



منطق رقمي و الكترونيات رقمية

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للاطالبة المبدعة
لينا الزيتاوي

إرادة - ثقة - تغيير

System \rightarrow Discrete (digital)

\rightarrow Analog

number systems:-

1- Decimal ($10 \in \{0, 1, 2, \dots, 9\}$) $\text{base } 10$ $\in \{0, 1, \dots, \infty\}$

2- Binary ($2 \in \{0, 1\}$) $0_{(2)}, 1, 10, 11, 100, 101, 111, 1000, 1001, 1010, 1100, 1101, 1110$

3- Octal (8)

$\in \{0, 1, 2, 3, 4, 5, 6, 7\}$ $0, 1, 2, 3, 4, 5, 6, 7, 10, 11, \dots$

4- Hexadecimal

$\in \{0, 1, 2, \dots, 9, A, B, \dots, F\}$ $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, \dots$

** conversions to Decimal

A. From Binary $\rightarrow \sum \leq 2^n * x$; n: Placement

x: value at n.

ex:-

$$\begin{array}{c} 2^2 \ 2^1 \ 2^0 \\ 101_{(2)} \end{array} \rightarrow 2^2 * 1 + 0 * 2^1 + 1 * 2^0$$

$$4 + 0 + 1 = 5_{(10)}$$

note:-

$$2^0 = 1$$

ex:- $2^0 \quad 2^{-1} \quad 2^{-2}$
 $0.11_{(2)} \rightarrow 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$
 $= 0 + \frac{1}{2} + \frac{1}{4}$
 $= \frac{3}{4}_{(10)}$

note
 اي اشارة قبل للفاصلة بعد امان
 0 عن

اي اشارة بعد للفاصلة بعد ا
 -1 عن

ex:- $2^2 \quad 2^1 \quad 2^0 \quad 2^{-1} \quad 2^{-2} \quad 2^{-3}$
 $111.011_{(2)} \rightarrow 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 1 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$
 $4 + 2 + 1 + 0 + \frac{1}{4} + \frac{1}{8} = 7 \frac{3}{4}$

B) From Octal $\rightarrow \sum 8^n \times X$, n : Placement
 X : value ~~at~~ at n

ex:- $8^2 \quad 8^1 \quad 8^0$
 $573_8 \rightarrow 8^2 \times 5 + 7 \times 8^1 + 3 \times 8^0 =$
 $320 + 56 + 1 \times 3 =$
 $320 + 56 + 3 = 379_{(10)}$

$8^0 \quad 8^{-1} \quad 8^{-2}$
 $0.36_8 \rightarrow 8^0 \times 0 + 8^{-1} \times 3 + 8^{-2} \times 6$
 $0 + \frac{3}{8} + \frac{6}{64} = \frac{15}{32}_{(10)}$

$8^1 \quad 8^0 \quad 8^{-1} \quad 8^{-2}$
 $23.42_8 \rightarrow 8^1 \times 2 + 3 \times 8^0 + 4 \times 8^{-1} + 2 \times 8^{-2}$
 $16 + 3 + \frac{4}{8} + \frac{2}{32}$
 $19 + \frac{17}{32} = 19 \frac{17}{32}$

C. From Hexadecimal $\rightarrow \sum 16^n * X$, $n = \text{Placement}$
 $X = \text{Value at } n.$

ex: $DAD_{(16)} \rightarrow 16^2 * 13 + 16^1 * 10 + 16^0 * 13$

$= 3501$

#	A	B	C	D	E	F
D	10	11	12	13	14	15

$0.C8_{(16)} \rightarrow$

$16^0 * 0 + 12 * 16^{-1} + 8 * 16^{-2}$

same process

Just بفر Base

$= 0 + \frac{8}{4} + \frac{1}{32}$

$1A.D8 \rightarrow 16^0 * 10 + 1 * 16^1 + 13 * 16^{-1} + 8 * 16^{-2}$

octan and Hexa
are extensions
to Binary.

$16 + 10 + \frac{26}{32} + \frac{1}{32} = 26 \frac{27}{32}$

** conversion From Decimal.

A. To Binary $\rightarrow X_{(10)} \rightarrow 1 < X < 0 * 2$
 $1 * 1 > 1 \div 2$

ex: $6_{(10)} \rightarrow 110_{(2)}$

$2 \div 6$ (remainder) \rightarrow in 0/1 الباقي

3 | 0 \rightarrow LSB ; ~~last~~ least significant Bit to
 1 | 1
 0 | 1 MSB ; Most significant Bit From

بوقف عند
zero

B. To Hexadecimal $\rightarrow X_{16} \rightarrow 1 < X < 0 \quad * 16$

$$|X| > 1 \div 16$$

ex- $190_{(10)} \rightarrow BE_{(16)} \rightarrow \text{chars}$

$16 \div 190$ Remainder:

11

14 $\rightarrow E$

\rightarrow LSD

EX1 + B * 16

14 + 11 * 16

0

11 $\rightarrow B$

\rightarrow MSD

190

conversion From Binary to octal

EX: 101 . 110

5 . 6

EX: 011 . 110

3 . 6

EX: 001101 . 110100

15 . 64

Binary	octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

conversion From Octal to Binary :-

EX :-

37,02 ₍₈₎ → Binary

011 111 . 000 010 ₍₂₎

conversion From Binary to Hexad

least significant bit

EX :- 0011011 . 11101000 ₍₂₎
1 B . E 8

MSB	Binary	Hexa
	0 0 0 0	0
	0 0 0 1	1
	0 0 1 0	2
	0 0 1 1	3
	0 1 0 0	4
	0 1 0 1	5
	0 1 1 0	6
	0 1 1 1	7
	1 0 0 0	8
	1 0 0 1	9
	1 0 1 0	A
	1 0 1 1	B
	1 1 0 0	C
	1 1 0 1	D
	1 1 1 0	E
	1 1 1 1	F

conversion From Hexa to Binary.

EX :-

ABED . DAD _(H) → Binary.

1010 1011 1110 1101 . 1101 1010 1101 ₍₂₎

Conversion From DECimal to Binary.

EX:- 0,25 → Binary

0,25		= 0.01 ₍₂₎
0,5	0	more.
0,0	1	less

EX:-

0,3 → Binary

1,2 = 0.0100₍₂₎

0,3		
0,6	0	هو زي نسبة الورد
0,2	1	دور زي بوجف
0,4	0	عند رقم 000
0,8	0	بطلبو
0,16	1	

EX:- هونو بنسجل كل رقم بحال

22,125
(10)

22

11 0 less

5 1

2 1

1 0

0 1 most

22 → 10110

22,125 → 10110,001

24,125 0,125

0,25 0 most

0,5 0

0,1 00 → 1 less

conversion From Decimal to Octal

EX:-

مثال ۱۰۰ کی ۸ کی شکل

۱۰۰ (10) → Octal
 ۱۰۰ کی ۸ کی شکل

$$100_{(10)} \rightarrow 144_{(8)}$$

8 ÷	100	
	12	4 least s d
	8	4
	0	1 most s digit

انگیزا کی شکل
 اسی

$$1 \times 8^2 + 4 \times 8^1 + 4 \times 8^0 = 100 \checkmark$$

EX:-

0,06125 (10) → Octal

	0,5	0 most
	0	4 least

$$0,06125_{(10)} \rightarrow 0,04_{(8)}$$

EX:-

366,2 (10) → Octal

مثال ۳۶۶ کی ۸ کی شکل

8 ÷	366	
	45	6 least
	5	5
	0	5 most

$$366_{(10)} \rightarrow 556$$

$$366,2_{(10)} \rightarrow 556,364$$

8 ×	0,2	
	0,6	1 most
	0,8	4
	0,4	digit
	0,2	3

conversion From Decimal to Hexa decimal

EX 2-

$\div 16$ 170 \rightarrow Hexa
(10)

170

170 \rightarrow $16^1 16^0$
AA

10 ~~10~~ A

0 ~~10~~ A

EX 3

0.03125

\rightarrow Hexa

(10)

$16^{-1} 16^{-2}$

$\times 16$

0.03125 \rightarrow 0.08₍₁₆₎
(10)

0.03125

0.5

0

0

8

EX 4-

1.1 \rightarrow Hexa

$\div 16$

(10)

بقسم الرقم

1.1 \approx 1.19₍₁₆₎

1

0.1

0.6 1

0.6 9

0.6 9

⋮
1

* K	1000	2^{10}	1024
M	1000,000	2^{20}	$1024 * 1024$
G	1000,000,000	2^{30}	$1024 * 1024$

* First complement for ~~Octan~~ Octan and Hexa are extential to Binary.

→ First complement for → one's complement - 1st complement

0100 → 1011
 11010 → 00101

→ second complement - two's complement - 2nd complement.

2's complement = 1st comp + 1

Now, Binary addition.

0	1	1	1	00
+	0	+	1	0111
0	0	1	1	0110 +
$0_{(2)}$	$1_{(2)}$	$10_{(2)}$	$11_{(2)}$	0101

Find the second complement →

0100 → 1st comp + 1

000
 1011
 1 +
 1110

110011 → 1st com
 001100
 1 +
 001101

To Have the 2nd complement with one step:

we use one step
because the controller
coef has infinity loop
and this cause
to unstable system.

① Having the first (one 1) * going from Right to
leading one

② ~~leave~~ leave all the numbers before it the same

③ make other number to 1st complement.

0100 → 1100
↑
leading
one

Binary coded Decimal (BCD)

bio no

Decimet	BCD	exg- Express the Following using binary value and BCD		
0	0 0 0 0			
1	0 0 0 1			
2	0 0 1 0			
3	0 0 1 1	value	Binary	BCD
4	0 1 0 0	3 ₍₁₀₎	11 ₍₂₎	0011
5	0 1 0 1	5 ₍₁₀₎	101 ₍₂₎	0101
6	0 1 1 0	6 ₍₁₀₎	110 ₍₂₎	0110
7	0 1 1 1	7 ₍₁₀₎	111 ₍₂₎	0111
8	1 0 0 0	8 ₍₁₀₎	1000 ₍₂₎	1000
9	1 0 0 1	9 ₍₁₀₎	1001 ₍₂₎	1001

10	20
5 0	10 0
2 1	5 0
1 0	2 1
0 1	1 0
	0 1

Gray-code Decimal equivalent

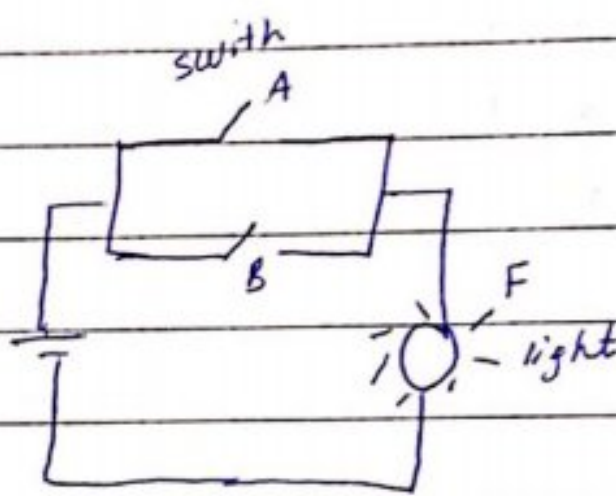
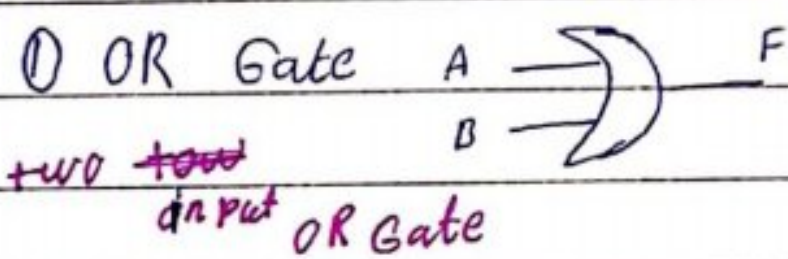
0000	0
0001	1
0011	2
1001	4
1000	15

ex:- express the following number in Gray and BCD code

value	BCD code	Gray
20 ₍₁₀₎	0010 0000 BCD	0011 0000 Gray
10 ₍₁₀₎	0001 0000 BCD	0010 0000 Gray

Binary

Basic Gates



truth table

A	B	F	0 → OFF	1 → ON	$F = A \oplus B$
OFF	OFF	OFF			↓ OR operator.
OFF	ON	ON			
ON	OFF	ON			
ON	ON	ON			



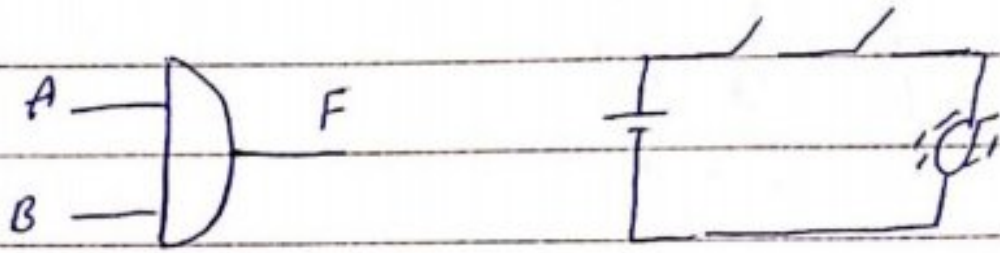
$F = A + B + C$

Three input OR Gate.

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

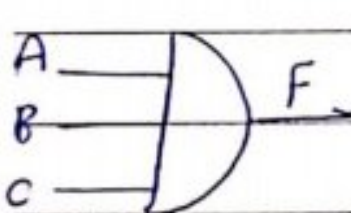
Five Apple

② AND Gate



truth table

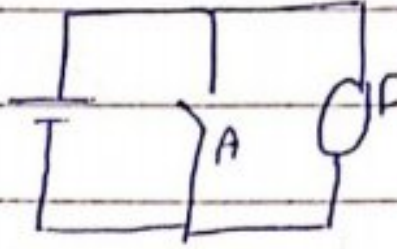
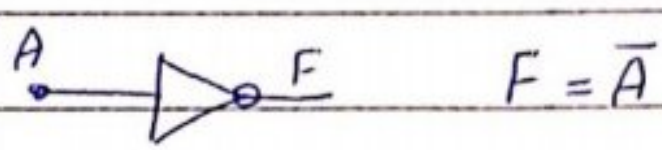
A	B	F	$F = A \odot B$
0	0	0	↳ AND operator
0	1	0	
1	0	0	
1	1	1	



A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

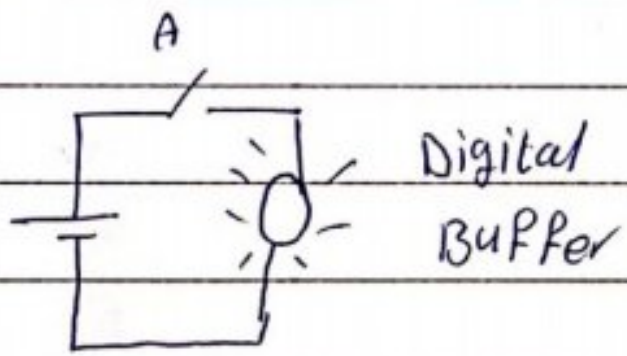
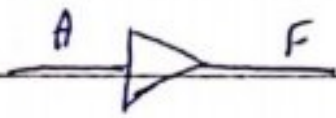
$F = A \cdot B \cdot C$

③ NOT Gate inverter

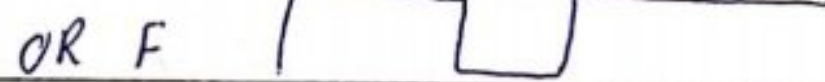
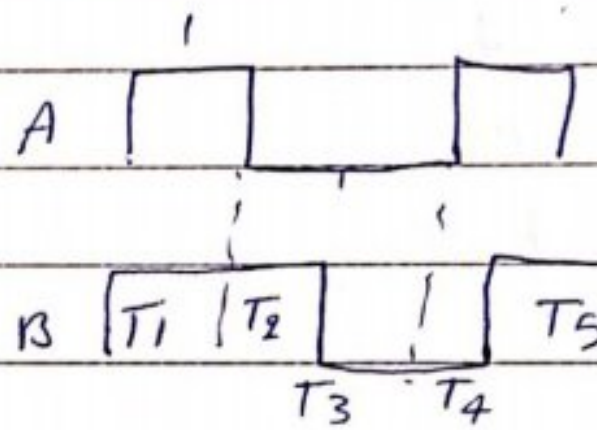


A	F
0	1
1	0

④ Buffer Gate



A	B
0	0
1	1



7 March 2024.

* Boolean Theorems and Properties.

$$S = \{0, 1\}$$

(A) Closure

(1) closure with respect to the OR operator :- "+"

(2) " " " " " " AND " " " " " "

(B) Identity element

(1) 0 → "+"

X digital variable (0, 1)

$$x + 0 = 0 + x = x \quad \langle\langle \text{OR} \rangle\rangle$$

(2) 1 → "."

$$x \cdot 1 = 1 \cdot x = x \quad \langle\langle \text{AND} \rangle\rangle$$

(C) Commutative

$$x + y = y + x$$

$$x \cdot y = y \cdot x = yx$$

y and x

(D) Distributive

$$x \cdot (y + z) = x \cdot y + x \cdot z$$

$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

$$\textcircled{e} \quad x + \bar{x} = 1$$

$$x \cdot \bar{x} = 0$$

↓ Nrl

Ⓝ EX in the De Morgan

$$\begin{aligned} \underline{1} \quad \overline{A+B} &= \bar{A} \cdot \bar{B} \\ &= \bar{A} \cdot \bar{B} = \bar{A} \bar{B} \end{aligned}$$

Ⓣ De Morgan Theorem :-

$$(x+y)' = x' \cdot y'$$

$$\underline{2} \quad \overline{\bar{A} \bar{B}} = \bar{\bar{A} + \bar{B}} = A + B$$

$$(x \cdot y)' = x' + y'$$

$$\underline{3} \quad \overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

$$\underline{4} \quad \overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$

ⓑ Absorption

$$x + xy = x$$

$$x(1+y) =$$

$$x(1) = x$$

$$\begin{array}{c} \xrightarrow{x \text{ جازم}} \\ \underbrace{x + xy + xy}_{\text{الاصح}} \end{array}$$

$$x(x+y) = x + xy = x$$

$$x * x = x$$

$$\begin{aligned} \underline{5} \quad \overline{A(B+C)} &= \bar{A} + \overline{B+C} \\ &= \bar{A} + (\bar{B} \cdot \bar{C}) \\ &= \bar{A} + \bar{B} \bar{C} \end{aligned}$$

() الإداوية

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10/March/2024

$$* \overline{AB + CD} = \overline{A + B + C + D}$$

$$* \overline{AB + CD} = (\overline{AB + CD})' = \overline{AB} \cdot \overline{CD}$$
$$(\overline{A + B}) \cdot (\overline{C + D})$$

$$* \overline{(A + B)(C + D)} = \overline{(A + B)} + \overline{(C + D)}$$

$$= (\overline{A} \cdot \overline{B}) + (\overline{C} \cdot \overline{D}) = (\overline{A} \cdot B) + (C \cdot \overline{D})$$

Properties and theorem :-

$$\textcircled{a} A(A + B) = \underline{AA} + AB = A + AB = A(1 + B)$$
$$= A(1) \quad 1$$
$$= A$$

$$\textcircled{b} A(\overline{A} + AB) = \underline{A\overline{A}} + AAB$$
$$= 0 + AB$$
$$= AB$$

$$\textcircled{c} BC + B\overline{C} = B(\underline{C + \overline{C}}) = B(1) = B$$
$$1$$

$$\textcircled{d} A(A + \overline{A}B) = \underline{AA} + \underline{A\overline{A}B} = A + 0 = A$$
$$A \quad 0$$

Five Anshu

Number representation:-

1 sign - magnitude representation

Ex:- Express the following number sign - mag rep - in bit format

a) +3 (10) 0 0 1 1

"0" -> +
"1" -> -

b) -3 (10) 1 0 1 1

c) -6 (10) 1 1 1 0

d) +8 (10) can't be rep in 4 bit bit 4 bit 8

e) -13 (10) 1 0 0 0 1 1 0 1

2 First complement representation.

Positive as is

negative 1's complement

EX- Express the following numbers in 1'st complement rep- in 4 bit format

(a) +3 ~~00100~~ 0011

(b) -3 1100
 +6 0110

ہونے پر بغلیے الرقم

(c) -6 1001

③ Two's complement Representation-

Positive as is

negative 2's complement

EX-

(a) +3 0011

عند اول واحد من اليمين

(b) -3 1101

بوكيف ابي عدد وبتبج
الاشارة

Ⓟ

Homework

Ranges of 2's complement, no sign-mag.:-

$$-(2^{n-1}); + (2^{n-1}-1)$$

CH1, 8, 9, 12
14, 16, 17

CH2, 5, 7, 11
14, 16, 21

Ranges of 1's comple. repr + sign-mag.

$$-(2^{n-1}-1) : (+ (2^{n-1}-1))$$

22, 23, 24
31, 32, 34
37, 39, 42
43, 45, 47

n: number of bit's

50, 51, 54

ex: For 4-bit

CH3: 1, 2, 5, 8
12, 18, 20
22, 33

$$-(2^{4-1}) : + (2^{4-1}-1) \text{ 2's}$$

$$-8 : +7 \text{ 2's}$$

$$-7 : +7 \text{ 1's + S-M}$$

CH4: 4, 5, 7
9, 10, 12
13, 17, 20, 22
31, 34, 40
43, 47
50

For 8-bit

$$-128 : +127 \text{ 2's}$$

$$-127 : +127 \text{ 1's, S-M}$$

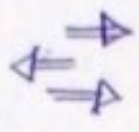
CH5: 2, 5, 10, 12
14, 15, 28, 31
32, 34

EX: Express the following using 8-bit format in:

(a) sign - magnitude representation

(b) First complement repr

(c) second " "



① $+33_{(10)} \rightarrow$ Binary (2)

33
 16 1 LSR
 8 0
 4 0
 2 0
 1 0
 0 1 mss

$+33 = 100001_{(2)}$

Ⓐ 00100001
 ↳
 8bit

Ⓑ 1's complement
 00100001

Ⓒ 2's complement
 00100001 ليس موجب
 موجب

Ⓐ $-33_{(10)}$

Ⓐ sign-mag. repr

1010001
 -

Ⓑ 1's complement repr
 11011110

Ⓒ 2's "
 11011111 ليس موجب
 negative.

Ⓒ $-99_{(10)}$

99
 49 1
 24 01
 12 0
 6 0
 3 0
 1 1
 0 1

1100011 Ⓐ sign magnitude
 1100011

Ⓑ 10011100

Ⓒ 10011101

EX: Represent $-8_{(10)}$ in 2's complement in 4 bit Format

$$-2^{n-1} : +(2^{n-1}-1)$$

$$-8 : +7$$

1000

$$8 = 1000$$

(10) (2)

EX: Find the decimal value for the 1's complement representation number

a) $0111 = +7_{(10)}$

$$\begin{array}{r} 1010 = -5_{10} \\ 0101 \end{array}$$

$$0000 = 0$$

$$1111 = 0$$

negative zero

Two values between zero.

EX: 2's complement representation

$$0000 = 0$$

$$1111 = -1$$

$$0001$$

$$0101 = 5$$

Positive value

complement

$$1010 = -6$$

$$0110$$

subtraction using 1's compl. repr...

$$3_{10} - 2_{10} = 3 + (-2)$$

$$+ 3 \quad \overset{1}{0} \overset{1}{0} \overset{1}{1}$$

$$- 2 \quad \overset{1}{1} \overset{1}{0} \overset{1}{1} \quad 0010 \text{ comp. repr.}$$

$$\underline{1 \quad 0000}$$

$$\quad \quad \quad \underline{1}$$
$$0001$$

$$-3_{10} + 2_{10} =$$

$$-3 \quad \overset{1}{1} \overset{1}{0} \overset{1}{0}$$

$$+2 \quad \overset{1}{0} \overset{1}{0} \overset{1}{1} \overset{1}{0}$$

$$\underline{1110}$$

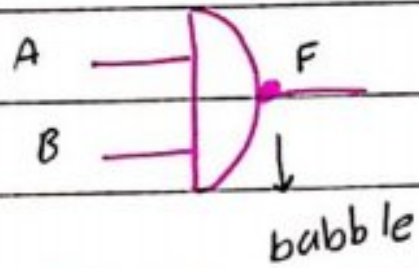
Two's complement subtraction:-

$$\begin{array}{r}
 4 \quad 4 \quad 0 \quad 1 \quad 0 \quad 0 \\
 -3 \quad -3 \quad 1 \quad 1 \quad 0 \quad 1 \\
 \hline
 1 \quad 0 \quad 0 \quad 0 \quad 1 \\
 \hline
 \text{بصلا}
 \end{array}$$

$$\begin{array}{r}
 3 \quad 3 \quad 0 \quad 0 \quad 1 \quad 1 \\
 -4 \quad -4 \quad 1 \quad 1 \quad 0 \quad 0 \\
 \hline
 1 \quad 1 \quad 1 \quad 1
 \end{array}$$

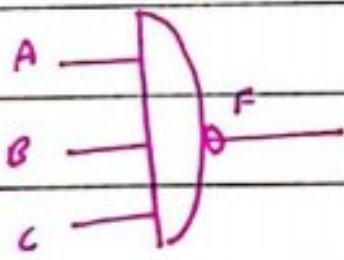
Logical Gate (continued)

⑤ NAND Gate = AND . NOT



A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

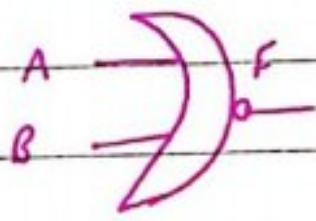
$$F = \overline{AB}$$



A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

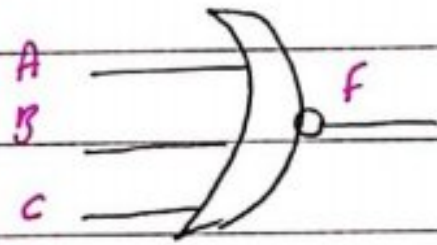
$$F = \overline{ABC}$$

⑥ NOR Gate = OR. Not



A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

$$F = \overline{A+B}$$



A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$$F = \overline{A+B+C}$$

⑦ Exclusive OR (XOR)

No more than two input

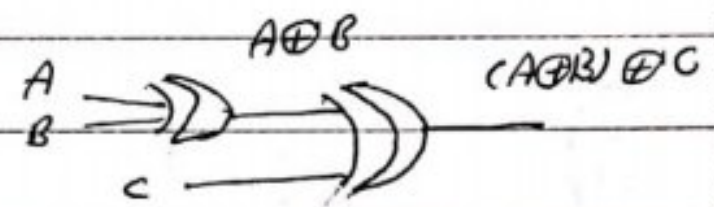


A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

$$F = A\bar{B} + \bar{A}B$$

$$= A \oplus B$$

XOR



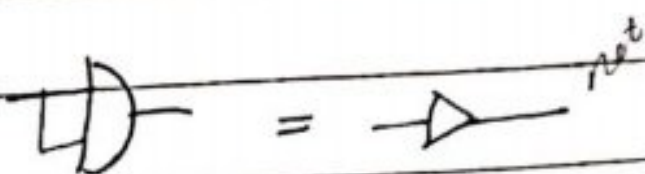
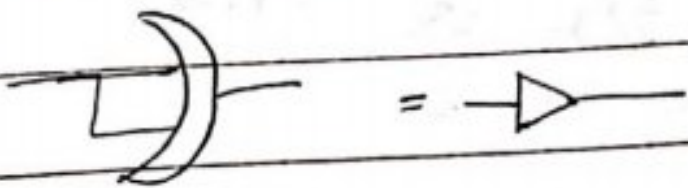
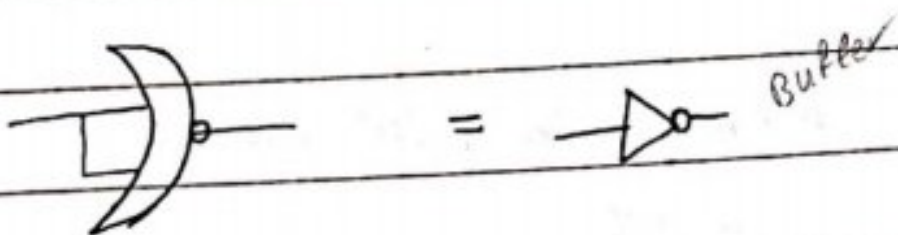
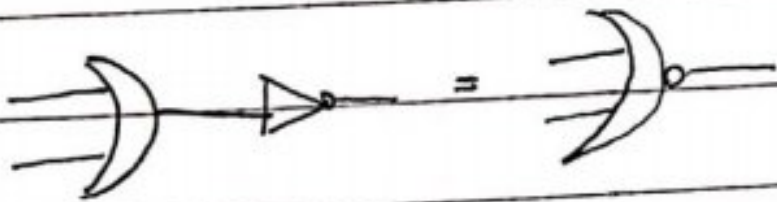
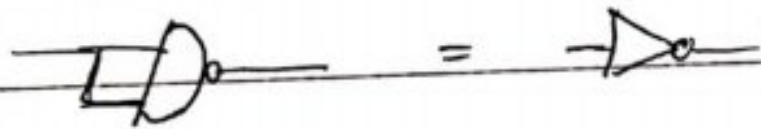
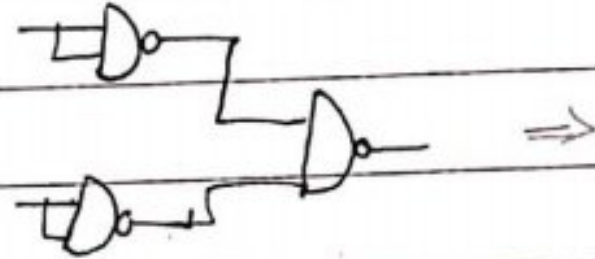
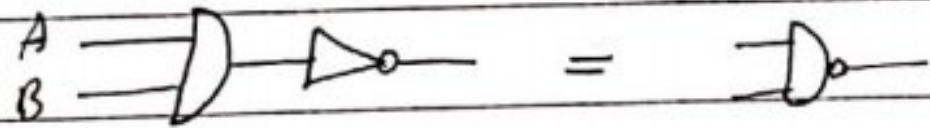
⑧ Exclusive NOR (X NOR) = XOR . NOT



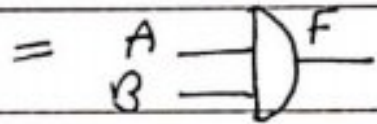
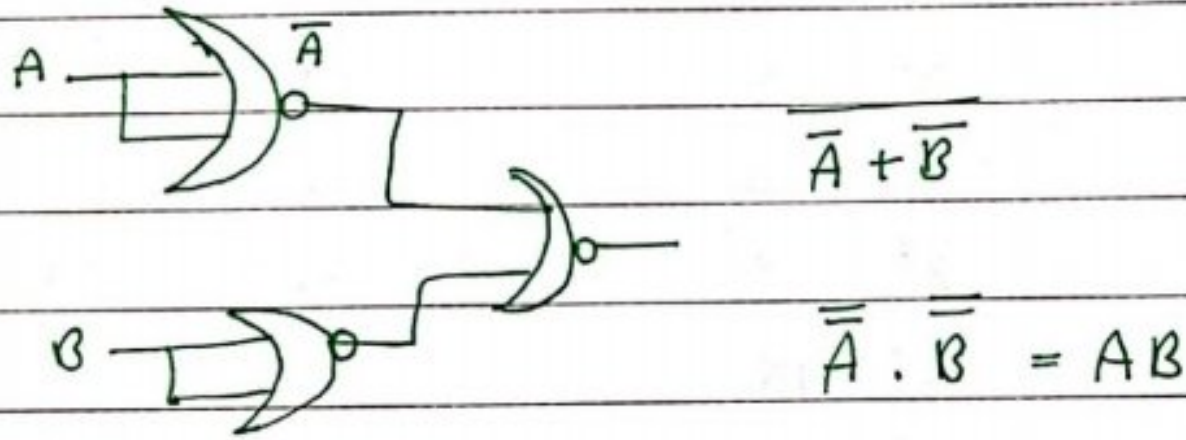
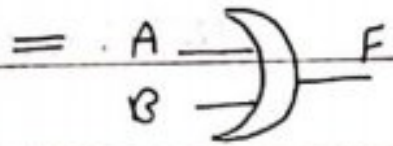
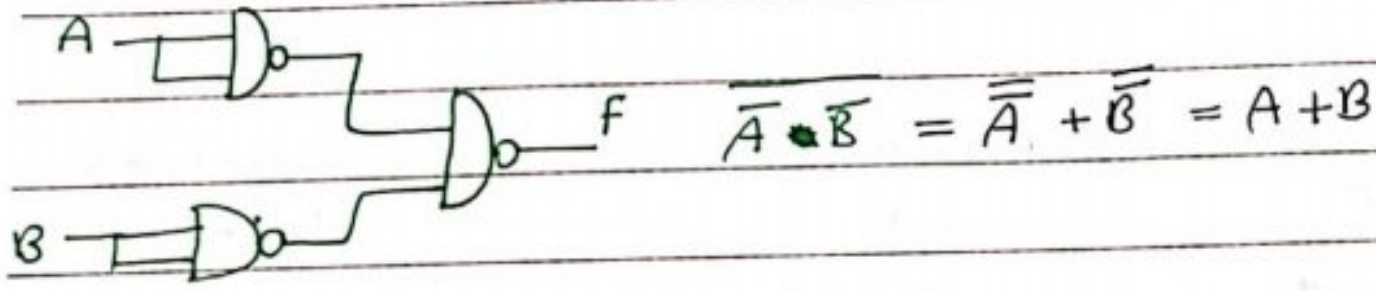
A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

$$F = AB + \bar{A}\bar{B}$$

$$= A \odot B$$



EX:-



Ex:- Determine the range for 4-bit and 8-bit in all these number representations

	sign - mag. mag. rep	1's complement rep	2's complement rep
n = 4 4 bit	-7 : +7	-7 : +7	-8 : +7
n = 8 8 bit	-127 : +127	-127 : +127	-128 : +127

$$-(2^{n-1}-1) : +(2^{n-1}-1)$$

2^0	1	2^7	128
2^1	2	2^8	256
2^2	4	2^9	512
2^3	8	2^{10}	1024
2^4	16		
2^5	32		
2^6	64		

CH2/V7: convert each of the following numbers to 8421 BCD :-
 $2^3 2^2 2^1 2^0$

a) $10_{(10)} \rightarrow$ BCD

0001 0000
 (BCD)

Express each Decimal number as an 8-bit in the 1's comple. Form:-

1) -34

3 4

$34 = 00100010_{(2)}$

1 7 0

8 1

11011101

4 0

2 0

1 0

2's comp.

0 1

2) -68

68

3 4 0

1 7 0

$68 = 01000100$

8 1

$-68 = 10111100$

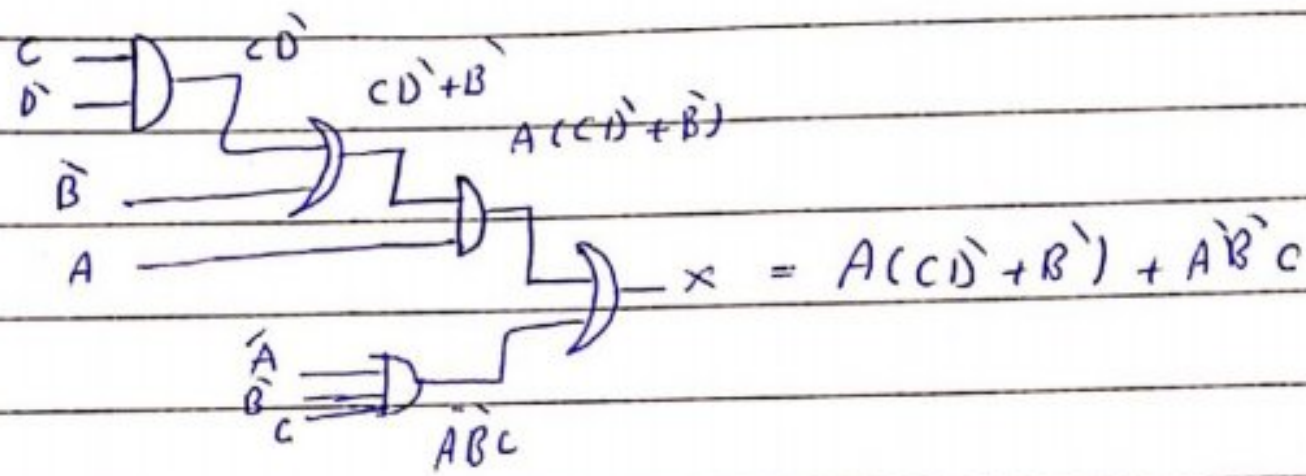
4 0

(10)

2 0

1 0

0 1



∴ इसका ड्रा

variables :-

xy AB ← Two जोड़ variable

xyz ABC

wxyz ABCD

Two - variable Karnaugh Map (K-Map)

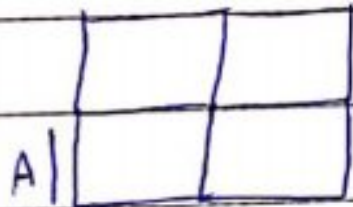
A B

0 0

0 1

1 0

1 1



Number	x	y	z	Term	Designation
0	0	0	0	$x'y'z'$	m_0
1	0	0	1	"	m_1
2	0	1	0	"	m_2
3	0	1	1	"	

Maxterm

Term	Designation
$x+y+z'$	M_0
$x+y'+z'$	M_2
$x+y'+z$	M_3

26/ March/ 2024

<< Min terms >>

<< Max term >>

Number	X	Y	Z	Term	Designation	Term	Designation
0	0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
1	0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
2	0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
3	0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
4	1	0	0	$x y'z'$	m_4	$x'+y+z$	M_4
5	1	0	1	$x y'z$	m_5	$x'+y+z'$	M_5
6	1	1	0	$x y z'$	m_6	$x'+y'+z$	M_6
7	1	1	1	$x y z$	m_7	$x'+y'+z'$	M_7

<< min term >>

<< Max term >>

X	Y	term	Res	term	Res
0	0	$x'y'$	m_0	$x+y$	M_1
1	0	$x'y$	m_1	$x+y'$	M_2
2	1	$x y'$	m_2	$x'+y$	M_3
3	1	$x y$	m_3	$x'+y'$	M_4

Ex: Implement $F(x,y) = x'y' + x'y$ in all

Possible representation

$$\Rightarrow x'(y'+y) = x' \cdot 1$$

$$F(x,y) = m_0 + m_1 = M_2 + M_3 = (x'+y) + (x'y)$$

sum of product	product of sum	1	0	1	1	X	Y	F
						0	0	1
						0	1	1
						1	0	0

Verification $(x'+y) \cdot (x'+y') = x' + xy' + x'y + yy' = x' + 0 + 0 + 1 = x'$

Five Apple

Ex: Represent the Following Functions using all possible representation:

A	B	C	F
0	0	0	0
0	0	1	1 ←
0	1	0	0
0	1	1	1 ←
1	0	0	1 ←
1	0	1	1 ←
1	1	0	0
1	1	1	0

② $F(A, B, C) = m_1 + m_3 + m_4 + m_5$

③

$\Sigma(1, 3, 4, 5)$

④

$= A'B'C + A'BC + A'B'C' + AB'C$

⑤

$F(A, B, C) = M_1 + M_2 + M_6 + M_7$

$= \Pi(0, 2, 6, 7)$

$= (A + B + C) \cdot (A'B' + C) \cdot (\bar{A} + \bar{B} + C) \cdot (A + \bar{B} + \bar{C})$

EX 2

$$F(x,y) = \overset{m_0}{x'y'} + \overset{m_1}{xy}$$

x	y	F(x,y)
0	0	1
0	1	1
1	0	0
1	1	0

EX 3 - Express $F(A,B) = A$ in truth table

non standard POS term

$$= A(B+B')$$

$$= AB + AB'$$

$m_2 \ m_3$

$$= \Sigma(2,3)$$

A	B	F(A,B)
0	0	0
0	1	0
1	0	1
1	1	1

EX 4 (a) Express $F(A,B) = (A+B) \cdot (A'+B')$

$M_1 \quad M_2$

POS \rightarrow Maxterm

(b) Express in Minterm

$$= \prod(1,3)$$

$$= \Sigma(0,2)$$

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

EX: Express $F(x,y,z) = (x+y) \cdot (x+z) \cdot y$

$$= (x+y+z) \cdot (x+y+z) \cdot (x+y+z) \cdot (x+y+z)$$

$$\cdot (x+y+z) \cdot (x+y+z) \cdot (x+y+z) \cdot (x+y+z)$$

M_0 M_1 M_2 M_3 M_4 M_5

100 1001 101

$$= \prod (0,1,2,3,4,5,6) = \sum (7) = (\prod (7)) = (\sum (0,1,2,3,4,5,6))$$

x y z F

0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

EX 2

$$F(A,B,C) = \sum (0,1,2,3,4,5,6) = 1$$

$$F(A,B,C) = \prod (0,1,2,3,4,5,6,7) = 0$$

Q. Express $F(A, B, C) = A + B^c C^c$ in all possible representation

standard, \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark non standard term

minterm

1 0 0 0 1 0

$$= \sum (2, 4, 5, 6, 7, 2, 6)$$

$$= \sum (2, 4, 5, 6, 7) = \prod (0, 1, 3) = (\sum (0, 1, 3))'$$

$$= (\prod (2, 4, 5, 6, 7))'$$

$$= A^c B^c C^c + A^c B^c C + A B^c C^c$$

$$+ A B^c C + A B C$$

$$= (A + B + C) \cdot (A + B + C') \cdot (A + B' + C)$$

Two - Variable K-map

A	B	F
0	0	0
0	1	0
1	0	1
1	1	1

	B	
A	m_0	m_1
0	0	0
1	1	1

Ex: simplify using K-map

A	B	F
0	0	0
0	1	1
1	0	0
1	1	1

	B	
A	m_0	m_1
0	0	1
1	0	1

adjacent = neighbor

$F = B$

EX: simplify

(a)

	B	
A	m_0	m_1
0	1	1
1	0	0

$F = \bar{A}$

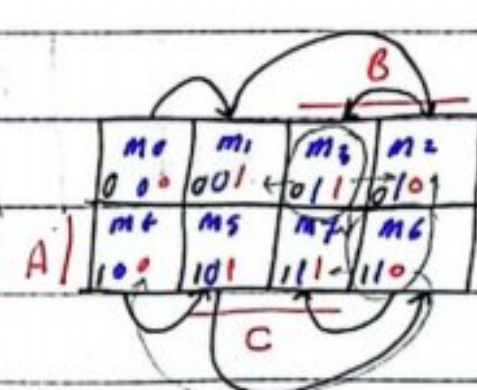
(b)

	B	
A	m_0	m_1
0	0	1
1	1	0

$F = AB' + A'B$
 $= A \oplus B$

Three - Variable K-map

A	B	C	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



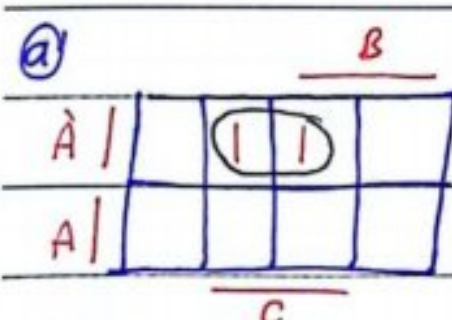
الترتيب مهم
 $A \rightarrow B \rightarrow C$
 * ترتيبهم لا يتماشى
 كلمة بنيتها

variable numbers واحد ال variable *

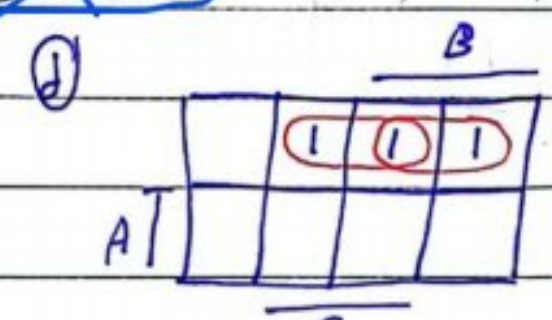
لازم
 الواحدات للسهولة
 داخل كل دائرة (كلها كبرتتها افضل)

Ex: Simplify

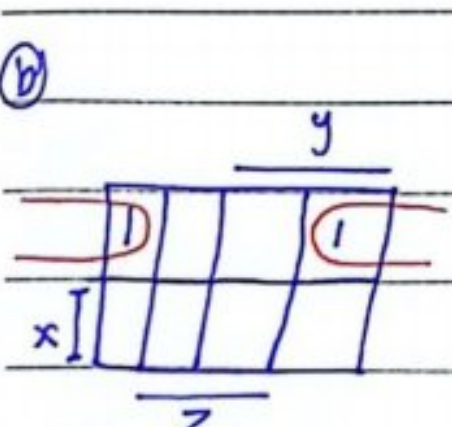
$2^0, 2^1, 2^2, 2^3, 2^4$
~~الدوائر للسهولة بنيتها~~ 1, 2, 4, 8, 16



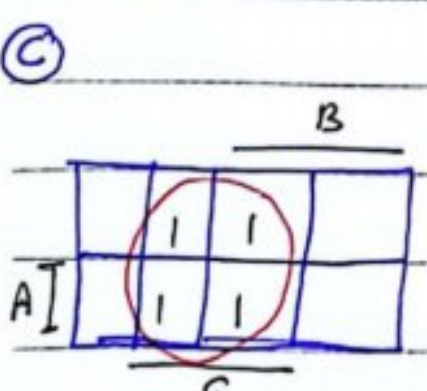
$F(A,B,C) = \bar{A} \bar{B} C$
 $= \bar{A} C$



$F(A,B,C) = \bar{A} C + \bar{A} B$



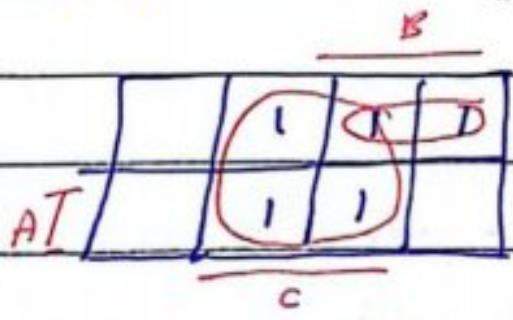
$F(x,y,z) = \bar{x} \bar{z}$



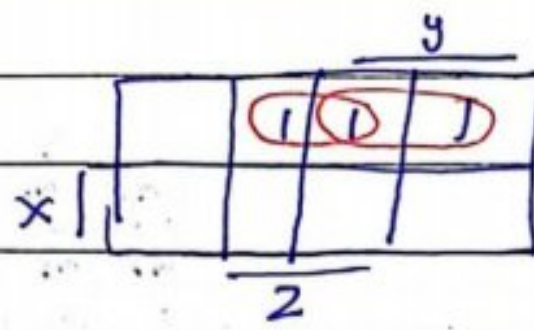
$F(A,B,C) = C$

2 April 2024

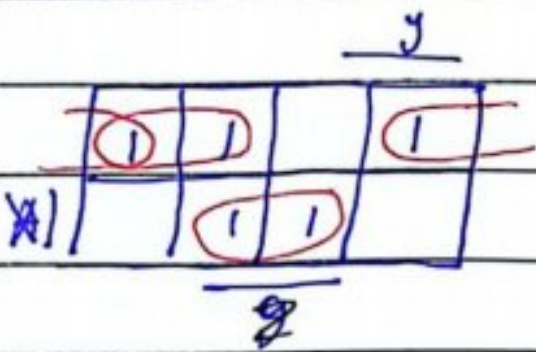
Ex:-



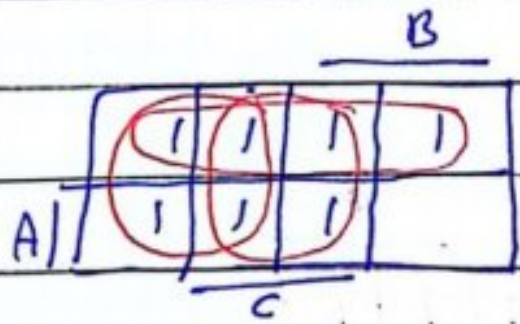
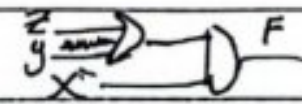
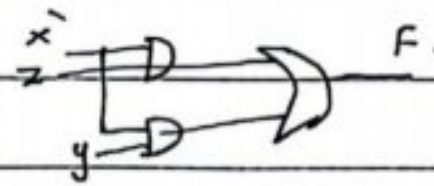
$$F = C + A'B$$



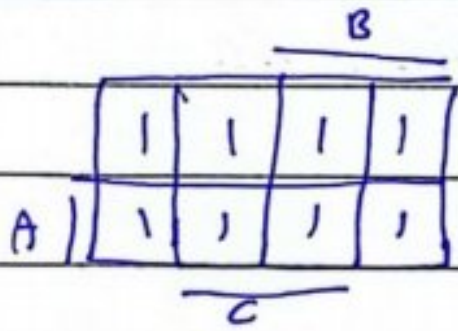
$$F = x'z + x'y = x'(z+y)$$



$$F = xy + x'z + x'y'$$



$$F = B + C + A'$$

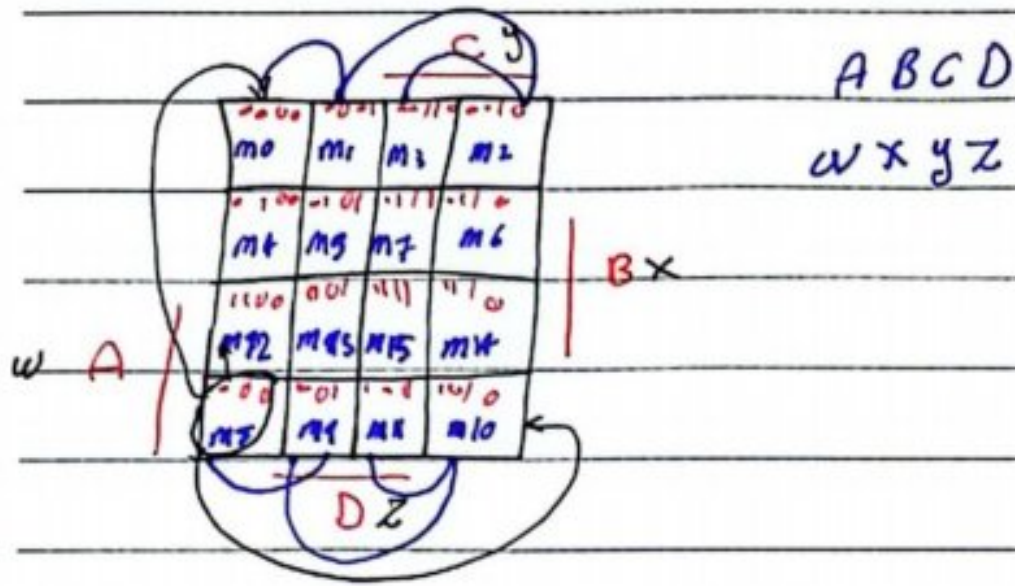


$$F = 1$$

وہو کا نئے کلمہ zero

$$F = 0$$

Four - Variable K-map 2^4

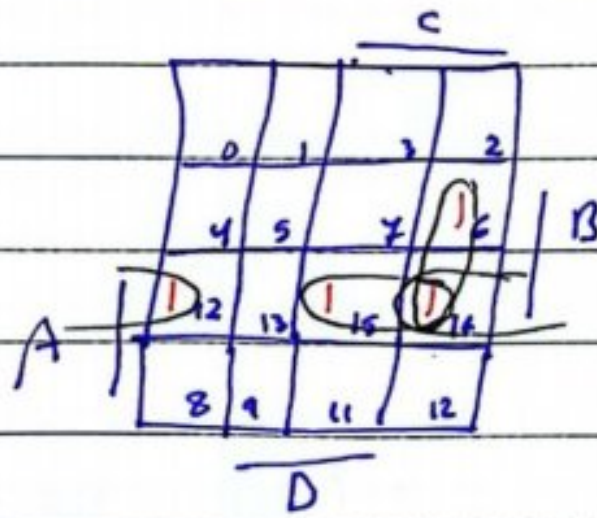


Ex:- $F(A, B, C, D)$ main meize using K-map

$$F(A, B, C, D) = ABC\bar{D} + ABC + \bar{A}BCD$$

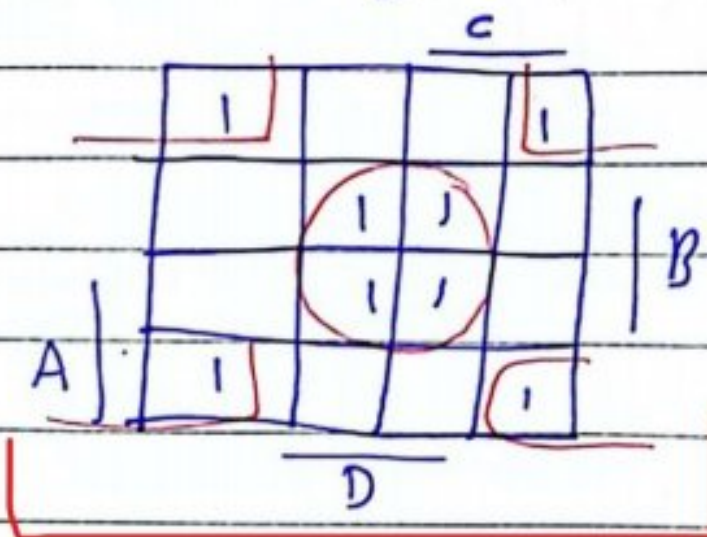
$$\begin{matrix} 1 & 1 & 0 & 0 & & 1 & 1 & 1 & & 0 & 1 & 1 & 0 \\ & & & & & m_{12} & & m_{13} + m_{14} & & & m_6 & & \end{matrix}$$

$$= \sum (6, 12, 14, 15)$$



$$F = AB\bar{D} + ABC + BCD$$

Ex:- simplify



$$F = \bar{B}\bar{D} + BD = B \odot D$$

وینے کا نتیجہ تباہل با عرف
 انہا یا XOR او XNOR

Rules for minimization

① cover all ones using circles and ~~non zero~~

must have

② circles are $2^0, 2^1, 2^2, 2^3 \dots$ ones

1 2 4 8

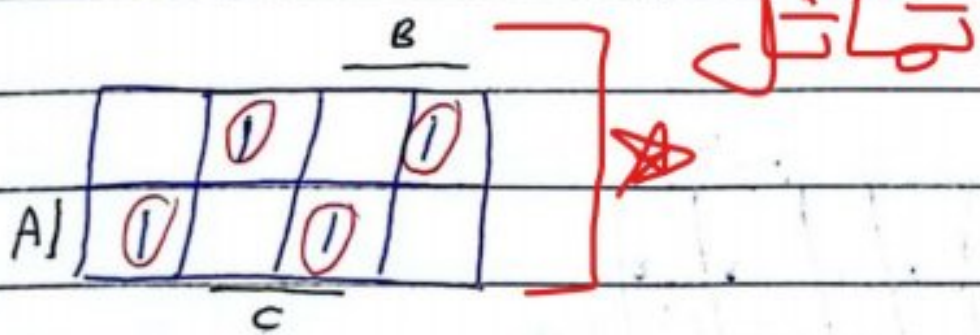
ای دائرہ زائدہ
[کل ال one's مشمولہ
دونہا اصلاً]

③ Minimize the number of circles

④ Maximize the size of each circles

لازم تکبیراً
قدر الامکان

Ex:-



hint:- $(A \odot B)' = A \oplus B$
 $(A \oplus B)' = A \odot B$

$$F = A \bar{B} \bar{C} + A \bar{B} C + ABC + A \bar{B} C$$

$$= A (\bar{B} \bar{C} + BC) + A (\bar{B} C + BC)$$

$$= A (B \odot C) + A (B \oplus C)$$

$$= A (B \oplus C)' + A (B + C)$$

$$= A \oplus B \oplus C$$



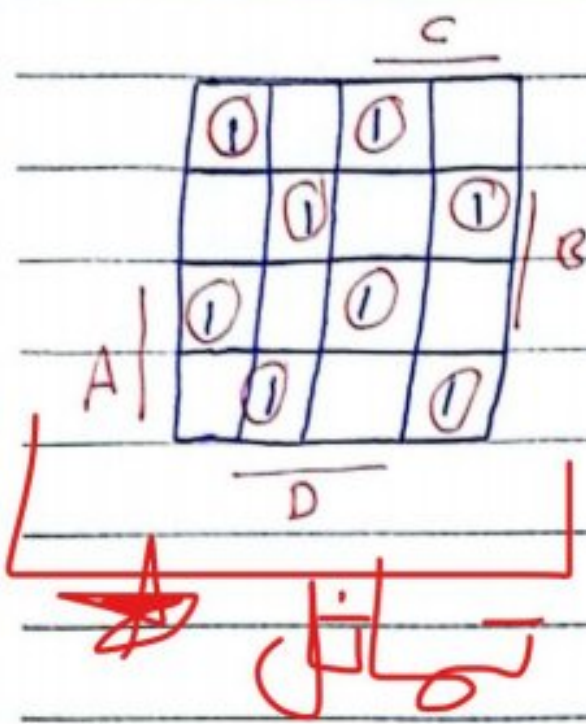
تذکرہ XOR و XNOR کا اہم

غیر 2-input

کو بولکہ اکثر لازم مستخدم

ہل چرکے

Exo- simplify



$$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABC\bar{D}$$

$$= \bar{A}\bar{B}(C\bar{D} + CD) + \bar{A}B(C\bar{D} + C\bar{D}) + AB(C\bar{D} + CD) + AB(C\bar{D} + C\bar{D})$$

$$= \bar{A}\bar{B}(C\odot D) + \bar{A}B(C\oplus D) + AB(C\odot D) + \bar{A}B(C\oplus D)$$

$$= (\bar{A}\bar{B} + \bar{A}B)(C\odot D) + (\bar{A}B + \bar{A}B)(C\oplus D)$$

$$= (A\odot B)(C\odot D) + (A\oplus B)(C\oplus D)$$

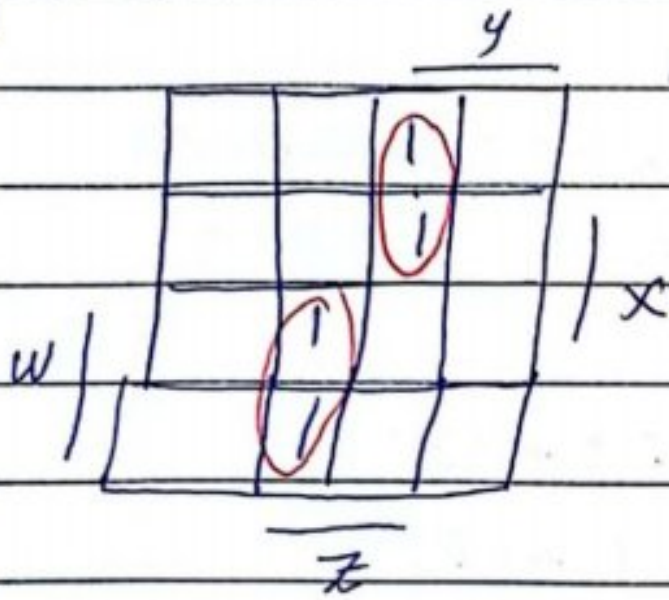
$$= (A\odot B)(C\odot D) + \overline{(A\oplus B)(C\oplus D)}$$

$$= A\odot B\odot C\odot D$$



تمثال

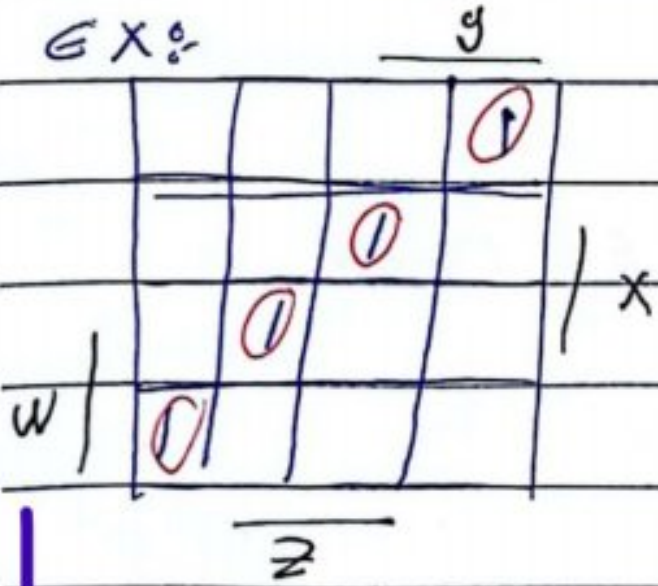
Ex:-



simplify.

$$\begin{aligned}
 F &= w \oplus w y z + w' y z \\
 &= (w y' + w z) z \\
 &= (w \oplus y) z
 \end{aligned}$$

Ex:-



$$F = \underline{A \bar{B} \bar{C} \bar{D}} + \underline{A B \bar{C} \bar{D}} + \underline{A' B C D} + \underline{A' B' C D'}$$

$$= A \bar{C} (\bar{B} \bar{D} + B D) + A' C (B D + D' \bar{D})$$

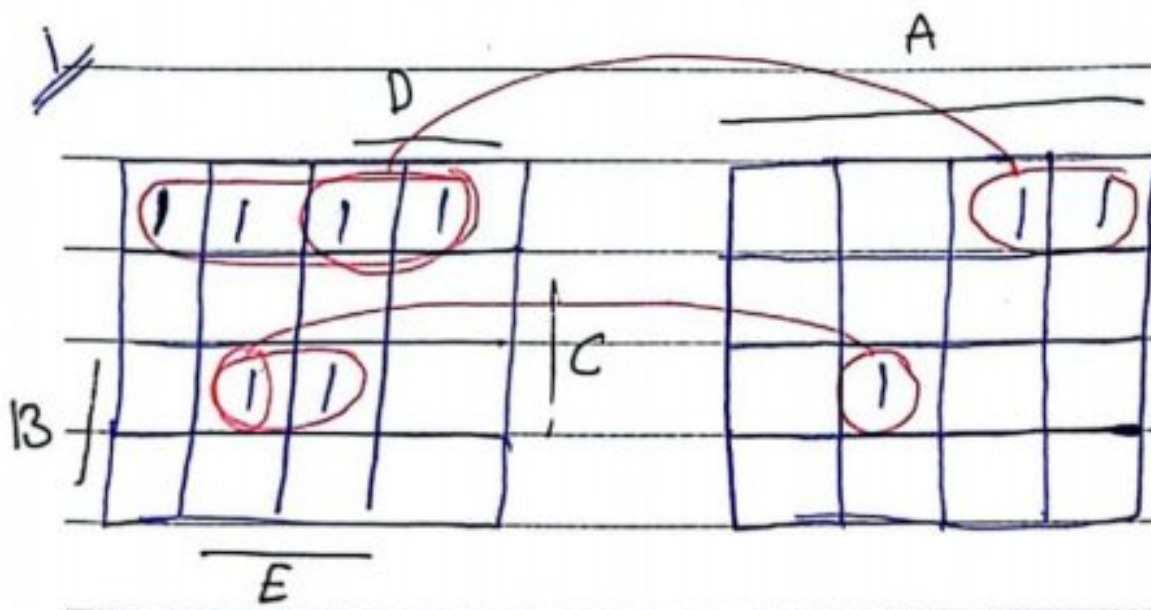
$$= (A \bar{C} + A' C) (B \oplus D)$$

$$= (A \oplus C) (B \oplus D)$$

Five variable K-map :-

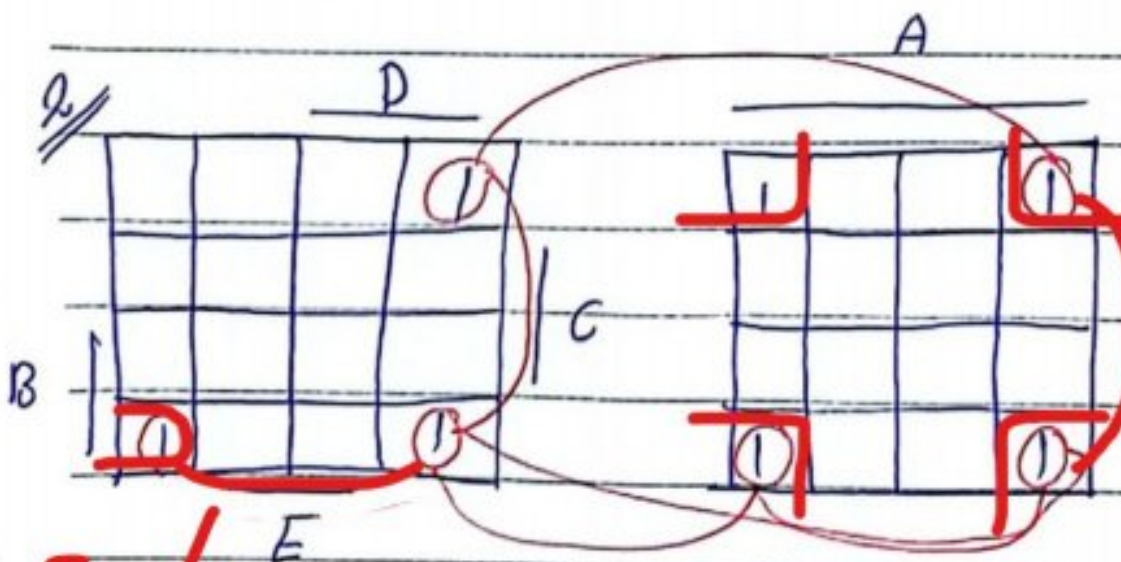
				A			
D				D			
m ₀ 00000	m ₁ 00001	m ₂ 00011	m ₃ 00010	m ₆ 10000	m ₇ 10001	m ₁₄ 10011	m ₁₅ 10010
m ₄ 00100	m ₅ 00101	m ₇ 00111	m ₆ 00110	m ₂₀ 10100	m ₂₁ 10101	m ₂₇ 10111	m ₂₂ 10110
m ₁₂ 01100	m ₁₃ 01101	m ₁₅ 01111	m ₁₄ 01110	m ₂₈ 11100	m ₂₉ 11101	m ₃₁ 11111	m ₃₀ 11110
m ₈ 01000	m ₉ 01001	m ₁₁ 01011	m ₁₀ 01010	m ₂₄ 11000	m ₂₅ 11001	m ₂₇ 11011	m ₂₆ 11010
E				E			

For ex- simplify :-



F(A, B, C, D, E)

$$= \bar{A}\bar{B}\bar{C}' + \bar{B}\bar{C}'D + BC\bar{D}E$$



~~F = A\bar{C}'E' + \bar{C}'DE' + BC\bar{E}'~~

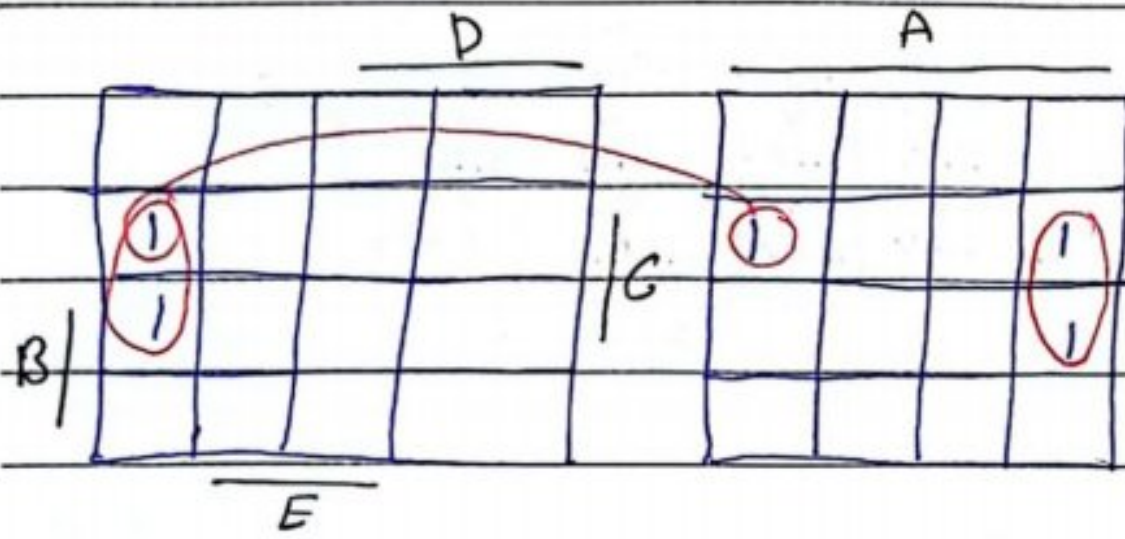
غلط

F = A\bar{C}' + \bar{C}' + BC

غلط

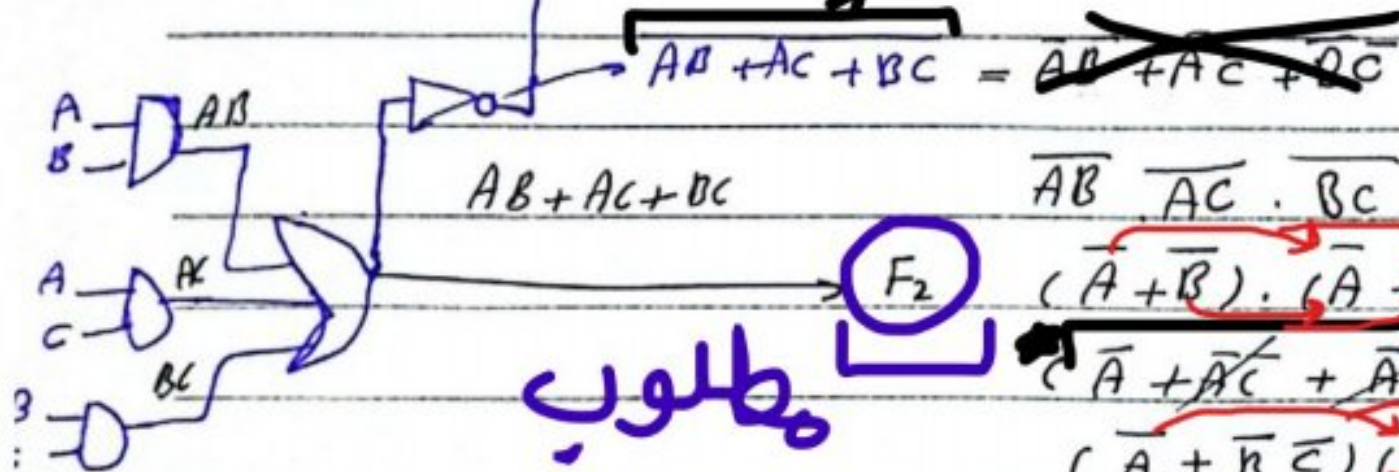
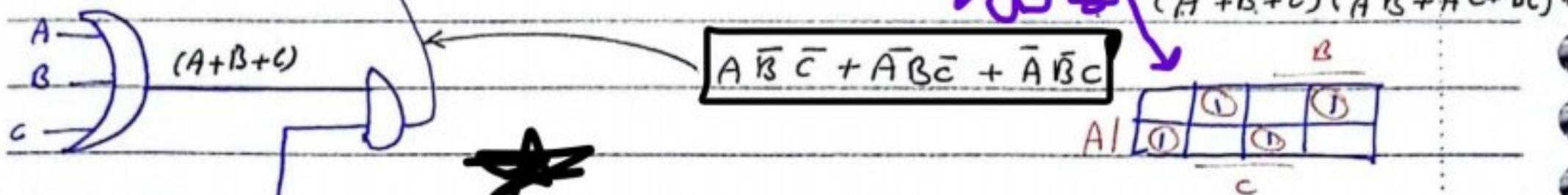
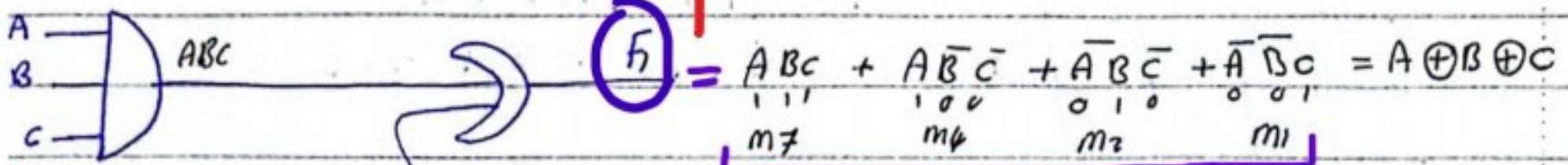
EX8- simplify $F(A,B,C,D,E) = \overset{00100}{m_4} A^1 B^1 C^0 D^0 E^1 + \overset{00100}{m_4+m_{12}} A^1 C^0 D^1 E^1 + \overset{10100}{m_{20}} A^0 B^1 C^0 D^1 E^1 + \overset{11110}{m_{30}} ABCDE^1 + \overset{10111}{m_{22}} AB^1 C^0 D^1 E^1$

$\Sigma (4, 12, 20, 22, 30)$



Digital circuits → combinational circuits (no memory)
 → sequential " (with memory)

معماری



مطلوب

$\bar{A}\bar{B} \cdot \bar{A}\bar{C} \cdot \bar{B}\bar{C}$
 $(\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{C}) \cdot (\bar{B} + \bar{C})$
 $(\bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C}) \cdot (\bar{B} + \bar{C})$
 $(\bar{A} + \bar{B}\bar{C})(\bar{B} + \bar{C})$
 $\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

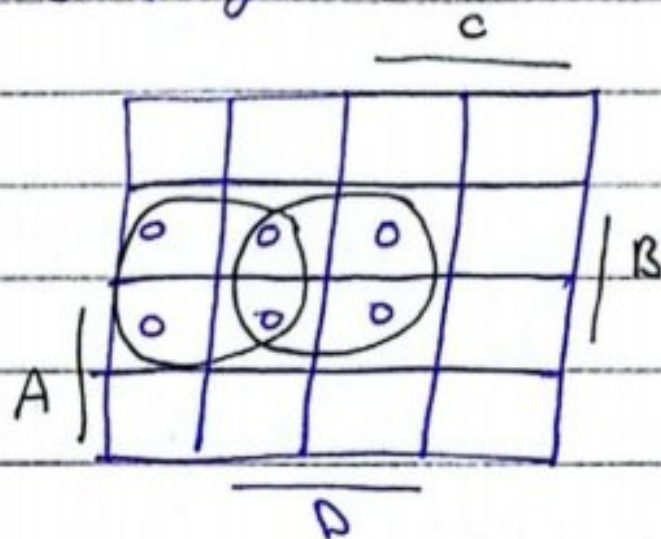
اذا اردك
 تصف كرد انجا
 ايسط صورة
 مخلصا بار K-map
 ايسط
 صورة

$A \oplus B \oplus C$
 $(A\bar{B} + A\bar{B}) \oplus C$
 $(A\bar{B} + A\bar{B}) \cdot C + (\bar{A}\bar{B} + \bar{A}\bar{B}) \cdot C$
 $(A\bar{B} + A\bar{B}) \cdot C + (\bar{A}\bar{B} + \bar{A}\bar{B}) \cdot C$
 $A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$

A	B	C	D	w	x	y	Z
0	0	0	0	0	0	1	1
0	0	0	1	0	0	1	1
0	0	1	0	0	1	0	0
0	0	1	1	0	1	0	1
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	1	0
1	0	0	1	1	0	1	1
1	1	0	0	1	1	0	0

K-map (mateam)

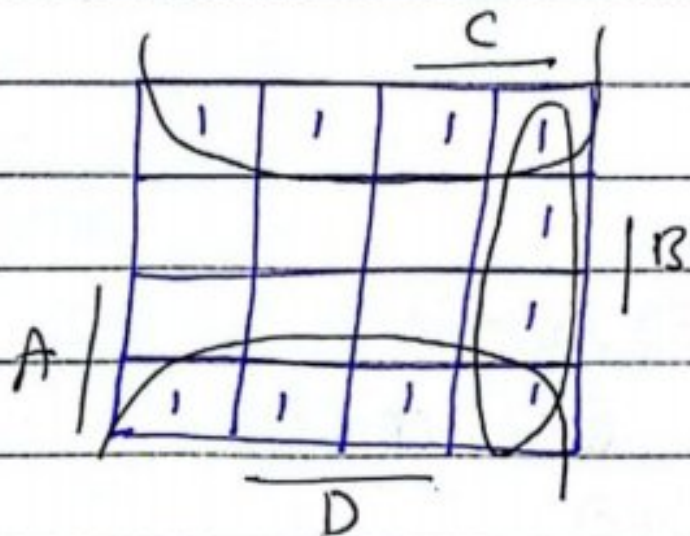
EX: simplify



$$F = (B' + D') \cdot (C'D' + C)$$

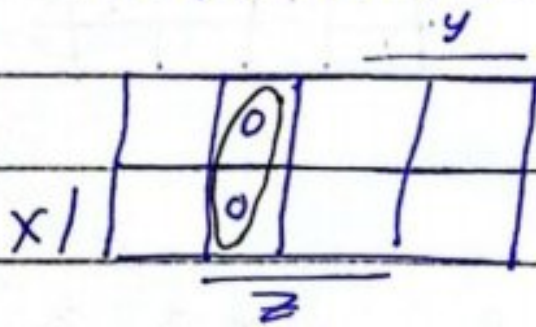
$$B'D' + D'C + B'D + C'D'$$

max term

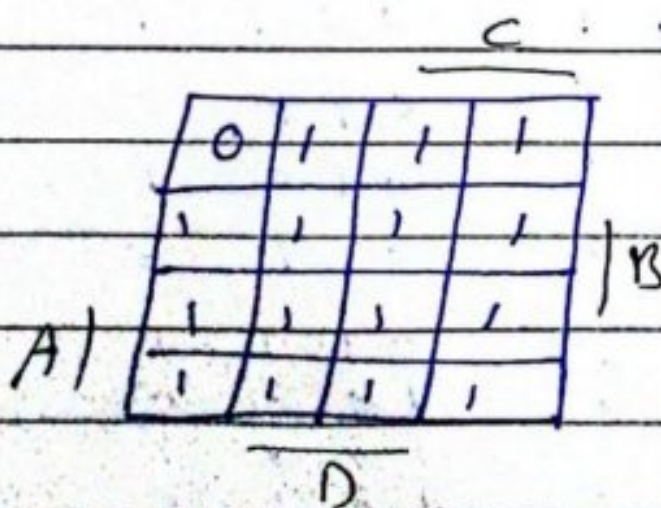


$$F = B + CD'$$

EX: simplify using maxterm:-



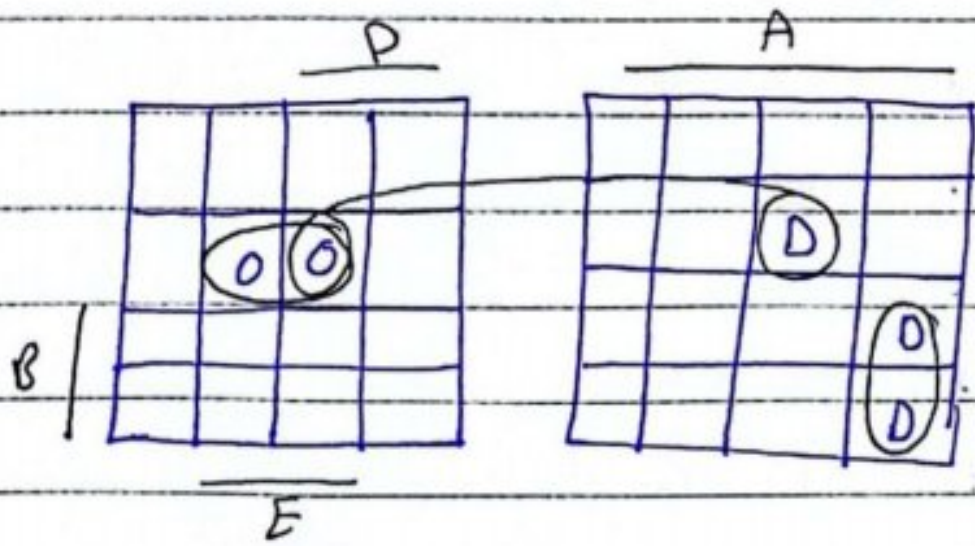
$$F = (z' + y)$$



$$F = (A+B + C + D)$$

لو ما حدد لك
بصير في سؤال
الـ max اسهل
واصح

ex:- simplify using K-map in Max term



$$F = (A + B + \bar{C} + \bar{E}) \cdot (B + \bar{C} + D + \bar{E}) \cdot (A + \bar{B} + \bar{D} + E)$$

التبسيط
الشروط

EX:- Design Excess 3 / 0-9 system

out Put

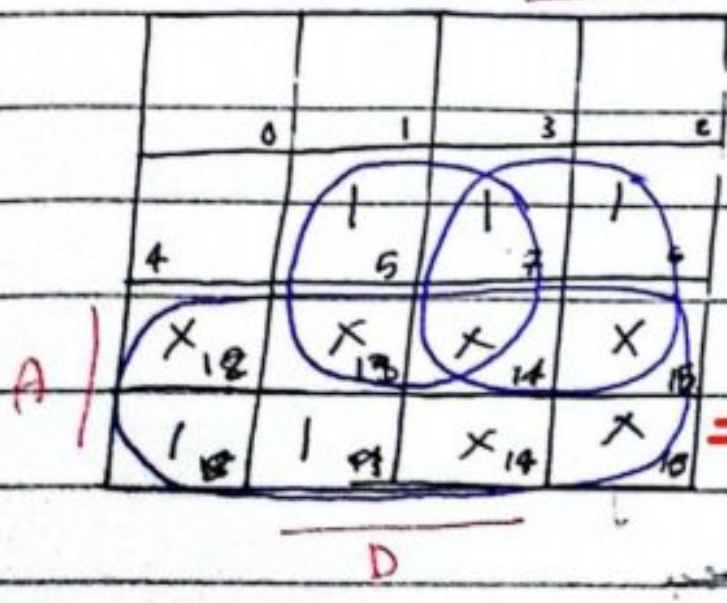
state	is	A	B	C	D	w	x	y	z
0	0	0	0	0	0	0	0	1	1
1	0	0	0	1	1	0	1	0	0
2	0	0	1	0	0	0	1	0	1
3	0	0	1	1	1	0	1	1	0
4	0	1	0	0	0	0	1	1	1
5	0	1	0	1	1	1	0	0	0
6	0	1	1	0	0	1	0	0	1
7	0	1	1	1	1	1	0	1	0
8	1	0	0	0	0	1	0	1	1
9	1	0	0	1	1	1	1	0	0
10	1	0	1	0	0	x	x	x	x
11	1	0	1	1	1	x	x	x	x

not used

sol:-

we don't care because not used
 Design care system

~~$W(A,B,C,D) = \sum (5, 6, 7, 8, 9) + \sum_x (10, 11, 12, 13, 14, 15)$~~

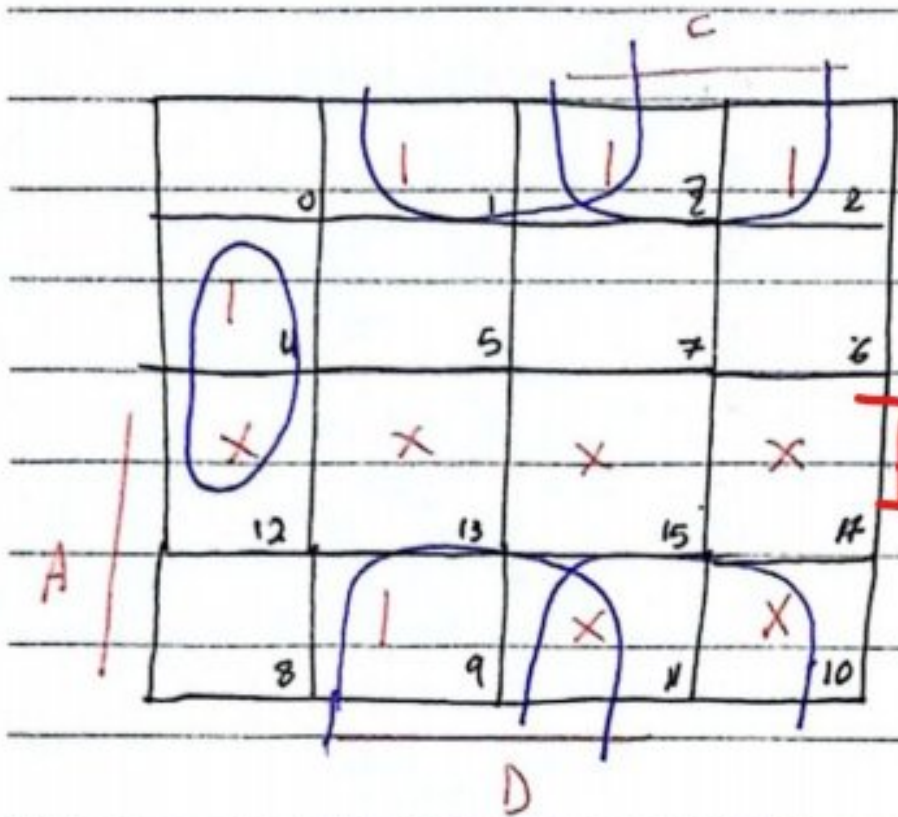


~~$X(A,B,C,D) = \sum (1, 2, 3, 4, 9) + \sum_x (10, 11, 12, 13, 14, 15)$~~

B is ABCD
 $= A + BD + BC$

variables C, B, A, D

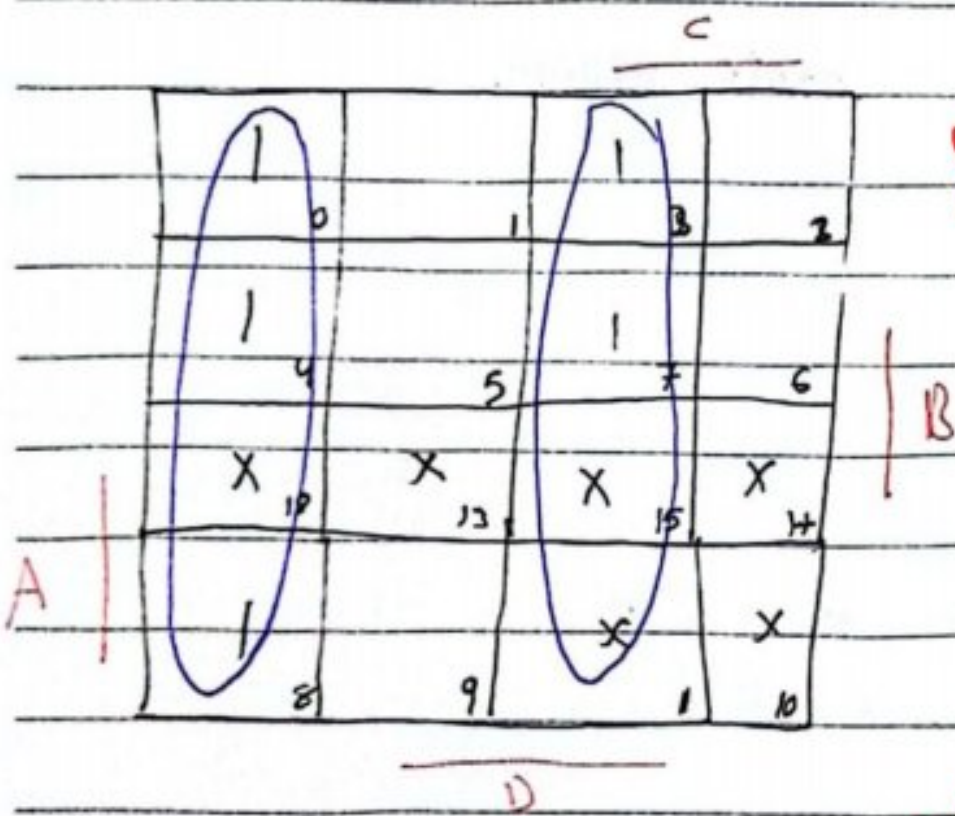
w, x, y, z input



$$X = BC'D + B'D + BC$$

$$*y(A,B,C,D) = \sum (0,3,4,7,8) + \sum_x (10,11,12,13,14,15)$$

كل ال one's
اشهر



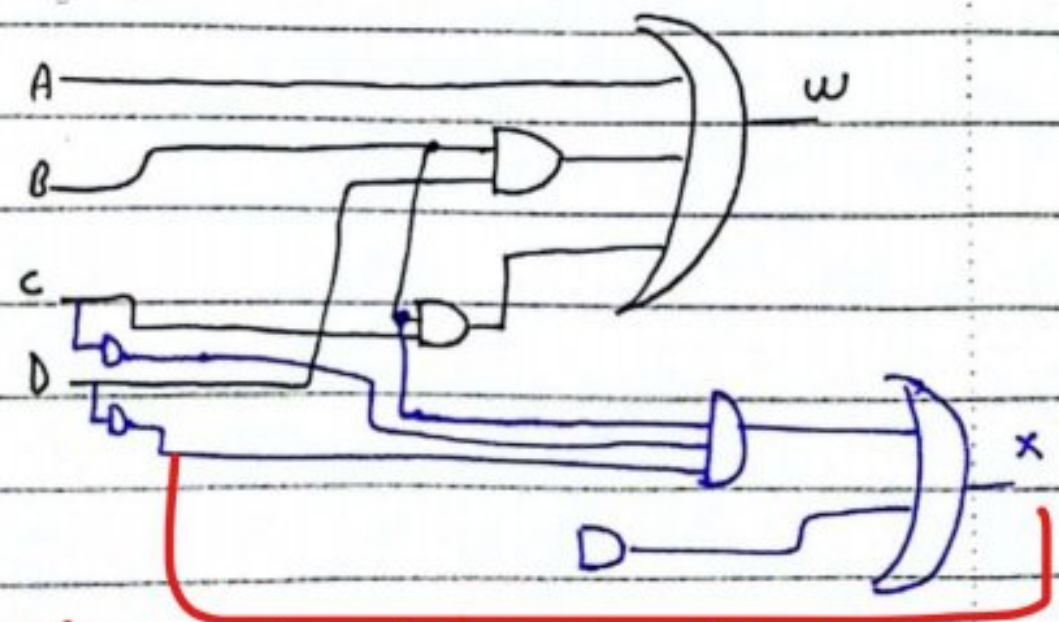
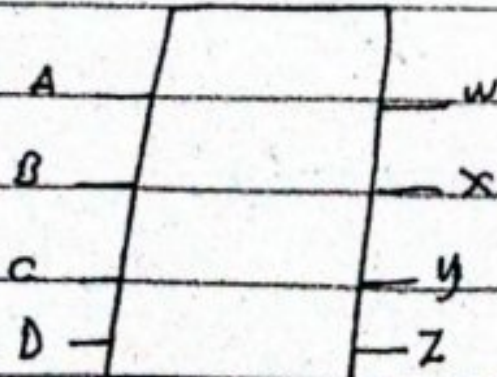
$$Y = C'D + CD = C \odot D$$

قدرة اطلعها
من اشارة لان
في علاقة بينها

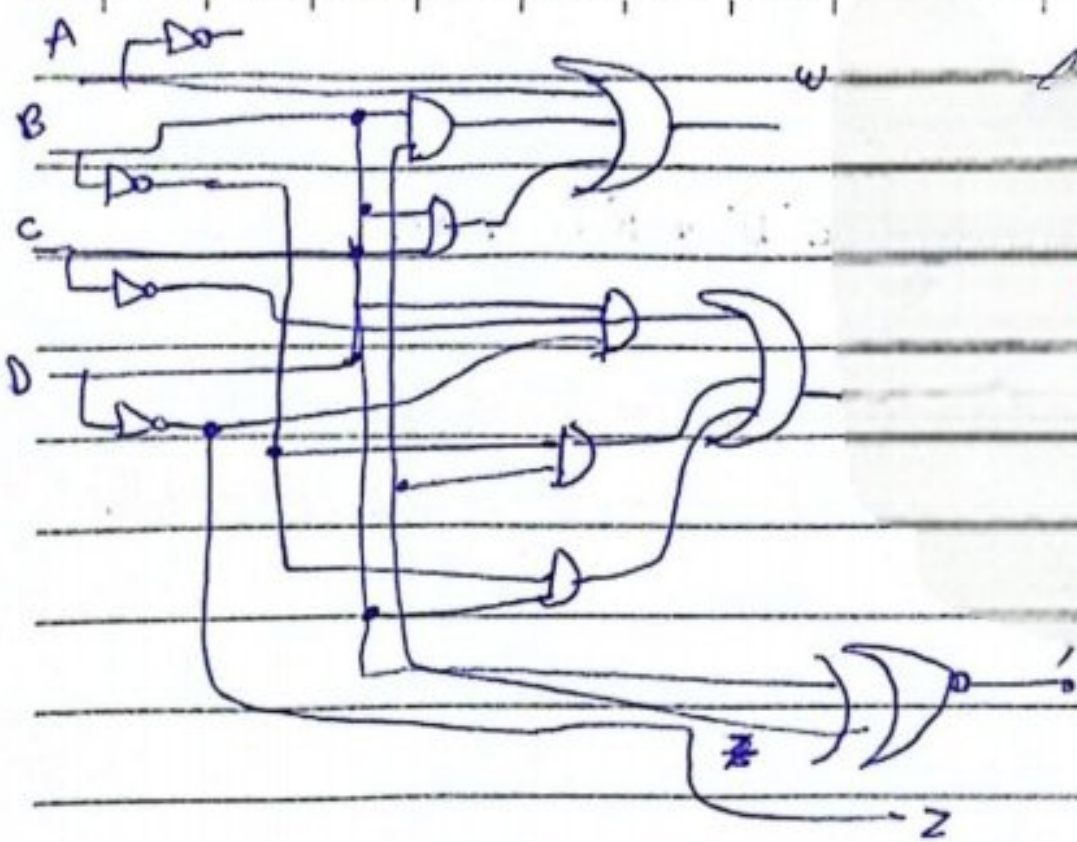
by inspection $Z(A,B,C,D) = D'$

لما
D و C
لا يكون
D like

D و

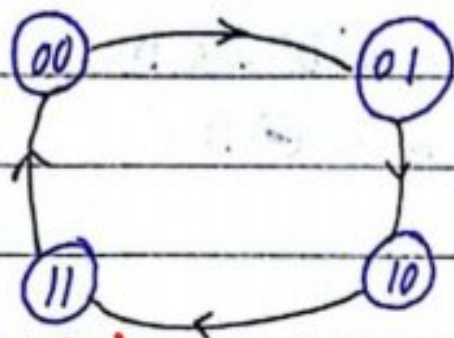


ال مطلوب



EX8- Design a 2-bit up counter

state diagram 1



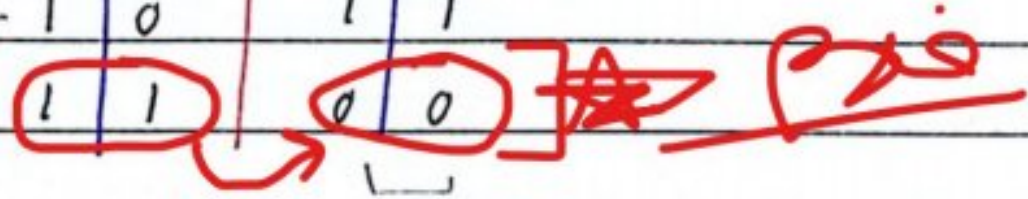
2

A	B	F ₁	F ₂
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

3

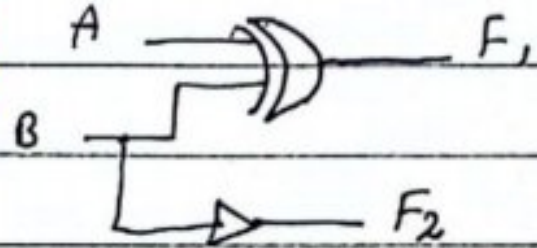
$F_1 = A \oplus B$
 $F_2 = B'$

لقد بنا علاقة مباشرة

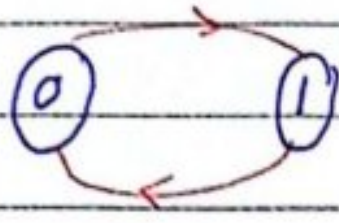


ماد يوجد من الاعداد

4



EX:- Design 1 bit up counter



state Diagram

A	F
0	1
1	0

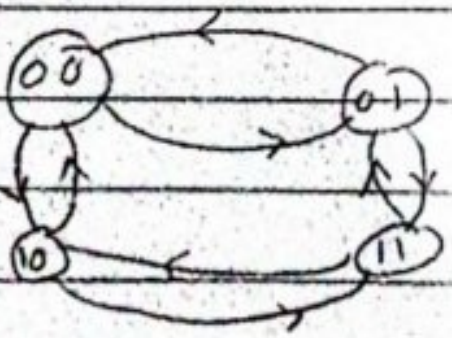


H.W Design 1 bit UP and down counter counter

clk rst

A	B	F	up	down
0	0	X	X	X
1	X	X	X	X
↑	↑	↑	↑	↑
0	0	0	0	0

A	UP	down	F	0	1
0	0	1	0	0	1
0	1	0	1	0	0
1	0	0	---	---	---
1	1	1	0	0	1
			1	0	1
			0	0	1

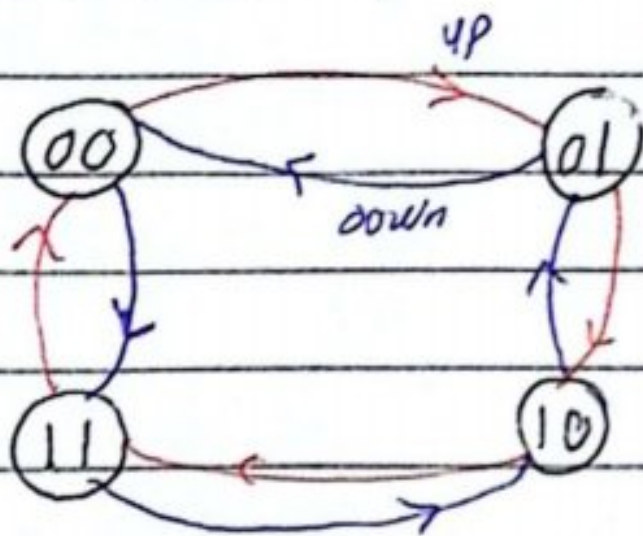


0	1	X	X	0
1	1	X	X	0
1	0	X	X	X

EX:- Design Twobit up/down counter

Sol:-

① Find the state diagram



let $x=0$ up
 $x=1$ down
 state diagram

② Find state table

By inspection

A	B	x	F ₁	F ₂
0	0	0 up	0	1
0	0	1 down	1	1
0	1	0 up	1	0
0	1	1 down	0	0
1	0	0 up	1	1
1	0	1 down	0	1
1	1	0	0	0
1	1	1	1	0

7 cyclin path, *

③ Find the function

$F_1 = \sum (1, 2, 4, 7)$

		B	
		0	1
A	0	1	1
	1	1	
		C	

$F_2 = \sum (A'B'C) + (A'BC') + (AB'C) + (ABC)$

$$= (A'B + AB)C + (A'B + AB')C'$$

$$= (A \odot B)C + (A \oplus B)C'$$

note:-

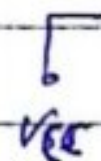
$$= (A \oplus B) \bar{C} + (A \oplus B) C$$

F₂ ground

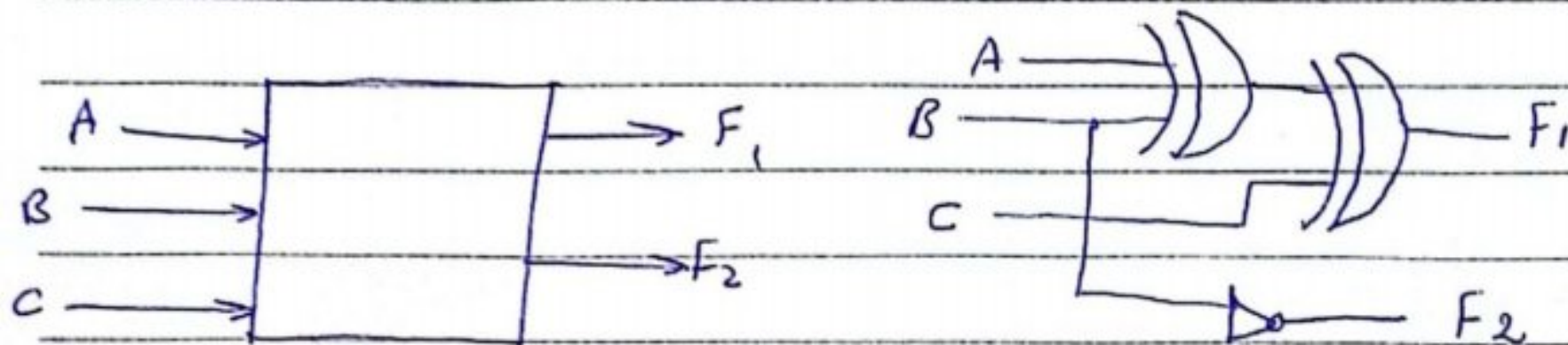


$$= A \oplus B \oplus C$$

F₂ source



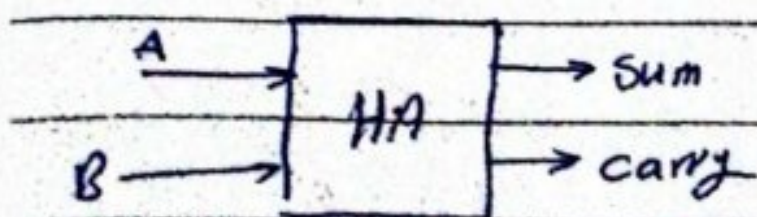
⊕ Draw the cct:-



* Binary Addition

$$\begin{array}{r}
 0011 + 1 \\
 \hline
 0110 \\
 \hline
 1001 \\
 \hline
 11
 \end{array}$$

* Half Adder (H.A)

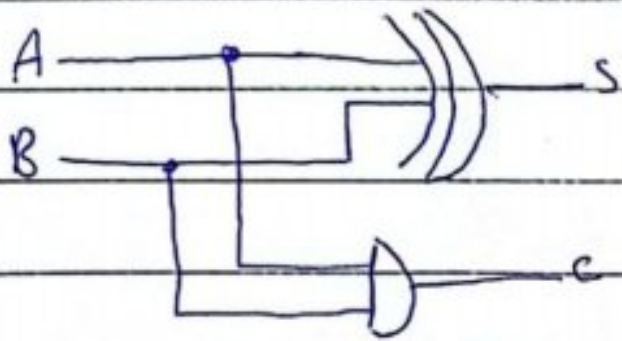


EX:- Design Half Adder

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = A \oplus B$$

$$C = A \cdot B$$

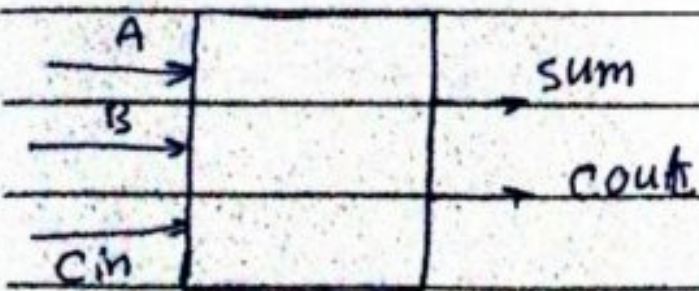


EX:- Add

$$\begin{array}{r} \text{Carry in} \\ 0110 \\ + 1011 \\ \hline 01 \end{array}$$

$$\begin{array}{r} 0110 \\ + 0111 \\ \hline 1101 \\ \text{Carry out} \end{array}$$

~~Full Adder~~ Full Adder (F.A)

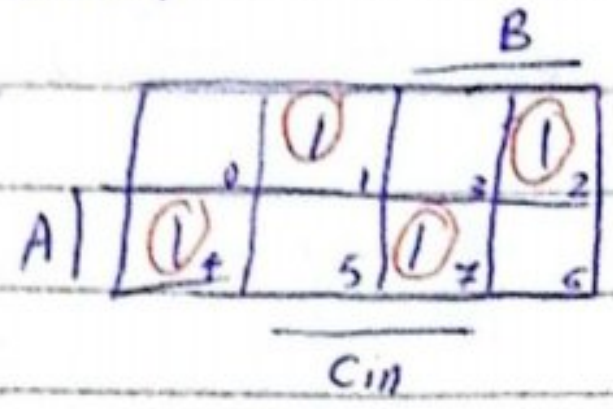


A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	2	0
0	1	1	3	1
1	0	0	4	0
1	0	1	5	1
1	1	0	6	1
1	1	1	7	1

K-map

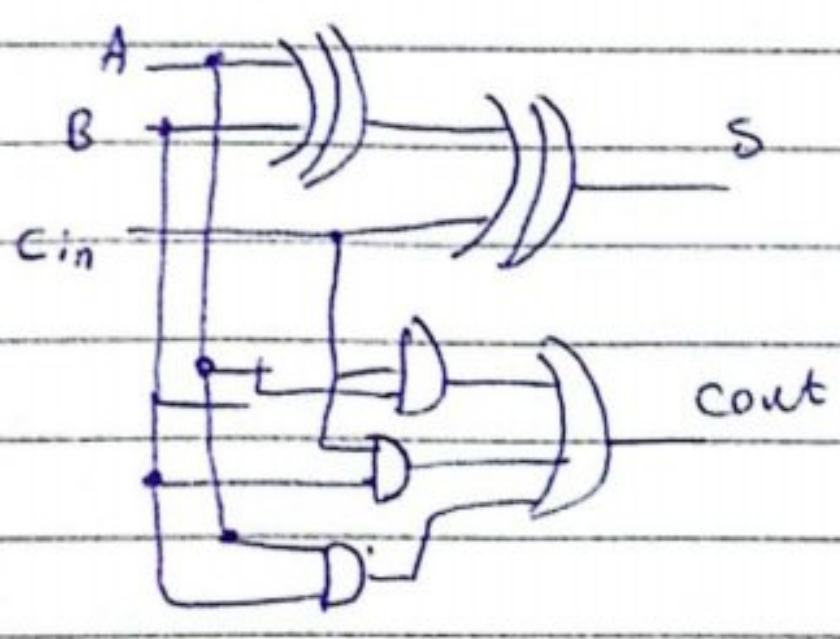
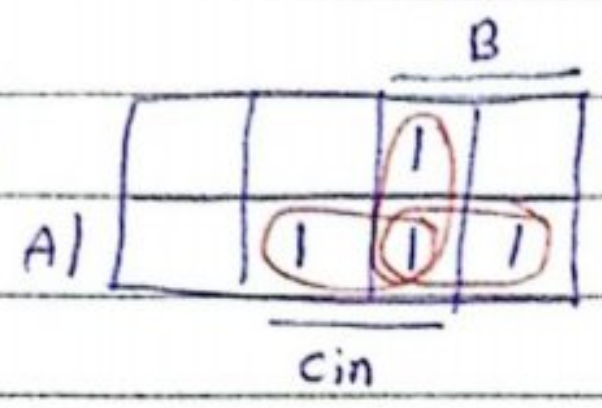
$$S = \sum(1, 2, 4, 7)$$

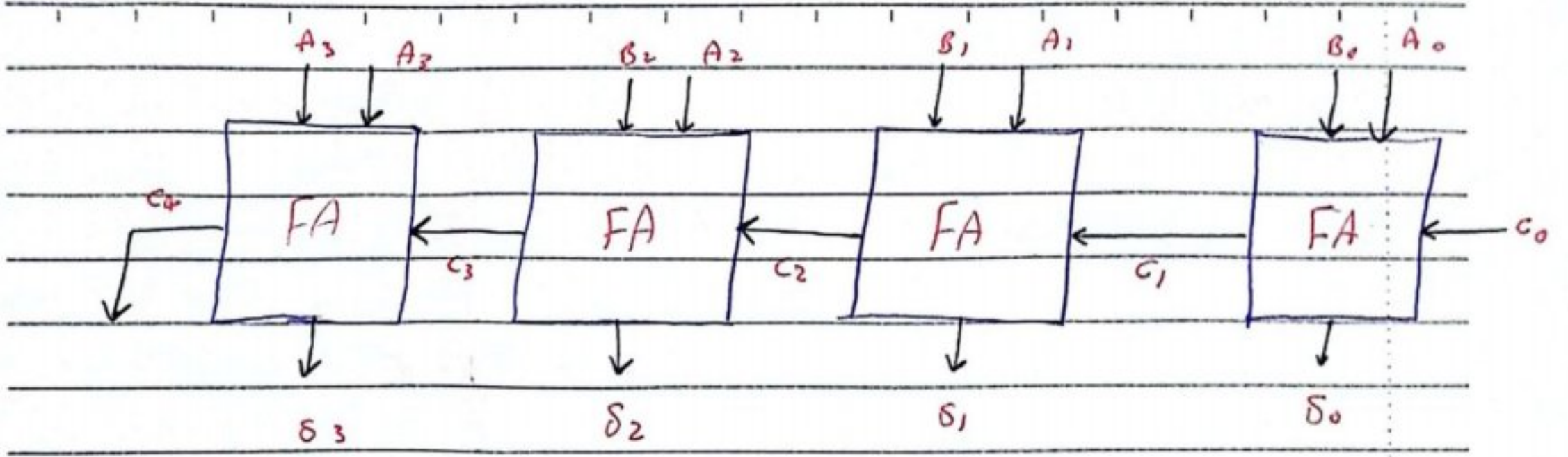
$$C_{out} = \sum(3, 5, 6, 7)$$



$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = A \cdot C_{in} + B \cdot C_{in} + A \cdot B$$





c_4	c_3	c_2	c_1	A_0	B_0	$\leftarrow c_0$
A_2	A_2	A_1	A_0	B_0	B_0	$\leftarrow c_0$
B_2	B_2	B_1	B_0	B_0	B_0	$\leftarrow c_0$
S_3	S_2	S_1	S_0	S_0	S_0	S_0

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(*)

		1		1
1			1	

$$F_1 = A'B'x + A'Bx' + AB'x + ABx$$

$$= A'(B'x + Bx') + A(B'x' + Bx)$$

$B \oplus x$ $B \oplus x$

~~consensus~~

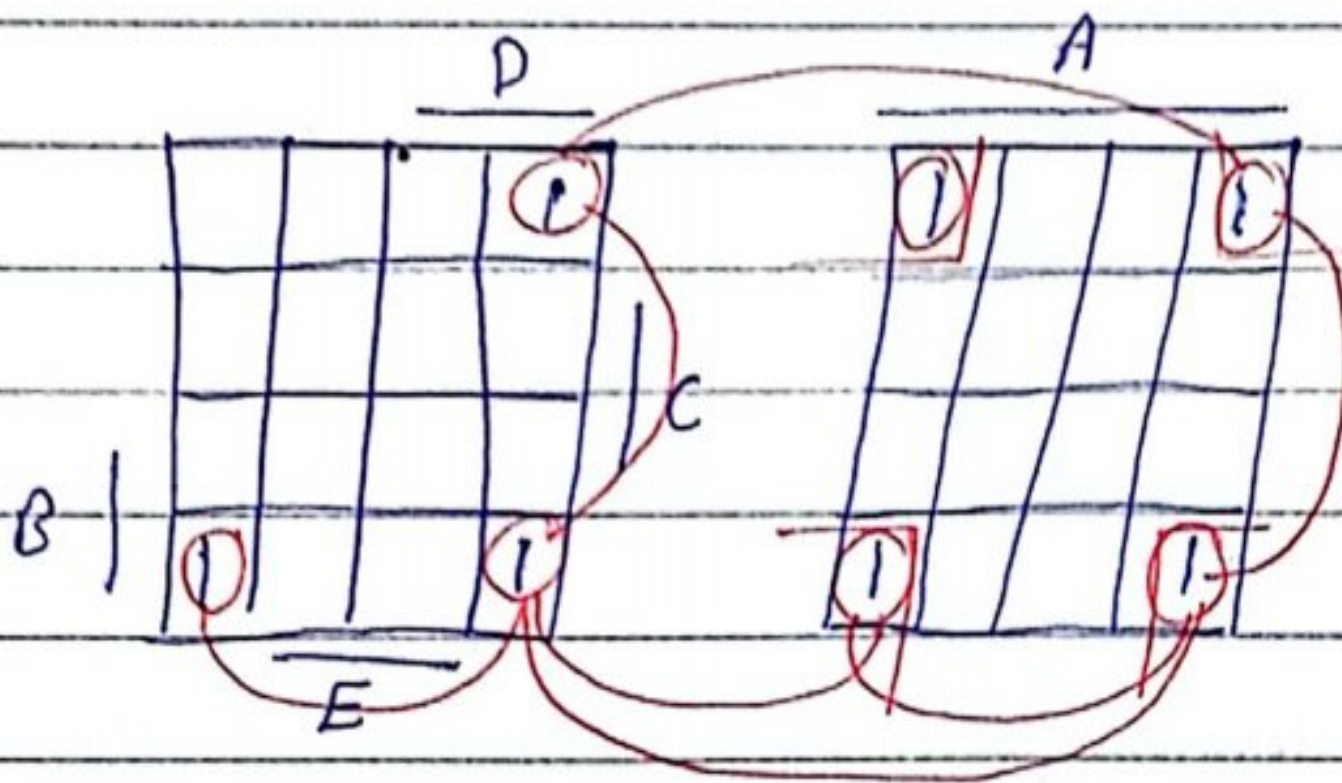
$$= A'(B \oplus x) + A(B \oplus x)$$

$$A'(B \oplus x) + A(B \oplus x)$$

$$= A \odot (B \oplus x)$$

$$= A \oplus B \oplus x$$

(*)



prime implicants *

$$F = AC'E + BC'E$$

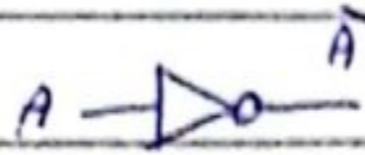
$$+ CDE$$

EX:-

⇒ be here now

Design a one bit up down counter

A	X	F ₁
0	0	1
0	1	1
1	0	0
1	1	0



*

$$B \oplus 0 = B\bar{0} + \bar{B}0 = B$$

$$B \cdot 1 + \bar{B}0 = B$$

$$B \oplus 1 = B\bar{1} + \bar{B}1 = \bar{B}$$

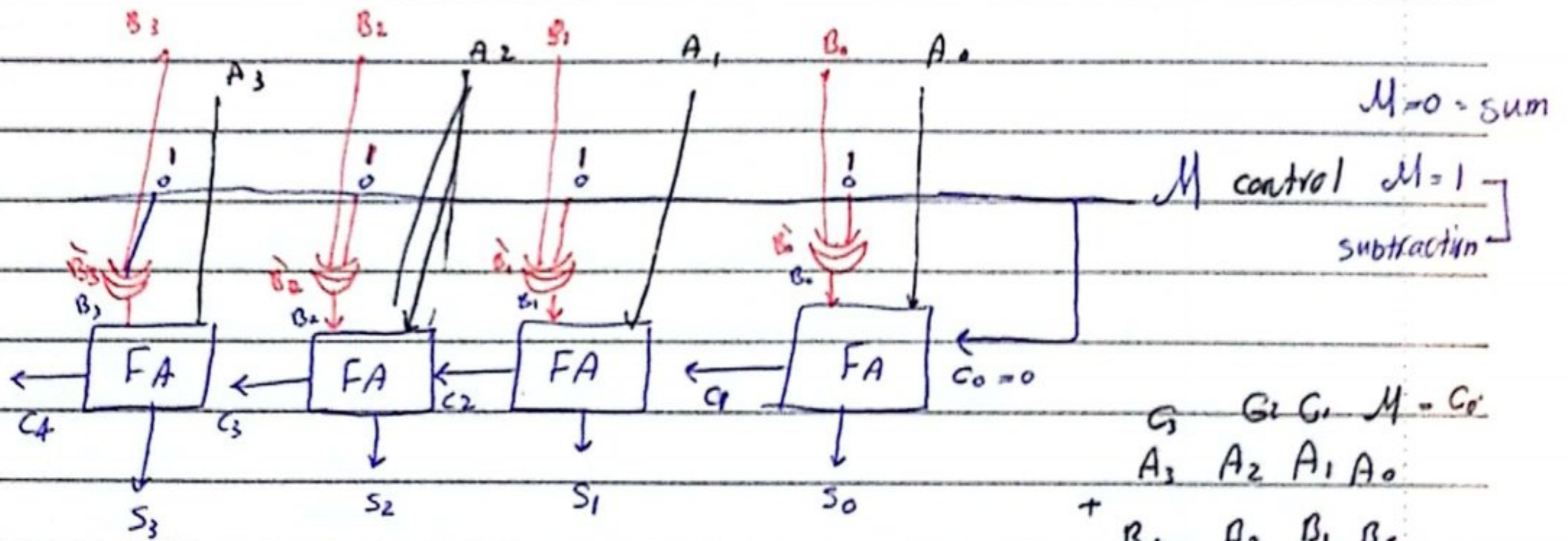


Figure 1

M=0 ⇒ A+B

A + (B̄ + 1)

M=1

FA = Full Adder

This is 4 bit adder/subtractor Figure 1

23/ April / 2024

BCD Addition

$$\begin{array}{r}
 0011 \\
 + 0100 \leq 9 \\
 \hline
 0111
 \end{array}
 \qquad
 \begin{array}{r}
 0101_{(BCD)} \\
 + 0110_{(BCD)} \\
 \hline
 1011 \geq 9 \\
 + 0110
 \end{array}$$

BCD

$$0001 + 0001_{(BCD)}$$

مغالبه سے اجتناب

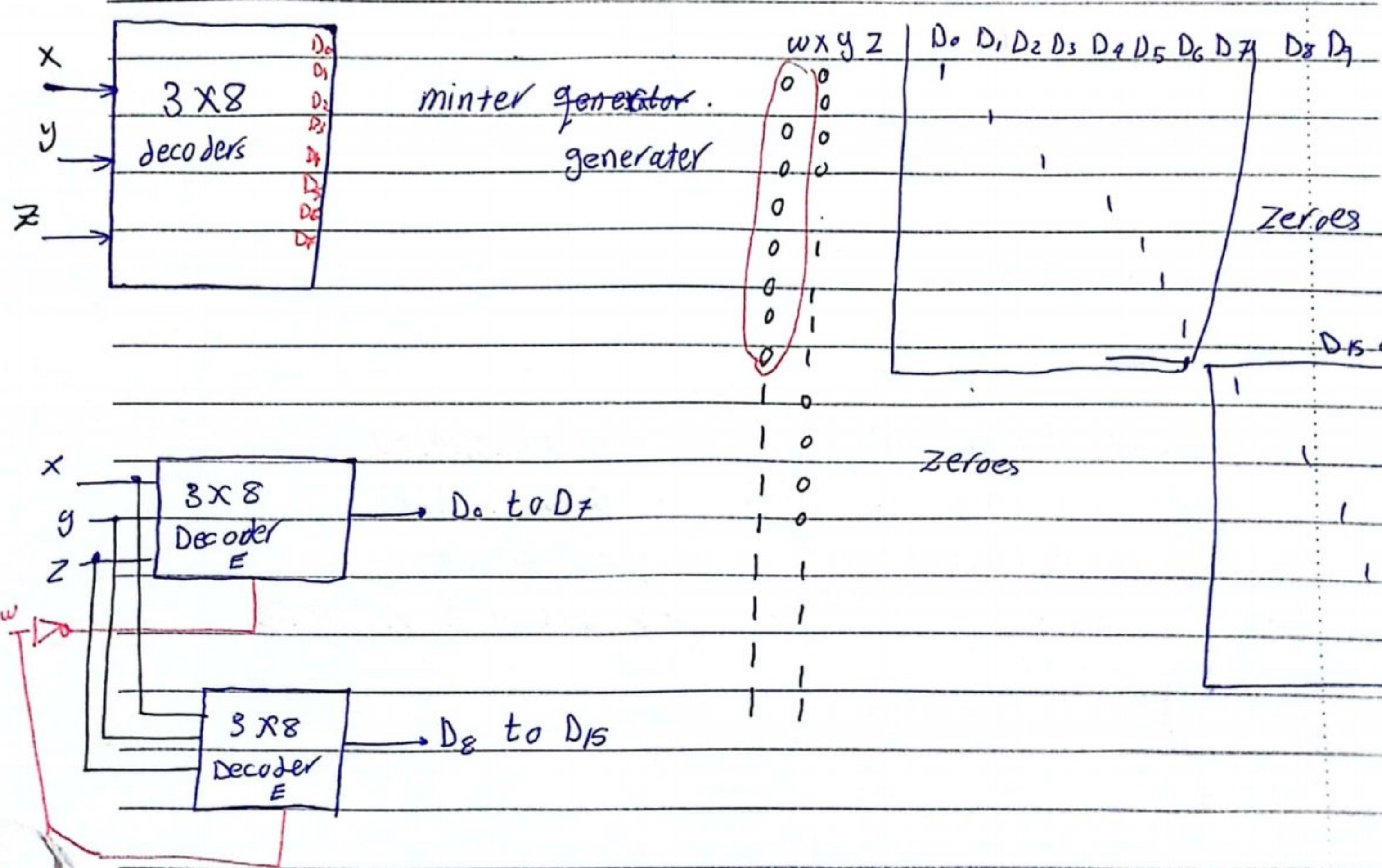
Exo. Add

$$\begin{array}{r}
 0010 \\
 + 0110 \\
 \hline
 1000 \quad 1001 \quad \checkmark
 \end{array}
 \qquad
 \begin{array}{r}
 0110 \quad 0110 \\
 + 0100 \quad 0101 \\
 \hline
 1011 \\
 + 0110 \\
 \hline
 1011 \quad 0001 \\
 + 0110 \\
 \hline
 0001 \quad 0001 \quad 0001
 \end{array}$$

علاقہ جمع
سے اجتناب
تو لگی

X	Y	Z	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

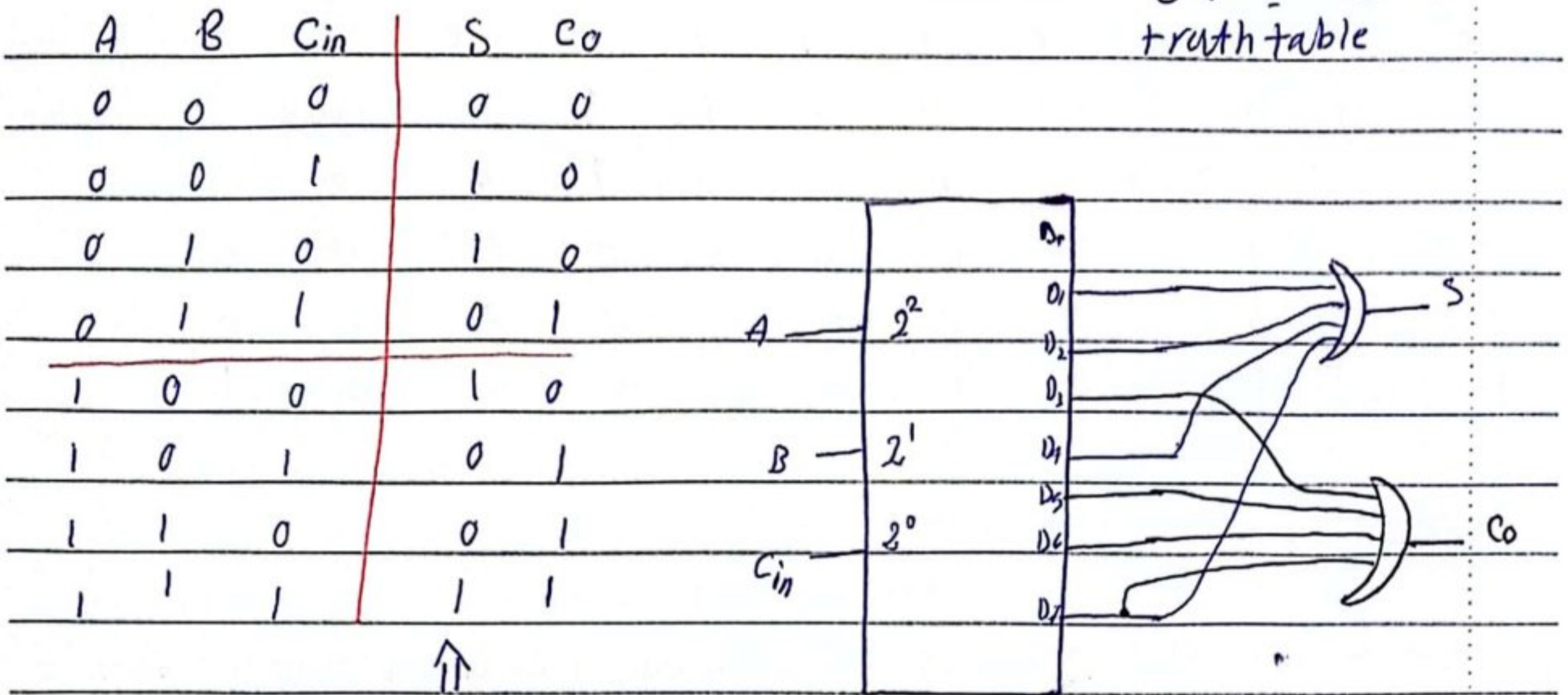
Decoders:-



Function implementation using Decoder

Ex:- implement Full adder using a decoder

اول اشرح بطرح
truth table



$$S = \sum(1, 2, 4, 7)$$

$$C_0 = \sum(3, 5, 6, 7)$$

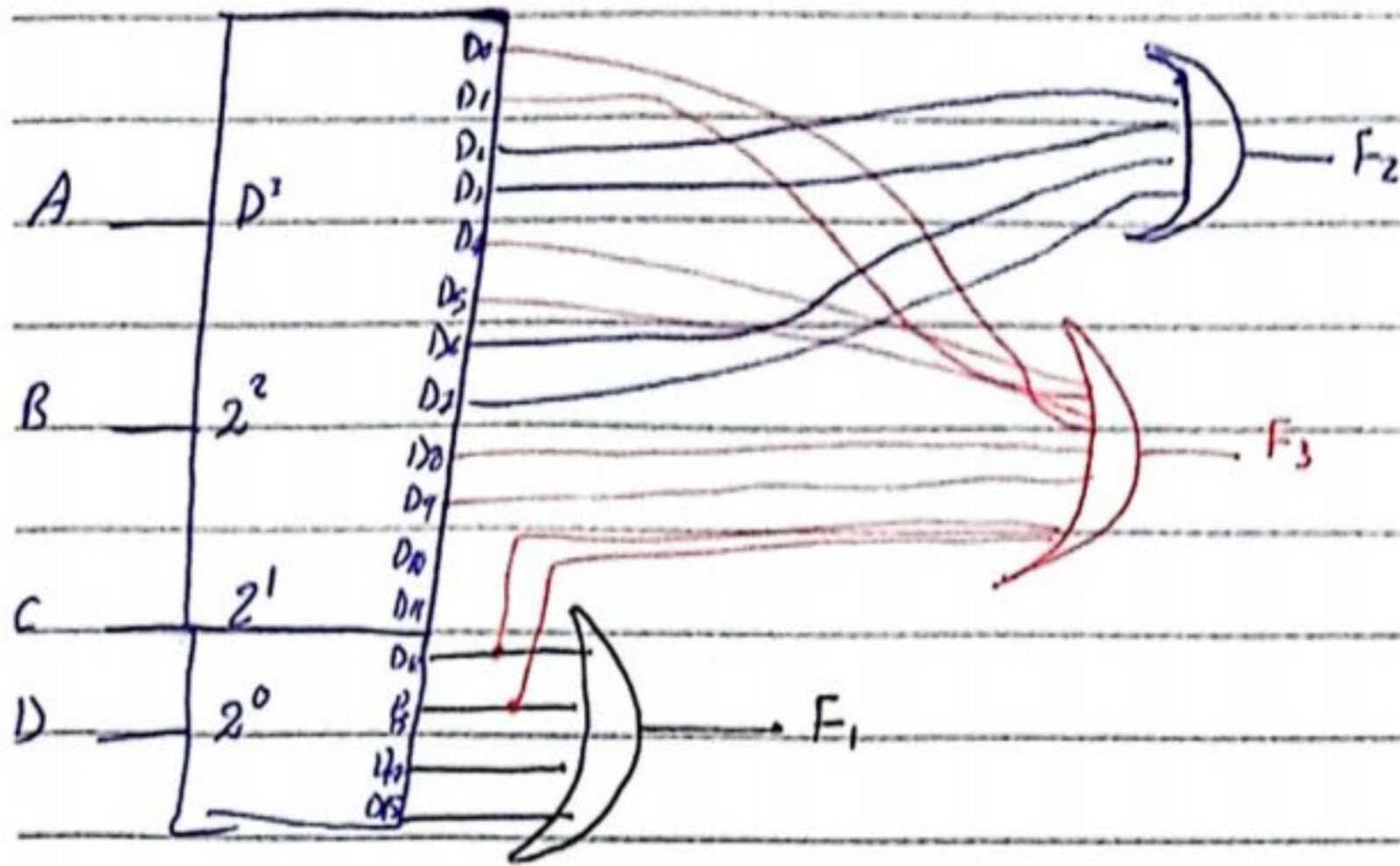
ex: implement $F_1(A, B, C, D) = AB = \sum(12, 13, 14, 15)$

$F_2(A, B, C, D) = A^0 B^0 C^1 D^0 = \sum(2, 3, 6, 7)$

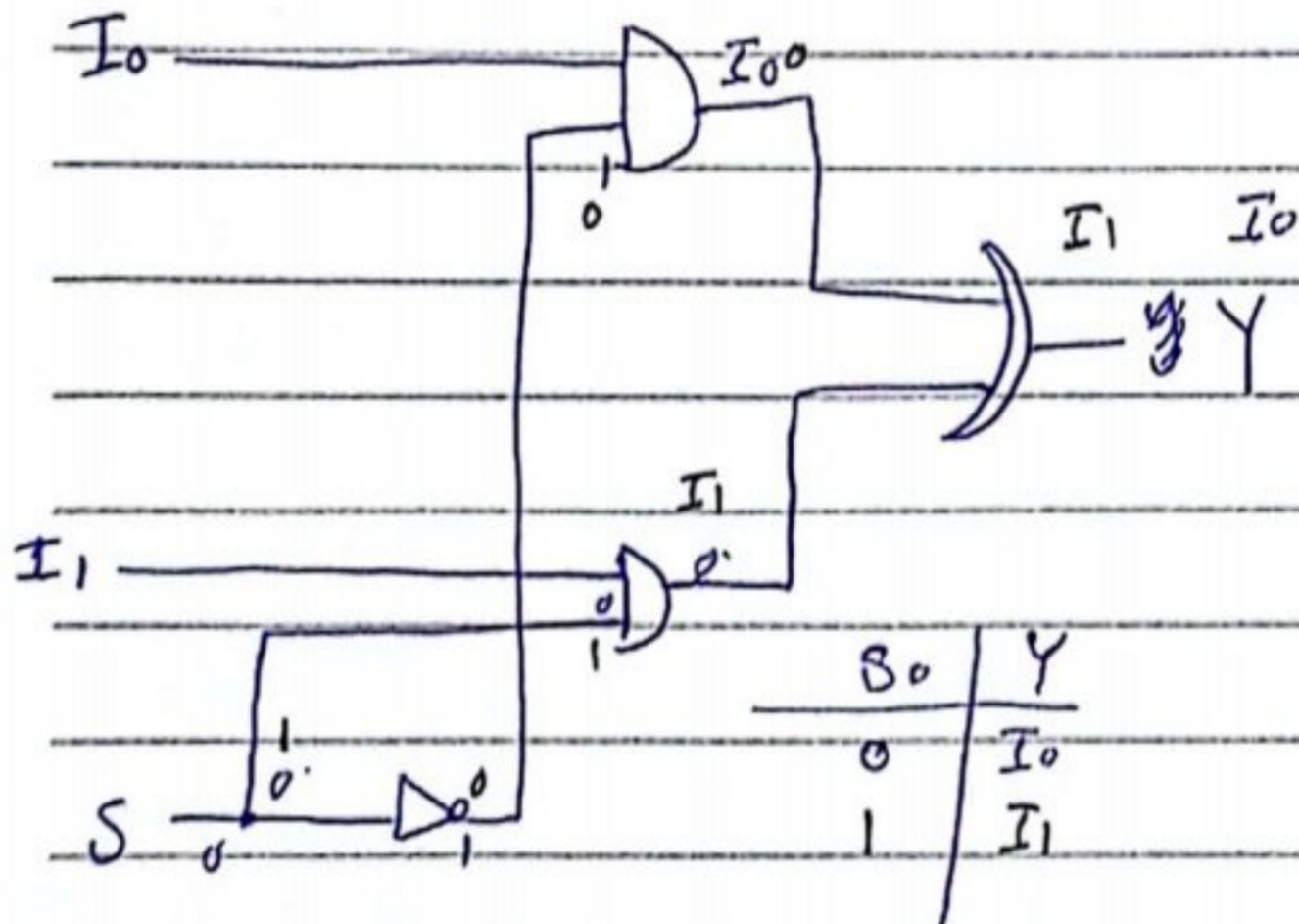
$F_3(A, B, C, D) = \bar{C}$ using OR decoder

$\sum(0, 1, 4, 5, 8, 9, 12, 13)$





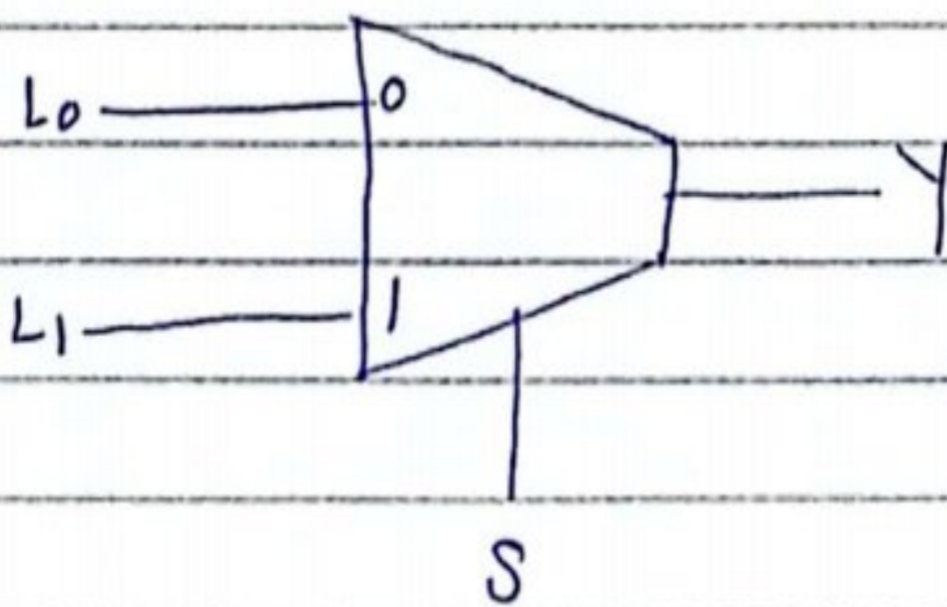
Multiplexer (MUX)



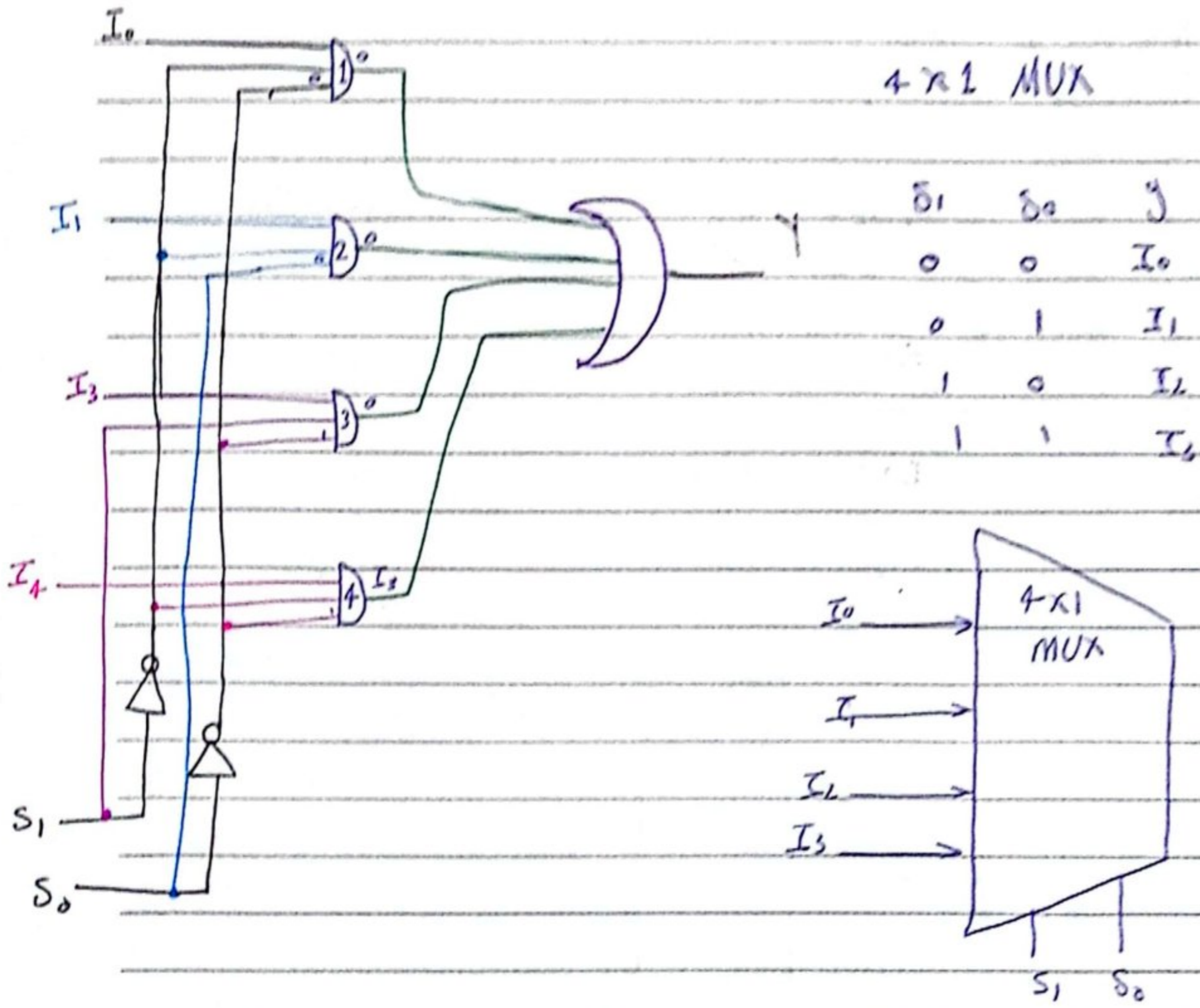
- 2 x 1
- 4 x 1
- 8 x 1
- 16 x 1

S ₀	Y
0	I ₀
1	I ₁

many input L one output



25 APRIL 2024



Design using MUX

	S_1	S_0	C	F_1	F_2
I_0	0	0	0	0	1
I_1	0	1	0	1	0
I_2	1	0	0	0	0
	1	1	0	1	0
	1	1	1	1	1

$F_1 = C$
 $F_2 = 1$
 $F_1 = C$
 $F_2 = 0$
 $F_1 = 0$
 $F = C$
 $F_1 = 0$
 $F = C$

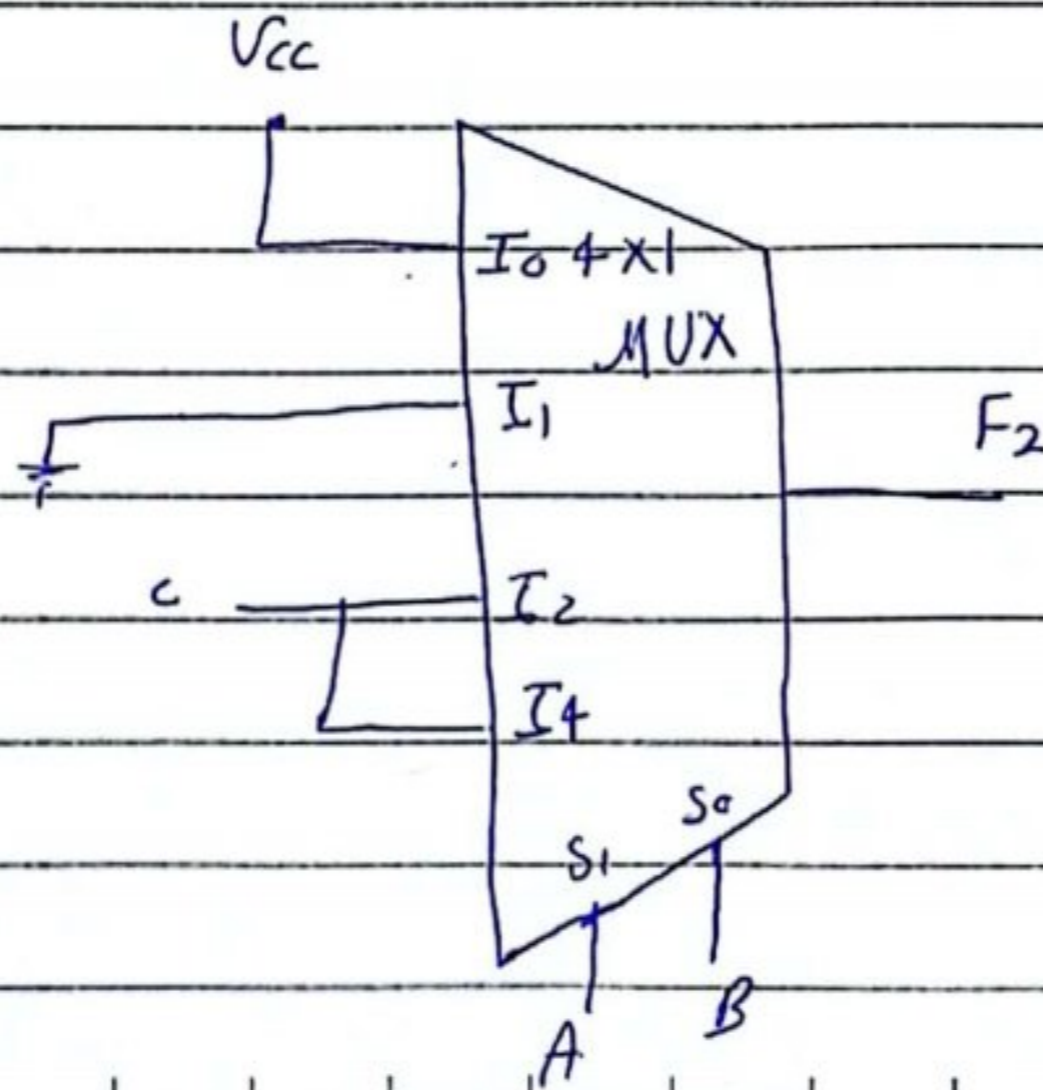
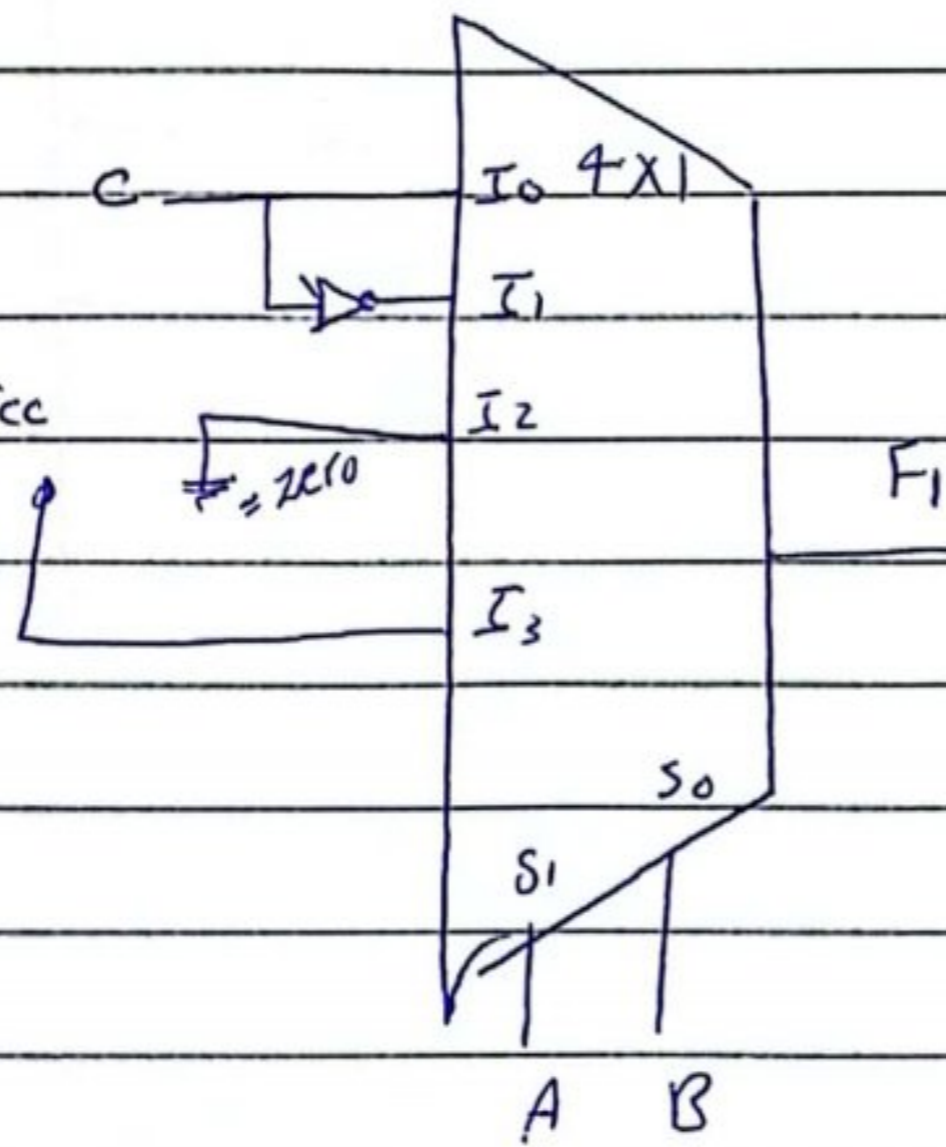
Function

إذا كان C متساويًا

إذا كان C متساويًا

C

إذا كان C متساويًا



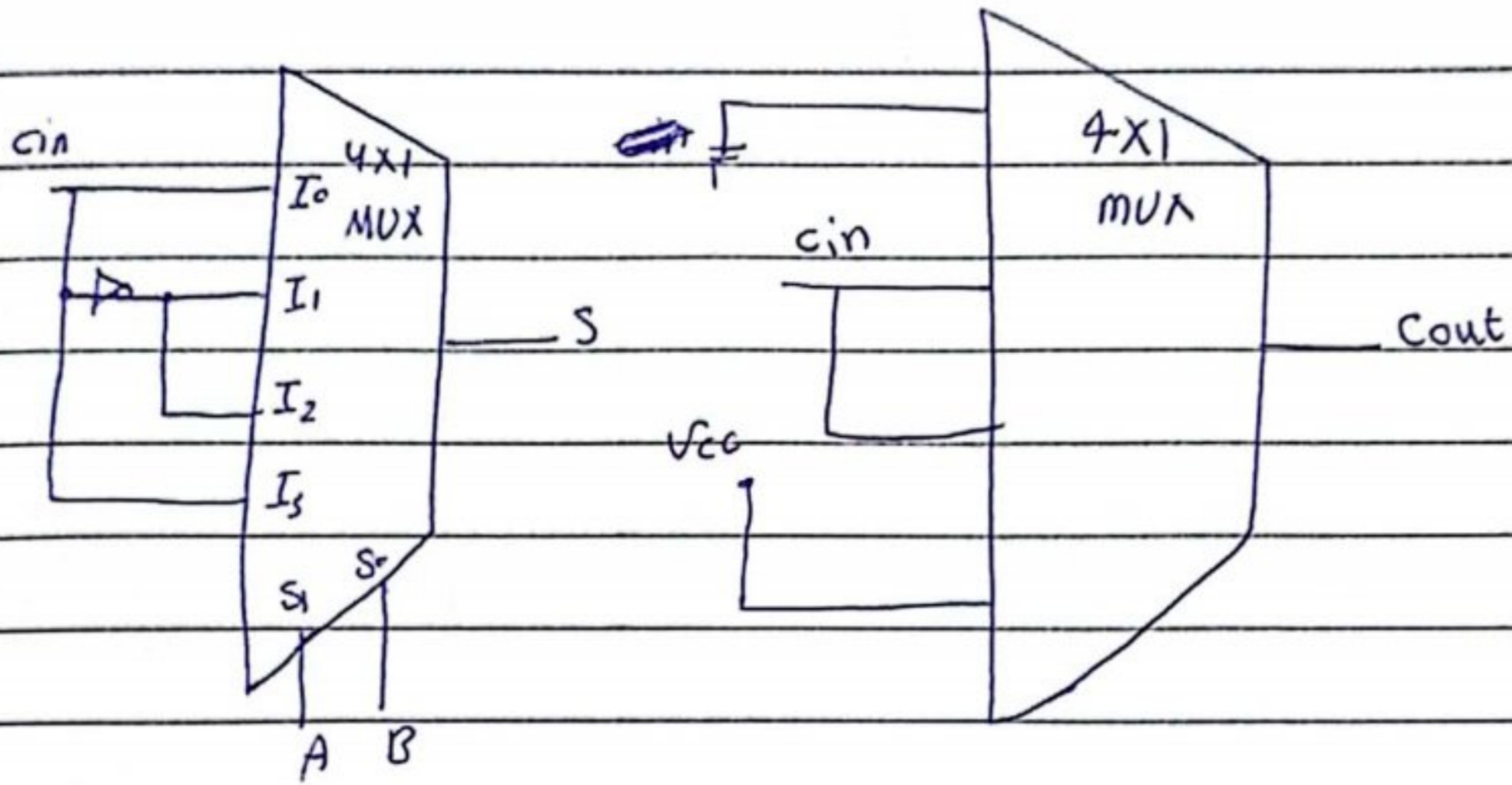
	input	selection lines	output
2x1 MUX	2	1	1
4x1 MUX	4	2	1
8x1 MUX	8	3	1
16x1 MUX	16	4	1

F.A

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

digit 1003

Design a FA using MUX (S)

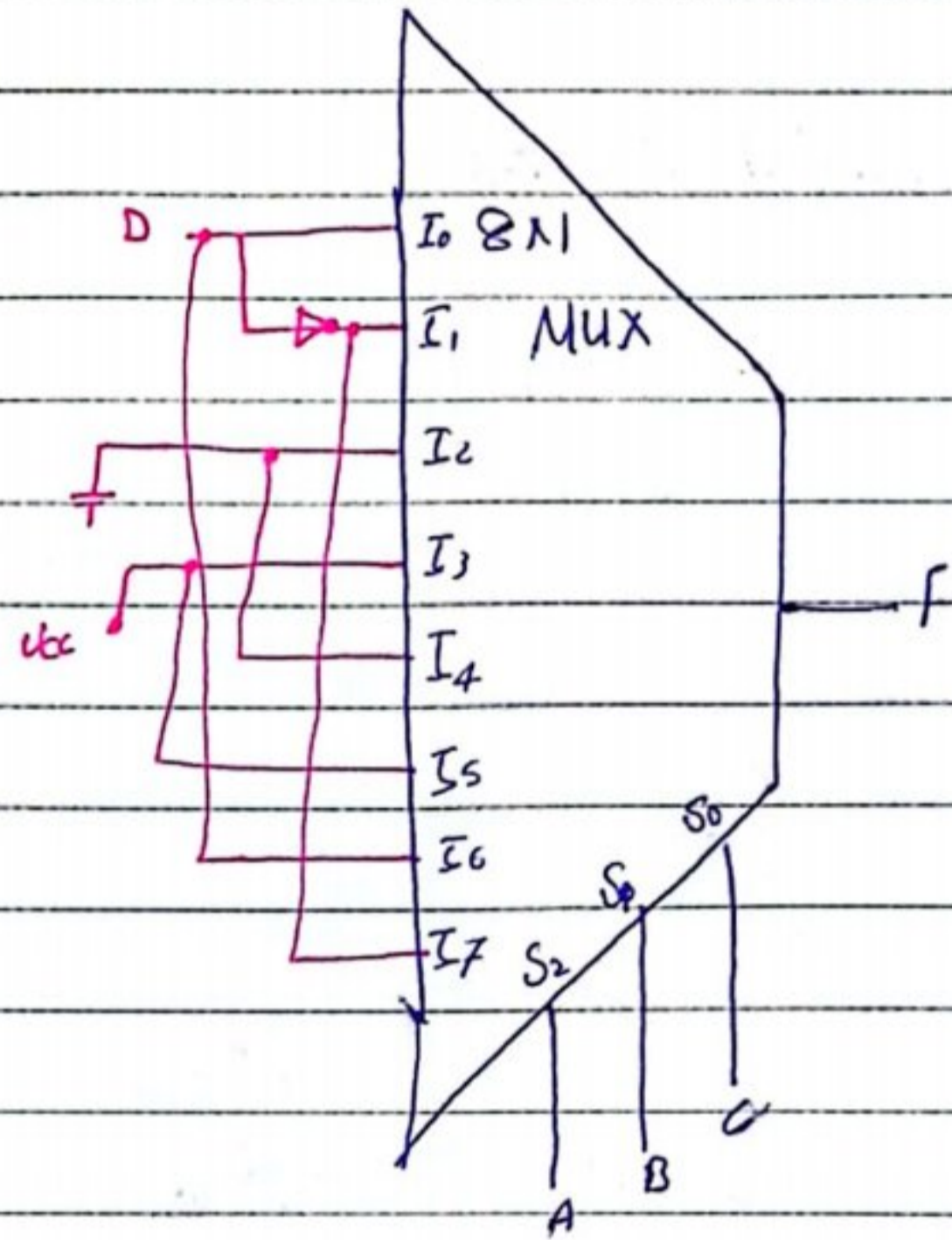


$V_{CC} = 1$
ground = 0

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Exg Design a using a MUX

S_2	S_1	S_0	D	F_1	
0	0	0	0	0	
0	0	0	1	1	$F = D$ 0
0	0	1	0	1	$F = \bar{D}$ 1
0	0	1	1	0	
0	1	0	0	0	$F = 0$ 2
0	1	0	1	0	
0	1	1	0	1	$F = 1$ 3
0	1	1	1	1	
1	0	0	0	0	$F = 0$ 4
1	0	0	1	0	
1	0	1	0	1	$F = 1$ 5
1	0	1	1	1	
1	1	0	0	0	$F = D$ 6
1	1	0	1	1	
1	1	1	0	1	$F = \bar{D}$ 7
1	1	1	1	0	



Digital circuits → combinational (no memory)
 → sequential (memory)

Sequential circuits

① set/Reset/Flipflop

not used

1 bit

8 byte

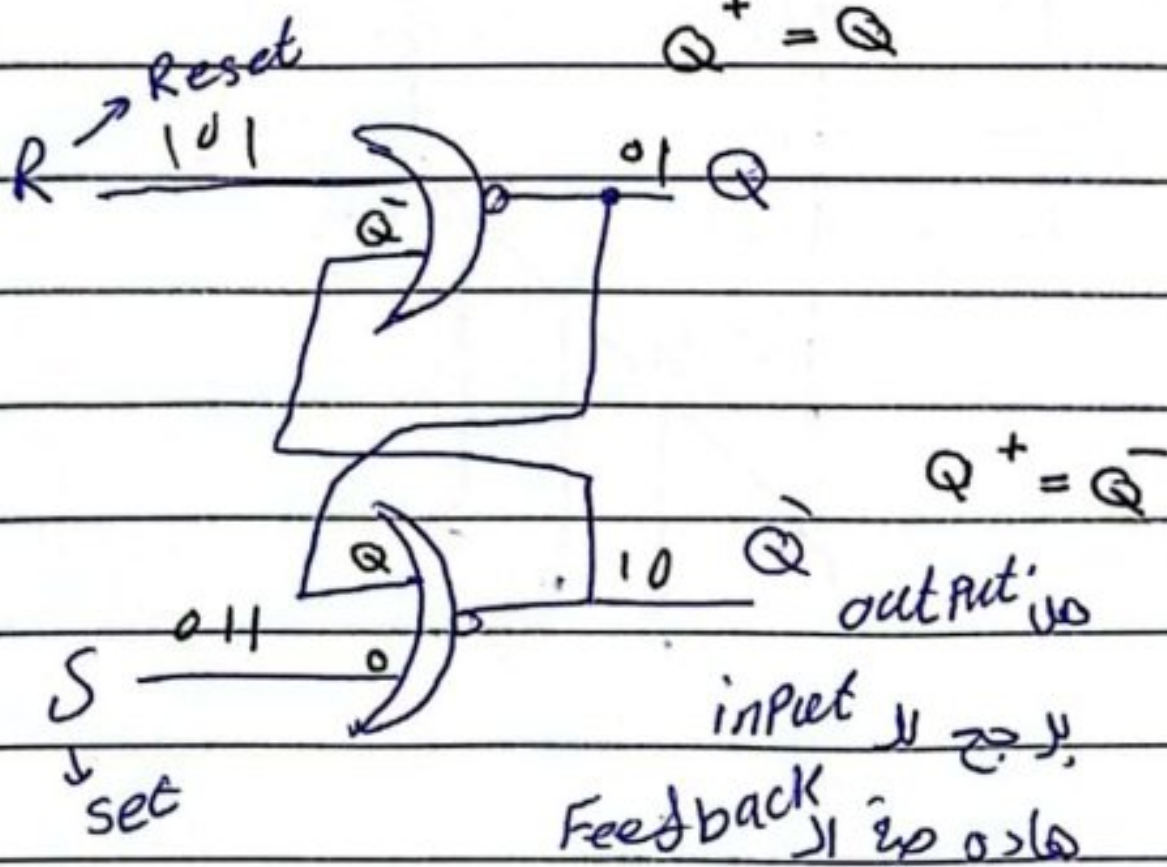
16 word

S R F / F

Q = output

32 Double word

64 Double Double word.

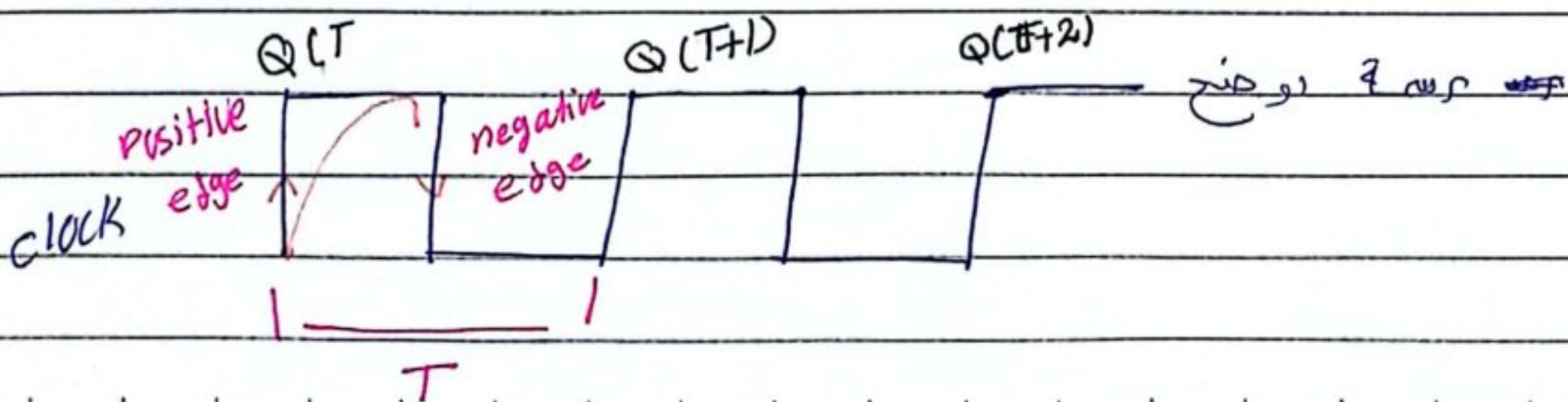
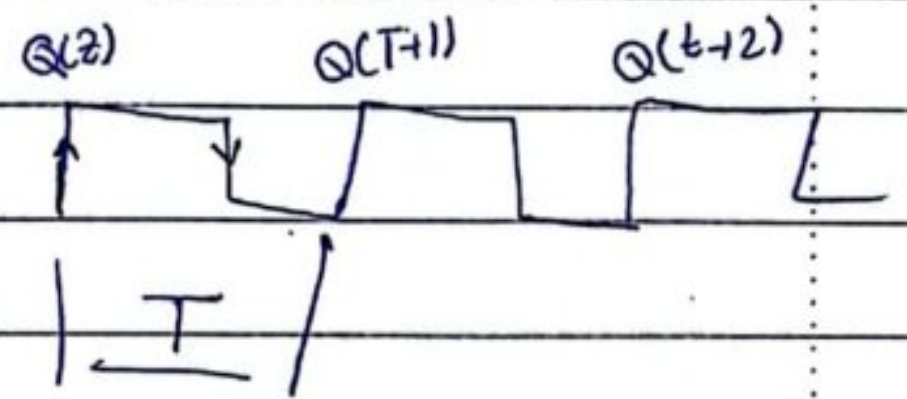
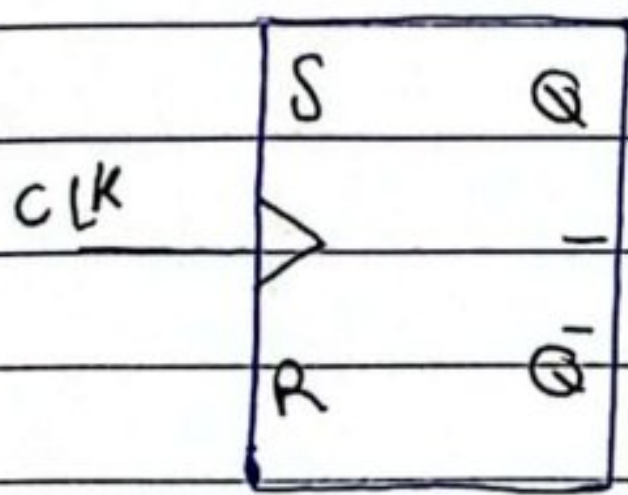


1 set to high
 0 reset to low

S	R	Q ⁺	NOR
0	0	Q no change	
0	1	0 Reset	
1	0	1 set	
1	1	X an determinat	

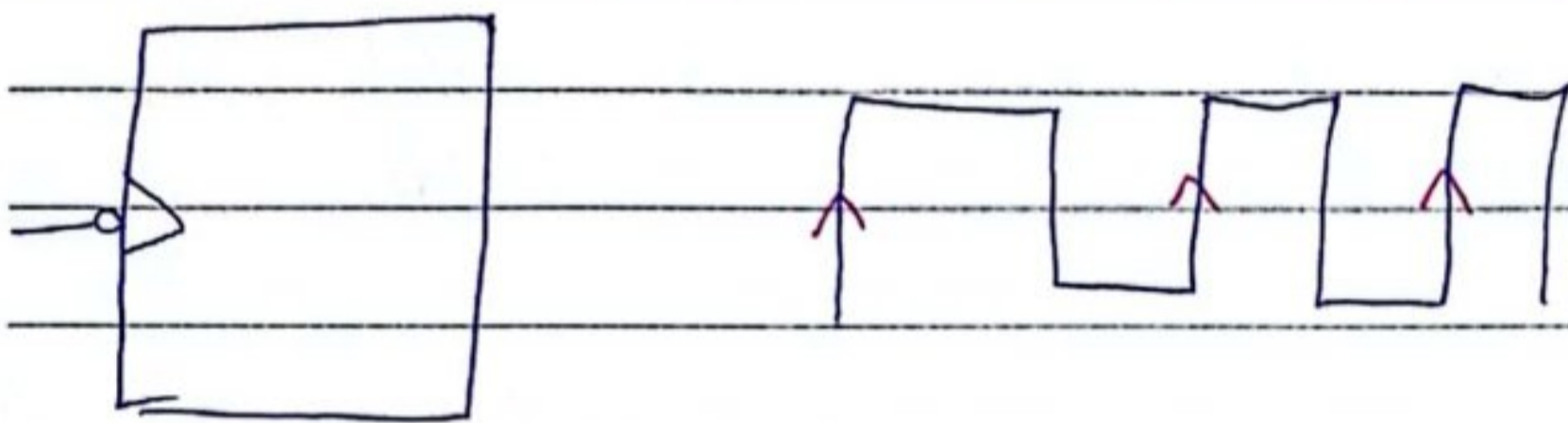
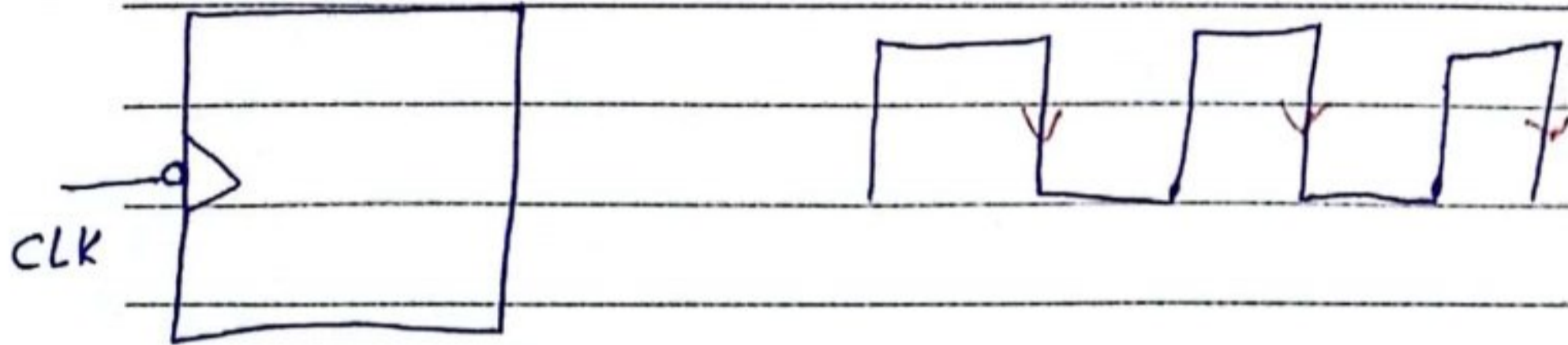
Date

سimplified



Flip / Flop ↔ Latch

يعني لما نفتح الباب ونغلقه
يسمى latch



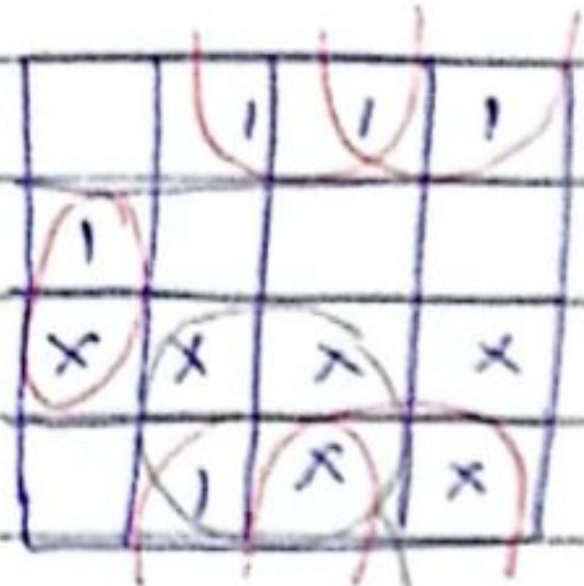
30 April 2024

$$\text{CH5: 12/a) } x = \bar{A}B + DC + \overline{(A+B)}(ACD + \bar{B}E)$$

$$= \bar{A}B + DC + (\bar{A}\bar{B})(\overbrace{ACD}^{A \cdot 2010} + \bar{B}E) \quad \bar{0} \times \bar{0} = \bar{0}$$

$$= \bar{A}B + DC + \bar{A}\bar{B}\bar{E}$$

Ex:-



خارج اعمه دائرة 4
4-4

لدي 0 في الـ 4

note

$$\begin{aligned} \text{⊗ } AB + ABC &= AB(1 + C) \\ &= AB \end{aligned}$$

characteristics

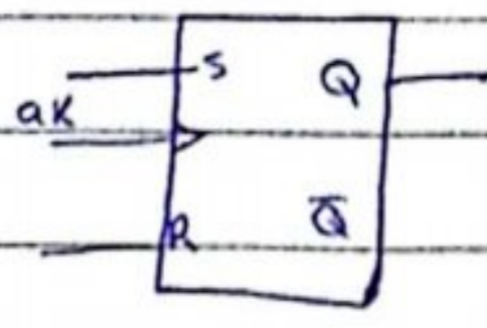
state equation

* name

symbol

truth table

1 SR/F

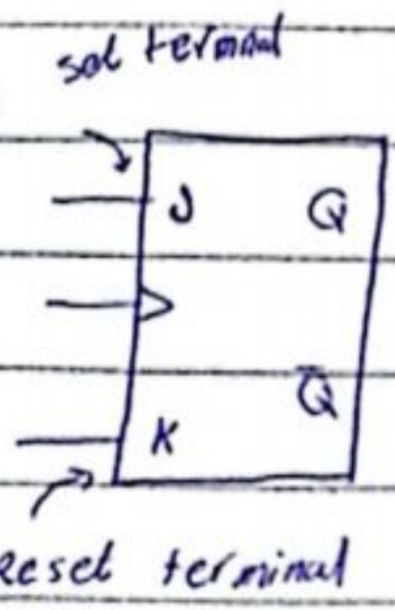


S	R	Q ⁺
0	0	Q no change
0	1	0 Reset
1	0	1 set
1	1	X not value

Function with

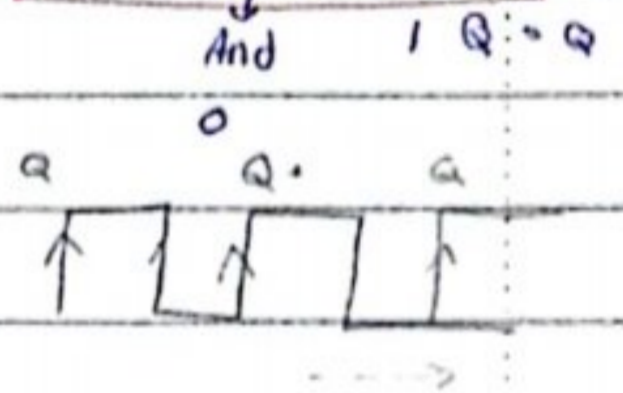
don't care

2 JK/F

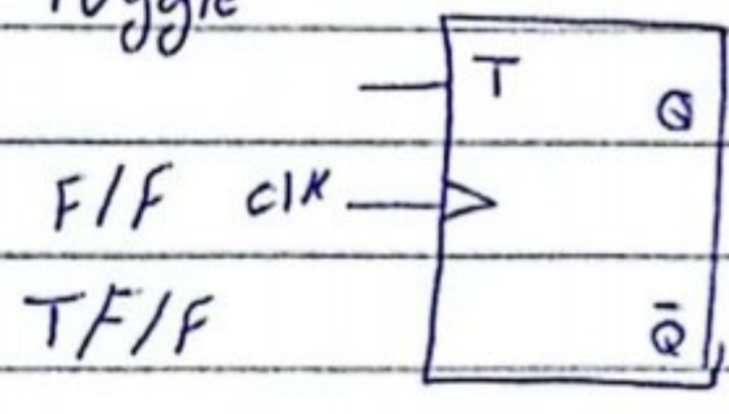


J	K	Q ⁺
0	0	Q No change
0	1	0 Reset
1	0	1 set
1	1	\bar{Q} toggle

$$Q^+ = J\bar{Q} + \bar{K}Q$$



3 Toggle

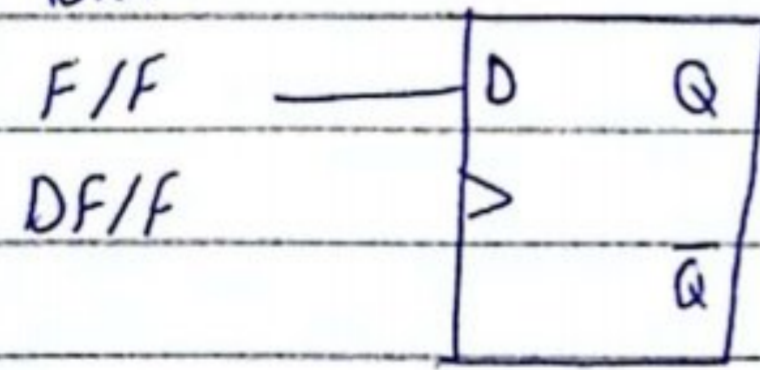


T	Q ⁺
0	Q
1	\bar{Q}

$$Q^+ = T \oplus Q$$

$$T\bar{Q} + \bar{T}Q$$

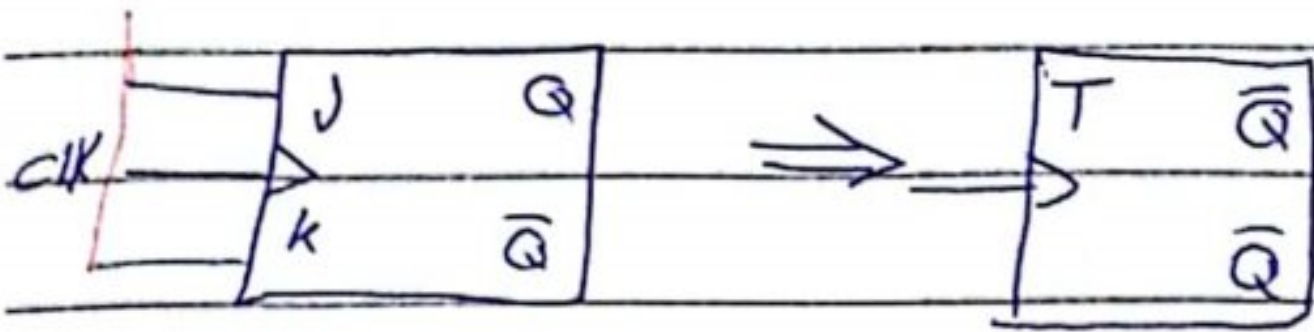
4 Data



D	Q ⁺
0	0
1	1

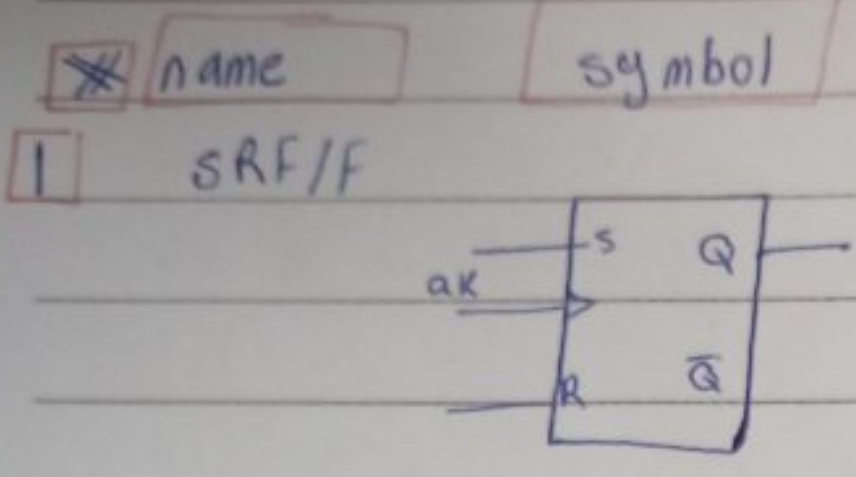
$$Q^+ = D$$

How make JKFF \rightarrow TFF



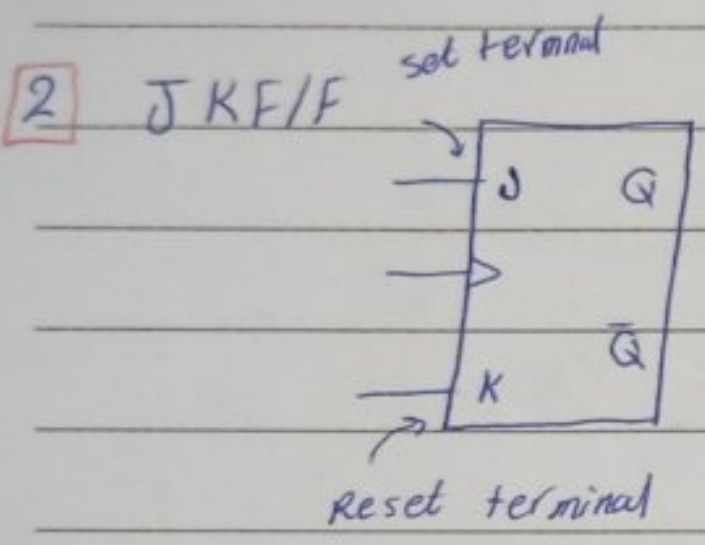
2024 / May / 5

characteristics
equation



S	R	Q ⁺
0	0	Q no change
0	1	0 Reset
1	0	1 set
1	1	x not value

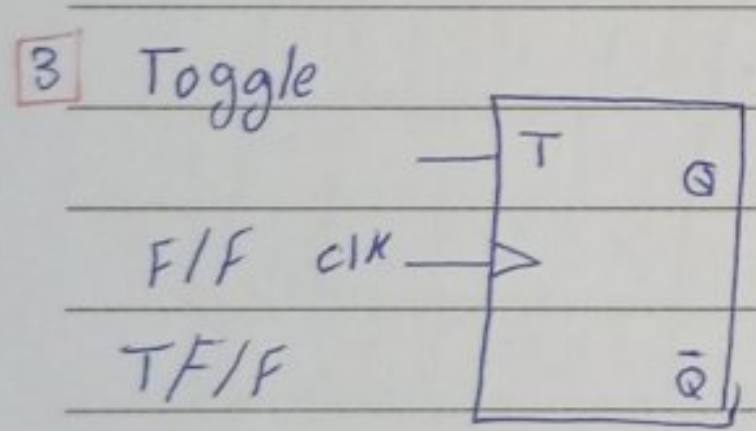
Function
don't care



J	K	Q ⁺
0	0	Q No Change
0	1	0 Reset
1	0	1 set
1	1	Q-bar toggle

$$Q^+ = J\bar{Q} + KQ$$

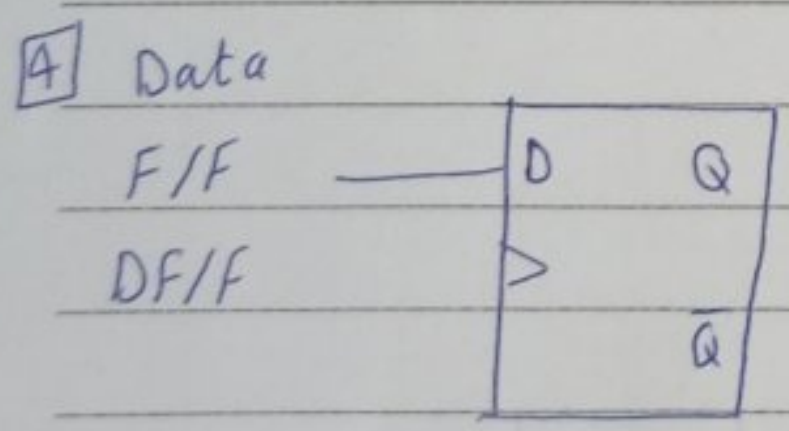
And $1\bar{Q} = \bar{Q}$



T	Q ⁺
0	Q
1	Q-bar

$$Q^+ = T \oplus Q$$

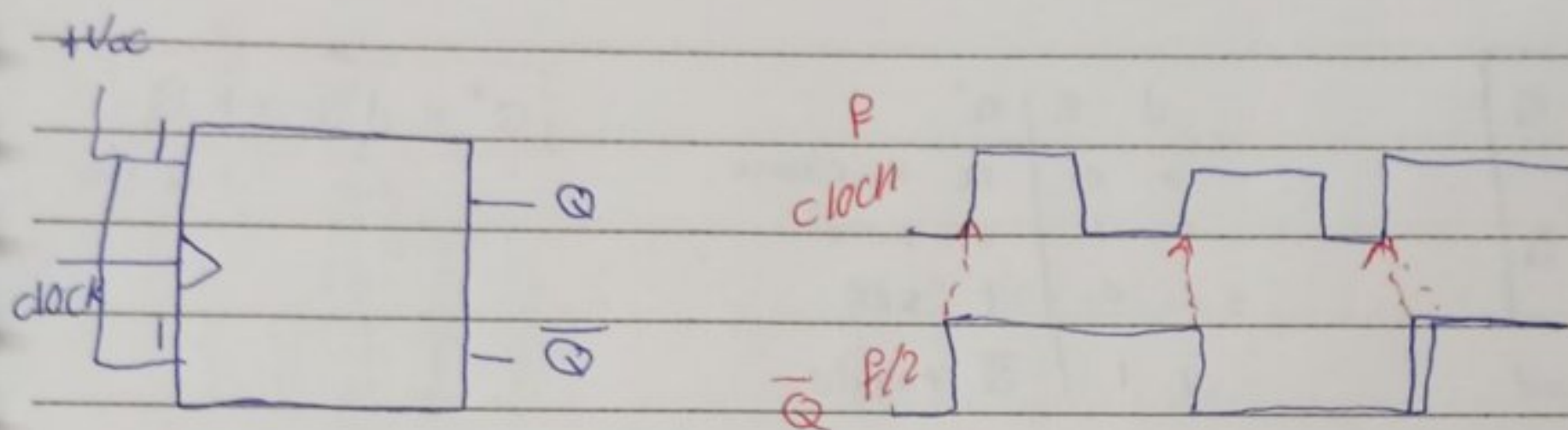
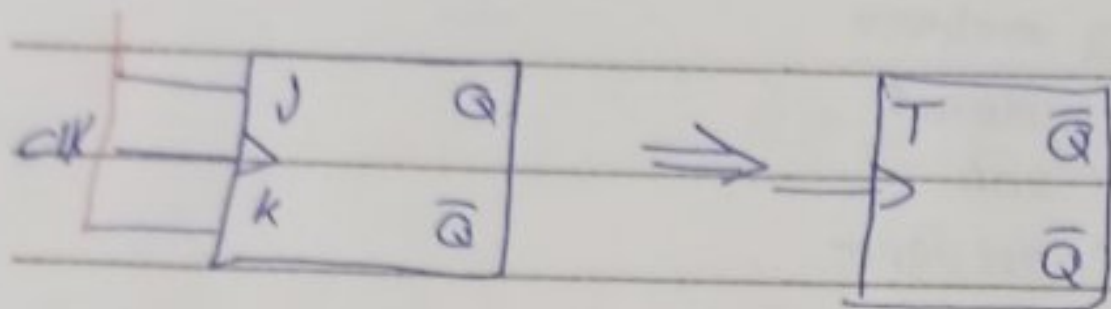
$$T\bar{Q} + \bar{T}Q$$



D	Q ⁺
0	0
1	1

$$Q^+ = D$$

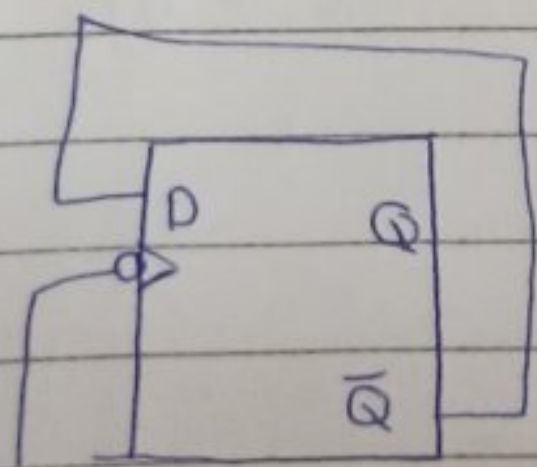
How make JKFF \rightarrow TFF



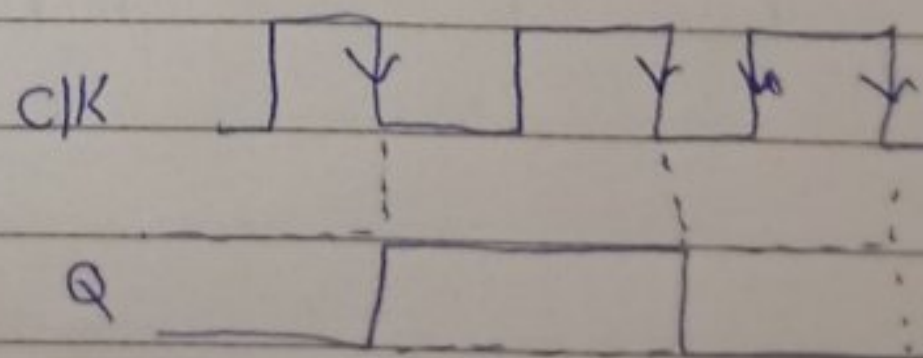
$$Q^+ = \overline{Q}$$

Div der by 2

J	K	Q^+
0	0	Q
0	1	0
1	0	1
1	1	\overline{Q}



$$Q^+ = D = \overline{Q}$$



Divi der by 2

CH5 / 12 :-

$$a) x = \bar{A}B + DC + (\bar{A} + \bar{B})(ACD + \bar{B}E)$$

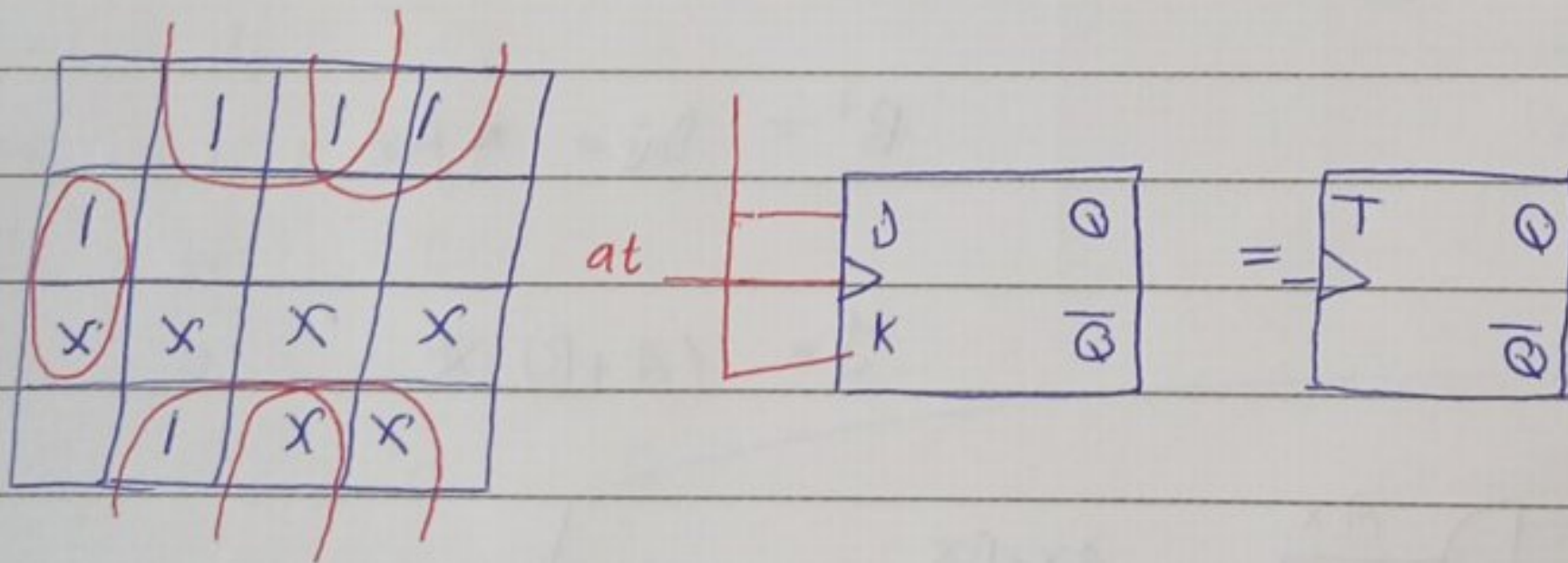
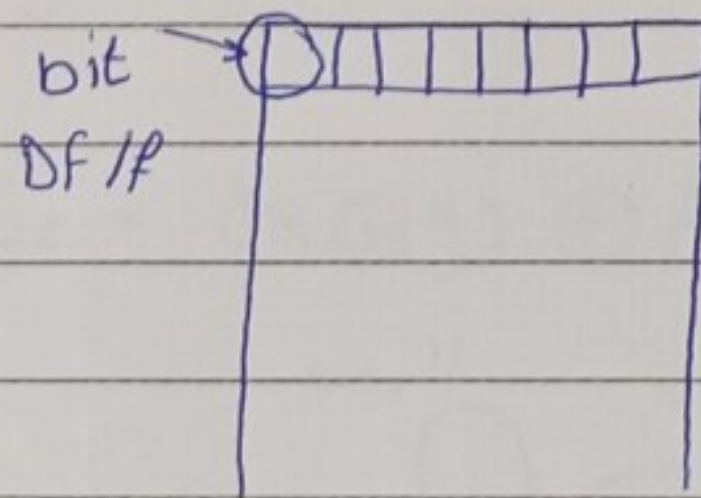
$$= \bar{A}B + DC + (\bar{A}\bar{B})(ACD + \bar{B}E)$$

$$= \bar{A}B + \frac{DC}{A} + \bar{A}\bar{B}E$$

$$= \underline{AB} + \underline{ABC}$$

#

$$= \underline{ABC(1+C)}$$



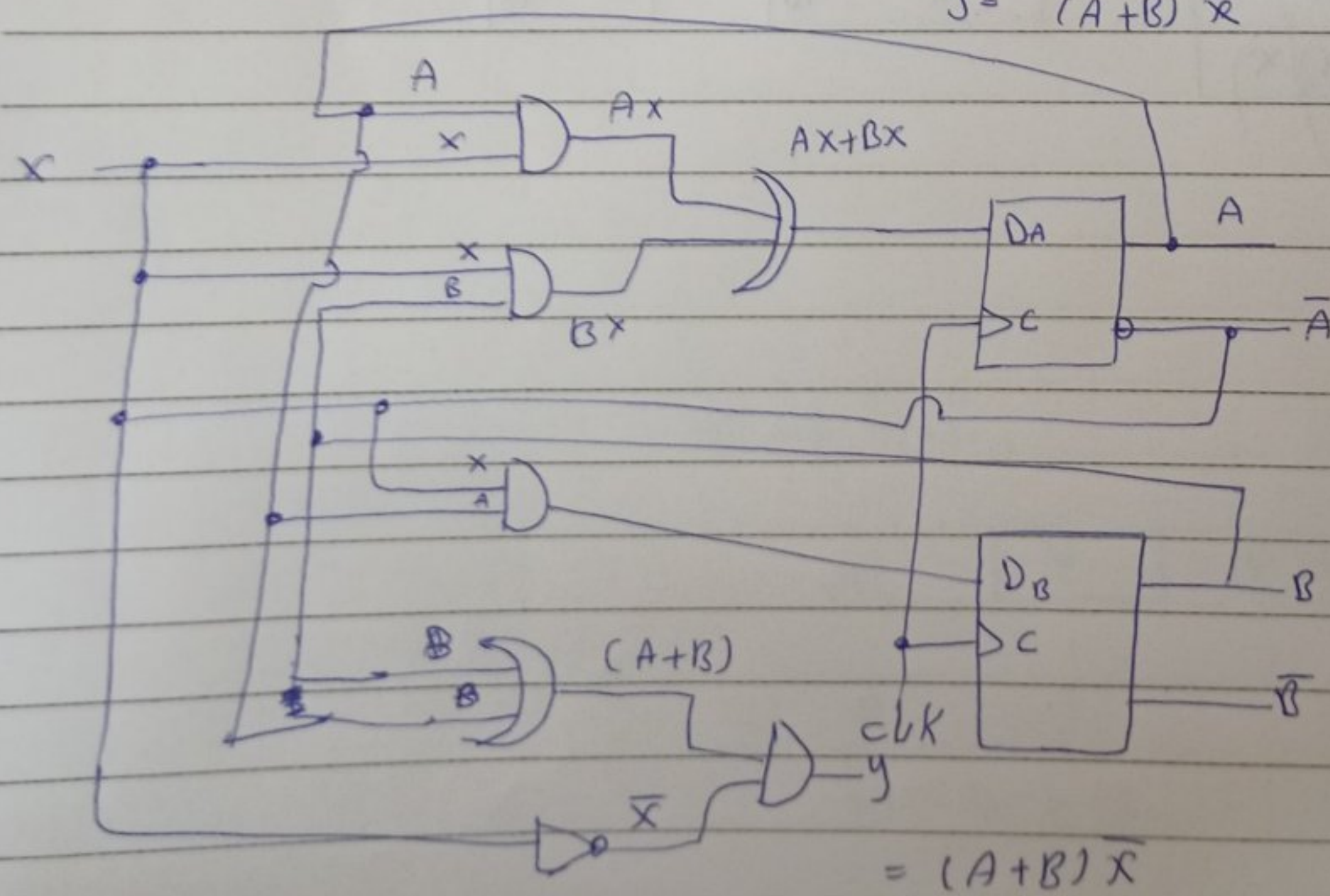
Analysis with DFF

A	B	x	A ⁺	B ⁺	y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	1	1	1	0

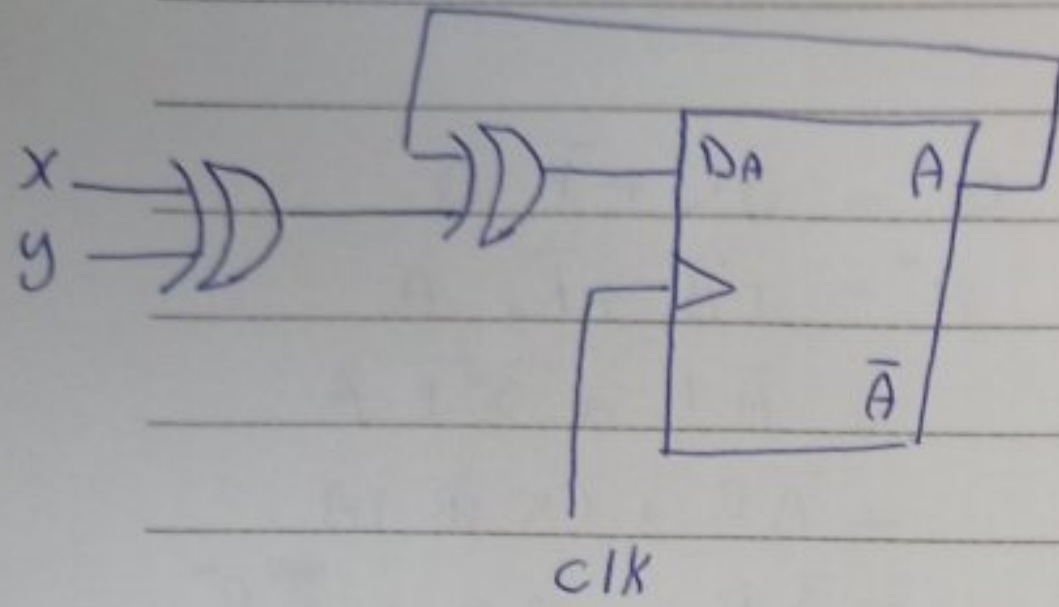
$$A^+ = D_A = AX + BX = (A+B)X$$

$$B^+ = D_B = AX$$

$$y = (A+B)\bar{x}$$



EX 8-

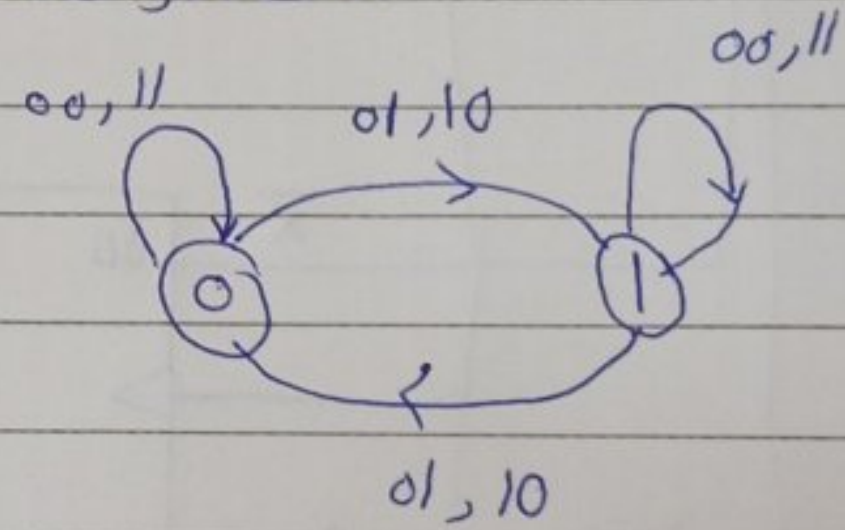


		Q
0	0	0
0	1	1
1	0	1
1	1	0

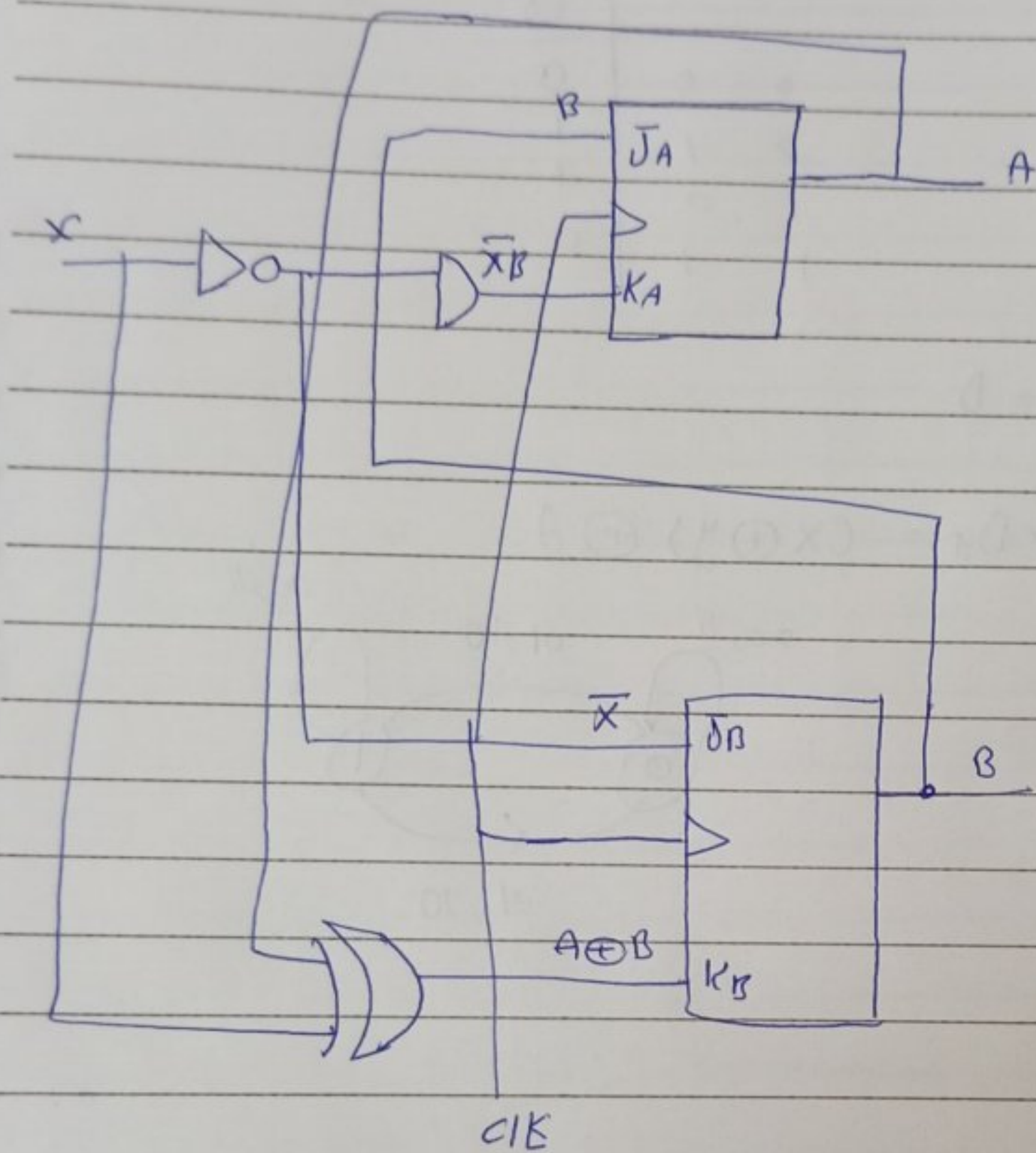
$$Q^+ = D$$

$$A^+ = D_A = (x \oplus y) \oplus A$$

A	x	y	A ⁺
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



Analysis with JK F/P



$$\begin{aligned}
 Q^+ &= J\bar{Q} + \bar{K}Q \\
 A^+ &= J\bar{A} + \bar{K}_A A \\
 &= \bar{A}B + \bar{X}BA \\
 &= \bar{A}B + (X + \bar{B})A \\
 &= \bar{A}B + AX + \bar{A}\bar{B}
 \end{aligned}$$

$$\begin{aligned}
 B^+ &= J_B \bar{B} + \bar{K}_B B \\
 &= \bar{X}B + \overline{(A \oplus X)}B
 \end{aligned}$$

$$= \bar{B}\bar{X} + (A \odot X)B$$

$$\begin{aligned}
 B^+ &= \bar{B}\bar{X} + (B \odot X)B \\
 &= \bar{B}\bar{X} + (AX + A\bar{X})B \\
 &= \bar{B}\bar{X} + ABX + \bar{A}B\bar{X}
 \end{aligned}$$

$\begin{matrix} 000 & 111 & 010 \\ 1 & 1 & 0 \end{matrix}$

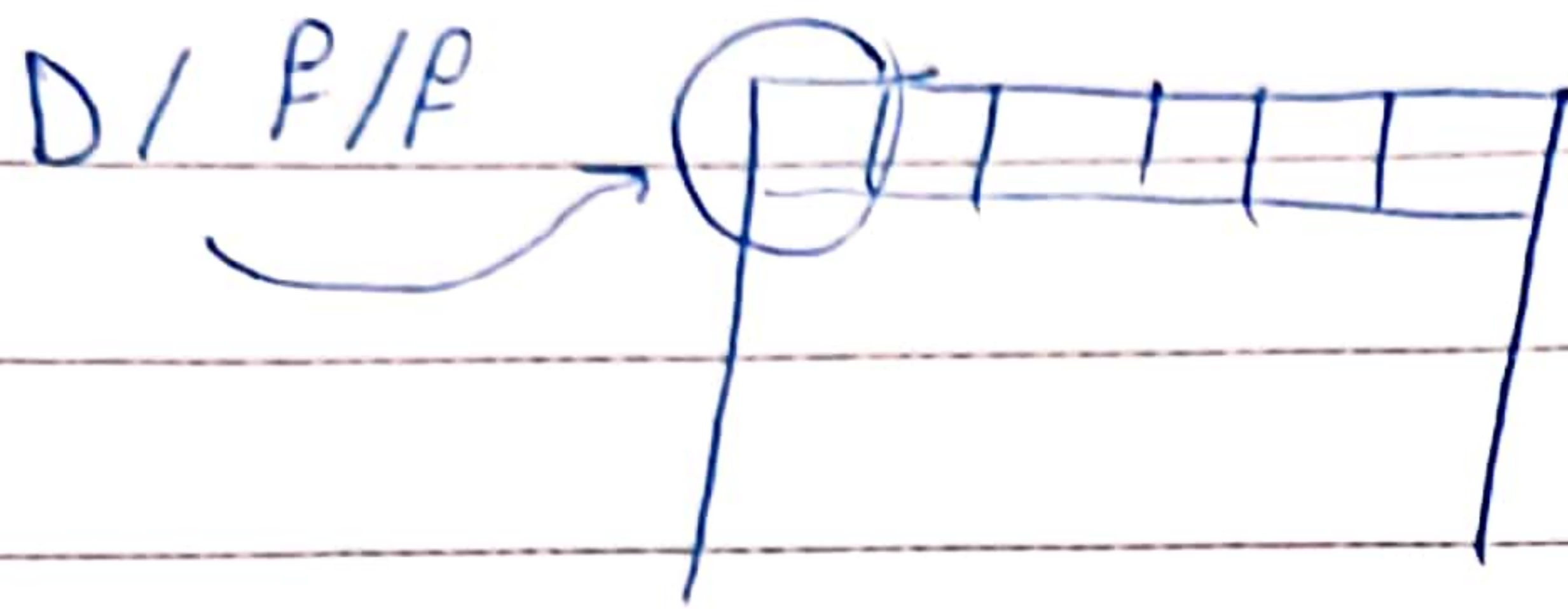
$$\sum (0, 2, 4, 7)$$

	\bar{X}			
	\bar{B}		B	
\bar{A}	1	1	1	1
A	1	1	1	1

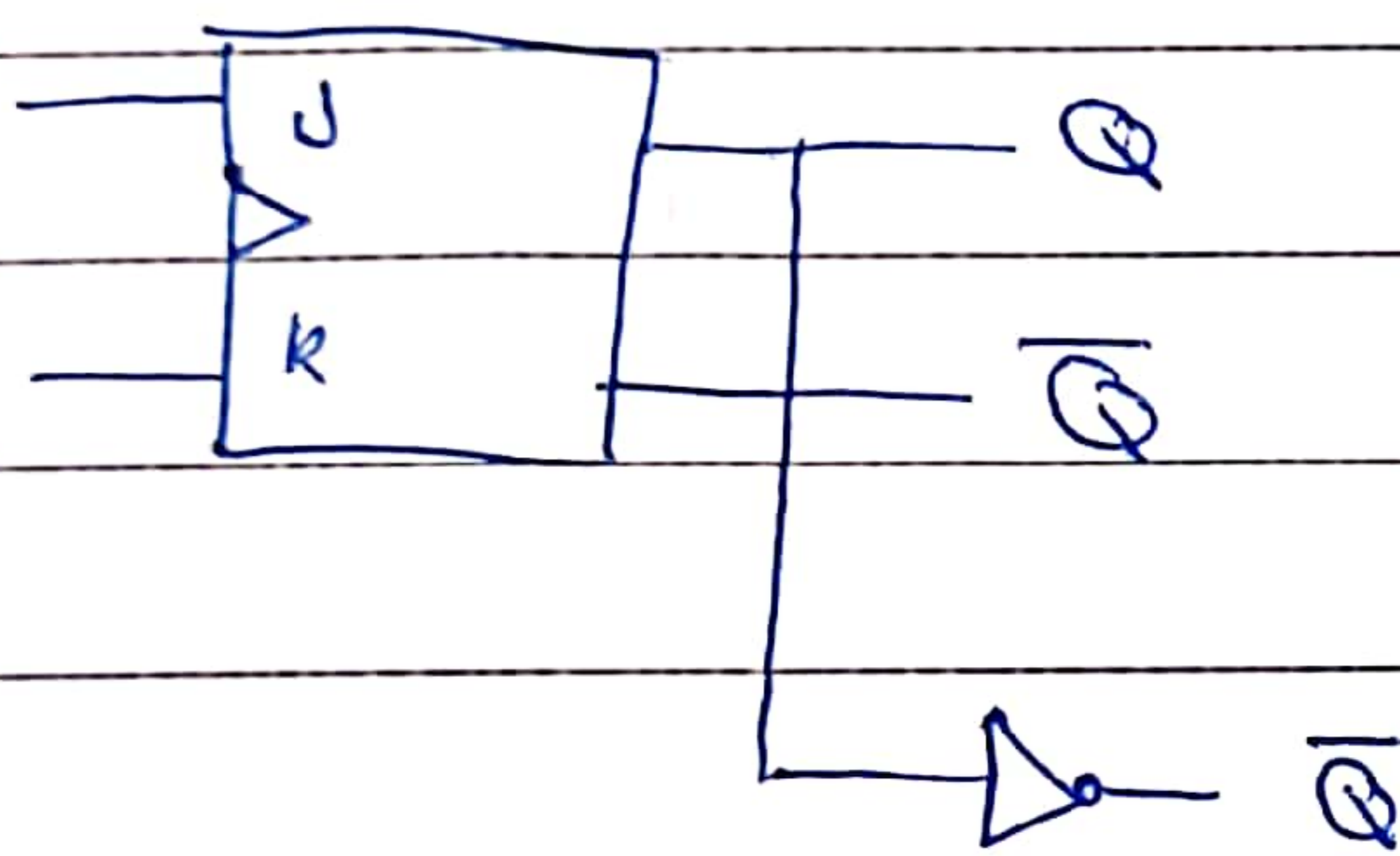
$$= \sum (2, 3, 4, 5, 7)$$

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Exo- bit



J	K	Q ⁺
0	0	Q
0	1	0
1	0	1
1	1	\bar{Q}



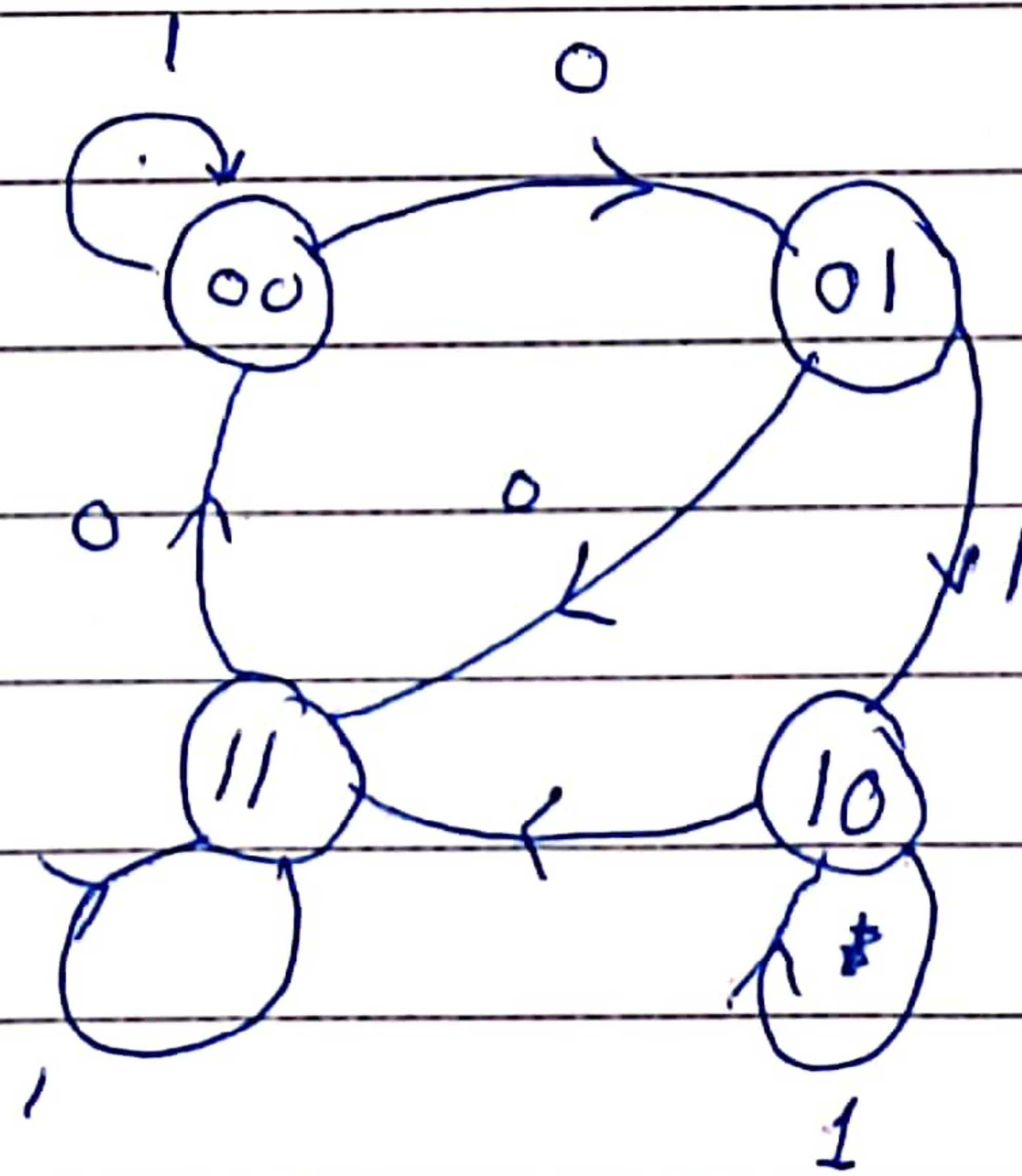
Exo- $A^+ = \bar{A}B + A\bar{B} + Ax$

$\Sigma(2, 3, 4, 5, 7)$

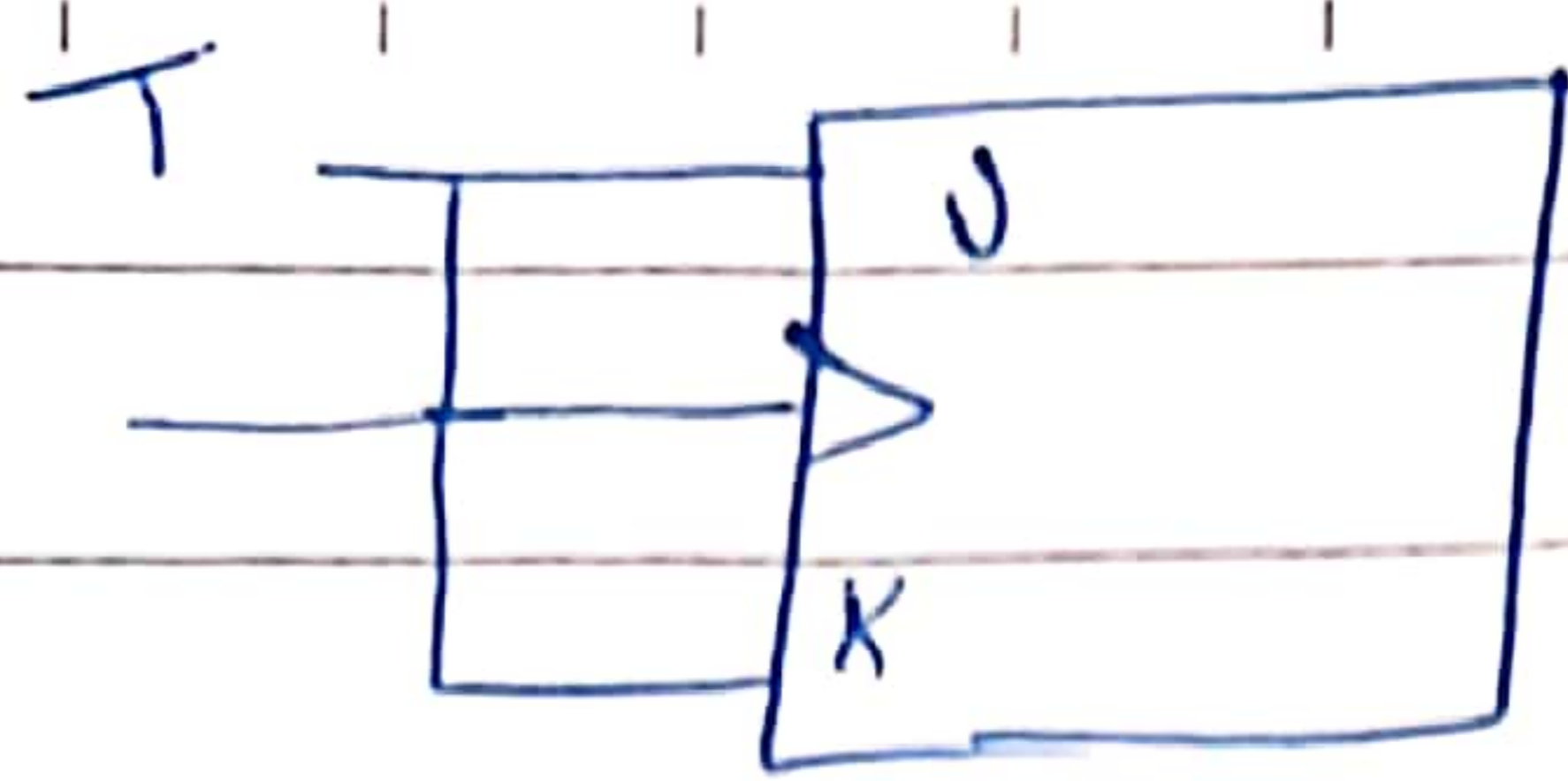
$B^+ = \bar{B}\bar{x} + ABx + \bar{A}B\bar{x}$

$= \Sigma(0, 2, 4, 7)$

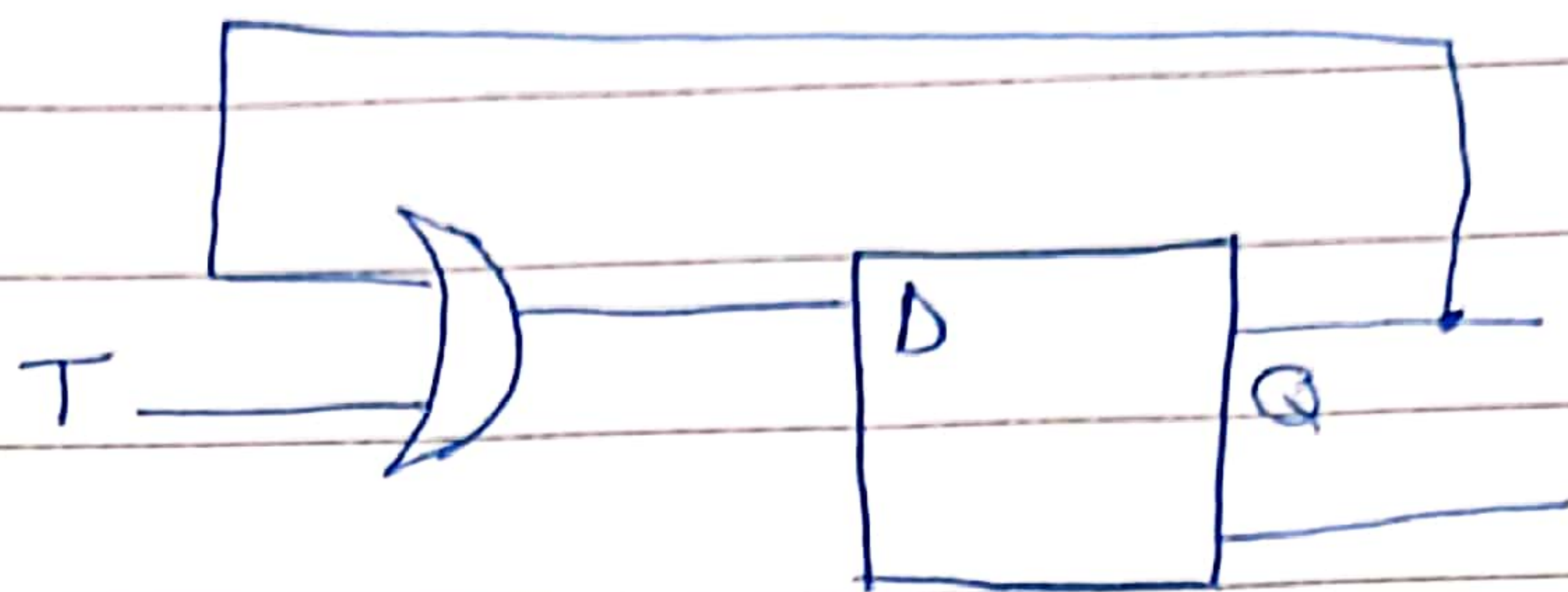
A	B	x	A ⁺	B ⁺
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1



⊗ B x̄



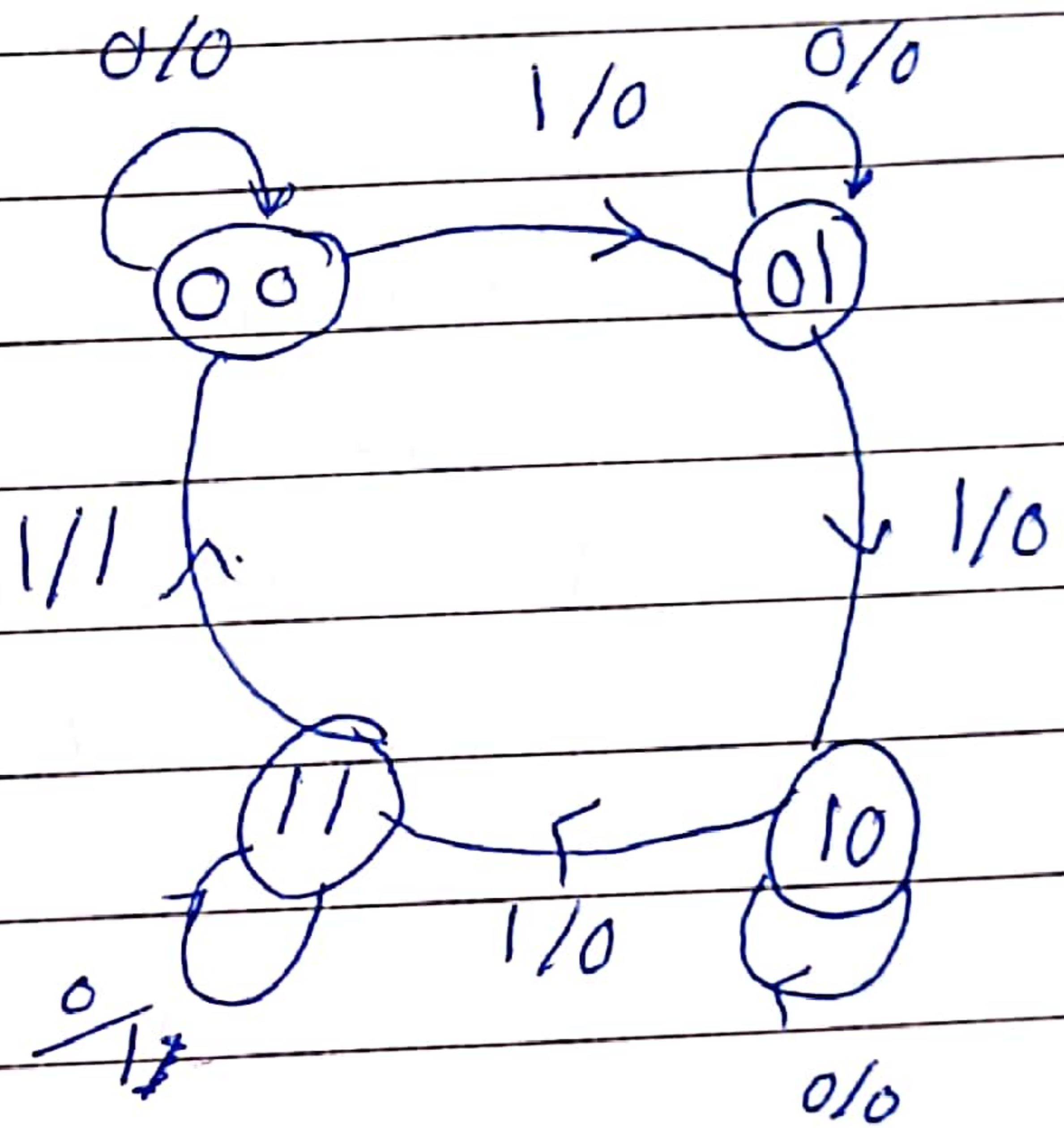
J	K	Q ⁺
0	0	Q
0	1	0
1	0	1
1	1	Q̄

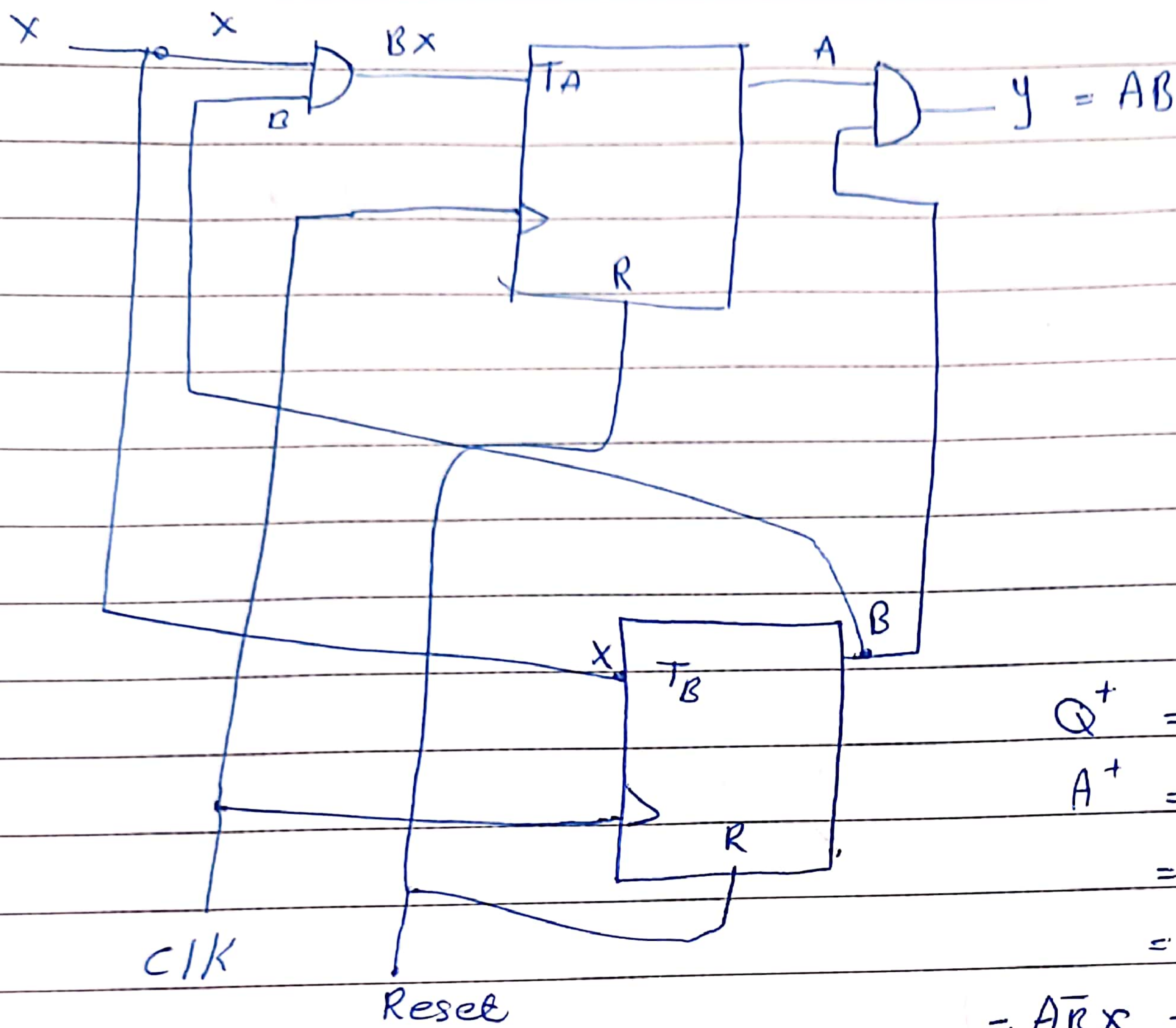


$$Q^+ = D = T \oplus Q$$

⊗ Analysis with T F/P

A	B	x	A ⁺	B ⁺	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1





$$\begin{aligned}
 Q^+ &= T \oplus Q \\
 A^+ &= T_A \oplus A \\
 &= BX \oplus A \\
 &= A \oplus BX \\
 &= A\bar{B}X + \bar{A}BX \\
 &= A(\bar{B} + X) + \bar{A}BX \\
 &= A\bar{B} + AX + \bar{A}BX
 \end{aligned}$$

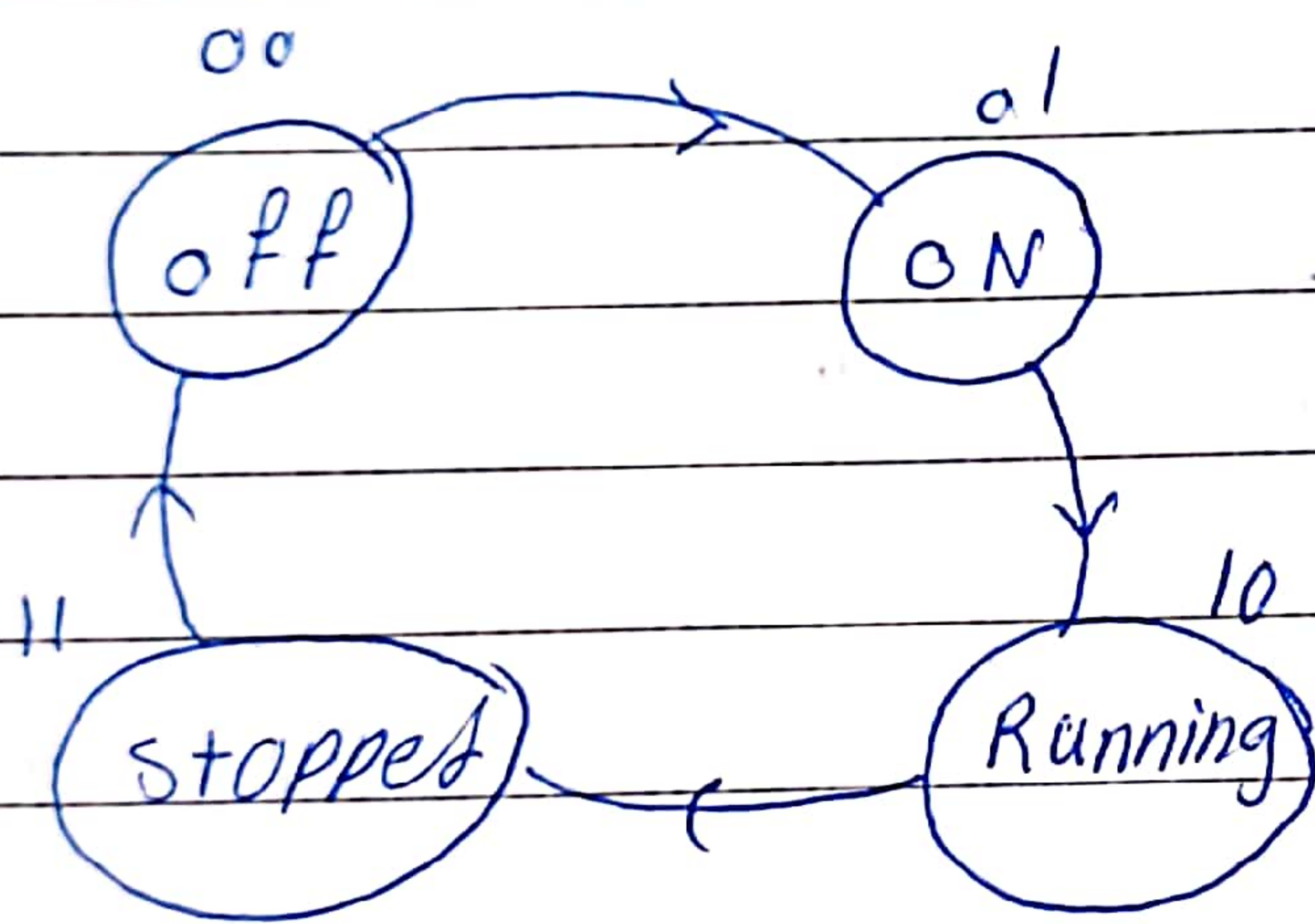
Design of sequential circuit :-

EX:- Design the following machines using seq. digital cct using

- ① D F/P
- ② T F/P
- ③ JK F/P
- ④ Draw the cct.

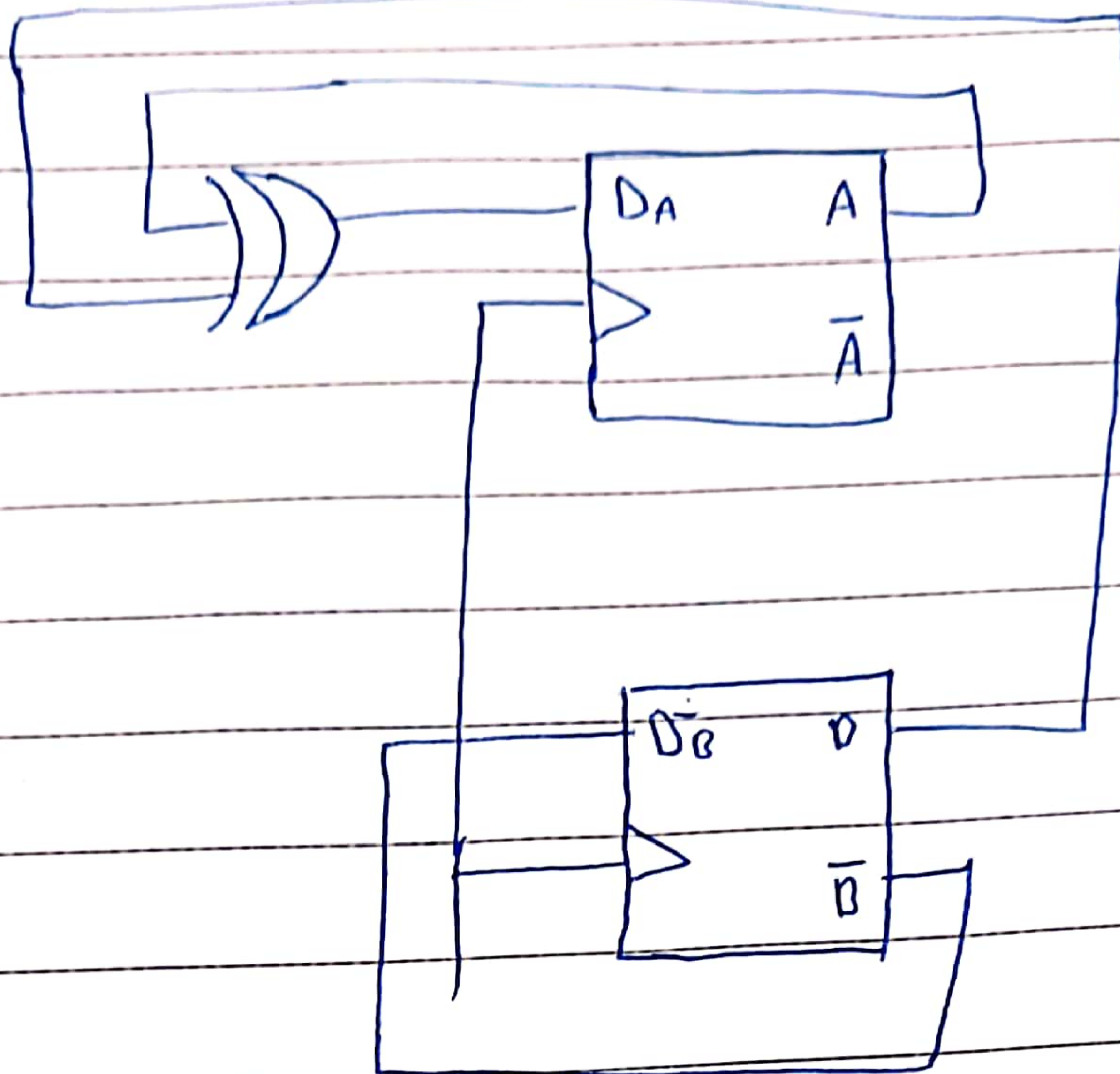
Procces

- ① Draw the state digram
- ② coding
- ③ state table
- ④ Draw the cct.

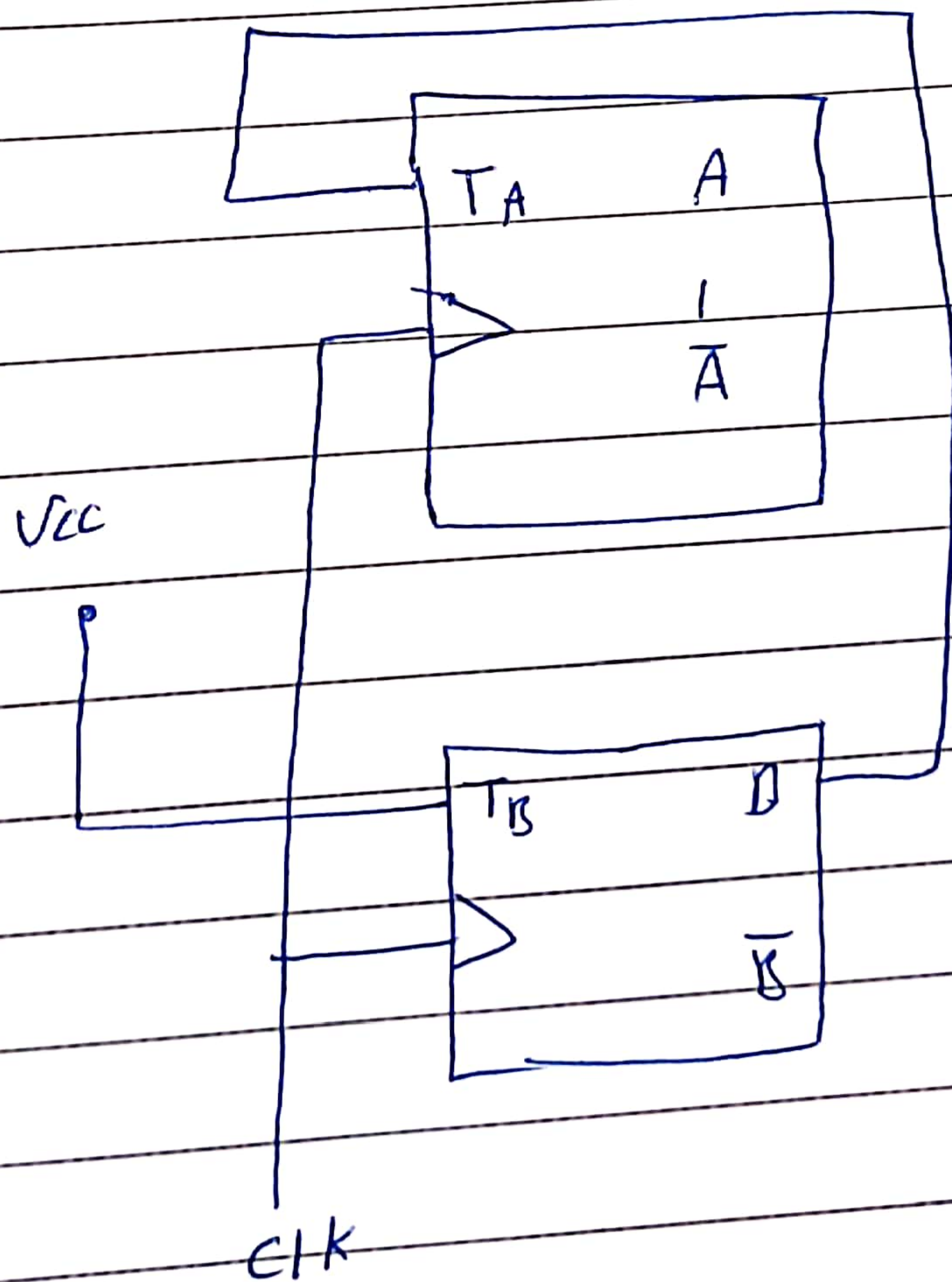


		D		T		JK					
A	B	A^+	B^+	D_A	D_B	T_A	T_B	J_A	K_A	J_B	K_B
0	0	0	1	0	1	0	1	0	x	1	x
0	1	1	0	1	0	1	1	1	x	x	1
1	0	1	1	1	1	0	1	x	0	1	x
1	1	0	0	0	0	1	1	x	1	x	1
				\uparrow +	\overline{B}	\overline{B}	1	\uparrow \overline{B}	\uparrow \overline{B}	1	1

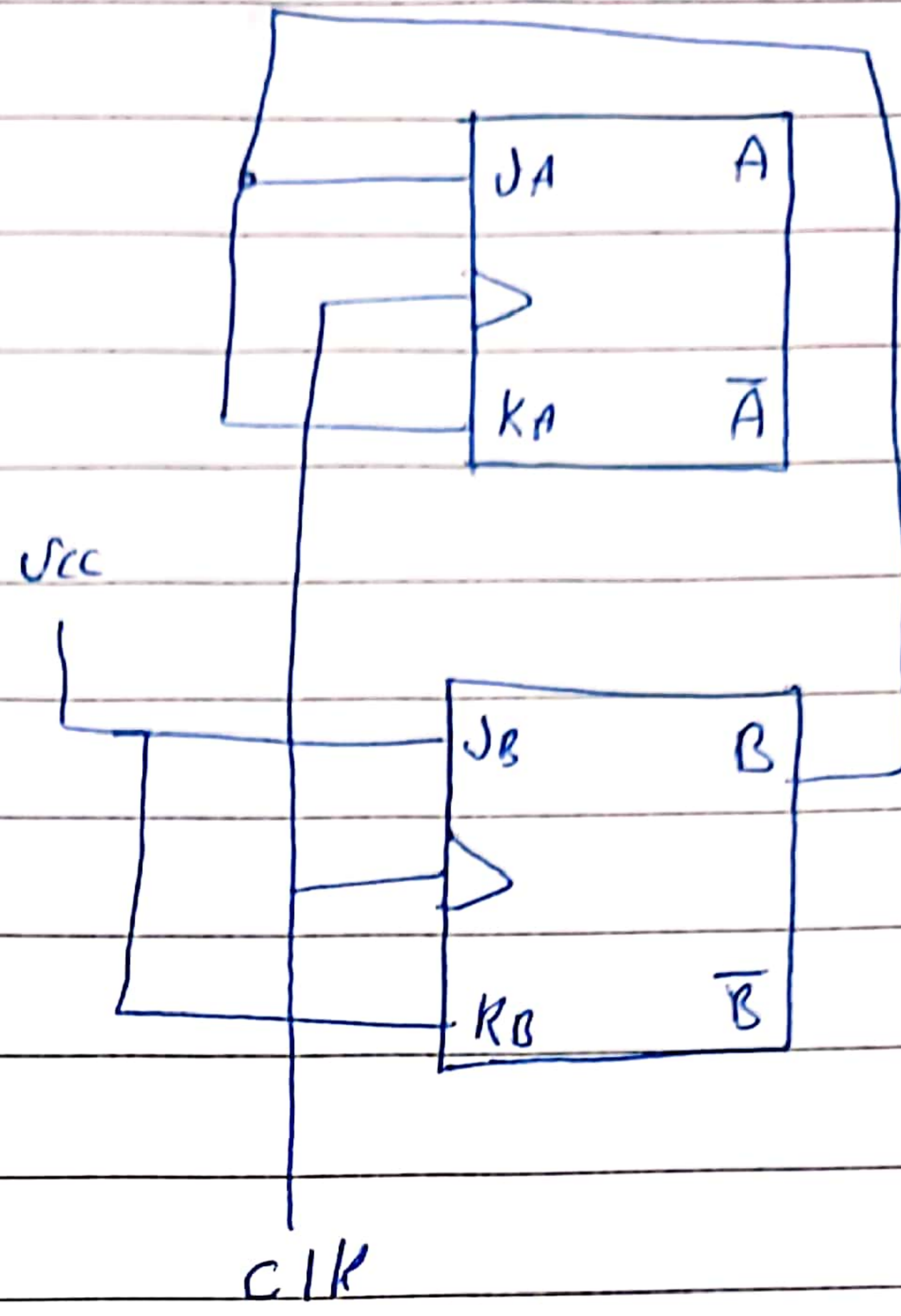
①



②

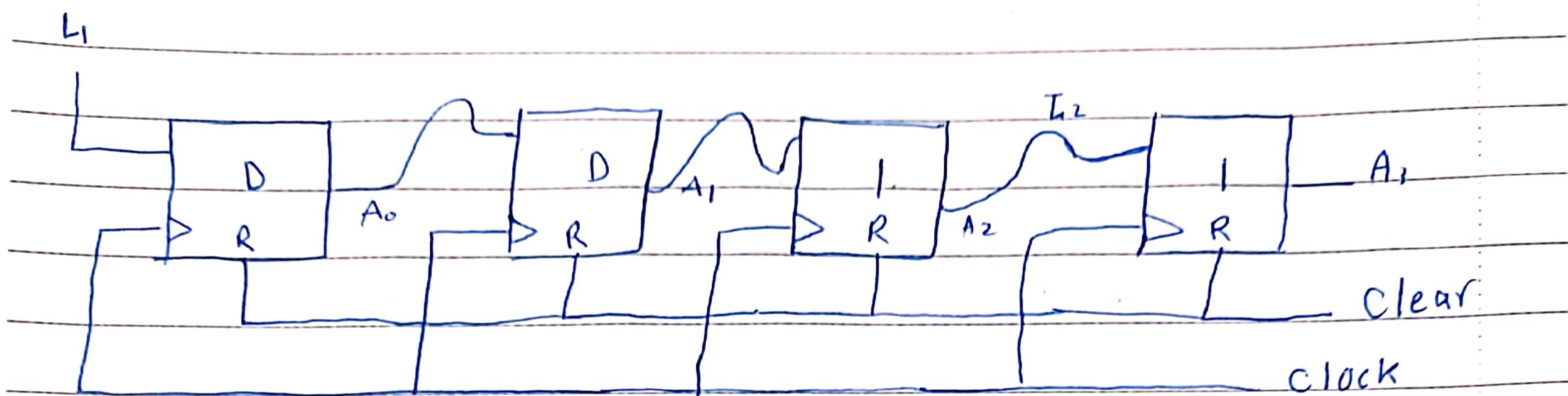


3



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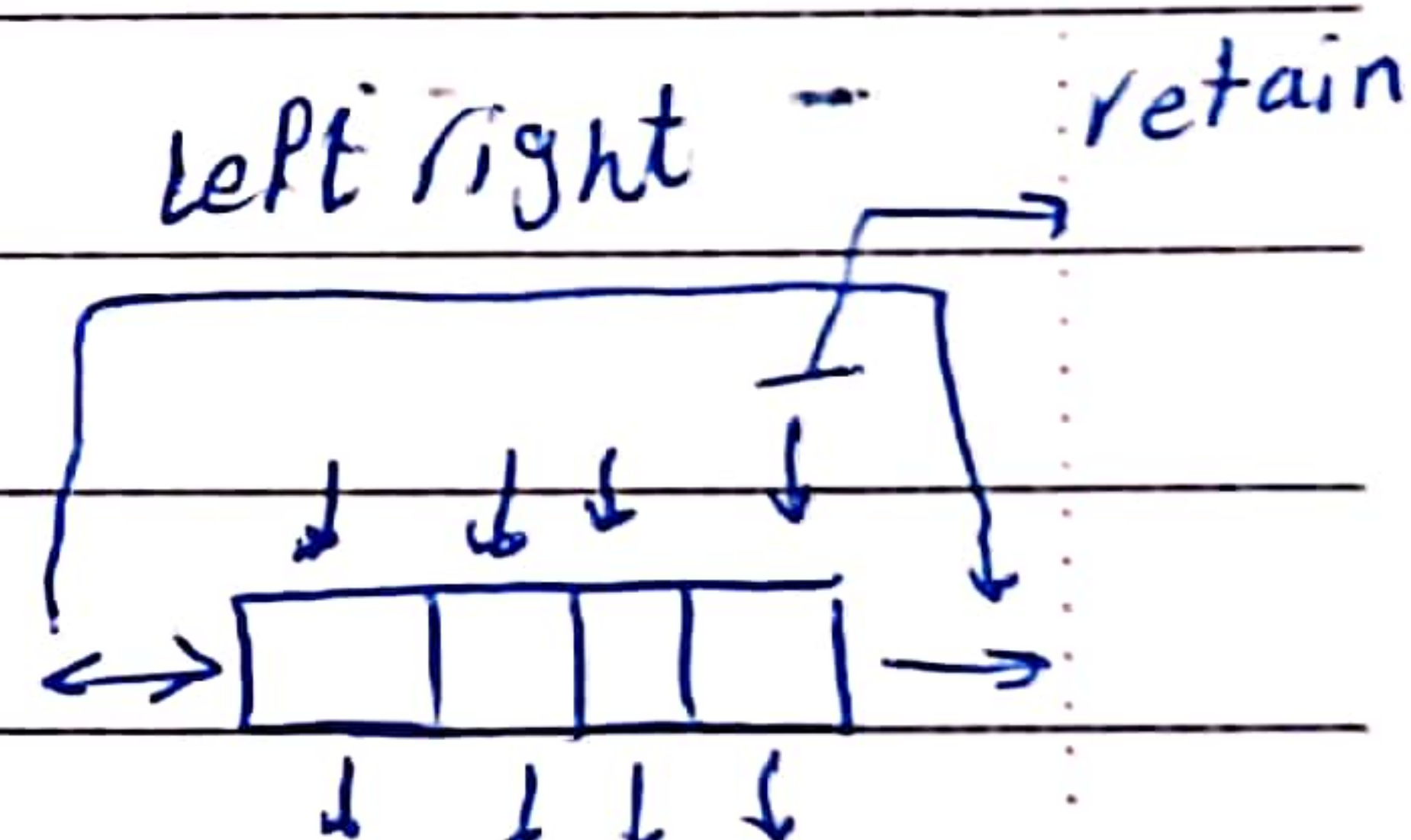
(*)



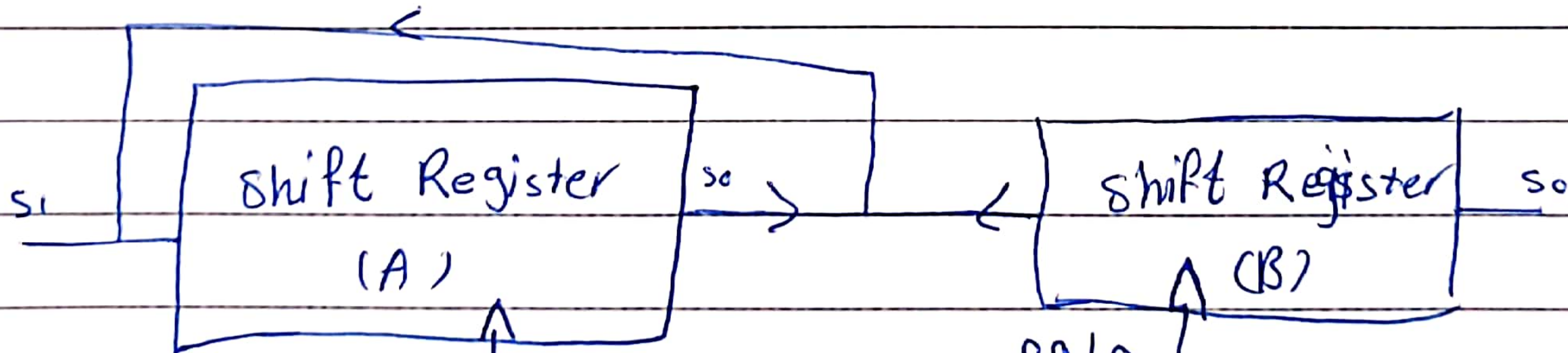
serial serial
4 - Bit Register Parallel-in Parallel-out

0011 → 010111

1 1
1 1 1 1
0 1 1 0 1 1
0 0 1 1 0 0 1 1



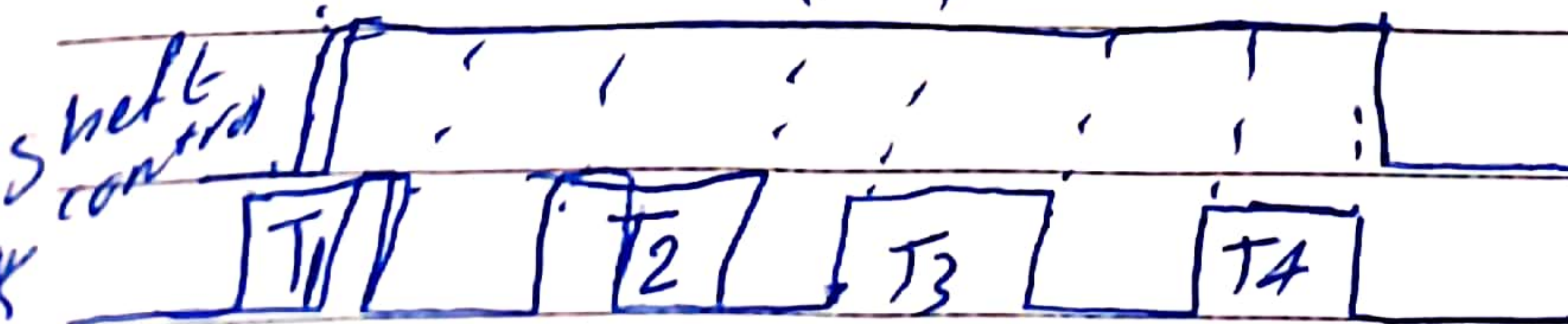
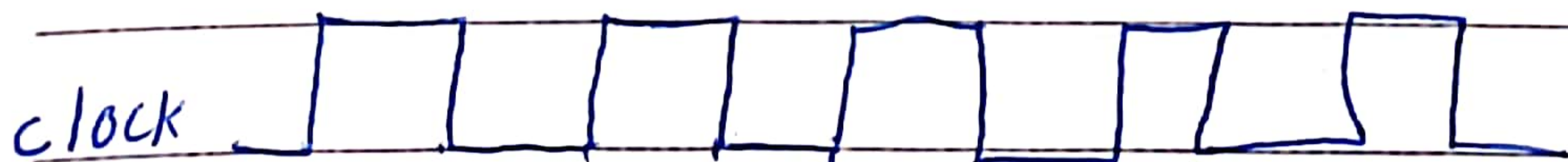
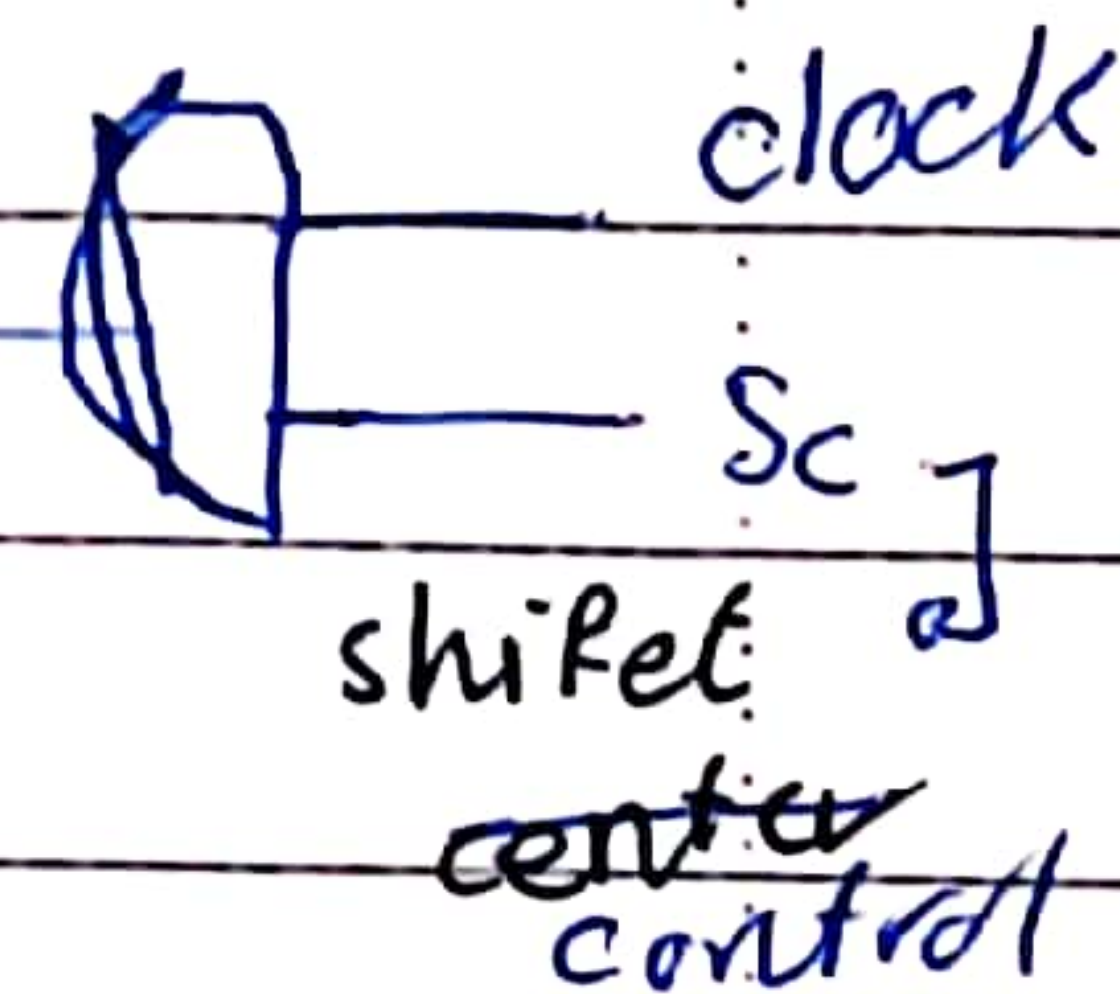
(*)



So = serial output
Si = " input
Sc = " control

T ₁	1101	0010
T ₂	1110	0001
T ₃	0111	1101
T ₄	1011	0110
		1011

COPY AB



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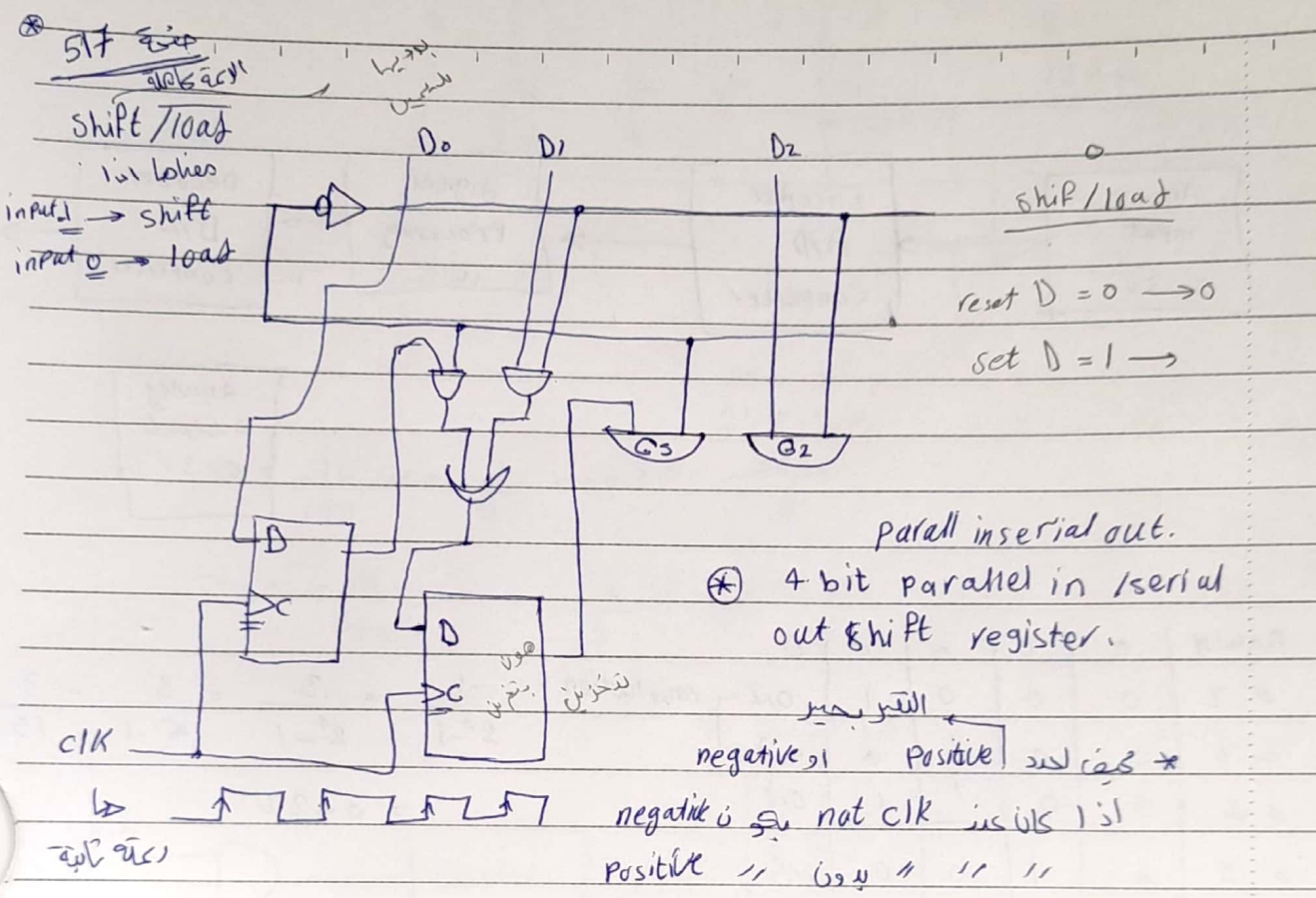
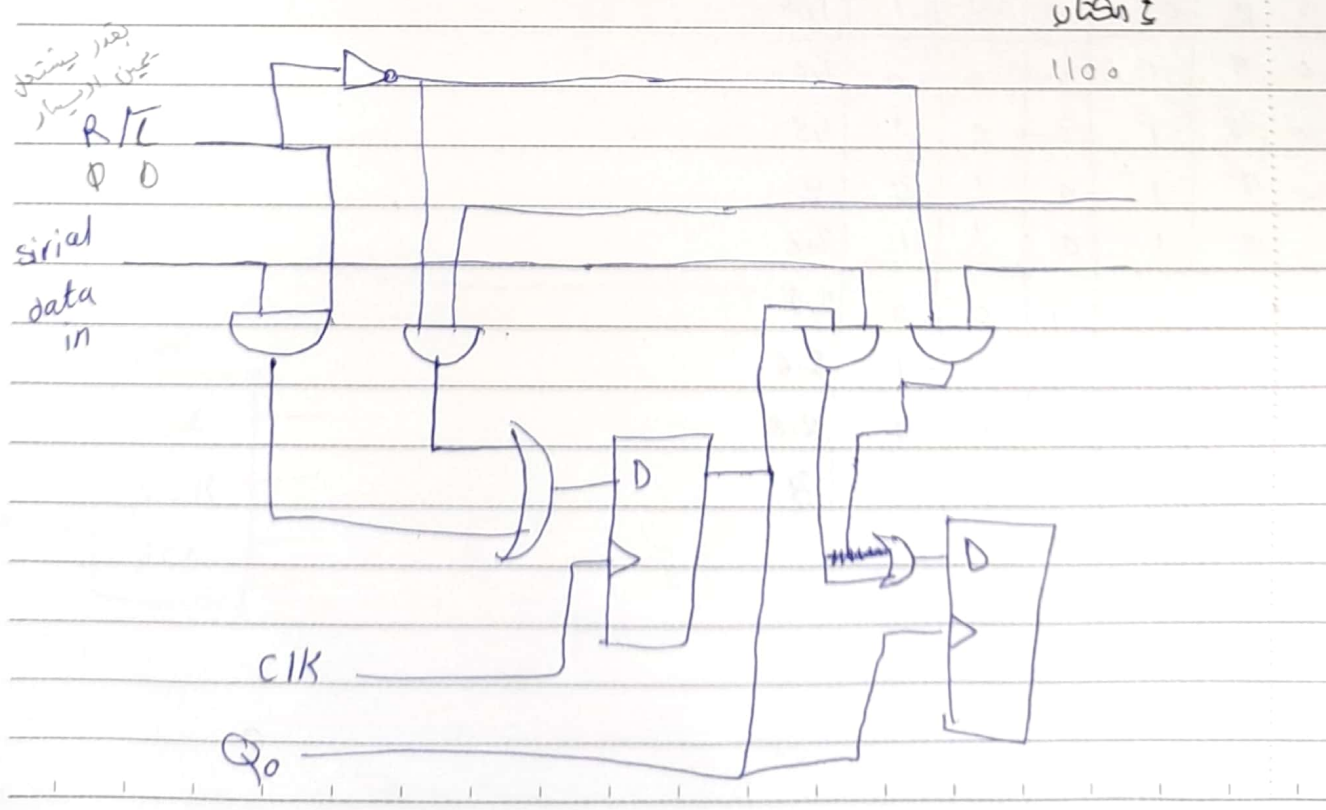
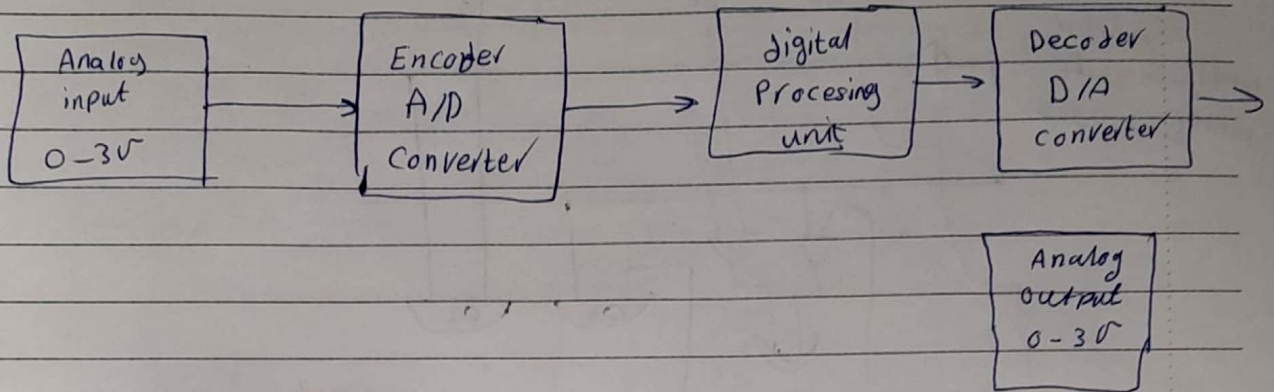


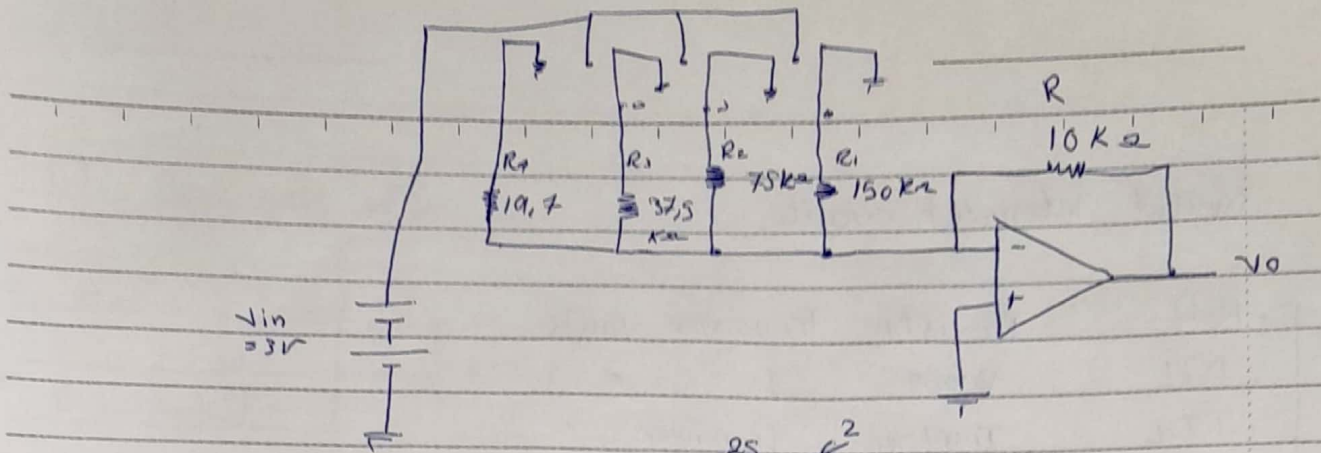
Figure 19 : Four-bit bidirectional register

523 مفتوح الاجوده
في كتابي
1100
delay!!!



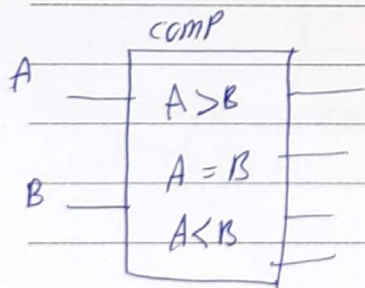
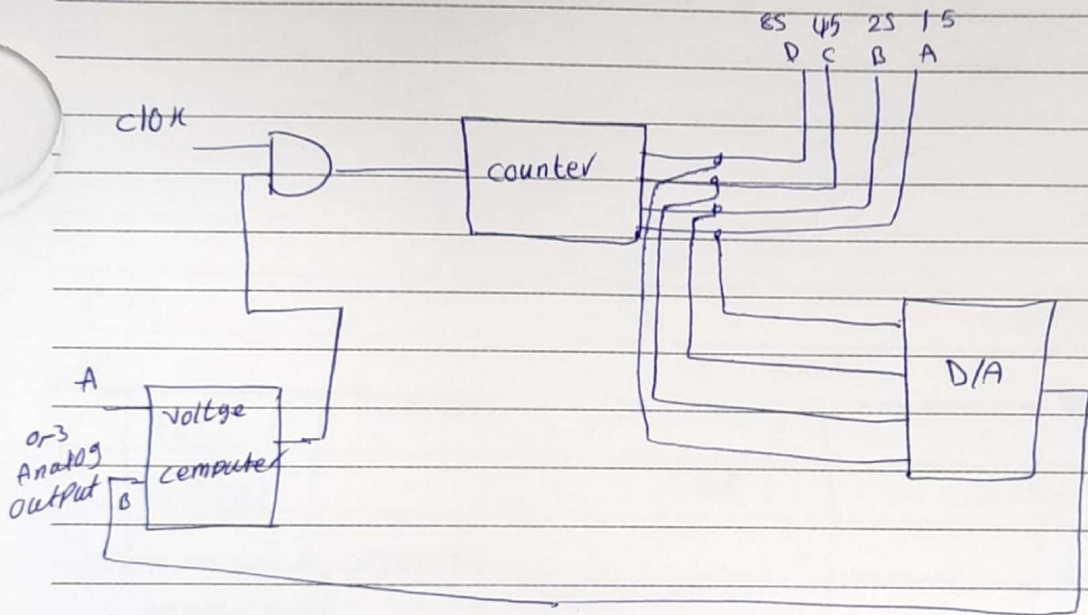


Row	0	0	0	0	0	
" 2	0	0	0	1	0,2	resolution $\frac{3}{2^n-1} = \frac{3}{2^4-1} = \frac{3}{16-1} = \frac{3}{15}$ $= 0,2 V$
" 3	0	0	1	0	0,4	
" 4	0	0	1	1	0,6	
" 5	0	1	0	0	0,8	
" 6	0	1	0	1	1	
" 7	0	1	1	0	1,2	
" 8	0	1	1	1	1,4	
" 9	1	0	0	0	1,6	
" 10	1	0	0	1	1,8	
" 11	1	0	1	0	2	
" 12	1	0	1	1	2,2	
" 13	1	1	0	0	2,4	
" 14	1	1	0	1	2,6	
" 15	1	1	1	0	2,8	
" 16	1	1	1	1	3	



$$V_o = \frac{-10 \times 3}{150} = -\frac{1}{5} = -0.2V$$

$$V_o = \frac{-10 \times 3}{50k} = -\frac{3}{5}V = -0.6V$$



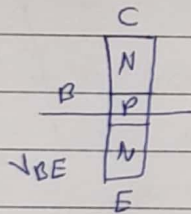
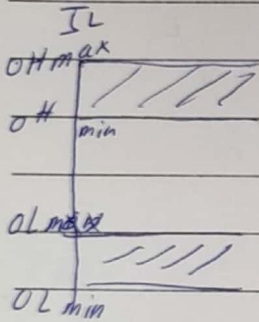
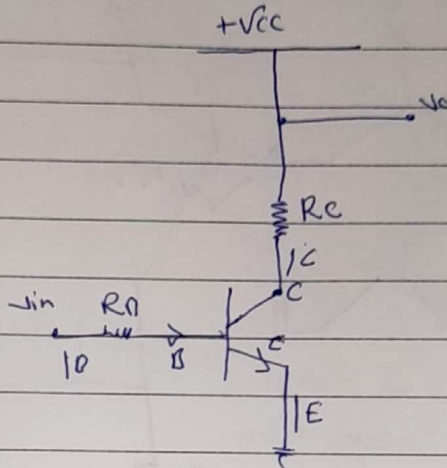
Digital Integrated circuits

BJT	• RTL	Resistor Transistor logic
	• DTL	Diode " "
	• TTL	Transistor - Transistor "
	• ECL	Emitter - coupled "

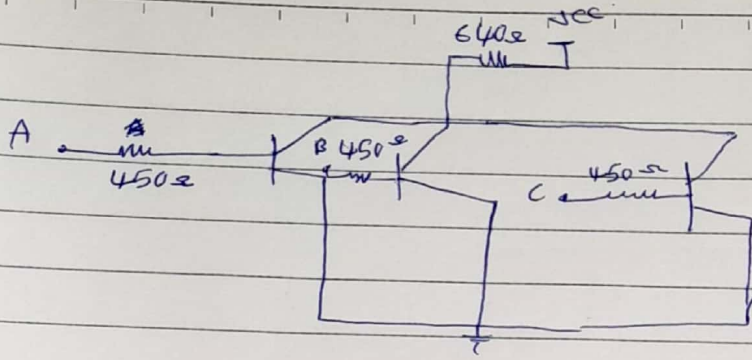
[• MOS	Metal - oxide semiconductor
	• CMOS	complementary - metal - oxide semiconductor.

The Fan-out of the gate is calculated from the ratio I_{OH}/I_{IH} or I_{OL}/I_{IL} whichever is smaller, for example the standard TTL gates have the following values

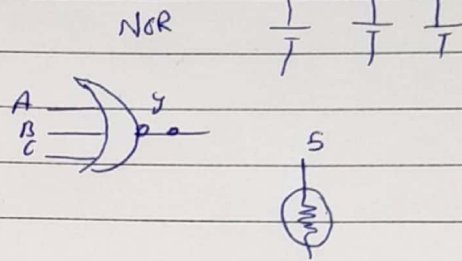
- $I_{OH} = 400 \mu A$
- $I_{IH} = 40 \mu A$ } 10
- $I_{OL} = 16 \text{ mA}$
- $I_{IL} = 1.6 \text{ mA}$ } 10



	Region	VBE (volts)	VCE (volts)	current Relationship
→	cutoff	0.6	open ckt	$I_B = I_C = 0$
	Active	0.6-0.7	> 0.8	$I_C = \beta_{FE} I_B$
↳	saturation	0.7-0.8	0.2	$I_B \gg I_{CS}$



$$Y = (A+B+C)'$$

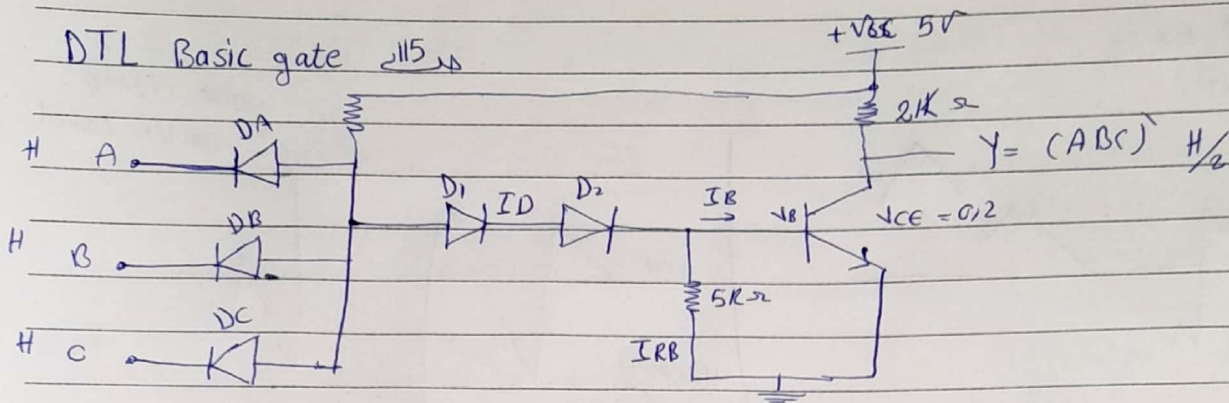


RTL

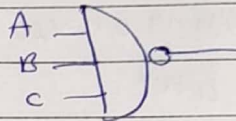
A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

23/May/2024

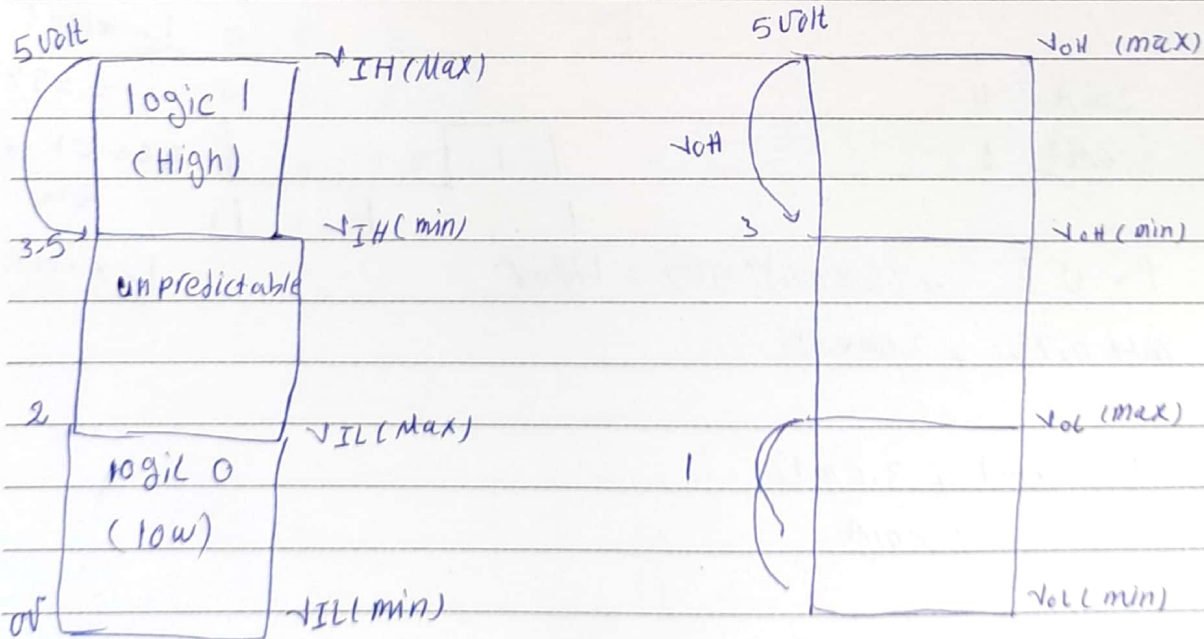
DTL Basic gate ≈ 115



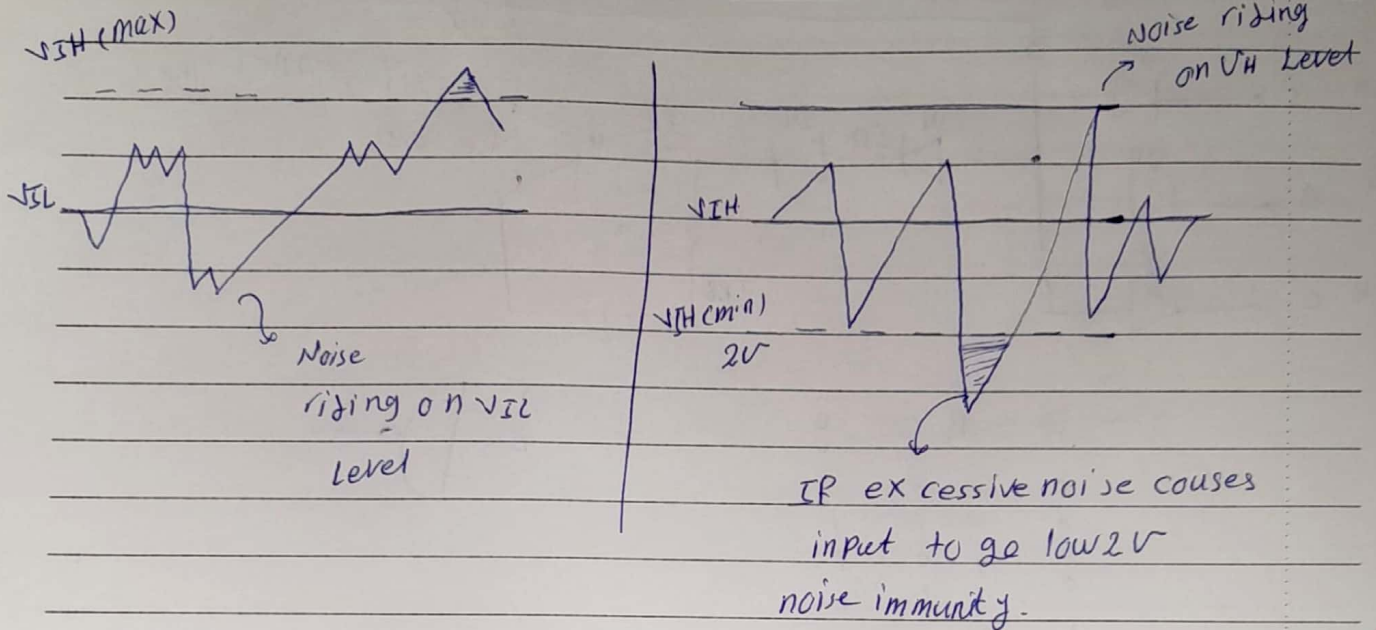
A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



TTL

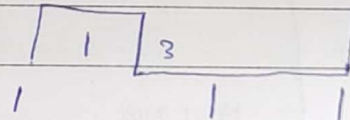


IP Ex:-



EX:- A certain gate draws 2mA when it's output is high and 3.6mA when the output is low, what is the avg. power dissipation if $V_{CC} = 5V$ and the gate is operated on 50% duty cycle
75%

2mA H
3.6mA L



$$\frac{1}{4} \times 100\% = 25\%$$

$$DC = \frac{ON}{Total} \times 100$$

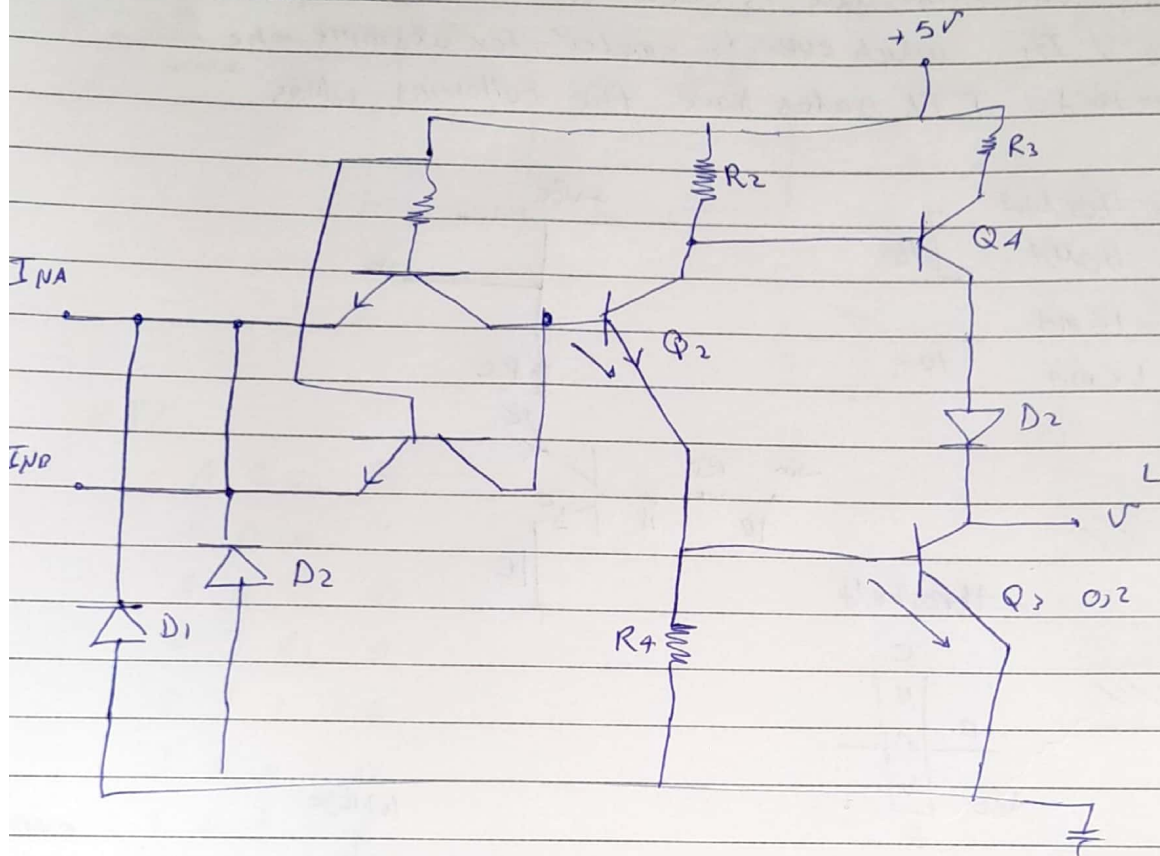
$$P = VI = (2.8mA)(5V) = 140mW$$

$$\frac{1}{2} \times 100\% = 50\%$$

$$2mA \times 0.75 + 3.6mA \times 0.25$$

$$\frac{1}{2} (2mA + 3.6mA) = 2.8mA$$

TTL NAND Gate



A	B	y
0	0	1
0	1	1
1	0	1
1	1	0

