \sin(-t) \cos(-t)]$$

$$x_o(t) = \frac{1}{2} [\cancel{\cos(t)} + \sin(t) + \sin(t) \cos(t) + \sin(t) - \cancel{\cos(t)} + \sin(t) \cos(t)]$$

$$x_o(t) = \frac{1}{2} [2 \sin(t) + 2 \sin(t) \cos(t)]$$

$$x_o(t) = \sin(t) + \sin(t) \cos(t)$$

$$\text{even} + \text{even} = \text{even}$$

$$\text{odd} + \text{odd} = \text{odd}$$

$$\text{even} + \text{odd} = \text{comp nither E nor O}$$

$$\text{even} * \text{even} = \text{even}$$

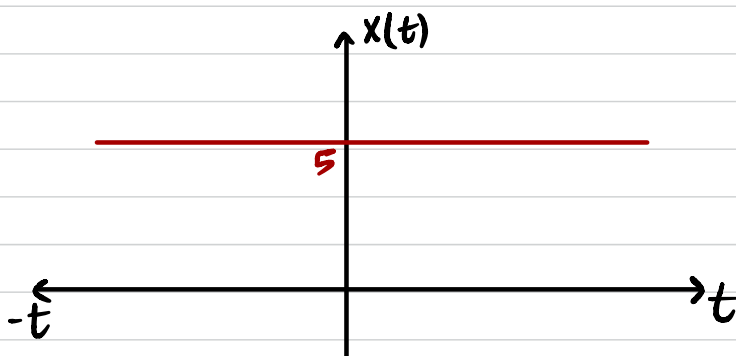
$$\frac{1}{\text{even}} = \text{even}$$

$$\text{even} * \text{odd} = \text{odd}$$

$$\frac{1}{\text{odd}} = \text{odd}$$

$$\text{odd} * \text{odd} = \text{even}$$

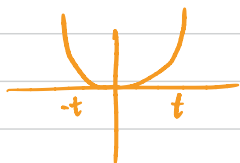
* a constant is even because when drawn, it's symmetric around the y-axis.



continuous signals:-

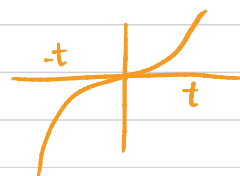
* if we know that the signal $x(t)$ is **even** then:-

$$\Rightarrow \int_{-t}^t x(t) dt = 2 \int_0^t x(t) dt$$



* if we know that the signal $x(t)$ is **odd** then:-

$$\Rightarrow \int_{-t}^t x(t) dt = 0$$



Discrete signals:-

* if we know that the signal $x[n]$ is even then:-

$$\Rightarrow \sum_{-n_0}^{n_0} x_e[n] = 2 \sum_0^{n_0} x_e[n]$$

* if we know that the signal $x[n]$ is odd then:-

$$\Rightarrow \sum_{-n_0}^{n_0} x_o[n] = 0$$

Conjugate Symmetric:-

(special case of even and odd)

→ complex signal

$$Z(t) = \underbrace{a(t)}_{\text{Real}} + j \underbrace{b(t)}_{\text{img}}$$

★ how do i know that the signal is a conjugate symmetric?

$$Z(-t) = Z^*(t)$$

$$Z(-t) = a(t) - j b(t)$$

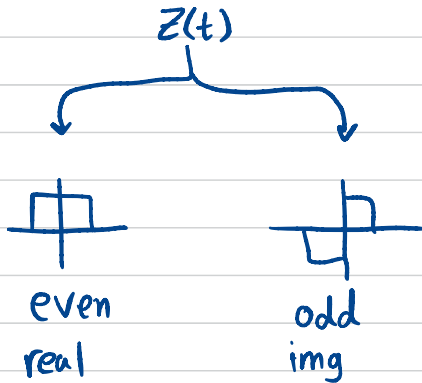
$$Z(t) = a(t) + j b(t)$$

even

odd

∴ if $Z(t) = \text{even} + j \text{odd} \rightarrow \text{conjugate symmetric.}$

or

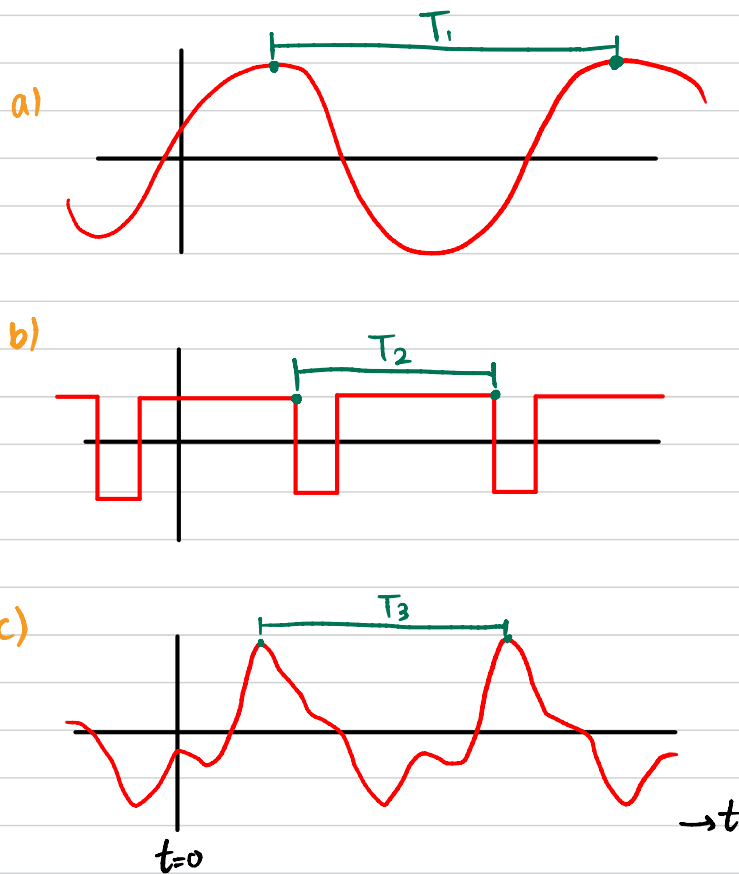


Euler identity:-

$$e^{j\theta} = \underbrace{\cos\theta}_{\text{even}} + j \underbrace{\sin\theta}_{\text{odd}}$$

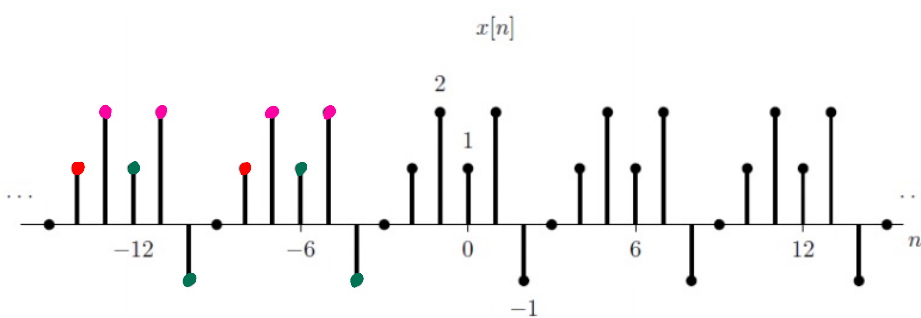
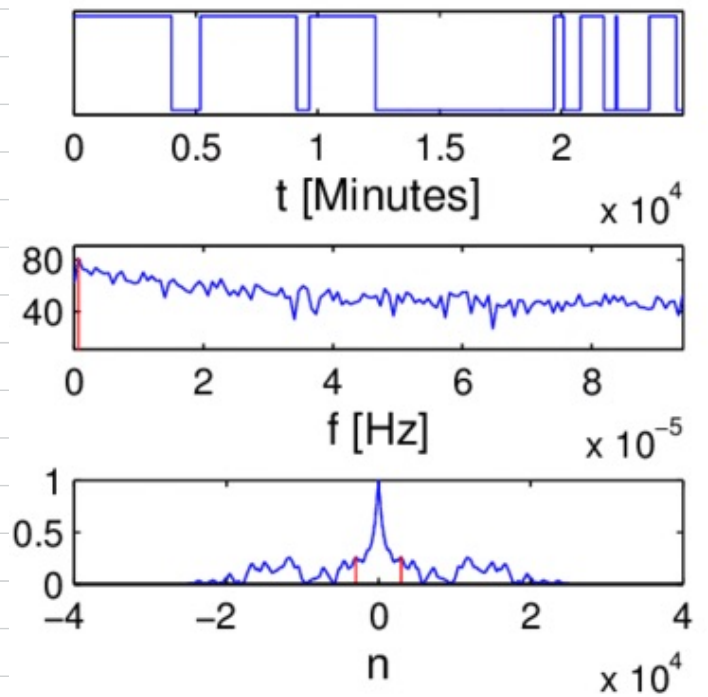
∴ Euler's identity is a conjugate symmetric.

► Periodic and Nonperiodic signals :-

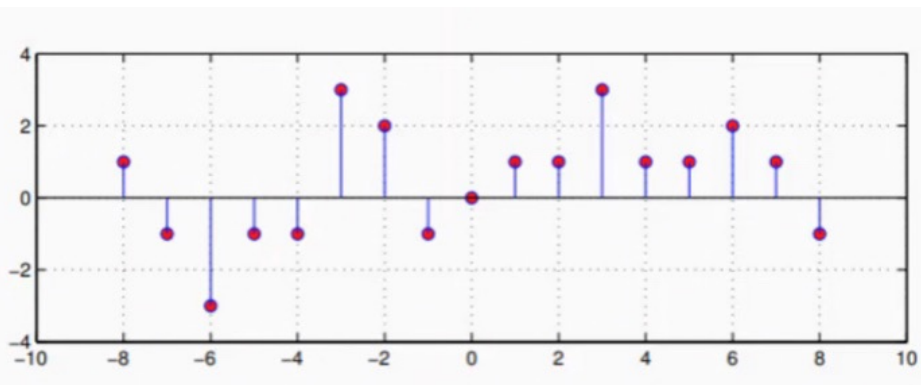


periodic cT ↗

nonperiodic cT ↗



periodic DT ↗



nonperiodic DT ↗

A signal $x(t)$ is said to be periodic signal with a period "T" if there is a positive non-zero value of "T" such that $x(t+T) = x(t)$ for all of t . usually T has multi values.

⇒ minimum possible value for "T" → fundamental period

↳ that satisfy $x(t) = x(t+T)$ and is denoted by " T_0 "

Example: $x(t) = \sin(t)$; check if it is periodic or not.

$$x(t+mT) = \sin(t+mT) = \sin(t)\cos(mT) + \cos(t)\sin(mT)$$

The goal is to get the new function to look like the original one.

so when $\cos(mT) = 1$ & $\sin(mT)$ will do so

$$\cos(mT) = 1 \rightarrow \text{when } T = \cancel{0}, 2\pi, 4\pi, 6\pi \dots$$

↳ but T can't be zero because if it was the signal would be non-periodic.

$$\sin(mT) = 0 \rightarrow \text{when } T = \cancel{0}, \pi, 2\pi, 3\pi, 4\pi \dots$$

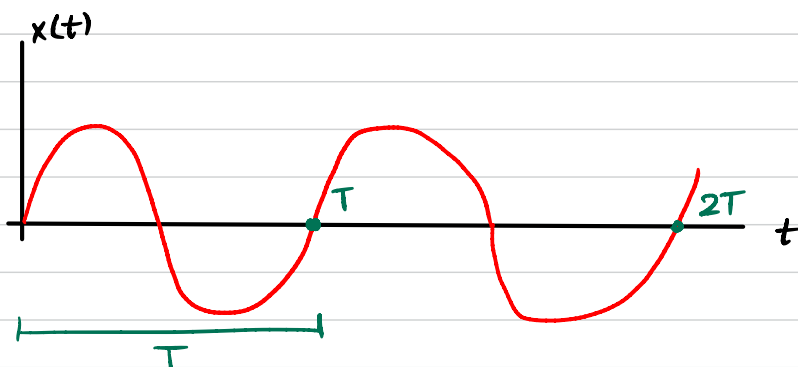
The signal is periodic

and the fundamnent period = 2π

Example: $x(t) = e^t$

$$x(t+mT) = e^{t+mT} = e^t \cdot e^{mT}$$

for the signal to go back to the original signal (e^t), e^{mT} must = 1 $\therefore T=0$ but $T=0$ only for a non-periodic signal $\therefore$ The signal is non-periodic.



$$T = \frac{2\pi}{\omega}$$

T : fundamental periodic of one complete cycle of $x(t)$

T : positive (time) [sec]

f : fundamental frequency. $= \frac{1}{T}$ [Hz], [cycle/sec]

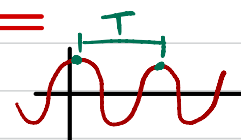
ω : angular frequency $= 2\pi f = \frac{2\pi}{T}$ [rad/sec]

$$T\omega = 2\pi$$

The fundamental period (T_0): smallest positive value of T that satisfy the equation $x(t) = x(t+T)$

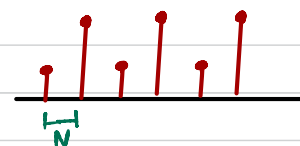
★ if $x(t) = x(t+mT)$ $\therefore$ signal is periodic

$\rightarrow$ if not then it is nonperiodic. (CT)



if $x[n] = x[n+mN]$ $\therefore$ signal is periodic

$\rightarrow$ if not then it is nonperiodic. (DT)



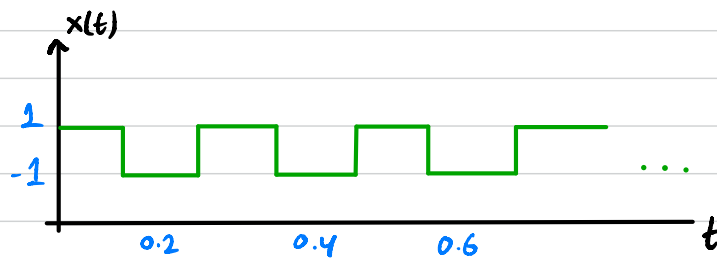
* N must be positive and integer.

Fundamental period (N_0): smallest positive value of N that satisfy the equation $x[n] = x[n+N]$

$$\omega = \frac{2\pi}{N} \text{ angular frequency}$$

* A periodic signal is said to be periodic if it repeats itself after a regular interval of time.

Example 1:-



$$T = 0.2 \text{ sec}$$

$$f = \frac{1}{T} = \frac{1}{0.2} = 5 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi(5) = 10\pi \text{ rad/s}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

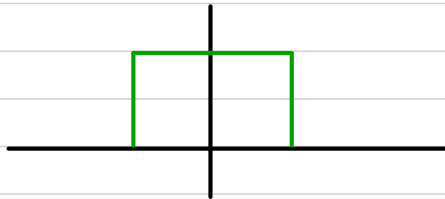
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos^2(\alpha t) = \frac{1}{2}(1 + \cos 2\alpha t)$$

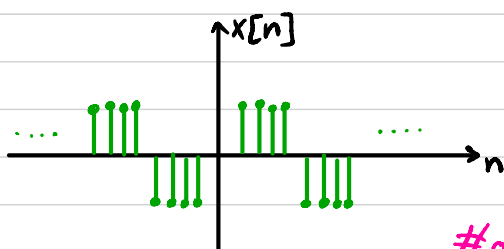
$$\sin^2(\alpha t) = \frac{1}{2}(1 - \cos 2\alpha t)$$

Example 2:-



non periodic
because there is
no repetition.

Example 3:-



$$N = 8$$

$$\omega = \frac{2\pi}{N} = \frac{2\pi}{8} = \frac{\pi}{4} \text{ (rad/sample)}$$

of samples that
are being repeated.

Example 4: $x(t) = \cos^2(2\pi t)$

$$\Rightarrow \frac{1}{2}(1 + \cos 4\pi t)$$

$$\omega = 4\pi \quad T = \frac{2\pi}{\omega} = \frac{1}{2} \text{ sec}$$

$$f = 2 \text{ Hz}$$

Example 5:- $x(t) = \cos(2\pi t)$; check if it's periodic or not.

$$x(t) = x(t+mT) \rightarrow \cos(2\pi(t+mT))$$

$$= \cos(2\pi t + 2\pi mT)$$

$$= \cos(2\pi t) \cos(2\pi mT) - \sin(2\pi t) \sin(2\pi mT)$$

$$= \cos(2\pi t) \quad \therefore x(t) = x(t+mT)$$

$\therefore$ signal is periodic.

Example: $e^{-10t} \quad t > 0$; periodic or non periodic?

$$x(t+mT) = e^{-10(t+mT)} = \underbrace{e^{-10t}}_{\substack{\text{signal} \\ \text{itself}}} \cdot \underbrace{e^{-10mT}}_{\neq 1}$$

$m: 0, 1, 2, 3, \dots$
 T must not be 0 if periodic

we need to check if $e^{-10t} = e^{-10(t+mT)}$ if yes it's periodic

$$x(t) \neq x(t+mT)$$

$\therefore$ non periodic.

Example: $x(t) = \cos(e^{2\pi t})$

$$x(t+mT) = \cos(e^{2\pi t} \cdot \underbrace{e^{2\pi mT}}_{\text{circled}})$$

non periodic

The same

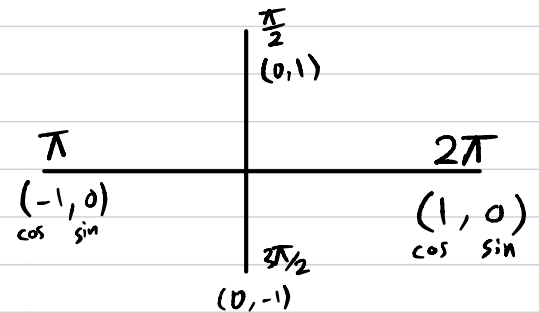
Example : $X(t) = e^{\sin(2\pi t)}$

$X(t+mT) = e^{\sin(2\pi t + 2\pi mT)}$

✓ periodic

$T = \frac{2\pi}{2\pi} = 1$

$\sin(2\pi t) \cos(2\pi mT) + \cos(2\pi t) \sin(2\pi mT)$
 $\xrightarrow{\text{green arrow}} \sin(2\pi t) \cos(2\pi mT) + \cos(2\pi t) \sin(2\pi mT)$
 $\xrightarrow{\text{pink arrow}} \sin(2\pi t)$



Example :- $t e^{\sin 2\pi t}$
periodic

$(t+mT)$

non periodic

since $t \neq t+mT$ because T can't be 0 if it was periodic

* $T \neq 0$ if the signal is periodic

$\therefore$ The signal is non-periodic

$\Rightarrow$ non periodic * periodic = non-periodic

periodic + periodic = periodic
 non-periodic + non-periodic = non-periodic
 periodic * periodic = periodic
 periodic * non-periodic = non-periodic
 non-periodic * non-periodic = non-periodic

Summation of periodic signals:-

Example: $x(t) = \cos(\overset{w_1}{8\pi t + 10^\circ}) - \cos(\overset{w_2}{4\pi t}) - 2\overset{w_3}{\sin(6\pi t)}$

Signal 1 Signal 2 Signal 3

* we don't care about the phase shift and amplitude.

★ Step 1: find all T_s

$$T_1 = \frac{2\pi}{w_1} \quad T_2 = \frac{2\pi}{w_2} \quad T_3 = \frac{2\pi}{w_3}$$
$$T_1 = \frac{2\pi}{8\pi} = \frac{1}{4} \quad T_2 = \frac{2\pi}{4\pi} = \frac{1}{2} \quad T_3 = \frac{2\pi}{6\pi} = \frac{1}{3}$$

★ Step 2: find the ratios between $T_1, T_2, T_3 \dots$

make one T fixed and by convention we choose T_1 .
i.e. we find $\frac{T_1}{T_2}, \frac{T_1}{T_3}, \frac{T_1}{T_4} \dots$

$$\frac{T_1}{T_2} = \frac{1/4}{1/2} = \frac{1}{2}$$

$$\frac{T_1}{T_3} = \frac{1/4}{1/3} = \frac{3}{4}$$

The result ratio is in the form of
 $\frac{\text{integer}}{\text{integer}} \therefore$ signal is periodic

★ Step 3: find T

* To find T for the whole signal we take the fixed T from step 2 (here T_1) and multiply it by k .

$$\begin{aligned} T &= T_1 * k \\ &= \frac{1}{4} * \text{LCM}(2, 4) \\ &= \frac{1}{4} * 4 \\ T &= 1 \text{ sec} \end{aligned}$$

* obviously a periodic signal because $T \neq 0$.

k : LCM (Least Common Multiple) between the Denominators of the ratios in step 2.

$$\text{LCM}(2, 3, 4) = 12 \quad \text{why?}$$

2: 2, 4, 6, 8, 10, 12

3: 3, 6, 9, 12

4: 4, 8, 12

$\therefore \text{LCM} = 12$

Example: $x(t) = \cos 4t + \sin \pi t$

$$\omega_1 = 4 \quad \omega_2 = \pi$$

Step 1: $T_1 = \frac{2\pi}{4} \quad T_2 = \frac{2\pi}{\pi}$

Step 2: $\frac{T_1}{T_2} = \frac{\pi/2}{2} = \frac{\pi}{4} \equiv \frac{\text{irrational}}{\text{integer}}$

$\therefore$ signal is non-periodic
since it's not $\frac{\text{integer}}{\text{integer}}$

Example: $x(t) = 1.3 \cos(\pi t + 9^\circ) + \cos(4t) - \sin(6\pi t)$

$$T_1 = \frac{2\pi}{\pi} = 2 \quad T_2 = \frac{2\pi}{4} = \frac{\pi}{2} \quad T_3 = \frac{2\pi}{6\pi} = \frac{1}{3}$$

$$\frac{T_1}{T_2} = \frac{2}{\pi/2} = \frac{4}{\pi} \quad \therefore \text{non-periodic } \left(\frac{\text{integer}}{\text{irrational}} \right)$$

Example: $x(t) = 0.2 e^{j(2t-\pi)} - 1.3 e^{j6t}$

* $e^{j\theta} = \cos \theta + j \sin \theta$

$$= 0.2 [\cos(2t-\pi) + j \sin(2t-\pi)] - 1.3 [\cos(6t) + j \sin(6t)]$$

This is a shift we
don't take it into
consideration.

$$T_1 = \frac{2\pi}{2} = \pi \quad T_2 = \frac{2\pi}{2} = \pi \quad T_3 = \frac{2\pi}{6} = \frac{1}{3}\pi$$

$$\frac{T_1}{T_2} = \frac{\pi}{\pi} = 1 \quad \frac{T_1}{T_3} = \frac{\pi}{\frac{1}{3}\pi} = 3$$

$$T_4 = \frac{2\pi}{6} = \frac{1}{3}\pi$$

$$\frac{T_1}{T_4} = 3 \quad \therefore \text{periodic}$$

$$\frac{T}{T} = \frac{T_1}{T_1} \neq k \rightarrow = 1$$

Example: $x(t) = \cos(2\pi t + 7^\circ) - 5.4 e^{j6\pi t}$

* signal is periodic

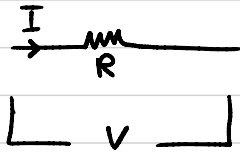
Example: $x(t) = \sin(2t) + 2e^{j2\pi t}$

* signal is non-periodic

Example: $x(t) = \cos t + \sin \sqrt{2} t$

* non-periodic

► Energy and power signal:-



$$p(t) = I^2(t) R = \frac{V^2(t)}{R}$$

instantaneous power

if $R=1 \rightarrow p(t) = I^2(t) = V^2(t) = x^2(t)$
"if Resistance is unit".

for any signal

$$\text{Energy} = E = \int_{-\infty}^{\infty} p(t) dt = \int_{-\infty}^{\infty} x^2(t) dt$$

$a + jb$ complex

$$|a + jb| = \sqrt{a^2 + b^2}$$

$$|a + jb|^2 = a^2 + b^2$$

phase $\angle a + jb = \tan^{-1}\left(\frac{b}{a}\right)$

$$\boxed{\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 dt} \quad \leftarrow \text{general form for all signals.}$$

average power:-

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

general form

if the signal is periodic:-

$$P = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

DT:-

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2$$

if signal is periodic:-

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

*Important Notes:-

★ energy signal: $x(t)$, $x[n]$
 $0 < E < \infty$ finite
and $P = 0$

Both conditions must be true to call it energy signals.

★ power signal:-
 $0 < P < \infty$ finite
and $E = \infty$

Both conditions must be true to call it power signals.

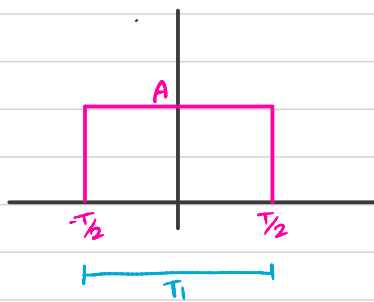
★ neither power nor Energy signals:-
if both of the conditions above weren't true.

Hint :- periodic or random $\Rightarrow$ power signal.
non-periodic or deterministic $\Rightarrow$ energy signal.

in most cases not all.

random $\rightarrow$ can't be predicted before it happens. $\rightarrow$ no pattern.
deterministic $\rightarrow$ can be predicted before it happens.

Example:- Energy or power signal?



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-T_1/2}^{T_1/2} |A|^2 dt$$

$$= A^2 t \Big|_{-T_1/2}^{T_1/2}$$

$$= A^2 \left(\frac{T_1}{2} + \frac{T_1}{2} \right)$$

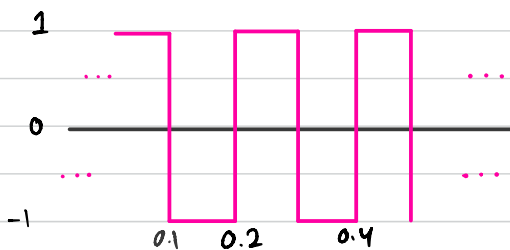
$$E = A^2 T_1 ; 0 < E < \infty$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |A|^2 dt$$

$\therefore$ The signal is an energy signal.

$$P = \lim_{T \rightarrow \infty} \frac{A^2 T_1}{T} = 0$$

Example:-



$$P = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{0.2} \left[\int_0^{0.1} |1|^2 dt + \int_{0.1}^{0.2} |-1|^2 dt \right]$$

$$= \frac{1}{0.2} \left[t \Big|_0^{0.1} + t \Big|_{0.1}^{0.2} \right] = \frac{0.1 + 0.1}{0.2} = \frac{0.2}{0.2} = 1 \quad 0 < P < \infty$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \infty$$

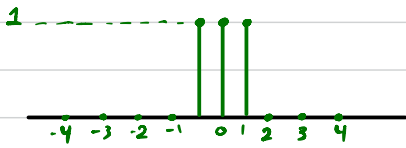
$\therefore$ Signal is power signal.



because of the
absolute and
square.

Example:-

non-periodic



$$E = \sum_{-\infty}^{\infty} |x[n]|^2 = \sum_{-1}^1 |x[n]|^2 = 1^2 + 1^2 + 1^2 = \underline{\underline{3}}$$

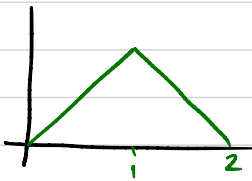
$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} * 3 = \underline{\underline{0}}$$

$\therefore$ energy signal

Example:-

$$x(t) = \begin{cases} t & 0 \leq t < 1 \\ 2-t & 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

non-periodic
because the
signal changes over
a period of time
and 0 everywhere else.



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^1 |t|^2 dt + \int_1^2 |2-t|^2 dt$$

$$\frac{1}{3} t^3 \Big|_0^1 + \frac{1}{3} (2-t)^3 \Big|_1^2 = \underline{\underline{\text{number}}}$$

$$P = \underline{\underline{0}}$$

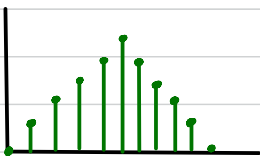
$\therefore$ energy signal.

Examples:-

$$x[n] = \begin{cases} n & 0 \leq n < 5 \\ 10-n & 5 \leq n \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

$$E = \sum_0^4 |n|^2 + \sum_5^{10} |10-n|^2$$

$$= (0+1+4+9+16) + (25+16+9+4+1+0) = 85$$



$$P = 0$$

$\therefore$ energy signal.

Example :-

$$x[n] = \begin{cases} \sin(\pi n) & -4 \leq n \leq 4 \\ 0 & \text{otherwise.} \end{cases} \quad \text{non-periodic}$$

$$E = \sum_{-4}^4 |\sin(\pi n)|^2$$

$$E = (-\sin(4\pi))^2 + (-\sin(3\pi))^2 + (-\sin(2\pi))^2 + (-\sin(\pi))^2 \\ + (\sin(0\pi))^2 + (\sin(\pi))^2 + (\sin(2\pi))^2 + (\sin(3\pi))^2 + (\sin(4\pi))^2 = 0$$

$$p = 0$$

$\therefore$ signal is neither power nor energy.

Example :-

$$x[n] = \begin{cases} \cos(\pi n) & -4 \leq n \leq 4 \\ 0 & \text{otherwise.} \end{cases} \quad \text{non-periodic}$$

$$E = \sum_{-4}^4 |\cos(\pi n)|^2$$

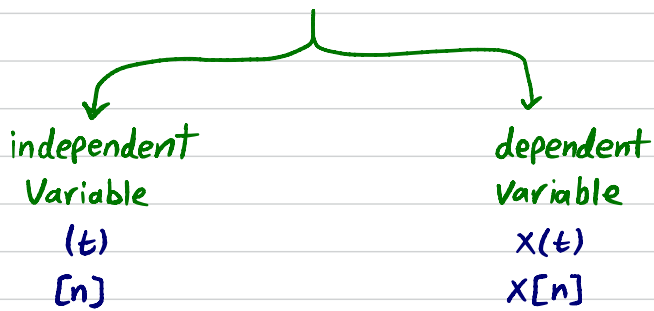
$$= (\cos 4\pi)^2 + (\cos 3\pi)^2 + (\cos 2\pi)^2 + (\cos \pi)^2 + (\cos 0)^2 \\ + (\cos \pi)^2 + (\cos 2\pi)^2 + (\cos 3\pi)^2 + (\cos 4\pi)^2$$

$$= 2 + 2 + 2 + 2 + 1 = 9$$

$$p = \lim_{N \rightarrow \infty} \frac{1}{2N+1} E = 0$$

$\therefore$ energy signal.

Basic operations on signals:-



1. time scale.
2. time reverse.
3. time shift.

The compression or expansion of a signal in time.

change only in the x-axis.

① Time scale:-

Original signal $\rightarrow x(t)$

new signal $\rightarrow y(t) = x(at)$

Scale number

$a > 1$

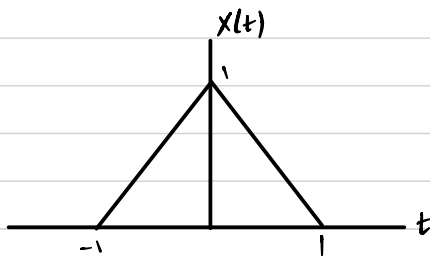
or

$0 < a < 1$

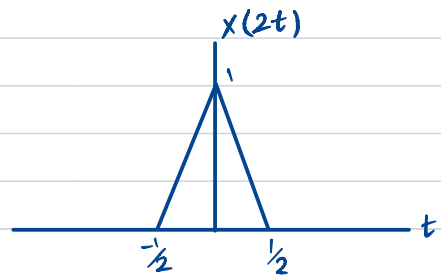
compress
(speed up)

expand
(speed down)

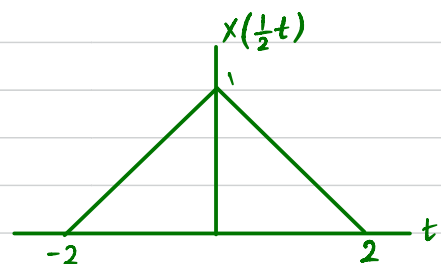
Example:-



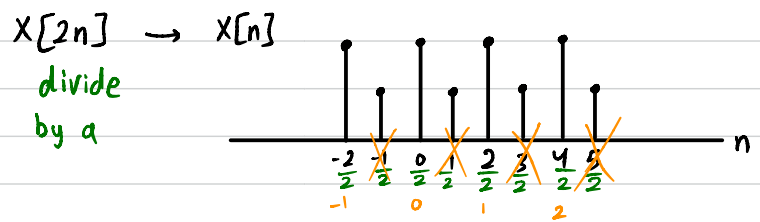
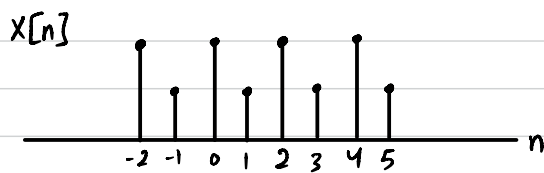
$x(2t) \rightarrow$
divide
 a



$x(\frac{1}{2}t) \rightarrow$
divide
 a



Example:-



n must always be integers so after scaling a sample like $\frac{-1}{2} = -0.5$ will be discarded

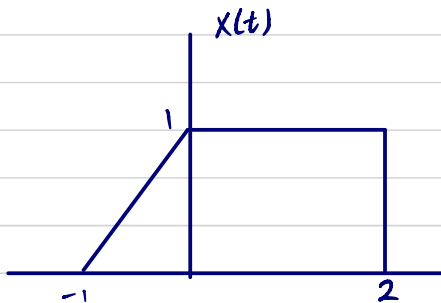
so in discrete we would lose information (samples).

(2) Time Reverse (reflection) :

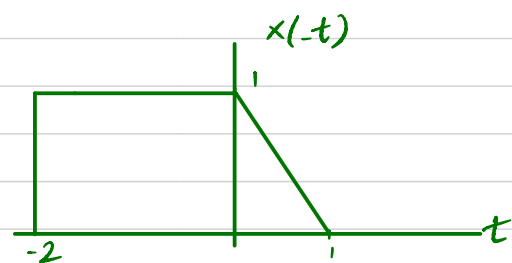
$$x(t)$$

$$y(t) = x(-t)$$

Example:-



$$y(t) = x(-t)$$

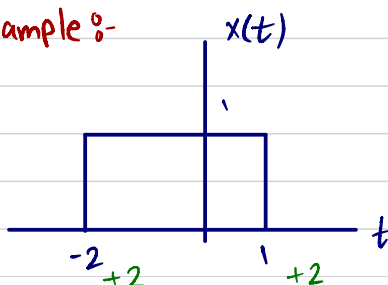


(3) Time Shift :

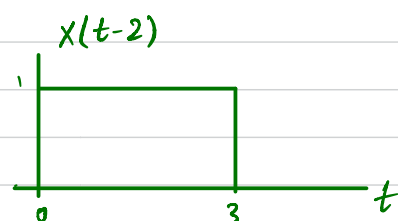
$$y(t) = x(t - t_0)$$

$t_0 \rightarrow +ve$ $x(t - t_0) \rightarrow$ shift to the right
 $t_0 \rightarrow -ve$ $x(t - t_0) = x(t + t_0) \rightarrow$ shift to the left

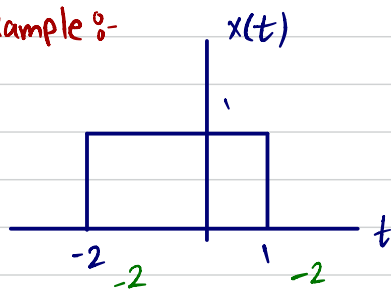
Example:-



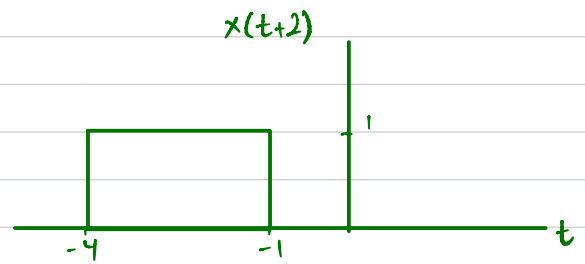
$$y(t) = x(t - 2)$$



Example:-



$$y(t) = x(t+2)$$



Precedence Rule:-

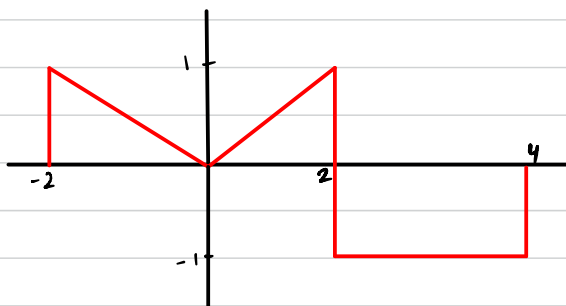
$$y(t) = x(at - t_0)$$

Steps :

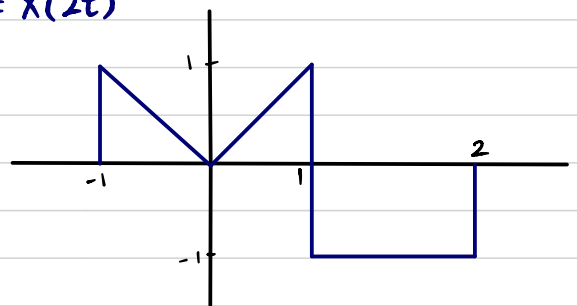
- ① Time shift.
- ② Time scale.
- ③ reflect / reverse.



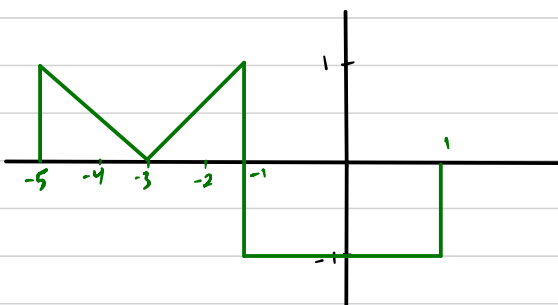
Example:-



$$y(t) = x(2t)$$



$$y(t) = x(t+3)$$



first

Operations on dependent variables $x(t)$:

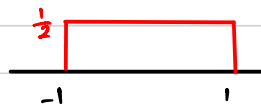
Amplitude scaling:

$$y(t) = ax(t)$$

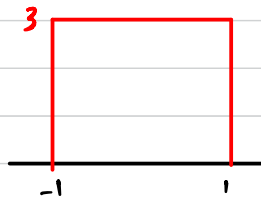
$0 < a < 1$ or $a > 1$
 $\downarrow$ $\downarrow$
 attenuation amplification



$$\frac{1}{2}x(t) \rightarrow$$



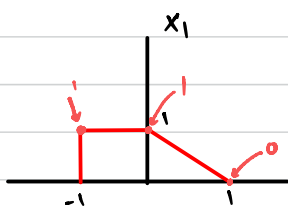
$$3x(t) \rightarrow$$



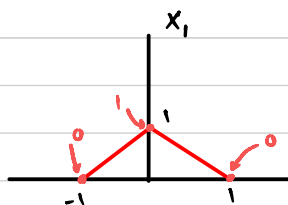
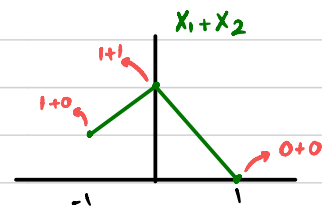
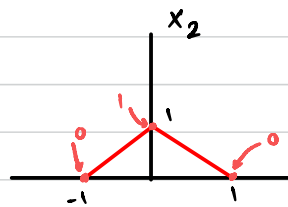
math operations:

a. addition/sub. $x_1(t) \pm x_2(t)$

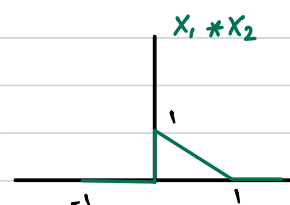
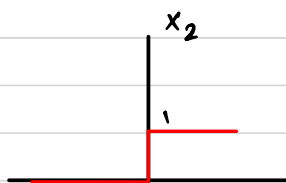
b. multiply/divide $\frac{x_1(t)}{x_2(t)}$, $x_1(t) * x_2(t)$



+



*



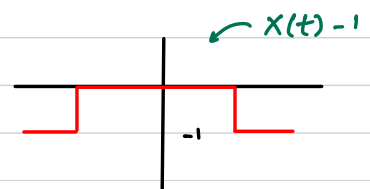
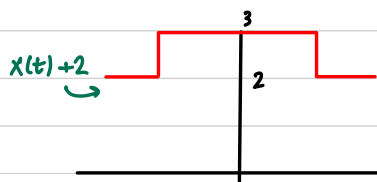
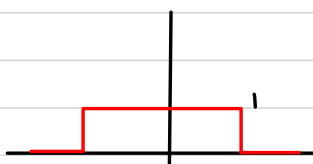
Amplitude shift.

(up or down)

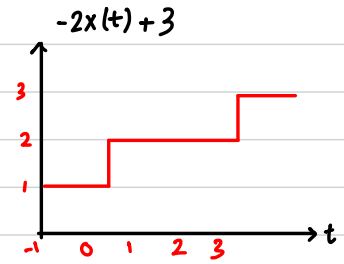
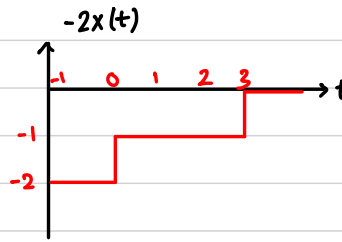
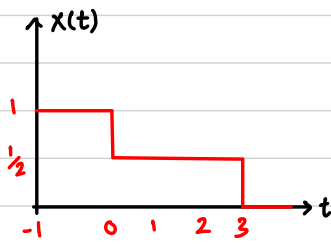
$$y(t) = ax(t) + b$$

+ve $\rightarrow$ up
-ve $\rightarrow$ down

ex:



Example: $y(t) = -2x(t) + 3$

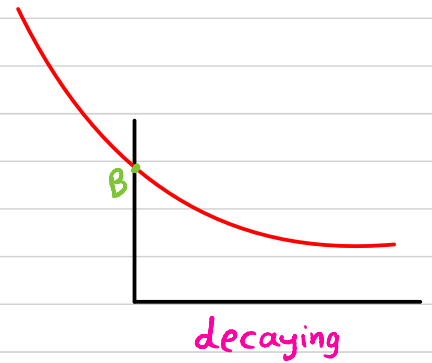


Elementary Signals:

① Exponential signals

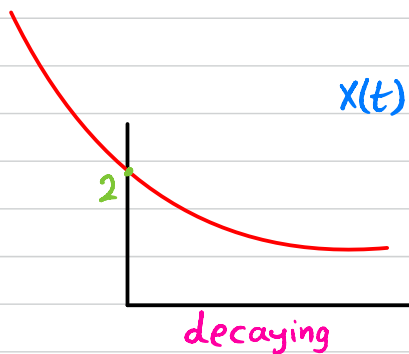
CT: $x(t) = Be^{at}$

B intersection point (y-axis).
a shape

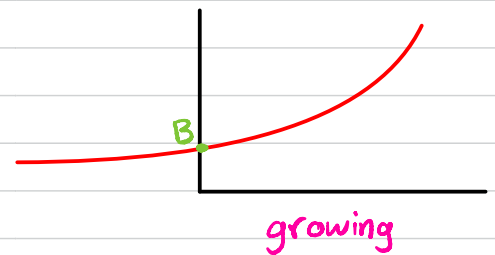


$a > 0$ growing
 $a < 0$ decaying

Example:



$$x(t) = 2e^{-3t}$$



$$x(t) = 2e^{3t}$$

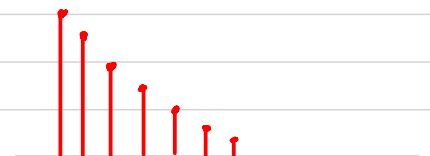
DT:

$$x[n] = Br^n$$

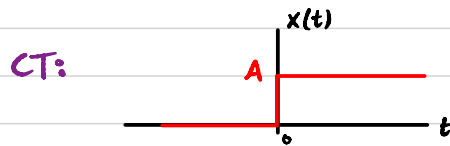
$$x[n] = 2(0.5)^n$$

$r > 1$ } growing exp.
 $r < -1$ }

$-1 < r < 1 \rightarrow$ decaying exp.

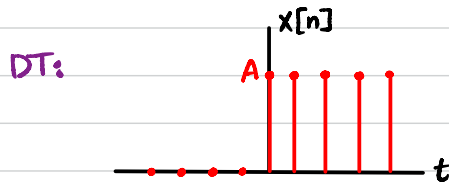


Step signal:-



$$u(t) = \begin{cases} 0 & t < 0 \\ A & t > 0 \end{cases}$$

← at $t=0 \rightarrow$ undefined
(slope = ∞).

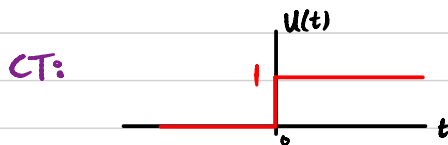


$$u[n] = \begin{cases} 0 & n < 0 \\ A & n \geq 0 \end{cases}$$

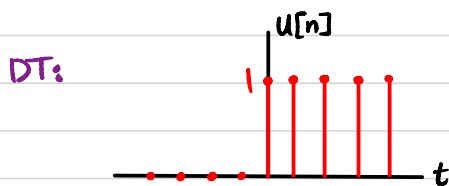
← at $n=0 \rightarrow$ defined
and $= A$.

Unit step:-

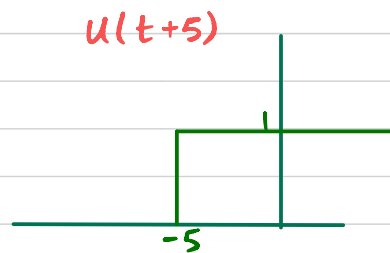
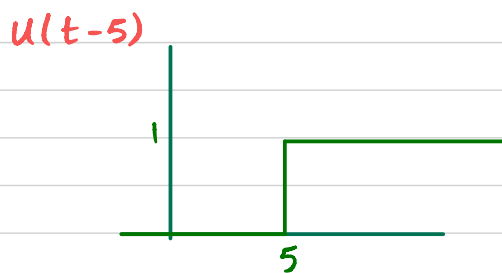
$A = 1$ (unit)



$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

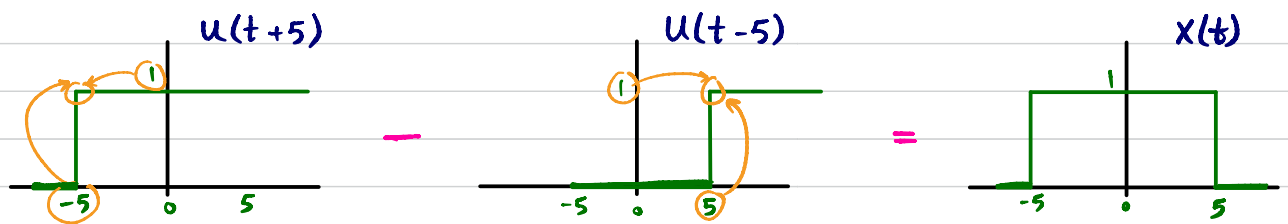


* Rectangular pulse *

$$x(t) = u(t - T_1) - u(t - T_2) \rightarrow \text{general rectangular pulse.}$$



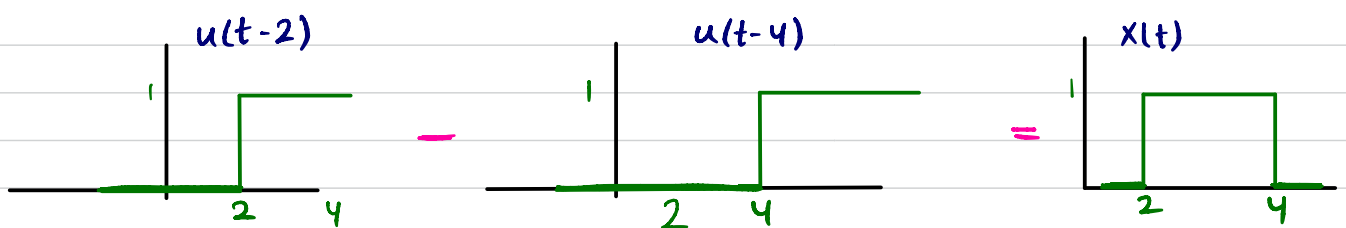
Ex 1



$$x(t) = u(t+5) - u(t-5) \uparrow$$

Before $-5 \rightarrow 0 - 0 = 0$
 at $-5 \rightarrow 1 - 0 = 1$
 at $0 \rightarrow 1 - 0 = 1$
 at $5 \rightarrow 1 - 1 = 0$

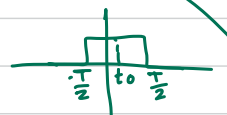
Ex 2



$$x(t) = u(t-2) - u(t-4) \uparrow$$

Before 2 $\rightarrow 0 - 0 = 0$
 at 2 $\rightarrow 1 - 0 = 1$
 at 4 $\rightarrow 1 - 1 = 0$

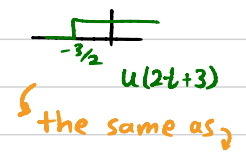
$$\text{rect}\left(\frac{t-t_0}{T}\right)$$



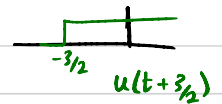
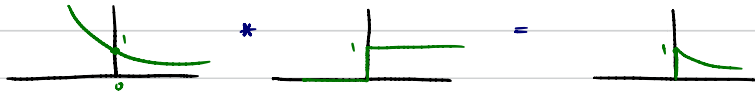
⇒ unit step properties:-

1. $u^2(t) = u^k(t) = u(t)$

2. $u(kt - t_0) = u(t - \frac{t_0}{k})$



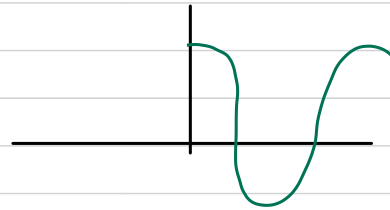
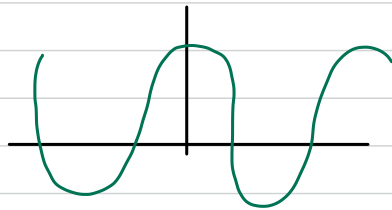
$y(t) = e^{-2t} \cdot u(t)$



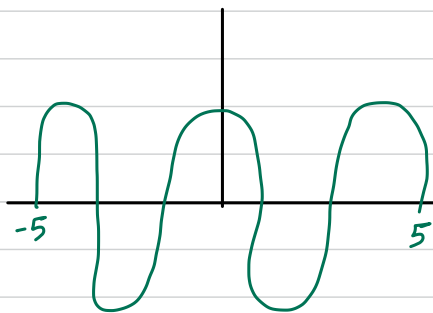
Before 0 → function $\neq$ 0 = 0
after 0 → function $\neq$ 1 = function.

Ex:-

$\cos(t) * u(t)$



Ex: $\cos(t) [u(t+5) - u(t-5)]$



★ Sinusoidal Signals

(CT) $x(t) = A \cos(\omega t + \phi)$ → periodic

$\downarrow$ amplitude $\downarrow$ angular frequency $\nwarrow$ phase shift

(DT) $x[n] = A \cos(\Omega n + \phi)$

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$\cos \theta = \frac{1}{2} [e^{j\theta} + e^{-j\theta}]$$

$$\sin \theta = \frac{1}{2j} [e^{j\theta} - e^{-j\theta}]$$

$N = \frac{2\pi}{\Omega} * m$ $N \Rightarrow$ must be integer to be periodic

→ The key to make N an integer.

★ Impulse signal (Dirac delta signal)

(DT) $\rightarrow \delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$



(CT) $\rightarrow \delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$



even

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$$

area at $(t=0) = 1$

to make the area = 1
the height must be $\frac{1}{\Delta}$ so area = $\Delta \cdot \frac{1}{\Delta}$



→ in impulse function Δ is very very small
therefore the height is very big (approaches ∞)

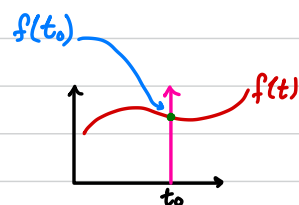
⊛ Dirac - Delta properties:-

① $\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$

② $\int_{-\infty}^{\infty} f(t) \delta(t - t_0) \cdot dt = f(t_0)$

→ integration is finding the area.

③ $\delta(at + b) = \frac{1}{|a|} \delta(t + \frac{b}{a})$



$x(t) \delta(t - t_0) = x(t_0) \delta(t - t_0)$

↳ no integration

stays

since it's = 1
when it's inside an integration

Delta (Impulse) → step

$\int_{-\infty}^{\infty} \delta(t) \cdot dt = u(t)$ $\nwarrow$ unit step

$\frac{d}{dt} u(t) = \delta(t)$

Example: $x(t) = \int_{-\infty}^{\infty} 2e^{4-3t} \cdot \delta(t-1) \cdot dt$

$$= 2e^{4-3(1)} = \boxed{2e}$$

Example: $x(t) = \sin t \delta(2t - \pi)$

$$= \sin t \cdot \frac{1}{|2|} \delta(t - \frac{\pi}{2})$$

Example: $x(t) = \int_{-\infty}^{\infty} 2e^{4-3t} \cdot \delta(2t+10) \cdot dt$

$$= \int_{-\infty}^{\infty} 2e^{4-3t} \cdot \frac{1}{|2|} \delta(t + \frac{10}{2}) \cdot dt$$

$$= \frac{1}{2} \cdot 2e^{4-3(-5)} = \boxed{e^{19}}$$

$$= \sin(\frac{\pi}{2}) \cdot \frac{1}{2} \cdot \delta(t - \frac{\pi}{2})$$

$$= \boxed{\frac{1}{2} \delta(t - \frac{\pi}{2})}$$

Example: $y(t) = 2t^2 \cdot \delta(t-3)$

$$y(t) = 2(3)^2 \cdot \delta(t-3)$$

no integration so it stays.

$$\boxed{y(t) = 18\delta(t-3)}$$

Example: $y(t) = \cos 4t \cdot \delta(2t - \pi)$

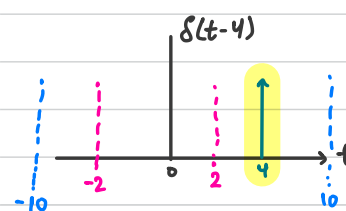
$$y(t) = \cos 4t \cdot \frac{1}{|2|} \cdot \delta(t - \frac{\pi}{2})$$

$$y(t) = \cos(4(\frac{\pi}{2})) \cdot \frac{1}{2} \cdot \delta(t - \frac{\pi}{2}) = \boxed{\frac{1}{2} \delta(t - \frac{\pi}{2})}$$

Example: $I = \int_{-2}^2 2t^2 \cdot \delta(t-4) \cdot dt = ?$

$$= \boxed{0}$$

shifts to the right by 4 which is outside the limits of integration.



Example: $I = \int_{-10}^{10} 2t^2 \cdot \delta(t-4) \cdot dt = ?$

Since $t=4$ is inside the integration limits, we can use the properties.

$$= 2(4)^2 = \boxed{32}$$

★ Rectangular Signal $\rightarrow \text{rect}(\frac{t}{T})$

$$\text{rect}(\frac{t}{T}) = \begin{cases} 1, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$



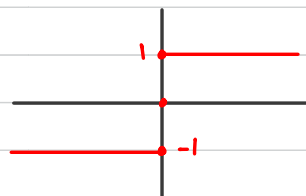
Ex: $\int_{-\infty}^{\infty} e^{\frac{t}{3}} \text{rect}(\frac{t}{3}) dt = \int_{-\frac{3}{2}}^{\frac{3}{2}} e^{\frac{t}{3}} dt = 3e$

$\rightarrow$ rect function just changes the limits of the integration.

★ Signum Signal $\rightarrow \text{Sgn}(t)$

$$\text{Sgn}(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$

odd signal



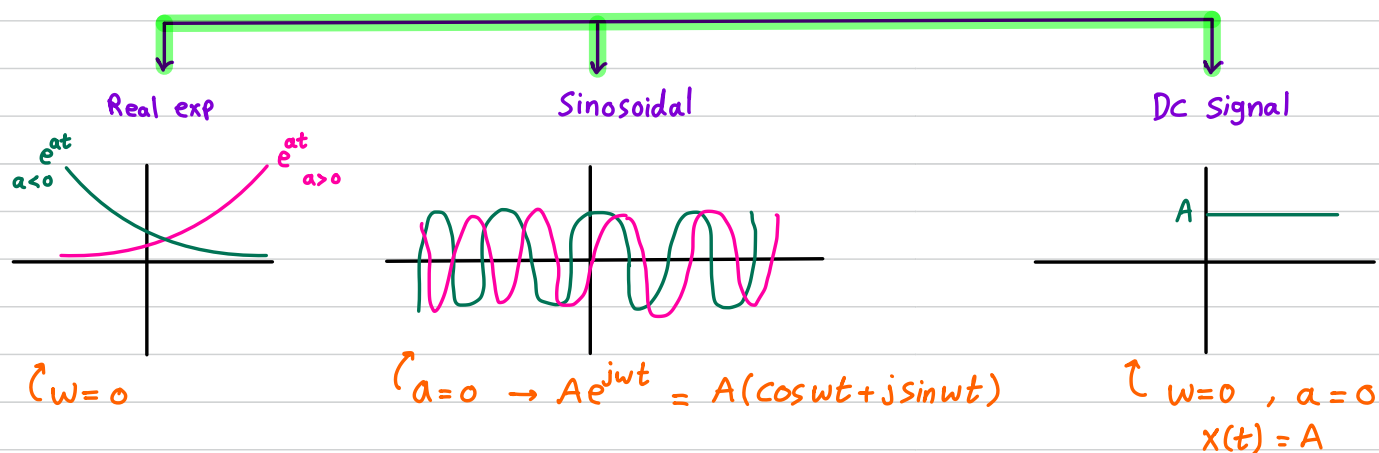
Ex: $\text{Sgn}(-3) = -1$

Ex: $\text{Sgn}(0) = 0$

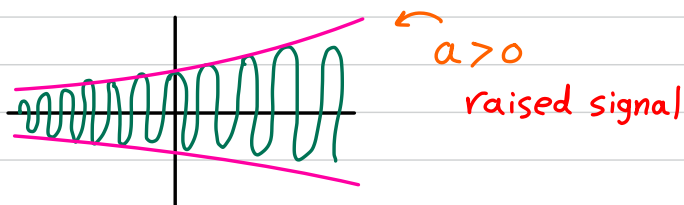
Ex: $\text{Sgn}(7) = 1$

Ex: $\text{Sgn}(\pi-3) = \text{Sgn}(0.14) = 1$

★ Complex exp. Signal $\rightarrow x(t) = Ae^{(a+jw)t}$



$\Rightarrow$ Damped or raised sinusoidal



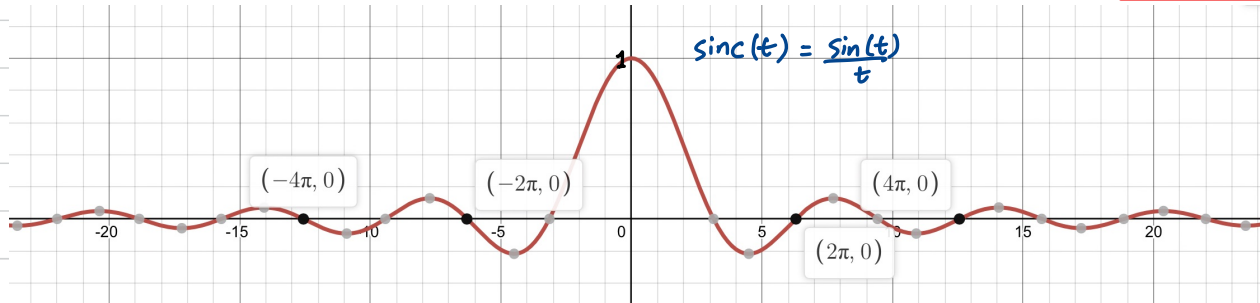
★ Sinc Signal $\rightarrow \text{Sinc}(t)$

$$X(t) = \frac{\sin(t)}{t} = \lim_{t \rightarrow 0} \text{sinc}(t) = 1$$

$$\text{Sinc}(at)$$

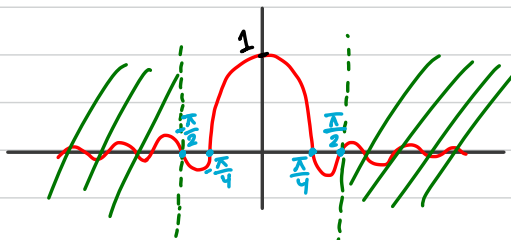
$$t = \pm \frac{n\pi}{a}$$

$$@ X(t) = 0$$



Example: $\text{sinc}(4t)$ $t = \pm \frac{n\pi}{4}$

Example: $\text{sinc}(4t) \text{rect}\left(\frac{t}{\pi}\right)$ $t = \pm \frac{n\pi}{4}$



only the signal between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ remains because of the $\text{rect}\left(\frac{t}{\pi}\right)$

$\downarrow T$
 $\rightarrow \begin{cases} 1, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$

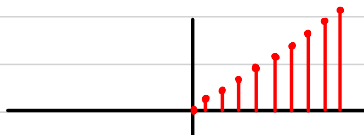
$T = \pi$

★ Ramp Signal

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



$$* \frac{d}{dt} r(t) = u(t)$$

$$* \frac{d^2}{dt^2} r(t) = \delta(t)$$

$$* \int_{-\infty}^t u \cdot dt = r(t)$$

Question: Using change of variables and the properties of the unit impulse function the value of $\int_{-\infty}^{\infty} \delta(at-b) \sin^2(t-4) \cdot dt$, and $a > 0$ is.

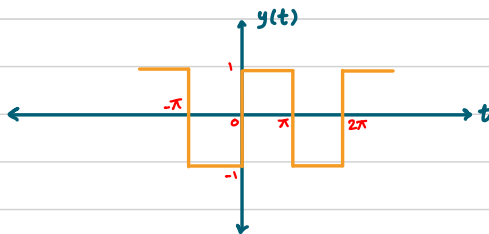
$$\int_{-\infty}^{\infty} \delta(at-b) \sin^2(t-4) \cdot dt = \int_{-\infty}^{\infty} \frac{1}{|a|} \delta\left(t - \frac{b}{a}\right) \sin^2(t-4) = \frac{1}{a} \sin^2\left(\frac{b}{a} - 4\right)$$

Question: Given a signal $f(t) = 4u(t+3)$. Evaluate $\int_{-10}^{10} f(t-1) \delta(-2t+6) dt$

$$\begin{aligned} \int_{-10}^{10} f(t-1) \frac{1}{|-2|} \cdot \delta(t-3) dt &= \frac{1}{2} f(3-1) = \frac{1}{2} f(2) \\ &= \frac{1}{2} (4u(5)) = 2 \end{aligned}$$

$\curvearrowright t=3$

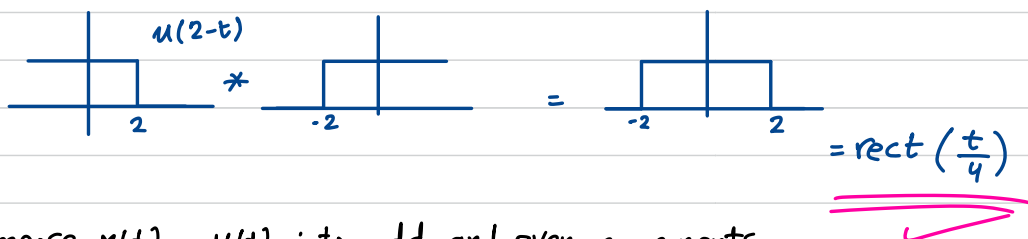
Question: The signal $y(t) = \text{sign}(\sin(t))$ has the shape:



Question: The signal $f(t) = u^2(2t-2) \delta(4-3t)$ can be expressed as:

$$f(t) = \frac{1}{|-3|} \cdot u^2(2t-2) \delta(t-3) = \frac{1}{3} \delta(t-3) \quad \checkmark$$

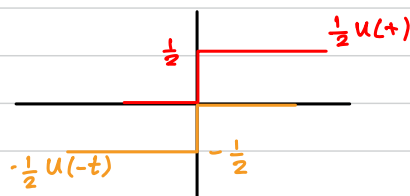
Question: The signal $x(t) = u(2-t)u(t+2)$



Question: Decompose $x(t) = u(t)$ into odd and even components

$$\begin{aligned} x_e(t) &= \frac{1}{2} [x(t) + x(-t)] \\ &= \frac{1}{2} [u(t) + u(-t)] \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} x_o(t) &= \frac{1}{2} [x(t) - x(-t)] \\ &= \frac{1}{2} [u(t) - u(-t)] \\ &= \frac{1}{2} \text{sgn}(t) \end{aligned}$$



$$\therefore u(t) = \frac{1}{2} + \frac{1}{2} \text{sgn}(t)$$

* Systems:

System properties:-

- Linear and non-linear
- Stable and non-stable
- Causal and non-causal
- memory and memoryless
- Invertible and non-invertible
- Time-Invariant and Time variant.

$$X(t) \rightarrow \boxed{H} \rightarrow y(t)$$

1 Stability :- (stable/unstable)

if system is **BIBO** $\rightarrow$ The system is stable

$\hookrightarrow$ Bounded Input - Bounded Output. For a stable system output should be bounded for bounded input for each and every instant of time

Bounded $\rightarrow$ from $-\infty$ to ∞ the amplitude of the signal is finite

* if we provide a Bounded input and the output turns out to be unbounded, we call the system unstable.

for example: dc signal $\rightarrow y(t)=6$ Bounded amplitude = 6 $\therefore$ finite



$\sin(t)$ and $\cos(t)$

Bounded amplitude is -1 to 1



$u(t)$ $\rightarrow$ unit step

Bounded amplitude is 0 or 1

ex: $y(t) = t \cdot x(t)$

$x(t) \xrightarrow{\text{input}} \text{Sys.} \rightarrow y(t) = t \cdot x(t) \xleftarrow{\text{output}}$
 $u(t) \xrightarrow{\text{input}} \text{Sys.} \rightarrow t \cdot u(t) = r(t) \xleftarrow{\text{unbounded}}$
Bounded $\therefore$ sys unstable goes to ∞

ex: $y(t) = x(t) + 2$

$$x(t) \rightarrow \text{sys.} \rightarrow y(t) = x(t) + 2$$

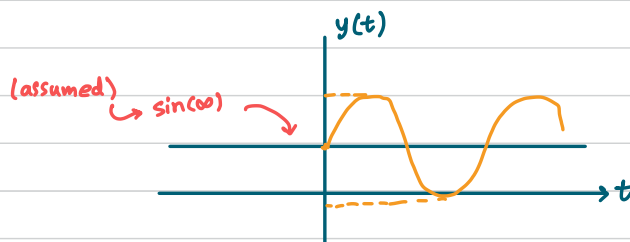
$$\text{DC} \rightarrow 4 \xrightarrow{\text{Bounded}} \text{sys.} \rightarrow y'(t) = 4 + 2 = 6 \xleftarrow{\text{Bounded}} \text{DC}$$

$\therefore$ system stable

Example: $y(t) = \int_{-\infty}^t x(\tau) d\tau$, $x(t) = \cos(t)$

$$y(t) = \int_{-\infty}^t \cos(\tau) d\tau = [\sin \tau]_{-\infty}^t = \sin t - \sin(-\infty)$$

$$= \sin t + \sin(\infty)$$

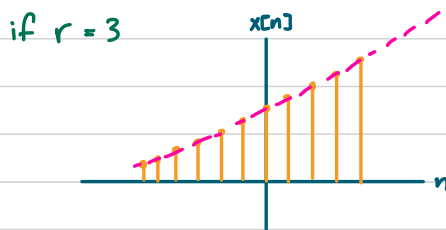


not defined ↖ can be assumed constant
 but $-1 \leq \sin(t) \leq 1$
 $-1 \leq \sin(\infty) \leq 1$ ✓
 $\therefore$ Bounded

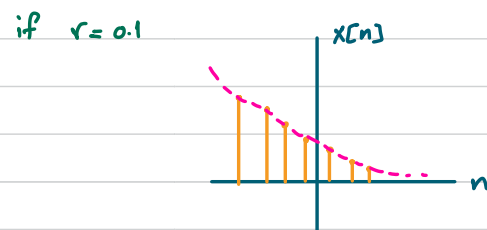
so $y(t)$ is Bounded, and input $\cos(t)$ is Bounded
 so bounded input bounded output $\therefore$ system stable.

Example: $y[n] = r^n x[n]$

$r \rightarrow$ growing, $r > 1$
 decreasing, $-1 < r < 1$



$\therefore$ system is unstable
 (goes to ∞).



$\therefore$ system is stable

★ $y_{ss} = \lim_{t \rightarrow \infty} y(t) = \square$

$\rightarrow$ finite number $\rightarrow$ stable
 $\rightarrow \infty \rightarrow$ unstable

2 memory :- (memory / memoryless)

if $y(t)$ and $x(t)$ depend on the same time $\rightarrow$ memoryless

if $y(t)$ and $x(t)$ depend on different time $\rightarrow$ memory

ex: $y(t) = x(t)$ $\rightarrow$ memoryless.

ex: $y(t) = x(t) + x^2(t)$ $\rightarrow$ memoryless

ex: $y(t) = x(t) + x(t^2)$ $\rightarrow$ memory

ex: $y(t) = e^{t+4} x(t)$ $\rightarrow$ memoryless. (we only care about what's inside x).

ex: $y(t) = 4(x(t+2)) + 5$ $\rightarrow$ memory. $\leftarrow$ Because $y(t) \rightarrow x(t+2)$

ex: $y(t) = \int_{-\infty}^{\infty} x(t) \cdot dt$ $\rightarrow$ memory. $\rightarrow$ different

ex: $y(t) = x(\sin t)$ $\rightarrow$ Always when there is integration.

$t = 0$

$$y(0) = x(\sin 0)$$

$$y(0) = x(0)$$

$t = \pi$

$$y(\pi) = x(\sin \pi)$$

$$y(\pi) = x(0)$$

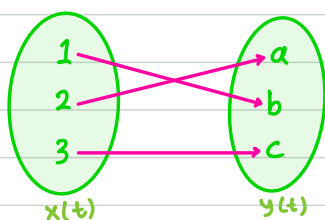
$\rightarrow$ different

$\rightarrow \therefore$ memory system.

3 Invertible / Non-Invertible

$$x(t) \rightarrow \boxed{H} \rightarrow y(t) \rightsquigarrow y(t) \rightarrow \boxed{H} \rightarrow x(t)$$

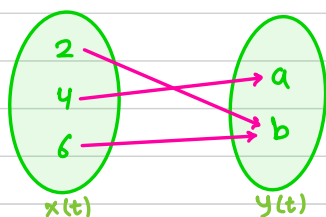
one to one mapping:



invertible ↗

→ for invertible system, there should be one to one mapping between input and output at each and every instant of time.

many to one mapping:



ex: $y(t) = x^2(t)$

$x(t)$	$y(t)$
-2	4
2	4

many to one mapping
∴ non-Invertible.

ex: $y(t) = |x(t)|$

$x(t)$	$y(t)$
-2	2
2	2
2j	2
-2j	2

many to one mapping
∴ non-Invertible.

ex: $y(t) = x(t) + 2$

$x(t)$	$y(t)$
0	2
1	3
-1	1

one to one mapping
∴ Invertible.

ex: $y(t) = 1.5x(t) - 5$

$$x(t) = \frac{y(t) + 5}{1.5} \rightarrow \text{inv}$$

$x(t)$	$y(t)$
1	-3.5
2	-2.5
0	-5

one to one mapping
∴ Invertible.

ex: $y(t) = \sin t \cdot x(t)$

$x(t)$	$y(t)$
$\delta(t)$	$y(t) = \sin t \cdot \delta(t) = \sin(0) \cdot \delta(t-0) = 0$
$2\delta(t)$	$y(t) = \sin t \cdot 2\delta(t) = 0$

$x(t) \cdot \delta(t-t_1) = x(t_1) \cdot \delta(t-t_1)$

many to one mapping
∴ non-Invertible.

ex: $y(t) = \sin(x(t))$

$x(t)$	$y(t)$
0	$\sin(0)=0$
π	$\sin(\pi)=0$
2π	$\sin(2\pi)=0$

many to one mapping
 $\therefore$ non-Invertable.

ex: $y(t) = x(t) u(t)$

non-Invertable.

4 Causality (Causal / non-Causal)

Causal system: output of the system is independent of future values of output.

ex: $y(t) = x(t)$ $\rightarrow$ Causal system, Because output depends only on the present not the future

Depends only on the:
 * present input.
 * past input.

ex: $y(t) = x(t) + x(t-1)$ $\rightarrow$ Causal system, Because output depends only on the present and past only.

non-Causal system: output of the system is dependent of future values of output.

ex: $y(t) = x(t+2)$ $\rightarrow$ non-Causal

Depends on:
 the future input.

ex: $x(t) + x(t-1) + x(t+1)$ $\rightarrow$ non-Causal

Note: if the output depends on at least 1 non-Causal input then the whole system is non-causal.

Example: $y(t) = x(3t)$

$$\begin{array}{ll} t = 0 & y(0) = x(0) \\ t = 1 & y(1) = x(3) \end{array}$$

↖ present output
↖ ∴ future input

∴ system non-causal

Example: $y(t) = \sin(t+1) \cdot x(t-1)$

$$t = 0 \quad y(0) = \sin(1) \cdot x(-1)$$

↖ present
↖ past

∴ system only depends on past inputs at all instances
So system is Causal.

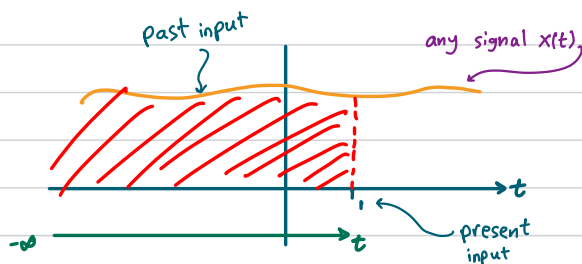
$$t = -1 \quad y(-1) = \sin(0) \cdot x(-2)$$

↖ present
↖ past

$$t = 1 \quad y(1) = \sin(2) \cdot x(0)$$

↖ present
↖ past

Example: $y(t) = \int_{-\infty}^t x(\tau) d\tau$



*The integration is the area under the curve, so in this case the area depends only on the past and present values of t .

∴ the system is causal.

$$\text{Example: } y(t) = \int_{-\infty}^{t+1} x(\tau) d\tau$$

↖ $x(t+1)$
@ $t=0 \rightarrow x(1)$
↖ $x(-\infty)$

↖ present
↖ future
↖ past

∴ system non-causal

$$\text{Example: } \int_{-\infty}^t x(3\tau) d\tau$$

↖ $x(3t)$
@ $t=1 \rightarrow x(3)$
↖ $x(-\infty)$

↖ present
↖ future
↖ past

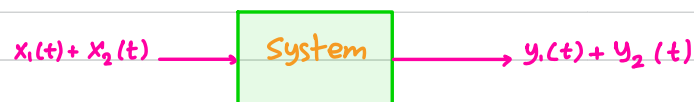
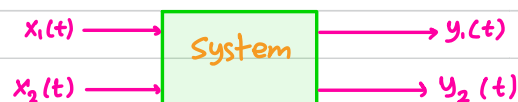
∴ system non-causal

5 Linearity (Linear / non-linear)

Linear systems: The system which follows the principle of superposition is known as a linear system.

Superposition:

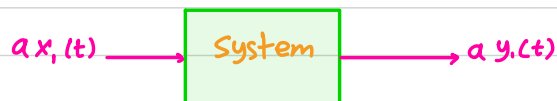
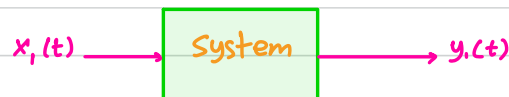
1. Additivity property:



* if we provide the input $x_1(t) + x_2(t)$ to the system and get an output of $y_1(t) + y_2(t)$ then the system is following the **additivity property**.

$$\text{if } F(x_1 + x_2) = F(x_1) + F(x_2)$$

2. Homogeneity property:



* if we multiply the input by a constant and the output of the system is also multiplied by the same constant then the system is following the **homogeneity property**.

$$\text{if } F(a \cdot x) = a \cdot F(x)$$

* only when the system is following both properties we say that it's linear.

Example: $y(t) = 4x(t)$

$$x(t) \rightarrow \boxed{4} \rightarrow y(t)$$

$$x_1(t) \rightarrow 4x_1(t) = y_1(t)$$

$$x_2(t) \rightarrow 4x_2(t) = y_2(t)$$

$$\begin{array}{l} ax_1(t) \rightarrow 4ax_1(t) = ay_1(t) \\ bx_2(t) \rightarrow 4bx_2(t) = by_2(t) \end{array} \Rightarrow ay_1(t) + by_2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

$$x_3(t) \rightarrow \boxed{4} \rightarrow y_3(t)$$

$$\begin{aligned} y_3(t) &= 4x_3(t) = 4(ax_1(t) + bx_2(t)) \\ &= 4ax_1(t) + 4bx_2(t) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

The same, $\therefore$ system is linear

Example: $y(t) = tx(t)$

$$x(t) \rightarrow \boxed{t} \rightarrow y(t)$$

$$x_1(t) \rightarrow tx_1(t) = y_1(t)$$

$$x_2(t) \rightarrow tx_2(t) = y_2(t)$$

$$\begin{array}{l} ax_1(t) \rightarrow atx_1(t) = ay_1(t) \\ + \quad bx_2(t) \rightarrow btx_2(t) = by_2(t) \end{array} +$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

$$x_3(t) \rightarrow \boxed{t} \rightarrow y_3(t)$$

$$\begin{aligned} y_3(t) &= tx_3(t) \\ &= t(ax_1(t) + bx_2(t)) \\ &= atx_1(t) + btx_2(t) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

The same $\therefore$ system is linear

Example: $y(t) = x^2(t)$

$$x(t) \rightarrow \boxed{\text{input}^2} \rightarrow y(t)$$

$$x_1(t) \rightarrow x_1^2(t) = y_1(t)$$

$$x_2(t) \rightarrow x_2^2(t) = y_2(t)$$

$$\begin{array}{l} ax_1(t) \rightarrow a^2x_1^2(t) = a^2y_1(t) \\ + \quad bx_2(t) \rightarrow b^2x_2^2(t) = b^2y_2(t) \end{array} +$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

$$\begin{aligned} y_3(t) &= x_3^2(t) = a^2x_1^2(t) + 2abx_1(t)x_2(t) + b^2x_2^2(t) \\ &= ay_1^2(t) + 2abx_1(t)x_2(t) + by_2^2(t) \end{aligned}$$

Not the same $\therefore$ system is non-linear

Example: $y[n] = 2x[n] + 3$

$$x_1[n] \rightarrow 2x_1[n] + 3 = y_1[n]$$

$$x_2[n] \rightarrow 2x_2[n] + 3 = y_2[n]$$

$$ax_1[n] \rightarrow a(2x_1[n] + 3) = ay_1[n]$$

$$bx_2[n] \rightarrow b(2x_2[n] + 3) = by_2[n]$$

$$x_3[n] = ax_1[n] + bx_2[n]$$

$$x_3[n] \rightarrow \boxed{2 \cdot \text{inp} + 3} \rightarrow y_3[n]$$

$$y_3[n] = 2x_3[n] + 3 = 2(ax_1[n] + bx_2[n]) + 3 \\ = 2ax_1[n] + 2bx_2[n] + 3$$

$$ay_1[n] + by_2[n]$$

Not the same,
 $\therefore$ system is non-linear.

Example: $y(t) = x(\sin t)$

$$x(t) \rightarrow \text{sys.} \rightarrow y(t) = x(\sin t)$$

$$x_1(t) \rightarrow x_1(\sin t) = y_1(t)$$

$$x_2(t) \rightarrow x_2(\sin t) = y_2(t)$$

$$ax_1(t) \rightarrow ax_1(\sin t) = ay_1(t)$$

$$bx_2(t) \rightarrow bx_2(\sin t) = by_2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

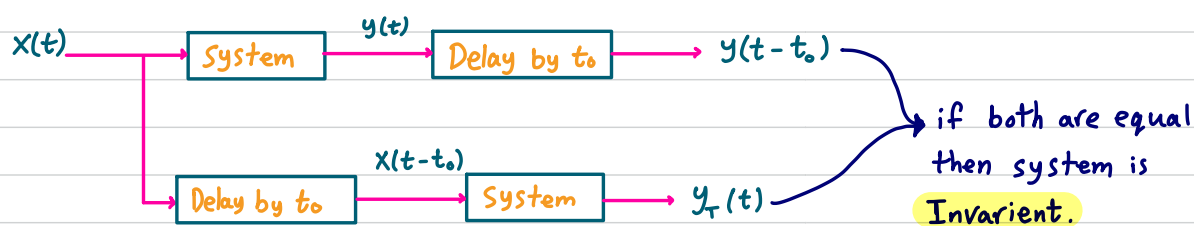
$$x_3(t) \rightarrow \text{sys.} \rightarrow y_3(t)$$

$$y_3(t) = ax_1(\sin t) + bx_2(\sin t) \\ = ay_1(t) + by_2(t)$$

$$ay_1(t) + by_2(t)$$

The same, $\therefore$ system is linear.

6 Time Variant and Invariant systems



Example: $y(t) = e^{-2t} x(t)$

$$y_T(t) = e^{-2t} x(t-t_0)$$

$$y(t-t_0) = e^{-2(t-t_0)} x(t-t_0)$$

} not the same $\therefore$ Time Variant system.

Example: $y(t) = \sin(t)$

$$y_T = \sin(t-t_0)$$

$$y(t-t_0) = \sin(t-t_0)$$

} the same $\therefore$ Time Invariant system.

Example: $y(t) = x(2t)$

$$x(t) \rightarrow \text{sys.} \rightarrow x(2t) = y(t)$$

$$y(t-t_0) = x(2(t-t_0)) = x(2t-2t_0)$$

$$x(t-t_0) \rightarrow \text{sys.} \rightarrow x(2t-t_0)$$

} not the same $\therefore$ system Variant.

The system scales the time.

NOTE: NOT $x(2(t-t_0))$, the system only perform time scaling on t not t_0 which is constant

Example: $y(t) = 2x(t) + 5$

$$y(t-t_0) = 2x(t-t_0) + 5$$

$$x(t-t_0) = 2x(t-t_0) + 5$$

} the same $\therefore$ Time Invariant system.

Question: The system described by $y(t) = t x(t-1) + x(2t)$ is

$\xrightarrow{\text{present}}$ $\xrightarrow{\text{past}}$ $\xrightarrow{\text{future}}$

with memory, non-causal, and Time Varying.

$$X(t-t_0) \rightarrow t X(t-t_0-1) + X(2t-t_0)$$

$$y(t-t_0) \rightarrow (t-t_0) X(t-t_0-1) + X(2t-2t_0)$$

not the same $\therefore$ Time Varying

Question: The system described by $y(t) = \cos(t+1)x(t-1)$ is

$$X_1(t) \rightarrow \cos(t+1)x_1(t-1) = y_1(t)$$

$$a X_1(t) \rightarrow a \cos(t+1)x_1(t-1) = a y_1(t)$$

$$X_2(t) \rightarrow \cos(t+1)x_2(t-1) = y_2(t)$$

$$b X_2(t) \rightarrow b \cos(t+1)x_2(t-1) = b y_2(t)$$

$$X_3(t) = a X_1(t) + b X_2(t)$$

$$y_3(t) = \cos(t+1)(a x_1(t-1) + b x_2(t-1))$$

$$= a \cos(t+1)x_1(t-1) + b \cos(t+1)x_2(t-1)$$

$$= a y_1(t) + b y_2(t)$$

the same
 $\therefore$ linear

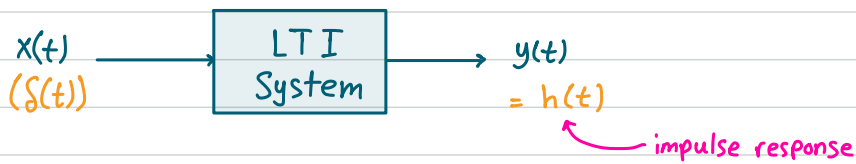
Linear, Causal, stable, Time variant

$$y(t-t_0) = \cos(t-t_0+1)x(t-t_0-1)$$

$$x(t-t_0) \rightarrow \cos(t+1)x(t-t_0-1)$$

not
the same

* LTI systems



DT:



$$x[n] * \delta[n] = x[n]$$

$$x[n] \delta[n - n_0] = x[n_0]$$

$$x[n] \delta[n+2] = x[-2] = 2$$

$$x[n] \delta[n+1] = x[-1] = -1$$

$$x[n] \delta[n] = x[0] = 2$$

$$x[n] \delta[n-1] = x[1] = 1$$

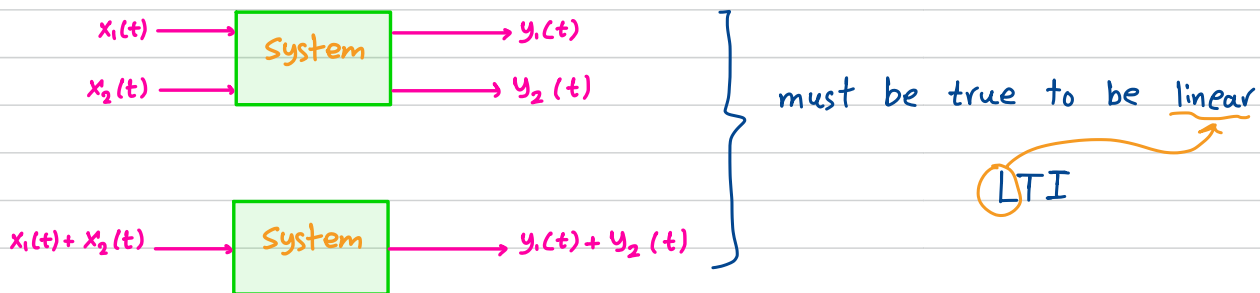
$$x[n] \delta[n-2] = x[2] = -2$$

$$x[n] = x[n] \delta[n+2] + x[n] \delta[n-1] + \dots$$

↻_{total}

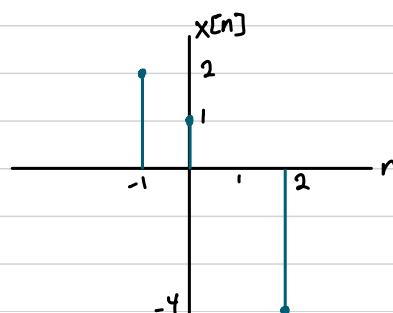
* any DT signal can be written in terms of impulse function.

In general: $x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k]$



Example: $x[n] = 2 \delta[n+1] + \delta[n] - 4 \delta[n-2]$

$\downarrow$ $\downarrow$ $\downarrow$
 $x[-1] = 2$ $x[0] = 1$ $x[2] = -4$





$$y[n] = H[x[n]]$$

The action that the system does on the input $x[n]$

$$\Rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$y[n] = H[x[n]]$ (not important) (Derivation)

$$y[n] = H\left[\sum_{k=-\infty}^{\infty} x[k] \delta[n-k]\right]$$

only in LTI

$$y[n] = \sum_{k=-\infty}^{\infty} H[x[n] \delta[n-k]]$$

$$= \sum_{k=-\infty}^{\infty} x[n] H[\delta[n-k]]$$

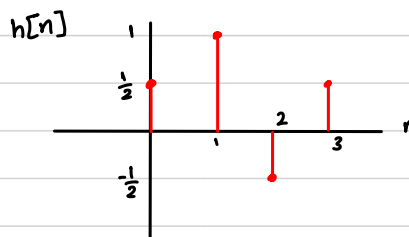
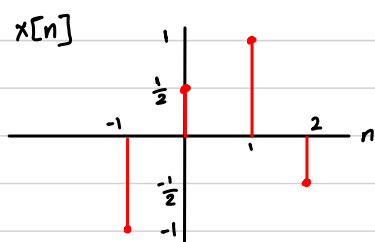
$h[n-k]$
impulse response

* Step response $\rightarrow$ The response when we have a step input

* Impulse response $\rightarrow$ The response when we have an Impulse input

$x[n] \rightarrow \boxed{\text{LTI}} \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] = x[n] * h[n]$
convolution

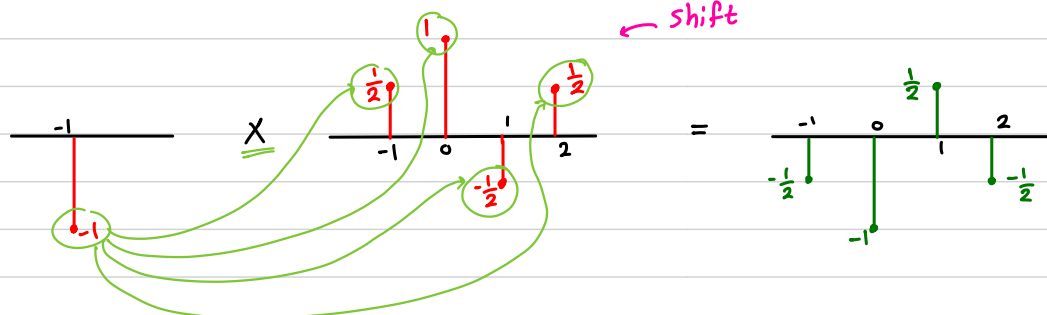
Example: Find $y[n]$, if $h[n]$ is an impulse response



$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] \rightarrow y[n] = \sum_{k=-1}^2 x[k] \cdot h[n-k]$$

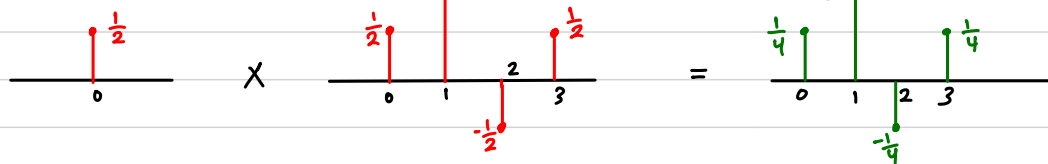
$\rightarrow k = -1 \quad x[-1] \delta[n+1] = -1$

$h[n+1]$
shift



→ $k=0$

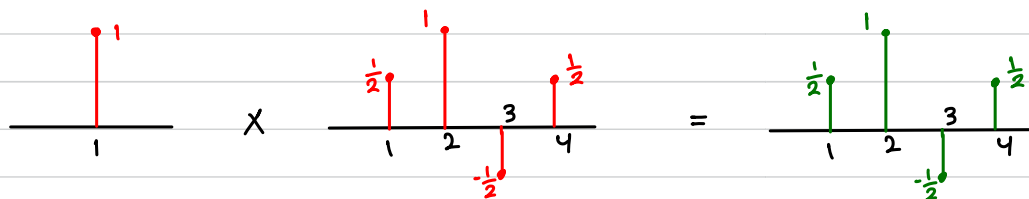
$$x[0] \delta[n] = \frac{1}{2}$$



→ $k=1$

$$x[1] \delta[n-1] = 1$$

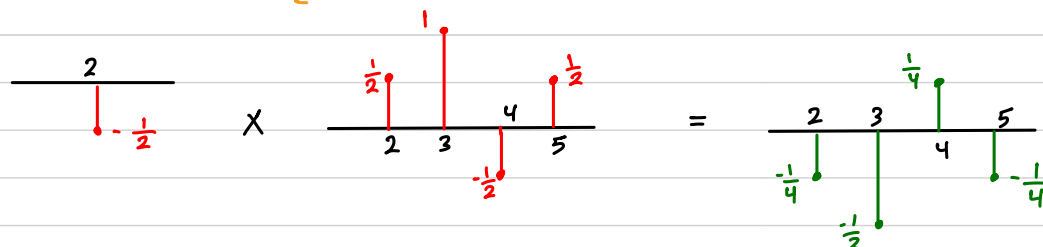
$$h[n-1]$$



→ $k=2$

$$x[2] \delta[n-2] = -\frac{1}{2}$$

$$h[n-2]$$



* now we sum all the values at each $[n]$
so we can draw the final $y[n]$

→ $y[n] =$

$$n=-1 \rightarrow -\frac{1}{2} + 0 + 0 + 0 = -\frac{1}{2}$$

$$n=0 \rightarrow -1 + \frac{1}{4} + 0 + 0 = -\frac{3}{4}$$

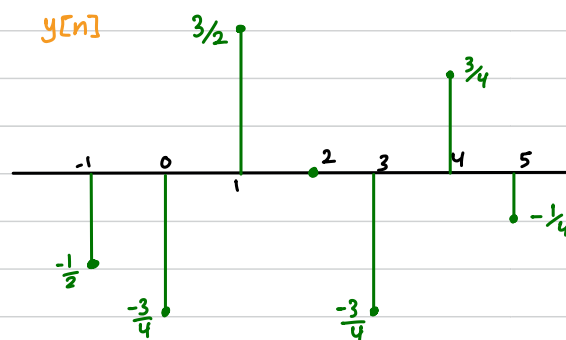
$$n=1 \rightarrow \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 0 = \frac{3}{2}$$

$$n=2 \rightarrow -\frac{1}{2} - \frac{1}{4} + 1 - \frac{1}{4} = 0$$

$$n=3 \rightarrow 0 + \frac{1}{4} - \frac{1}{2} - \frac{1}{2} = -\frac{3}{4}$$

$$n=4 \rightarrow 0 + 0 + \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$n=5 \rightarrow 0 + 0 + 0 - \frac{1}{4} = -\frac{1}{4}$$



Example: Given the output of LTI

$$\Rightarrow y[n] = x[n] + \frac{1}{2}x[n-1]$$

$$\text{let } x[n] = \delta[n]$$

$$h[n] = \delta[n] + \frac{1}{2}\delta[n-1]$$

impulse response

$$h[n] = \begin{cases} 1 & n=0 \\ \frac{1}{2} & n=1 \\ 0 & , \text{ otherwise} \end{cases}$$

Determine the output of the system due to the following input ?

$$x[n] = \begin{cases} 2 & n=0 \\ 4 & n=1 \\ -2 & n=2 \\ 0 & , \text{ otherwise} \end{cases}$$

$$x[n] = 2\delta[n] + 4\delta[n-1] - 2\delta[n-2] \rightarrow \boxed{\text{LTI}} \rightarrow y[n]$$

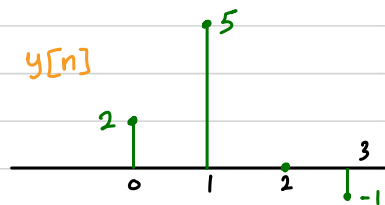
$$y[n] = 2h[n] + 4h[n-1] - 2h[n-2]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = 2[\delta[n] + \frac{1}{2}\delta[n-1]] + 4[\delta[n-1] + \frac{1}{2}\delta[n-2]] - 2[\delta[n-2] + \frac{1}{2}\delta[n-3]]$$

$$= 2\delta[n] + \delta[n-1] + 4\delta[n-1] + 2\delta[n-2] - 2\delta[n-2] - \delta[n-3]$$

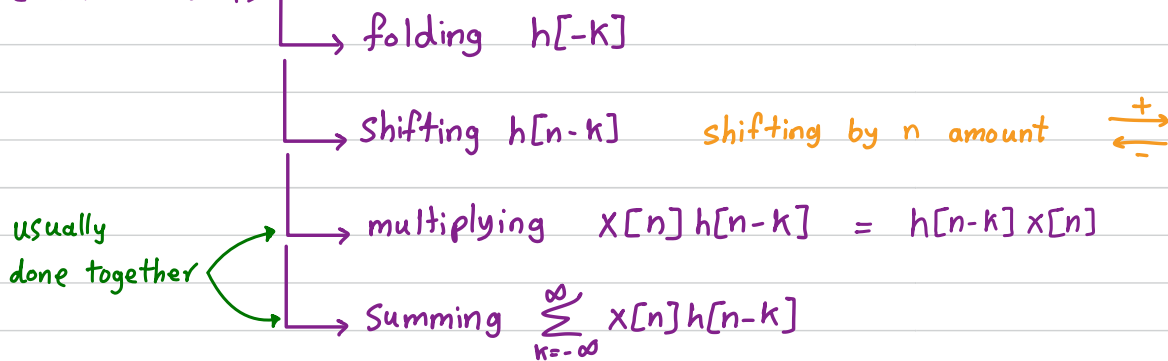
$$= 2\delta[n] + 5\delta[n-1] - \delta[n-3]$$



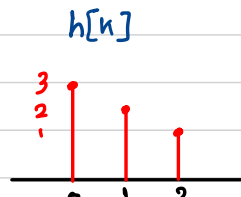
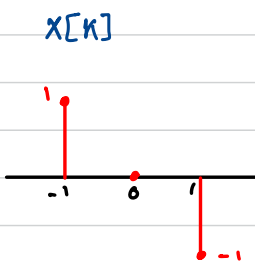
Convolution Sum:-

$$x[n] \xrightarrow{\text{LTI}} y[n] = \sum_{k=-\infty}^{\infty} x[n] h[n-k] = x[n] * h[n]$$

convolution steps:



Example: if the sys is LTI find $y[n]$ if

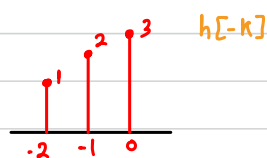


$$* y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = x[k] * h[k]$$

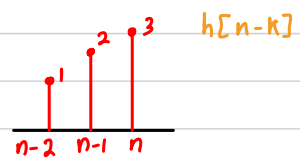
convolution

⇒

1 → folding

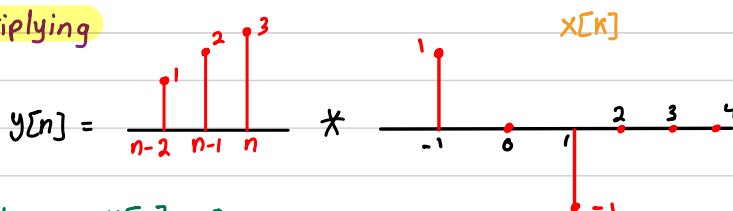


2 → shifting



← $-\infty$ and shift with (n)

3 → multiplying



$$n < -1 \rightarrow y[n] = 0$$

$$n = -1 \rightarrow y[n] = 3 * 1 = 3$$

$$n = 0 \rightarrow y[n] = 3 * 0 + 2 * 1 = 2$$

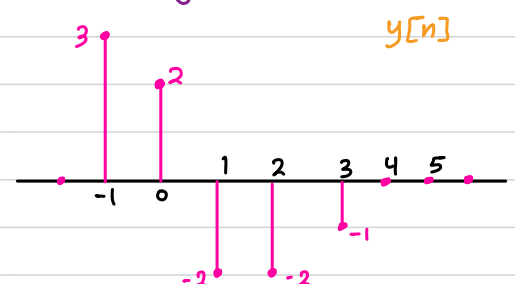
$$n = 1 \rightarrow y[n] = 3 * -1 + 2 * 0 + 1 * 1 = -2$$

$$n = 2 \rightarrow y[n] = 3 * 0 + 2 * -1 + 1 * 0 = -2$$

$$n = 3 \rightarrow y[n] = 3 * 0 + 2 * 0 + 1 * -1 = -1$$

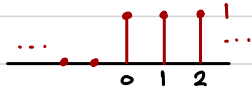
$$n > 3 \rightarrow y[n] = 0$$

4 → Summing

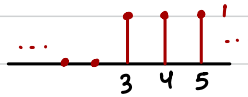


Example: Find $y[n] = u[n] * u[n-3]$
 $\hookrightarrow x[n]$ $\hookrightarrow h[n]$

$x[k]$

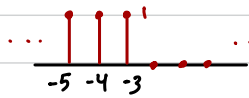


$h[k]$



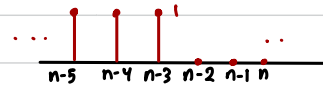
1 $\rightarrow$ folding

$h[-k]$

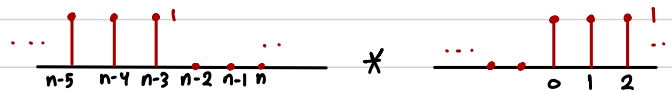


2 $\rightarrow$ shifting

$h[n-k]$



3 $\rightarrow$ multiplying



$$n-3 < 0 \quad n < 3 \quad y[n] = 0$$

$$n=3 \quad y[n] = 1 * 1 = 1$$

$$n=4 \quad y[n] = 1 * 1 + 1 * 1 = 2$$

$$n=5 \quad y[n] = 1 * 1 + 1 * 1 + 1 * 1 = 3$$

$$n=6 \quad y[n] = 1 * 1 + 1 * 1 + 1 * 1 + 1 * 1 = 4$$

$\vdots$

$$n < 3 \rightarrow y[n] = 0$$

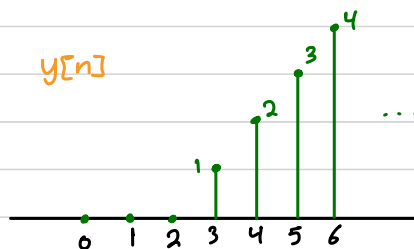
$$n \geq 3 \rightarrow y[n] = \sum_{k=0}^{n-3} x[n] * h[n-k]$$

$$= \sum_{k=0}^{n-3} 1 = n-3-0+1$$

$$= n-2$$

4 $\rightarrow$ Summing

$y[n]$



$$y[n] = \begin{cases} 0, & n < 3 \\ n-2, & n \geq 3 \end{cases}$$

In CT: $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

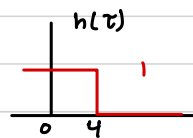
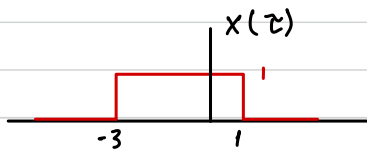
or $\int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$

impulse response

depends which one is easier to do

Commutative property

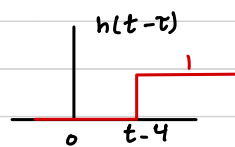
Example: $y(t) = \underbrace{[u(t+3) - u(t-1)]}_{x(t)} * \underbrace{u(-t+4)}_{h(t)}$



1 → Folding

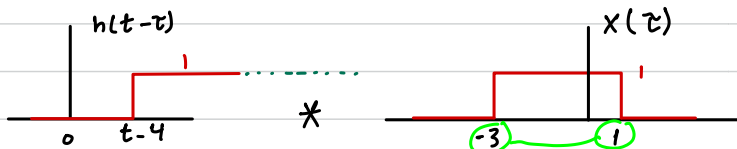


2 → Shifting

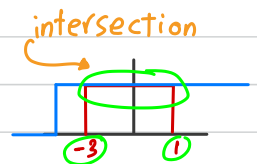


-T
add +ve num
shift
add -ve num
shift

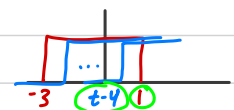
3 → Multiplying



$t-4 < -3$
 $t < 1$
 $y(t) = \int_{-3}^1 x(\tau) d\tau = \int_{-3}^1 d\tau = 1 - (-3) = 4$



$t-4 < 1$
 $t < 5$
 $y(t) = \int_{t-4}^1 d\tau = \tau \Big|_{t-4}^1 = 1 - (t-4) = 5-t$



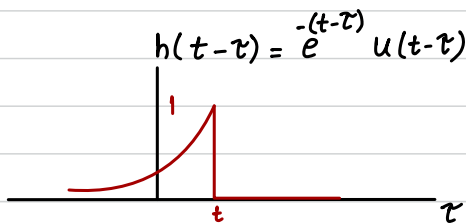
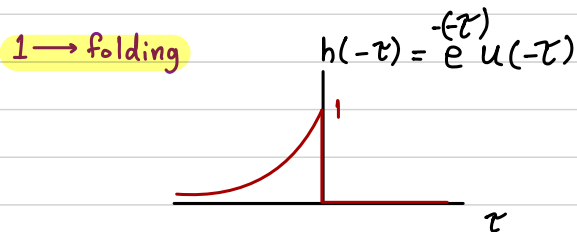
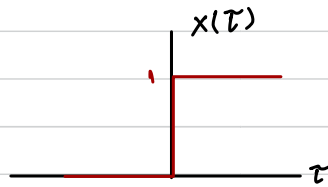
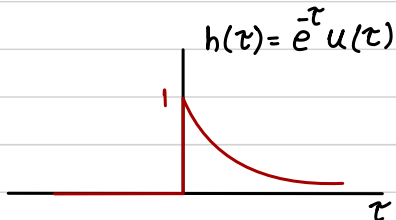
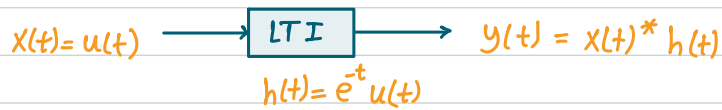
$t-4 > 1$
 $t > 5$
 $y(t) = 0$



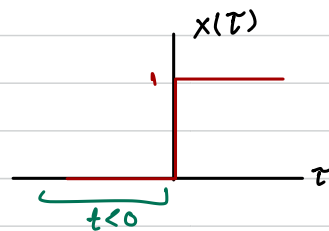
no intersection so zero

it will keep sliding from -3 to 1

Example: let the impulse response of LTI system be $h(t) = e^{-t}u(t)$, Find $y(t)$ if the input is $x(t) = u(t)$



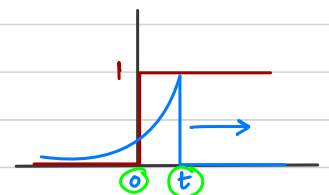
*



$t < 0 \quad y(t) = 0$

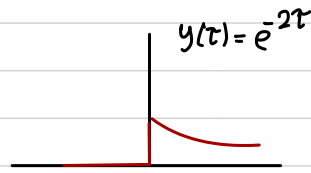
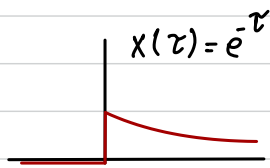
$t \geq 0 \quad y(t) = \int_0^t e^{-(t-\tau)} u(t-\tau) * 1 d\tau$

$$\begin{aligned}
 y(t) &= \int_0^t e^{-(t-\tau)} d\tau = e^{-(t-\tau)} \Big|_0^t \\
 &= e^{-(t-t)} - e^{-(t-0)} \\
 &= 1 - e^{-t}
 \end{aligned}$$

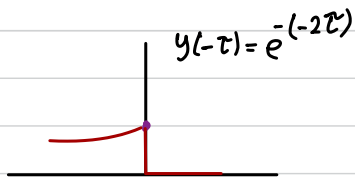


$$\therefore y(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-t} & t \geq 0 \end{cases}$$

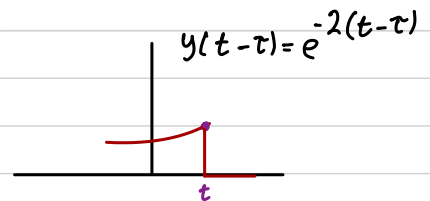
Example: Given two continuous time signals $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exists for $t > 0$, the convolution $z(t) = x(t) * y(t)$



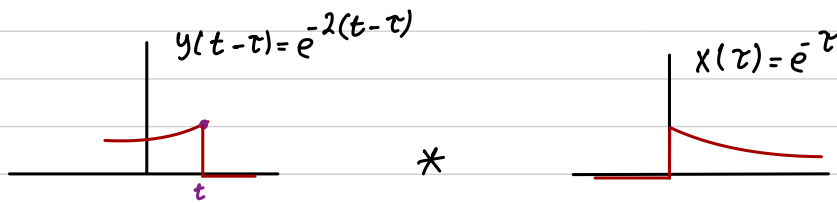
1 $\rightarrow$ folding



2 $\rightarrow$ shifting



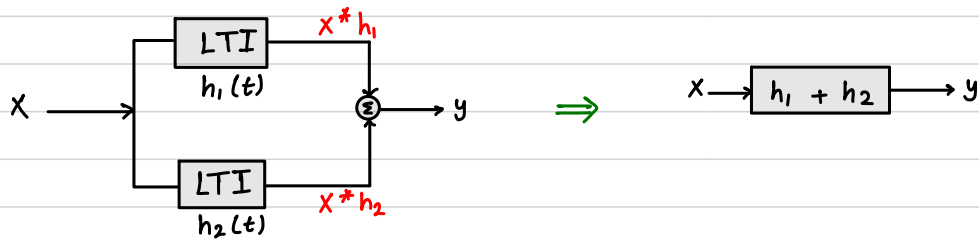
3 $\rightarrow$ Multiplying



$$\begin{aligned} y(t) &= \int_0^t e^{-\tau} \cdot e^{-2(t-\tau)} d\tau \\ &= \int_0^t e^{\tau} \cdot e^{-2t} d\tau \\ &= e^{-2t} \int_0^t e^{\tau} d\tau = e^{-2t} (e^t - 1) \\ &= e^{-t} - e^{-2t} \end{aligned}$$

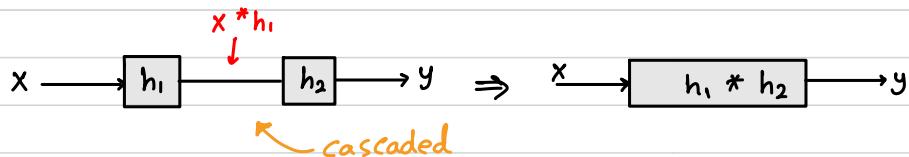
Interconnections of LTI systems

1. Distributive property



$$\begin{aligned} \therefore y &= (x * h_1) + (x * h_2) \\ &= x * (h_1 + h_2) \end{aligned}$$

2. Associative property



$$\begin{aligned} \therefore y &= x * h_1 * h_2 \\ &= (x * h_1) * h_2 \\ &= x * (h_1 * h_2) \end{aligned}$$

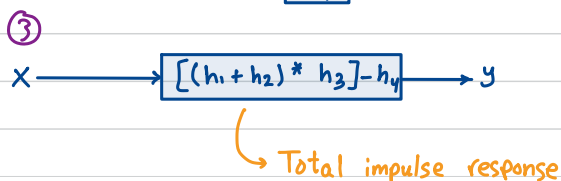
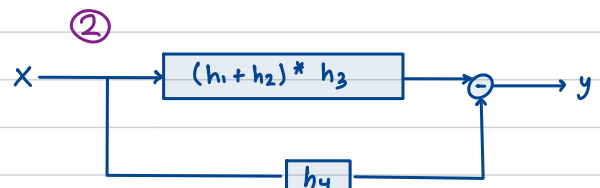
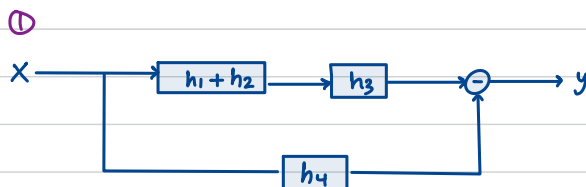
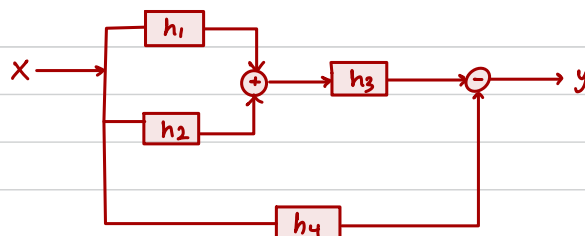
3. Commutative property

$$x * h = h * x$$

4. $\delta(t) * h(t) = h(t)$

$$\delta(t) * \text{something} = \text{something}$$

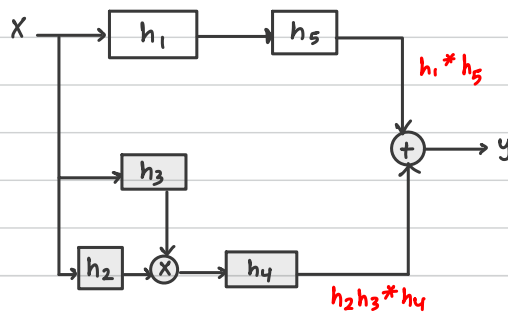
Example:



$$y = ([(h_1 + h_2) * h_3] - h_4) * x$$

response.

YEARS: In the system shown below. let $h_1(t) = h_2(t) = \delta(t-1)$, $h_3(t) = \text{Sign}(t)$, $h_4(t) = u(t)$, $h_5(t) = u(t-1)$. The total impulse response $h(t)$ of the system is?



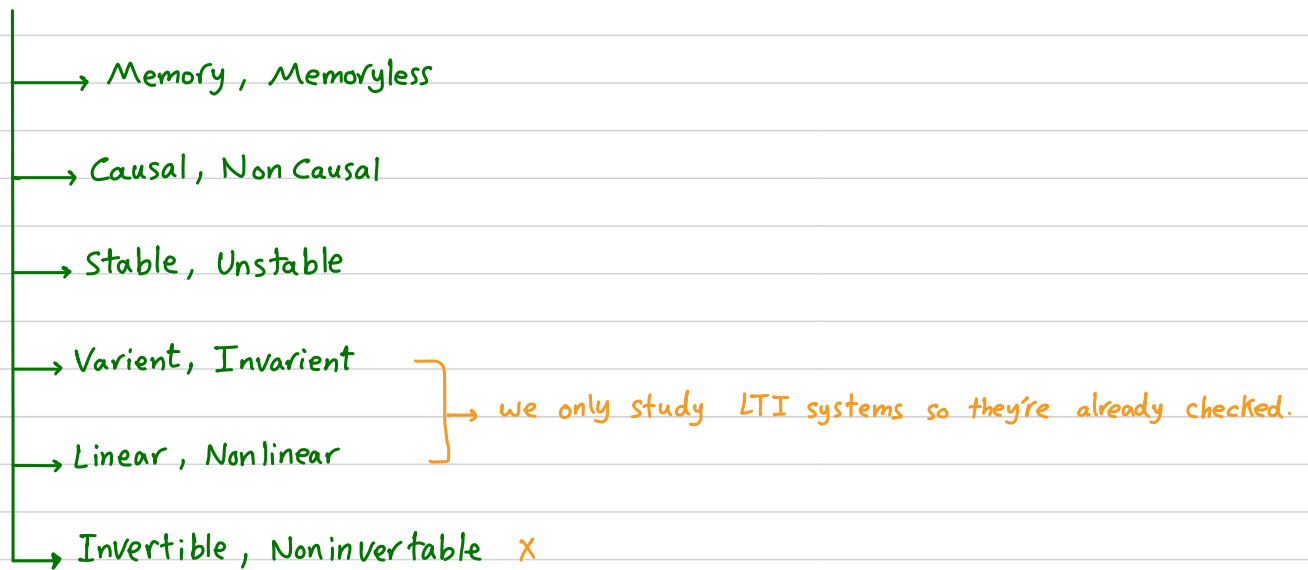
$$y = h_1 * h_5 + h_2 h_3 * h_4$$

$$= \delta(t-1) * u(t-1) + \delta(t-1) \text{Sign}(t) * u(t)$$

$$= \delta(t-1) * u(t-1) + u(t) * \delta(t-1)$$

$$y = u(t-2) + u(t-1)$$

LTI System properties



1. Memory, Memoryless

→ LTI is said to be Memoryless iff $\Rightarrow$ its impulse response is delta function.
 $h(t), h[n]$ $\hookrightarrow$ Time constant

$$h[n] = c \delta[n]$$

$$h(t) = c \delta(t) \quad ; c: \text{constant}$$

$$h(t) = 30 \delta(t) \quad \text{Memoryless}$$

$$h[n] = \delta[n] + 1 \quad \text{Memory}$$

$$h[n] = \sin[n+7] \delta[n] \quad \text{Memoryless}$$

$\hookrightarrow \sin[0+7]: \text{constant}$

2. Causal, Non Causal

$$\text{LTI causal} \rightarrow \begin{cases} h[n] = 0, & n < 0 \\ h(t) = 0, & t < 0 \end{cases}$$



Causal



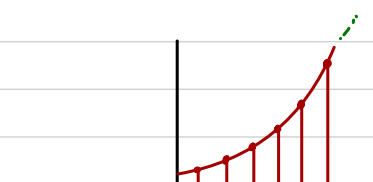
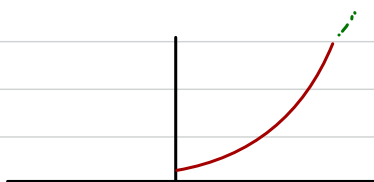
Non Causal

$\leftarrow$ no negative values

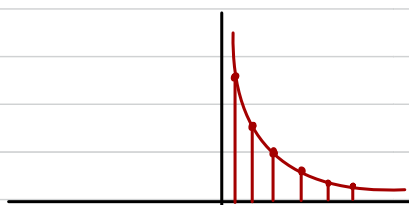
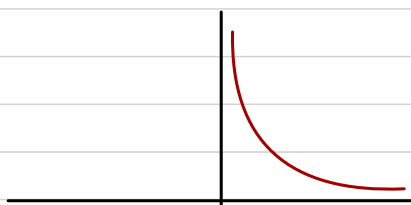
3. Stability

CT: $\int_{-\infty}^{\infty} h(\tau) d\tau \neq \pm \infty \rightarrow \therefore \text{stable}$

DT: $\sum_{k=-\infty}^{\infty} h[k] \neq \pm \infty \rightarrow \therefore \text{stable}$

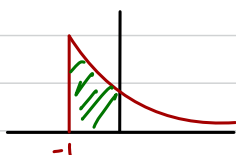


goes to $\infty \therefore$ unstable.



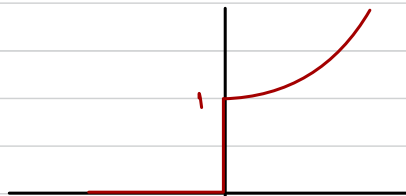
goes to 0 $\therefore$ stable.

Example: $h(t) = 2e^{-4t} u(t+1)$



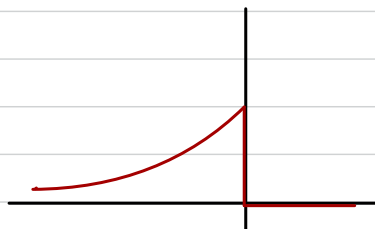
memory ; Because it's not in the format of $c\delta(t)$
 non-causal ; Because there are negative values.
 stable ; Because it goes to 0.

Example: $h(t) = 3.2e^{4t} u(t)$



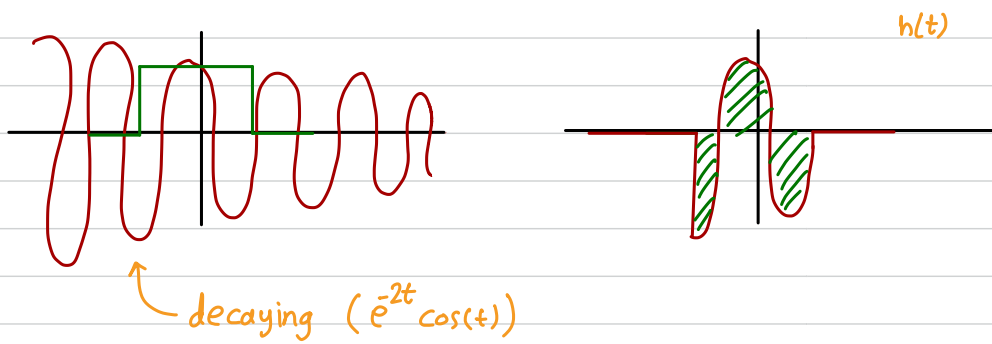
memory ; Because it's not in the format of $c\delta(t)$
 causal ; Because there are no negative values.
 Unstable ; Because it goes to ∞ .

Example: $h(t) = e^{4t} u(-t)$



memory ; Because it's not in the format of $c\delta(t)$
 non-causal ; Because there are negative values.
 stable ; Because area is going to be a number (not ∞).

Example: $h(t) = e^{-2t} \cos(t) \text{ rect}(\frac{t}{2})$



memory ; Because it's not in the format of $c \delta(t)$

non-causal ; Because there are negative values.

stable ; Because area is going to be a number (not ∞).

YEARS: let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (\frac{1}{2})^n u[n]$ and $g[n]$ is a causal sequence. if $y[0] = 1$ and $y[1] = \frac{1}{2}$, then $g[1] = ?$

$$y[n] = h[n] * g[n] = \sum_{k=-\infty}^{\infty} h[k] g[n-k]$$

$$y[n] = h[0]g[n] + h[1]g[n-1] + h[2]g[n-2] \dots$$

$$y[0] = 1 = h[0]g[0] + h[1]g[-1] + h[2]g[-2] \dots \quad g[-1] = g[-2] = 0 \quad \leftarrow \text{causal}$$

$$y[1] = 0.5 = h[0]g[1] + h[1]g[0] + h[2]g[-1] \dots$$

$$1 = h[0]g[0]$$

$$g[0] = \frac{1}{h[0]} = \frac{1}{\frac{1}{4}} = 4$$

$$0.5 = h[0]g[1] + h[1]g[0]$$

$$0.5 = \frac{1}{4}g[1] + \frac{1}{4} \cdot 4$$

$$g[1] = -2$$