تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

مواضيع خاصة في هندسة القدرة

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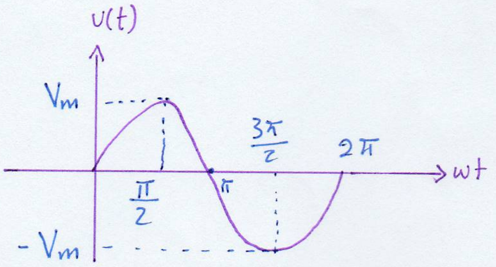
Monday: 5-2-2018

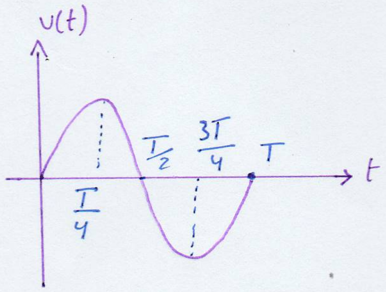
"Characteristics of Sinusoids"

$v(t) = V_m \sin(wt)$, where
V_m: amplitude
wt: argument
w: angular or radian frequency



⇒ Time Period of the function is 2π

* To demonstrate the concept of the period :



⇒ T: Period of the function

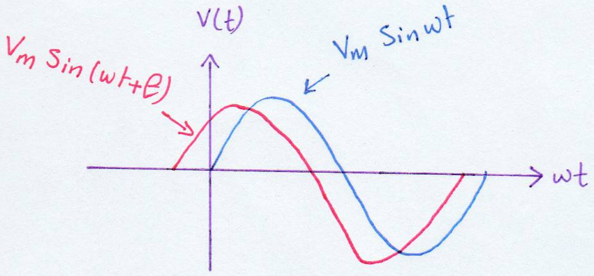
$f = \frac{1}{T}$

$w = 2\pi f$

* Leading / Lagging :

Assume $v_1(t) = V_m \sin(wt)$

$v_2(t) = V_m \sin(wt+\theta)$



* $V_m \sin(wt+\theta)$ is Leading $V_m \sin(wt)$
* $V_m \sin(wt)$ is Lagging $V_m \sin(wt+\theta)$
} Out of Phase

* If the phase angles are the same, sinusoids are said to be in phase.

* Two sinusoidal functions whose phases are to be compared :-

1) Both must be written as sine or cosine waves.
2) +Ve amplitude.
3) Same frequency

"Characteristics of Sinusoids"

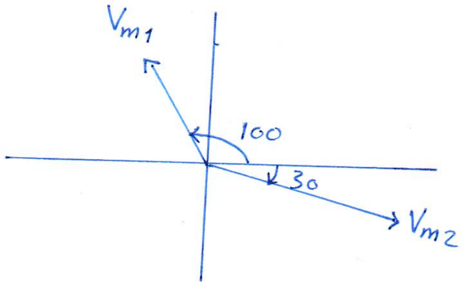
Monday: 5-2-2018

* $\sin(wt) = \cos(wt - 90^\circ)$

Ex $v_1(t) = V_{m1}\cos(5t + 10^\circ)$, $v_2(t) = V_{m2}\sin(5t - 30^\circ)$

$$v_1(t) = V_{m1}\cos(5t + 10^\circ) = V_{m1}\sin(5t + 10^\circ + 90^\circ) = V_{m1}\sin(5t + 100^\circ)$$

∴ $v_1(t) = V_{m1}\sin(5t + 100^\circ)$
$v_2(t) = V_{m2}\sin(5t - 30^\circ)$ } V_1 is Leading V_2 by 130°

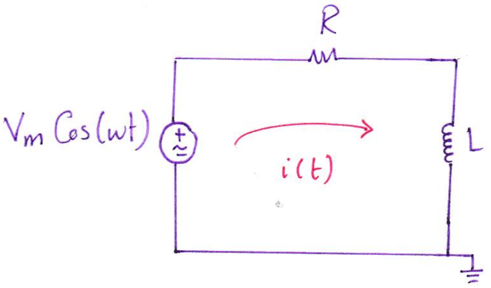


OR: $v_1(t) = V_{m1}\sin(5t - 260^\circ)$
$v_2(t) = V_{m2}\sin(5t - 30^\circ)$ } V_2 is Leading V_1 by 230°

"Forced Response to Sinusoidal Function"

* **Steady-State:** Refers to the condition that is reached after the transient or natural response has died out.

* RL Circuit:



$i(t)$: Forced Response.

$$i(t) = I_1\cos(wt) + I_2\sin(wt)$$

$$-V_m\cos(wt) + R \cdot i(t) + L\frac{di(t)}{dt} = 0$$

$$-V_m\cos(wt) + R\left(I_1\cos(wt) + I_2\sin(wt)\right) + L\left(-wI_1\sin(wt) + wI_2\cos(wt)\right) = 0$$

$$I_1 = \frac{RV_m}{R^2 + w^2L^2} , \quad I_2 = \frac{wLV_m}{R^2 + w^2L^2}$$

$$\therefore\ i(t) = \frac{RV_m}{R^2 + w^2L^2}\cos(wt) + \frac{wLV_m}{R^2 + w^2L^2}\sin(wt)$$

"Forced Response to Sinusoidal Function"

Monday: 5-2-2018

* To write $i(t)$ in the form : $i(t) = A\cos(\omega t - \theta)$

$$i(t) = I_1 \cos(\omega t) + I_2 \sin(\omega t)$$

$$A\cos(\omega t)\cos(\theta) + A\sin(\omega t)\sin(\theta) = \frac{R V_m}{R^2 + \omega^2 L^2}\cos(\omega t) + \frac{\omega L V_m}{R^2+\omega^2 L^2}\sin(\omega t)$$

⇒ After Solving for A & θ:

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cdot \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

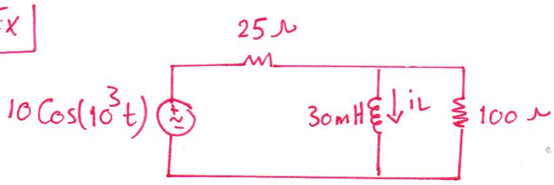
* ωL: Induced reactance of the inductor (Ω). Opposition that is offered by the inductor to passage of a sinusoidal current.

* Assuming that the passive sign convention is satisfied:

Inductor: I lags V by 90°

Capacitor: I Leads V by 90°

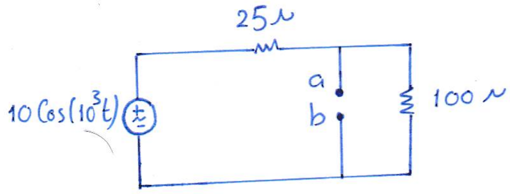
Ex



, Find $i_L(t)$?

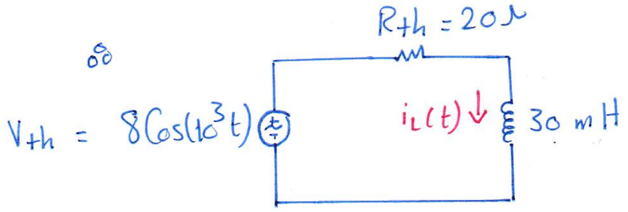
$$\Rightarrow i(t) = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

* using Thevenin:-



$$V_{th} = \left(\frac{100}{100+25}\right) 10\cos(10^3 t) = 8\cos(10^3 t) \quad , \quad R_{th} = 25 \,//\, 100 = 20\ \Omega$$

∴

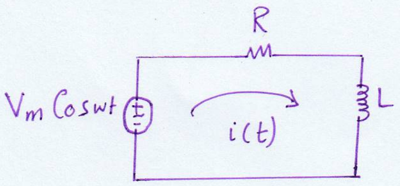


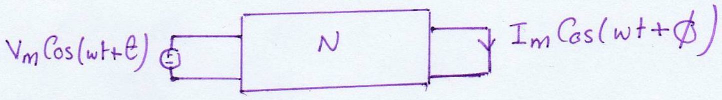
$$\Rightarrow i(t) = \frac{8}{\sqrt{20^2 + (30)^2}}\cos\left(10^3 t - \tan^{-1}\left(\frac{30}{20}\right)\right)$$

$$i_L(t) = 222\cos(10^3 t - 56.3^\circ)\ mA$$

"Complex Forcing Function"

Wednesday: 7-2-2018

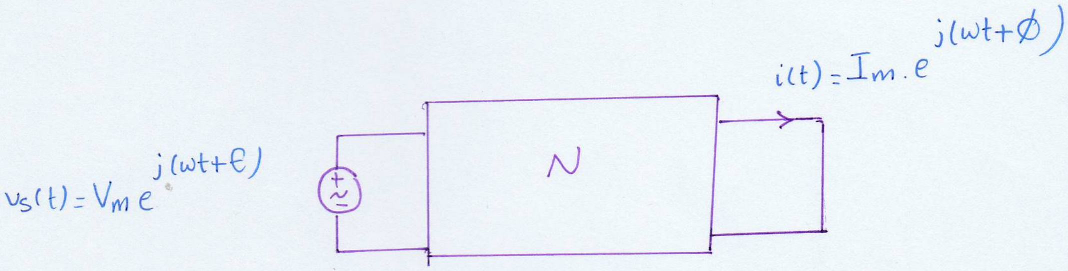




Sinusoidal Forcing Function	Forced Response	
$V_m \cos(wt+\theta)$	$I_m \cos(wt+\phi)$	← Real
$V_m \cos(wt+\theta-90^\circ)$	$I_m \cos(wt+\phi-90^\circ)$	
$V_m \sin(wt+\theta)$	$I_m \sin(wt+\phi)$	
$j V_m \sin(wt+\phi)$	$j I_m \sin(wt+\phi)$	← Imaginary

$\Rightarrow \quad v(t) = V_m \cos(wt+\theta) + j V_m \sin(wt+\theta)$

$V(t) = V_m e^{j(wt+\theta)} \quad \Rightarrow \quad i(t) = I_m \cdot e^{j(wt+\phi)}$



$\Rightarrow \quad V_m \cos(wt) = Re\{V_m \cos wt + j \sin wt\} = Re\{V_m \cdot e^{jwt}\}$

* The necessary format of the complex forcing function is:

$v_s(t) = V_m \cdot e^{jwt}$

$i(t) = I_m \cdot e^{j(wt+\phi)}$

"Complex Forcing Function"

Wednesday: 7-2-2018

$$\Rightarrow -V_s(t) + R\cdot i(t) + L\cdot\frac{di(t)}{dt} = 0$$

$$-V_m\cdot e^{j\omega t} + R\cdot I_m\, e^{j(\omega t+\phi)} + L\cdot\frac{d}{dt}\left(I_m\cdot e^{j(\omega t+\phi)}\right) = 0$$

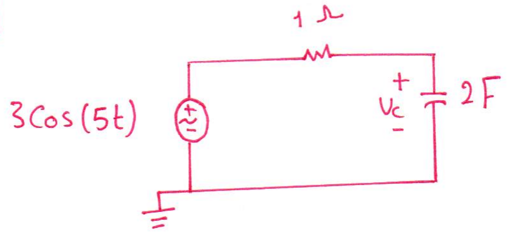
$$-V_m\cdot e^{j\omega t} + R\cdot I_m\cdot e^{j\omega t}\cdot e^{j\phi} + j\omega L\cdot I_m\, e^{j\omega t}\cdot e^{j\phi} = 0$$

$$-V_m + R\cdot I_m\cdot e^{j\phi} + j\omega L\cdot I_m\cdot e^{j\phi} = 0$$

$$I_m\cdot e^{j\phi} = \frac{V_m}{R + j\omega L} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\ \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\Rightarrow i(t) = I_m\cdot \cos(\omega t+\phi) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cdot \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

Ex



, Find $V_c(t)$?

$V_s(t) = 3e^{j5t}$ [Complex format of the forcing function]

$\Rightarrow V_c(t) = V_m\cdot e^{j5t}$ [Expected Forced response]

$$\Rightarrow -V_s(t) + i_c + V_c(t) = 0$$

$$-3e^{j5t} + c\frac{d}{dt}\cdot V_c(t) + V_m\, e^{j5t} = 0 \quad\Rightarrow\quad -3e^{j5t} + 2\,\frac{d}{dt}\left(V_m\cdot e^{j5t}\right) + V_m\cdot e^{j5t} = 0$$

$$-3e^{j5t} + 10jV_m\, e^{j5t} + V_m\cdot e^{j5t} = 0 \quad\Rightarrow\quad 10jV_m + V_m = 3$$

$$V_m(10j+1) = 3 \;\Rightarrow\; V_m = \frac{3}{1+10j}$$

$$|V_m| = \frac{3}{\sqrt{1^2+10^2}} = 29.85\ mV \quad , \quad \angle V_m = -\tan^{-1}\left(\frac{10}{1}\right) = -84.3^\circ$$

" Complex Forcing Function"

Wednesday: 7-2-2018

$$\Rightarrow v_c(t) = V_m \cdot e^{j5t} = 29.85 \cdot e^{-j84.3} \cdot e^{j5t} \ mV$$

$$v_c(t) = 29.85 \left(\cos(5t - 84.3^\circ)\right) \ mV$$

" The Phasor "

Current : $i(t) \rightarrow I$

Voltage: $v(t) \rightarrow V$

$* \ i(t) = I_m \cdot \cos(wt + \phi)$

$i(t) = Re\{ I_m \cdot e^{j(wt+\phi)} \}$

$\Rightarrow I = I_m \cdot e^{j\phi} = I_m \angle \phi \Rightarrow$ Polar Form

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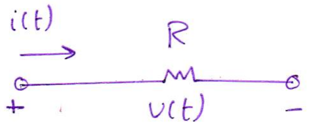
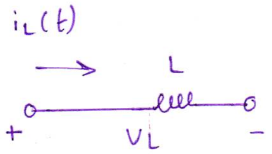
Ex) $V = 115 \angle -45^\circ$, $w = 500$ rad/sec.

$v(t) = 115 \cdot \cos(500t - 45^\circ)$

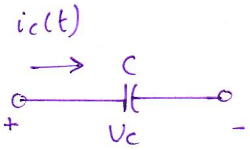
Time domain :



$v(t) = R \cdot i(t)$



$v_L(t) = \dfrac{L \, di(t)}{dt}$



$i_c(t) = C \cdot \dfrac{d \, V_c(t)}{dt}$

"The Phasor"

Wednesday: 7-2-2018

1) Resistor:

$$v(t) = V_m . e^{j(\omega t+\theta)}$$
$$i(t) = I_m . e^{j(\omega t+\phi)} \Rightarrow v(t) = R . i(t)$$

$$\Rightarrow V_m . e^{j(\omega t+\theta)} = R . I_m . e^{j(\omega t+\phi)}$$

$$V_m . e^{j\theta} = R . I_m . e^{j\phi} \Rightarrow V = R I$$

Ex) $v(t) = 8 \cos(100t - 50°) V$ is applied to $R = 4 \Omega$, Calculate $i(t)$ and I (Phasor)

$$\Rightarrow V = 8 \angle -50° \Rightarrow i(t) = \frac{v(t)}{R} = 2 \cos(100t - 50°) A$$

$$I = 2 \angle -50° \ A$$

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2) Inductor:

$$v_L(t) = L \frac{di}{dt} \Rightarrow v(t) = V_m . e^{j(\omega t+\theta)}, \quad i(t) = I_m . e^{j(\omega t+\phi)}$$

$$V_m . e^{j\omega t} . e^{j\theta} = j\omega L . I_m . e^{j\omega t} . e^{j\phi}$$

$$V_m . e^{j\theta} = j\omega L . I_m . e^{j\phi} \Rightarrow V_m \angle \theta = j\omega L I_m \angle \phi \Rightarrow V = (j\omega L) . I$$

Ex) $V = 8 \angle -50°$ is applied to a 4 H inductor, at $\omega = 100$ rad/sec.

Determine $i(t)$?

$$V = (j\omega L) I \Rightarrow I = \frac{V}{j\omega L} = \frac{8 \angle -50°}{j\,400} = -j0.02 \angle -50$$

$$\Rightarrow I = (1 \angle -90°)(0.02 \angle -50°) = 0.02 \angle -140° \ A$$

$\therefore$ $i(t) = 20 \cos(100t - 140°)$ mA

"The Phasor"

Wednesday: 7-2-2018

3) Capacitor

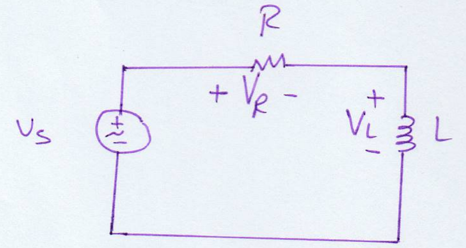
$$i_c(t) = C \cdot \frac{dv_c(t)}{dt}$$

$$I_m \cdot e^{j(wt+\phi)} = C \cdot \frac{d}{dt}\left(V_m \cdot e^{j(wt+\theta)}\right)$$

$$I_m \cdot e^{jwt} \cdot e^{j\phi} = jwC \cdot V_m \cdot e^{jwt} \cdot e^{j\theta} \Rightarrow I = (jwC) \cdot V$$

$$\therefore \; V = \left(\frac{1}{jwC}\right) \cdot I$$

* Solving RL circuit using the phasors:



$$-V_S + V_R + V_L = 0 \quad , \quad V_S = V_m \angle 0^\circ$$

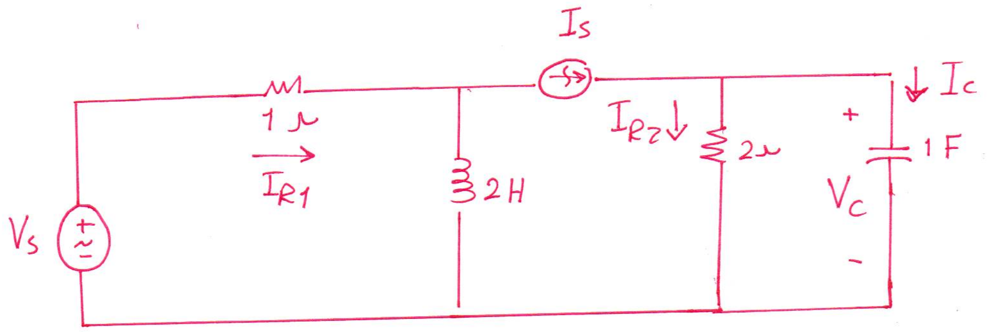
$$-V_m + RI + jwL \cdot I = 0$$

$$I = \frac{V_m}{R + jwL}$$

$$\therefore \; I = \frac{V_m}{\sqrt{R^2 + w^2L^2}} \angle -\tan^{-1}\left(\frac{wL}{R}\right)$$

Monday: 12_2_2018

"The Phasors"



given $w = 2$ rad/sec & $I_c = 2\angle 28^\circ$ A

Determine I_s (Phasor) and $i_s(t)$?

$$\Rightarrow I_s = I_{R2} + I_c$$

$$I_s = \frac{V_c}{R_2} + 2\angle 28^\circ$$

where $V_c = \dfrac{I_c}{jwc} = \dfrac{2\angle 28^\circ}{j \times 2 \times 1} = -j\angle 28 = 1\angle -62^\circ$

$$I_s = \frac{1\angle -62^\circ}{2} + 2\angle 28^\circ = 2.06\angle 13.96^\circ \text{ A}$$

& $i_s(t) = 2.06 \cos(2t + 13.96)$ A

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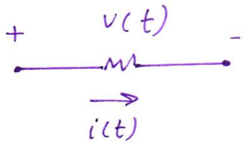
* The impedance:



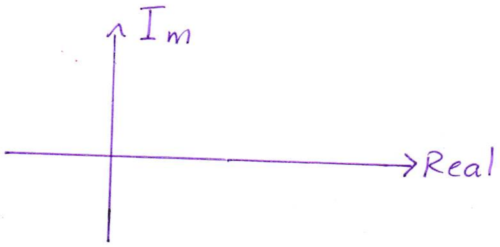
$v(t) = R.i(t) \longleftrightarrow V = RI \Rightarrow Z_R = R$

$V_c = \left(\frac{1}{jwc}\right) I \Rightarrow Z_c = \frac{1}{jwc}$

$V_L = (jwL) I \Rightarrow Z_L = jwL$

∴ $Z = R + jX$, R: resistance

X: reactance



"The Phasors"

Monday: 12-2-2018

Ex] $\omega = 10^4$ rad/sec

$L = 5mH \longrightarrow Z_L = j \times 10^4 \times 5 \times 10^{-3} = j50\,\Omega$

$C = 100\,\mu F \longrightarrow Z_C = \dfrac{1}{j\omega C} = \dfrac{-j}{10^4 \times 100 \times 10^{-6}} = -j\,\Omega$

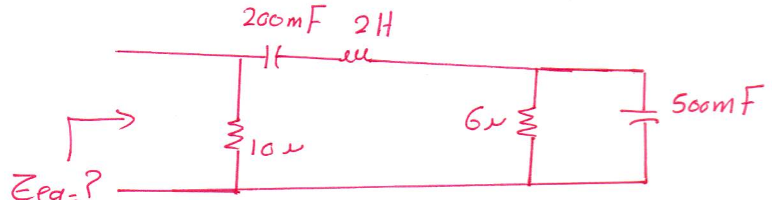
Find: 1) L-C series combination

$$Z_{eq} = j50 - j = j49\,\Omega$$

2) L-C Parallel Combination

$$Z_{eq} = \frac{j50 \times -j}{j50 - j} = -j1.02\,\Omega$$

Ex]



$\omega = 5$ rad/sec

$\Rightarrow$ For $C_1 = 200\,mF$

$$Z_{C_1} = \frac{-j}{5 \times 200 \times 10^{-3}} = -j\,\Omega$$

$\Rightarrow$ For $C_2 = 500\,mF$

$$Z_{C_2} = \frac{-j}{5 \times 500 \times 10^{-3}} = -j0.4\,\Omega$$

$\Rightarrow$ For $L = 2H$

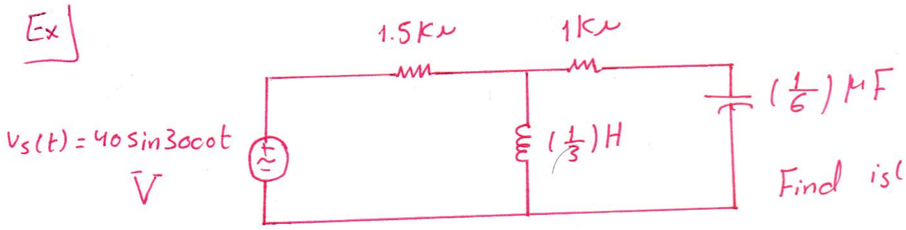
$$Z_L = j \times 5 \times 2 = j10\,\Omega$$

$$Z_{eq} = \left[\left[Z_{C_2} \,//\, 6\Omega \right] + Z_{C_1} + Z_L \right] // \,10\Omega$$

$$Z_{eq} = \frac{\left[\frac{-j0.4 \times 6}{6 - j0.4} - j + j10 \right] \times 10}{\left[\frac{-j0.4 \times 6}{6 - j0.4} - j + j10 \right] + 10}$$

$$Z_{eq} = 6.511 \angle 49.2^\circ\,\Omega$$

Ex]



Find $i_s(t)$

$v_s(t) = 40 \sin 3000t = 40 \cos(3000t - 90^\circ)\,V \Rightarrow V_s = 40 \angle -90^\circ\,V$

$\Rightarrow$ For $L = \frac{1}{3}H \longrightarrow Z_L = j\omega L = j \times 3000 \times \frac{1}{3} = j1000 = j1k\Omega$

$\Rightarrow$ For $C = \frac{1}{6}\mu F \rightarrow Z_C = \dfrac{-j}{3000 \times \frac{1}{6} \times 10^{-6}} = -j2k\Omega$

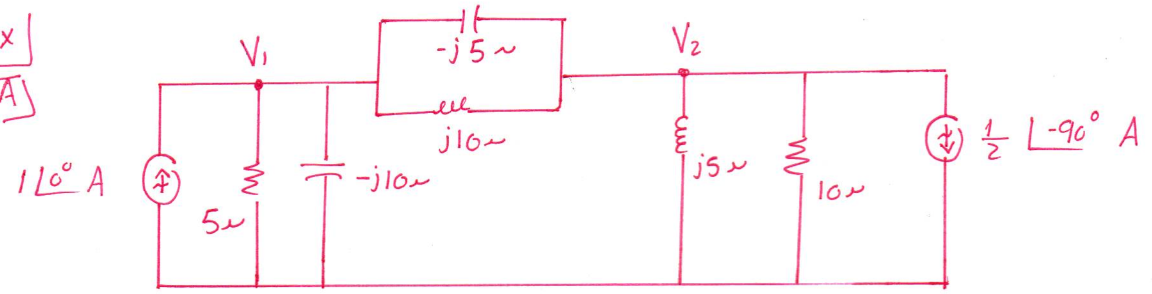
$\Rightarrow Z_{eq} = 1.5k\Omega + j1k\Omega \,//\, [1k\Omega - j2k\Omega] = 2 + j1.5 = 2.5 \angle 36.87^\circ\,k\Omega$

$\Rightarrow I_s = \dfrac{V_s}{Z_{eq}} = 16 \angle -126.9\,mA \Rightarrow i_s(t) = 16 \cos(3000t - 126.9^\circ)\,mA$

* The Admittance (Y):

$Y = G + jB$ (S) → simens , where G: Conductance & $Y = \frac{1}{Z} = \frac{1}{R + jX}$
B: Susceptance

Ex A



* To Find V_1 & V_2 we can write nodal equations:-

$$\frac{V_1}{5} + \frac{V_1}{-j10} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10} = 1\angle 0° \quad \text{-- } \boxed{1}$$

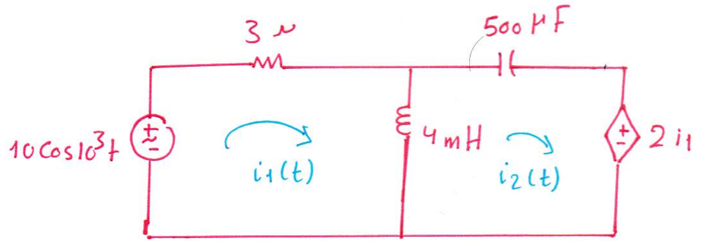
$$\frac{V_2}{10} + \frac{V_2}{j5} + \frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j10} = \frac{-1}{2}\angle -90° \quad \text{-- } \boxed{2}$$

then solve [1] & [2] to get:

$V_1 = A\angle\theta_1 \Rightarrow v_1(t) = A\cos(\omega t + \theta_1)$ V

$V_2 = B\angle\theta_2 \Rightarrow v_2(t) = B\cos(\omega t + \theta_2)$ V

Ex B



$\Rightarrow Z_L = j\omega L = j \times 10^3 \times 4 \times 10^{-3} = j4\,\Omega$

$Z_C = \frac{1}{j\omega C} = \frac{-j}{10^3 \times 500 \times 10^{-6}} = -j2\,\Omega$

* We Can Find I_1 & I_2 writing Mesh equations:-

$-10\angle 0° + 3I_1 + j4(I_1 - I_2) = 0$ -- [1]

$j4(I_2 - I_1) + (-j2)I_2 + 2I_1 = 0$ -- [2]

⇒ then solve [1] & [2] to achive I_1 & I_2

* Back to example A instead of writing nodal equations we can use other ways to Find V_1 & V_2:

1) Finding V_1 by superposition:

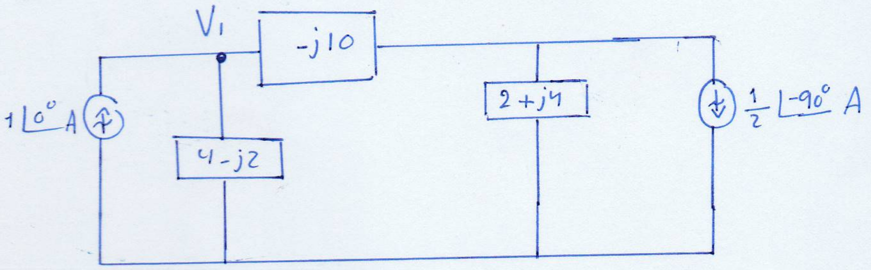
* First we can simplify the circuit to:

$5 \parallel -j10 = 4-j2$ ~

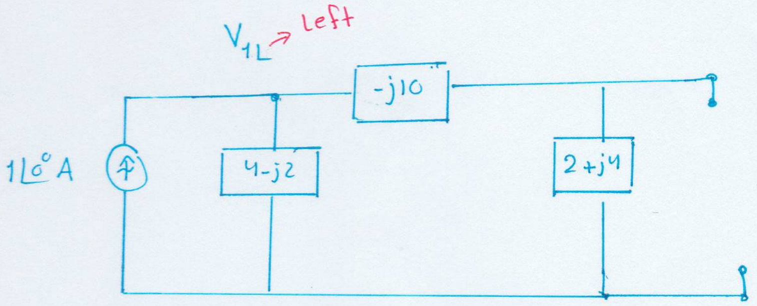
$-j5 \parallel j10 = -j10$ ~

$j5 \parallel 10 = 2+j4$ ~

$\Longrightarrow$



* V_1 due to the left source

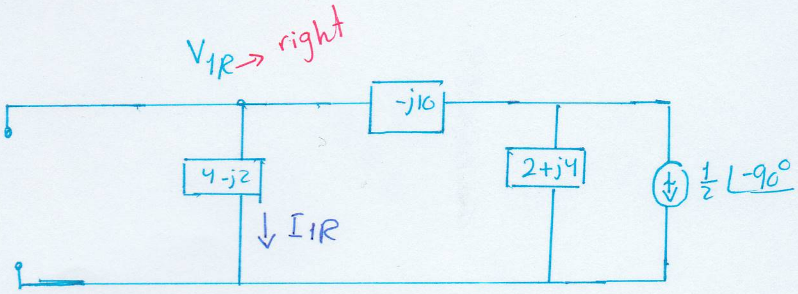


$V_{1L} = 1\angle 0^\circ \left[(4-j2) \parallel (2+j4-j10)\right]$

$V_{1L} = 1\angle 0^\circ \left[\frac{(4-j2)(2-j6)}{(4-j2)+(2-j6)}\right]$

$V_{1L} = 2-j2$ V

* V_1 due to the right source



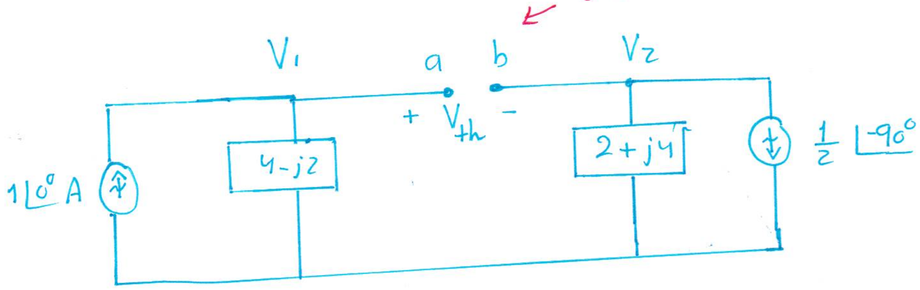
$I_{1R} = \frac{2+j4}{(2+j4)+(4-j2-j10)} \times \left(-\frac{1}{2}\angle -90^\circ\right)$

$I_{1R} = -0.2 - j0.1$ A

$V_{1R} = I_{1R}(4-j2) = (-0.2-j0.1)(4-j2) = -1$ V

$\therefore V_1 = V_{1L} + V_{1R} = (2-j2) + (-1) = 1-j2$ V

2) Finding V_1 using Thévenin Equivalent :-



$V_{o.c} = V_{th} = V_1 - V_2$

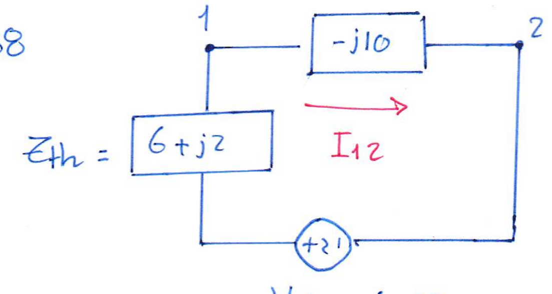
$V_{th} = (1\angle 0^\circ)(4-j2) - (-\frac{1}{2}\angle -90^\circ)(2+j4)$

$V_{th} = 6 - j3$ V

meant to be Zth

$\Rightarrow R_{th} = (4-j2) + (2+j4) = 6+j2\ \Omega$

∴



$I_{12} = \frac{V_{th}}{6+j2-j10}$

$I_{12} = 0.6 + j0.3$ A

↳ The current Flowing in the -j10 impedance

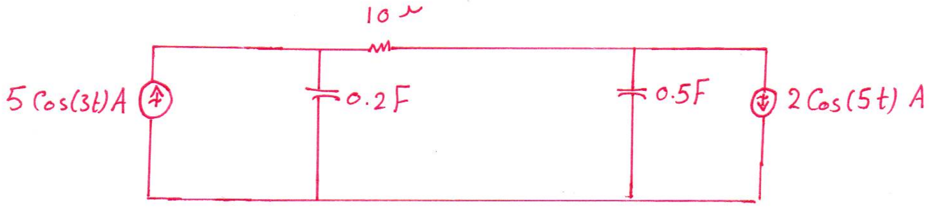
The Left Source

$I_1 = 1\angle 0^\circ - [0.6 + j0.3] = 0.4 - j0.3$ A

The current Flowing in the 4-j2 impedance.

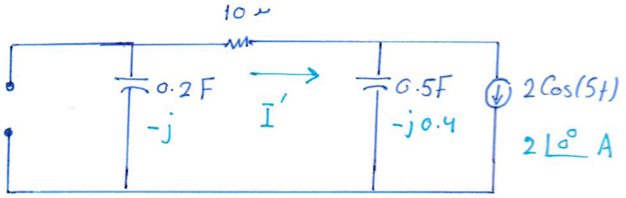
∴ $V_1 = I_1(4-j2) = (0.4-j0.3)(4-j2) = 1-j2$ V

"The Phasors"



Find the Power dissipated in the 10 Ω resistor?

* Kill the left source:



at w = 5 rad/sec :

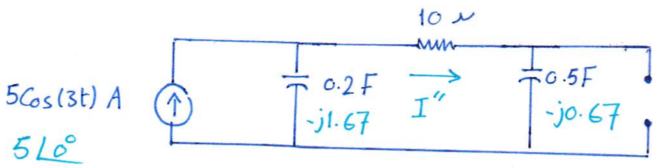
For $C_1 = 0.2F \Rightarrow Z_{C1} = \frac{1}{jwC_1} = \frac{-j}{wC_1} = \frac{-j}{5 \times 0.2} = -j\ \Omega$

For $C_2 = 0.5F \Rightarrow Z_{C2} = \frac{1}{jwC_2} = \frac{-j}{5 \times 0.5} = -j0.4\ \Omega$

$$\Rightarrow I' = \frac{-j0.4}{-j0.4 + (10 - j)} \times 2\angle 0^\circ = 79.23\angle -82.03\ mA$$

$i'(t) = 79.23\ Cos(5t - 82.03)\ mA$

* Kill the right source:



at w = 3 rad/sec :

For $C_1 = 0.2F \rightarrow Z_{C1} = \frac{-j}{3 \times 0.2} = -j1.67\ \Omega$

For $C_2 = 0.5F \rightarrow Z_{C2} = \frac{-j}{3 \times 0.5} = -j0.67\ \Omega$

$$\Rightarrow I'' = \frac{-j1.67}{-j1.67 + 10 - j0.67} \times 5\angle 0^\circ = 811.7\angle -76.86^\circ\ mA$$

$i''(t) = 811.7\ Cos(3t - 76.86^\circ)\ mA$

$\therefore\ P_{10\Omega} = (i'' + i')^2 \times 10$

$P_{10\Omega} = 10\,[\,79.23\ Cos(5t - 82.03^\circ) + 811.7\ cos(3t - 76.86^\circ)\,]^2\ \mu W$

"Phasor Diagram"

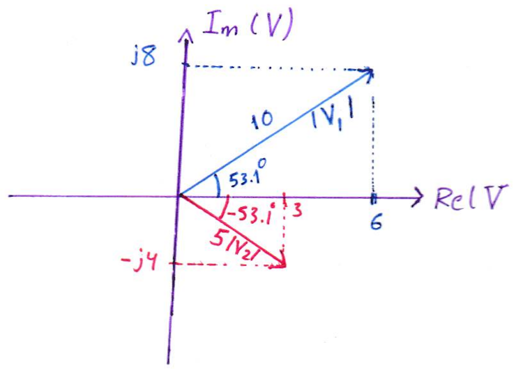
⇒ Phasor diagram: sketch plotted in the complex plane to determine the relationship between different circuit elements.

Assume: $V_1 = 6 + j8$

$|V_1| = \sqrt{6^2 + 8^2} = 10$

$\angle V_1 = \tan^{-1}\left(\frac{8}{6}\right) = 53.1^\circ \Rightarrow V_1 = 10\angle 53.1^\circ$

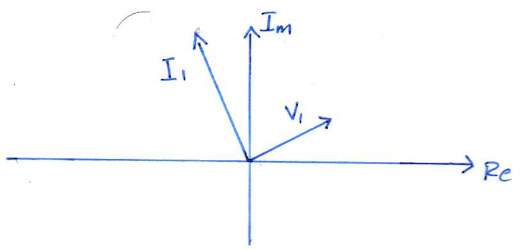
& $V_2 = 3 - j4 = 5\angle -53.1^\circ$



Ex| Let $V_1 = (6 + j8)$ V , $Y = 1 + j1$, Find I ?

$I_1 = Y V_1 = (1+j)(6+j8) = -2 + j14$ A

$= (\sqrt{2}\angle 45^\circ)(10\angle 53.1^\circ) = \sqrt{2}\, 10\angle 98.1^\circ$ A



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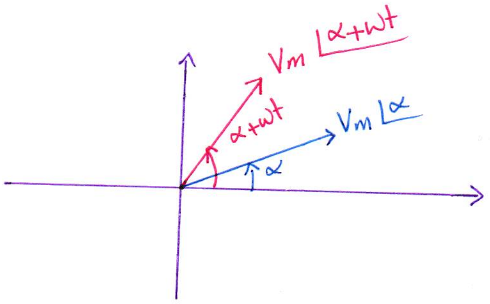
* Phasor Voltage: Let $V = V_m\angle\alpha$ (in frequency domain)

⇒ To go into the time domain, multiply by $e^{j\omega t}$.

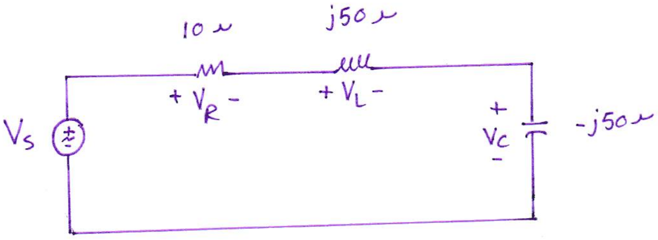
∴ $V = V_m\angle\alpha + \omega t$



⇒ Rotating line segment with the instanteneous position being with radians ahead of (ωt)

"Phasor diagrams of general circuits"

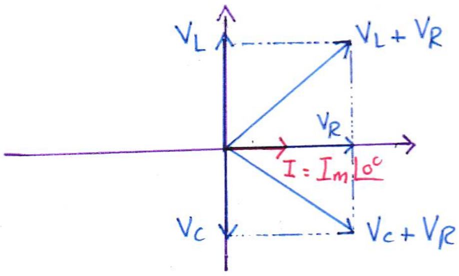
1) Series RLC Circuit :



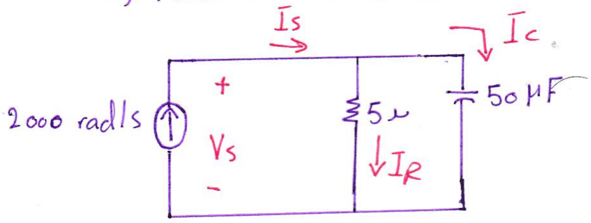
* Since same current is passing through all circuit elements, we choose it as a reference phase :-

$Z_L = j50$ & $Z_c = -j50$ → $I = I_m \angle 0°$ (reference phasor)

→ ($Z_c = -Z_L$) (Circuit is in resonance)



2) Parallel RC Circuit :



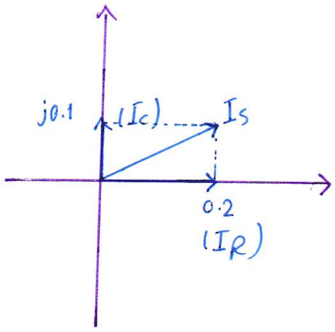
* Since both circuit elements have the same Voltage → Let $V = V_s = 1 \angle 0°$ Phasor Reference

$V = \frac{1}{jwc} I$

$\Rightarrow I_R = \frac{V}{R} = \frac{1 \angle 0°}{5} = 0.2\ A$

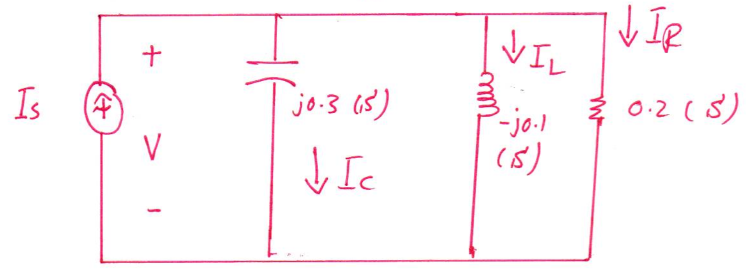
$\Rightarrow I_c = \frac{V}{1/jwc} = jwc = j \times 2000 \times 50 \times 10^{-6} = j0.1\ A$

$\therefore I_s = I_R + I_c = 0.2 + j0.1$



"Phasor Diagrams"

Ex



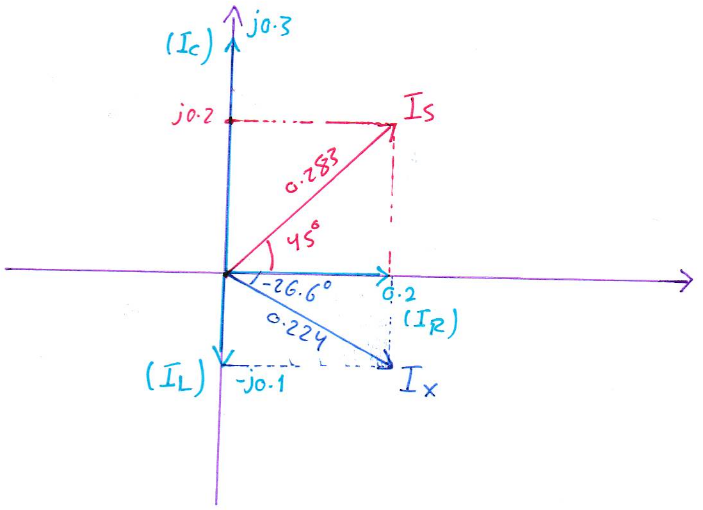
Draw the phasor diagram of the currents?

⇒ Let $V = 1\angle 0^\circ$ (reference Voltage)

∴ $I_R = 0.2 \times 1 = 0.2$ A

$I_L = -j0.1 \times 1 = 0.1\angle -90^\circ$ A ⇒

$I_c = j0.3 \times 1 = 0.3\angle 90^\circ$ A



⇒ $I_x = I_L + I_R = 0.2 - j0.1 = 0.224\angle -26.6^\circ$

⇒ $I_s = I_x + I_c = (0.2 - j0.1) + j0.3$

$I_s = 0.2 + j0.2 = 0.283\angle 45^\circ$

∴ I_s Leads I_R by 45°
I_s Leads I_c by -45°
I_s Leads I_x by $26.6^\circ + 45^\circ = 71.6^\circ$

} Result.

"Energy Storage in Inductor"

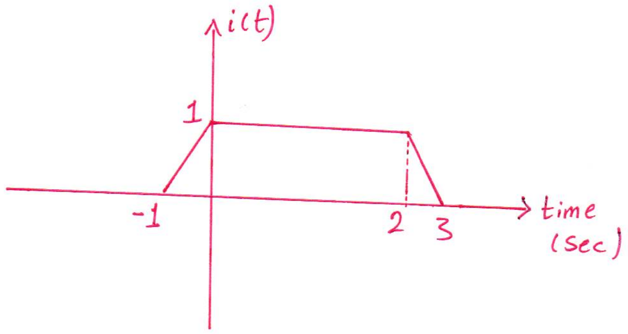
i(t) → L, + v(t) −

$$v(t) = L \cdot \frac{di}{dt}$$

* L: Is the inductance measured in Henry (H) or in Volt.Second/Ampere

"Energy Storage in Inductor"

Ex



if $L = 3H$ Determine & sketch $v(t)$?

* $i(t)$ can be divided into 5 main intervals:-

1) $t \leq -1s \Rightarrow \quad i(t) = 0 \quad \therefore \quad v(t) = L\frac{di}{dt} = 0$

2) $-1 \leq t \leq 0 \Rightarrow$ The current begins to increase at a rate of 1 Ampere/second

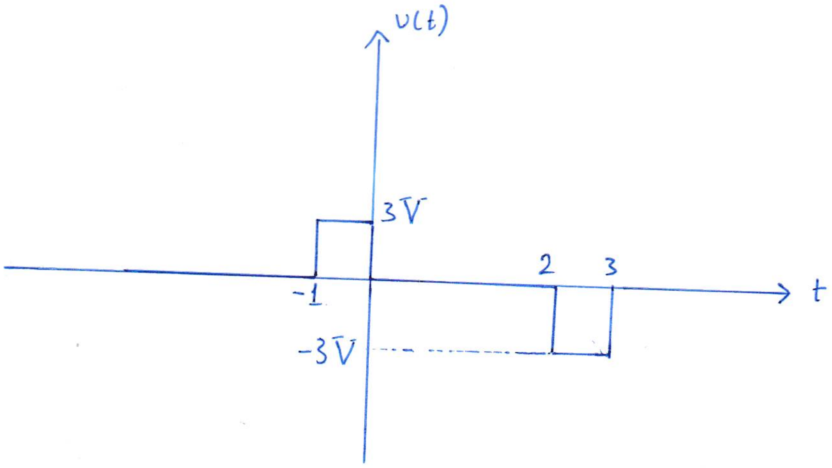
$\therefore \frac{di}{dt} = 1\,A/s \Rightarrow V(t) = 3H \times 1\frac{A}{s} = 3V$

3) $0 \leq t \leq 2 \Rightarrow$ The current is constant

$\therefore \frac{di}{dt} = 0$ & Hence $v(t) = 0$

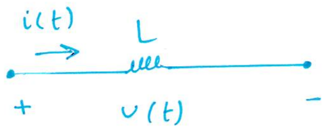
4) $2 \leq t \leq 3 \Rightarrow \frac{di}{dt} = -1\,A/s \quad \therefore \quad v(t) = -3V$

5) $t \geq 3s \Rightarrow \quad v(t) = 0$



"Energy Storage in Inductor"

Monday: 19-2-2018



$\Rightarrow \quad v(t) = L\,(di/dt)$

$\rightarrow \quad i(t) = \frac{1}{L} \int_{t_0}^{t} v(t')\,dt' + i(t_0)$

Ex) $L = 2H$, $v(t) = 6\cos(5t)$

$i(t = -\pi/2) = 1A$

Determine the resulting inductor current?

$\rightarrow \quad i(t) = \frac{1}{2} \int_{t_0}^{t} 6\cos(5t')\,dt' + i(t_0 = -\pi/2)$

$i(t) = \frac{1}{2}\left(\frac{6}{5}\right) \sin(5t') \Big]_{t_0}^{t} + 1A$

$i(t) = 0.6\,[\sin(5t) - \sin(5t_0)] + 1$

$i(t) = 0.6\sin(5t) - 0.6\sin(-2.5\pi) + 1$

$i(t) = 0.6\sin(5t) + 0.6 + 1$

$i(t) = 0.6\sin(5t) + 1.6 \ A$

---

* Energy Storage:

$\rightarrow \quad w_R = \int_{t_0}^{t} P(t')\,dt'$ (this energy is being stored in the magnetic field around the Coil)

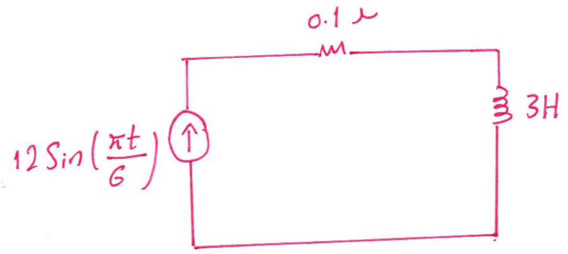
$\rightarrow \quad w_R(t) - w_R(t_0) = \frac{1}{2} L\cdot i^2(t) - \frac{1}{2} L\cdot i^2(t_0)$

* If we select t_0 such that no current flows in the inductor $\Rightarrow W_L(t) = \frac{1}{2} L\, i^2(t)$

"Energy Storage in Inductor" Monday: 19-2-2018

* Series RL Circuit powered by a sinusoidal.

Ex



1) Determine the maximum energy that can be stored in the inductor.

$\rightarrow w_L = \frac{1}{2} L . i^2(t)$

$w_L = \frac{1}{2} \times 3 \times 144 \sin^2\left(\frac{\pi t}{6}\right)$

$w_L = 216 \sin^2\left(\frac{\pi t}{6}\right)$ J.

$\rightarrow$ Max. Energy :-

At $t = 0$ sec $\rightarrow w_L = 0$ J

At $t = 3$ sec $\rightarrow w_L = 216 \sin^2(\pi/2) = 216$ (Maximum energy that can be stored in the inductor).

2) Determine the energy absorbed by the resistor.

$P_R(t) = i^2(t) \times R$

$P_R(t) = 144 \sin^2\left(\frac{\pi t}{6}\right) \times 0.1$

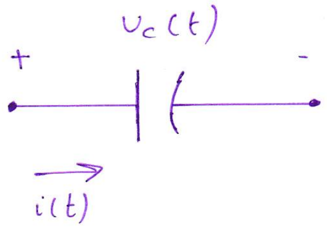
$P_R(t) = 14.4 \sin^2\left(\frac{\pi t}{6}\right)$ W

$w_R = \int_0^6 14.4 \sin^2(\pi t/6) = 14.4 \int_0^6 \left(\frac{1}{2}\left[1 - \cos\left(\frac{\pi t}{3}\right)\right]\right) dt$

$w_R = 43.2$ Joules

"Capacitors for Energy Storage"

Monday: 19-2-2018



$$i(t) = C.\left(\frac{dv}{dt}\right)$$

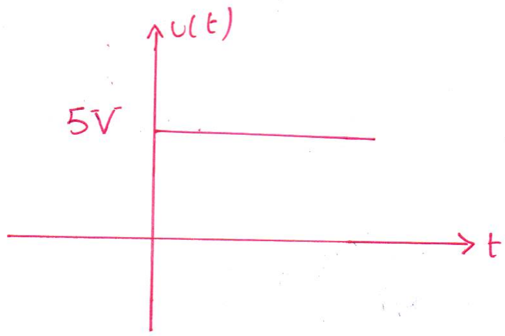
* The Capacitor acts as an open circuit to D.C.

$$\rightarrow v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0) = \frac{1}{C}\cdot q(t)$$

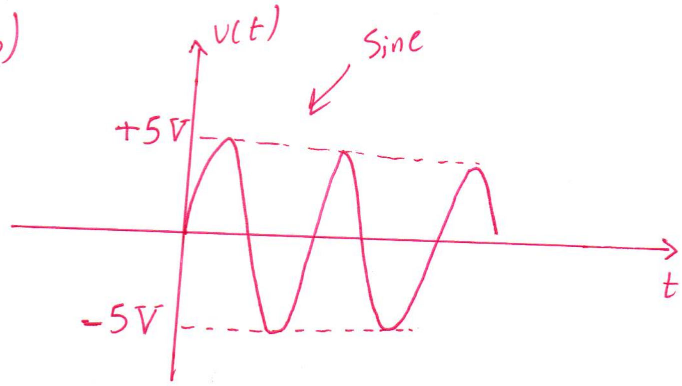
$q(t)$ ↙ charge

Ex) $C = 2F$, sketch the current waveforms for:

a)

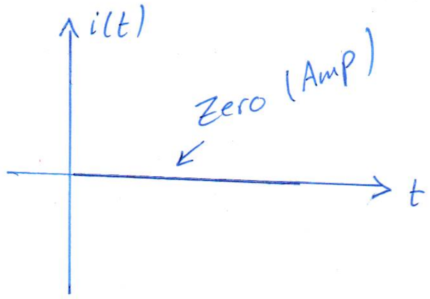


b)

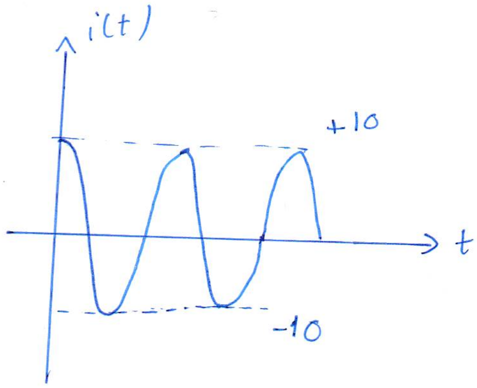


$$\rightarrow i(t) = C.\left(\frac{dv}{dt}\right)$$

a)



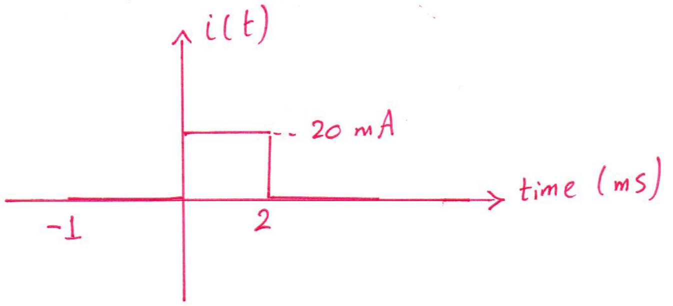
b)



* The derivative is cosine with same freq. but twice the amplitude.

"Capacitors For Energy Storage" Monday: 19-2-2018

Ex) * Determine and sketch the voltage waveform of a 5 μF Capacitor.



$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t').dt' + v(t_0)$$

1) For $t \leq 0$ $(-\infty, 0)$

$v(t) = v(t_0) = v(-\infty) = \text{Zero}$

2) For $(0 \leq t \leq 2)$ ms

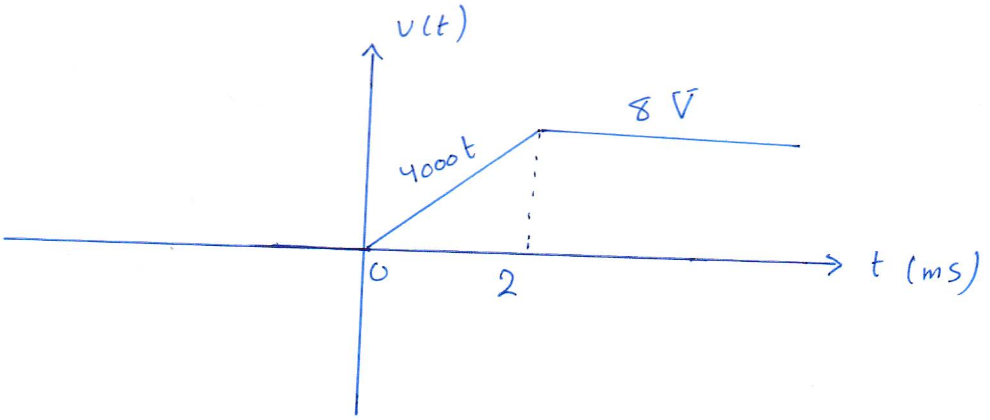
at $t_0 = 0 \rightarrow v(t_0) = v(0) = \text{Zero}$

$$v(t) = \frac{1}{C}\int_0^t (20 \times 10^{-3})dt' = \frac{20 \times 10^{-3}}{5 \times 10^{-6}} \int_0^t 1 dt'$$

$v(t) = 4000t \, V$, $0 \leq t \leq 2ms$

3) For $t \geq 2$ ms

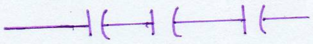
$v(t) = v(t_0) = v(2ms) = 4000 \times 0.002 = 8 \, V$



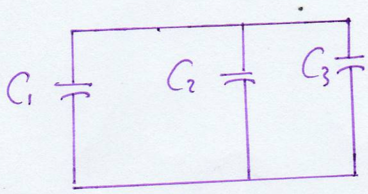
"Capacitors For Energy Storage"

Monday: 19-2-2018

* Combinations of capacitors.

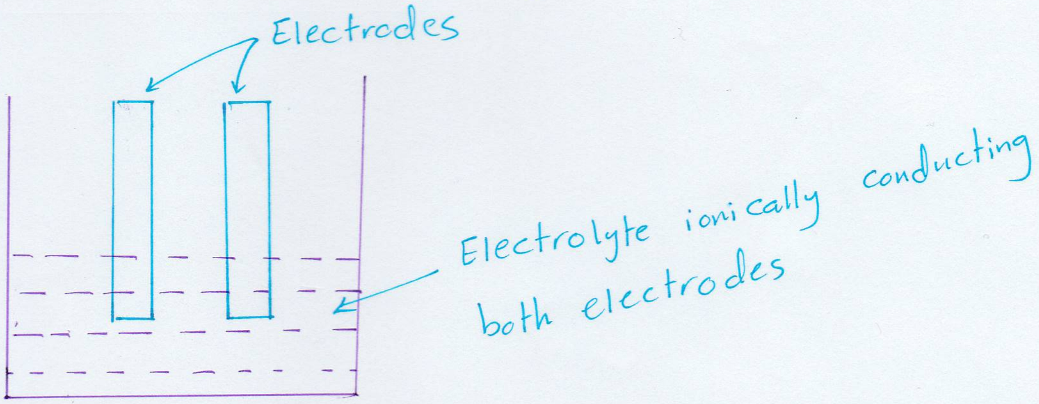


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



$$C_{eq} = C_1 + C_2 + C_3$$

* Super-Capacitors: Electrochemical energy storage devices.



* Supercapacitor Categories:-

1) Electrical Double Layer Capacitor (EDLC).
2) Pseudo Capacitor.
3) Hybrid Supercapacitor

1) EDLC: Charge storage is electrostatic at the electrode-electrolyte interface.

2) Pseudo Capacitor: Energy storage is based on faradic redox reactions.
at: 1) electrode-electrolyte interface.
2) within the bulk of the electrode material.

"Capacitors For Energy Storage"

* Fabrication Materials:

1) EDLCs: Carbon and it's derivatives
2) Pseudo Capacitors: Conducting Polymers and material oxides.

---

* Electrochemical Characterization Techniques:

are used to evaluate the performance of a supercapacitor.

→ Three Techniques:

1) Cyclic Voltmmetry (CV)
2) Charge-Discharge test (CD)
3) Electrochemical impedance spectroscopy. (EIS)

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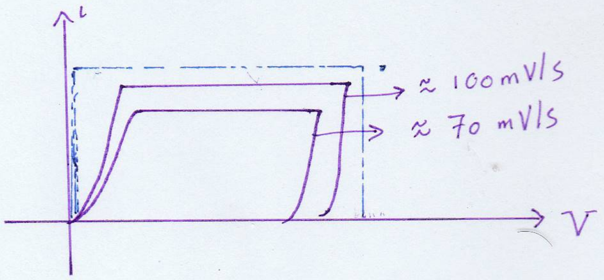
1) Cyclic Voltmmetry (CV):

measures the current of the working electrode versus the applied potential by cycling the potential at various scan rates (mV/sec).



$\rightarrow C = \frac{i_a + i_c}{2S}$ (mF/cm²)

where:
i_a: Cathodic Current
i_c: Anodic Current
S: Scan Rate (mV/sec)

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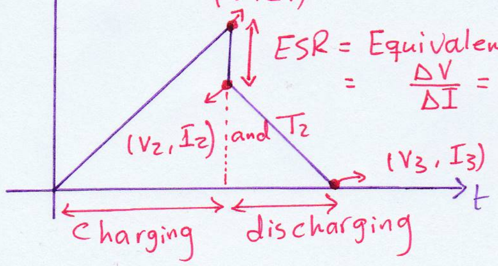
2) Charge_Discharge Technique (CD):

The charge-discharge profile is generated such that the cell voltage is plotted against the time.



ESR = Equivalent Series Resistance $= \frac{\Delta V}{\Delta I} = \frac{V_1 - V_2}{I_1 - I_2}$

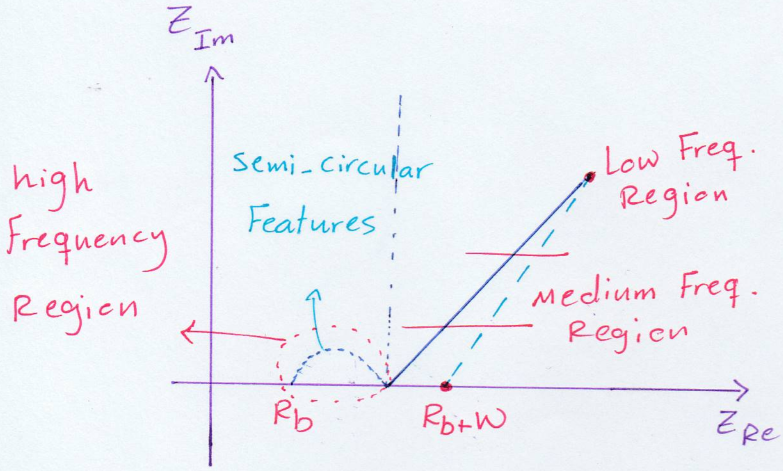
$\rightarrow C_s = I/(\Delta V/\Delta T)$

where:
$\Delta V = V_2 - V_3$
$\Delta T = T_3 - T_2$

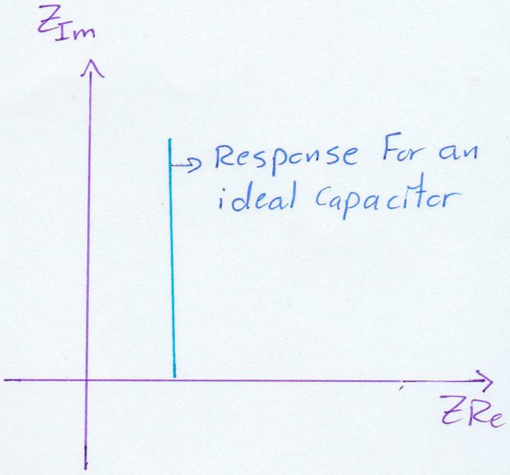
$\rightarrow$ Efficiency $= \frac{t_{Dis}}{t_{ch}} \times 100\%$

3) Electrochemical Impedance Spectroscopy (EIS):

used to evaluate resistive and capacitive properties of a supercapacitor.



"Response for a practical Capacitor"

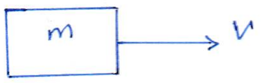


1) Low Frequency Region: Used to evaluate capacitance.
2) Medium Frequency Region: Used to evaluate the ion diffusion resistance (Warburg Impedance)
3) High Frequency Region: Used to evaluate the bulk and the charge transfer resistances of a supercapacitor.

CHP6: Wind Power

→ Only From (6.2-6.6)
where (6.2+6.6)
are selfstudy sections

* Lets Consider a packet of air of mass (m) moving with velocity (v) throughout an area (A):

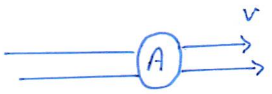


then → (Kinetic Energy "K.E") $= \frac{1}{2} m V^2$

→ Power = Energy per unit time

$$= \frac{1}{2} \left(\frac{\text{mass}}{\text{time}} \right) . v^2$$

→ Mass Flow Rate $(\dot{m}) = \dfrac{\text{mass passing through area (A)}}{\text{time}}$



→ $\dot{m} = \rho A v$, where: ρ: Air density $= 1.225\ Kg/m^3$ at $15^\circ C$ and 1 atm pressure.

A: Cross Sectional Area.

v: Windspeed normal to A.

∞ Power in the wind = $P_W = \frac{1}{2} \rho A v^3$ (watts)

" Wind Power "

* To calculate the power generated by a wind power system:

$$P_w = \frac{1}{2} \rho A v^3$$

* This equation is non-Linear → The average wind speed can't be used to predict the energy in a wind power system.

Ex) At 15°C and 1 atm, calculate the energy consumed in 1 m^2 in the following systems:

a) $v = 6$ m/s for 100 hours.

$$P_w = \frac{1}{2} . \rho . A . v^3 \rightarrow E = P . \Delta t$$

$$\therefore E = \frac{1}{2} . \rho . A . v^3 . \Delta t = \frac{1}{2}(1.225)(1)(6)^3(100) = 13230 \text{ Wh.}$$

b) $v = 3$ m/s For 50 hours.

$$E = \frac{1}{2}(1.225)(1)(3)^3(50) = 827 \text{ Wh.}$$

c) $v = 9$ m/s For 50 hours

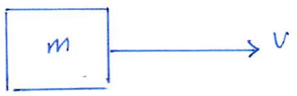
$$E = \frac{1}{2}(1.225)(1)(9)^3(50) = 22326 \text{ Wh.}$$

"WIND POWER SYSTEMS"

Monday: 26-2-2018

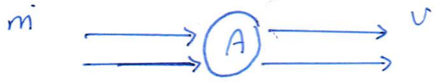
* Recall that:

$$K.E = \frac{1}{2} m v^2$$



$$\text{Power} = \text{Energy per unit time}$$

$$= \frac{1}{2} \left(\frac{\text{mass}}{\text{time}} \right) . v^2$$



$$\dot{m} = \frac{\text{mass passing through an area } A}{\text{time}}$$

* $\dot{m}$ is called mass flow rate.

$$\dot{m} = \rho A v$$

* Power in the wind, ⇒ $P_w = \frac{1}{2} . \rho . A . v^3$ (watts)

* specific power (watt/m^2) = $\frac{1}{2} . \rho . v^3$

* under 15° C and 1 atm pressure:

$$\rho = 1.225 \ Kg/m^3$$

Monday: 26-2-2018

⇒ The ideal gas Low is used to calculate the air density at conditions other than 15°C and 1 atm pressure.

$$PV = nRT$$

where:

- P: Absolute pressure [atm]
- V: Volume [m^3]
- n: Number of moles [mol]
- R: Ideal gas constant = 8.2056×10^{-5} [$m^3 . atm . K^{-1} . mol^{-1}$]
- T: Temperature in kelvin [°C + 273.15].

⇒ 1 atm = 101.325 KPa (Pa → Pascal.)

⇒ To calculate the air density:

$$\rho = \frac{P \times M.W \times 10^{-3}}{R . T}$$

where:

M.W: Molecular weight ; for air = 28.97 g/mol.

Ex| Calculate the air density at 30°C and 1 atm?

$$\rho = \frac{1 \times 28.97 \times 10^{-3}}{8.2056 \times 10^{-5} \times (30°C + 273.15)} = 1.165 \ Kg/m^3$$

"Altitude Correction For Air Density"

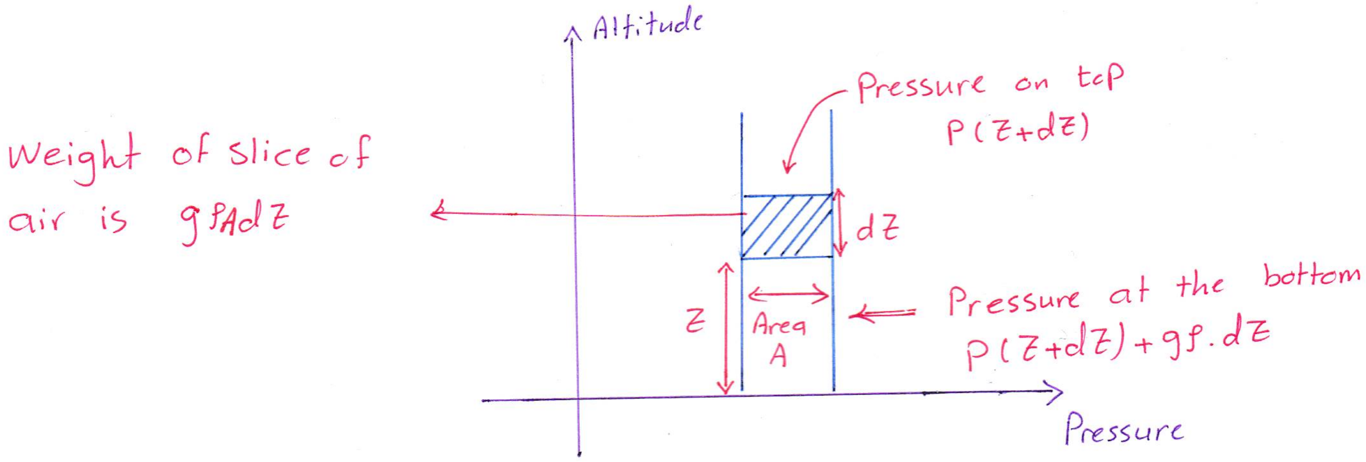
Monday: 26-2-2018

* Air density generally depends on:

  1) Pressure ⇒ A Function of altitude.

  2) Temperature.

* We need a correction to estimate wind power of sites above sea Level.



* Pressure on top = $P(Z+dZ)$

* Pressure on bottom = $P(Z+dZ) + g\rho dZ$

* The incremental pressure (dP) due to the incremental change in elevation:

$$dP = P(Z+dZ) - P(Z) = -g\rho dZ$$

$$\therefore \frac{dP}{dZ} = -g\rho = -g\left(\frac{P \times M.W \times 10^{-3}}{RT}\right)$$

where:

g: gravitational constant = 9.8 m/s^2

⇒ $P = P_o \cdot e^{-(1.185 \times 10^{-4})H}$ ⇒ where P_o: Reference measure at 1 atm.

" Altitude Correction For Air Density

Monday : 26-2-2018

Ex] Estimate the air density for the following cases:

1) 15°C and 2000 m
2) 5°C and 2000 m.

⇒ 1) $$\rho = \frac{P \times M.W \times 10^{-3}}{R\,T}$$

$$\Rightarrow P = P_o \cdot e^{-(1.185\times10^{-4})\times H} = 1\,atm \cdot e^{-(1.185\times10^{-4})\cdot 2000}$$

$$P = 0.789\ atm$$

$$\rho_{15^\circ C} = \frac{0.789 \times 28.97 \times 10^{-3}}{8.2056\times10^{-5} \times (15^\circ C + 273.15)} = 0.967\ kg/m^3$$

---

2) $\rho_{5^\circ C} = 1.00\ kg/m^3$

$$\Rightarrow \rho = 1.225 \cdot K_T \cdot K_A$$

K_T → Temperature Correction Factor

K_A → Pressure Correction Factor

"Altitude Correction For Air Density"

Monday: 26-2-2018

Ex Calculate the power density in a 10 m/s wind power system at 5°C and 2000 m height?

$K_T = 1.04$

$K_A = 0.789$

$$P_w = \frac{1}{2} \cdot \rho \cdot A \cdot v^3$$

Specific Power = Power density = $\frac{1}{2} \cdot \rho \cdot v^3$

$$\Rightarrow \rho = 1.225(1.04)(0.789) = 1.00 \ kg/m^3$$

$$\text{Power density} = \frac{1}{2}(1)(10)^3 = 500 \ W/m^2$$

"Impact of Tower Height"

* To characterize the coefficient of roughness on the wind speed:

$$\left(\frac{v}{v_o}\right) = \left(\frac{H}{H_o}\right)^{\alpha}$$

v: Windspeed at height of H

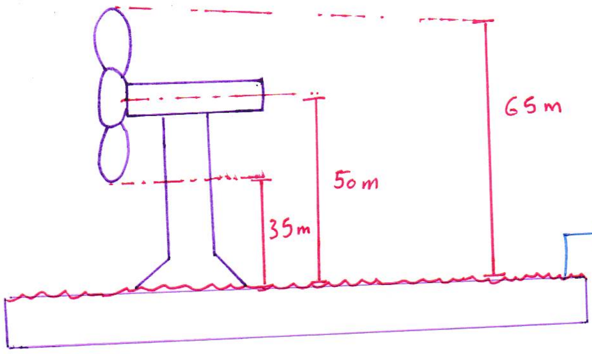
v_o: windspeed at height of H_o

α: Friction Coefficient.

* Rule: H_o is assumed to be 10 m, unless otherwise stated.

"Impact of Tower Height"

Ex



→ Surface with crops, hedges and shrubs

$\Rightarrow \alpha = 0.2$

⇒ Calculate the ratio P_{65} / P_{35}

P_{65}: Power in the wind at height of 65 m

P_{35}: Power in the wind at height of 35 m

$$\Rightarrow \frac{P_{65}}{P_{35}} = \left(\frac{65}{35}\right)^{3 \times 0.2} = 1.45$$

Exercise An anemometer is mounted on top of a surface with a 10 m height, shows a wind speed of 5 m/s.

Calculate: 1) Wind speed

2) Specific Power

at 50 m, Assume 15°C and 1 atm with $\alpha = 0.2$

1) $v_{50} = 5\left(\frac{50}{10}\right)^{0.2} = 6.9$ m/s

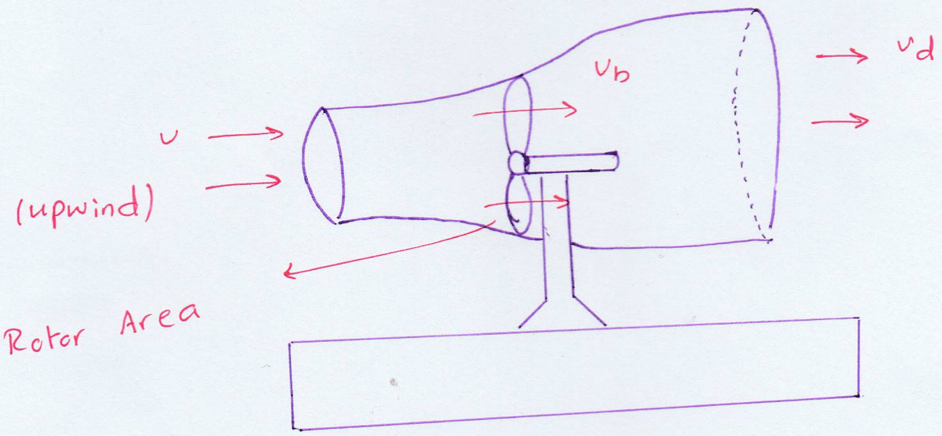
2) Specific Power = Power Per unit Area.

$$= \frac{1}{2} \cdot \rho \cdot v^3 = \frac{1}{2}(1.225)(6.9)^3 = 201 \text{ W/m}^2$$

"Maximum Rotor Efficiency"

Wednesday: 28-2-2018

Renewable Energy Technology	Fundemental Constraint
Heat Engines	Carnot Efficiency
Photovoltaic Materials	Bandgap of the material
Fuel Cells:	Gibbs Free Energy
Wind Power Systems:	Maximum Rotor Efficiency (Bet'z Efficiency) ⇒ Theoritically = 59.2 %



① Pressure ↓

② Velocity ↓

v: upwind speed

v_b: wind speed through blades

v_d: Downwind speed.

* An ideal power system reduces the speed to (1/3) of it's original value.

"Maximum Rotor Efficiency"

Wednesday: 28-2-2018

P_b: Power extracted by the blades:

$$P_b = \frac{1}{2} \cdot \dot{m} \, (v^2 - v_d^2) \quad ; \quad \dot{m} = \rho A v_b$$

* Assume that $v_b = \frac{v + v_d}{2}$

$$P_b = \frac{1}{2} \dot{m} \, (v^2 - v_d^2) = \frac{1}{2} \cdot \rho \cdot A \cdot \left(\frac{v + v_d}{2}\right)(v^2 - v_d^2) \Rightarrow \lambda = \frac{v_d}{v}$$

$$P_b = \frac{1}{2} \rho A \, (v^3 - \lambda^2 v^2) \left(\frac{v + \lambda v}{2}\right)$$

$$P_b = \frac{1}{2} \rho A v^3 \left[\frac{1}{2} (1 + \lambda)(1 - \lambda^2) \right]$$

$$\therefore \quad P_b = P_w \cdot \left[\frac{1}{2} (1 + \lambda)(1 - \lambda^2) \right]$$

↳ C_p: Rotor Efficiency

* To calculate the maximum rotor efficiency:

$$\frac{dc_p}{d\lambda} = 0 \rightarrow \lambda = 1/3$$

⇒ Substitute $\lambda = \frac{1}{3}$ into c_p:

$$c_p = \frac{1}{2} \left(1 + \frac{1}{3}\right)\left(1 - \frac{1}{9}\right) = 59.3\,\%$$

"Tip Speed Ratio (TSR)"

$$\Rightarrow TSR = \frac{\text{Rotor tip Speed}}{\text{Wind Speed}} = \frac{rpm * \pi D}{60 \, v}$$

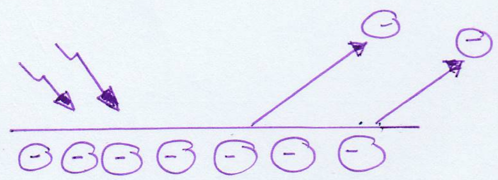
Where,

rpm: revolutions per minute

D: rotor diameter

v: upwind speed.

CH8: "Photoelectric effect".



* Energy of emitted photons depend on λ (not intensity).

* Photon dispertion: Photons having different wavelength, travel with different speed.

* How to calculate the energy of photon:

$$\Rightarrow E = h\nu = \frac{hc}{\lambda} \Leftarrow$$

E: Energy of photons

h: Plank's constant = 6.62×10^{-34}

ν: Frequency (HZ)

C: Speed of Light = 3×10^{8} m/s

$$\Rightarrow E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times \lambda}$$; $1ev = 1.6 \times 10^{-19}$ (J)

$$\Rightarrow E_{(ev)} = \frac{1.24}{\lambda\,(\mu m)}$$ ⇐ To calculate the energy in (ev)

"Photon's Energy"

Wednesday: 28.2.2018

Ex

$\lambda_1 = 400$ nm $\rightarrow$ $\lambda_1 = 0.4$ μm

$\lambda_2 = 800$ nm $\rightarrow$ $\lambda_2 = 0.8$ μm

1) Calculate the energy of photons at specified wavelengths.

$$E_1 (eV) = \frac{1.24}{\lambda_1} = \frac{1.24}{0.4} = 3.1 \ eV$$

$$E_2 (eV) = \frac{1.24}{\lambda_2} = \frac{1.24}{0.8} = 1.55 \ eV$$

2) Calculate the minimum bandgap energy a semiconductor should have to absorb light of 550 nm?

Min bandgap energy = Energy of photons

$\lambda = 550$ nm $= 0.55$ μm

$$E (eV) = \frac{1.24}{0.55} = 2.25 \ (eV)$$

* Blank $\rightarrow$ ideal الفوتونات اللي بيمتصها بطلعها

"Radiation Matrix"

1) Photon's Flux: Number of monochromatic light falling on a surface per meter square per second

$$\phi = \frac{H}{E} = \frac{H}{h\nu} = \frac{H\lambda}{hc}$$; H: Light intensity (W/m^2)

Current density $\leftarrow J = q\phi$ (A/m^2)

Current $\leftarrow I = q\phi A$ (A)

2) Irradient Power Density, H (W/m^2)

is the photon Flux multiply by the energy of photons.

$$H = \phi . E = \phi \left(\underbrace{\frac{hc}{\lambda}} \right)$$

Energy of photons (Joules)

$$H = q . \phi . \left(\underbrace{\frac{1.24}{\lambda}} \right)$$

q: electron charge

Energy of photons in (eV)

$*$ $1 \ eV = 1.602 \times 10^{-19}$ Joule

Ex) A monochromatic light source with

$\lambda = 0.54 \ \mu m = 540 \ nm$

$\phi = 5 \times 10^{20}$ photons / m^2. sec (photon's flux)

calculate power density H (W/m^2)

$$H = \frac{1.6 \times 10^{-19} \times 5 \times 10^{20} \times 1.24}{0.54} = 183 \ W/m^2$$

"Photon's Energy"

Wednesday: 14-3-2018

* Properties of Photons:

1) no mass, no charge.

2) They do NOT spontaneously decay in empty space.

3) Dispersion: Photon having different frequencies travel at different speeds.

4) Undergo scattering, change momentum.

* How to calculate the energy of photons?

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{1.602 \times 10^{-19}\,\lambda} \Rightarrow E(eV) = \frac{1.24}{\lambda[\mu m]}$$

E: Energy of photons

h: Plank's constant = 6.62×10^{-34}

C: Speed of the Light = 3×10^{8} m/s

λ: Wave length.

* Radiation Matrics

1) Photon Flux $\Phi = \frac{H}{E} = \frac{H}{h\nu}$

* Number of monochromatic photons falling on a surface per meter square per second.

$J = q.\Phi$ $(A.cm^{-2})$, $I = q.A.\Phi$ (A)

↳ Photon Flux

2) Irradiant Power Density $H\ (W.m^{-2})$

* is the photon Flux multiplied by the energy per photons
  ↳ Possible units: eV or Joules.

$$H\ (W/m^2) = \Phi \times \boxed{\frac{hc}{\lambda}}$$ → Joule

$$H\ (W/m^2) = q\,\Phi \times \boxed{\frac{1.24}{\lambda}}$$ → energy of photons in (eV).

$1\ eV = 1.602 \times 10^{-19}$ Joules.

---

3) Spectral irradiance $F(\lambda)$.

* Power per unit area (Power density) of a solar cell in a Particular wavelength.
* The unit of $F(\lambda)$ is $W.m^{-2}.\mu m^{-1}$

---

4) Spectral radiant Power $H\ (W.\mu m^{-1})$.

* Over all wavelength.
* The total power density emitted from a light source over all wave length.

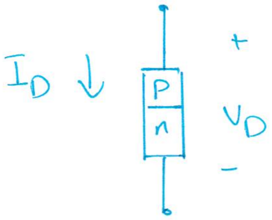
$$H\ (W.\mu m^{-1}) = \int_0^{\infty} F(\lambda)\, d\lambda$$ "Contineous radiant power"

$$H\ (W.\mu m^{-1}) = \sum_{i=1}^{N} F_i(\lambda) . \Delta\lambda$$ "Discrete radiant power"

"Electrical Characteristics of Photo voltaic Materials"

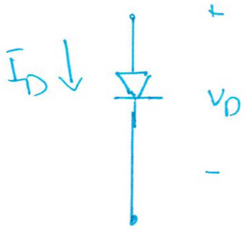
Wednesday:
14-3-2018

* The PN junction:



* If we try to send a current in the reverse direction, only a very small amount would flow.

$I_o = 10^{-12}\ A\ cm^{-2}$



and it's called: reverse saturation current
↳ Caused by thermally generated carriers.

* Shockly Equation:

used to represent the I-V characteristic curve of the pn-junction diode.

$$I_D = I_o \left(e^{qV_D/KT} - 1 \right)$$

I_o: Reverse saturation current

q: electronic charge = 1.602×10^{-19} C

V_D: Forward-bias voltage

k: Boltzman's Constant = 1.38×10^{-23} J/K

T: temperature in kelvin.

* At 25°C: $I_D = I_o \left(e^{38.9 V_D} - 1 \right) \Rightarrow V_D = 0.0257 \ln \left(\frac{I_D}{I_o} + 1 \right)$ (0.0257 ← 1/38.9)

Ex| Pn junction at 25°C , $I_o = 1\times10^{-9}\ A/cm^{-2}$, calculate V_D under the following conditions:

1) $I_D = 1A$
2) $I_D = 10\ A$

⇒ At $I_D = 1A$ ⇒ $V_D = 0.0257 \ln\left(1/10^{-9} + 1\right) = 0.532\ V$

⇒ At $I_D = 10\ A$ ⇒ $V_D = 0.0257 \ln\left(10/10^{-9} + 1\right) = 0.592\ V$

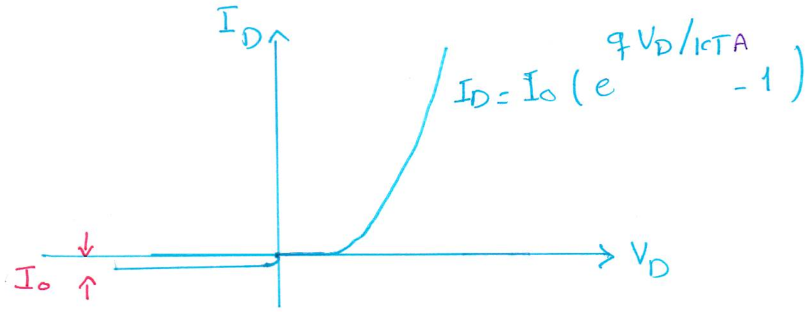
---

* Ideality Factory (A)

represents the different mechanisims that are responsible for moving the carriers across the junction.

⇒ $A = 1$ (if the transport process is purely diffusive)

⇒ $A = 2$ (if it is primarily recombination in the depletion region).

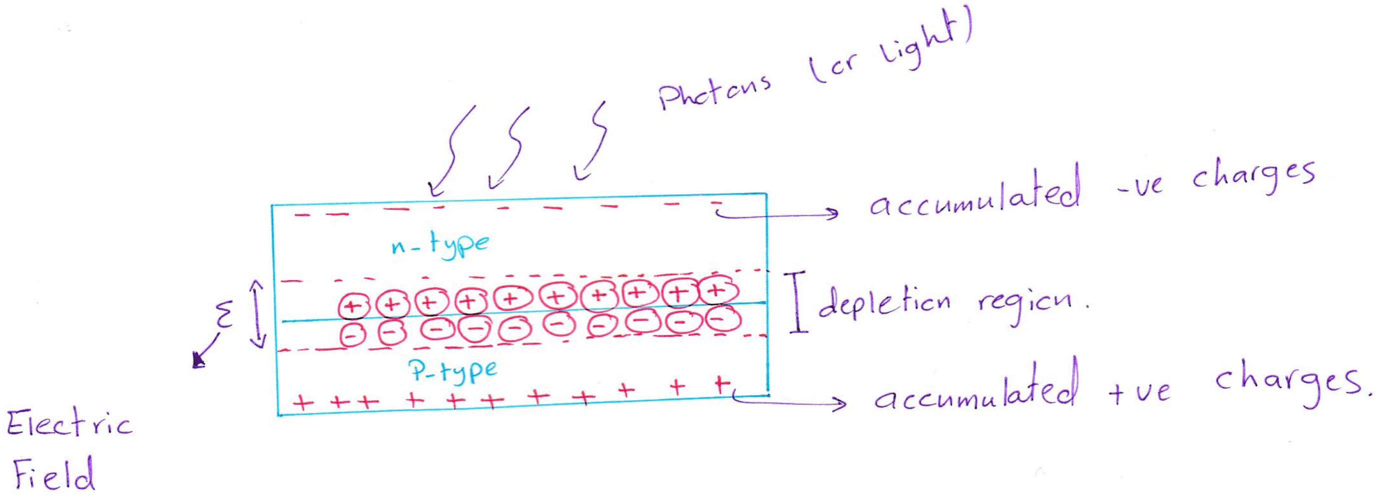


"Electrical characteristics of Photovoltaic Materials"

Wednesday:
14-3-2018

* The general photovoltaic cell.

* what happens when light falls on a pn junction?

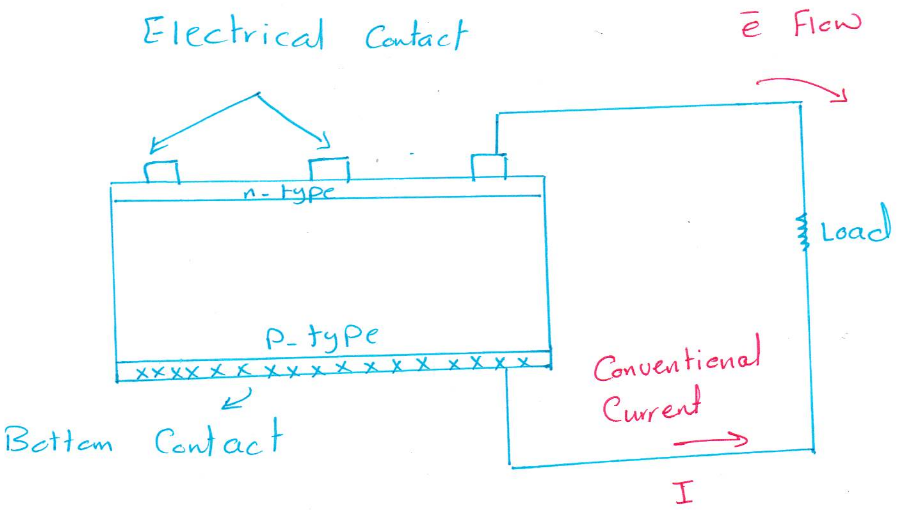


⇒ general principle of operation of PV cells:

1) when photons are being absorbed, electron-hole pair may be formed.

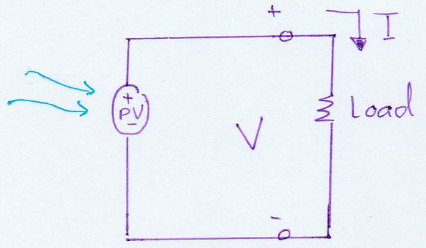
2) Electric field will push the holes into the P-type, and the electrons into n-type.

3) This will create a voltage that can be used to deliver current to the Load.

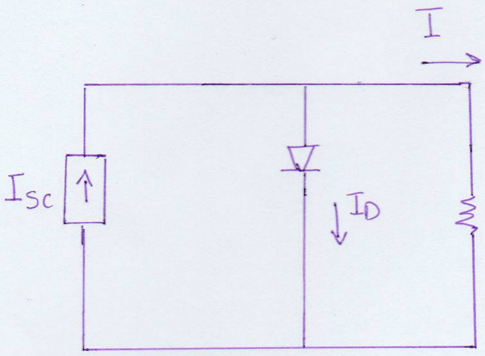


"Equivalent Circuit of Solar Cells"

Wednesday: 14-3-2018



* The equivalent circuit of a solar cell is represented by a real diode connected in parallel with an ideal current source.



⇒ under open circuit conditions:

$V_D = V_{o.c.}$, $I = Zero$ AT 25°C

$I_{sc} = I_o \left(e^{qV_{o.c}/KT} - 1\right) \Rightarrow V_{o.c} = 0.0257 \ln\left(I_{sc}/I_o + 1\right)$

This equation is used to calculate the open-circuit voltage.

⇒ KCL: $I = I_{sc} - I_D$

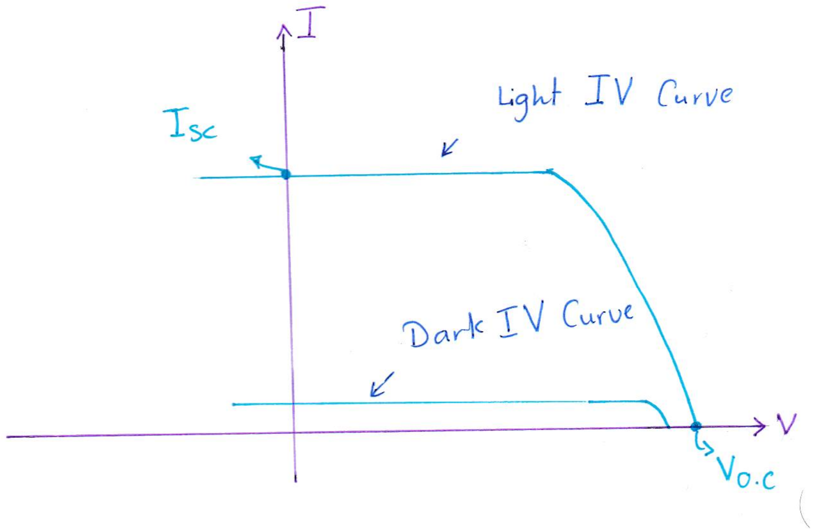
$I = I_{sc} - I_o\left(e^{qV_D/KT} - 1\right)$

under 25°C : $I = I_{sc} - I_o\left(e^{38.9 V_D} - 1\right)$

"I-V Characteristics under Dark and Light Conditions"

Wednesday: 14-3-2018

* The dark curve is just the diode turned upside-down.
* The light curve is the dark curve plus I_{sc}.



Ex| A PV cell with:
- Area = 100 cm²
- Reverse saturation current, $I_o = 10^{-12}\ A/cm^2$

Given: At full intensity, $I_{sc} = 40\ mA.cm^{-2}$ at 25°C.

Determine the open-cct Voltage:

1) At full Sun.
2) At 50 % sunLight.

---

1) At full Sun →

$A = 100\ cm^2 \Rightarrow I_{sc} = 40\ mA.cm^{-2} \times 100\ cm^2 = 4\ A$

$I_o = 10^{-12}\ A.cm^{-2} \times 100\ cm^2 = 10^{-10}\ A$

⇒ To calculate the open-cct voltage:

$$V_{o.c} = 0.0257 \ln\left(\frac{I_{sc}}{I_o} + 1\right) = 0.0257 \ln\left(4 \times 10^{10} + 1\right) = 0.627\ V$$

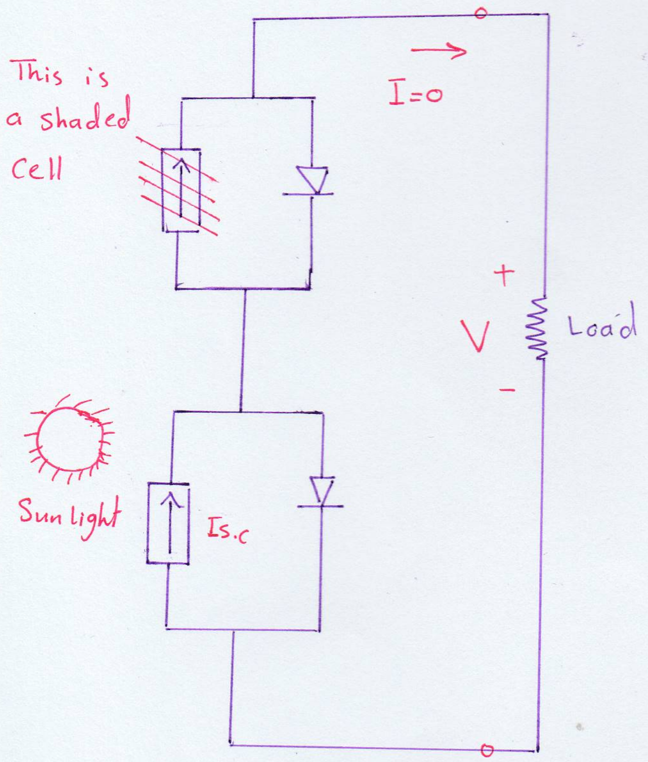
"I_V Characteristics"

Monday: 19-3-2018

2) At 50 % Sunlight ⇒

I_{sc} is reduced by 50 %, since I_{sc} is proportional to the solar intensity ⟹ $I_{sc} = 2A$

& $V_{o.c} = 0.0257 \ln(2 \times 10^{10} + 1) = 0.609 \, V$



⇒ under Shading:

(1) Short cct. current of the current source is Zero.

(2) Diode is reversed biased.

* Conclusion: No power will be delivered to the load if any of the cells is shaded.

* To improve the accuracy of the equivalent circuit:

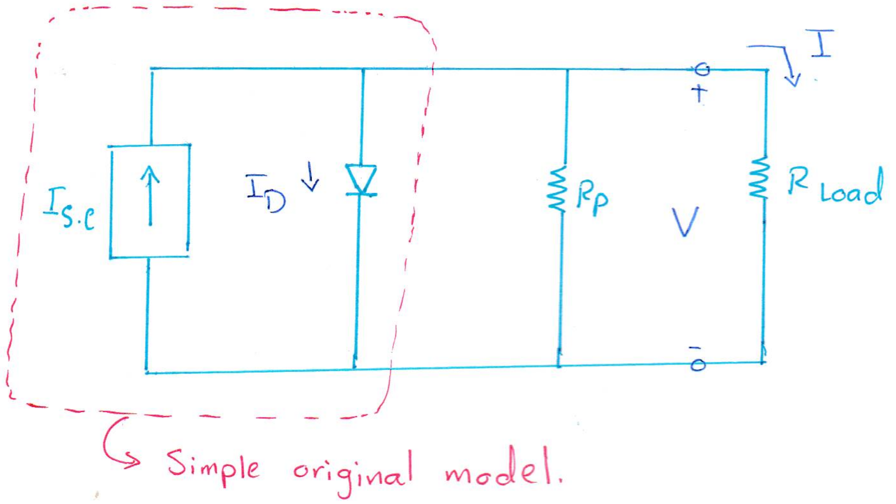
(1) Add Parallel leakage resistance. (R_p)

(2) Add Series resistance. (R_s)

"I-V Characteristics"

Monday: 19-3-2018

1) Parallel leakage resistance

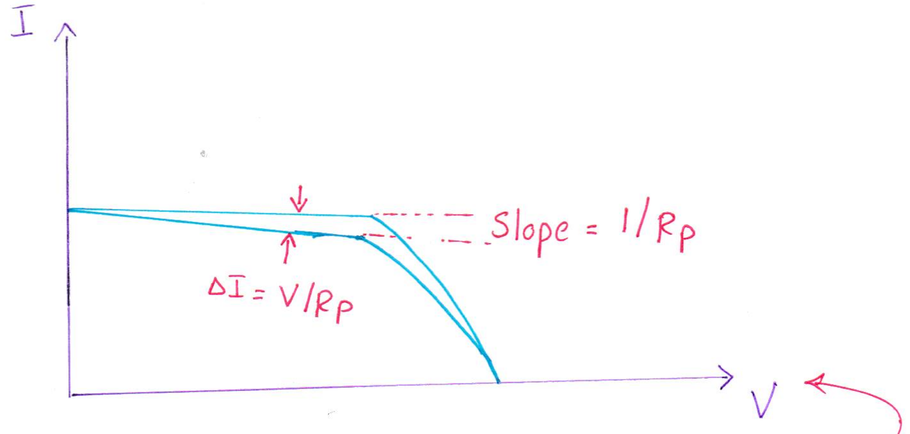


⇐ Eq. cct. with R_p

$$\Rightarrow \quad I = (I_{sc} - I_D) - \frac{V}{R_p}$$

This is the current of the original simple model.

* Conclusion: R_p reduces the load current of the ideal model by (V/R_p).



"Effect of R_p on the PV I-V Curve"

* Design Conditions:

To achieve losses of less than 1% due to R_p ⇒

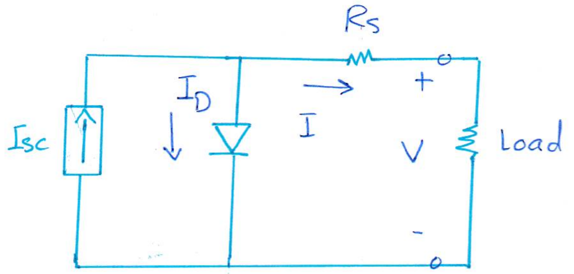
$$R_p > \frac{100\, V_{o.c}}{I_{s.c}}$$

"I-V Characteristics" — Monday: 19-3-2018

2) Series resistance. (R_s)

⇒ Where does the series resistance come from?

1. Contact resistance.
2. Resistance of the semi-conductor material.



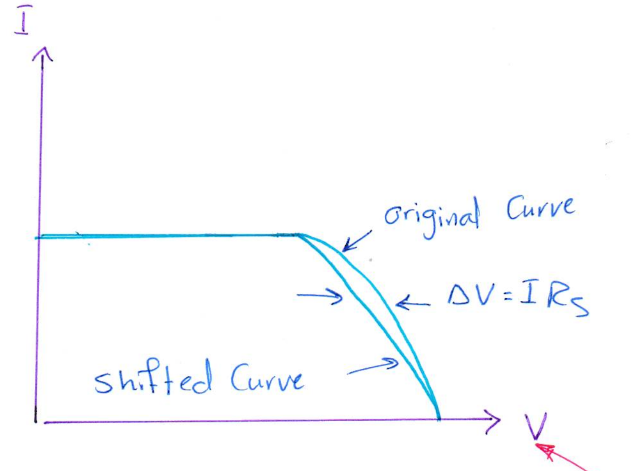
← Eq. Cct. with R_s

⇒ $I = I_{sc} - I_D$

$$I = I_{sc} - I_o\left(e^{qV_D/kT} - 1\right) = I_{sc} - I_o\left(e^{q(V+IR_s)/kT} - 1\right)$$

where, $V_D = V + IR_s$

↳ This equation can be interpreted as the original PV I-V curve with the voltage at any given current shifted to the left by $\Delta V = IR_s$.



"Effect of R_s on the PV I-V Curve"

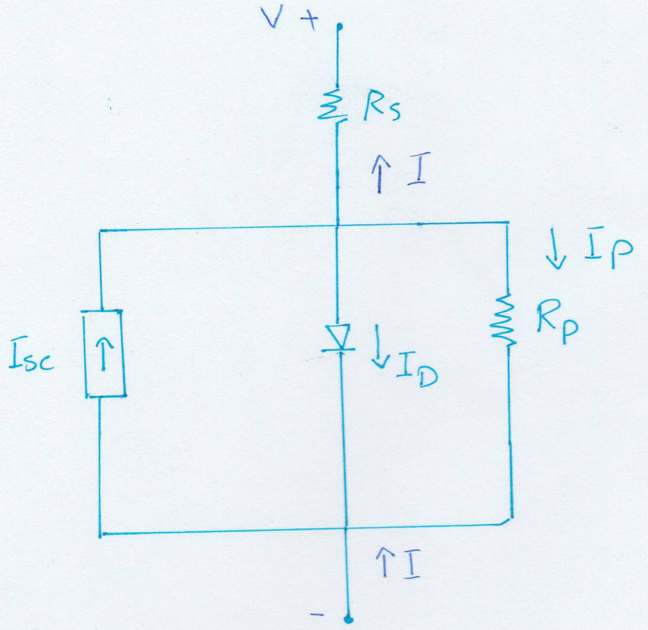
* Design Condition:

To achive losses of less 1% ⇒

$$R_s < \frac{0.01\, V_{oc}}{I_{sc}}$$

"I-V Characteristics"

* Generalization of the PV Equivalent Circuit Model.



$$\Rightarrow \quad I = I_{sc} - I_D - \frac{V_D}{R_p} = I_{sc} - I_o\left(e^{qV_D/KT} - 1\right) - \frac{V_D}{R_p}$$

At 25°C:

$$I = I_{sc} - I_o\left(e^{38.9V_D} - 1\right) - \frac{V_D}{R_p}$$

where, :

$$V_D = V + IR_s$$

$$\Rightarrow I = I_{sc} - I_o\left(e^{38.9(V+IR_s)} - 1\right) - \left(\frac{V + IR_s}{R_p}\right)$$

* The voltage of a single PV cell is 0.5 V.

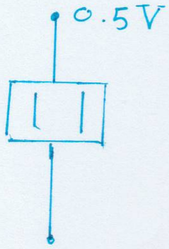
∞ Module: group of PV cells connected in series.

"Types of PV Modules"

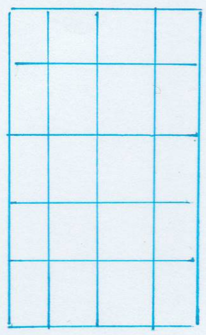
Wednesday: 21.3.2018

⇒ PV module: A group of photovoltaic cells connected in series, and encapsulated as one unit.

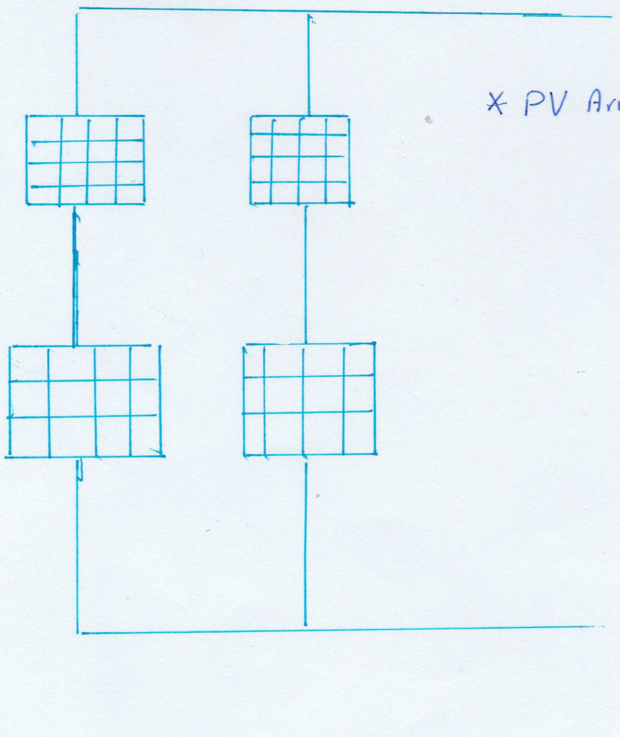
1) 12-V module (36 cells connected in series).
2) 24-V module (72 cells connected in series).



Single PV Cell.



PV module

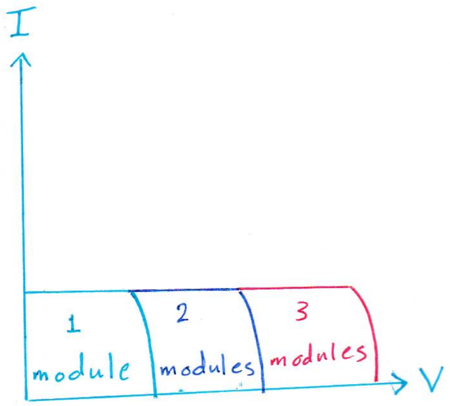


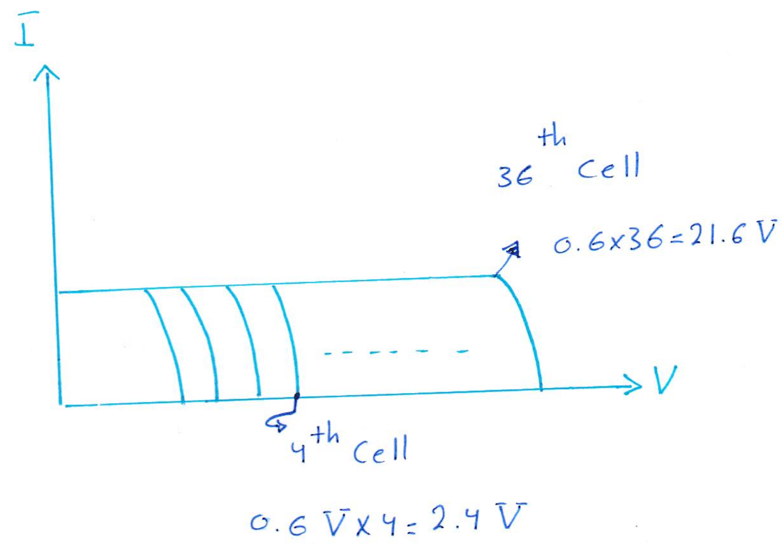
* PV Array: A group of PV modules connected together in series or in parallel.

"Types of PV Modules"

Wednesday: 21-3-2018

* Assume an individual cell voltage of 0.6 V.





* To calculate the overall voltage of the module:

$$\Rightarrow V = n\,(V_D - I R_s)$$

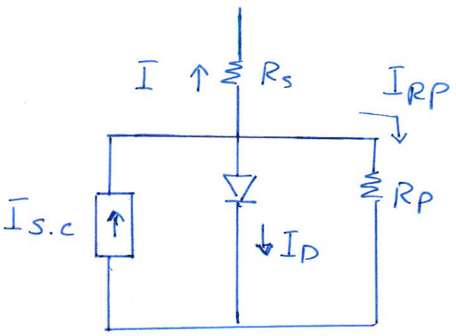
where, n :- Number of Cells.

Ex| A PV module that consists of 36 cells connected in series.

Given: $I_{s.c} = 3.4\,A$, $R_P = 6.6\,\Omega$

$I_o = 6\times10^{-6}\,A$, $R_s = 0.005\,\Omega$

$V_D = 0.5\,V$, $T = 25^\circ C$ ↳ Standard Temp.



Find:

1) Current of the PV module?

$$I = I_{s.c} - I_D - I_{RP} = I_{sc} - I_o\left(e^{qV_D/kT} - 1\right) - \frac{V_D}{R_P}$$

$$\Rightarrow I = 3.4 - (6\times10^{-6})\left(e^{38.9\times0.5} - 1\right) - \frac{0.5}{6.6} = 3.16\ A$$

" Types of PV Modules " Wednesday: 21-3-2018

2) Calculate the overall voltage of the module?

$$V_{module} = n(V_D - I R_s)$$

$$= 36(0.5 - 3.16 \times 0.005)$$

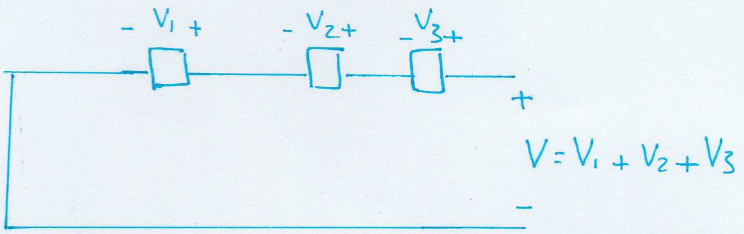
$$V_{module} = 17.43 \ V$$

3) Power?

$$Power = V_{module} \cdot I = (17.43)(3.16) = 55 \ Watts$$

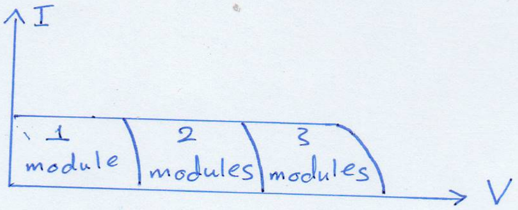
---

1) PV modules Connected in series



* String: Series group of modules

⇒ Series Connection is used to increase the voltage



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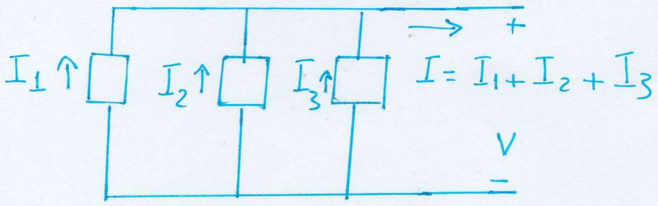
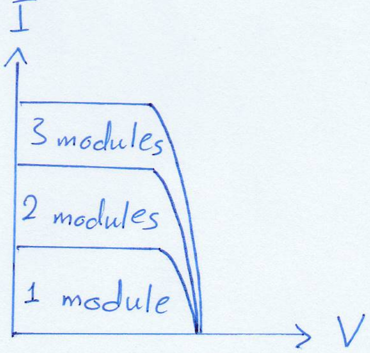
2) PV modules Connected in parallel.





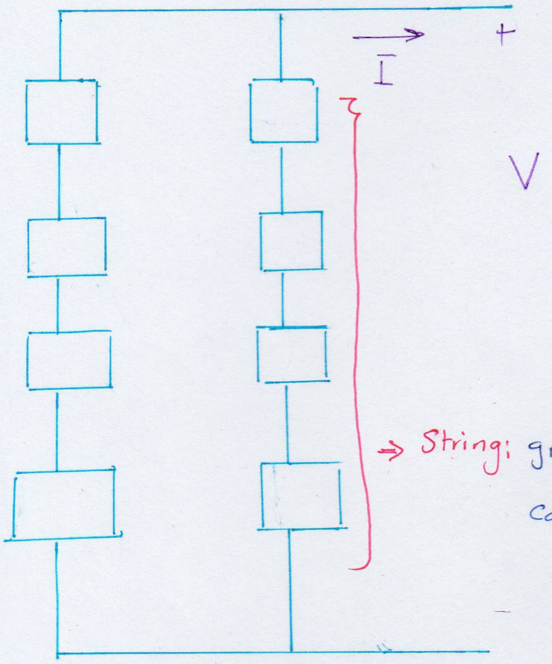
⇒ Parallel Connection is used to increase the current

"Types of PV Modules"

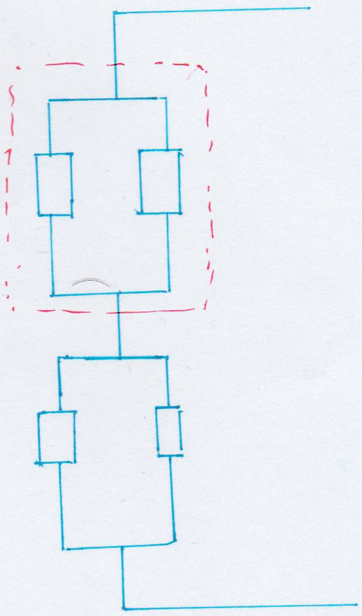
Wednesday: 21-3-2018

* To enhance the power requirement

⇒ Series and parallel combinations of PV modules may be used.



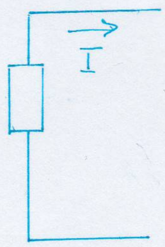
⇒ String: group of PV modules connected in series



This connection is more efficient because if an entire String is removed from service we can still deliver whatever voltage needed by the load.

1) Open - Circuit

PV module

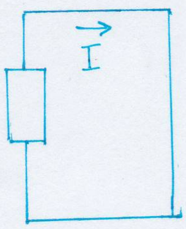


$I = Zero$

$V = V_{o.c}$

$P = VI = 0$

2) Short - Circuit



$I = I_{sc}$

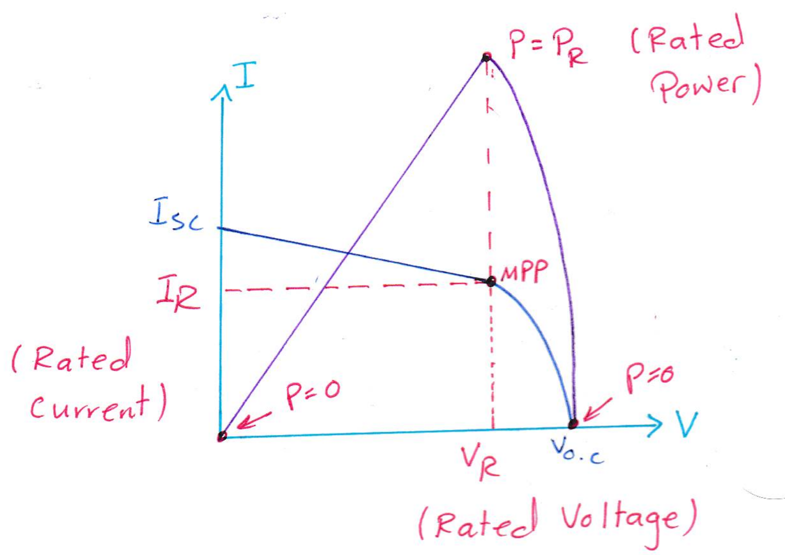
$V = 0$

$P = VI = 0$

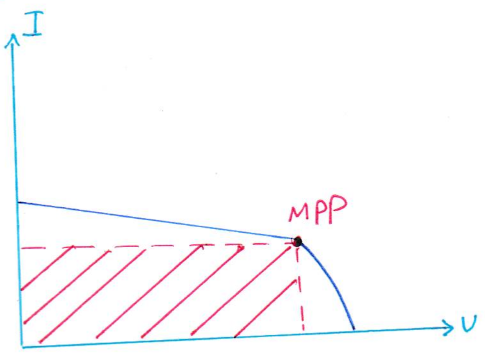
"Types of PV Modules"

Wednesday: 21-3-2018

* If a load is connected to the PV modules, a combination of current and voltage will result ⇒ Power



OR



* The biggest possible area beneath the rectangle

---

* $$\text{Fill Factor} = \frac{\text{Power at MPP}}{V_{o.c} * I_{s.c}} = \frac{V_R I_R}{V_{o.c} I_{s.c}}$$

⇒ (70-75) % : Crystelline Silicon Solar cells.

⇒ (50 - 60) % : Multi-junction amorphous Si.

---

* To enable fair comparisons of one PV module to another, standard test conditions (STC) have been established:

1) 1- Sun (1 KW/m²) ⇒ insolation or solar radiation.
2) AM = 1.5 ⇒ air mass ratio
3) T = 25°C ⇒ Temperature of the cell itself

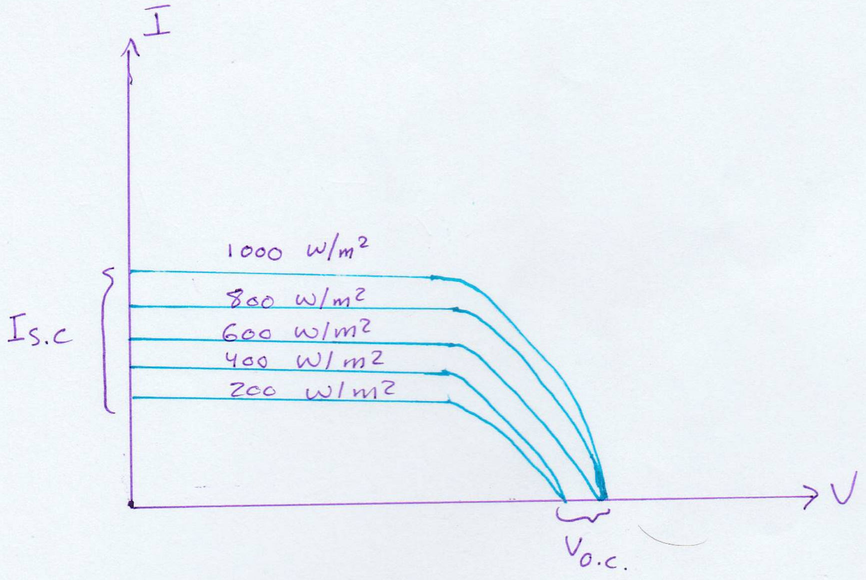
⇒ Air Mass Ratio (AM):

The path of the length taken by the sun rays through the atmosphere to reach a spot on the ground, divided by the path length directly overhead.

"Impacts of Temperature and Insolation on I.V Curves"

Monday: 26-3-2018

⇒ "Impact of Insolation on PV IV Curves"



* When insolation is reduced:

1) I_{sc} is also reduced following the same proportion.

2) $V_{o.c.}$ voltage also decreases following a logarthmic relationship (Less impact)

⇒ "Impact of Temperature on PV I.V Curves"

1) When temperature increases, the open-circuit voltage ($V_{o.c.}$) decreases significantly.

⇒ For crystalline Si Solar cells, the $V_{o.c.}$ is reduced by 0.37 % for each 1°C rise in temp.

2) The short circuit current increases only slightly.

⇒ For crystalline Si Solar cells I_{sc} increases by 0.05 % for each 1°C rise in temp.

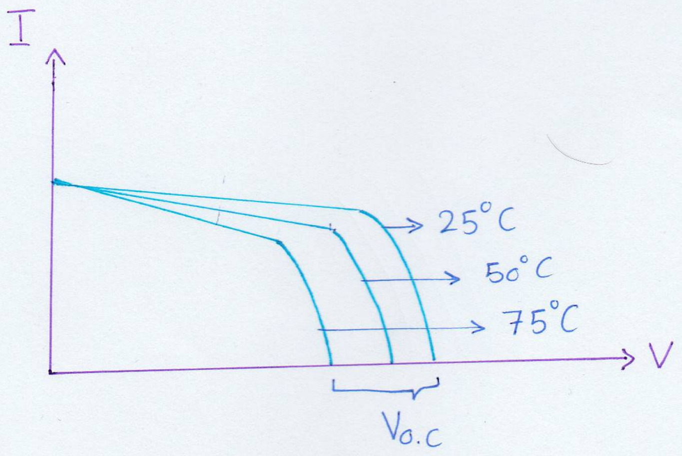
"Impacts of Temperature and Insolation on I-V Curves"

Monday:
26-3-2018

* For crystalline Si solar cells:

For each 1°C rise in cell temperature ⇒

1) $V_{o.c}$ is reduced by 0.37 %
2) $I_{s.c}$ increases by 0.05 %
3) Maximum power is reduced by 0.5 %



* To help system designers account for changes in Temp ⇒

NOCT [nominal operating cell temperature] is the temperature of a PV module when:

1) Ambient Temp. is 20°C.
2) Solar insolation of 0.8 KW/m²
3) Windspeed is 1 m/s

$$\Rightarrow \quad T_{cell} = T_{ambient} + \left(\frac{NOCT - 20°}{0.8}\right) \cdot S$$

$$T_{cell} = T_{ambient} + \gamma \left(\frac{insolation}{1\ kW/m^2}\right)$$

γ: Proportionality Factor (25°C - 35°C)

Ex| Consider a 150-W PV module under 1-sun condition,

Given:
1) ambient = 30°C
2) NOCT = 47°C
3) $V_{o.c}$ = 42.8 V at 25°C

} Assuming it Crystalline Si

Find :

① cell temperature.

$$T_{cell} = T_{ambient} + \left(\frac{NOCT - 20^\circ}{0.8}\right) \times S$$

$$T_{cell} = 30^\circ C + \left(\frac{47 - 20}{0.8}\right) \times 1$$

$$T_{cell} = 64^\circ C$$

② Open-Circuit Voltage

$$V_{o.c} = 42.8 \left[1 - 0.0037 \left(64^\circ c - 25^\circ c \right) \right]$$

$$V_{o.c} = 36.7 \; V$$

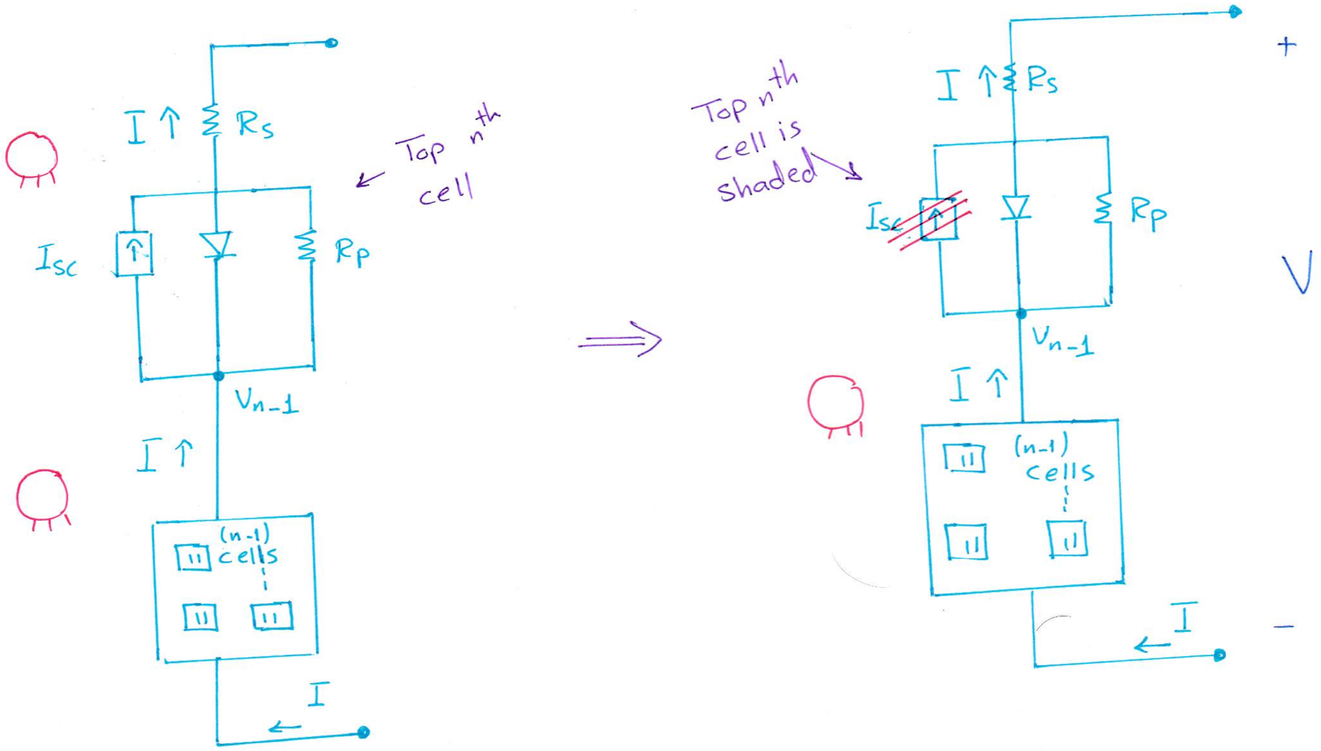
③ Maximum Power

$$P = 150 \left[1 - 0.005 \left(64^\circ - 25^\circ \right) \right]$$

$$P = 121 \text{ Watts.}$$

"Physics of Shading"

Monday: 26-3-2018



⇒ Let's consider the case when the bottom (n-1) cells still have full sun, and still somehow their original current I:

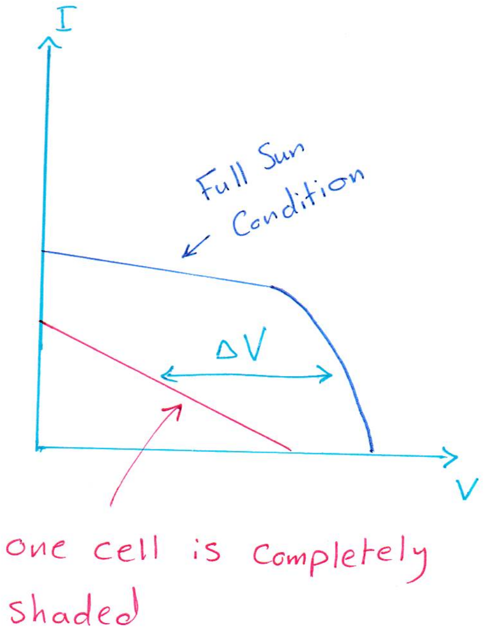
$$V_{SH} = V_{n-1} - I(R_p + R_s)$$

$$V_{SH} = \left(\frac{n-1}{n}\right)V - I(R_p + R_s)$$

$$V_{SH} = \left(1 - \frac{1}{n}\right)V - I(R_p + R_s)$$

$$\Delta V = V - V_{SH}$$

$$\Rightarrow \Delta V = \frac{V}{n} + I(R_p + R_s)$$



if $R_p \gg R_s \Rightarrow \Delta V = \frac{V}{n} + I \cdot R_p$

"Physics of Shading"

Monday: 26-3-2018

Ex) Consider a 36 cell photovoltaic module with:

$R_p = 6.6\ \Omega$, $R_s = 0.005\ \Omega$, $I = 2.14$ A , $V = 19.41$ V

1) Calculate the new module output voltage and power.

$$\Delta V = \frac{V}{n} + I(R_p + R_s)$$

$$\Delta V = \frac{19.41}{36} + 2.14\,(6.6 + 0.005) = 14.66\ V$$

$$V_{module} = 19.41 - 14.66 = 4.75\ V$$

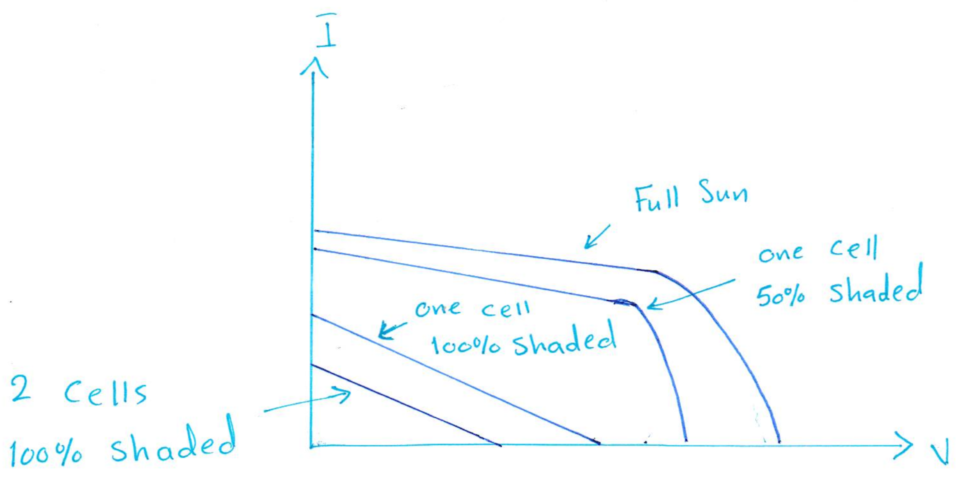
$$P_{module} = (4.75)(2.14) = 10.1\ \text{Watts}$$

2) Calculate the voltage drop across the shaded cell.

$$V_c = I(R_p + R_s) = 2.14\,(6.6 + 0.005) = 14.14\ V$$

3) Calculate the power of the shaded cell.

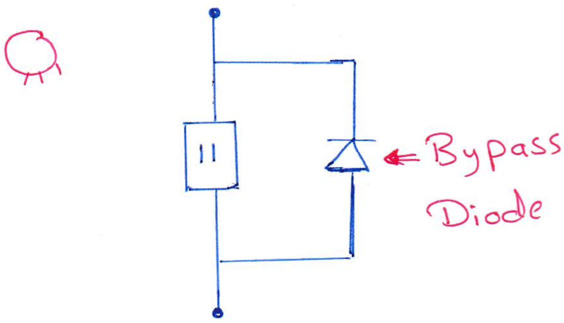
$$P = VI = (14.14)(2.14) = 30.2\ \text{Watts}$$



"Physics of Shading"

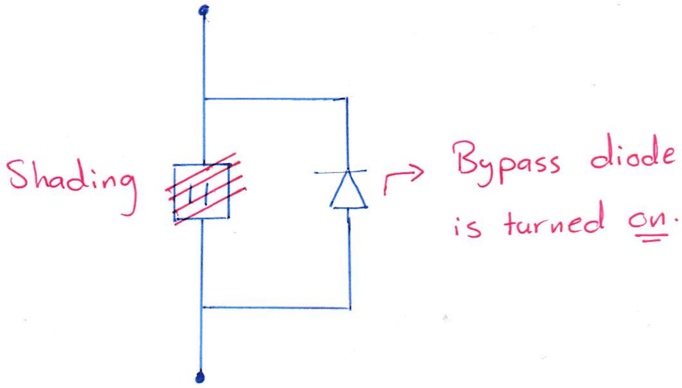
Monday: 26-3-2018

* Bypass diode: used to mitigate the impact of shading.



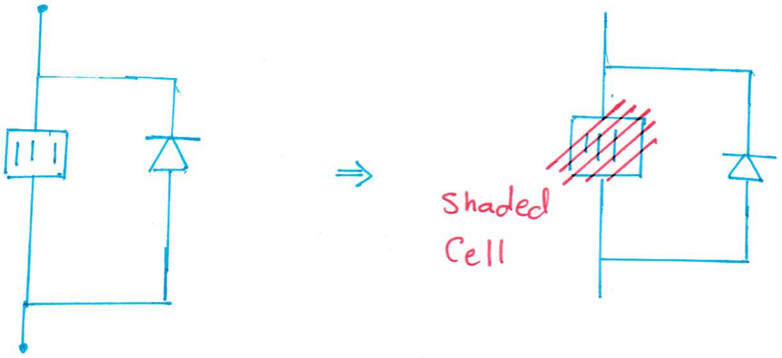
⇒ When a Solar cell is in the sun, there is a voltage rise across the cell, So the bypass diode is cut-off.

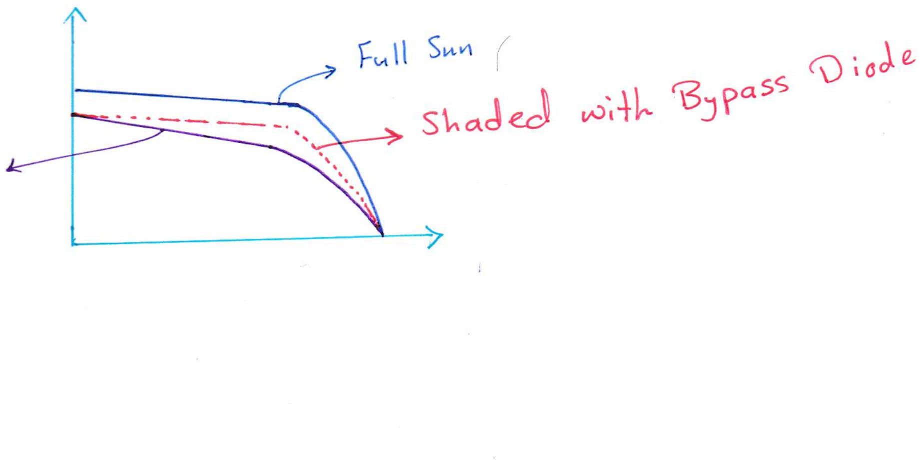
* In case of shading:



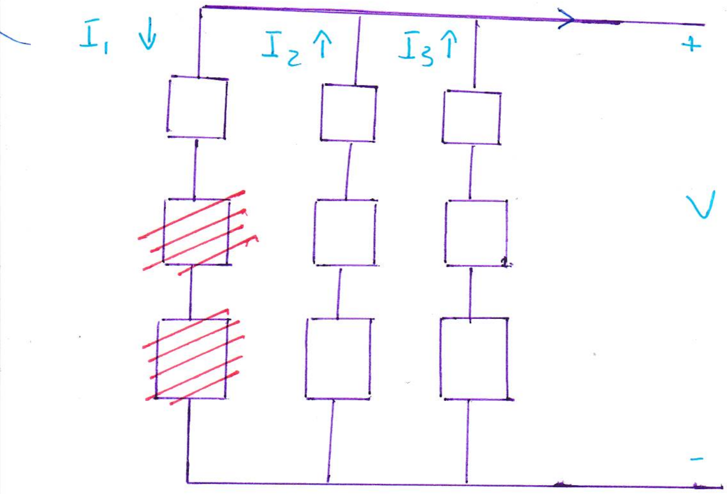
⇒ The bypass diode controls the voltage drop across the shaded cell, Limiting it to 0.6 V instead of the rather large drop that may occur without it.

Recall: By Pass Diode

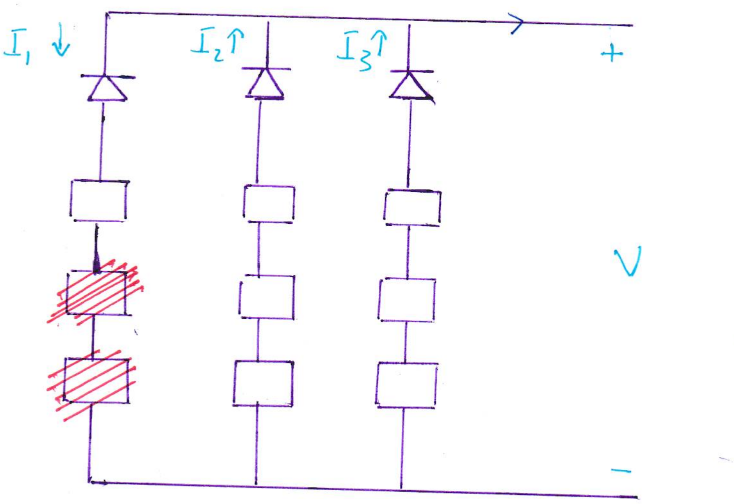




$I = (I_2 + I_3) - I_1$



$I = (I_2 + I_3) - \cancel{I_1}^{\,0}$



* Blocking diodes can be used to mitigate the Problem of shading

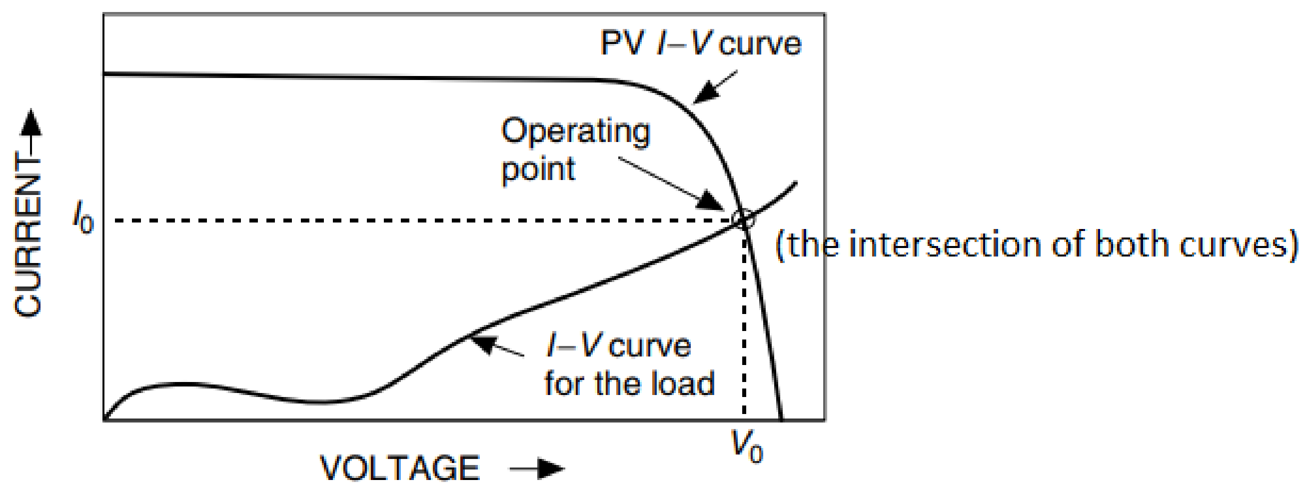
Major Types of PV Systems

1) PV systems that feed power directly into the utility grid

2) Stand-alone PV systems that charge batteries

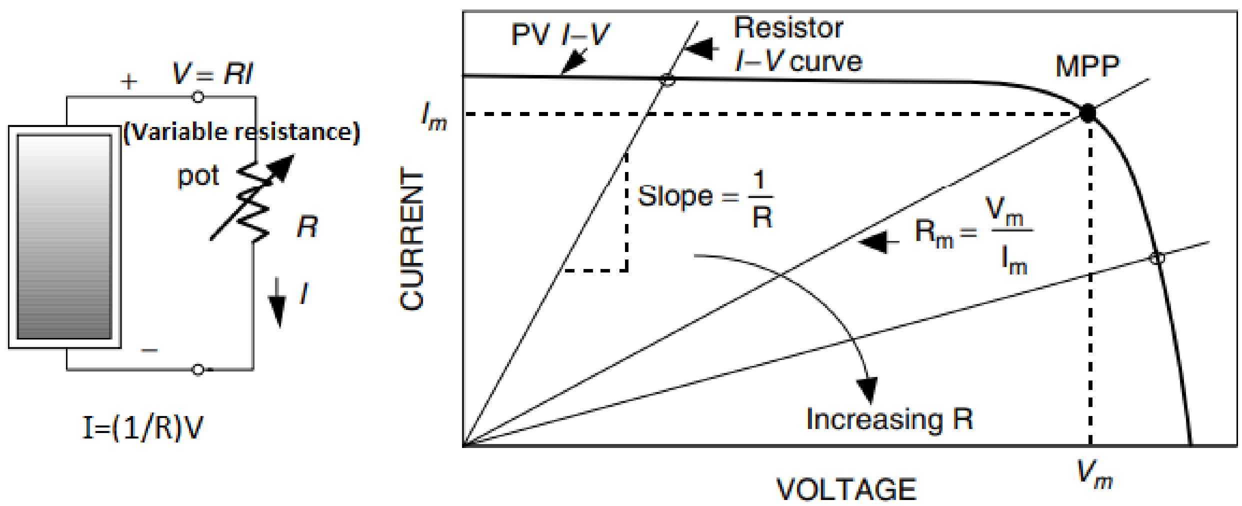
3) Applications in which the load is directly connected to the PV module (such as pumping systems).

Operating Point of PV Systems



- **Simple resistive load:**

By using a variable resistance called a potentiometer as a load and then varying its resistance ; pairs of current and voltage can be obtained which can be plotted to generate the PV I-V curve.



V_m & I_m: Voltage and current at maximum power point.

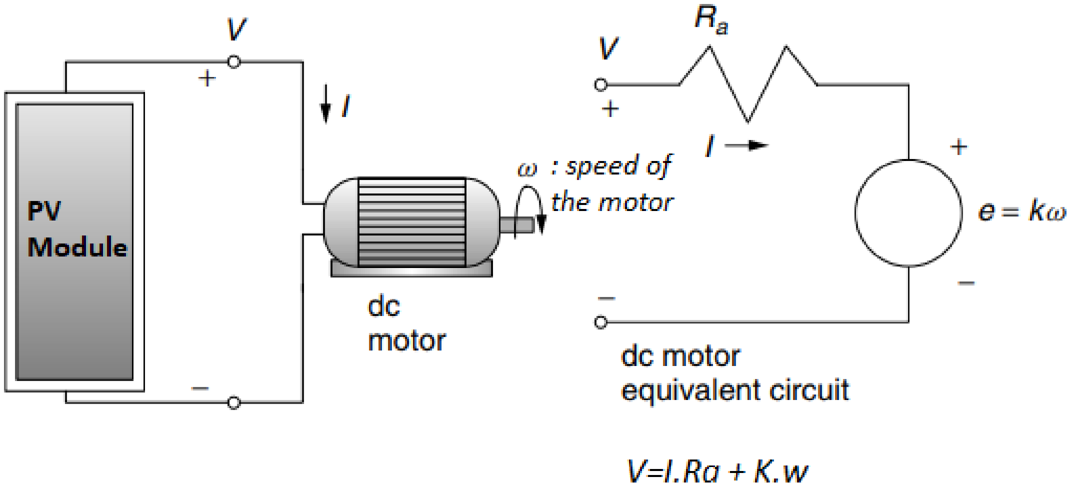
V_R & I_R: Rated values of the voltage and current under Shaded Test Condition.

STC: 1-Sun, 25 °C, AM=1.5

Electrical Model of a Permanent Magnet DC Motor

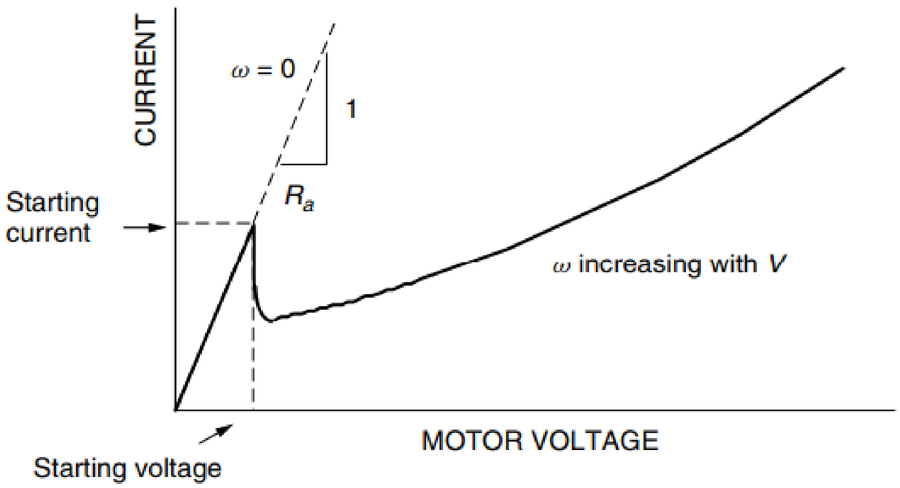
➢ Principle of operation:

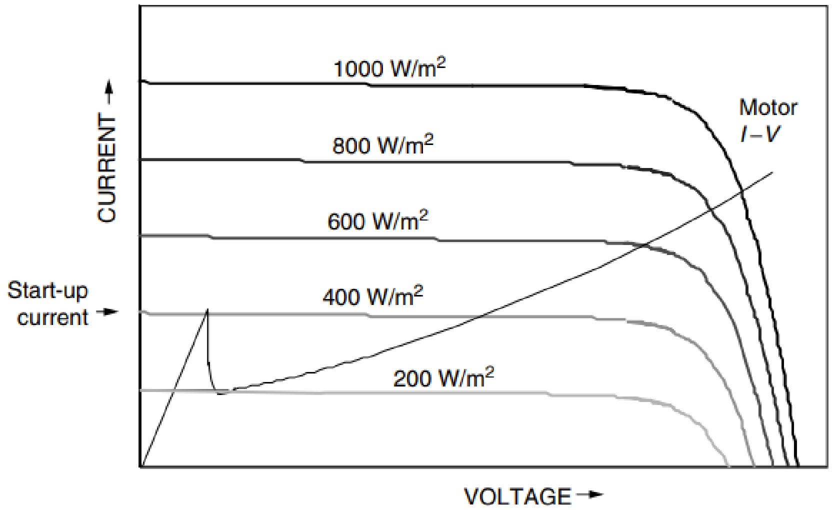
Once the motor starts spinning, it develops back electromotive (emf) force which is a voltage proportional to the speed of the motor.



- Relationship between torque requirement and armature current:

As the torque requirement increase, the motor slows slightly which drops the back emf, allowing more armature current to flow.

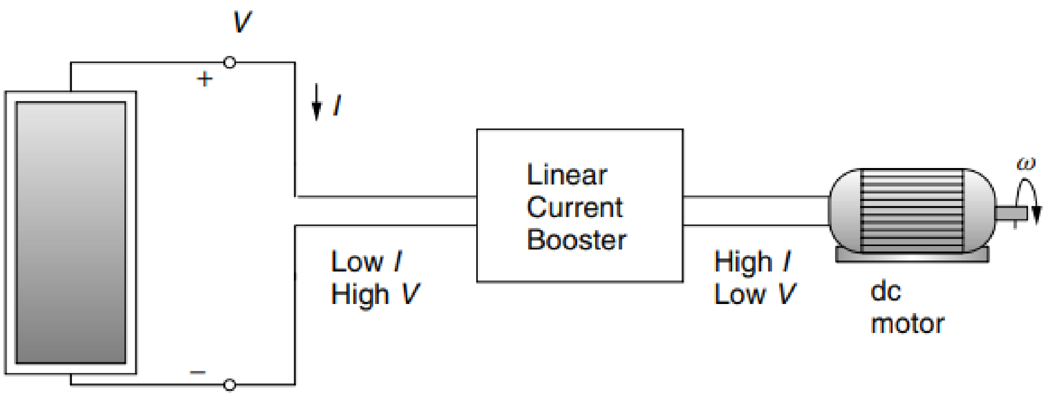




- The DC motor requires $400\ w/m^2$ to break the motor loose from static friction.
- Once the motor start spinning, it requires $200\ w/m^2$ to keep running.

Linear Current Booster

LCB is used to switch the current-voltage relationship



Batteries for Energy Storage

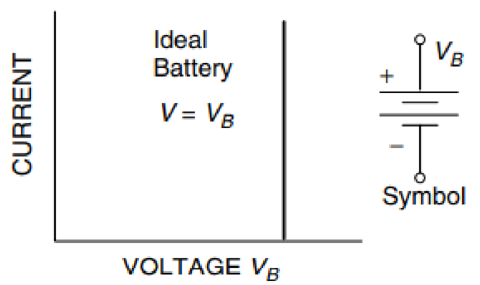
➢ Grid connected systems:

Utility lines themselves can be used to store electrical energy.

➢ Off- grid systems:

Batteries are used to store electrical energy.

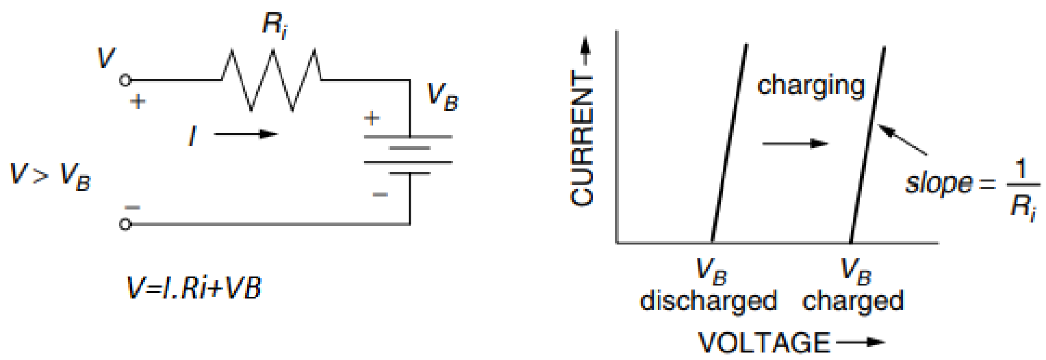
I-V Curve for an Ideal Battery



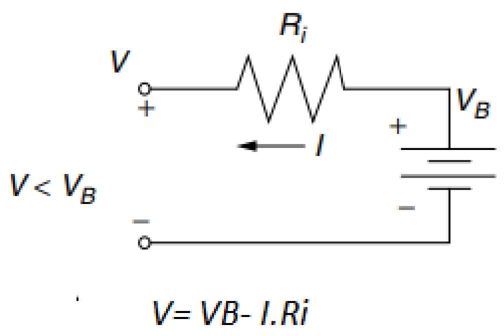
The voltage remains constant no matter how much current is drown

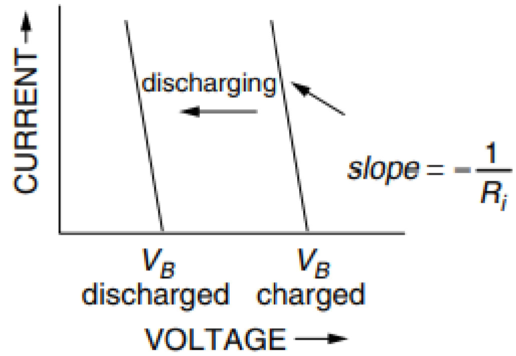
Equivalent Circuit of a Real Battery

1) Charging cycle:



2) Discharging cycle





Open circuit voltage of the battery depends on:

1- Battery temperature

2- State of charge

3- How long the battery has been resting without any current flow

Example: Consider a nearly depleted **12** V lead-acid battery has an O.C. voltage of **11.7** V and an internal resistance of **0.03 Ω** :

a. What voltage would a PV module operate at if it is delivering **6** A to the battery?

$V = I.Ri+VB= (6)(0.03)+11.7= 11.88\ V$

b. If **20** A is drawn from a fully charged battery with open-circuit voltage **12.7** V, what voltage would the PV module operate at?

Discharge cycle: $V = VB-I.Ri= 12.7-(20)(0.03)= 12.1\ V$

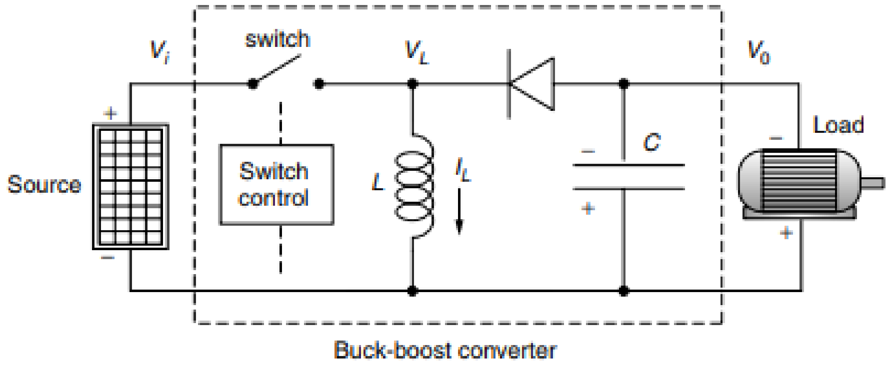
Buck-Boost Converter

- Boost converter

Is a commonly used circuit to step up the voltage from a DC source.

- Buck Converter

To step down the voltage from a DC source.

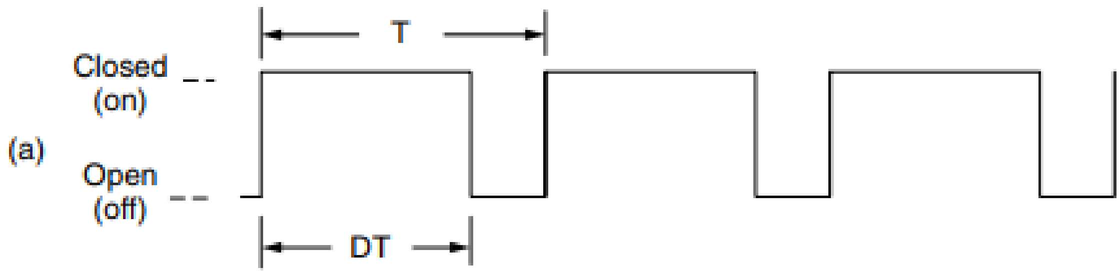


This cct. Depends on the energy balance in the magnetic field of the inductor

➢ When the switch is closed	➢ When the switch is open
1) All of source current goes through the inductor, since the diode blocks any flow to the rest of the circuit.	1) The inductor current now goes through the capacitor, diode and the load.
2) During this portion of the cycle, energy is being added to the magnetic field of the inductor.	2) Magnetic energy begins to collapse.

➢ Duty of the switch

The duty cycle controls the relationship between the input and the output voltages of the buck-boost converter .



$$\frac{Vo}{Vin} = \frac{D}{1-D}$$ D: Duty Cycle

Example: Under certain ambient conditions, a PV module has its maximum power point at Vm = 17 volts and Im = 6 A. What duty cycle should an MPPT have if the module is delivering power to a 10 ohm resistance?

$$P = Vm.Im = (17)(6) = 102\ Watts$$

$$P = \frac{Vout^2}{R} \rightarrow 102 = \frac{Vout^2}{10} \rightarrow Vout = 31.9\ V$$

$$\frac{Vout}{Vin} = \frac{31.9}{17} = \frac{D}{1-D} \rightarrow D = 0.65$$

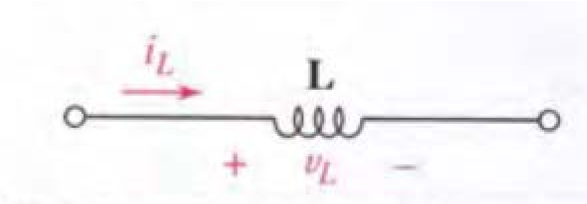
The duty cycle is the time fraction during which the switch is closed, (0<D<1).

Instantaneous Power

$P(t) = v(t) \times i(t)$

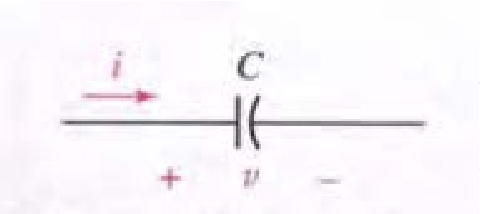
For a resistive load : $P(t) = i^2(t) \times R = v^2(t)/R$

For an inductive load: $P(t) = \frac{1}{L} v(t) \int_{-\infty}^{t} v(t')dt'$



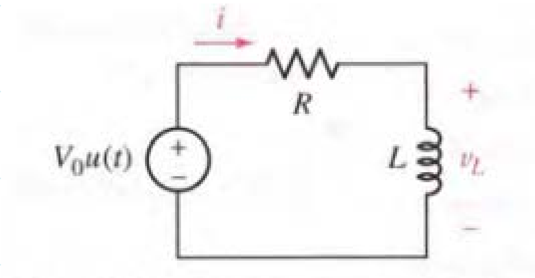
$$v(t) = L \frac{di}{dt}$$

For a capacitive load:



$$i(t) = c \frac{dv}{dt}$$

Example: The expected formula of the forced response is: $i(t) = Vo/R\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)$



1- The instantaneous power of the source.

2- Instantaneous power absorbed by the resistor

3- Instantaneous power absorbed by the inductor

1- Instantaneous power of the source.

$$P(t) = v(t)i(t) = [Vo.u(t)][\frac{Vo}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)]$$

$$P(t) = \frac{Vo^2}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)$$

2- Instantaneous power absorbed by the resistor

$$P(t) = i^2(t).R = \left[\frac{Vo}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)\right]^2.R$$

$$P(t) = \frac{Vo^2}{R}\left[\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)\right]^2$$

3- Instantaneous power absorbed by the inductor

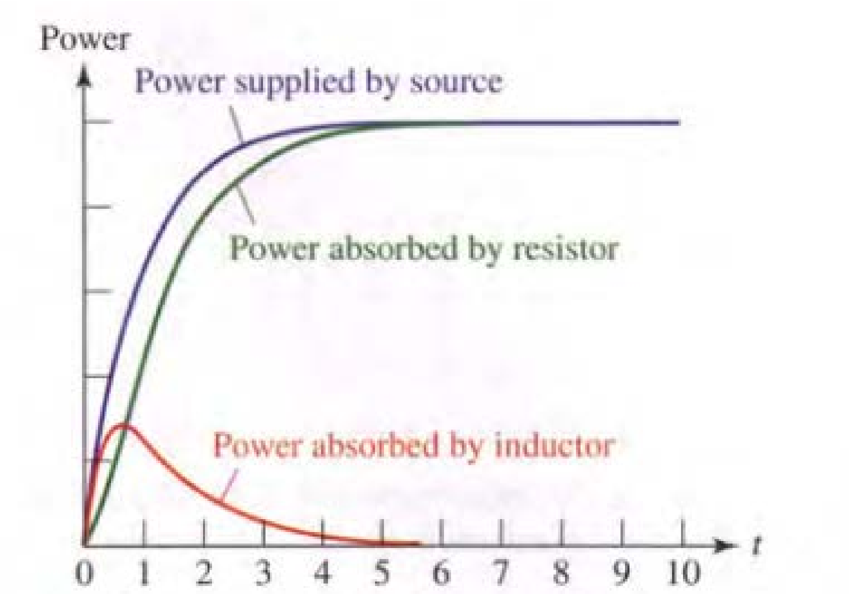
$$V(t) = L\frac{di}{dt}\ ,\ \ i(t) = \frac{Vo}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)$$

$$\frac{di}{dt} = \frac{Vo}{R}\left(0 + \frac{R}{L}e^{-\left(\frac{R}{L}\right)t}\right)u(t) = \frac{Vo}{L}e^{-\left(\frac{R}{L}\right)t}\ u(t)$$

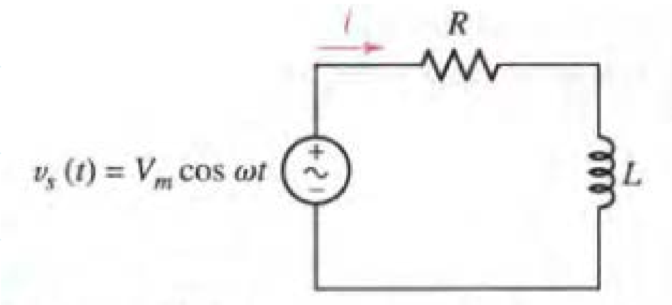
$$L\frac{di}{dt} = \ Vo.e^{-\left(\frac{R}{L}\right)t}\ u(t)$$

$$P(t) = v(t)i(t) = [Vo.e^{-\left(\frac{R}{L}\right)t}\ u(t)\ .\ [\frac{Vo}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)]$$

$$P(t) = \frac{Vo^2}{R}e^{-\left(\frac{R}{L}\right)t}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)$$



Sinusoidal Source Excitation



$$i(t) = Im\,.\cos\,(wt + \emptyset)$$

$$Im = \frac{Vm}{\sqrt{R^2 + w^2L^2}} \quad , \quad \emptyset = tan^{-1}(\frac{wL}{R})$$

$$P(t) = v(t)i(t)$$

$$P(t) = Vm\,.\cos\,(wt) \times Im\,.\cos\,(wt + \emptyset)$$

$$P(t) = \frac{1}{2}Vm.Im.(cos(\emptyset) + cos(2wt + \emptyset))$$

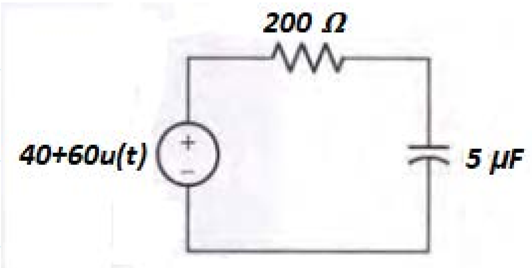
تكامله صفر

$$P(t) = \frac{1}{2}Vm.Im.cos(\emptyset) + \frac{1}{2}Vm.Im.cos(2wt + \emptyset)$$

ثابت

$$P = \frac{1}{2}Vm.Im.cos(\emptyset)$$

Example:



Determine the power absorbed by the **200 Ω** resistor and the **5 μF** capacitor at t= **1.2** ms

Voltage across the R-C combination:

$$At\ t = 0^- \rightarrow u(t) = 0 \rightarrow v(t) = 40 + 60(0) = 40V$$

$$At\ t = 0^+ \rightarrow u(t) = 1 \rightarrow v(t) = 40 + 60(1) = 100V$$

Voltage across the capacitor cannot change instantaneously, so:

$$At\ t = 0^+ \rightarrow Voltage\ across\ R = 60V \rightarrow Io = \frac{60}{200} = 300\ mA$$

$$i(t) = 300\ e^{-\frac{t}{\tau}}\ mA$$

$$\tau = RC = 200 \times 5 \times 10^{-6} = 1\ ms$$

$$i(t = 1.2\ ms) = 300\ e^{-\frac{1.2}{1}} = 90.36\ mA$$

$$P, 200\Omega = i(1.2\ ms)^2 . R = (90.36 \times 10^{-30})^2 \times 200 = 1.633\ Watts$$

$$vc(t) = 100 - 60e^{-\frac{t}{\tau}} \rightarrow vc(t = 1.2\ ms) = 100 - 60e^{\frac{1.2}{1}} = 81.93\ V$$

$$Pc(t = 1.2ms) = vc(t = 1.2ms). ic(t = 1.2ms) = (81.93)(90.36 \times 10^{-3}) = 7.4\ Watts$$

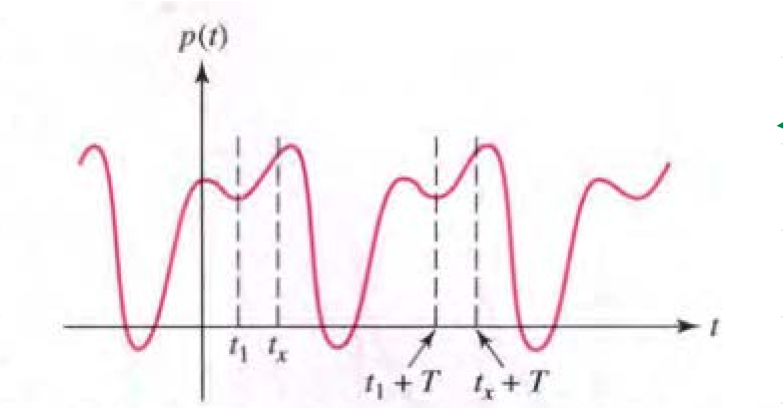
Average Power

$$P = \frac{1}{t2 - t1}\int_{t1}^{t2} p(t)dt$$

Average Power → P; Instantaneous Power → $p(t)$

We may define a periodic function f(t) mathematically by requiring that:

$f(t) = f(t + T)$



← This is a periodic function

P_1: average power at time instant t1

P_x: average power at time instant tx

$P = \frac{1}{t2-t1}\int_{t1}^{t2} p(t)dt$

$$P = \frac{1}{T}\int_{t1}^{t1+T} p(t)dt$$

$$P = \frac{1}{T}\int_{tx}^{tx+T} p(t)dt$$

$$P = \frac{1}{nT}\int_{tx}^{tx+nT} p(t)dt \qquad ;\ n = 1, 2, 3, \ldots$$

Average Power in Sinusoidal Steady State

$v(t) = Vm.\cos(wt + \theta)$, $i(t) = Im.\cos(wt + \emptyset)$

$$P(t) = v(t).i(t) = \frac{1}{2}Vm.Im.\cos(\theta - \emptyset) + \frac{1}{2}Vm.Im.\cos(2wt + \theta + \emptyset)$$

This term is constant (independent on t)

This term has a cyclic variation at twice the applied frequency

$$P = \frac{1}{2}Vm.Im.\cos(\theta - \emptyset)$$

To calculate the average power

Example: $v(t) = 4\cos(\frac{\pi t}{6})$ is applied to an impedance of $Z = 2\angle 60°$, find:

1- The average power

2- Expression for the instantaneous power for the corresponding phasor voltage

$$V = 4\angle 0°$$

$$Z = 2\angle 60°$$

$$I = \frac{4\angle 0°}{2\angle 60°} \rightarrow i(t) = 2\cos\left(\frac{\pi t}{6} - 60°\right)$$

1- Average power

$$P = \frac{1}{2} Vm.Im.\cos(\theta - \emptyset) = \frac{1}{2}(4)(2)\cos(60) = 2\ Watts$$

2- Instantaneous power

$$v(t) = 4\cos\left(\frac{\pi t}{6}\right) \quad , \quad i(t) = 2\cos\left(\frac{\pi t}{6} - 60°\right)$$

$$p(t) = v(t).i(t) = \frac{1}{2}(4)(2)\left(\cos\left(\frac{2\pi t}{6} - 60°\right) + \cos(60°)\right) = 2 + 4\cos\left(\frac{\pi t}{3} - 60°\right) Watts$$

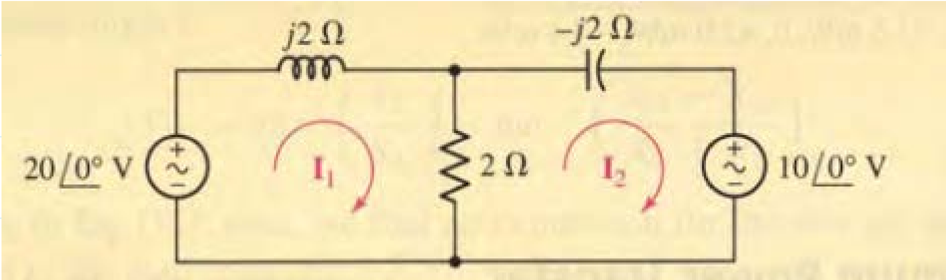
Average Power Absorbed by a purely resistive element

For a purely resistive element: $(\theta - \emptyset) = Zero$, voltage and current waveforms are in phase

$$P = \frac{1}{2} Vm.Im.\cos(\theta - \emptyset) = \frac{1}{2} Vm.Im = \frac{1}{2} Im^2.R = \frac{1}{2}\frac{Vm^2}{R}$$

The average power for a purely reactive element is zero ($Px = 0 \rightarrow (\theta - \emptyset) = 90°$)

Example: Determine the average power associated with all circuit elements.



$I1 = 11.18\angle -63.43°$

$I2 = 7.07\angle -45°$

$I, 2\Omega = I1 - I2 = 5\angle -90°$

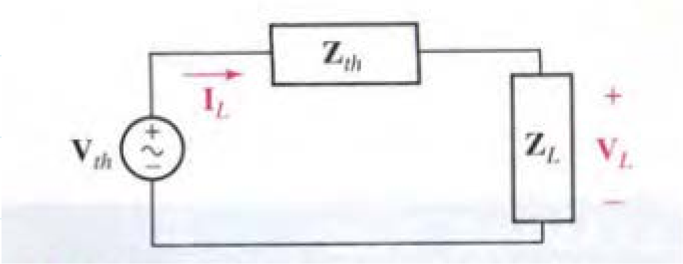
$Px(inductor) = 0$, $Px(capacitor) = 0$

$$P, 2\Omega = \frac{1}{2} Im^2 . R = \frac{1}{2}(5)^2(2) = 25\, Watts$$

$$P, 20\angle 0° = \frac{1}{2} Vm . Im . cos(\theta - \emptyset) = \frac{1}{2}(20)(11.18)cos(0 + 63.43) = 50\, Watts$$

$$P, 10\angle 0° = \frac{1}{2} Vm . Im . cos(\theta - \emptyset) = \frac{1}{2}(10)(7.07)cos(0 + 45) = 25\, Watts$$

Power Transfer Theorem



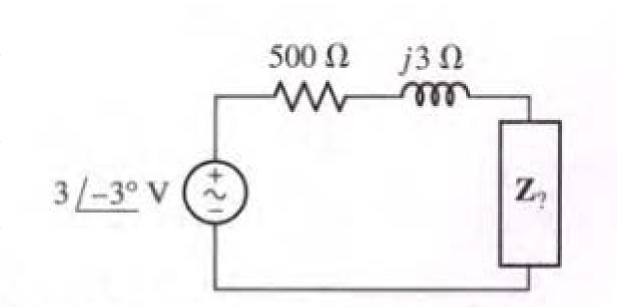
$Zth = Rth + j\, Xth$

$Zth^* = Rth - j\, Xth$

$ZL = RL + j\, XL$

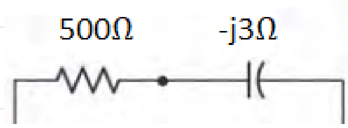
The maximum power transfer is satisfied when: $ZL = Zth^*$

Example: Determine the element value at which the maximum power is delivered



$$v(t) = 3\cos(100t - 3°)V$$

$$ZL = Zth^{*} = 500 - j3\ \Omega$$



This load can be simulated by connecting a ***500 Ω*** resistor in series with a capacitor Having an impedance of $-j3\ \Omega$

$$Zc = \frac{1}{jwC} \rightarrow -j3 = -\frac{j}{100C} \rightarrow \frac{1}{100C} = 3 \rightarrow C = 3.33\ mF$$

Average Power for Non-Periodic Functions

i(t) sum of several sinusoids having different periods and arbitrary amplitudes:

$$i(t) = I_{m1}.\cos(w_1 t) + I_{m2}.\cos(w_2 t) + \cdots + I_{mn}.\cos(w_n t)$$

$$P = \frac{1}{2}\left(I_{m1}^2 + I_{m2}^2 + \cdots + I_{mn}^2\right).R$$

Example: Determine the average power delivered to a 4-ohm resistor by the periodic current: a- $i(t) = 2\cos(10t) - 3\cos(20t)$, b- $i(t) = 2\cos(10t) - 3\cos(10t)$

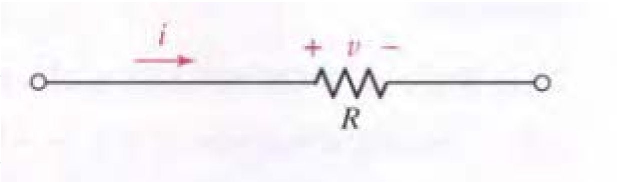
a- $P = \frac{1}{2}(2^2)(4) + \frac{1}{2}(3^2)(4) = 8 + 18 = 26\ Watts$

b- $i(t) = 2\cos(10t) - 3\cos(10t) = -\cos(10t)$

$$P = \frac{1}{2}((-1)^2)(4) = 2\ Watts$$

Effective Value:

RMS (root mean square) value: The average power delivered to an R-ohm resistor by a periodic current $i(t)$.



$$P = \frac{1}{T}\int_0^T i^2(t).R\,dt = \frac{R}{T}\int_0^T i^2(t)\,dt = I_{eff}^2.R$$

$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$ ← Root mean square, or effective value of the current

- For a sinusoidal waveform:

$$i(t) = I_m cos(wt + \emptyset)$$

$$I_{eff} = \frac{I_m}{\sqrt{2}} = 0.707\,I_m$$

The average power can be calculated either:

1- In terms of maximum values (amplitudes)

$$P = \frac{1}{2}V_m.I_m.cos(\theta - \emptyset) = \frac{1}{2}V_m.I_m = \frac{1}{2}I_m^2.R = \frac{1}{2}\frac{V_m^2}{R}$$

2- In terms of the effective values

$$P = V_{eff}.I_{eff}.cos(\theta - \emptyset) = V_{eff}.I_{eff} = I_{eff}^2.R = \frac{V_{eff}^2}{R}$$

Effective value is a measure of the effectiveness of a voltage source in delivering to a resistance load

Effective Values for a Multiple Frequency Circuit

$$P = I_{eff}^2 . R$$

$$P = (I_{1,eff}^2 + I_{2,eff}^2 + \cdots + I_{n,eff}^2) . R$$

$$I_{eff}^2 = \sqrt{I_{1,eff}^2 + I_{2,eff}^2 + \cdots + I_{n,eff}^2}$$

Apparent Power and Power Factor

The average power can be calculated either:

1- In terms of maximum values (amplitudes)

2- In terms of the effective values

$$P = V_{eff} . I_{eff} . cos(\theta - \emptyset)$$

The product of the effective values of the voltage and current is called the apparent power

$$\text{Power Factor } (PF) = \frac{\text{Average real power}}{\text{Apparent power}} = \frac{P}{V_{eff} . I_{eff}}$$

General Definition

In sinusoidal case:

$PF = cos(\theta - \emptyset)$, PF angle: The angle by which the voltage leads the current

$cos(\theta - \emptyset)$ cannot have a magnitude greater than unity → The average real power can never be greater than the apparent power

1- Purely resistive load (unity PF)

Voltage and current are in phase → $(\theta - \emptyset) = 0°$ → $PF = cos(\theta - \emptyset) = 1$

2- Purely reactive load

$(\theta - \emptyset) = \pm 90°$ → $PF = cos(\theta - \emptyset) = 0$

3- Between the two extremes

$PF = 0.5$ → $cos(\theta - \emptyset) = 0.5$

→ $(\theta - \emptyset) = 60$, Inductive load

→ $(\theta - \emptyset) = -60$, Capacitive load

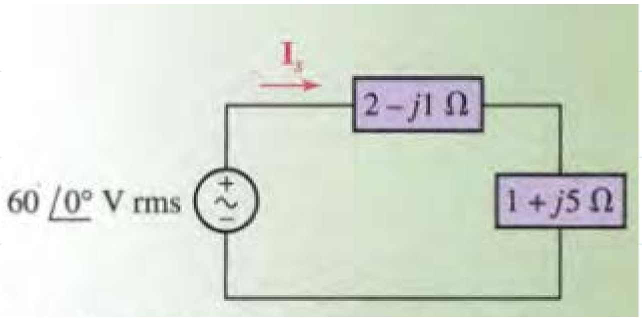
Leading and lagging PF:

The terms leading or lagging PF is determined based on the phase angle of the current with respect to the phase angle of the voltage

Inductive load: PF is lagging

Capacitive load: PF is leading

Example: For the network, determine the following:



1- Average power delivered to each of the two loads

2- Apparent power supplied by the source

3- Power factor (PF)

$$P = I_{eff}^2 . R$$

$$Z_{tot} = (2 - j1) + (1 + j5) = 3 + j4\ \Omega$$

$$I_{eff} = \frac{60(rms)}{3+4j} = 12\angle -53.13^\circ\ (A)(rms)$$

1- $P(2 - j1) = (12)^2 .(2) = 288\ Watts$

$P(1 + j5) = (12)^2 .(1) = 144\ Watts$

$P_{tot} = 288 + 144 = 432\ Watts$

2- $|S| = V_{eff} . I_{eff} = (60)(12) = 720\ VA$

3- $PF = \frac{P}{V_{eff} . I_{eff}} = \frac{432}{720} = 0.6\ lagging$

Complex Power

$\boldsymbol{S = P + jQ}$

$\boldsymbol{P}$: Average real power (Watts)

$\boldsymbol{Q}$: Reactive power (VAR)→ it represents the rate of energy transfer into and out of reactive load Components (such as inductors and capacitors).

The magnitude of the complex power $|\boldsymbol{S}|$ is called the apparent power.

Complex power analysis:

$V_{eff} = V_{eff}\angle\theta \quad , \quad I_{eff} = I_{eff}\angle\emptyset$

The average power delivered to the network:

$P = V_{eff}.I_{eff}.\cos(\theta - \emptyset) = V_{eff}.I_{eff}.Re(e^{j(\theta-\emptyset)})$

$P = Re[(V_{eff}.e^{j(\theta)})(I_{eff}.e^{-j\emptyset})]$

This quantity is the complex conjugate of the phasor current

This quantity represents the phasor voltage

$P = Re(V_{eff}\,.I^{*}_{eff})$

$S = P + jQ$

$$S = V_{eff} . I^{*}_{eff} = V_{eff} . I_{eff} . e^{j(\theta - \emptyset)} = V_{eff} . I_{eff} . \angle(\theta - \emptyset)$$

$P = V_{eff} . I_{eff} . cos(\theta - \emptyset)$, Average real power

$Q = V_{eff} . I_{eff} . sin(\theta - \emptyset)$, Reactive power

To summarize:

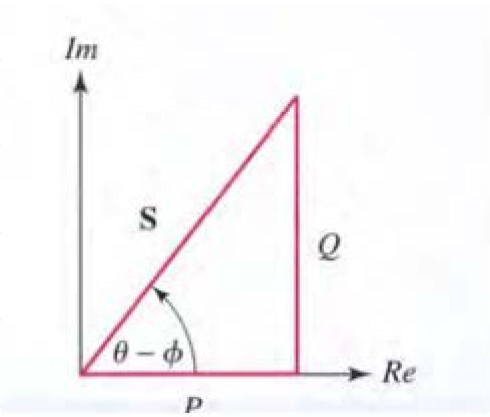
Symbol	Standard Name	Formula	Units
P	Average real power	$V_{eff} . I_{eff} . cos(\theta - \emptyset)$	Watt
Q	Reactive power	$V_{eff} . I_{eff} . sin(\theta - \emptyset)$	VAR
S	Complex power	$S = P + jQ = V_{eff} . I_{eff} . e^{j(\theta - \emptyset)}$	VA
$\|S\|$	Apparent power	$\|S\| = V_{eff} . I_{eff}$	VA

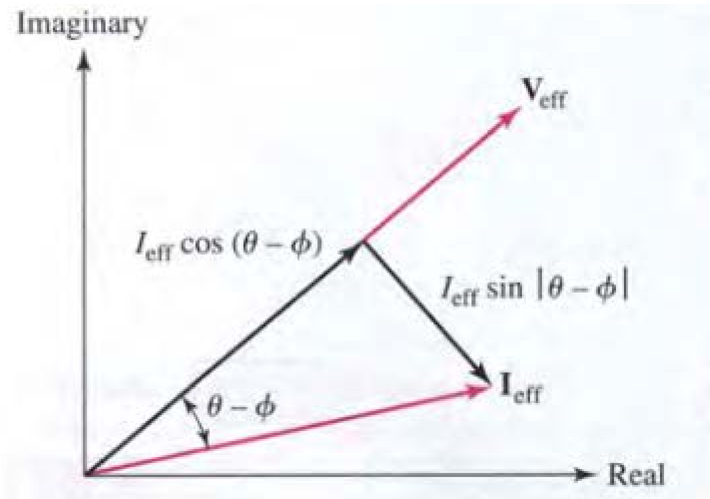
Power Triangle

Is used to represent the complex power

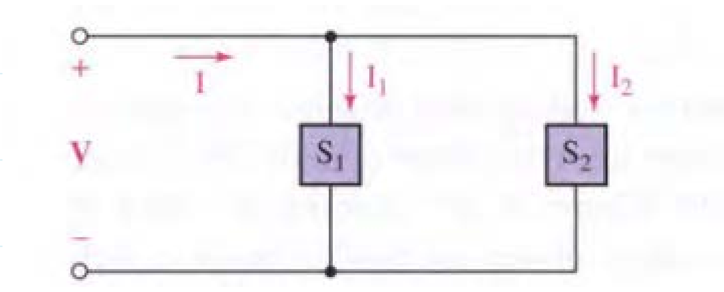
1) $(\theta - \emptyset) > 0$: This corresponds to an inductive load with a lagging PF

2) $(\theta - \emptyset) < 0$: This corresponds to a capacitive load with a leading PF





Power Factor Correction



$$\boldsymbol{S = V.I^*}$$

$$\boldsymbol{S = V(I_1 + I_2)^*}$$

$$\boldsymbol{S = V.I_1^* + V.I_2^*}$$

Example: An industrial consumer is operating a **50** KW induction motor at a Lagging PF of **0.8,** the source voltage is **230** V (rms), if the customer wishes to increase the PF to **0.95** lagging specify a suitable solution.

To increase the power factor from **0.8** to **0.95,** a purely reactive device must be connected in parallel with the induction motor

S_1: The complex power associated with the induction motor

$$S_1 = \frac{50\ KW}{0.8} \angle cos^{-1}(0.8) = 50 + j37.5\ KVA$$

- After PF correction: PF is increased to **0.95** ⟶ $S = S_1 + S_2$

$$S = \frac{50\ KW}{0.95} \angle cos^{-1}(0.95) = 50 + j16.43\ KVA$$

$$S_2 = S - S_1 = -j21.07\ KVA$$

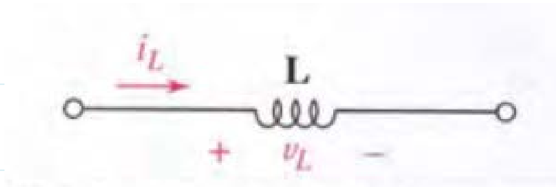
$$I_2^* = \frac{S_2}{V} = \frac{-j21.07}{230} = -j91.6\ A$$

$$I_2 = j91.6\ A$$

$$Z_2 = \frac{V}{I_2} = \frac{230}{j91.6} = -j2.51\ \Omega$$

The production of a magnetic flux by a current.

The flux being proportional to the current in linear inductors



$$v(t) = L\frac{di}{dt}$$

Physical meaning of this equation

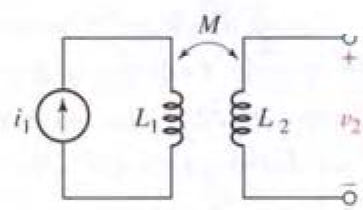
The production of a voltage by a time-varying magnetic field, the voltage being proportional to the time rate of the magnetic field

Coefficient of Mutual Inductance

1- A current flowing in one coil produces a magnetic flux about that coil and also about a second coil nearby

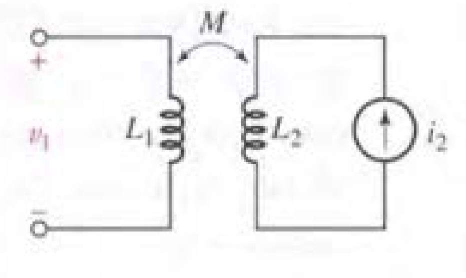
2- The time varying magnetic flu surrounding the second coil produces an open circuit voltage that is proportional to the time rate of change of the current flowing through the first coil.

M_{21}: Voltage response produced at L_2 due to current source at L_1



$$v_2(t) = M_{21}\frac{di_1(t)}{dt}$$

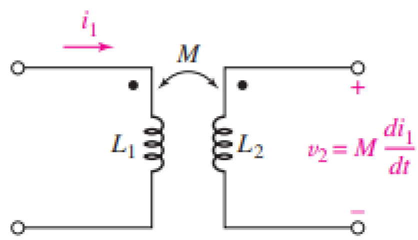
$$v_1(t) = M_{12}\frac{di_2(t)}{dt}$$

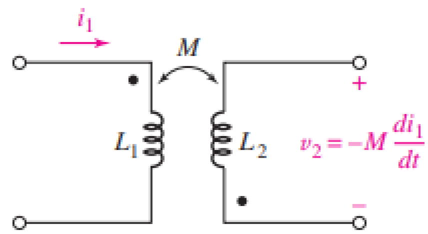


$$M_{12} = M_{21} = M$$

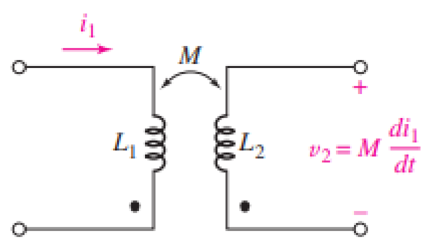
Rule # 1: Current is entering the dotted terminal

If it is entering the dotted terminal, it will produce an Open circuit voltage across L_2 with a positive voltage reference at the dotted terminal of L_2

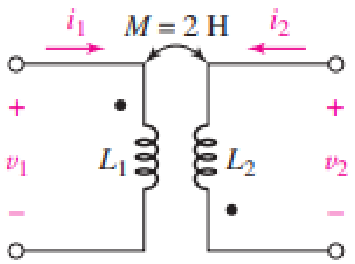




Rule # 2: Current is entering the un-dotted terminal



Example: For the circuit shown determine:



1- v_1 if $i_2 = 5\, sin(45t)$ and $i_1 = 0$

2- v_2 if $i_1 = -8e^{-t}$ and $i_2 = 0$

$$v_1(t) = -M\frac{di_2}{dt} = -2 \times 5 \times 45 \times cos(45t) = -450\, cos(45t)\; V$$

$$v_2(t) = -M\frac{di_1}{dt} = -2 \times -8 \times -e^{-t} = -16\, e^{-t}\; V$$

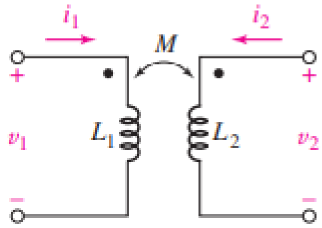
Combined Mutual and self-Induction Voltage

Dot convection is satisfied:

$$v_1(t) = L_1\frac{di_1}{dt} + M\underline{\frac{di_2}{dt}}$$

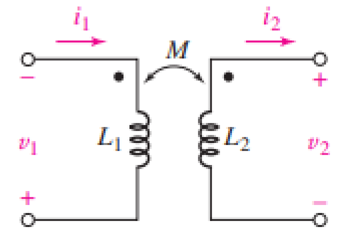
$$v_2(t) = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Voltage due to mutual inductance



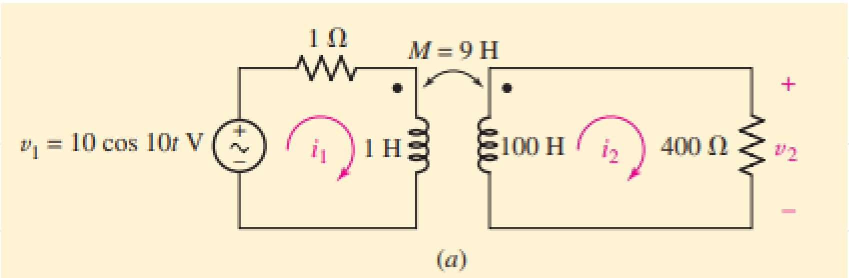
$$v_1(t) = -L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2(t) = -L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$



Dot convection is satisfied, passive sign convection is not satisfied

Example: Determine the ratio of the output voltage v_2 to the source voltage v_1 (expressed in phasor)



(a)

For $L_1 = 1\ H \rightarrow jwL_1 = j(10)(1) = j10\ \Omega$

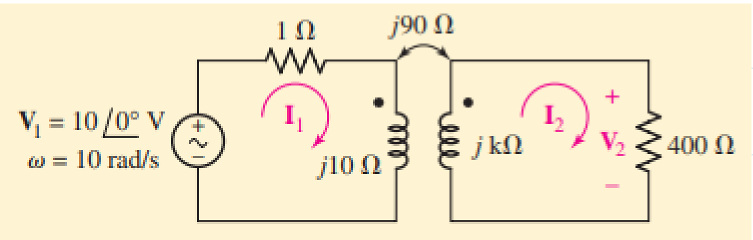
For $L_2 = 100\ H \rightarrow jwL_2 = j(10)(100) = j1000\ \Omega$

Inductance representation in frequency domain

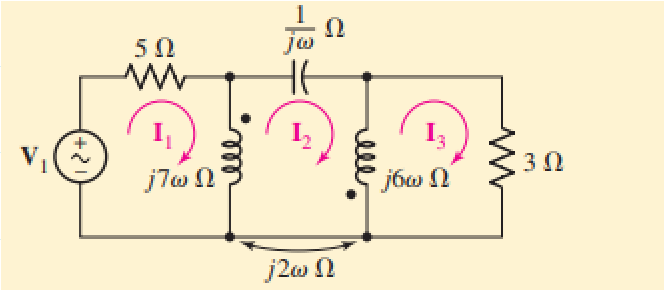
Mesh #1: $(1 + j10)I_1 - j90\ I_2 = V_s = 10\angle 0°$

Mesh #2: $(400 + j1000)I_2 - j90\ I_1 = 0$

$$\frac{V_2}{V_1} = \frac{400 I_2}{10}$$



Example: Construct the appropriate mesh equations



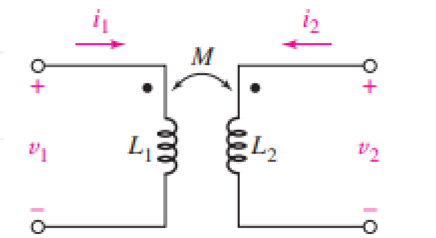
Mesh #1: $5I_1 + 7jw(I_1 - I_2) + 2jw(I_3 - I_2) = V_1$

Mesh #2: $7jw(I_2 - I_1) + 2jw(I_2 - I_3) + \frac{1}{jw} I_2 + 6jw(I_2 - I_3) + 2jw(I_2 - I_1) = 0$

Mesh #3: $6jw(I_3 - I_2) + 2jw(I_1 - I_2) + 3 I_3 = 0$

Energy Considerations:

To calculate the energy stored in a magnetically coupled system:



$$w(t) = \frac{1}{2}L_1 i_1^2(t) + \frac{1}{2}L_2 i_2^2(t) + M[i_1(t).i_2(t)]$$

$$M \le \sqrt{L_1 L_2}$$

Coupling coefficient **(k):** The degree to which M approaches its maximum value described by: $k = \frac{M}{\sqrt{L_1 L_2}}$ since $M \le \sqrt{L_1 L_2}$ → $0 \le k \le 1$

When **k** is close to unit, coils are said to be tightly coupled

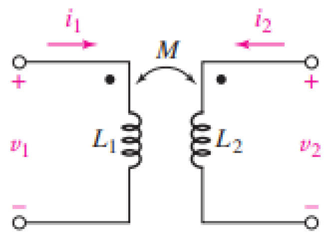
How can we obtain large values of **k** ?

1- Coils are physically close to each other

2- Coils are wound or oriented to provide a large common magnetic flux

3- The use of high permeability materials to concentrate and localize the magnetic flux

Example: Consider the following magnetically coupled system:



$L_1 = 0.4\ H$
$L_2 = 2.5\ H$
$k = 0.6$
$i_1 = 4i_2 = 20\cos(500t - 20°)mA$

Determine:

1- $v_1(0)$

$$v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$M = k\sqrt{L_1L_2} = 0.6\sqrt{(0.4)(0.6)} = 0.6\ H$$

$$\frac{di_1}{dt} = -(0.02)(500)\,sin(500t - 20°) = -10\,sin(500t - 20°)$$

$$L_1\left(\frac{di_1}{dt}\right) = -0.4(10)sin(500t - 20°) = -4\,sin(500t - 20°)$$

$$i_2(t) = \frac{1}{4}i_1(t) = 0.005\,cos(500t - 20°)$$

$$\frac{di_2(t)}{dt} = -(0.005)(500)\,sin(500t - 20°) = -2.5\,sin\,(500t - 20°)$$

$$M\left(\frac{di_2}{dt}\right) = -(0.6)(2.5)\,sin(500t - 20°) = -1.5\,sin(500t - 20°)$$

$$v_1(t) = -5.5\,sin(500t - 20°)$$

$$v_1(0) = -5.5\,sin(-20°) = 1.881\,V$$

2- The energy stored in the system at t=0

$$w(t) = \frac{1}{2}L_1 i_1^2(t) + \frac{1}{2}L_2 i_2^2(t) + M[i_1(t).i_2(t)]$$

$$i_1(0) = \big((0.02)\,cos(-20°)\big) = 0.0188\,A \quad , \quad i_2(0) = \big((0.005)\,cos(-20°)\big) = 0.0047\,A$$

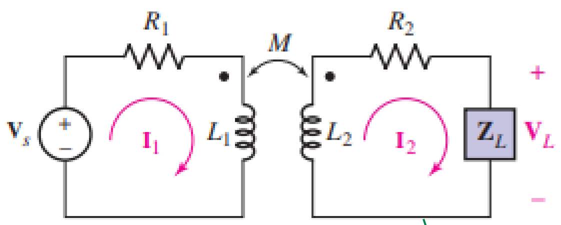
$$w(0) = \frac{1}{2}(0.4)(0.0188)^2 + \frac{1}{2}(2.5)(0.0047)^2 + 0.6[(0.0188)(0.0047)] = 151.2\,\mu J$$

The Linear Transformer

1- The linear transformer is considered to be an excellent model for devices used at radio frequency or higher frequency

2- The linear transformer implies that no magnetic material is employed because it may cause a non-linear flux versus current relationship

Basic construction of the linear transformer:



The basic construction of a linear transformer

Primary mesh

Secondary mesh

The mesh that contains the voltage source is called Primary mesh

L_1:Primary inductance

L_2:Secondary inductance

Input Impedance of the linear transformer:

$$Z_{in} = \frac{V_s}{I_s}$$

$Z_{11} = R_1 + jwL_1$, $Z_{22} = R_2 + jwL_2 + Z_L$

Mesh #1: $(R1 + jwL_1)I_1 - jwM\, I_2 = Vs$

Mesh #2: $(R_2 + jwL_2 + Z_L)I_2 - jwM\, I_1 = 0$

Transformer mesh equations:

$Z_{11}I_1 - jwMI_2 = V_s$ [1]

$Z_{22}I_2 - jwMI_1 = 0$ [2]

From [2] → $I_2 = \frac{jwMI_1}{Z_{22}}$

Substituting in [1] → $Z_{11}I_1 - jwM\frac{jwM}{Z_{22}}I_1 = V_s$

$$Z_{in} = \frac{V_s}{I_1} = Z_{11} + \boxed{\frac{w^2M^2}{Z_{22}}}$$

$\frac{w^2M^2}{Z_{22}}$ → Reflected impedance

Z_{in} → Input impedance of a linear transformer